

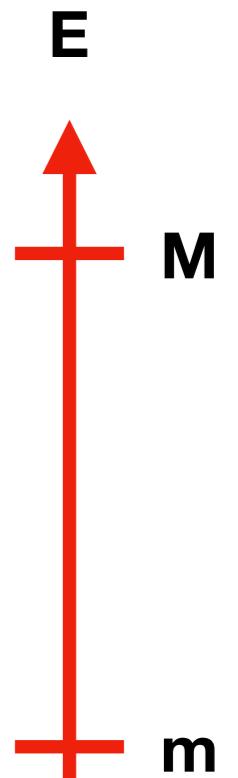
Discussion

- 1) very basic introduction to the hierarchy problem**
- 2) some ‘controversial’ statements**

**Andreas Weiler
(TUM)**

Hierarchy problem

- No hierarchy problem without hierarchy
- M (high scale) $\gg m$ (low scale)
- *Relevant operator unprotected by symmetry*
will be sensitive to high scale.



Quantum effects

$$m \bar{\psi} \psi$$

Relevant operator in SM, UV insensitive
b/c protected by (chiral) symmetry

$$m^2 \phi^* \phi \quad \sqrt{-g} \Lambda_{cc}^4$$

relevant UV sensitive operators in SM

$$\delta m_{QM} \approx \cancel{m} \frac{g^2}{16\pi^2} \log \frac{\Lambda}{m}$$

No problem!

QM effects $\sim m$ (low scale)

$$\delta m_{QM}^2 \approx \frac{g^2}{16\pi^2} \cancel{\Lambda}^2$$

Problem!

QM effects \sim Highest scale

Quantum effects 2

$$m_h^2(\mu) \phi^* \phi$$

Leading log effects in “pure SM” (no other physics above m_h):

$$\beta_{m_h^2} = \frac{dm_h^2}{d \log \bar{\mu}} = \frac{3m_h^2}{8\pi^2} \left(2\lambda + y_t^2 - \frac{3g^2}{4} - \frac{g'^2}{4} \right)$$

No hierarchy problem: corrections are $\sim m_h^2$!

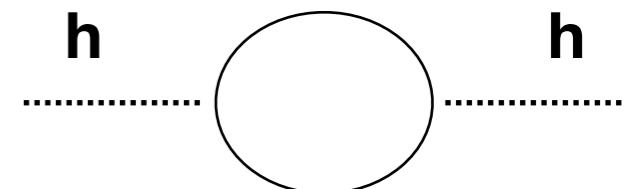
“No hierarchy problem without hierarchy”

Quantum effects 3

$$m_h^2(\mu) \phi^* \phi \quad y \phi \bar{\psi} \psi + M \bar{\psi} \psi$$

Add heavy Dirac fermion of mass $\textcolor{green}{M} \gg \textcolor{red}{m}_h$ with coupling y

$$\begin{aligned} \delta m_h^2 &= \text{Re } \Pi_{hh}|_{p^2=m_h^2} = \frac{y^2}{2(4\pi)^2} \text{Re} [\Delta_\epsilon + (m_h^2 - 4M^2)B_0(m_h; M, M) - 2A_0(M)] \\ &= \frac{y^2}{2(4\pi)^2} \left(\Delta_\epsilon + (6M^2 - m_h^2) \log \frac{m_h^2}{\bar{\mu}^2} + f(m_h, M) \right), \end{aligned}$$



$$\beta_{m_h^2} = \frac{d m_h^2(\bar{\mu})}{d \log \bar{\mu}} = \frac{y^2}{(4\pi)^2} (m_h^2 - \boxed{6M^2}) + \dots$$

Quantum effects 3

$$m_h^2(\mu) \phi^* \phi$$

$$y \phi \bar{\psi} \psi + M \bar{\psi} \psi$$

Add heavy Dirac fermion of mass $M \gg m_h$ with coupling y

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$$m_h^2(m_h) \approx m_h^2(M) - \mathcal{O}(1) M^2 \log \frac{M}{m_h}$$

Need to tune vs. to get $m_h^2(m_h) \ll M^2$

Solutions

- Protect $m_h^2 \phi^* \phi$ by some symmetry
 - Susy: chiral symmetry. Super-multiplet (**higgs,higgsino**). Exact susy guarantees equal mass of fermion and higgs boson (also at quantum level).
 - Composite Higgs: shift symmetry $\phi \rightarrow \phi + \alpha$
- Make dynamical and select $(\Lambda^2 - \varphi) \phi^* \phi$ vacuum (relaxion)

Fine-tuning “measure”

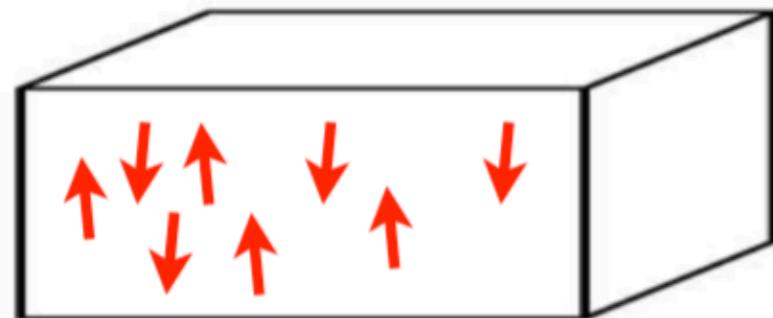
- We want to quantify the severity of this cancellation.
Factors of $O(1)$ are irrelevant. Logs are not.

$$\log \frac{\text{GUT}}{\text{TeV}} \approx 30$$

- Which measure?
- “I know it, when I see it” *
- We have experimentally performed this tuning many times...

* Supreme court Justice Potter Stewart describing his threshold test for obscenity (1964)

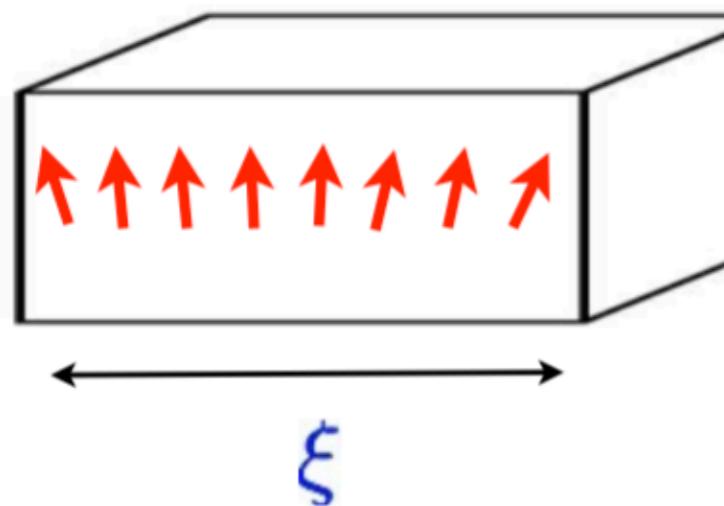
For generic T ferromagnet is not a critical point:



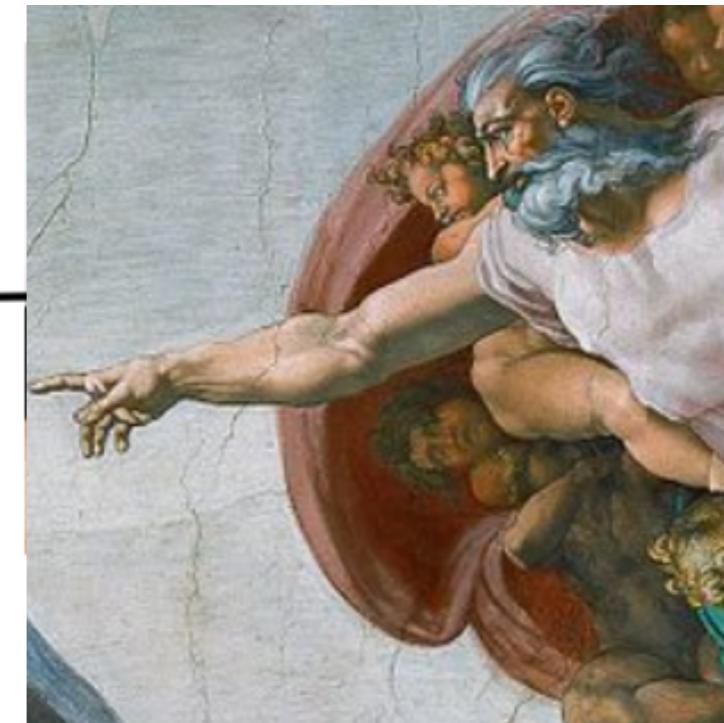
Landau,Ginzburg

$$\Lambda^{-1} \longleftrightarrow a \quad - \text{atomic spacing}$$
$$m^{-1} \longleftrightarrow \xi \quad - \text{correlation length}$$
$$\Lambda \gg m \quad \Longleftrightarrow \quad \xi \gg a \quad - \text{critical point}$$

$T \rightarrow T_c$ requires to finetune the temperature:



$$T \approx T_c$$



Correlation length \gg atomic spacing

Barbieri Giudice

For the tuning calculation, we will use the Barbieri-Giudice measure [2], reformulated in terms of $m_h^2 \approx (125 \text{ GeV})^2$ instead of m_Z^2 [5], in order to better take into account the radiative corrections to the Higgs quartic:

$$\Delta_{M^2} = \left| \frac{\partial \log m_h^2}{\partial \log M^2} \right| \quad (1.1)$$

where M^2 is a UV mass-squared parameter (e.g. μ^2 , M_3^2 , or $m_{Q_3}^2$). When multiple sources of tuning are present, we take the maximum tuning as our measure, $\Delta = \max_{\{M_i^2\}} \Delta_{M_i^2}$.

**Example from recent paper [http://arxiv.org/abs/
arXiv:1611.05873](http://arxiv.org/abs/arXiv:1611.05873)**

Example:

$$m_h^2(m_h) \approx m_h^2(M) - \frac{6y^2}{16\pi^2} M^2 \log \frac{M}{m_h}$$

$$\Delta_{BG} = \frac{\partial \log m_h^2}{\partial \log M^2} = \frac{M^2}{m^2} \frac{\partial m_h^2}{\partial M^2}$$

Here:

$$\Delta_{BG} \approx \frac{M^2}{m_h^2} \frac{6y^2}{16\pi^2} \ln \frac{M}{m_h}$$

This makes sense. Roughly: $\ln_{10} \Delta_{BG}$ corresponds to # digits which need to cancel in UV parameters vs. quantum.

Natural EWSB in Susy

Fine-tuning of (Higgs mass)²

MSSM, NMSSM, GNMSM ...

$$\frac{m_{Higgs}^2}{2} = -|\mu|^2 + \dots + \delta m_H^2$$

Higgsinos

Barbieri/Guidice,

1loop

$$\delta m_H^2|_{stop} = -\frac{3}{8\pi^2} y_t^2 \left(\underline{m_{U_3}^2 + m_{Q_3}^2} + |A_t|^2 \right) \log \left(\frac{\Lambda}{\text{TeV}} \right)$$

stops, sbottom_L

2loop

$$\delta m_H^2|_{gluino} = -\frac{2}{\pi^2} y_t^2 \left(\frac{\alpha_s}{\pi} \right) \underline{|M_3|^2} \log^2 \left(\frac{\Lambda}{\text{TeV}} \right)$$

gluino

Minimally natural susy

sparticle
masses

$$\delta m_H^2$$

<<

≈ 1000 GeV

2loop

≈ 500 GeV

1loop

≈ 250 GeV

tree

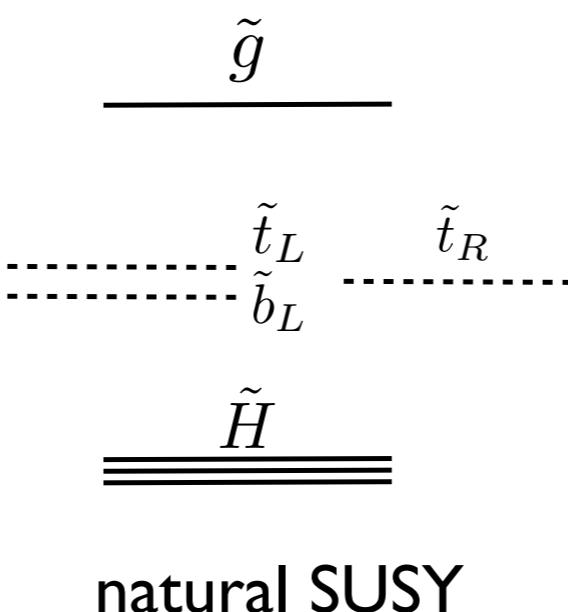
vs

Current Limits
(13 TeV, 36 fb $^{-1}$)

2020 GeV

1050 GeV

145 GeV



O(1) departure from
natural expectation!

(Non?)Alternatives

- Howie's measure. "EW fine-tuning" (no logs, cancellations?)

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2.$$

$$\Sigma_u^u(\tilde{t}_{1,2}) = \frac{3}{16\pi^2} F(m_{\tilde{t}_{1,2}}^2) \times \left[f_t^2 - g_Z^2 \mp \frac{f_t^2 A_t^2 - 8g_Z^2 (\frac{1}{4} - \frac{2}{3}x_W) \Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \right]$$

- Focus points (Hyung Do Kim will show what's wrong with them)

EW Fine-tuning

2% change in UV parameter

-> 100 % change in EWfine-tuning measure

Case A			Case B		
$m_{H_u}^2(M_{\text{GUT}})$	μ	Δ_{EW}	$m_{H_u}^2(M_{\text{GUT}})$	μ	Δ_{EW}
1.03×10^7	150	9.04	2.73×10^7	150	15.4
1.02×10^7	250	18.8	2.72×10^7	250	24.1
1.00×10^7	400	42.4	2.70×10^7	400	49.5

Table 1: An illustration of the insensitivity of the EWFT fine-tuning measure Δ_{EW} to $m_{H_u}^2(M_{\text{GUT}})$. For case **A** the NUHM2 parameters are $m_0 = 2.5$ TeV, $m_{1/2} = 400$ GeV, $\tan \beta = 10$, $m_A = 1$ TeV, while for case **B** we have, $m_0 = 4$ TeV, $m_{1/2} = 1$ TeV, $\tan \beta = 15$, $m_A = 2$ TeV. For both cases, we take $A_0 = -1.6m_0$. The numbers in the Table are in GeV units.

<https://arxiv.org/pdf/1212.2655v1.pdf> , Baer et. al

Fine-tuned fine-tuning measure?

- Hyung Do's slide

Focus point RG equation

uses the following equation

$$m^2(Q) = e^{-cy_t^2 d \log \frac{\Lambda^2}{Q^2}} m^2(\Lambda)$$

thus it is sensitive to y_t and cutoff $\frac{\Lambda}{Q}$
 GUT
 EW
 vs $\tan \beta$