Analytic resummation: motivation and practical examples

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DESY Students seminar

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Presentation Plan

1. Motivation
2. Simple example: Jet mass resummation
3. Other applications
4. Conclusion

Presentation Plan

Motivation

What is resummation?

- Suppose one wants to compute a QCD observable
- We first try a **fixed-order expansion** in the strong coupling α_s

$$\langle O \rangle = \sum_n \alpha_s^n c_n$$

Fixed-order $\rightarrow$ truncating this series at a given n

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- Problem: cases where c_n is not "well behaved"
  $\rightarrow$ FO expansion **does not converge**
- Happens when there is a strong hierarchy between scales
  $\rightarrow$ c_n enhanced by large logarithms (more on that later)
  e.g. Jet mass at boosted regimes $m \ll p_t$

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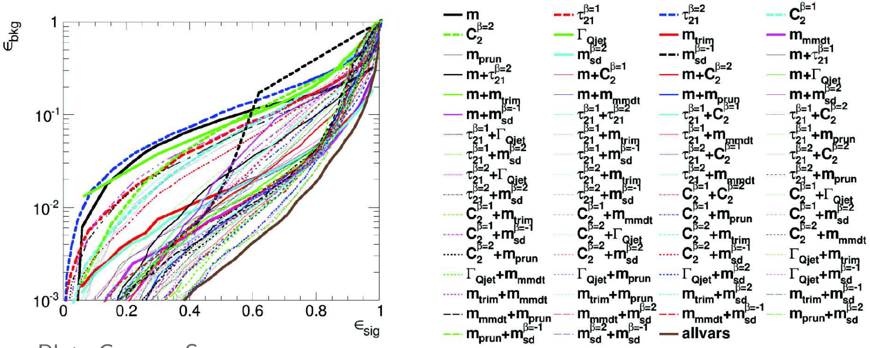
  e.g. Jet mass at boosted regimes $m \ll p_t$
- Need an **all-order (in α_s) resummed calculation**

Why do we need resummation? **Better understanding**

- Parton shower Monte Carlo generators are very useful tools, but numerically costly and the physical message is not always clear



Plot: Gregory Soyez

- Example: ROC curves for different jet substructure methods

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- Compute robust **uncertainty bands**
  $\rightarrow$ Correct assessment of the higher orders corrections we are neglecting

Disclaimer

1. The goal is not to present a formal course
2. Mostly centered in **jet substructure techniques** in boosted regimes
   $\rightarrow$ Resummation techniques have many different applications
3. There are many different techniques/formalisms for resummation
   - "Plain" perturbative QCD (used in following example)
   - Effective field theories
     $\rightarrow$ Soft-Collinear Effective Field (SCET)
     $\rightarrow$ Non-Relativistic QCD (NRQCD)
   - ...

In the end, all these formalisms are equivalent

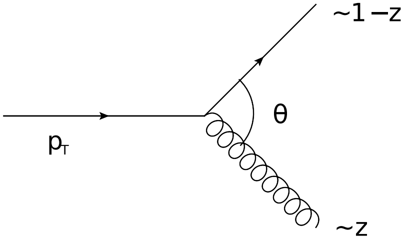
Limitations

- Resummation rarely describe all phase space
  $\rightarrow$ needs to be **matched to a fixed-order (FO) calculation**
  $\rightarrow$ one needs to avoid double-counting of the terms
  $\rightarrow$ this is a non-trivial process
- Limitation of the analytical approach is the poor understanding of **non-perturbative effects** (underlying event and hadronization)
- In practice : **FO + Resum + Parton Shower**
  $\rightarrow$ Need an external Monte Carlo generator

Presentation Plan

Mass distribution

- Integrated distribution for the jet mass m;
- Simplest case: jet with only one emission $q \to q + g$
  $\to$ p_T is the jet transverse momentum
  $\to$ R is the characteristic jet radius



$$m^2 \simeq p_T^2 z(1-z)\theta^2$$

- **Boosted jets** regime $\to p_T \gg m$

Mass distribution

- From Feynman rules, we can write the distribution as

$$\Sigma(> m^2) \;\; = \;\; \frac{\alpha_s}{2\pi}\int_0^{R^2}\frac{d\theta^2}{\theta^2}\int_0^1 dz P_{qg}(z)\Theta\left(p_T^2 z(1-z)\theta^2 - m^2\right)$$

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- For **boosted jets** $\rightarrow \rho \ll 1$

$$\Sigma(\rho) \simeq \frac{\alpha_s C_F}{\pi}\left[\frac{1}{2}\log\left(\frac{1}{\rho}\right)^2 - \frac{3}{4}\log\left(\frac{1}{\rho}\right) + \mathcal{O}(1)\right].$$

Mass distribution

- For **higher orders** in α_s
  $\rightarrow$ terms like $\left[\alpha_s \log(1/\rho)^2\right]^n$
- **Boosted jets**
  $\rightarrow \rho = m^2/(R^2 p_T^2) \ll 1$
  $\rightarrow \alpha_s \log(1/\rho)^2 \sim 1$
  $\rightarrow$ fixed order expansion in $\mathcal{O}(\alpha_s^n)$ does not converge.

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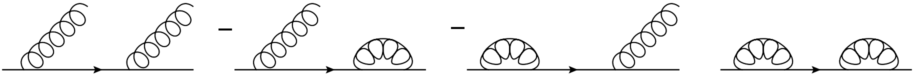
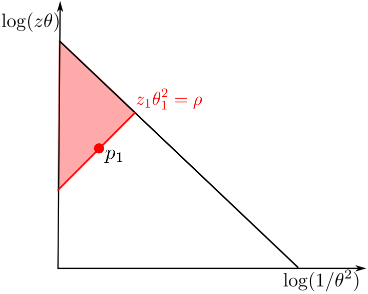
What can we do?

- **Resummation at all orders** supposing $\rho \ll 1$.
- Only interested at the dominating term $\sim \alpha_s \log(1/\rho)^2$
  $\rightarrow$ Leading Logarithm (LL)
- Virtual emissions $\rightarrow$ cancel out soft and collinear divergences.



Resumed results

- We suppose independent emissions
  $\rightarrow$ constraints as an exponential factor.



- For a jet of a given mass:

$$\begin{aligned} \Sigma(\rho) &= 1 - e^{-R(\rho)} \\ R(\rho) &= \frac{\alpha_s}{2\pi} \int_0^1 \frac{d\theta^2}{\theta^2} \int_0^1 dz P_i(z) \\ &\times \Theta(z\theta^2 - \rho) \end{aligned}$$

- This exponent is the Sudakov radiator

Resumed results

- If we take only the dominant logarithm terms

$$\Sigma(\rho) = 1 - \frac{\alpha_s C_F}{\pi} \exp\left[-\frac{\alpha_s C_F}{2\pi} \log\left(\frac{1}{\rho}\right)^2\right]$$

$\rightarrow$ leading-logarithm (LL) accuracy

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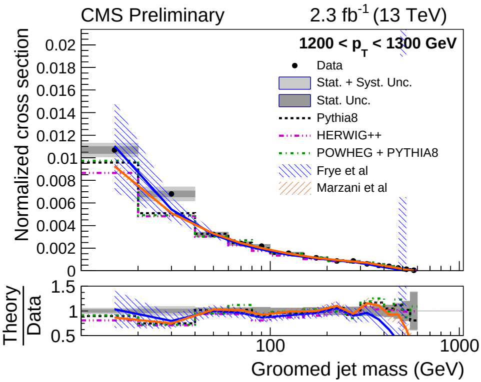
Comparison with experiment

- Jet mass distribution for a groomed jet (with modified MassDrop Tagger)



Plot : CMS-PAS-SMP-16-010

NNLL + LO : Frye, Larkoski, Schwartz, Yan (2016)

(N)LL + NLO : Marzani, LS, Soyez (2017)

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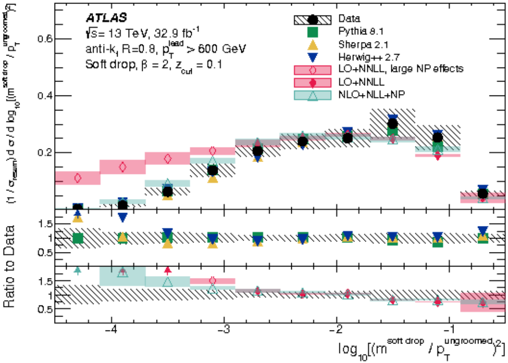
- Jet mass distribution for a groomed jet (with SoftDrop)



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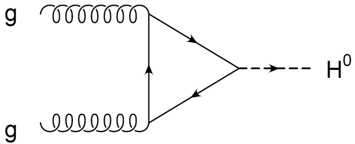
Presentation Plan

q_T resummation

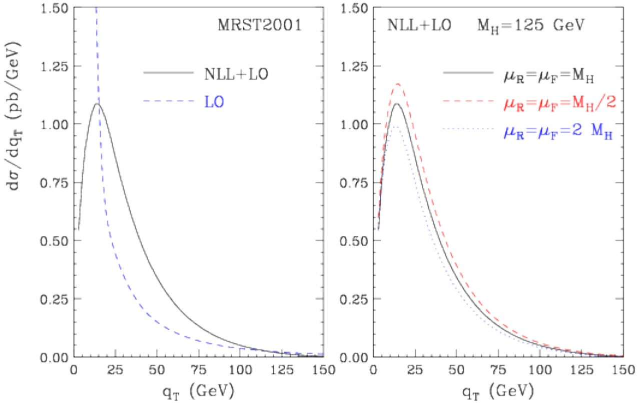
- Transverse momentum (q_T) of the Higgs boson (produced via gluon fusion);



- Resummation dominates at the small q_T region $\rightarrow q_T \ll M_H$
- Logarithmic enhancements in the form $\sim \alpha_s^m \log\left(\frac{M_H^2}{q_T^2}\right)^{2m}$

q_T resummation

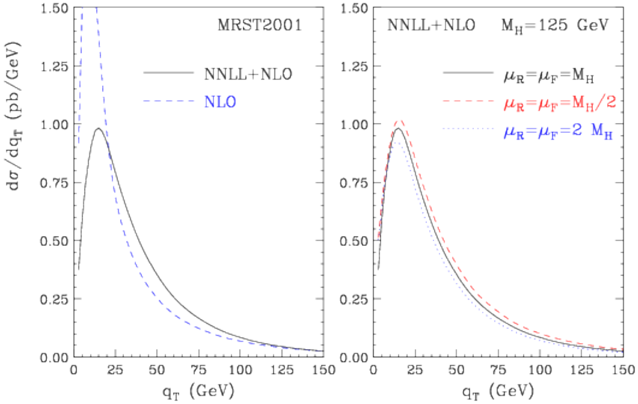
- q_T distribution at LO + NLL



From arXiv:hep-ph/0302104

q_T resummation

- q_T distribution at NLO + NNLL



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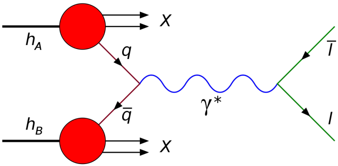
Threshold resummation

- For Drell-Yan pair production, the threshold variable is $\tau = M^2/s$
  $\rightarrow$ M is the lepton pair invariant mass
  $\rightarrow$ s is the squared center-of-mass energy of boson



- Correction from soft gluon radiation in the form of logarithm enhanced terms $\alpha_s^n \log(1-\tau)^{2n}$

- Introduced in the 80s, using Mellin transformations

General features

- Resummed physical observables can be presented in the general form

$$\begin{aligned}\langle O \rangle &\simeq 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \dots \\ &\simeq \exp\left(L g_1(\alpha_s L) + g_2(\alpha_s L) + \dots\right)\end{aligned}$$

- In regions were $\alpha_s L^2 \sim 1$ truncating the series at a given order $\mathcal{O}(\alpha_s^2)$ does not converge
- In these regions, we **need resumation**
  $L g_1(\alpha_s L)$ is leading-logarithm (LL)
  $g_2(\alpha_s L)$ is next-to-leading logarithm (NLL) ...

Presentation Plan

Conclusion

- Resummation is needed when there is a strong hierarchy between scales, fixed-order calculation is not enough
- Resummation allow for **better accuracy**
  $\rightarrow$ Parton showers are leading-logarithm
  $\rightarrow$ Resummed calculation can be systematically improved
- Analytical expressions allow **better understanding** of observables
- Computation of robust **uncertainty bands**
- Limitations of analytical approach $\rightarrow$ non-perturbative effects

Backup

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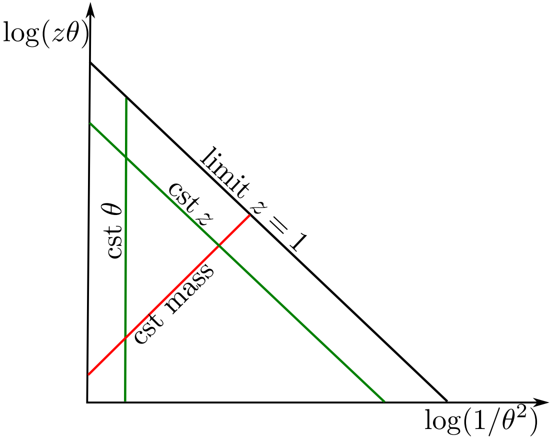
Lund Diagrams



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