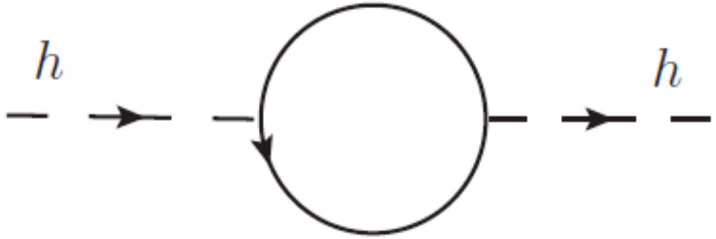
Universal Imprints of a PNGB Higgs Boson

Workshop on "Confronting Naturalness: from LHC to Future Colliders"

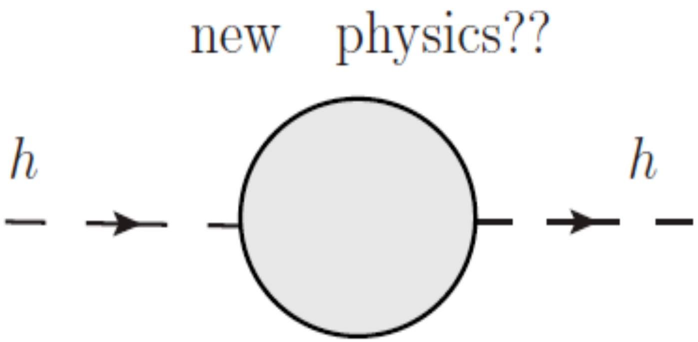
Ian Low, Argonne/Northwestern/CERN

April 25, 2018

The discovery of 125 GeV Higgs sharpens the question of Naturalness:

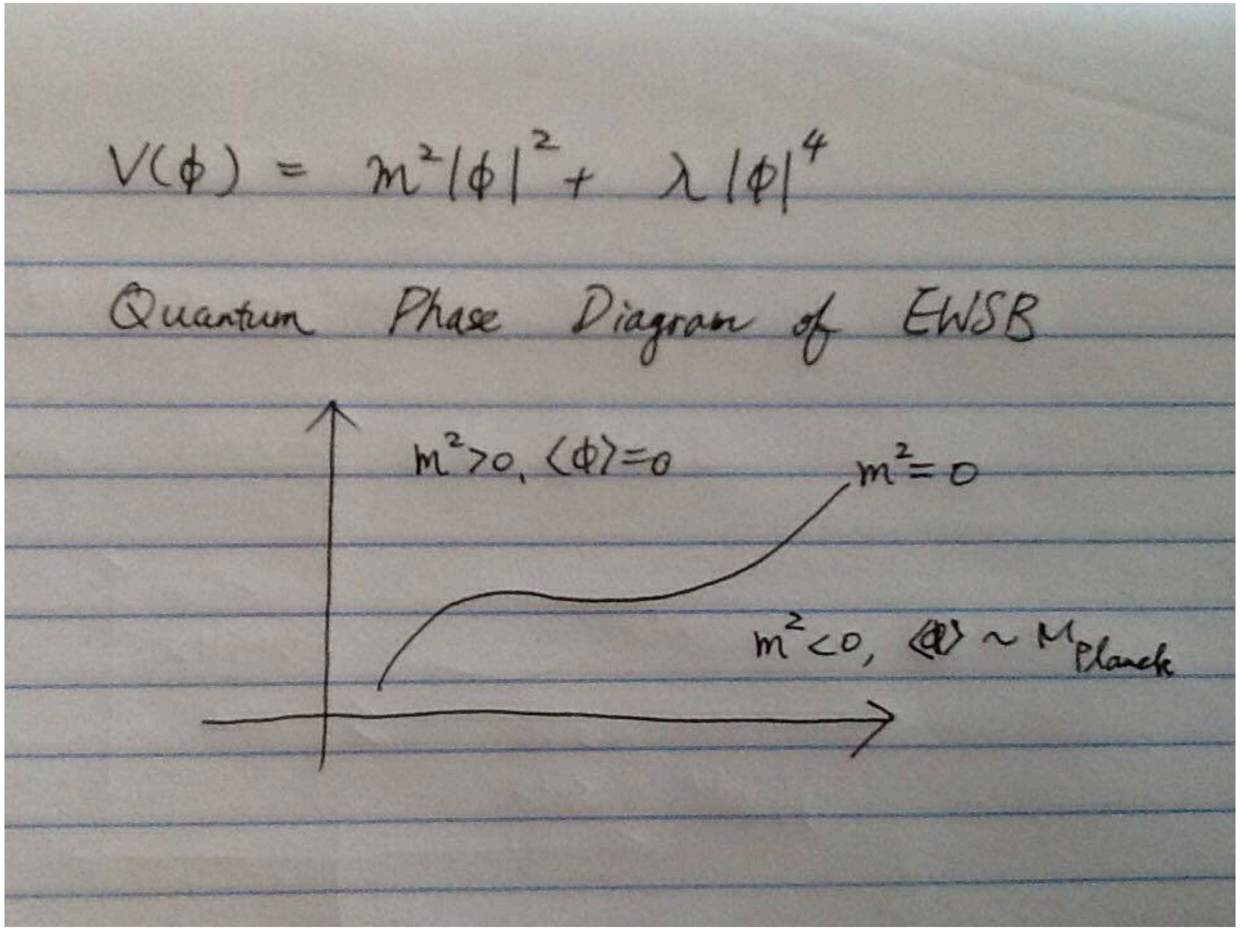


$$\propto -\frac{3\lambda_t^2}{8\pi^2}\Lambda^2$$

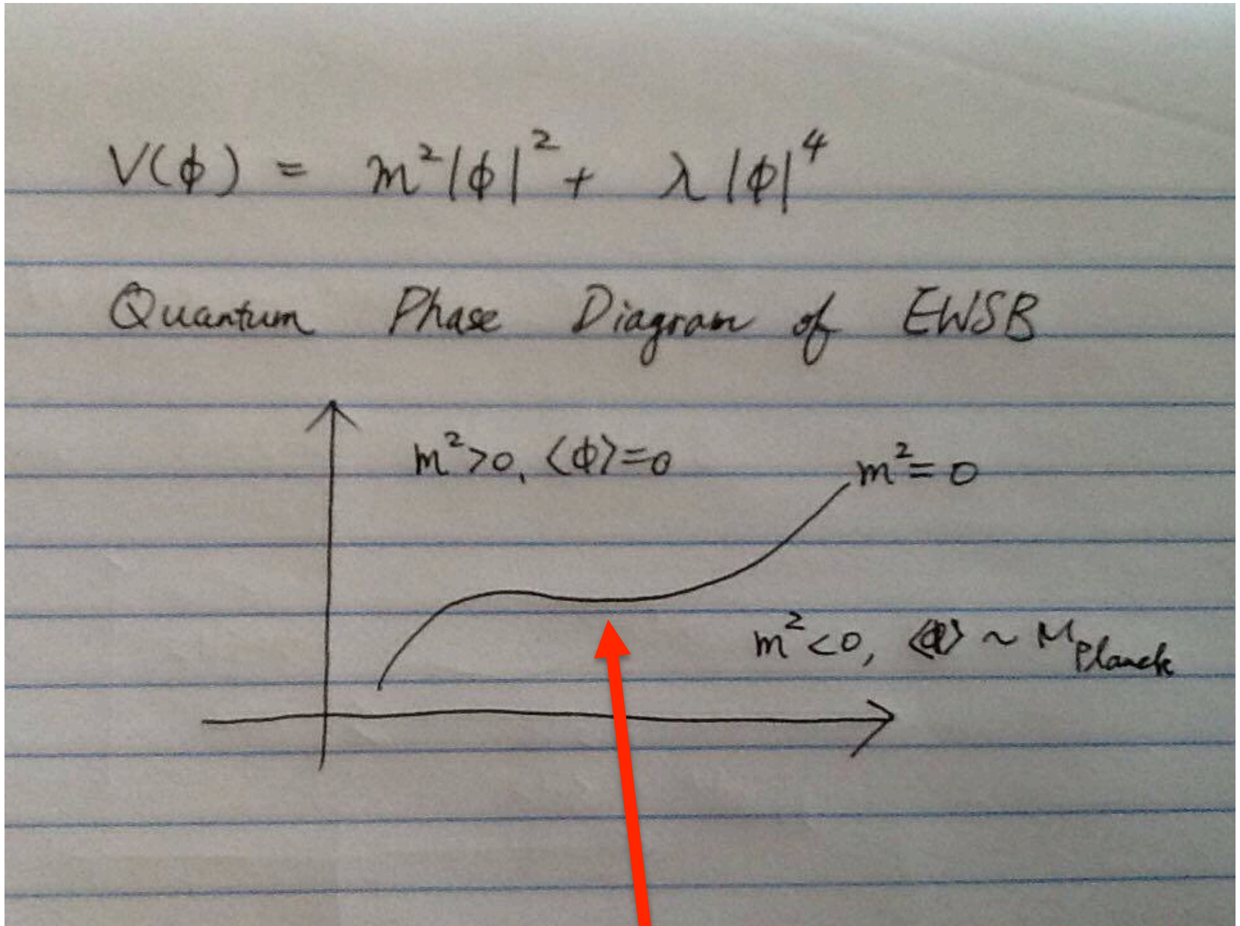


OR

One can rephrase the Naturalness problem in terms of quantum criticality:



One can rephrase the Naturalness problem in terms of quantum criticality:



Mh=125 GeV. We are sitting extremely close to the criticality. **WHY??**

One popular (appealing) possibility -- the critical line is a locus of enhanced symmetry.

This could be why the electroweak vacuum likes to sit, practically, on top of the critical line.

One popular (appealing) possibility -- the critical line is a locus of enhanced symmetry.

This could be why the electroweak vacuum likes to sit, practically, on top of the critical line.

Theorists could come up with (pretty much) only two examples of such enhanced symmetries:

- Bosonic symmetry: the (spontaneously broken) global symmetry.
  The Higgs is a pseudo-Nambu-Goldston boson and the model goes by the name of "composite Higgs models."

One popular (appealing) possibility -- the critical line is a locus of enhanced symmetry.

This could be why the electroweak vacuum likes to sit, practically, on top of the critical line.

Theorists could come up with (pretty much) only two examples of such enhanced symmetries:

- Bosonic symmetry: the (spontaneously broken) global symmetry.

  The Higgs is a pseudo-Nambu-Goldston boson and the model goes by the name of "composite Higgs models."

- Fermionic symmetry: the (broken) supersymmetry.

- supersymmetric theories are all built upon a minimal lagrangian -- the MSSM:

$$W_{\rm MSSM} = \overline{u}\mathbf{y_u} Q H_u - \overline{d}\mathbf{y_d} Q H_d - \overline{e}\mathbf{y_e} L H_d + \mu H_u H_d$$

$$\begin{aligned}
\mathcal{L}_{\rm soft}^{\rm MSSM} \;=\; & -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + {\rm c.c.}\right) \\
& - \left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d + {\rm c.c.}\right) \\
& -\widetilde{Q}^\dagger\,\mathbf{m_Q^2}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m_L^2}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m_{\overline{u}}^2}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m_{\overline{d}}^2}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m_{\overline{e}}^2}\,\widetilde{\overline{e}}^\dagger \\
& - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (bH_u H_d + {\rm c.c.})\,.
\end{aligned}$$

This is the minimal lagrangian the makes standard model supersymmetric.

On the other hand, the theory space of a composite Higgs looks huge:

$\mathcal{G}$	$\mathcal{H}$	C	N_G	$\mathbf{r}_{\mathcal{H}} = \mathbf{r}_{SU(2)\times SU(2)}$ $(\mathbf{r}_{SU(2)\times U(1)})$	Ref.
SO(5)	SO(4)	✓	4	$\mathbf{4} = (\mathbf{2}, \mathbf{2})$	[11]
SU(3) × U(1)	SU(2) × U(1)		5	$\mathbf{2}_{\pm 1/2} + \mathbf{1}_0$	[10, 35]
SU(4)	Sp(4)	✓	5	$\mathbf{5} = (\mathbf{1}, \mathbf{1}) + (\mathbf{2}, \mathbf{2})$	[29, 47, 64]
SU(4)	$[SU(2)]^2 \times U(1)$	✓*	8	$(\mathbf{2}, \mathbf{2})_{\pm 2} = 2 \cdot (\mathbf{2}, \mathbf{2})$	[65]
SO(7)	SO(6)	✓	6	$\mathbf{6} = 2 \cdot (\mathbf{1}, \mathbf{1}) + (\mathbf{2}, \mathbf{2})$	–
SO(7)	G_2	✓*	7	$\mathbf{7} = (\mathbf{1}, \mathbf{3}) + (\mathbf{2}, \mathbf{2})$	[66]
SO(7)	SO(5) × U(1)	✓*	10	$\mathbf{10}_0 = (\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}) + (\mathbf{2}, \mathbf{2})$	–
SO(7)	$[SU(2)]^3$	✓*	12	$(\mathbf{2}, \mathbf{2}, \mathbf{3}) = 3 \cdot (\mathbf{2}, \mathbf{2})$	–
Sp(6)	Sp(4) × SU(2)	✓	8	$(\mathbf{4}, \mathbf{2}) = 2 \cdot (\mathbf{2}, \mathbf{2})$	[65]
SU(5)	SU(4) × U(1)	✓*	8	$\mathbf{4}_{-5} + \bar{\mathbf{4}}_{+5} = 2 \cdot (\mathbf{2}, \mathbf{2})$	[67]
SU(5)	SO(5)	✓*	14	$\mathbf{14} = (\mathbf{3}, \mathbf{3}) + (\mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{1})$	[9, 47, 49]
SO(8)	SO(7)	✓	7	$\mathbf{7} = 3 \cdot (\mathbf{1}, \mathbf{1}) + (\mathbf{2}, \mathbf{2})$	–
SO(9)	SO(8)	✓	8	$\mathbf{8} = 2 \cdot (\mathbf{2}, \mathbf{2})$	[67]
SO(9)	SO(5) × SO(4)	✓*	20	$(\mathbf{5}, \mathbf{4}) = (\mathbf{2}, \mathbf{2}) + (\mathbf{1} + \mathbf{3}, \mathbf{1} + \mathbf{3})$	[34]
$[SU(3)]^2$	SU(3)		8	$\mathbf{8} = \mathbf{1}_0 + \mathbf{2}_{\pm 1/2} + \mathbf{3}_0$	[8]
$[SO(5)]^2$	SO(5)	✓*	10	$\mathbf{10} = (\mathbf{1}, \mathbf{3}) + (\mathbf{3}, \mathbf{1}) + (\mathbf{2}, \mathbf{2})$	[32]
SU(4) × U(1)	SU(3) × U(1)		7	$\mathbf{3}_{-1/3} + \bar{\mathbf{3}}_{+1/3} + \mathbf{1}_0 = 3 \cdot \mathbf{1}_0 + \mathbf{2}_{\pm 1/2}$	[35, 41]
SU(6)	Sp(6)	✓*	14	$\mathbf{14} = 2 \cdot (\mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{3}) + 3 \cdot (\mathbf{1}, \mathbf{1})$	[30, 47]
$[SO(6)]^2$	SO(6)	✓*	15	$\mathbf{15} = (\mathbf{1}, \mathbf{1}) + 2 \cdot (\mathbf{2}, \mathbf{2}) + (\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3})$	[36]

Table 1: Symmetry breaking patterns $\mathcal{G} \to \mathcal{H}$ for Lie groups. The third column denotes whether the breaking pattern incorporates custodial symmetry. The fourth column gives the dimension N_G of the coset, while the fifth contains the representations of the GB's under $\mathcal{H}$ and $SO(4) \cong SU(2)_L \times SU(2)_R$ (or simply $SU(2)_L \times U(1)_Y$ if there is no custodial symmetry). In case of more than two SU(2)'s in $\mathcal{H}$ and several different possible decompositions we quote the one with largest number of bi-doublets.

Bellazzni, Csaki and Serre:1401.2457

Construction of effective Lagrangians for composite Higgs bosons relies on the CCWZ formalism:

Structure of Phenomenological Lagrangians. I*

S. COLEMAN

Harvard University, Cambridge, Massachusetts 02138

AND

J. WESS† AND BRUNO ZUMINO

New York University, New York, New York 10003

(Received 13 June 1968)

Structure of Phenomenological Lagrangians. II*

CURTIS G. CALLAN, JR. AND SIDNEY COLEMAN

Harvard University, Cambridge, Massachusetts 02138

AND

J. WESS† AND BRUNO ZUMINO

New York University, New York, New York 10003

(Received 13 June 1968)

- CCWZ is a very geometrical approach:

  One decides on a nonlinearly realized group G, and a subgroup H of G that is linearly realized.
  We say G is the broken group and H the unbroken group:

- The “pions” are the coordinates on the coset manifold G/H, and the action of the full group G on pions is complicated and nonlinear!

$$\xi = e^{i\Pi/f}, \quad \Pi = \pi^a X^a,$$

$$g\,\xi = \xi'\,U(g,\xi)\;, \quad U(g,\xi) \in H$$

- CCWZ is a very geometrical approach:

  One decides on a nonlinearly realized group G, and a subgroup H of G that is linearly realized.

  We say G is the broken group and H the unbroken group:

- The "pions" are the coordinates on the coset manifold G/H, and the action of the full group G on pions is complicated and nonlinear!

$$\xi = e^{i\Pi/f}, \quad \Pi = \pi^a X^a,$$

$$g\,\xi = \xi'\,U(g,\xi)\ , \quad U(g,\xi) \in H$$

$$\Pi' = \Pi'(\Pi, g)$$

No one dared asking/working out what this mess is.

CCWZ thus looked for objects that have "simple" transformation properties under the action of G.

These are contained in the Cartan-Maurer one-form:

$$\xi^\dagger \partial_\mu \xi \;=\; i\mathcal{D}^a_\mu X^a + i\mathcal{E}^i_\mu T^i \equiv i\mathcal{D}_\mu + i\mathcal{E}_\mu$$

They are the "Goldstone covariant derivative" and the "associated gauge field",

$$\mathcal{D}_\mu \to U\mathcal{D}_\mu U^{-1} \;, \quad \mathcal{E}_\mu \to U\mathcal{E}_\mu U^{-1} - (\partial_\mu U)U^{-1}$$

upon which the complete effective lagrangian can be built (apart from the topological terms)

$$\mathcal{L}_{eff} = \frac{f^2}{2}\mathrm{Tr}\mathcal{D}_\mu \mathcal{D}^\mu + \cdots$$

In composite Higgs models, CCWZ gives the effective action right below the cutoff scale:

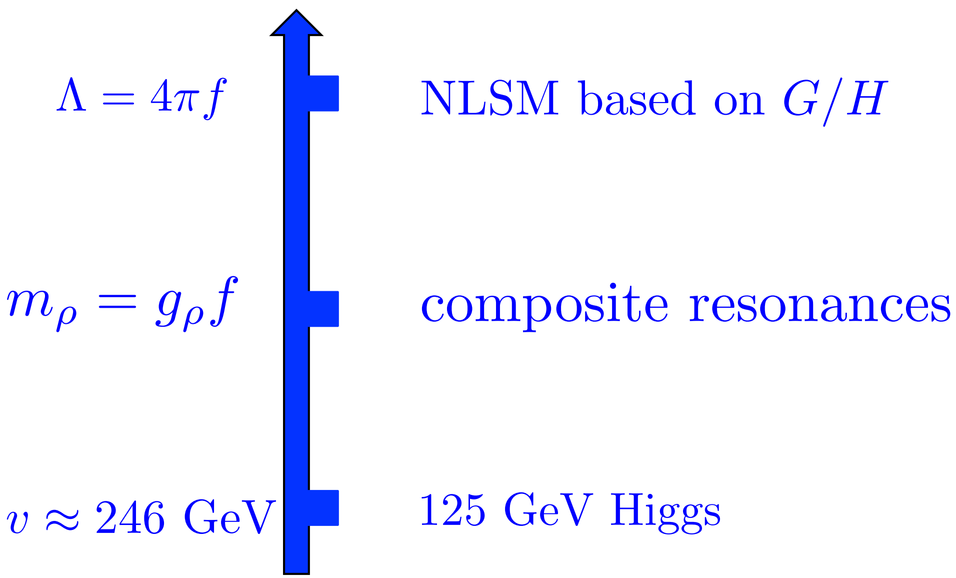
$$S_{\rm NLSM} = \int d^4x\, \Lambda^2 f^2\, \mathcal{L}\left(\frac{\pi}{f}, \frac{\partial}{\Lambda}\right) = \int d^4x\, \mathcal{L}^{(2)} + \cdots \qquad \Lambda = 4\pi f$$

But composite Higgs models often have composite resonances at the scale “f”:

In composite Higgs models, CCWZ gives the effective action right below the cutoff scale:

$$S_{\rm NLSM} = \int d^4x\, \Lambda^2 f^2\, \mathcal{L}\left(\frac{\pi}{f}, \frac{\partial}{\Lambda}\right) = \int d^4x\, \mathcal{L}^{(2)} + \cdots \qquad \Lambda = 4\pi f$$

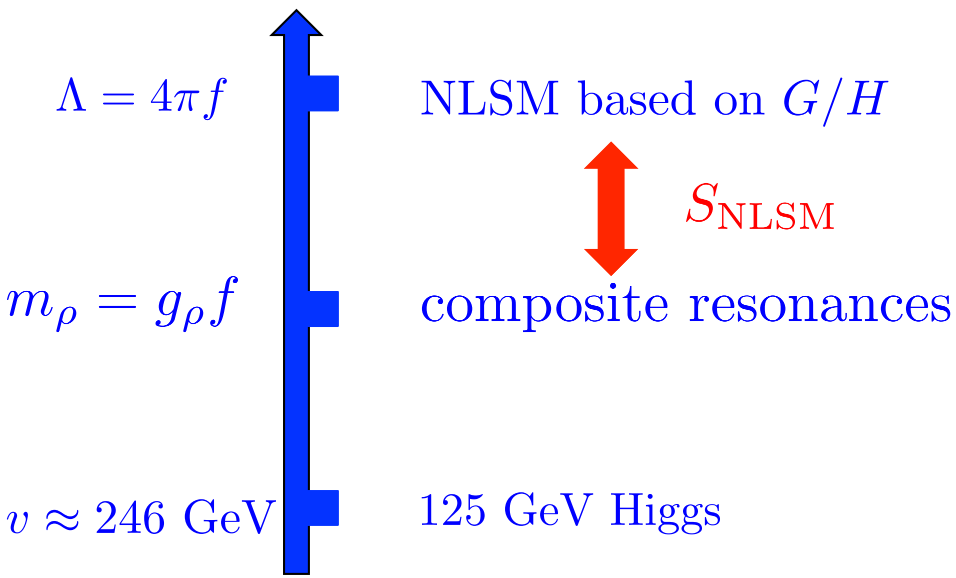
But composite Higgs models often have composite resonances at the scale "f" :



In composite Higgs models, CCWZ gives the effective action right below the cutoff scale:

$$S_{\mathrm{NLSM}} = \int d^4x\, \Lambda^2 f^2\, \mathcal{L}\left(\frac{\pi}{f}, \frac{\partial}{\Lambda}\right) = \int d^4x\, \mathcal{L}^{(2)} + \cdots \qquad \Lambda = 4\pi f$$

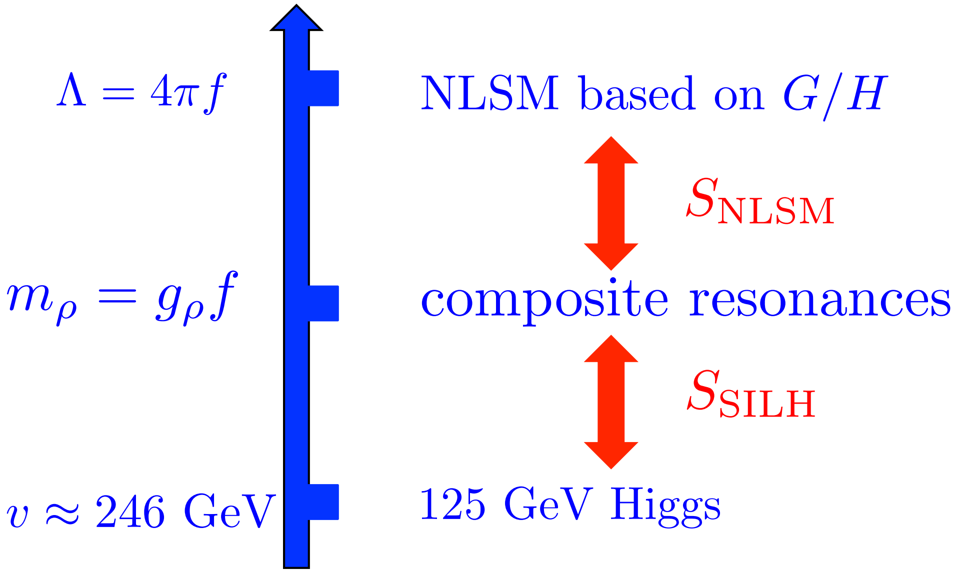
But composite Higgs models often have composite resonances at the scale “f” :



In composite Higgs models, CCWZ gives the effective action right below the cutoff scale:

$$S_{\rm NLSM} = \int d^4x\, \Lambda^2 f^2\, \mathcal{L}\left(\frac{\pi}{f}, \frac{\partial}{\Lambda}\right) = \int d^4x\, \mathcal{L}^{(2)} + \cdots \qquad \Lambda = 4\pi f$$

But composite Higgs models often have composite resonances at the scale “f” :



- At low energies, CCWZ is “matched” to the SILH lagrangian:

$$
\begin{aligned}
\mathcal{L}_{\rm SILH} = & \boxed{\frac{c_H}{2f^2}\partial^\mu(H^\dagger H)\partial_\mu(H^\dagger H) + \frac{c_T}{2f^2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right)\left(H^\dagger \overleftrightarrow{D}_\mu H\right)} \\
& -\frac{c_6\lambda}{f^2}(H^\dagger H)^3 + \left(\frac{c_y y_f}{f^2}H^\dagger H \bar{f}_L H f_R + \text{h.c.}\right) \\
& +\frac{ic_W g}{2m_\rho^2}\left(H^\dagger \sigma^a \overleftrightarrow{D}_\mu H\right)(D_\nu W^{\mu\nu})^a + \frac{ic_B g'}{2m_\rho^2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)(\partial_\nu B^{\mu\nu}) \\
& +\frac{ic_{HW} g}{16\pi^2 f^2}(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu} + \frac{ic_{HB} g'}{16\pi^2 f^2}(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu} \\
& +\frac{c_\gamma g'^2}{16\pi^2 f^2}\frac{g^2}{g_\rho^2}H^\dagger H B_{\mu\nu}B^{\mu\nu} + \frac{c_g g_s^2}{16\pi^2 f^2}\frac{y_t^2}{g_\rho^2}H^\dagger H G_{\mu\nu}G^{\mu\nu},
\end{aligned}
$$

Giudice, Grojean, Pomarol, Rattazzi: hep-ph/0703164

NLSM contribution to SILH coefficients for some of the composite Higgs models:

- SU(5)/SO(5) Littlest Higgs: $c_H^{(\sigma)} = \frac{1}{4}\,, \quad c_T^{(\sigma)} = -\frac{1}{16}$

- SO(5)/SO(4) minimal composite Higgs (MCHM): $c_H^{(\sigma)} = 1\,, \quad c_T^{(\sigma)} = 0$

- SO(9)/SO(5)xSO(4) littlest Higgs: $c_H^{(\sigma)} = \frac{1}{12}\,, \quad c_T^{(\sigma)} = 0$

- $\frac{SU(5)}{SO(5)} \times \frac{[SU(2)\times U(1)]_L \times [SU(2)\times U(1)]_R}{[SU(2)\times U(1)]_V}$ T-parity: $c_H^{(\sigma)} = \frac{1}{6}\,, \quad c_T^{(\sigma)} = 0$

c_T is dictated by the custodial symmetry. However, c_H is different for different coset.

CCWZ is extremely powerful, but it adopts a “top-down” perspective, which requires knowing the broken group “G” is in the UV.

It also implies each G/H gives a different effective Lagrangian!

Each time a young hot shot comes up with a new composite Higgs model, we need to work out the experimental consequences all over again.

CCWZ is extremely powerful, but it adopts a “top-down” perspective, which requires knowing the broken group “G” is in the UV.

It also implies each G/H gives a different effective Lagrangian!

Each time a young hot shot comes up with a new composite Higgs model, we need to work out the experimental consequences all over again.

This begs the question:

Are there universal predictions of a composite Higgs boson that are independent of the symmetry-breaking pattern?

To this end, let's recall that NLSM is all about the presence of non-trivial vacua:

Goldstone bosons are long-range degrees of freedom that connect different vacua!

It then seems a little odd that their interactions would care about the broken group G in the UV!

The IR perspective was pursued vigorously in the context of pions in the ‘60s by Adler, Nambu, Goldstone, Weinberg, etc.

This body of work was collectively known as “soft pion theorems,” although a significant part of them does not depend on the particular symmetry breaking pattern!

- one particularly important “soft-pion” theorem is the Adler’ s zero condition:

on-shell scattering amplitudes of Goldstone bosons must vanish in the limit the momentum of one Goldstone boson is taken off-shell and soft.

- often this is over-simplified as saying “the Goldstone boson is derivatively coupled.”

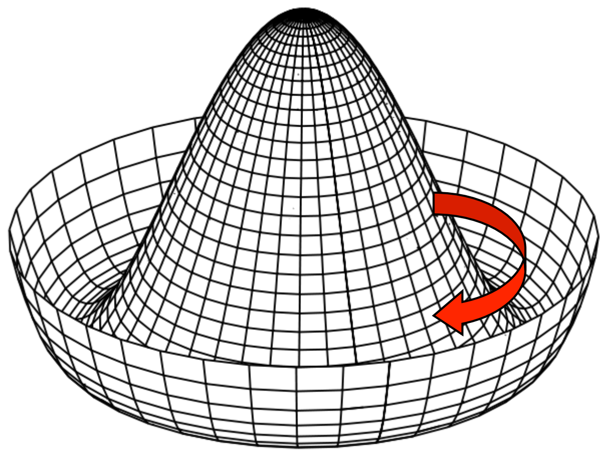
it is an over-simplification because it doesn’t do justice to the full power of the Adler’s zero condition.

I would advocate promoting Adler's zero condition to be the **defining property** of Goldstone bosons:

Nambu-Goldstone bosons are defined by the Adler's zeros and their transformation property under the unbroken group in the IR.

The Adler's zero is a direct consequence of nontrivial degenerate vacua.

Recall the different vacua are related by a rotation in the broken direction:



$$e^{i\theta}|\theta_0\rangle = |\theta_0 + \theta\rangle$$

Excitations along the broken direction gives the Goldstone boson,

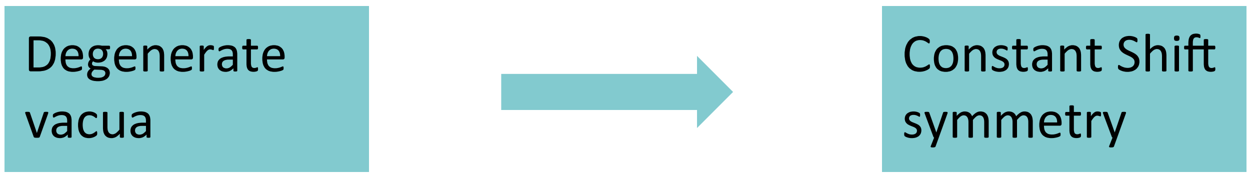
$$e^{i(\rho(x)+\theta)}|\theta_0\rangle = e^{i\rho(x)}|\theta_0 + \theta\rangle$$

But the physics is invariant whether one chooses $|\theta_0\rangle$ or $|\theta_0 + \theta\rangle$

NLSM possesses a constant "shift symmetry"!

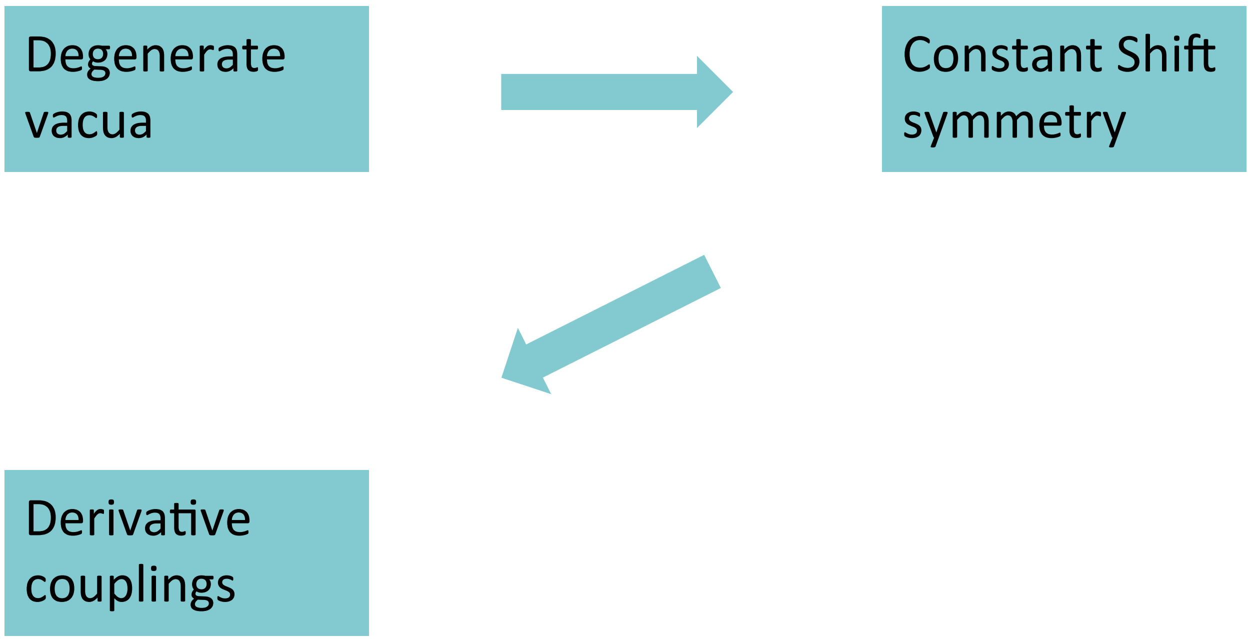
The Adler's zero is a direct consequence of nontrivial degenerate vacua.

The usual reasoning:



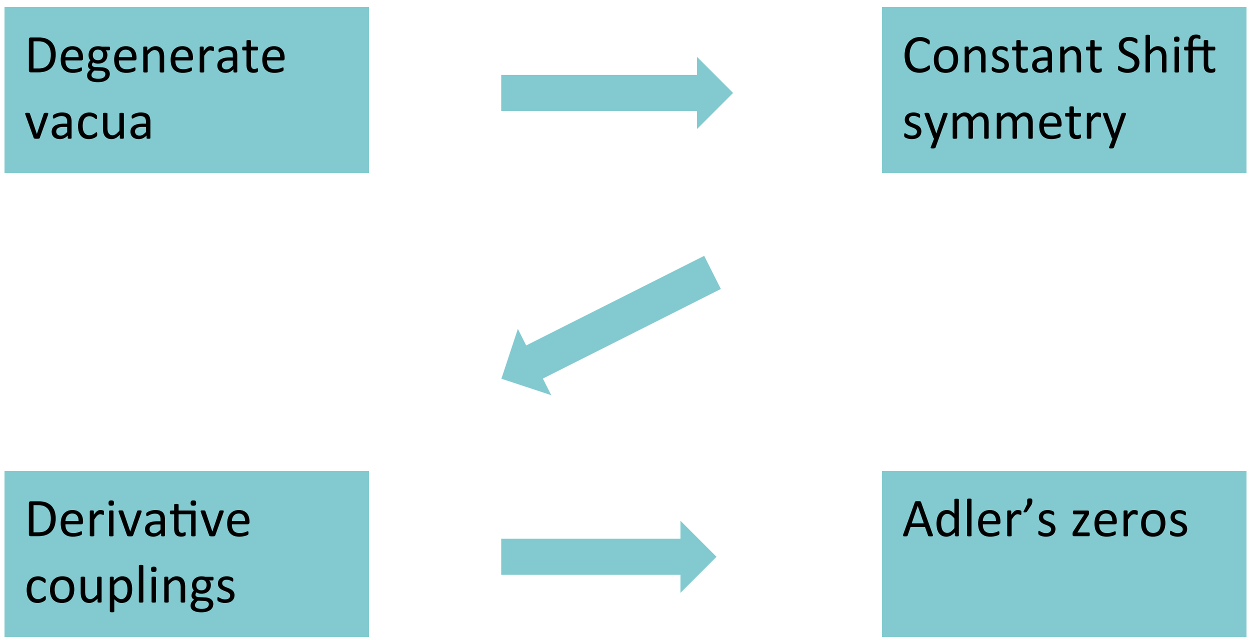
The Adler's zero is a direct consequence of nontrivial degenerate vacua.

The usual reasoning:



The Adler's zero is a direct consequence of nontrivial degenerate vacua.

The usual reasoning:



For an arbitrary coset G/H, consider 4-pt scattering amplitudes among Goldstones of the same “flavor”,

- Adler’s zero condition forbids a constant term!

$$\mathcal{A}(\pi^a\pi^a \to \pi^a\pi^a) = a\ (p_1 \cdot p_2) + b\ (p_1 \cdot p_3) + c\ (p_2 \cdot p_3)$$

For an arbitrary coset G/H, consider 4-pt scattering amplitudes among Goldstones of the same "flavor",

- Adler's zero condition forbids a constant term!

$$\mathcal{A}(\pi^a\pi^a \to \pi^a\pi^a) = a\ (p_1 \cdot p_2) + b\ (p_1 \cdot p_3) + c\ (p_2 \cdot p_3)$$

- Bose symmetry implies *a* = *b* = *c* !

$$\mathcal{A}(\pi^a\pi^a \to \pi^a\pi^a) = \mathcal{O}(p^4)$$

For an arbitrary coset G/H, consider 4-pt scattering amplitudes among Goldstones of the same "flavor",

- Adler's zero condition forbids a constant term!

$$\mathcal{A}(\pi^a\pi^a \to \pi^a\pi^a) = a\ (p_1\cdot p_2) + b\ (p_1\cdot p_3) + c\ (p_2\cdot p_3)$$

- Bose symmetry implies *a* = *b* = *c* !

$$\mathcal{A}(\pi^a\pi^a \to \pi^a\pi^a) = \mathcal{O}(p^4)$$

The argument can be generalized to n-pt amplitudes to show that $O(p^2)$ term always vanishes!

$$\begin{aligned}\mathcal{A}(\pi^a\pi^a \cdots \to \pi^a\pi^a \cdots) &= a(p_1 + \cdots + p_n)^2 + \mathcal{O}(p^4)\\ &= \mathcal{O}(p^4)\end{aligned}$$

- The effective Lagrangian, when all other Goldstones are turned-off, is very simple:

$$\mathcal{L}(\pi^i = 0, i \neq a) = \frac{1}{2}\partial_\mu \pi^a \partial \pi^a + \mathcal{O}(\partial^4)$$

- At the Lagrangian level, this can be achieved by requiring a constant "shift symmetry":

$$\pi^a \to \pi^a + \varepsilon^a$$

- The derivative of pion has simpler transformation under the shift symmetry:

$$\partial_\mu \pi^a \to \partial_\mu \pi^a$$

We have learned a simple yet powerful statement that is universal in NLSM:

For any coset G/H, self-interactions among Goldstones of the same flavor are fixed by Adler's zero condition and Bose symmetry, and must have the form:

$$\mathcal{L}(\pi^i = 0, i \neq a) = \frac{1}{2}\partial_\mu \pi^a \partial \pi^a + \mathcal{O}(\partial^4)$$

The goal --

Construct an effective Lagrangian satisfying the following two properties:

- The Lagrangian for Goldstone bosons of the same flavor reduces to

$$\mathcal{L}(\pi^i = 0, i \neq a) = \frac{1}{2}\partial_\mu \pi^a \partial \pi^a + \mathcal{O}(\partial^4)$$

when all other flavors are turned off.

The goal --

Construct an effective Lagrangian satisfying the following two properties:

- The Lagrangian for Goldstone bosons of the same flavor reduces to

$$\mathcal{L}(\pi^i = 0, i \neq a) = \frac{1}{2}\partial_\mu \pi^a \partial \pi^a + \mathcal{O}(\partial^4)$$

  when all other flavors are turned off.

- Invariance under the unbroken group H.

When there are multiple flavors of Goldstones, higher order terms appear in the shift symmetry.

- Let's consider two flavors of goldstones transforming as a complex scalar under an unbroken U(1):

$$\phi = (\pi_1 + i\pi_2)/\sqrt{2} \quad \to \quad e^{i\alpha}\ \phi$$

- The shift symmetry at NLO can be written as

$$\phi \mapsto \phi' = \phi + \epsilon - \frac{c_1}{f^2}(\phi^*\epsilon)\phi - \frac{c_2}{f^2}(\epsilon^*\phi)\phi$$

- When we turn off one of the two flavors , we must return to the single flavor case, π_i -> $\pi_i + \varepsilon_i$,

$$\phi \mapsto \phi' = \phi + \epsilon - \frac{c_1}{f^2}(\phi^*\epsilon - \epsilon^*\phi)\phi$$

This is the generalization of constant shift symmetry:

$$\phi \mapsto \phi' = \phi + \epsilon - \frac{c_1}{f^2}(\phi^* \epsilon - \epsilon^* \phi)\phi$$

The question:

What is the Lagrangian that is invariant under the generalized shift symmetry?

We can do it by brute force, or we can try to be a little more clever…

We can do it by brute force, or we can try to be a little more clever…

Let's look for objects that have simple transformation properties under the general shift symmetry:

$$\mathcal{D}_\mu \phi \mapsto \mathcal{D}_\mu \phi' = e^{i\alpha\, u(\phi,\epsilon)/f}\ \mathcal{D}_\mu \phi$$

Then the effective Lagrangian can be built straightforwardly.

- By demanding the Adler's zero and invariance under the unbroken U(1), we can write down

$$\mathcal{D}_\mu \phi = \partial_\mu \phi - \frac{d_1}{f^2}(\phi\, \partial_\mu \phi^* - \partial_\mu \phi\ \phi^*)\phi$$

- the form is again fixed by reducing to the single flavor case:

$$\mathcal{D}\phi\big|_{\pi_2=0} = \partial_\mu \pi_1$$

When all is said and done, the leading two-derivative Lagrangian can be obtained:

$$\begin{aligned}\mathcal{L}^{(2)} &= \mathcal{D}_\mu\phi^*\mathcal{D}^\mu\phi \\ &= \partial_\mu\phi^*\partial^\mu\phi - \frac{c_1}{f^2}|\partial_\mu\phi^*\ \phi - \partial_\mu\phi\ \phi^*|^2 + \mathcal{O}(1/f^4)\end{aligned}$$

which is invariant under

$$\phi \mapsto \phi' = \phi + \epsilon - \frac{c_1}{f^2}(\phi^*\epsilon - \epsilon^*\phi)\phi$$

The surprise is this procedure can be continued order-by-order in 1/f:

$$\mathcal{D}_\mu \phi = \partial_\mu \phi + \phi \frac{\partial_\mu \phi^* \, \phi - \partial_\mu \phi \, \phi^*}{2|\phi|^2} \left(1 - \frac{\tilde{f}}{|\phi|} \sin \frac{|\phi|}{\tilde{f}} \right)$$

$$\mathcal{L}^{(2)} \; = \; \mathcal{D}_\mu \phi \mathcal{D}^\mu \phi = \partial_\mu \phi^* \partial^\mu \phi - \frac{|\partial_\mu \phi^* \, \phi - \partial_\mu \phi \, \phi^*|^2}{4|\phi|^2} \left(1 - \frac{\tilde{f}^2}{|\phi|^2} \sin^2 \frac{|\phi|}{\tilde{f}} \right)$$

Low: 1412.2145
Low: 1412.2146

The surprise is this procedure can be continued order-by-order in 1/f:

$$\mathcal{D}_\mu\phi = \partial_\mu\phi + \phi\frac{\partial_\mu\phi^*\,\phi - \partial_\mu\phi\,\phi^*}{2|\phi|^2}\left(1 - \frac{\tilde{f}}{|\phi|}\sin\frac{|\phi|}{\tilde{f}}\right)$$

$$\mathcal{L}^{(2)} = \mathcal{D}_\mu\phi\mathcal{D}^\mu\phi = \partial_\mu\phi^*\partial^\mu\phi - \frac{|\partial_\mu\phi^*\,\phi - \partial_\mu\phi\,\phi^*|^2}{4|\phi|^2}\left(1 - \frac{\tilde{f}^2}{|\phi|^2}\sin^2\frac{|\phi|}{\tilde{f}}\right)$$

- We managed to derive the effective Lagrangian without referring to any UV coset!

- There is only one undetermined parameter in the end, which corresponds to the overall normalization of f:

$$\tilde{f} = f/\sqrt{c_1}$$

Low: 1412.2145
Low: 1412.2146

- the sign of c_1 is not fixed:
  a positive sign implies a compact G/H (suppression), while a negative sign implies a non-compact G/H (enhancement).

- if UV completion is a concern, $c_1 > 0$ and the sign of the dim-6 operator is negative.

$$\tilde{f} = f/\sqrt{6c_1}$$

- one could introduce another object that transforms non-homogeneously like a gauge field:

$$\mathcal{E}_\mu \mapsto e^{-iu}\mathcal{E}_\mu e^{iu} - ie^{-iu}\partial_\mu e^{iu} = \mathcal{E}_\mu + \partial_\mu u(\phi, \epsilon)$$

$$\mathcal{E}_\mu = \frac{i}{\alpha}\frac{\partial_\mu\phi^*\,\phi - \partial_\mu\phi\,\phi^*}{|\phi|^2}\sin^2\frac{|\phi|}{2\tilde{f}}$$

Let's pause for a moment and reflect on what's happened…

We derived the two-derivative lagrangian for a complex Goldstone boson charged under an unbroken U(1):

$$\mathcal{L}^{(2)} = \mathcal{D}_\mu\phi\mathcal{D}^\mu\phi = \partial_\mu\phi^*\partial^\mu\phi - \frac{|\partial_\mu\phi^*\,\phi - \partial_\mu\phi\,\phi^*|^2}{4|\phi|^2}\left(1 - \frac{\tilde{f}^2}{|\phi|^2}\sin^2\frac{|\phi|}{\tilde{f}}\right)$$

The only assumptions are

1. The Adler's zero condition.
2. There exists an unbroken U(1).

As such, this is the universal lagrangian among all NLSM containing a complex Goldstone!

We can check against the universality using explicit examples:

$$
\begin{aligned}
SU(2)/U(1) \to\ & |\partial_\mu \phi|^2 - \frac{1}{3f^2}\left|\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*\right|^2 + \frac{8}{45f^4}\left|\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*\right|^2|\phi|^2 \\
& - \frac{16}{315f^6}\left|\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*\right|^2|\phi|^4 + \cdots \ , \qquad (3.18)
\end{aligned}
$$

$$
\begin{aligned}
SU(5)/SO(5) \to\ & |\partial_\mu \Phi|^2 - \frac{1}{48f^2}\left|\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*\right|^2 + \frac{1}{1440f^4}\left|\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*\right|^2|\Phi|^2 \\
& - \frac{1}{80640f^6}\left|\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*\right|^2|\Phi|^4 + \cdots \ , \qquad (3.19)
\end{aligned}
$$

At the first glance the two Lagrangians do not look the same…

We can check against the universality using explicit examples:

$$SU(2)/U(1) \to |\partial_\mu \phi|^2 - \frac{1}{3f^2}\left|\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*\right|^2 + \frac{8}{45f^4}\left|\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*\right|^2|\phi|^2$$
$$-\frac{16}{315f^6}\left|\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*\right|^2|\phi|^4 + \cdots \ , \tag{3.18}$$

$$SU(5)/SO(5) \to |\partial_\mu \Phi|^2 - \frac{1}{48f^2}\left|\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*\right|^2 + \frac{1}{1440f^4}\left|\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*\right|^2|\Phi|^2$$
$$-\frac{1}{80640f^6}\left|\Phi^*\partial_\mu\Phi - \Phi\partial_\mu\Phi^*\right|^2|\Phi|^4 + \cdots \ , \tag{3.19}$$

At the first glance the two Lagrangians do not look the same…

But upon f -> 4f in SU(2)/U(1) case, the two become identical!

This approach can be generalized to a general unbroken group H in the IR.

We assume a set of scalars furnishing a linear representation under a simple Lie group H:

$$\pi^a(x) \to \pi^a(x) + i\alpha^i (T^i)_{ab} \pi^b(x)$$

It is convenient to choose a basis where all generators are purely imaginary (and hence anti-symmetric!)

$$(T^i)^T = -T^i \text{ and } (T^i)^* = -T^i$$

Requiring

- The Adler's zero condition
- Unbroken H-invariance is linearly realized

We can derive the effective Lagrangian to all orders in 1/f:

$$\mathcal{L} = \frac{1}{2}[F_2(\mathcal{T})^2]_{ab}\ \partial_\mu \pi^a \partial^\mu \pi^b$$

$$\mathcal{T}_{ab} = (T^i)_{ar}(T^i)_{sb}\pi^r\pi^s$$

$$F_2(\mathcal{T}) = \frac{\sin\sqrt{\mathcal{T}}}{\sqrt{\mathcal{T}}}$$

The Lagrangian is invariant under the shift symmetry:

$$\pi^{a\,\prime} = \pi^a + [F_1(\mathcal{T})]_{ab}\ \varepsilon^b$$

$$F_1(\mathcal{T}) = \sqrt{\mathcal{T}}\cot\mathcal{T}$$

IL and Zhewei Yin: 1709.08639
IL and Zhewei Yin: 1804.08629

Requiring

- The Adler's zero condition
- Unbroken H-invariance is linearly realized

We can derive the effective Lagrangian to all orders in 1/f:

$$\mathcal{L} = \frac{1}{2}[F_2(\mathcal{T})^2]_{ab}\ \partial_\mu \pi^a \partial^\mu \pi^b$$

$$\mathcal{T}_{ab} = (T^i)_{ar}(T^i)_{sb}\pi^r\pi^s$$

$$F_2(\mathcal{T}) = \frac{\sin\sqrt{\mathcal{T}}}{\sqrt{\mathcal{T}}}$$

The Lagrangian is invariant under the shift symmetry:

$$\pi^{a\,\prime} = \pi^a + [F_1(\mathcal{T})]_{ab}\ \varepsilon^b$$

$$F_1(\mathcal{T}) = \sqrt{\mathcal{T}}\cot\mathcal{T}$$

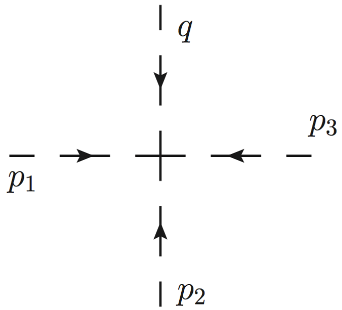
All this is achieved using only IR data, without recourse to a coset G/H!

IL and Zhewei Yin: 1709.08639
IL and Zhewei Yin: 1804.08629

In essence, the IR approach is “boostrapping” the NLSM amplitudes from Adler’s zeros:

Starting from a lower point amplitudes, construct the higher point amplitudes such that the Adler’s zero is satisfied, by introducing the necessary higher point vertices.

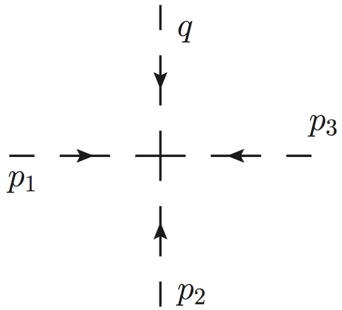
Starting from 4-pt amplitude:



$$-\frac{1}{f^2}s_{24}$$

$$s_{ij} = (p_i + p_j)^2$$
$$P_{ijk}^2 = (p_i + p_j + p_k)^2$$

Starting from 4-pt amplitude:

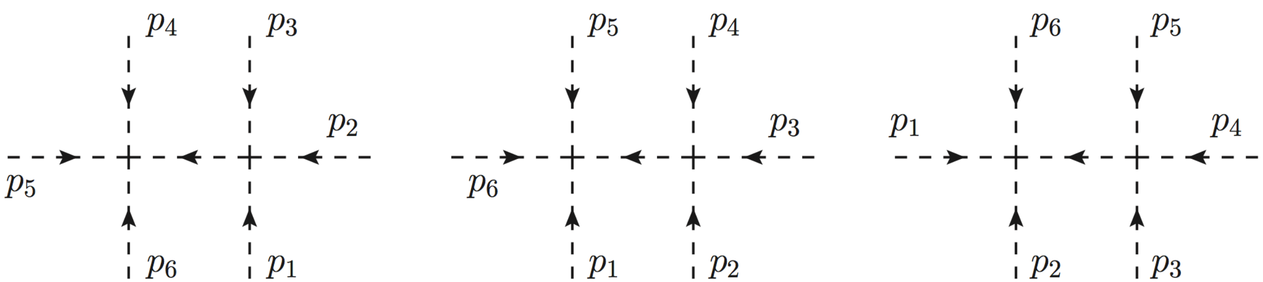


$$-\frac{1}{f^2}s_{24}$$

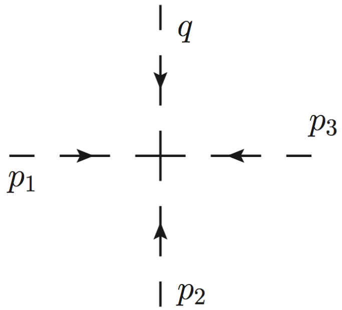
$$s_{ij} = (p_i + p_j)^2$$
$$P^2_{ijk} = (p_i + p_j + p_k)^2$$

The 6-pt amplitude built out of 4-pt amplitude doesn't have the correct soft limit (Adler's zero),

$$-\frac{1}{f^4}\left[\frac{(s_{12}+s_{23})(s_{45}+s_{56})}{P^2_{123}} + \frac{(s_{23}+s_{34})(s_{16}+s_{56})}{P^2_{234}} + \frac{(s_{34}+s_{45})(s_{16}+s_{12})}{P^2_{345}}\right]$$



Starting from 4-pt amplitude:



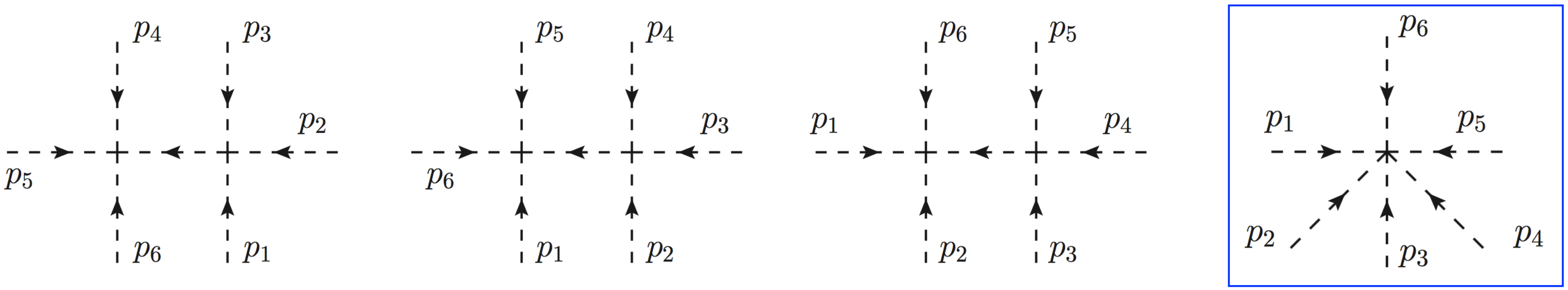
$$-\frac{1}{f^2}s_{24}$$

$$s_{ij} = (p_i + p_j)^2$$
$$P^2_{ijk} = (p_i + p_j + p_k)^2$$

The 6-pt amplitude built out of 4-pt amplitude doesn't have the correct soft limit (Adler's zero), unless a 6-pt vertex is added:

$$-\frac{1}{f^4}\left[\frac{(s_{12}+s_{23})(s_{45}+s_{56})}{P^2_{123}} + \frac{(s_{23}+s_{34})(s_{16}+s_{56})}{P^2_{234}} + \frac{(s_{34}+s_{45})(s_{16}+s_{12})}{P^2_{345}}\right]$$

$$+\frac{1}{f^4}\left(s_{12}+s_{23}+s_{34}+s_{45}+s_{56}+s_{16}\right)$$



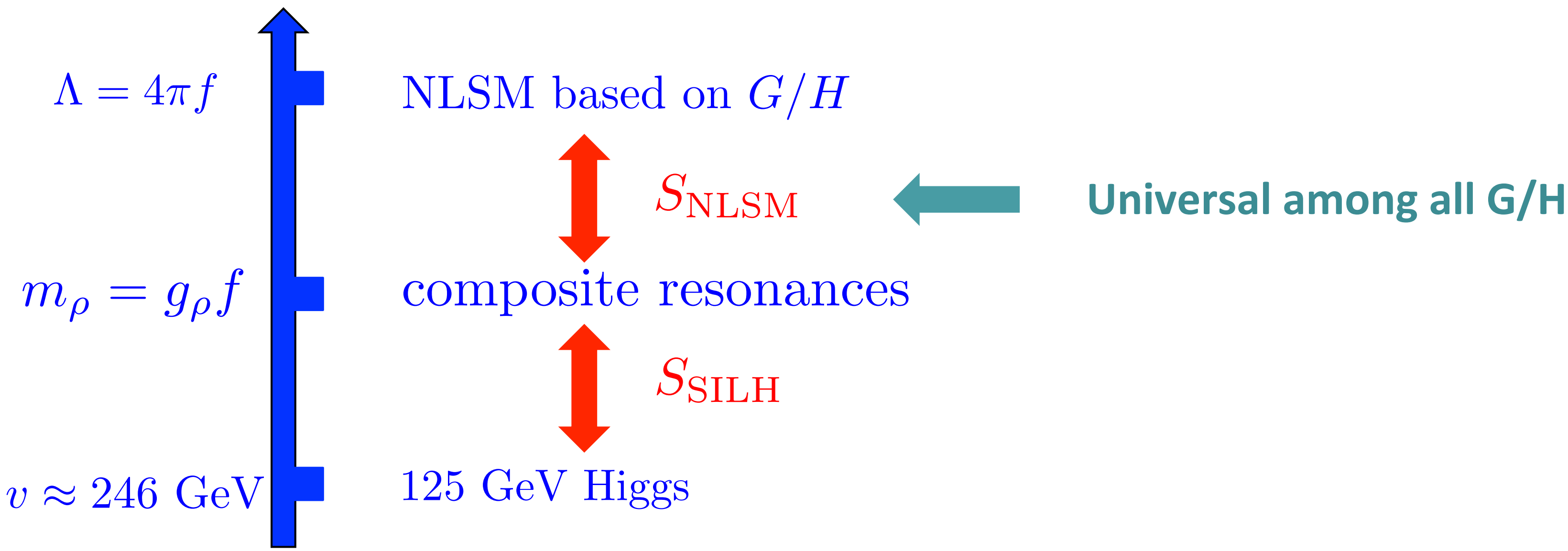
One common ingredient among all composite Higgs models are

- The unbroken group H contains an SO(4).

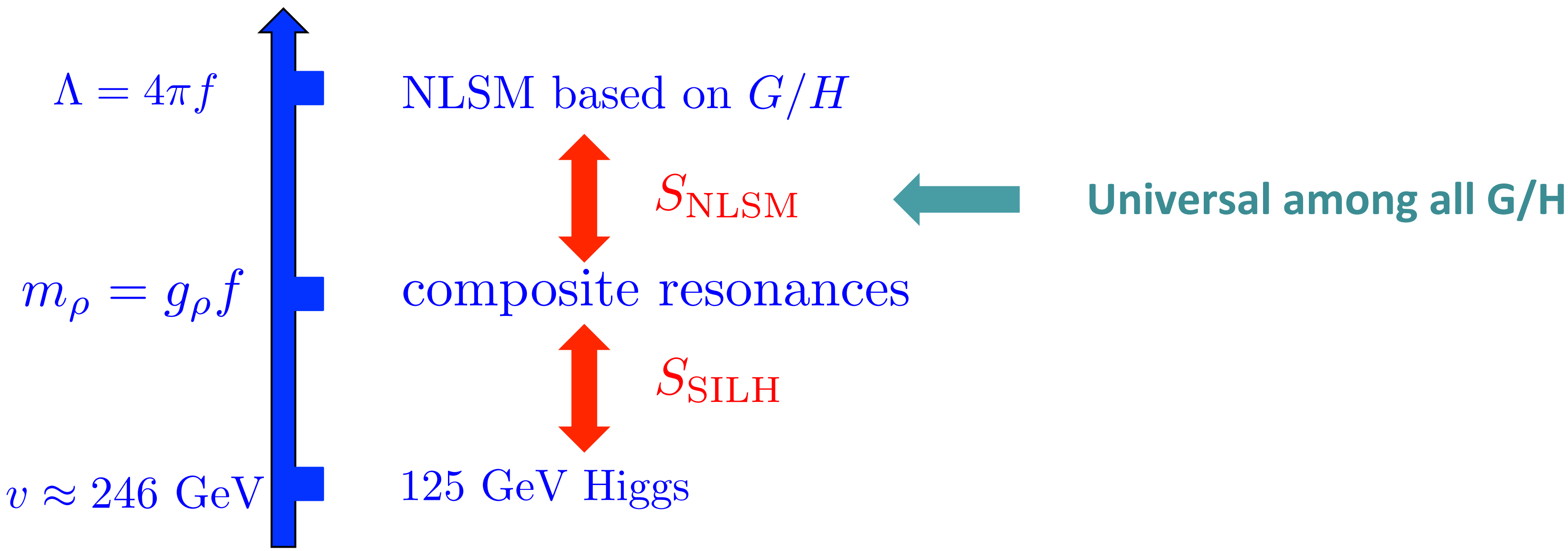
- The 125 GeV Higgs transforms as the fundamental representation of SO(4).

We conclude that the NLSM Lagrangian involving the 125 GeV Higgs in all composite Higgs models is **universal**, up to the normalization of the “f”.

But this is only half of the story.



But this is only half of the story.



Recall that the composite resonances are model-dependent. How can we make a statement on the EFT below the scale “f”??

Here we are "rescued" by the most important insights from SILH:

1. Each extra Goldstone leg is weighted by a factor $1/f$. For instance the addition of two Higgs doublet legs involves the factor $H^\dagger H/f^2$.

2. Each extra derivative is weighted by a factor $1/m_\rho$. When the SM subgroup is weakly gauged, the replacement $\partial_\mu \to \partial_\mu + iA_\mu \equiv D_\mu$ is in order; this same rule implies that each extra insertion of a gauge field strength $F_{\mu\nu} = -i[D_\mu, D_\nu]$ is weighted by a factor $1/m_\rho^2$.

3. Higher-dimensional operators that violate the symmetry of the σ-model must be suppressed by the same (weak) coupling associated to the corresponding renormalizable interaction in the SM Lagrangian (*e.g.*, Yukawa couplings y_f and quartic Higgs coupling λ).

Giudice, Grojean, Pomarol, Rattazzi: hep-ph/0703164

Here we are "rescued" by the most important insights from SILH:

1. Each extra Goldstone leg is weighted by a factor $1/f$. For instance the addition of two Higgs doublet legs involves the factor $H^\dagger H/f^2$.

2. Each extra derivative is weighted by a factor $1/m_\rho$. When the SM subgroup is weakly gauged, the replacement $\partial_\mu \to \partial_\mu + iA_\mu \equiv D_\mu$ is in order; this same rule implies that each extra insertion of a gauge field strength $F_{\mu\nu} = -i[D_\mu, D_\nu]$ is weighted by a factor $1/m_\rho^2$.

3. Higher-dimensional operators that violate the symmetry of the σ-model must be suppressed by the same (weak) coupling associated to the corresponding renormalizable interaction in the SM Lagrangian (*e.g.*, Yukawa couplings y_f and quartic Higgs coupling λ).

Giudice, Grojean, Pomarol, Rattazzi: hep-ph/0703164

Here we are “rescued” by the most important insights from SILH:

1. Each extra Goldstone leg is weighted by a factor $1/f$. For instance the addition of two Higgs doublet legs involves the factor $H^\dagger H/f^2$.

2. Each extra derivative is weighted by a factor $1/m_\rho$. When the SM subgroup is weakly gauged, the replacement $\partial_\mu \to \partial_\mu + iA_\mu \equiv D_\mu$ is in order; this same rule implies that each extra insertion of a gauge field strength $F_{\mu\nu} = -i[D_\mu, D_\nu]$ is weighted by a factor $1/m_\rho^2$.

3. Higher-dimensional operators that violate the symmetry of the σ-model must be suppressed by the same (weak) coupling associated to the corresponding renormalizable interaction in the SM Lagrangian (*e.g.*, Yukawa couplings y_f and quartic Higgs coupling λ).

Giudice, Grojean, Pomarol, Rattazzi: hep-ph/0703164

But otherwise the nonlinearity structure of the CCWZ Lagrangian is preserved in SILH, even after integrating out the composite resonances!

But otherwise the nonlinearity structure of the CCWZ Lagrangian is preserved in SILH, even after integrating out the composite resonances!

Instructions from SILH:

- Construct the CCWZ Lagrangian based on G/H.

- Below the scale "f", after all resonances have been integrated out, make the replacement:

$$\Lambda = 4\pi f \to m_\rho = g_\rho f$$

But otherwise the nonlinearity structure of the CCWZ Lagrangian is preserved in SILH, even after integrating out the composite resonances!

Instructions from SILH:

- Construct the CCWZ Lagrangian based on G/H.

- Below the scale "f", after all resonances have been integrated out, make the replacement:

$$\Lambda = 4\pi f \to m_\rho = g_\rho f$$

In the end, **SILH inherits the universal nonlinearity from CCWZ**:

$$\mathcal{L} = \frac{m_\rho^4}{g_\rho^2}\left[\mathcal{L}^{(0)}(U, \Phi, \partial/m_\rho) + \frac{g_\rho^2}{(4\pi)^2}\mathcal{L}^{(1)}(U, \Phi, \partial/m_\rho) + \frac{g_\rho^4}{(4\pi)^4}\mathcal{L}^{(2)}(U, \Phi, \partial/m_\rho) + \ldots\right]$$

The leading two-derivative universal Lagrangian in the unitary gauge:

$$\mathcal{L}^{(2)} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{g^2 f^2}{4}\sin^2(\theta + h/f) \times \left(W_\mu^+ W^{-\mu} + \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu \right)$$

$$= \frac{1}{2}\partial_\mu h \partial^\mu h + \left[1 + 2\sqrt{1-\xi}\,\frac{h}{v} + (1-2\xi)\,\frac{h^2}{v^2} + \cdots \right] \times \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \right) \; ,$$

$$\sin\theta \;\equiv\; v/f$$

$$\xi \equiv v^2/f^2$$

The leading two-derivative universal Lagrangian in the unitary gauge:

$$
\begin{aligned}
\mathcal{L}^{(2)} &= \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{g^2 f^2}{4}\sin^2(\theta + h/f) \\
&\quad \times \left(W_\mu^+ W^{-\mu} + \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu \right) \\
&= \frac{1}{2}\partial_\mu h \partial^\mu h + \left[1 + 2\sqrt{1-\xi}\,\frac{h}{v} + (1-2\xi)\,\frac{h^2}{v^2} + \cdots \right] \\
&\quad \times \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \right) ,
\end{aligned}
$$

$$\sin\theta \equiv v/f$$

$$\xi \equiv v^2/f^2$$

Recall that $\sin\theta$ is related to the normalization of “f” and thus coset-dependent.

Once it’s measured, the rest of the $h^n VV$ couplings are fully determined!

It is clear now what the strategy is:

Measure one single parameter "sin θ" in hVV coupling as the input, and use it to predict other $h^n VV$ couplings, $n \geq 2$.

For examples, if we define in the SILH Lagrangian

$$g_{nh}^{(2)} \left(\frac{h}{v}\right)^n \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu\right)$$

Then one universal prediction of the Higgs nonlinearity is

$$\frac{g_h^{(2)}}{g_{2h}^{(2)}} = \frac{1-2\xi}{\sqrt{1-\xi}}$$

But this is just one of the many universal predictions.

The overarching goal should be to "over-constrain" the predictions using as many observables as possible.

We can get more observables by going to VV -> nh scattering with n>3. But the rate obviously works against us, even in a future high energy collider.

But this is just one of the many universal predictions.

The overarching goal should be to "over-constrain" the predictions using as many observables as possible.

We can get more observables by going to VV -> nh scattering with n>3. But the rate obviously works against us, even in a future high energy collider.

Alternatively, we can stay with VV -> nh channel, with $n \leq 2$, by exploiting angular observables. However, this requires going to $O(p^4)$ in the SILH Lagrangian.

There are 11 operators at O(p^4). We focus on those 6 that do not contain the epsilon tensors:

$$
\begin{aligned}
O_1 &= \left(d^a_\mu d^{\mu a}\right)^2 , \qquad O_2 = (d^a_\mu d^a_\nu)^2 \ , \\
O_3 &= \left[\left(E^L_{\mu\nu}\right)^i\right]^2 - \left[\left(E^R_{\mu\nu}\right)^i\right]^2 \ , \\
O_4^\pm &= -i\, d^a_\mu d^b_\nu \left[(f^{+L}_{\mu\nu})^i\, T^i_L \pm (f^{+R}_{\mu\nu})^i\, T^i_R\right]_{ab} \ , \\
O_5^+ &= \left[(f^-_{\mu\nu})^a\right]^2 , \qquad O_5^- = \left[(f^{+L}_{\mu\nu})^i\right]^2 - \left[(f^{+R}_{\mu\nu})^i\right]^2
\end{aligned}
$$

Contino et. al.: 1109.1570

These operators were enumerated previously, but not computed.

Like the two-derivative Lagrangian, they can also be expressed entirely using only IR data:

$$d_\mu^a(\pi, \partial) = [\mathcal{F}_2(\mathcal{T})]_{ab}\partial_\mu \pi^b \ ,$$

$$E_\mu^i(\pi, \partial) = \frac{1}{f^2}\partial_\mu \pi^a [\mathcal{F}_4(\mathcal{T})]_{ab}(T^i\pi)^b$$

$$(f_{\mu\nu}^-)^a = [\mathcal{F}_2(\mathcal{T})]_{ab}(T^i\pi)^b\, F_{\mu\nu}^i$$

$$(f_{\mu\nu}^+)^i = \frac{1}{f^2}F_{\mu\nu}^j(T^j\pi)^a[\mathcal{F}_4(\mathcal{T})]_{ab}(T^i\pi)^b$$

$$\mathcal{F}_2(\mathcal{T}) = \frac{\sin\sqrt{\mathcal{T}}}{\sqrt{\mathcal{T}}} \qquad \mathcal{F}_4(\mathcal{T}) = \frac{2}{\mathcal{T}}\sin^2\frac{\sqrt{\mathcal{T}}}{2}$$

Da Liu, IL, Zhewei Yin: 1805.nnnnn

We then worked out the complete predictions in hVV couplings:

Interaction $\mathcal{I}_i^h$	C_i^h (NL)	C_i^h (D6)
$2m_W^2 \frac{h}{v} W_\mu^+ W^{-\mu}$	$\sqrt{1-\xi}$	$1-\frac{1}{2}c_H\xi$
$m_Z^2 \frac{h}{v} W_\mu^+ W^{-\mu}$	$\sqrt{1-\xi}$	$1-\frac{1}{2}c_H\xi$
(1) $\frac{h}{v} W_\mu^+ \mathcal{D}^{\mu\nu} W_\nu^-$	$4(-2c_3+c_4^-)$ $+4c_4^+\cos\theta$	$2(c_W+c_{HW})$
(2) $\frac{h}{v} W_{\mu\nu}^+ W^{-\mu\nu}$	$-4\left(c_4^+-2c_5^+\right)\cos\theta$ $-4(c_4^-+2c_5^-)$	- $2c_{HW}$
(3) $\frac{h}{v} Z_\mu \mathcal{D}^{\mu\nu} Z_\nu$	$\frac{4c_{2\theta_W}}{c_{\theta_W}^2}\left(-2c_3+c_4^-\right)$ $+\frac{4}{c_{\theta_W}^2}c_4^+\cos\theta$	$2c_W+c_{HW}$ $+2t_{\theta_W}^2(c_B+c_{HB})$
(4) $\frac{h}{v} Z_{\mu\nu} Z^{\mu\nu}$	$-\frac{2}{c_{\theta_W}^2}\left(c_4^+-2c_5^+\right)\cos\theta$ $-\frac{2}{c_{\theta_W}^2}\left(c_4^-+2c_5^-\right)c_{2\theta_W}$	-$(c_{HW}+t_{\theta_W}^2 c_{HB})$
(5) $\frac{h}{v} Z_\mu \mathcal{D}^{\mu\nu} A_\nu$	$8\left(-2c_3+c_4^-\right)t_{\theta_W}$	$2t_{\theta_W}(c_W+c_{HW})$ $-2t_{\theta_W}(c_B+c_{HB})$
(6) $\frac{h}{v} Z_{\mu\nu} A^{\mu\nu}$	-4 $\left(c_4^-+2c_5^-\right)t_{\theta_W}$	-$t_{\theta_W}(c_{HW}-c_{HB})$

TABLE I: Single Higgs coupling coefficients C_i^h for the non-linearity case (NL) and the purely dimension-six case (D6). Here

Da Liu, IL, Zhewei Yin: 1805.nnnnn

And hhVV couplings:

Interaction $\mathcal{I}_i^{h^2}$	$C_i^{h^2}$ (NL)	$C_i^{h^2}$ (D6)
$m_W^2 \frac{h^2}{v^2} W_\mu^+ W^{-\mu}$	$1-2\xi$	$1-2c_H\xi$
$\frac{1}{2} m_Z^2 \frac{h^2}{v^2} W_\mu^+ W^{-\mu}$	$1-2\xi$	$1-2c_H\xi$
(1) $\frac{h^2}{v^2} W_\mu^+ \mathcal{D}^{\mu\nu} W_\nu^-$	$2(-2c_3+c_4^-)\cos\theta$ $+2c_4^+\cos 2\theta$	$c_W + c_{HW}$
(2) $\frac{h^2}{v^2} W_{\mu\nu}^+ W^{-\mu\nu}$	$-2\left(c_4^+ - 2c_5^+\right)\cos 2\theta$ $-2\left(c_4^- + 2c_5^-\right)\cos\theta$	- c_{HW}
(3) $\frac{h^2}{v^2} Z_\mu \mathcal{D}^{\mu\nu} Z_\nu$	$\frac{2c_{2\theta_W}}{c_{\theta_W}^2}\left(-2c_3+c_4^-\right)\cos\theta$ $+\frac{2}{c_{\theta_W}^2} c_4^+ \cos 2\theta$	$c_W + c_{HW}$ $+t_{\theta_W}^2(c_B + c_{HB})$
(4) $\frac{h^2}{v^2} Z_{\mu\nu} Z^{\mu\nu}$	$-\frac{1}{c_{\theta_W}^2}\left(c_4^+ - 2c_5^+\right)\cos 2\theta$ $-\frac{c_{2\theta_W}}{c_{\theta_W}^2}\left(c_4^- + 2c_5^-\right)\cos\theta$	$-\frac{1}{2}(c_{HW} + t_{\theta_W}^2 c_{HB})$
(5) $\frac{h^2}{v^2} Z_\mu \mathcal{D}^{\mu\nu} A_\nu$	$4t_{\theta_W}\left(-2c_3+c_4^-\right)\cos\theta$	$t_{\theta_W}(c_W + c_{HW})$ $-t_{\theta_W}(c_B + c_{HB})$
(6) $\frac{h^2}{v^2} Z_{\mu\nu} A^{\mu\nu}$	$-2t_{\theta_W}\left(c_4^- + 2c_5^-\right)\cos\theta$	$-\frac{1}{2} t_{\theta_W}(c_{HW} - c_{HB})$
(7) $\frac{(\partial_\nu h)^2}{v^2} W_\mu^+ W^{-\mu}$	$16\xi c_1$	
(8) $\frac{\partial^\mu h \partial^\nu h}{v^2} W_\mu^+ W_\nu^-$	$16\xi c_2$	
(9) $\frac{(\partial_\nu h)^2}{v^2} Z_\mu Z^\mu$	$\frac{8}{c_{\theta_W}^2}\xi c_1$	
(10) $\frac{\partial_\mu h \partial_\nu h}{v^2} Z^\mu Z^\nu$	$\frac{8}{c_{\theta_W}^2}\xi c_2$	

TABLE II: The coupling coefficients $C_i^{h^2}$ involved two Higgs boson for the non-linearity case (NL) and the purely dimension-six case (D6).

Da Liu, IL, Zhewei Yin: 1805.nnnnn

And hhVV couplings:

Interaction $\mathcal{I}_i^{h^2}$	$C_i^{h^2}$ (NL)	$C_i^{h^2}$ (D6)
$m_W^2 \frac{h^2}{v^2} W_\mu^+ W^{-\mu}$	$1-2\xi$	$1-2c_H\xi$
$\frac{1}{2} m_Z^2 \frac{h^2}{v^2} W_\mu^+ W^{-\mu}$	$1-2\xi$	$1-2c_H\xi$
(1) $\frac{h^2}{v^2} W_\mu^+ \mathcal{D}^{\mu\nu} W_\nu^-$	$2(-2c_3+c_4^-)\cos\theta$ $+2c_4^+\cos 2\theta$	$c_W + c_{HW}$
(2) $\frac{h^2}{v^2} W_{\mu\nu}^+ W^{-\mu\nu}$	$-2\left(c_4^+ - 2c_5^+\right)\cos 2\theta$ $-2\left(c_4^- + 2c_5^-\right)\cos\theta$	- c_{HW}
(3) $\frac{h^2}{v^2} Z_\mu \mathcal{D}^{\mu\nu} Z_\nu$	$\frac{2c_{2\theta_W}}{c_{\theta_W}^2}\left(-2c_3+c_4^-\right)\cos\theta$ $+\frac{2}{c_{\theta_W}^2} c_4^+ \cos 2\theta$	$c_W + c_{HW}$ $+t_{\theta_W}^2(c_B + c_{HB})$
(4) $\frac{h^2}{v^2} Z_{\mu\nu} Z^{\mu\nu}$	$-\frac{1}{c_{\theta_W}^2}\left(c_4^+ - 2c_5^+\right)\cos 2\theta$ $-\frac{c_{2\theta_W}}{c_{\theta_W}^2}\left(c_4^- + 2c_5^-\right)\cos\theta$	$-\frac{1}{2}(c_{HW} + t_{\theta_W}^2 c_{HB})$
(5) $\frac{h^2}{v^2} Z_\mu \mathcal{D}^{\mu\nu} A_\nu$	$4t_{\theta_W}\left(-2c_3+c_4^-\right)\cos\theta$	$t_{\theta_W}(c_W + c_{HW})$ $-t_{\theta_W}(c_B + c_{HB})$
(6) $\frac{h^2}{v^2} Z_{\mu\nu} A^{\mu\nu}$	$-2t_{\theta_W}\left(c_4^- + 2c_5^-\right)\cos\theta$	$-\frac{1}{2} t_{\theta_W}(c_{HW} - c_{HB})$
(7) $\frac{(\partial_\nu h)^2}{v^2} W_\mu^+ W^{-\mu}$	$16\xi c_1$	
(8) $\frac{\partial^\mu h \partial^\nu h}{v^2} W_\mu^+ W_\nu^-$	$16\xi c_2$	
(9) $\frac{(\partial_\nu h)^2}{v^2} Z_\mu Z^\mu$	$\frac{8}{c_{\theta_W}^2}\xi c_1$	
(10) $\frac{\partial_\mu h \partial_\nu h}{v^2} Z^\mu Z^\nu$	$\frac{8}{c_{\theta_W}^2}\xi c_2$	

TABLE II: The coupling coefficients $C_i^{h^2}$ involved two Higgs boson for the non-linearity case (NL) and the purely dimension-six case (D6).

Da Liu, IL, Zhewei Yin: 1805.nnnnn

And TGC couplings:

Interaction $\mathcal{I}_i$	Coefficients C_i^{3V}	TGC
$-\frac{t_{\theta_W}}{2} W^{(3)}_{\mu\nu} B^{\mu\nu}$	$8c_5^+$	$\hat{S}$
$igc_{\theta_W} W^+_\mu W^{-\mu\nu} Z_\nu$ $+h.c.$	$\frac{2}{c^2_{\theta_W}}\left(-2c_3+c_4^-\right)\cos\theta$ $+\frac{2}{c^2_{\theta_W}}c_4^+$	δg_1^Z
$igc_{\theta_W} W^{+\mu} W^{-\nu} Z_{\mu\nu}$	$\frac{2}{c^2_{\theta_W}}\left(2c_3-c_4^-\right)\cos\theta$ $-\frac{2}{c^2_{\theta_W}}(c_4^+ c_{2\theta_W}+4c_5^+ s^2_{\theta_W})$	$\delta\kappa_Z$
$ieW^{+\mu}W^{-\nu}A_{\mu\nu}$	$-4\left(c_4^+-2c_5^+\right)$	$\delta\kappa_\gamma$
$ig\frac{h}{v}W^+_\mu W^{-\mu\nu} Z_\nu$ $+h.c.$	$-\frac{4}{c_{\theta_W}}c_4^+\cos\theta$ $+\frac{1}{c_{\theta_W}}(2c_3-c_4^-)(1+3\cos 2\theta)$ $-\frac{16}{c_{\theta_W}}(c_3+c_5^- -c_5^+\cos\theta)c^2_{\theta_W}$	
$ie\frac{h}{v}W^+_\mu W^{-\mu\nu} A_\nu$ $+h.c.$	$-16(c_3+c_5^- -c_5^+\cos\theta)$	
$ig\frac{h}{v}W^{+\mu}W^{-\nu}Z_{\mu\nu}$	$\frac{2}{c_{\theta_W}}(4c_3-2c_4^+)\cos^2\theta$ $+\frac{2}{c_{\theta_W}}(-2c_3+c_4^-)\sin^2\theta$ $+16c_{\theta_W}(c_3+c_5^-)$ $-\frac{4}{c_{\theta_W}}(4c_5^+ + c_4^+ c_{2\theta_W})\cos\theta$	
$ie\frac{h}{v}W^{+\mu}W^{-\nu}A_{\mu\nu}$	$16(c_3+c_5^-)-8c_4^+\cos\theta$	
$ig' W^+_{[\mu} W^-_{\nu]} B^\mu \frac{\partial^\nu h}{v}$	$-8c_4^+\frac{\cos 2\theta}{\sin\theta}$ $+8\frac{\cos\theta}{\sin\theta}(2c_3-c_4^- +3c_3\sin^2\theta)$	

TABLE III: Triple gauge boson couplings.

Da Liu, IL, Zhewei Yin: 1805.nnnnn

There are many predictions:

$$\frac{C_5^{h^2}}{C_5^h} = \frac{1}{2}\cos\theta, \qquad \frac{C_6^{h^2}}{C_6^h} = \frac{1}{2}\cos\theta$$

$$\frac{C_1^{h^2} - \frac{C_5^{h^2}}{2\tan\theta_W}}{C_1^h - \frac{C_5^h}{2\tan\theta_W}} = \frac{\cos 2\theta}{2\cos\theta}, \qquad \frac{C_2^{h^2} - \frac{C_6^{h^2}}{\tan\theta_W}}{C_2^h - \frac{C_6^h}{\tan\theta_W}} = \frac{\cos 2\theta}{2\cos\theta}$$

$$\frac{C_3^{h^2} - \frac{C_5^{h^2}\cos 2\theta_W}{2\tan\theta_W}}{C_3^h - \frac{C_5^h\cos 2\theta_W}{2\tan\theta_W}} = \frac{\cos 2\theta}{2\cos\theta}, \quad \frac{C_4^{h^2} - \frac{C_6^{h^2}\cos 2\theta_W}{2\tan\theta_W}}{C_4^h - \frac{C_6^h\cos 2\theta_W}{2\tan\theta_W}} = \frac{\cos 2\theta}{2\cos\theta}$$

$$\frac{C_1^h - \frac{C_5^h}{2\tan\theta_W}}{2\delta g_1^Z\cos^2\theta_W - \frac{C_5^h\cos\theta}{2\tan\theta_W}} = \cos\theta$$

Da Liu, IL, Zhewei Yin: 1805.nnnnn

This is in sharp contrast with SMEFT with arbitrary coefficients,

$$\mathcal{O}_W = \frac{ig}{2}\left(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H\right) D^\nu W^a_{\mu\nu}, \quad \mathcal{O}_B = \frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right) \partial^\nu B_{\mu\nu}$$

$$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}, \quad \mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$$

$$-\frac{C_1^h - \frac{C_5^h}{2\tan\theta_W}}{2\delta\tilde{g}_1^Z \cos^2\theta_W + \frac{C_5^h \cos\theta}{2\tan\theta_W}} \sim 1 - \frac{1}{2}\frac{c_W + c_{HW} - c_B - c_{HB}}{c_W + c_{HW} + c_B + c_{HB}}\xi$$

While universal nonlinearity predicts

$$-\frac{C_1^h - \frac{C_5^h}{2\tan\theta_W}}{2\delta\tilde{g}_1^Z \cos^2\theta_W + \frac{C_5^h \cos\theta}{2\tan\theta_W}} = \cos\theta = 1 - \frac{1}{2}\xi + \cdots$$

Da Liu, IL, Zhewei Yin: 1805.nnnnn

Concluding Remarks:

- The Adler's zero should be taken as the defining property of Nambu-Goldstone bosons. Goldstone nonlinear interactions are universal among a common unbroken group H in the IR.
- All composite Higgs models contain a common universal Lagrangian (the symmetry-preserving part.)
- The universal nonlinearity predicts all Higgs couplings to VV with only one free parameter, reflecting the normalization of "f".
- Testing these relations should be among the top priorities in future experimental programs involving the 125 GeV Higgs.