

Higgs mass predictions im MSSM: impact of bottom contributions

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Supersymmetry and the Higgs sector

Features:

- Solves hierarchy problem.
- Can predict the SM-like Higgs mass.
- Gauge coupling unification at $\sim 10^{16}$ GeV.
- Possible connection to super-gravity models and string theory.
- Natural candidate for DM.

No SUSY particles have been found so far \Rightarrow SUSY particles could be heavy! ($\gtrsim 1$ TeV)

Higgs potential of the MSSM

$$V_H^{MSSM} = V_{Higgs}^{MSSM} + V_{breaking}^{MSSM},$$

$$V_{Higgs}^{MSSM} = \frac{1}{8}(g_1^2 + g_2^2)(|\mathcal{H}_1|^2 - |\mathcal{H}_2|^2)^2 + \frac{1}{2}g_2^2|\mathcal{H}_1^\dagger \mathcal{H}_2|^2 + |\mu|^2(|\mathcal{H}_1|^2 + |\mathcal{H}_2|^2),$$

$$V_{Higgs}^{breaking} = \tilde{m}_1^2|\mathcal{H}_1|^2 + \tilde{m}_2^2|\mathcal{H}_2|^2 + (m_{12}^2 \mathcal{H}_1 \cdot \mathcal{H}_2 + \text{h.c.})$$

$$\mathcal{H}_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix} \quad \mathcal{H}_2 = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \end{pmatrix}$$

$$\tan \beta \equiv t_\beta = \frac{v_2}{v_1}, \quad v = \sqrt{v_1^2 + v_2^2} = 174 \text{ GeV}$$

- Higgs sector at tree-level determined by only two variables: M_A and $\tan \beta$
- Mass of the SM-like Higgs can be predicted

Higgs potential of the MSSM

At tree level:

$$(\phi_1, \phi_2) \xrightarrow{\alpha} (h, H), \quad (\chi_1, \chi_2) \xrightarrow{\beta} (A, G^0)$$

$$m_{h,H}^2 = \frac{1}{2} \left(M_A^2 + M_Z^2 \mp \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta} \right)$$

If $M_A \gg M_Z$:

$$m_h^2 \approx M_Z^2 c_{2\beta}^2 < (125 \text{ GeV})^2 \text{ (tree level)}$$

Quantum corrections:

$$M_h^2 = m_h^2 + \Delta m_h^2, \quad M_h^2 \lesssim (135 \text{ GeV})^2$$

Uncertainties:

$$\Delta M_h^{theo} \gtrsim (1 \dots 2) \text{ GeV}$$

$$\Delta M_h^{exp} = 0.24 \text{ GeV [PDG - 2017]}$$

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$$\Delta M_h^{ILC} = 0.014 \text{ GeV **talk by Junping Tian**}$$

Radiative corrections: Feynman Diagrammatic Approach

Higgs masses at a given order = real part of poles of propagator matrix

$$\hat{\Gamma}_{hHA}(p^2) = i \begin{pmatrix} p^2 - m_h^2 + \hat{\Sigma}_{hh}(p^2) & \hat{\Sigma}_{hH}(p^2) & \hat{\Sigma}_{hA}(p^2) \\ \hat{\Sigma}_{Hh}(p^2) & p^2 - m_H^2 + \hat{\Sigma}_{HH}(p^2) & \hat{\Sigma}_{HA}(p^2) \\ \hat{\Sigma}_{Ah}(p^2) & \hat{\Sigma}_{AH}(p^2) & p^2 - m_A^2 + \hat{\Sigma}_{AA}(p^2) \end{pmatrix}$$

CP-violating case \Rightarrow 3x3 matrix

Advantages

- Precise prediction if $M_{\text{Susy}} \sim m_t$
- Includes logarithmic, non-logarithmic, suppressed terms $\mathcal{O}(\nu^2/M_{\text{Susy}}^2)$

Problems

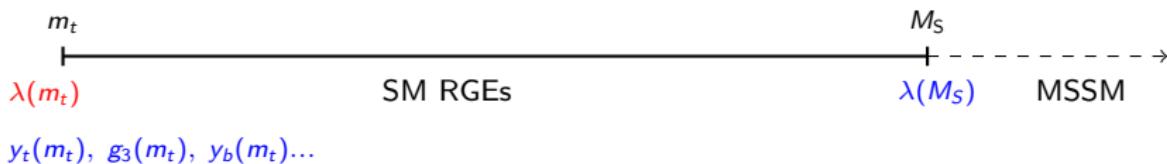
Large logarithms which need to be resummed if $M_{\text{Susy}} \gg m_t$.

For $M_A \sim M_{\text{Susy}} \gg \nu$:

$$M_h^2 = m_h^2 - \hat{\Sigma}_{hh}^{1L}(m_h^2) + \mathcal{O}(\alpha_t^2) \approx m_h^2 + \frac{3}{\pi} \alpha_t m_t^2 \log \frac{M_S^2}{m_t^2}, \quad M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$$

Radiative corrections: EFT Approach

Haber, Hempfling 1993, Carena, Quiros, Wagner 1995



- Below M_S is the SM as an effective field theory, $M_h^2 = 2\lambda(m_t)v^2$
 - Effects of SUSY particles are reflected in the boundary conditions for $\lambda(M_S)$:

$$\lambda(M_S) = \frac{1}{4}(g_1^2 + g_2^2) \cos^2 2\beta + \underbrace{\Delta_{t,b}\lambda + \Delta_{\text{heavy H}}\lambda + \Delta_{\text{EWino}}\lambda + \dots}_{\text{Threshold corrections from SUSY particles}}$$

- ↪ $M_Z \ll M_\chi = M_1 = M_2 = |\mu| \leq M_s \Rightarrow \text{Split-model}$
 $E.Bagnaschi, G.F.Giudice, P.Slavich, A.Strumia 2014; H.Bahl, W.Hollik 2016$
 - ↪ $M_A \simeq m_t \ll M_s \Rightarrow \text{THDM}$
 $Haber, Hempfling 1993; Lee, Wagner 2015$

Radiative corrections: EFT Approach

Renormalization group equation (RGE) for coupling g :

$$\frac{dg}{d \log Q^2} = \beta_g = k \beta_g^{(1)} + k^2 \beta_g^{(2)} + \dots, \quad k = \frac{1}{16\pi^2}$$

Solution:

$$g(m_t) = g(M_S) - \sum_{n=1}^{\infty} k^n \sum_{j=0}^{\infty} \beta_g^{(n,j)}(M_S) (-1)^j \frac{L^{j+1}}{(k+1)!}, \quad L = \log \frac{M_S^2}{m_t^2},$$

$$\beta_g^{(n,j)}(M_S) \sim \mathcal{O}(k^j)$$

$$\lambda(m_t) = \lambda(M_S) - k \beta_\lambda^{(1)}(M_S) L + k \beta_\lambda^{(1,1)}(M_S) \frac{L^2}{2} - k^2 \beta_\lambda^{(2)}(M_S) L + \dots$$

Radiative corrections: EFT Approach

$\beta_g^{(1)} \rightarrow (kL)^n, n = 1, 2, \dots \rightarrow$ Leading Logs (LL)

$\beta_g^{(2)} \rightarrow k(kL)^n, n = 1, 2, \dots \rightarrow$ Next – to – Leading Logs (NLL)

$\beta_g^{(3)} \rightarrow k^2(kL)^n, n = 1, 2, \dots \rightarrow$ NNLL

...

1L threshold \rightarrow NLL

2L threshold \rightarrow NNLL

...

Thresholds used in this work:

E. Bagnaschi, G. F. Giudice, P. Slavich and A. Strumia 2014

$$\Delta_t \lambda^{1L} = 6kh_t^4 s_\beta^4 \hat{X}_t^2 \left(1 - \frac{\hat{X}_t^2}{12}\right), \quad \Delta_b \lambda^{1L} = 6kh_b^4 c_\beta^4 \hat{X}_b^2 \left(1 - \frac{\hat{X}_b^2}{12}\right)$$

where $h_{t,b}$ are MSSM Yukawa couplings ($h_t = \frac{y_t}{s_\beta}$, $h_b = \frac{y_b}{c_\beta|1+\Delta_b|}$),

$X_t = A_t - \mu/t_\beta$, $X_b = A_b - \mu t_\beta$, $\hat{X}_{t,b} = X_{t,b}/M_{\text{Susy}}$.

Radiative corrections: Hybrid Approach

- EFT calculations allow to resum large logarithms
→ should be accurate for high SUSY scale M_{Susy}
- miss however terms $\sim \frac{v}{M_{\text{Susy}}}$
- diagrammatic calculation expected to be more accurate for low M_{Susy} (\lesssim few TeV)

Idea: Combine both!

$$M_h^2 = (M_h^2)^{\text{FO}} + \Delta M_h^2$$

$$\Delta M_h^2 = (\Delta M_h^2)^{\text{EFT}}(A_i^{\overline{\text{MS}}}) - (\Delta M_h^2)^{\text{EFT, non-log}}(A_i^{\text{OS}}) - (\Delta M_h^2)^{\text{FO, logs}}(A_i^{\text{OS}})$$

- Have to avoid double-counting of 1L and 2L logarithms as well as non-logarithmic terms
- EFT uses $\overline{\text{DR}}$, FO uses $\overline{\text{DR}}/\text{OS} \Rightarrow$ parameter conversion needed

Fixed order vs EFT. Example

Expansion of fixed-order result:

$$\begin{aligned} -\hat{\Sigma}_{h_0 h_0}^{1L+2L, \alpha_t, \alpha_t^2} = & + \frac{3\alpha_t m_t^2}{\pi} \log \frac{M_S^2}{m_t^2} + \frac{3\alpha_t^2 m_t^2}{16\pi^2} \log \frac{M_S^2}{m_t^2} \left(-14 + 6 \log \frac{M_S^2}{m_t^2} + 8\hat{X}_t^2 + 6\hat{X}_t^4 - \hat{X}_t^6 \right) + \\ & + \text{nonlog} + \mathcal{O}\left(\frac{m_t^2}{M_S^2}\right) \end{aligned}$$

Fixed order vs EFT. Example

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EFT result:

$$M_h^2 = m_{h,\text{tree}}^2 + \frac{3\alpha_t m_t^2}{\pi} \log \frac{M_S^2}{m_t^2} + \frac{3\alpha_t^2 m_t^2}{16\pi^2} \log \frac{M_S^2}{m_t^2} \left(-14 + 6 \log \frac{M_S^2}{m_t^2} + 6\hat{X}_t^2 + 6\hat{X}_t^4 - \hat{X}_t^6 \right) + \dots$$

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Equation on the loop-corrected mass, $M_A \gg m_Z$:

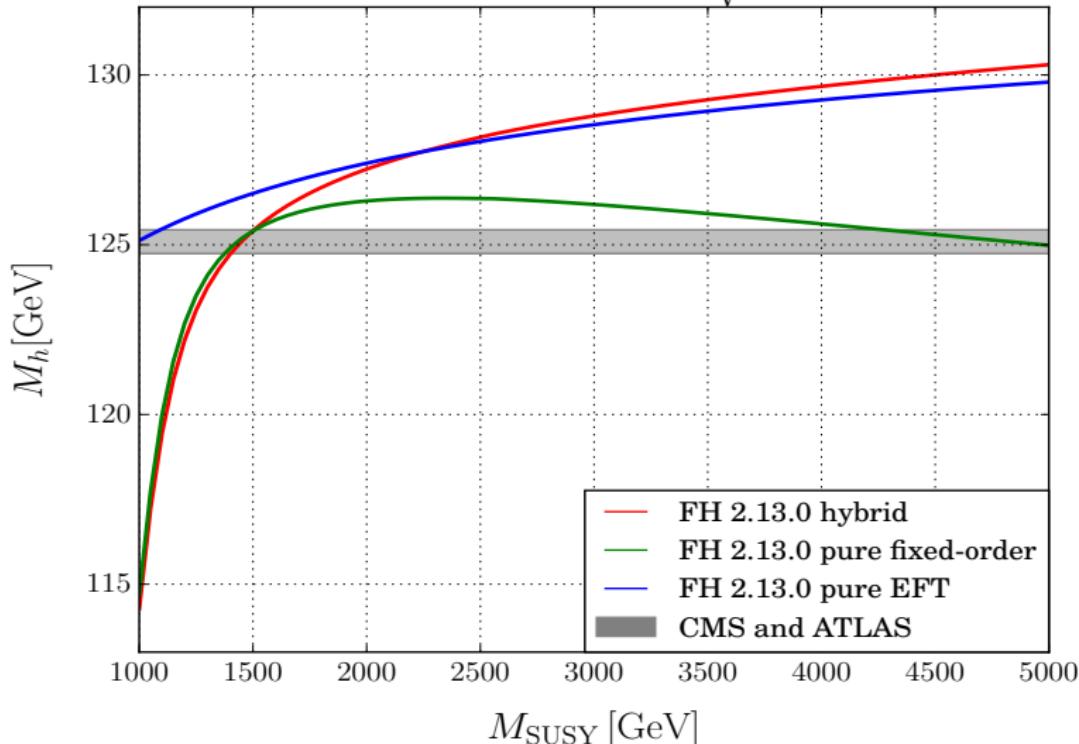
$$M_h^2 = m_{h,\text{tree}}^2 - \hat{\Sigma}_{h_0 h_0}(M_h^2)$$

Iterative Solution:

$$\begin{aligned}
 M_h^2 &= m_{h,\text{tree}}^2 - \hat{\Sigma}_{h_0 h_0}^{1L+2L}(m_{h,\text{tree}}^2) + \hat{\Sigma}_{h_0 h_0}^{1L}(m_{h,\text{tree}}^2) \hat{\Sigma}_{h_0 h_0}^{1L}(m_{h,\text{tree}}^2)' + \dots \\
 \hat{\Sigma}_{h_0 h_0}^{1L}(m_{h,\text{tree}}^2) \hat{\Sigma}_{h_0 h_0}^{1L}(m_{h,\text{tree}}^2)' &= \frac{3\alpha_t^2 m_t^2}{16\pi^2} \log \frac{M_S^2}{m_t^2} \left(-2\hat{X}_t^2 \right)
 \end{aligned}$$

Fixed order vs EFT. Example

$$M_A = M_{\text{SUSY}}/8, M_{\tilde{Q}_3} = 3M_{\text{SUSY}}, X_t^{\text{DR}} = \sqrt{6M_{\tilde{Q}_3}M_{\tilde{U}_3}}, \tan \beta = 8$$



Δ_b corrections in the MSSM.

Tree-level **MSSM = type II 2HDM**:

$$\mathcal{L}_y = -y_u^{ij} \bar{u}_R^i Q_j^T \epsilon H_u + y_d^{ij} \bar{d}_R^i Q_j^T \epsilon H_d + \text{h.c.}$$

Quantum corrections when $M_{SUSY} \gg v \Rightarrow \text{MSSM} = \text{general 2HDM}$

$$\mathcal{L}_{y,d}^{\text{eff}} = y_{d_i} \bar{d}_R^i Q_i^T \epsilon H_d - \tilde{y}_d^{ij} \bar{d}_R^i Q_j^T H_u^* + \text{h.c.}$$

Relation between physical mass and Yukawa coupling is modified:

$$y_{d_i} = \frac{m_{d_i}}{v_d} \Rightarrow y_{d_i} = \frac{m_{d_i}}{v_d(1 + \Delta_{d_i})},$$

$$\Delta_{d_i} = -\epsilon_i t_\beta = -\frac{2\alpha_s}{3\pi} M_{\tilde{g}} \mu t_\beta C_0(M_{\tilde{g}}, m_{\tilde{d}_1^i}, m_{\tilde{d}_2^i}) + \dots$$

$M_{\tilde{g}} = \mu = m_{\tilde{d}_L^i} = m_{\tilde{d}_R^i}$:

$$\Delta_{d_i} \simeq \text{sgn}(M_{\tilde{g}} \mu) \frac{\alpha_s}{3\pi} t_\beta$$

Large $\tan \beta$ & $\text{sgn}(M_{\tilde{g}} \mu) < 0$:

- ↪ Bottom Yukawa is enhanced.
- ↪ $\hat{X}_b^2 \sim t_\beta^2$, 2-loop next-to-leading logs are polynomials in \hat{X}_b .
- ↪ **Corrections from bottom-type quarks can be relevant!**

Δ_b corrections in the MSSM.

- SO(10) GUT theory requires $\tan \beta \simeq \frac{m_t}{m_b}$
L.J. Hall, R. Rattazzi and U. Sarid 1994, P. Langacker and N. Polonsky 1994, ...
- Neutral Higgs-mediated FCNC operators within the MSSM:

$$\mathcal{O}_{\Delta B=2} \sim \frac{y_b^2}{M_h^2} \left[\frac{\alpha_s \tan \beta}{3\pi} \frac{\mu}{M_{\text{Susy}}} \right]^2 \sim \frac{y_b^2}{M_h^2} \Delta_b^2$$

$$\mathcal{O}_{\Delta S=2} \sim \frac{y_b^2}{M_h^2} \left[\frac{\alpha_s \tan \beta}{3\pi} \frac{\mu}{M_{\text{Susy}}} \right]^4 \sim \frac{y_b^2}{M_h^2} \Delta_b^4$$

C.Hamzaoui, M.Pospelov, M.Toharia 1998

- $b \rightarrow s\gamma$: The next-to-leading order is $\tan \beta$ -enhanced compared to the leading order.

M.Carena, D.Garcia, U.Nierste, C.Wagner 2000

FeynHiggs code

- Code for calculation of masses, mixings, decay rates, branching ratios, electroweak precision observables and flavour observables in the MSSM.
- Written in Fortran. Can be called from Mathematica or used as a library for Fortran/C++ code. Current version 2.14.1.
- Works with real and complex parameters.
- Mixed $\overline{\text{DR}}$ /OS renormalization scheme.
- **Fixed order:** Full 1L corrections and 2L corrections $\mathcal{O}(\alpha_s \alpha_t, \alpha_s \alpha_b, (\alpha_t + \alpha_b)^2)$ in the limit ($g_1, g_2 \rightarrow 0, p^2 \rightarrow 0$)
- **EFT:** Hybrid approach with full LL, NLL and $\mathcal{O}(\alpha_s \alpha_t)$ NNLL resummation (assuming $M_A = M_{\text{Susy}}$). Independent intermediate electroweakino and gluino thresholds are allowed.

Parameter conversion.

Fixed order

- $\hat{X}_{t,b}^{\text{OS}}$ is used in 1,2-loop self-energies
- \hat{y}_b is used in 1,2-loop self-energies

A.Dedes, G.Degrassi, P.Slavich 2003

$$\hat{y}_b = \frac{y_b^{\text{RGE}}}{|1 + \Delta_b|} (1 + \delta_b) + \delta y_b, \quad \delta y_b = y_b^{\overline{\text{DR}}} - y_b^{\text{OS}}$$

$\delta_b, \delta y_b$ involve large logs!

EFT

- $\hat{X}_{t,b}^{\overline{\text{DR}}}$ is used in 1-loop thresholds
- $y_b^{\overline{\text{MS}}} = \frac{\sqrt{2m_b}}{\bar{v}}$ is used in the numerical solution of the RGEs.

Parameter conversion.



$$\hat{X}_t^{\overline{\text{DR}}} = \hat{X}_t^{\text{OS}} \left[1 + \left(\frac{\alpha_s}{\pi} + \frac{3}{16\pi} \left(\alpha_b \left(1 + \hat{X}_b^2 \right) - \alpha_t \left(1 - \hat{X}_t^2 \right) \right) \right) \log \frac{M_S^2}{m_t^2} \right]$$

$$\hat{X}_b^{\overline{\text{DR}}} = \hat{X}_b^{\text{OS}} \left[1 + \left(\frac{\alpha_s}{\pi} + \frac{3}{16\pi} \left(\alpha_t \left(1 + \hat{X}_t^2 \right) - \alpha_b \left(1 - \hat{X}_b^2 \right) \right) \right) \log \frac{M_S^2}{m_t^2} \right]$$

- $\hat{y}_b = \frac{y_b^{\text{RGE}}}{|1+\Delta_b|} + \delta y_b$ is used in the fixed-order result.

Parameter conversion $\hat{y}_b \rightarrow y_b^{\text{RGE}}$ (see backup):

$$\frac{y_b^{\text{RGE}}}{|1 + \Delta_b|} = \hat{y}_b (1 + \delta_1 k + \delta_2 k L)$$

Subtraction terms

$$\lambda^{2L,LL}(m_t) = \left(9(\hat{y}_b^2 - y_t^2)^2(\hat{y}_b^2 + y_t^2) - 48g_3^2(\hat{y}_b^4 + y_t^4) \right) (kL)^2 \\ = (-48g_3^2\hat{y}_b^4 + 9\hat{y}_b^6 - 9\hat{y}_b^4y_t^2 - 48g_3^2y_t^4 - 9\hat{y}_b^2y_t^4 + 9y_t^6)(kL)^2$$

$$\lambda^{2L,NLL}(m_t) = 6y_t^4 \left[g_3^2 \left(8\hat{X}_t^2 - \frac{4}{3}\hat{X}_t^4 \right) + \left(\frac{3\hat{X}_t^2}{2} - \frac{\hat{X}_t^4}{4} \right) \left(y_b^2 (1 + \hat{X}_b^2) - y_t^2 (1 - \hat{X}_t^2) \right) + \left(3y_t^2 + 3\hat{y}_b^2 - 16g_3^2 \right) \left(\hat{X}_t^2 - \frac{1}{12}\hat{X}_t^4 \right) - \left(5y_t^2 - y_b^2 - \frac{16}{3}g_3^2 \right) \right] k^2 L + \\ + 6\hat{y}_b^4 \left[g_3^2 \left(8\hat{X}_b^2 - \frac{4}{3}\hat{X}_b^4 \right) + \left(\frac{3\hat{X}_b^2}{2} - \frac{\hat{X}_b^4}{4} \right) \left(y_t^2 (1 + \hat{X}_t^2) - \hat{y}_b^2 (1 - \hat{X}_b^2) \right) + \left(3y_t^2 + 3\hat{y}_b^2 - 16g_3^2 \right) \left(\hat{X}_b^2 - \frac{1}{12}\hat{X}_b^4 \right) - \left(3\hat{y}_b^2 - y_t^2 - \frac{16}{3}g_3^2 \right) \right] k^2 L \\ \delta\lambda = 24\hat{y}_b^4 k^2 L (\delta_1 + \delta_2 L) + 24\hat{y}_b^4 k^2 (\delta_1 + \delta_2 L) \hat{X}_b^2 \left(1 - \frac{\hat{X}_b^2}{12} \right)$$

$$\hat{X}_t \equiv \frac{X_t^{\text{OS}}}{M_S}, \quad \hat{X}_b \equiv \frac{X_b^{\text{OS}}}{M_S}, \quad k = \frac{1}{16\pi^2}, \quad L \equiv \log \frac{M_S^2}{m_t^2}$$



Resummation procedure in FeynHiggs.

```
loglevel = 1
```

- Single scale scenario: $M_A = M_{1,2,3} = |\mu| = M_{\tilde{Q}_3} = M_{\tilde{U}_3} = M_{\tilde{D}_3} = M_{\text{Susy}}$
- $\mathcal{O}(\alpha_s, \alpha_t)$ LL and NLL resummation
- Input values @ m_t^{pole} : $y_b^{\text{RGE}} = \frac{\sqrt{2m_b}}{v}$, y_t^{RGE} , g_3^{RGE} .
- Analytical solution of 1L RGEs: $g_3(M_{\text{Susy}})$, $y_t(M_{\text{Susy}})$, $y_b(M_{\text{Susy}})$.
- $\lambda(M_{\text{Susy}}) = \Delta_t \lambda^{1L} + \Delta_b \lambda^{1L}$, 2L numerical running $M_{\text{Susy}} \rightarrow m_t$.
- $\{y_t(m_t), y_b(m_t), g_3(m_t)\} \rightarrow \{y_t^{\text{RGE}}(m_t), y_b^{\text{RGE}}(m_t), g_3^{\text{RGE}}(m_t)\}$,
2L numerical running $m_t \rightarrow M_{\text{Susy}}$.
- $\lambda(M_{\text{Susy}}) = \Delta_t \lambda^{1L} + \Delta_b \lambda^{1L}$, 2L numerical running $M_{\text{Susy}} \rightarrow m_t$.

Boundary conditions both @ m_t and @ M_{Susy} are satisfied!

Numerical results.

Single scale scenario:

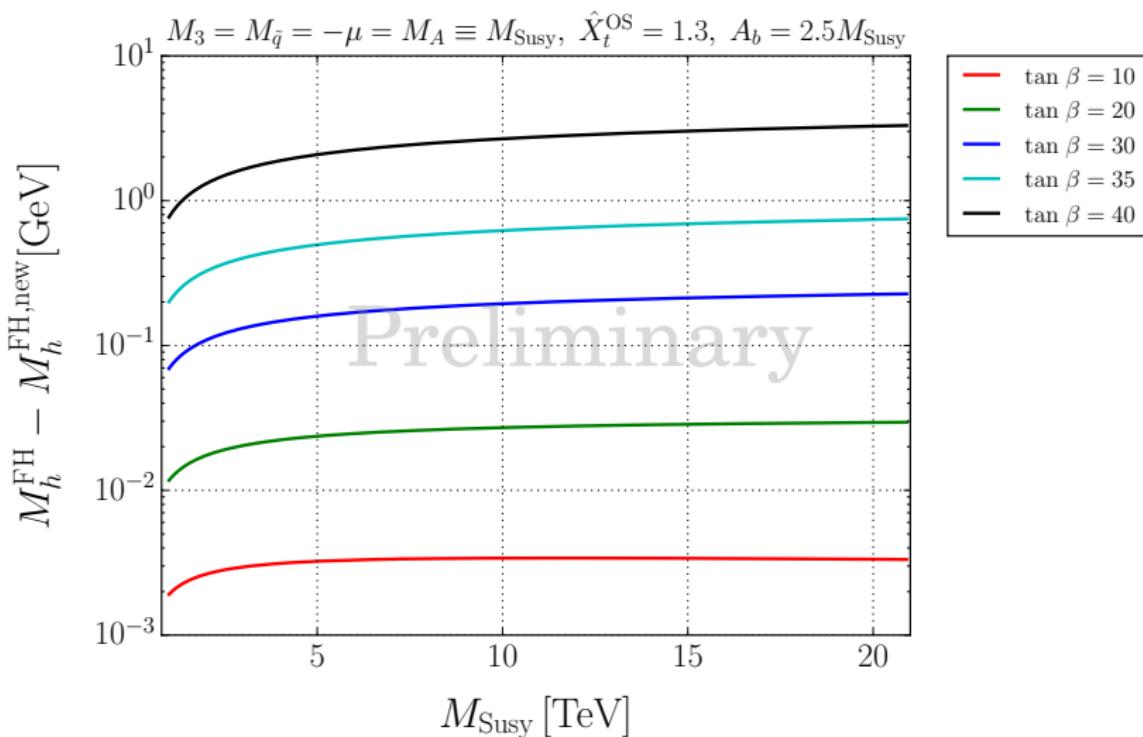
$$M_A = M_{1,2,3} = |\mu| = M_{\text{Susy}}$$

$$A_b = 2.5 \ M_{\text{Susy}}$$

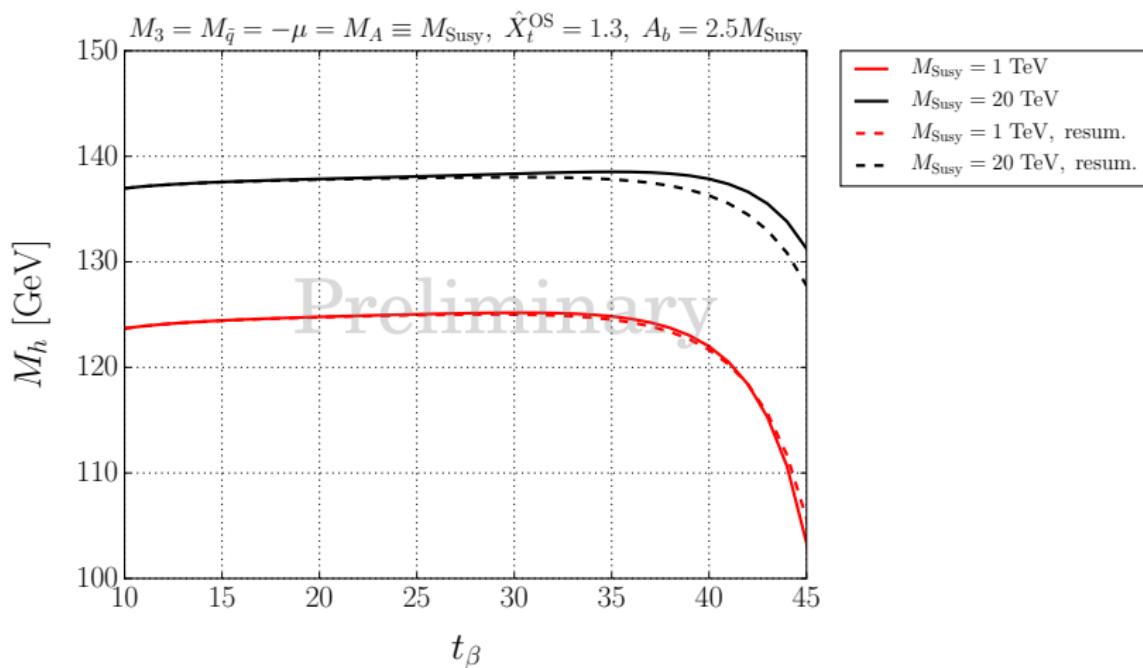
$$M_{\tilde{Q}_3} = M_{\tilde{U}_3} = M_{\tilde{D}_3} = \dots = M_{\text{Susy}}$$

$$m_t^{\text{pole}} = 173.5 \text{ GeV [PDG - 2017]}$$

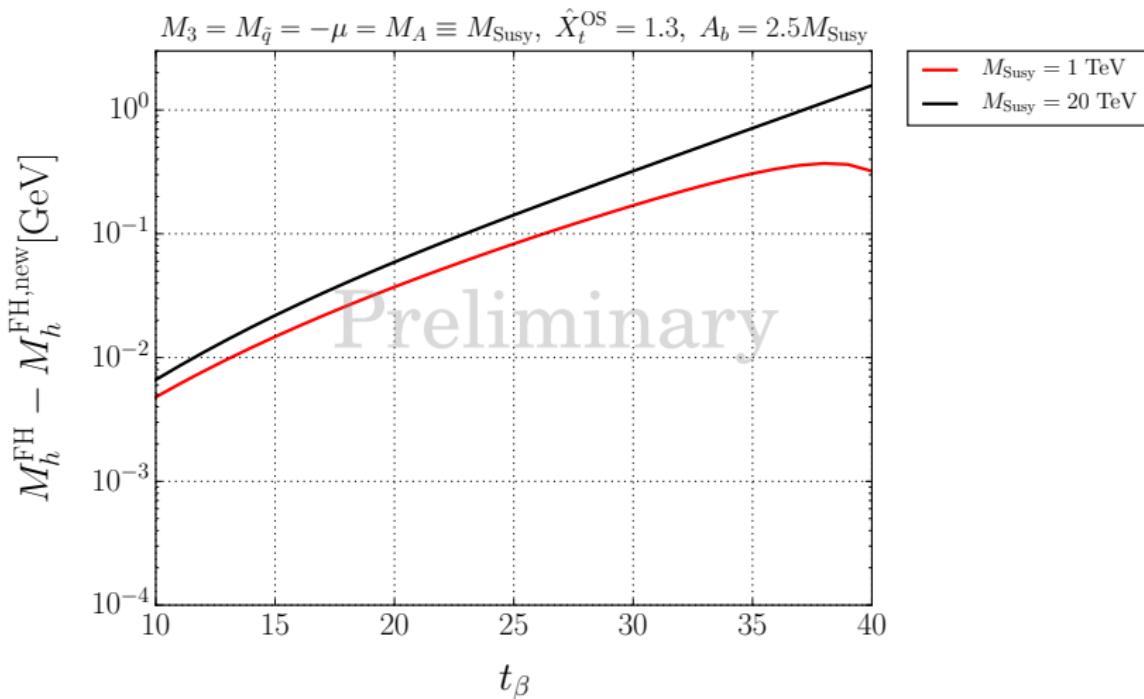
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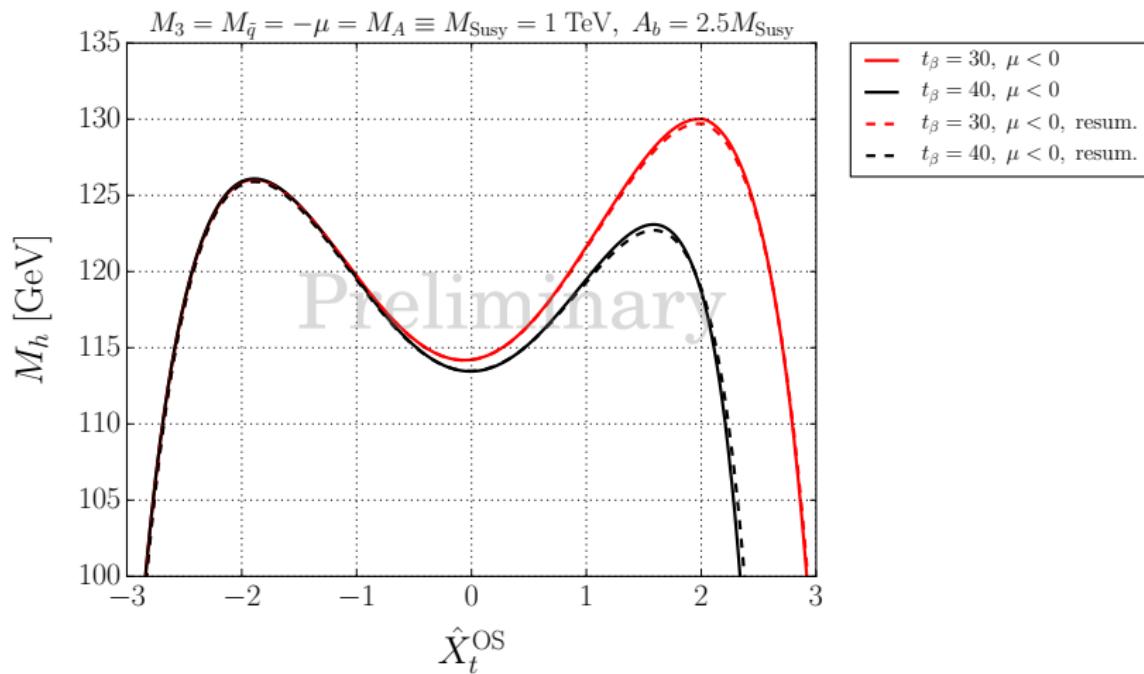
Numerical results.



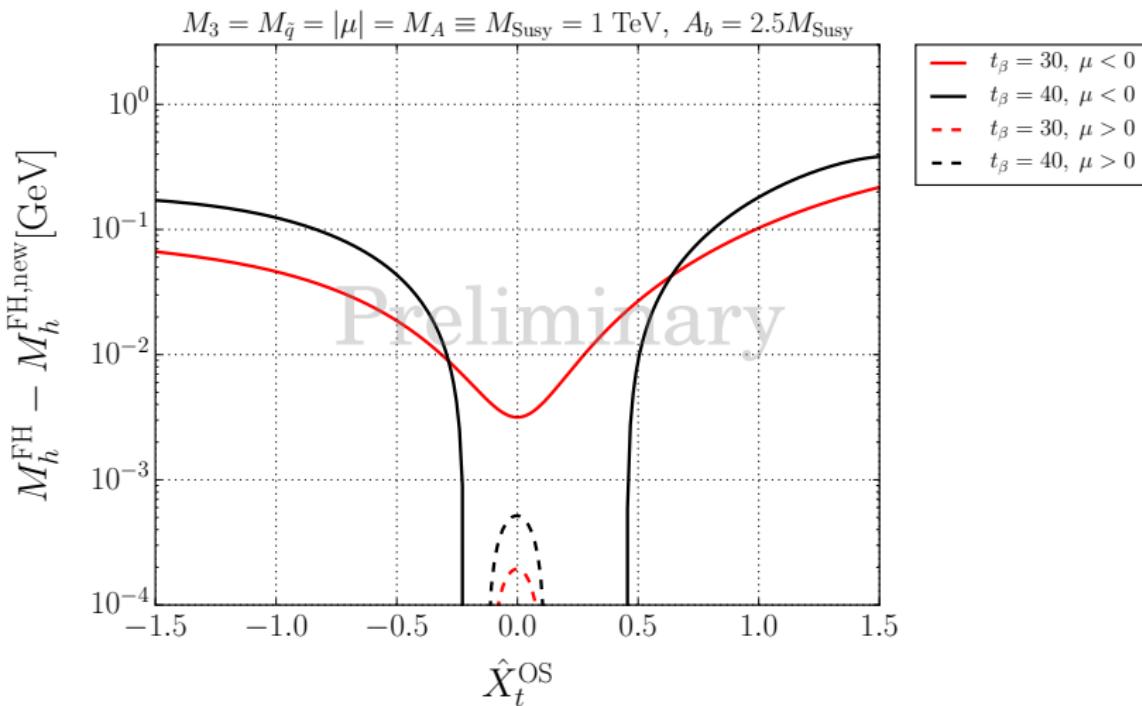
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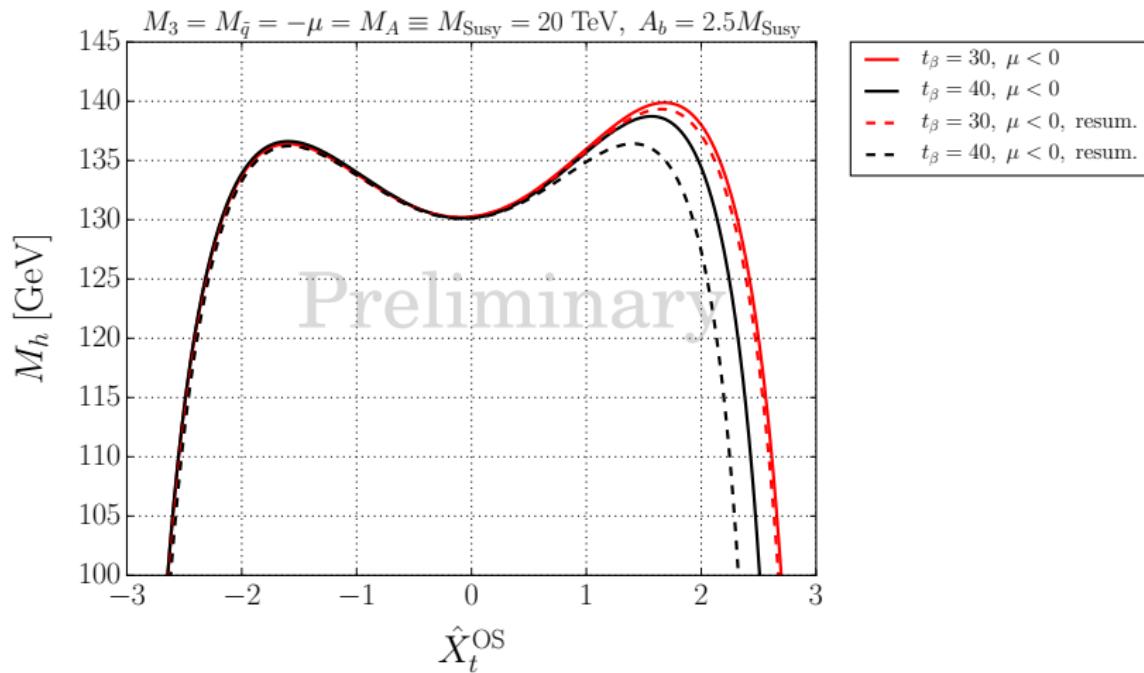
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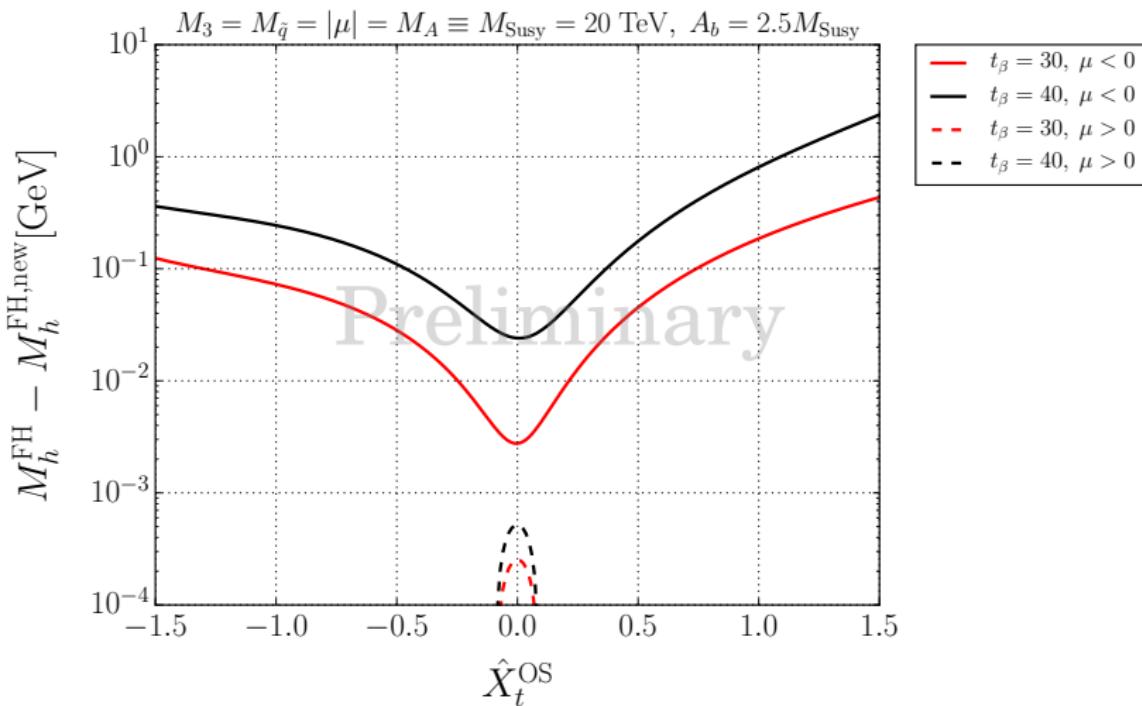
Numerical results.



Numerical results.



Numerical results.



Summary and Outlook

What is done so far?

- LL,NLL resummation of $\mathcal{O}(\alpha_b)$, $\mathcal{O}(\alpha_b\alpha_s, \alpha_b\alpha_t)$ contributions is implemented into FeynHiggs code.
- Preliminary results seem to indicate that effect can be fairly large $\sim \mathcal{O}(1 \text{ GeV})$ in the large $\tan \beta$ regime

What comes next?

- ↪ Implementation of resummation of $\sim m_b^2 \alpha_{EW} \log \frac{M_S^2}{m_t^2}$.
- ↪ Implementation of τ -lepton contributions in the hybrid approach.
- ↪ Implementation of NNLL resummation of $\mathcal{O}(\alpha_b)$, $\mathcal{O}(\alpha_b\alpha_s, \alpha_b\alpha_t)$ contributions.

Backup slides

Δ_b corrections in the MSSM. t_β resummation.

Bottom mass counter-term in the OS scheme:

L.Hofer, U.Nierste, D.Scherer 2009.

$$\delta m_b = \frac{m_b}{2} [\Sigma_b^{LL}(m_b^2) + \Sigma_b^{RR}(m_b^2)] + \Sigma_b^{RL}(m_b^2),$$

$$\Sigma_b(p) = \not{p} [\Sigma_b^{LL}(p^2)P_L + \Sigma_b^{RR}(p^2)P_R] + \Sigma_b^{RL}(p^2)P_L + \Sigma_b^{LR}(p^2)P_R$$

When $t_\beta \rightarrow \infty$: $\delta m_b = \Sigma_b^{RL}(m_b^2) \simeq \Sigma_b^{RL}(0) := m_b \Delta_b = m_b \epsilon_i t_\beta \Rightarrow \delta y_b = y_b \epsilon_i t_\beta$.

- ① δy_b is UV finite when $t_\beta \rightarrow \infty$
- ② There are no t_β -enhanced contributions to δy_b from genuine multiloop diagrams *M.Carena, D.Garcia, U.Nierste, C.Wagner 2000.*

$$\delta y_b^{(1)} = y_b \epsilon_i t_\beta, \quad \delta y_b^{(2)} = \delta y_b^{(1)} \epsilon_i t_\beta, \dots, \delta y_b^{(n+1)} = \delta y_b^{(n)} \epsilon_i t_\beta,$$

$$y_b^{(0)} = y_b + \delta y_b^{(1)} + \delta y_b^{(2)} + \dots = y_b (1 + \epsilon_i t_\beta + (\epsilon_i t_\beta)^2 + \dots) = \frac{y_b}{1 - \epsilon_i t_\beta}$$

2L RGEs in the gaugeless limit

D.Buttazzo, G.Degrassi, P.P.Giardino, G.F.Giudice, F.Sala, A.Salvio, A.Strumia 2013

$$\begin{aligned}\frac{dg_3}{d \log Q^2} &= \frac{1}{2} g_3^3 k \left[-7 + k \left(-26g_3^2 - 2y_t^2 - 2y_b^2 \right) \right] \\ \frac{dy_t}{d \log Q^2} &= \frac{1}{2} y_t k \left[\frac{9}{2} y_t^2 + \frac{3}{2} y_b^2 - 8g_3^2 + k \left(6y_t^2 \left(6g_3^2 - 2y_t^2 - \frac{11}{24} y_b^2 - \lambda \right) + \right. \right. \\ &\quad \left. \left. + y_b^2 \left(-\frac{1}{4} y_b^2 + 4g_3^2 \right) + \frac{3}{2} \lambda^2 - 108g_3^4 \right) \right] \\ \frac{dy_b}{d \log Q^2} &= \frac{1}{2} y_b k \left[\frac{9}{2} y_b^2 + \frac{3}{2} y_t^2 - 8g_3^2 + k \left(6y_b^2 \left(6g_3^2 - 2y_b^2 - \frac{11}{24} y_t^2 - \lambda \right) + \right. \right. \\ &\quad \left. \left. + y_t^2 \left(-\frac{1}{4} y_t^2 + 4g_3^2 \right) + \frac{3}{2} \lambda^2 - 108g_3^4 \right) \right] \\ \frac{d\lambda}{d \log Q^2} &= k \left[6 \left(\lambda^2 + \lambda y_t^2 + \lambda y_b^2 - y_t^4 - y_b^4 \right) + \frac{k}{2} \left(y_t^2 (60y_t^4 - 3y_t^2 \lambda + 80g_3^2 \lambda \right. \right. \\ &\quad \left. \left. - 64g_3^2 y_t^2 - 72\lambda^2 - 42y_b^2 \lambda - 12y_t^2 y_b^2) + y_b^2 (-72\lambda^2 - 3y_b^2 \lambda + 80g_3^2 \lambda + 60y_b^4 - \right. \right. \\ &\quad \left. \left. - 12y_b^2 y_t^2 - 64y_b^2 g_3^2) - 78\lambda^3 \right) \right]\end{aligned}$$

Analytical solutions to 1L RGEs

$\sim y_b^2$ terms are neglected!

$$\frac{g_3^2(M_{\text{Susy}})}{g_3^2(m_t)} = \frac{1}{1 + 7k \log \frac{M_{\text{Susy}}^2}{m_t^2}}$$

$$\frac{y_t^2(M_{\text{Susy}})}{y_t^2(m_t)} = \frac{2g_3^2(m_t)}{(2g_3^2(m_t) - 9y_t^2(m_t)) \left(1 + 7k \log \frac{M_{\text{Susy}}^2}{m_t^2}\right)^{\frac{8}{7}} + 9y_t^2(m_t) \left(1 + 7k \log \frac{M_{\text{Susy}}^2}{m_t^2}\right)}$$

$$\frac{y_b^2(M_{\text{Susy}})}{y_b^2(m_t)} = \frac{\left(1 + 7k \log \frac{M_{\text{Susy}}^2}{m_t^2}\right)^{-\frac{8}{7}}}{\left(1 - \frac{9y_t^2(m_t)}{2g_3^2(m_t)} \left(1 - \left(1 + 7k \log \frac{M_{\text{Susy}}^2}{m_t^2}\right)^{-\frac{1}{7}}\right)\right)^{\frac{1}{3}}}$$

$\nu_{\text{OS}} \rightarrow \nu_{\overline{\text{MS}}}$ conversion

D.Buttazzo, G.Degrassi, P.P.Giardino, G.F.Giudice, F.Sala, A.Salvio, A.Strumia 2013

$$\begin{aligned} v_{\overline{\text{MS}}}^2 &= v_{\text{OS}}^2 + \frac{3}{(4\pi)^2} \left[m_t^2 - 2A_0(m_t^2) - \frac{1}{6} (2m_w^2 + m_z^2 + M_h^2) + \right. \\ &+ \frac{m_w^2}{M_h^2 - m_w^2} A_0(M_h^2) + \left(\frac{4}{3} - \frac{c_w^2}{s_w^2} \right) A_0(m_z^2) + \\ &\left. + \left(\frac{11}{3} + \frac{c_w^2}{s_w^2} - \frac{M_h^2}{M_h^2 - m_w^2} A_0(m_w^2) \right) \right] \end{aligned}$$

Two-loop \hat{y}_b at large M_{Susy}

$$\begin{aligned}\delta_b &= \frac{4}{3} g_s^2 k \left(\frac{A_b}{M_{\text{Susy}}} - \log \frac{M_{\text{Susy}}^2}{m_t^2} \right) + \frac{y_t^2 k}{2} \left(\frac{7}{4} - \frac{5}{2} \log \frac{M_{\text{Susy}}^2}{m_t^2} \right) + \\ &+ \frac{y_b^2 k}{2 c_\beta^2} \left(\frac{3}{4} - 3 \log \frac{M_{\text{Susy}}^2}{m_t^2} \right) \\ \frac{\delta v}{v} &= -\frac{k}{4} \left[y_t^2 \left(3 + \left(\frac{A_t}{M_{\text{Susy}}} \right)^2 \right) + \frac{y_b^2}{c_\beta^2} \right]\end{aligned}$$

Two-loop \hat{y}_b at large M_{Susy}

$$\begin{aligned}
 \delta y_b = & \frac{8}{3} g_s^2 k \log \frac{M_{\text{Susy}}^2}{m_t^2} + y_t^2 k \left[\frac{5}{4} + \frac{4\pi}{3\sqrt{3}} - \left(\frac{1}{2} - \frac{3}{4} \frac{A_t^2}{M_{\text{Susy}}^2} \right) \log \frac{M_{\text{Susy}}^2}{m_t^2} + \right. \\
 & + \frac{5}{4} \frac{A_t^2}{M_{\text{Susy}}^2} + \frac{\pi}{3\sqrt{3}} \frac{A_t A_b}{M_{\text{Susy}}^2} - \\
 & - \frac{1}{4} \frac{A_t}{M_{\text{Susy}}} \left(\frac{A_t}{M_{\text{Susy}}} + \frac{2\bar{m}_b}{\bar{m}_t c_\beta} \right) \left(1 + \frac{A_t}{M_{\text{Susy}}} \frac{\bar{m}_t c_\beta}{\bar{m}_b} \right) \log \left| \frac{A_t}{M_{\text{Susy}}} + \frac{\bar{m}_b}{\bar{m}_t c_\beta} \right| - \\
 & - \frac{1}{4} \frac{A_t}{M_{\text{Susy}}} \left(\frac{A_t}{M_{\text{Susy}}} - \frac{2\bar{m}_b}{\bar{m}_t c_\beta} \right) \left(1 - \frac{A_t}{M_{\text{Susy}}} \frac{\bar{m}_t c_\beta}{\bar{m}_b} \right) \log \left| \frac{A_t}{M_{\text{Susy}}} - \frac{\bar{m}_b}{\bar{m}_t c_\beta} \right| \Big] + \\
 & + \frac{y_b^2}{c_\beta^2} \left[\frac{7}{4} - \frac{A_b^2}{M_{\text{Susy}}^2} \left(\frac{3}{2} - \frac{\pi}{\sqrt{3}} \right) - \log \frac{2\bar{m}_b}{\bar{m}_t c_\beta} - \frac{11}{4} \log \frac{M_{\text{Susy}}^2}{m_t^2} - \right. \\
 & - \frac{1}{4} \left(1 + \frac{A_t}{M_{\text{Susy}}} \frac{\bar{m}_t c_\beta}{\bar{m}_b} \right) \log \left| \frac{A_t}{M_{\text{Susy}}} + \frac{\bar{m}_b}{\bar{m}_t c_\beta} \right| - \\
 & \left. - \frac{1}{4} \left(1 - \frac{A_t}{M_{\text{Susy}}} \frac{\bar{m}_t c_\beta}{\bar{m}_b} \right) \log \left| \frac{A_t}{M_{\text{Susy}}} - \frac{\bar{m}_b}{\bar{m}_t c_\beta} \right| \right]
 \end{aligned}$$

δ_1 and δ_2 coefficients.

$$\begin{aligned}
\delta_1 = & \frac{4}{3} g_s^2 \left(\frac{A_b}{M_{\text{Susy}}} - 1 \right) + y_t^2 \left[\frac{3}{8} - \frac{4\pi}{3\sqrt{3}} - \frac{A_t^2}{M_{\text{Susy}}^2} - \frac{\pi}{3\sqrt{3}} \frac{A_t A_b}{M_{\text{Susy}}^2} + \right. \\
& + \frac{1}{4} \frac{A_t}{M_{\text{Susy}}} \left(\frac{A_t}{M_{\text{Susy}}} + \frac{2\bar{m}_b}{\bar{m}_t c_\beta} \right) \left(1 + \frac{A_t}{M_{\text{Susy}}} \frac{\bar{m}_t c_\beta}{\bar{m}_b} \right) \log \left| \frac{A_t}{M_{\text{Susy}}} + \frac{\bar{m}_b}{\bar{m}_t c_\beta} \right| + \\
& + \frac{1}{4} \frac{A_t}{M_{\text{Susy}}} \left(\frac{A_t}{M_{\text{Susy}}} - \frac{2\bar{m}_b}{\bar{m}_t c_\beta} \right) \left(1 - \frac{A_t}{M_{\text{Susy}}} \frac{\bar{m}_t c_\beta}{\bar{m}_b} \right) \log \left| \frac{A_t}{M_{\text{Susy}}} - \frac{\bar{m}_b}{\bar{m}_t c_\beta} \right| \Big] + \\
& + \frac{y_b^2}{c_\beta^2} \left[-1 + \frac{A_b^2}{M_{\text{Susy}}^2} \left(\frac{3}{2} - \frac{\pi}{\sqrt{3}} \right) + \log \frac{2\bar{m}_b}{\bar{m}_t c_\beta} + \right. \\
& + \frac{1}{4} \left(1 + \frac{A_t}{M_{\text{Susy}}} \frac{\bar{m}_t c_\beta}{\bar{m}_b} \right) \log \left| \frac{A_t}{M_{\text{Susy}}} + \frac{\bar{m}_b}{\bar{m}_t c_\beta} \right| + \frac{1}{4} \left(1 - \frac{A_t}{M_{\text{Susy}}} \frac{\bar{m}_t c_\beta}{\bar{m}_b} \right) \log \left| \frac{A_t}{M_{\text{Susy}}} - \frac{\bar{m}_b}{\bar{m}_t c_\beta} \right| \Big] \\
\delta_2 = & -4g_s^2 - \frac{3}{4} y_t^2 \left(1 + \frac{A_t^2}{M_{\text{Susy}}^2} \right) + \frac{5y_b^2}{4c_\beta^2}
\end{aligned}$$