Higgscitement: Cosmological Dynamics of Fine Tuning

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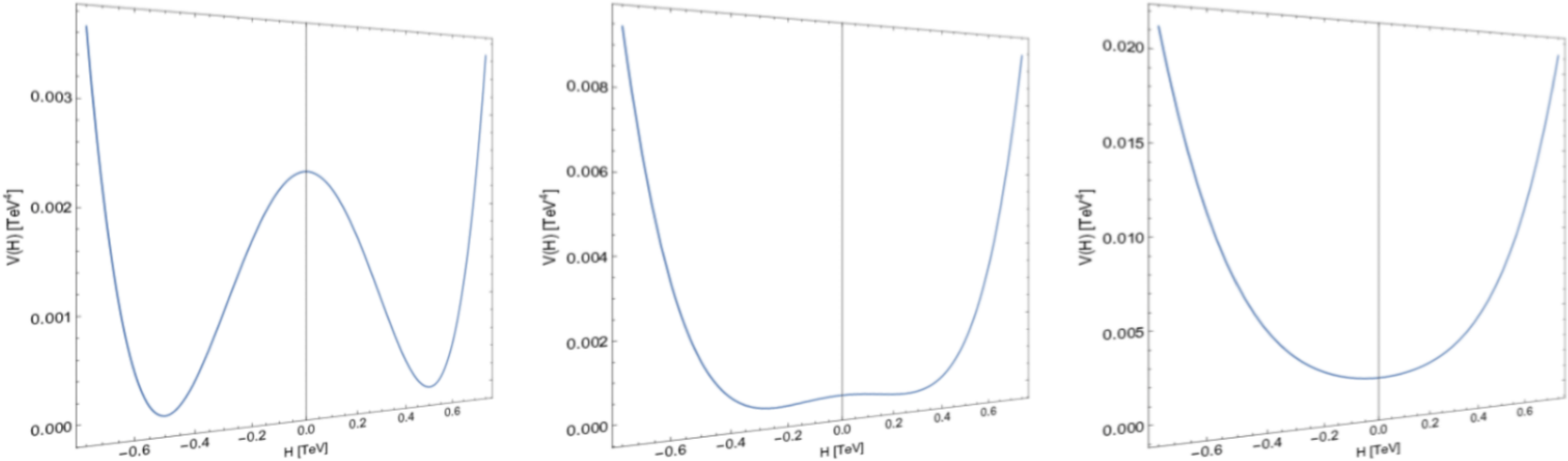
Based on 1802.00444 with Mustafa Amin, JiJi Fan, and Kaloian Lozanov

A Fine-Tuned Higgs?



Is our universe precariously balanced between unbroken EWSB and badly broken EWSB? If so, how can we tell?

We want to suggest some possible new ways to think about these questions.

An Opinion

No new physics at the LHC is not a crisis.

It's probably more of a small accident.

We're unlucky enough to live in a corner of the multiverse where we didn't get our SUSY (or other) discovery yet.

So What?

One easy answer is to sit back and wait for more data.

We can also plan for higher-energy colliders if the LHC isn't quite enough.

But another question is: could there be a *positive* signal of fine-tuning?

Fine-tuning in *Field Space*

The idea of tuning in *theory space* is too abstract to do much with. If heavy particles coupled differently to the Higgs, our vacuum would be very different. But we can't change how particles couple.

Or can we? Couplings depend on VEVs.

In the early universe, various scalar fields could have had large VEVs, so effective couplings were different.

Could have had unbroken EWSB or much more badly broken EWSB.

Even better, could have *dynamics*, fine-tuning in *time*.

Well motivated theories supply lots of good candidates for large variations in field space: saxions, moduli, D-flat directions.

Let's explore what can happen!

Coupling a modulus to the Higgs

Consider a coupling linear in the modulus:

$$V(\phi, H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + M^2 \frac{\phi}{f}\left(H^\dagger H - \frac{v^2}{2}\right) + \frac{1}{2} m_\phi^2 \phi^2 + \cdots$$

Higgs mass term depends on the modulus value.
Global minimum at $H/\sqrt{2} = v$, $\phi = 0$.

Scales: μ **: Standard Model Higgs mass param**

f **: Modulus field range (~ Planck?)**

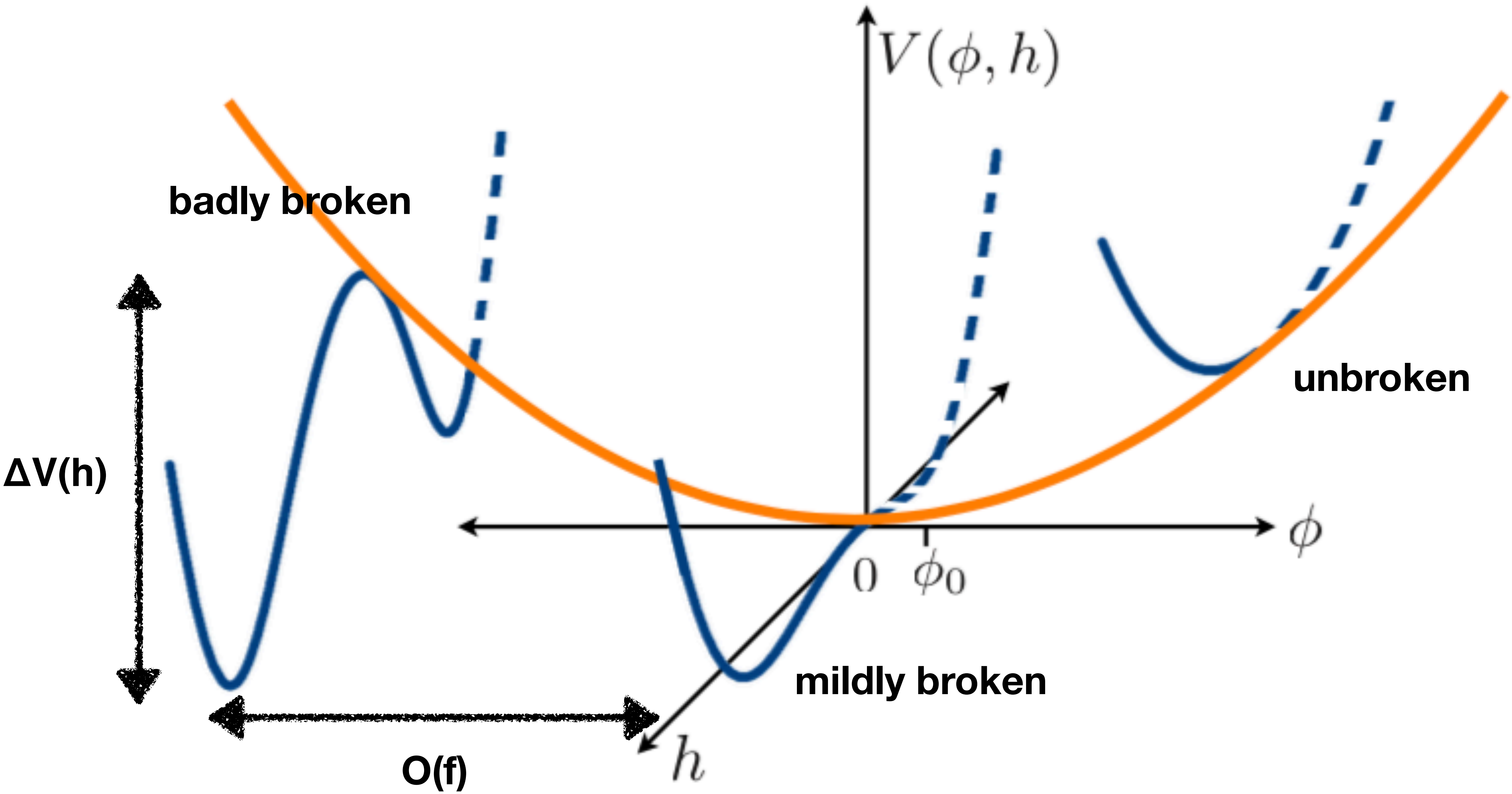
M **: "Natural" Higgs mass param (~ 100s TeV?)**

m_ϕ **: Modulus mass (~ 100s TeV?)**

Possible hierarchies: $\mu \ll m_\phi \lesssim M \ll f$

(Worth considering other variations too)

Modulus-Higgs potential



Fine tuning is the coincidence between the minimum of the ϕ potential and the point of marginal EWSB.

More on Fine-Tuning

The notion of fine-tuning we're using here is really the same as the standard one where we talk about loop corrections, at least up to 4pi's.

For instance, if the modulus has a SUSY breaking VEV:

$$\langle \boldsymbol{X} \rangle = X_0 + F_{X,0}\theta^2, \quad \text{where} \quad X_0 \sim m_{\rm pl}, \quad F_{X,0} \sim m_{3/2} m_{\rm pl}.$$

then it affects soft terms through Planck-suppressed operators:

$$\int d^4\theta \frac{\xi_{XZ}}{m_{\rm pl}^2} \boldsymbol{X}^\dagger \boldsymbol{X} \boldsymbol{Z}^\dagger \boldsymbol{Z} \supset \xi_{XZ} \frac{|F_X|^2}{m_{\rm pl}^2} Z^\dagger Z,$$

but also, if X deviates from its minimum, these soft terms shift:

$$\frac{2\xi_{XZ}\mathrm{Re}(F_{X,0} m_X)}{m_{\rm pl}^2} \mathrm{Re}(X) Z^\dagger Z.$$

All of our structure fits nicely in SUSY with *M, m*$_\phi$ set by SUSY-breaking scale. Fine-tuning the Higgs VEV below the SUSY breaking scale can be done, and will naturally occur for *some* choice of ϕ; for that choice to be the ground state is where the tuning goes.

Oscillating between EWS and EWSB

Ignoring backreaction, the modulus starts oscillating when Hubble is below its mass. Assuming a modulus-dominated universe,

$$\phi(t) \approx \frac{\xi_\phi f}{m_\phi t} \cos(m_\phi (t - t_0)).$$

The Higgs will flip between tachyonic and not tachyonic if

$$|M^2 \phi(t)/f| > |\mu^2|$$

This flipping stops when

$$m_\phi t \gtrsim \xi_\phi \frac{M^2}{\mu^2}$$

But M^2/μ^2 **is a measure of tuning!**

The number of EW-flipping oscillations probes fine tuning.

Tachyonic particle production

As the modulus oscillates, if mϕ is at least a little bit small compared to *M*, the Higgs has time to respond.

That is, there is a tachyonic particle production process when the Higgs flips to the tachyonic side, converting modulus energy into the Higgs energy.

Tachyonic resonance efficiency parameter:

$$q \equiv \frac{M^2}{m_\phi^2} \gg 1$$

The problem of backreaction

As the modulus oscillates, if mϕ is at least a little bit small compared to *M*, the Higgs has time to respond.

That is, there is a tachyonic particle production process.

This potentially depletes energy from the modulus. But: create too many Standard Model particles, and they backreact.

Simple estimate: the process shuts off once

$$\rho_{\mathrm{SM}} \sim \rho_\phi$$

Crudely, can think of this as the quartic

$$\lambda h^4 \sim \lambda \langle h^2 \rangle h^2$$

turning into a positive mass for the Higgs (more discussions later)

Numerics

Saying what happens after backreaction occurs is difficult.

Use a modified version of LatticeEasy (Felder, Tkachev ’00).

These are *classical field theory* calculations on a lattice with stochastic initial conditions.

They are valid only for a limited range of times. Power transferred to small scales eventually invalidates the calculation.

Still, we can learn at least a couple of useful parametric statements from the results.

For some parameters, the dynamics are violent, the modulus fragments, and we get an interesting *interacting phase*.

> ***This scenario is similar to “tachyonic preheating”: Dufaux, Felder, Kofman, Peloso, Podolsky, hep-ph/0602144.***

Results: fragmentation and equation of state

Fragmentation of the modulus due to back-reaction is controlled by

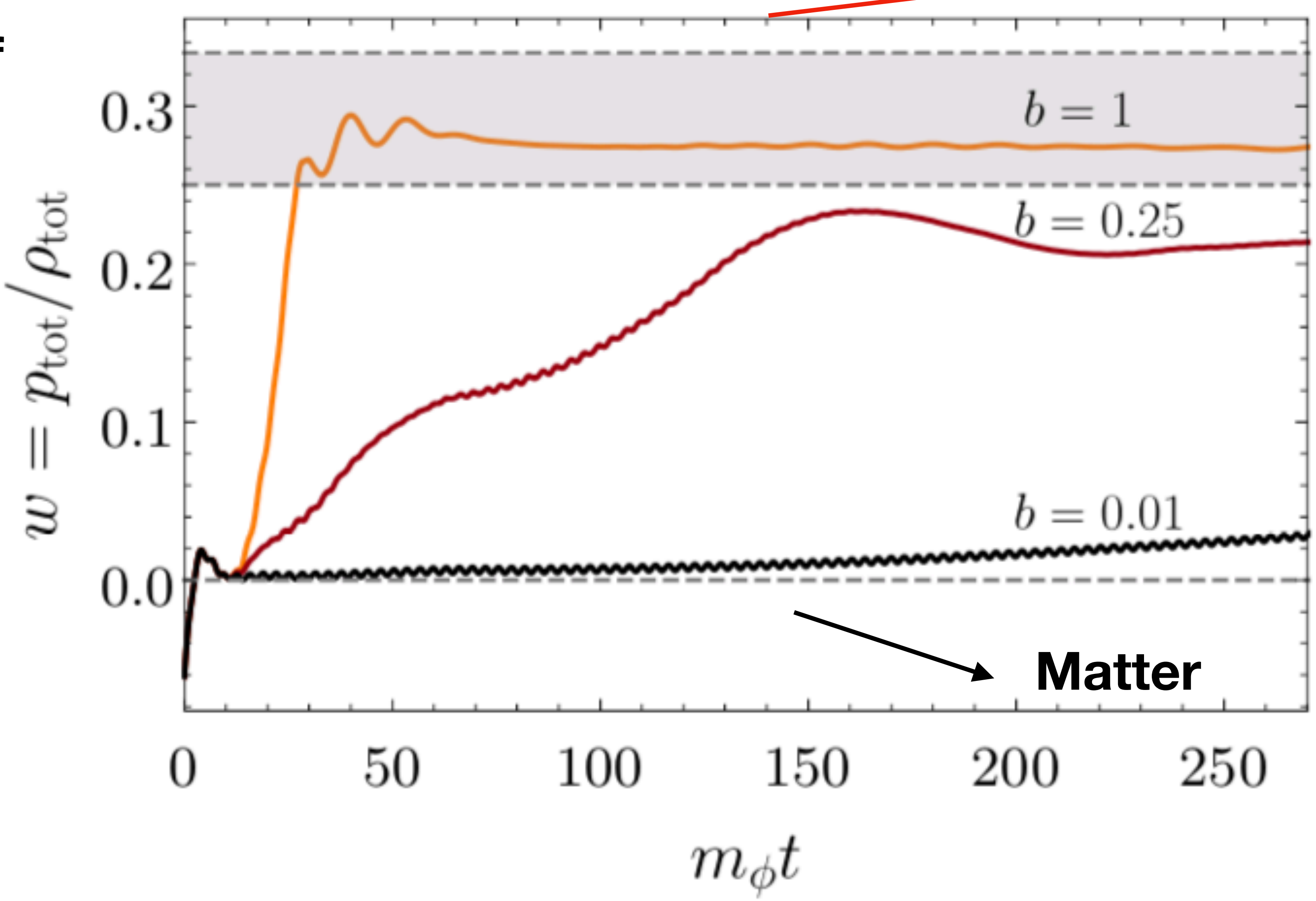
potential is positive definite

$$b \equiv \frac{M^4}{2\lambda f^2 m_\phi^2} \leq 1$$

Radiation

equation of state

Full fragmentation

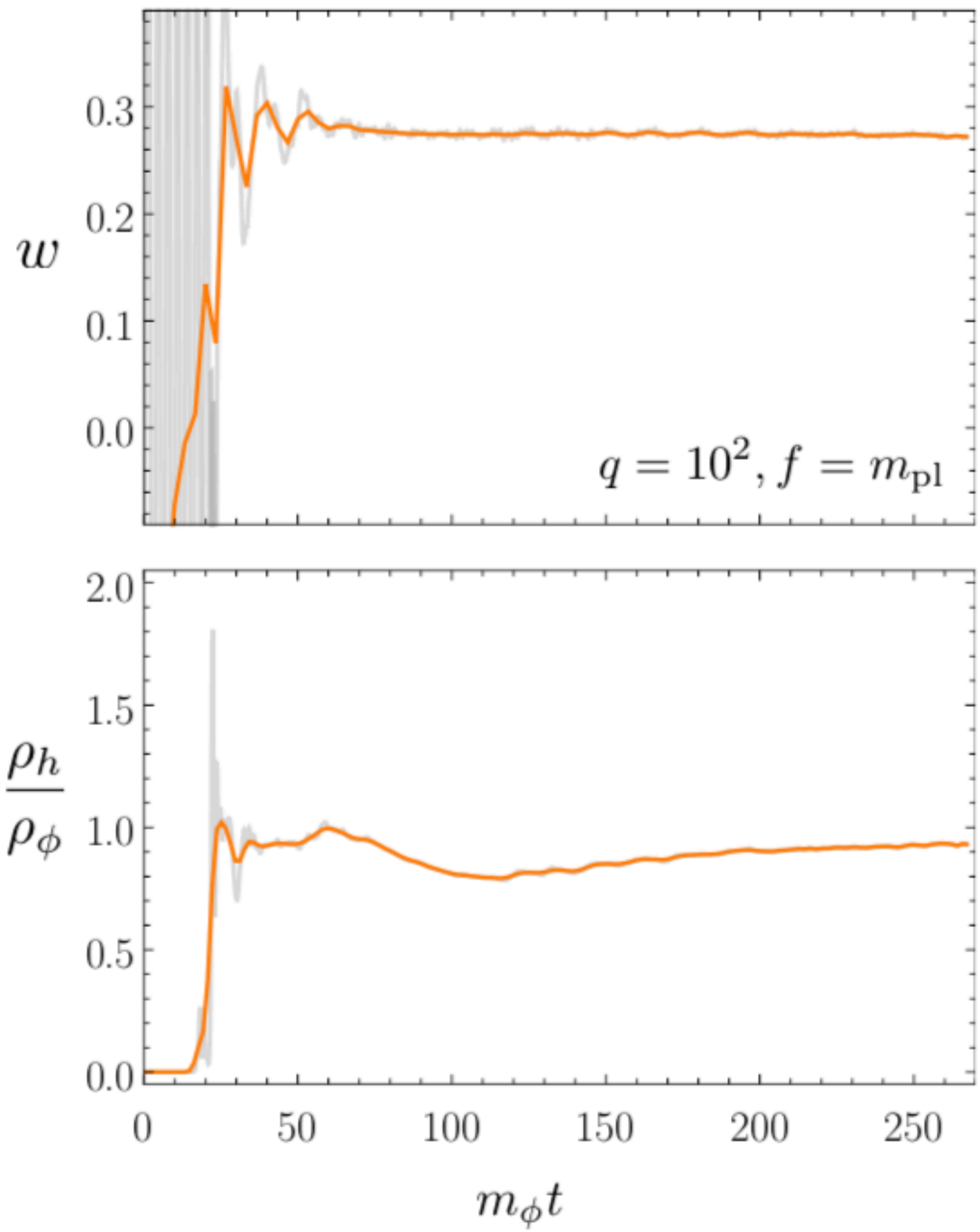


No fragmentation

Matter

Time

Full fragmentation

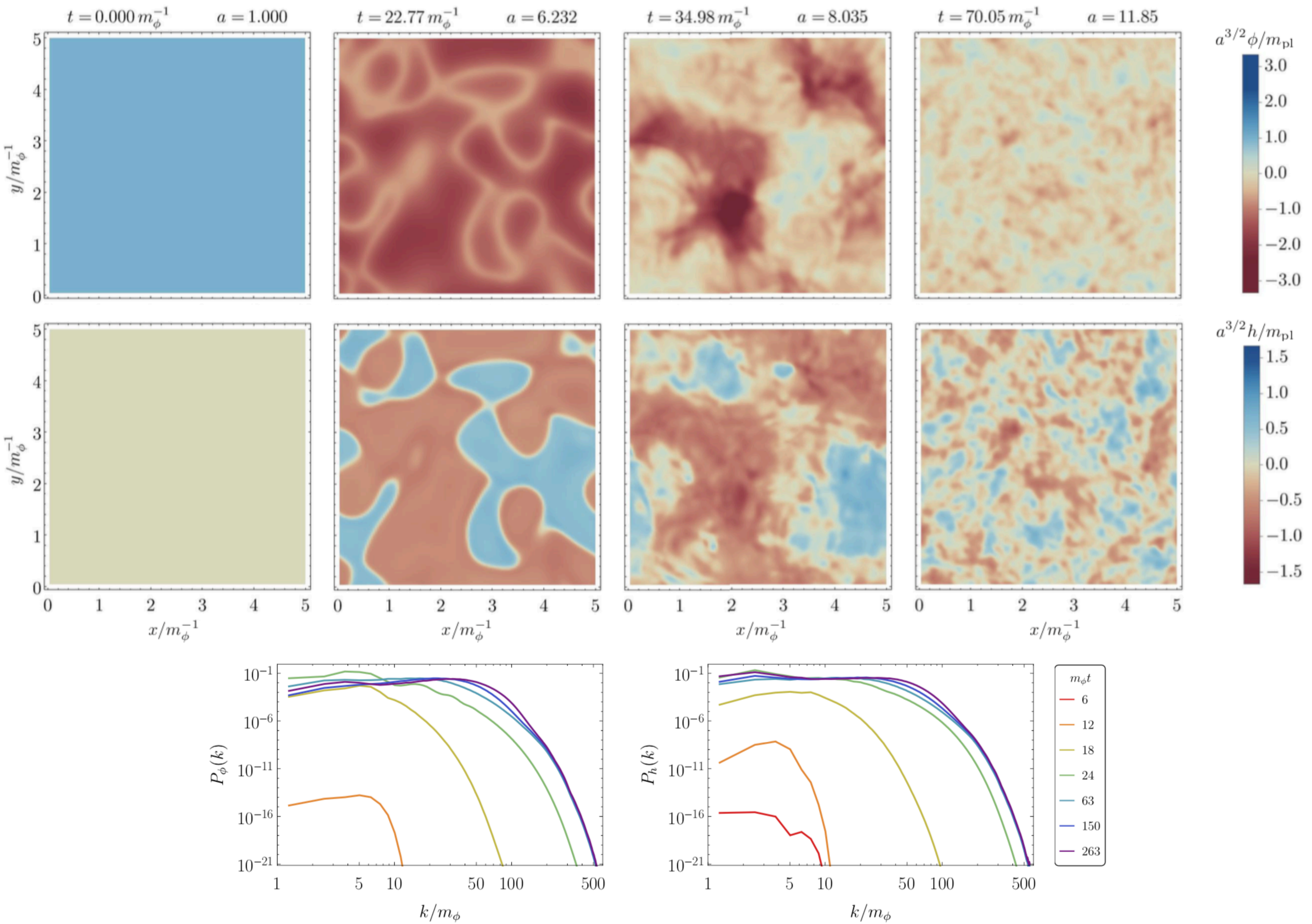


Coupled phase: neither matter domination nor radiation domination.

The modulus and the lighter field remain at comparable energy density.

$$\rho(h)/\rho(\phi) \approx 1$$

Evolution of the Fields

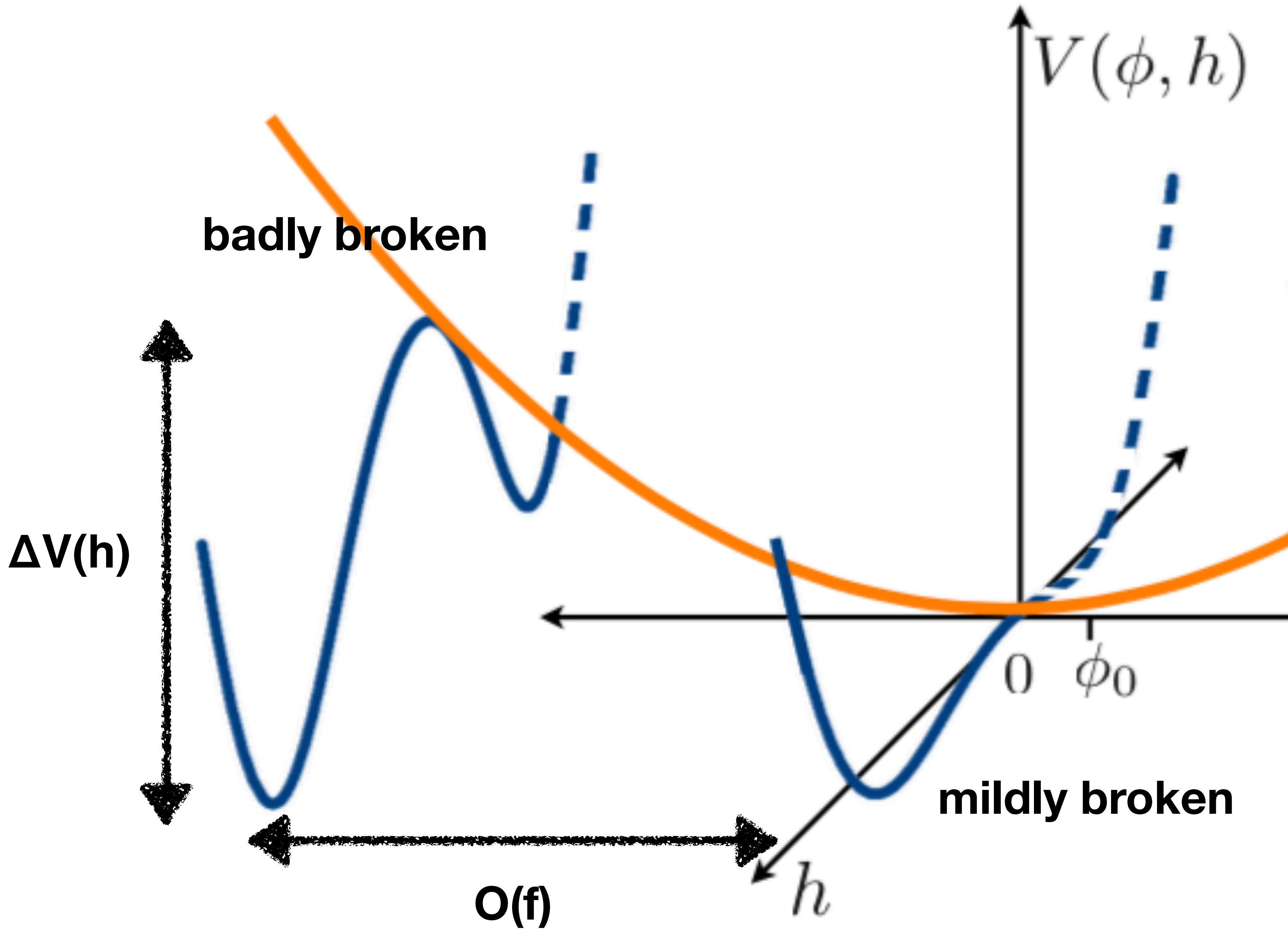


Large dynamical effect?

What the numerics tells us is that we need ΔV(h) ~ V(ϕ).

$$\Delta V(h) \sim M^4/\lambda$$
$$V(\phi) \sim m_\phi^2 f^2$$

$$\frac{M^4}{\lambda} \sim m_\phi^2 f^2$$
$$\Rightarrow b \equiv \frac{M^4}{2\lambda f^2 m_\phi^2} \sim 1$$



Summary of the numerical results

Backreaction efficiency parameter: $$b \equiv \frac{M^4}{2\lambda f^2 m_\phi^2} \leq 1$$

Tachyonic resonance efficiency parameter: $$q \equiv M^2/m_\phi^2$$

$$b \sim 1, q \gg 1 : \quad w \approx 1/3$$

$\downarrow$

Efficient conversion of modulus energy into Higgs (radiation)

Parametrics: Can We Get an Effect?

What the numerics are showing is that to get a significant period of coupled, out-of-equilibrium modulus/Higgs dynamics, we need

$$M^4 \sim \lambda m_\phi^2 f^2 \qquad\qquad \left(M^2 \frac{\phi}{f} H^\dagger H\right)$$

This could be satisfied in:

$$a) m_\phi \lesssim M \ll f \sim M_{\rm pl}, \lambda \ll 1$$
$$b) m_\phi \ll M \ll f \sim M_{\rm pl}, \lambda \sim 1$$

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For a), small quartics can arise along D-flat directions in SUSY.

If we think in the full SUSY 2HDM, the Higgs getting a large VEV can be $H_u = H_d$. This is the possibility we'll discuss in the most detail.

Gravitational Wave Production

Easther, Lim '06; Amin, Hertzberg, Kaiser, Karouby '14

Violent dynamics, like fragmenting the modulus field, produces GW background with amplitude

$$\Omega_{\rm gw}(f_0) \sim \Omega_{r0}\delta_\pi^2\beta^2,$$

***IF* the universe remains radiation dominated after GW production until the usual matter-radiation equality**

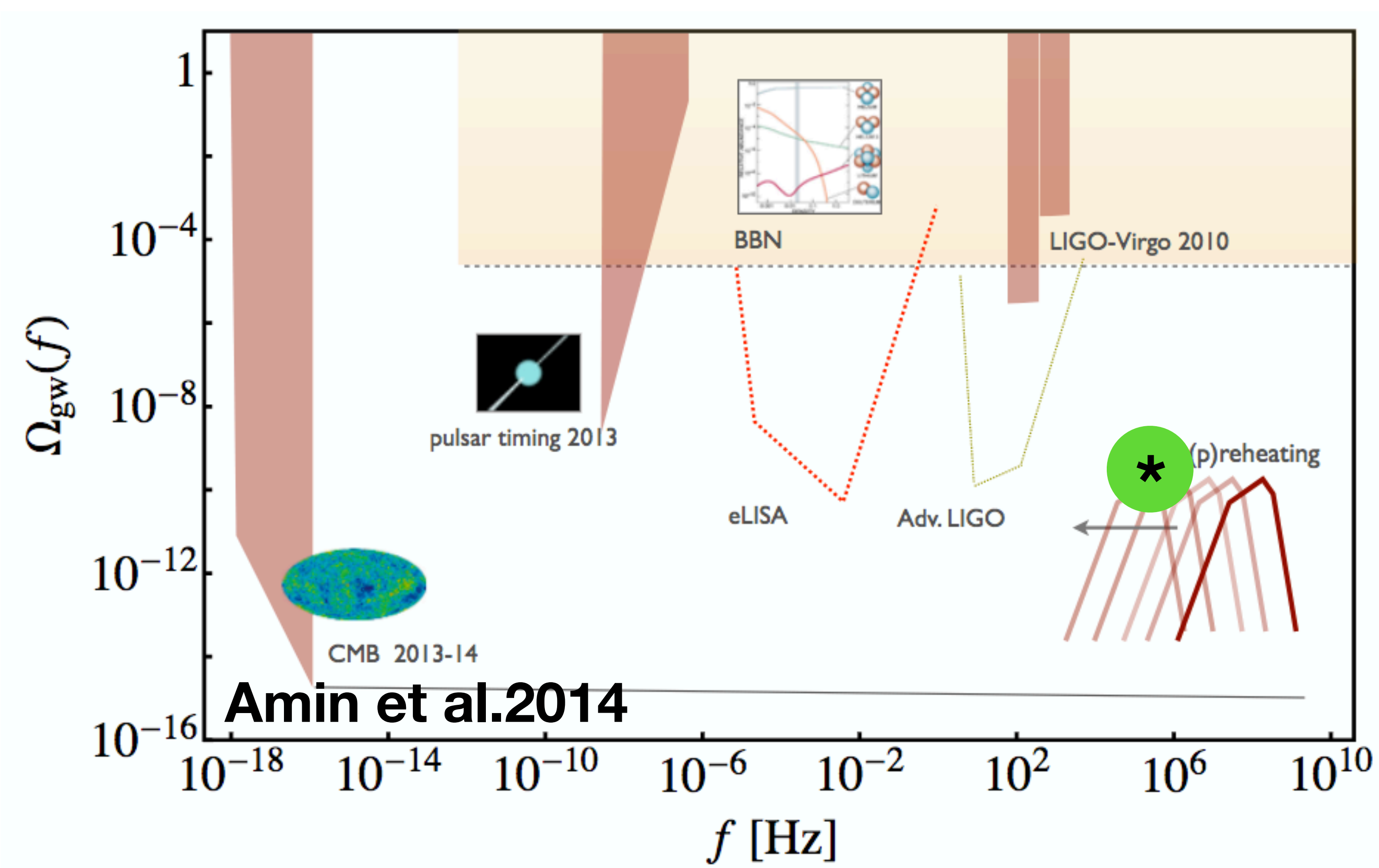
δ_π **: fraction of energy in quadrupoles ($\sim 10^{-1}$)**

β **: relation between GW peak wavelength and Hubble ($\sim 10^{-1}$)**

Gravitational Waves from Moduli

If **the out-of-equilibrium dynamics immediately converts all of the moduli to radiation, these simple estimates yield ($\beta \sim q^{-1/2}$):**

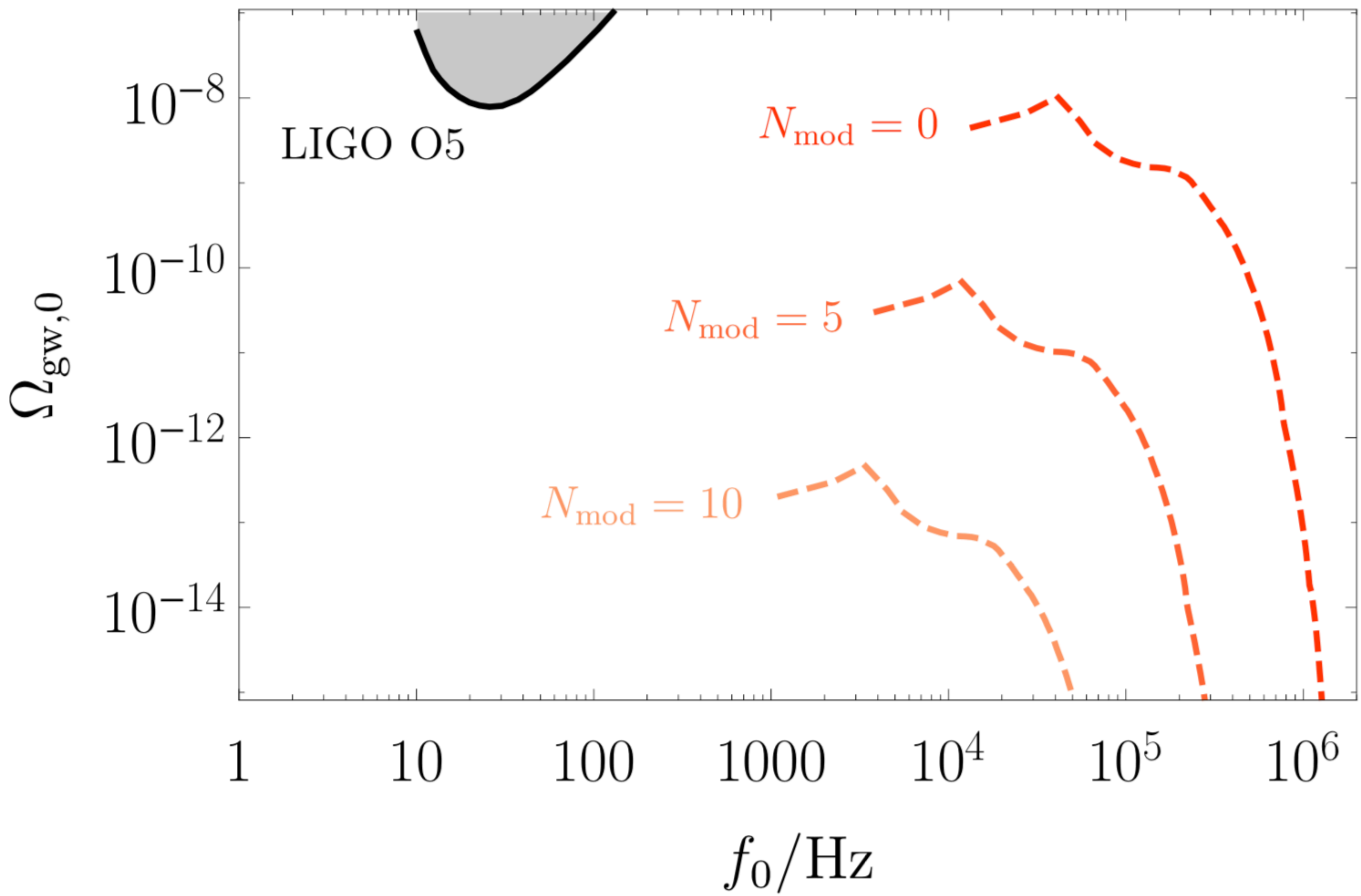
$$f_0 \sim \frac{a_{\rm osc}}{a_0}\beta^{-1}H_{\rm osc} \sim 10^5\beta^{-1}\,{\rm Hz}\left(\frac{m_\phi}{10^5\,{\rm TeV}}\right)^{1/2}$$



This frequency is above the LIGO band. Need new technologies (Akutsu et al. ’08; Goryachev, Tobar ’14; Arvanitaki and Geraci ’12).

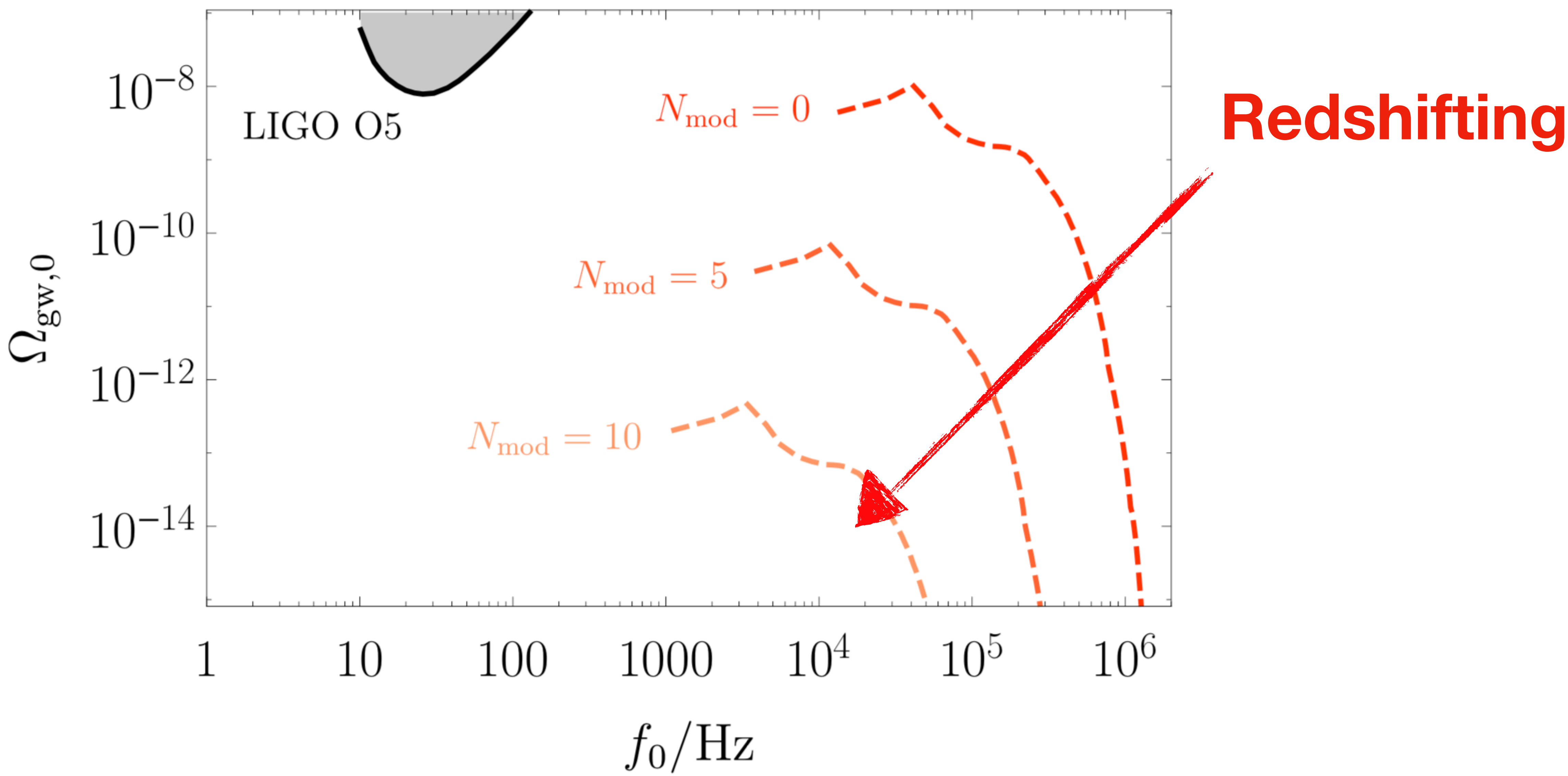
The *amplitude* isn’t terrible, and astrophysical backgrounds are low at high frequencies.

Numerical GW Spectrum



computed with HLattice (Z. Huang '11)

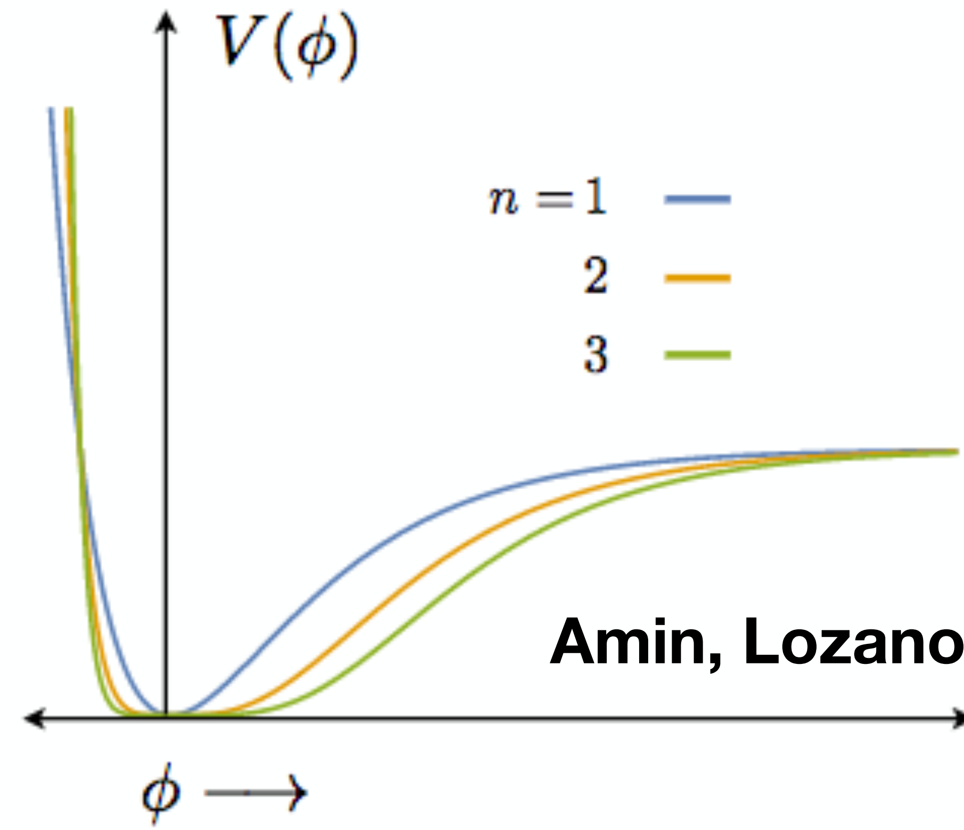
Numerical GW Spectrum



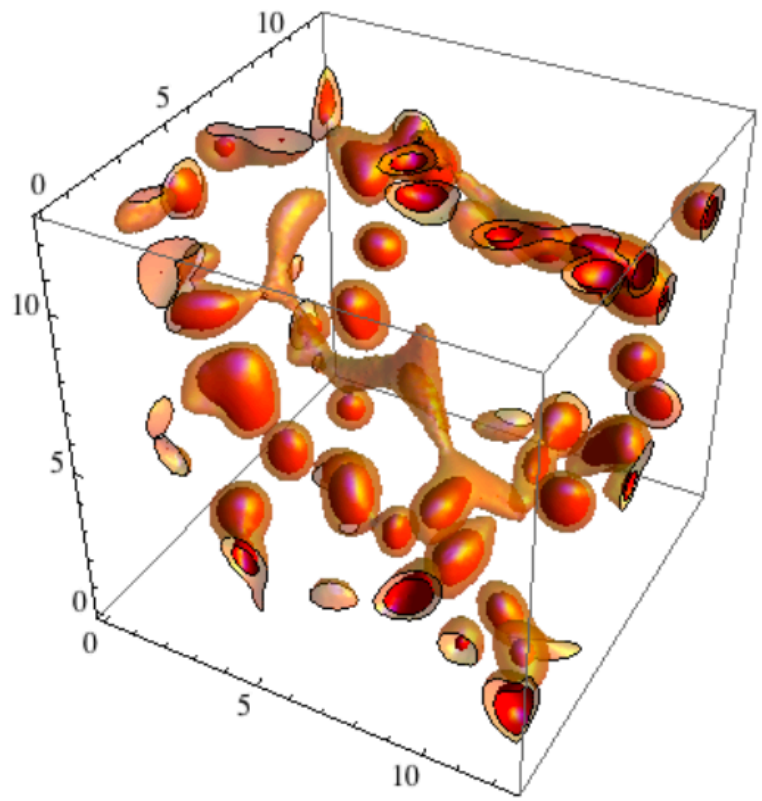
A difficulty is that we do *not* expect the moduli will instantly decay fully into radiation. From the numerics we expect an extended phase of $w \sim 0.3$, possibly reverting to standard moduli cosmology at some time.

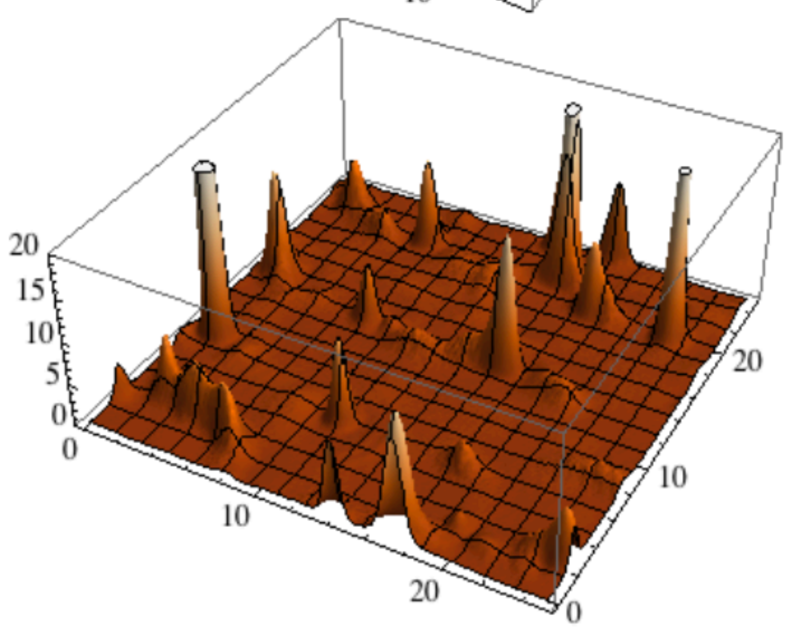
This means *more redshift*: smaller f and smaller Ω_{gw}.

One More Ingredient: Oscillons



Amin, Lozanov ’17





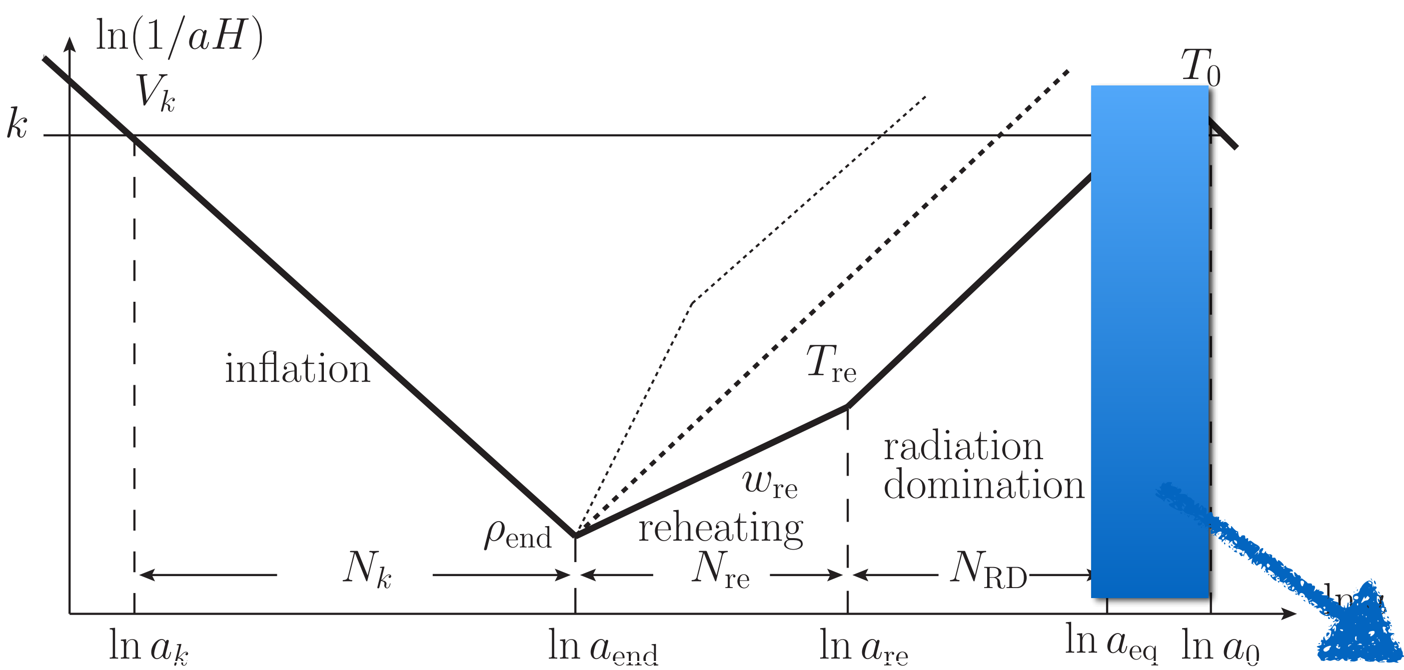
The shapes of potentials that arise for moduli can lead to formation of “oscillons”—localized lumps of oscillating field.

This could change our story in interesting ways, as the modulus doesn’t redshift inside the oscillon. More mass sign flipping and less backreaction?

No conclusions yet! Need more studies.

Amin, Easther, Finkel, Flauger, Hertzberg ’11

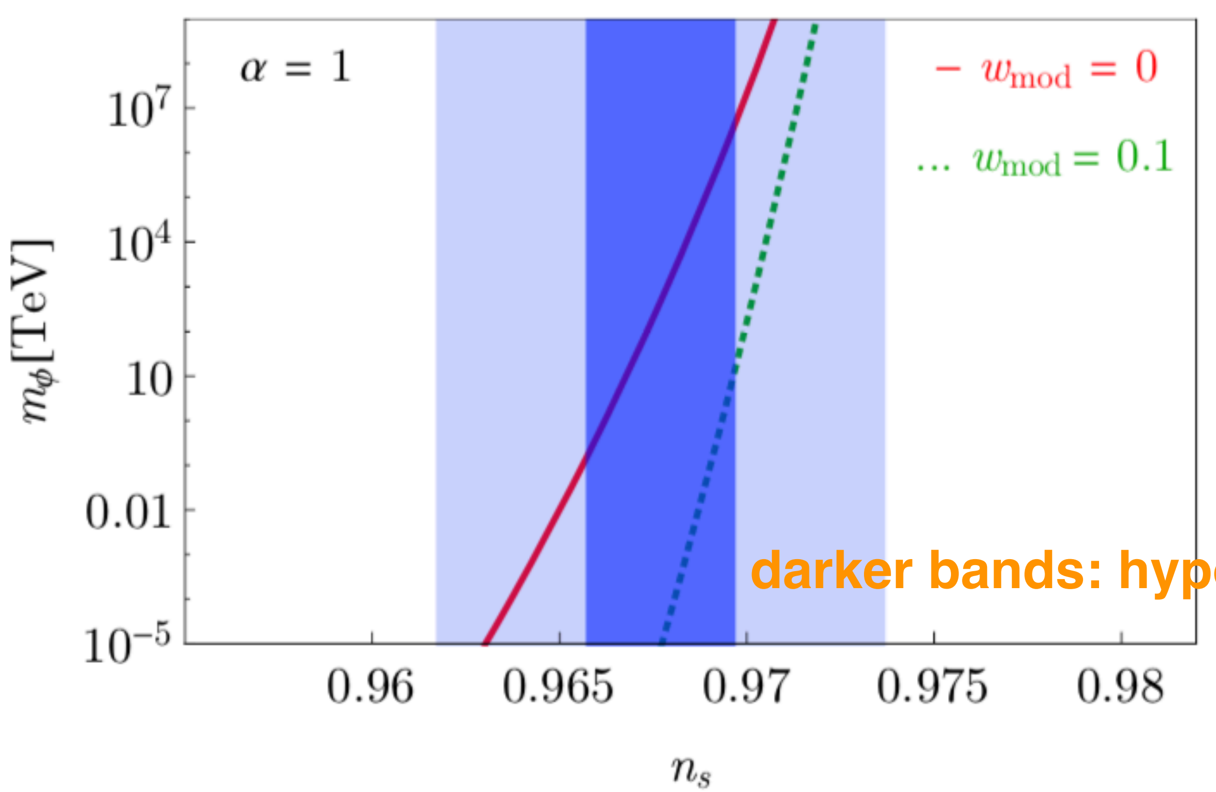
(n_s, r) and the Time Interval After Inflation

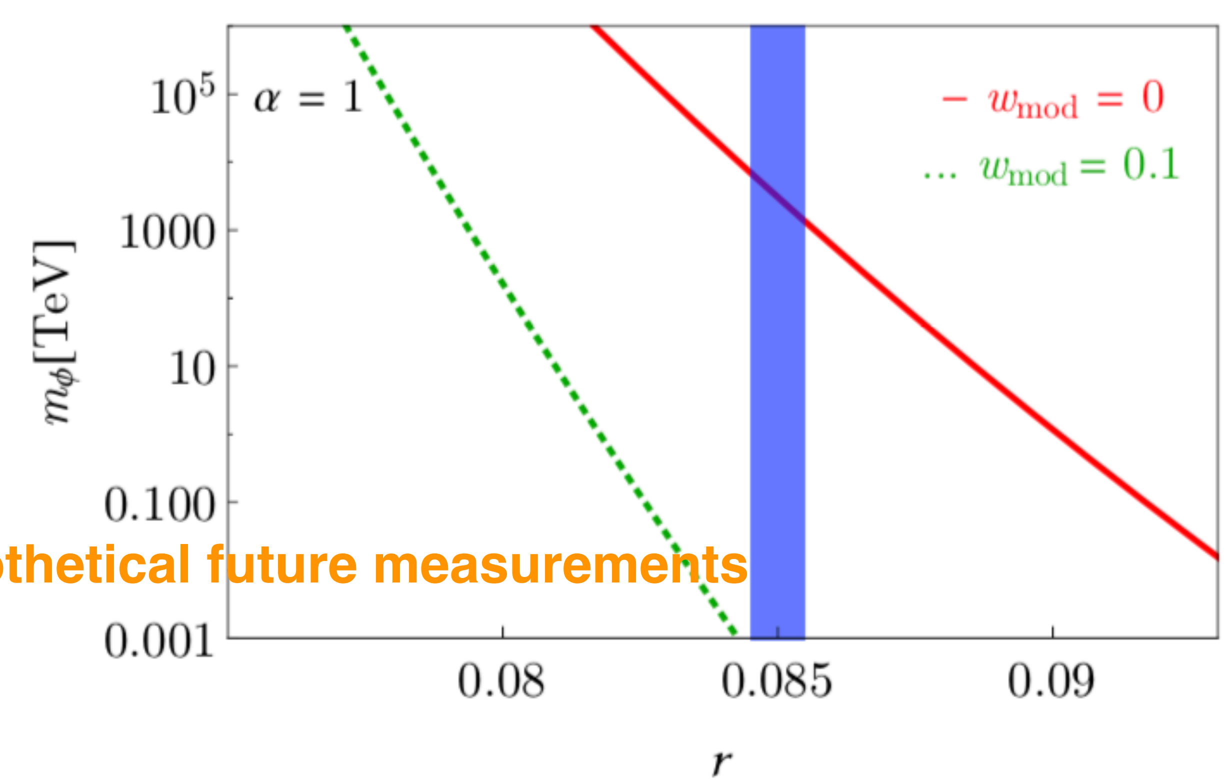


Given a cosmological history,
N_k related to the total number of e-folds between end of inflation and today; energy density during inflation related to energy density today.

Liddle, Leach '03
Easther, Galvez, Ozsoy, Watson '13

early-time matter domination





darker bands: hypothetical future measurements

For some inflation models, *disfavors* extended period of moduli domination (Dutta, Maharana)

More realistic model: SUSY

How to achieve small Higgs quartic? $m_\phi \lesssim M \ll f \sim M_{\rm pl}, \lambda \ll 1$

Reminder:

The tree-level MSSM has a Higgs quartic coupling from D-terms, completely fixed by the Higgs' electroweak representations:

$$V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (bH_u^0 H_d^0 + \text{c.c.})$$
$$+ \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2$$

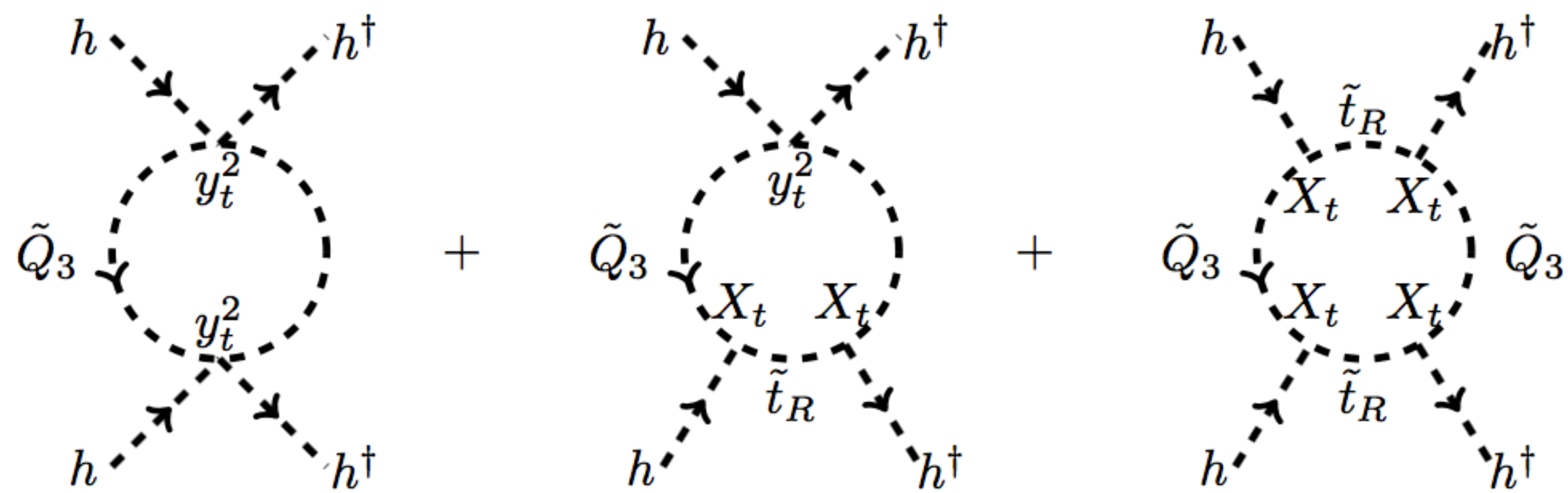
Notice the D-flat direction: $|H_u^0| = |H_d^0|$

The Higgs quartic coupling

In addition to the tree-level potential,

$$V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (bH_u^0 H_d^0 + \text{c.c.})$$
$$+ \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2$$

a SUSY-breaking contribution to the Higgs quartic comes from loops of stops:



$$V_{1-\text{loop}} \approx \frac{3y_t^4}{16\pi^2}(H_u^\dagger H_u)^2 \left[\log\frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2}\left(1 - \frac{1}{12}\frac{X_t^2}{m_{\tilde{t}}^2}\right)\right]$$

Non-vanishing along the D-flat direction. Does it stop us?

EWSB Along the Flat Direction

Suppose there is a tachyonic direction pointing along the flat direction, that is, that we have

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} |\mu|^2 + m_{H_u}^2 & -b \\ -b & |\mu|^2 + m_{H_d}^2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2 - 2b < 0$$

How large will the Higgs VEV be? At first, you would expect to be stopped by the loop-level quartic coupling:

$$V_{1-\mathrm{loop}} \approx \frac{3y_t^4}{16\pi^2} (H_u^\dagger H_u)^2 \left[\log \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2} \left(1 - \frac{1}{12} \frac{X_t^2}{m_{\tilde{t}}^2} \right) \right]$$

But importantly, the stop mass here is the geometric mean of the *physical* stop masses,

$$m_{\tilde{t}}^2 \approx m_{Q_3, \bar{u}_3}^2 + y_t^2 |H_u^0|^2$$

and as we move far out along the flat direction the stop and top become degenerate:

$$\langle H_u^0 \rangle \gg M_{\mathrm{soft}} \Rightarrow m_{\tilde{t}} \approx m_t$$

Approximate SUSY suppresses the quartic by a factor of M_{soft}^2/H^2, allowing Higgs VEVs much larger than soft masses!

Higher-Dimension Operators Lifting the Flat Direction

Flat directions should always be lifted at very large field values.

Kähler corrections are compatible with VEVs of order the cutoff:

$$\int d^4\theta \frac{X^\dagger X}{\Lambda^4} (H_u^\dagger H_u)^2 \to \frac{m_{\rm soft}^2}{\Lambda^2} (H_u^\dagger H_u)^2$$

Superpotential terms at first glance appear more dangerous.

$$\int d^2\theta \left(\mu H_u \cdot H_d + \frac{1}{M} (H_u \cdot H_d)^2 \right)$$

gives rise to quartics:

$$\frac{\mu^\dagger}{M} (H_u^\dagger H_u)(H_u \cdot H_d) + \ldots \Rightarrow \langle h \rangle \sim \sqrt{\mu M}$$

but given that some spurion forbids the mu term we expect

$$\frac{1}{M} \lesssim \frac{\mu}{\Lambda^2} \Rightarrow \langle h \rangle \sim \Lambda$$

Summary

Cosmology allows us to see the effects of fine-tuning in *field space*.

Time-dependent VEVs of moduli explore regions where the Higgs potential can be very different than in our late-time universe.

This can lead to a *coupled dynamical evolution* of the modulus and the Higgs, with exotic equation of state w near 1/3.

The modulus can fragment and produce *gravitational waves*.

However, that requires unusual parameter choices, for instance *tiny quartic couplings*.

In SUSY, such tiny quartics occur when venturing out along the *D-flat directions*! The fact that our universe is tuned might make it easy to access such regions of field space.

Thank you!