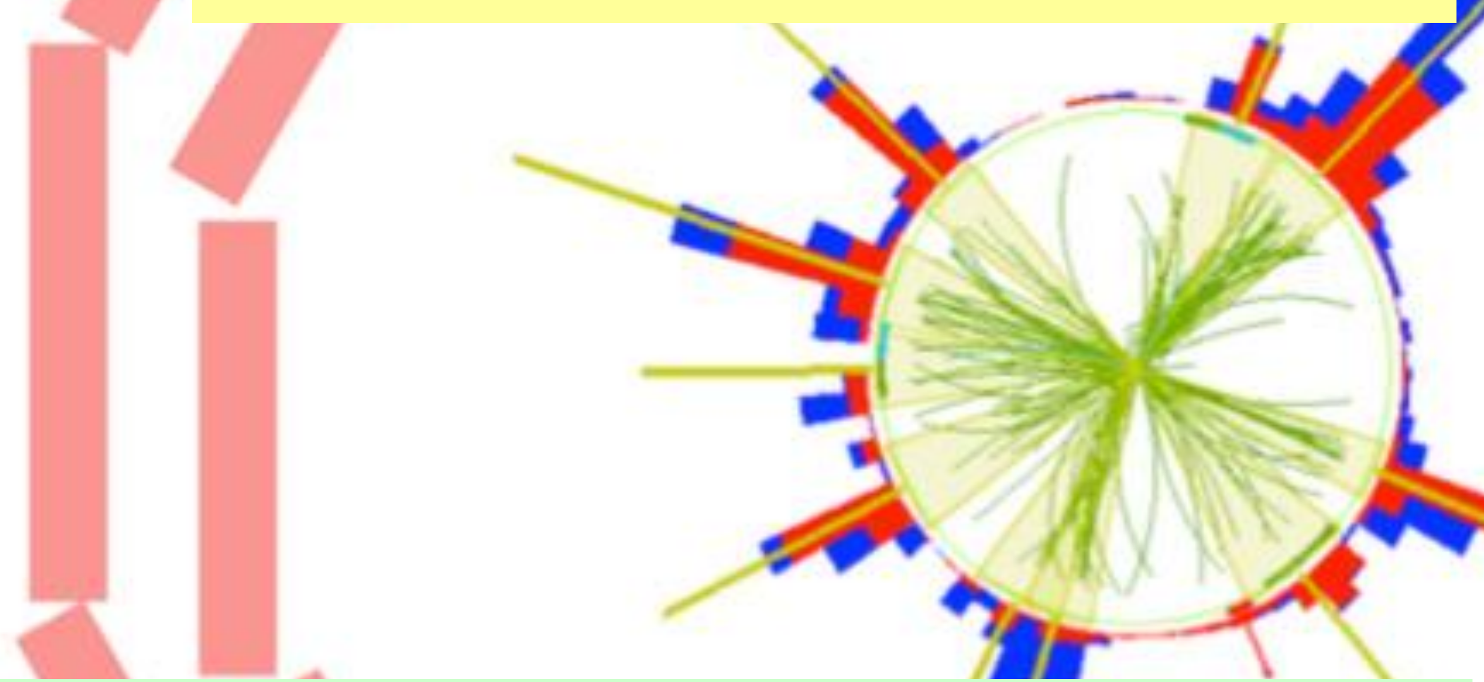
Scattering Amplitudes and Precision Theory for the LHC



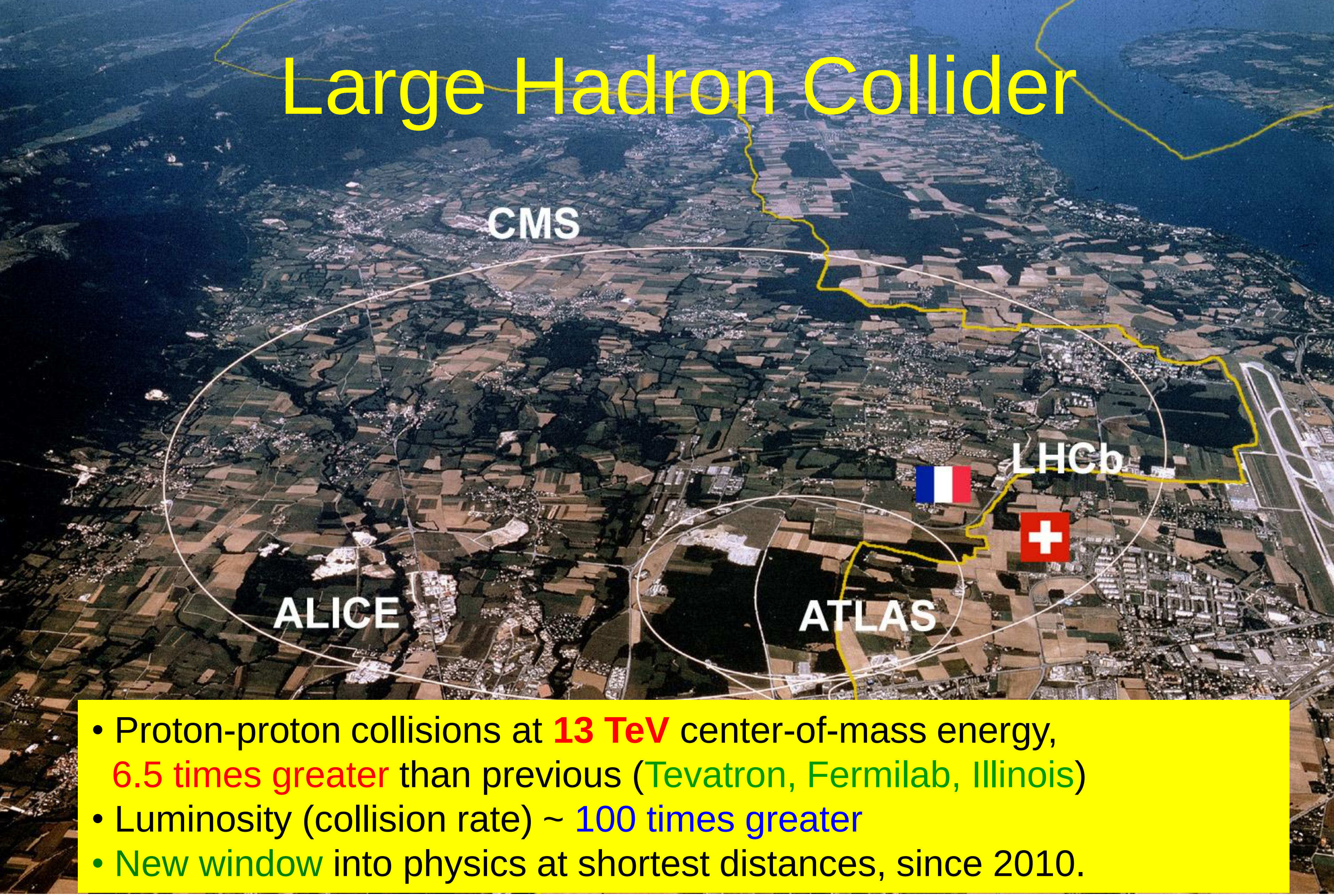
Lance Dixon (SLAC)

Concluding Colloquium

Graduate College on Mass, Spectrum & Symmetry

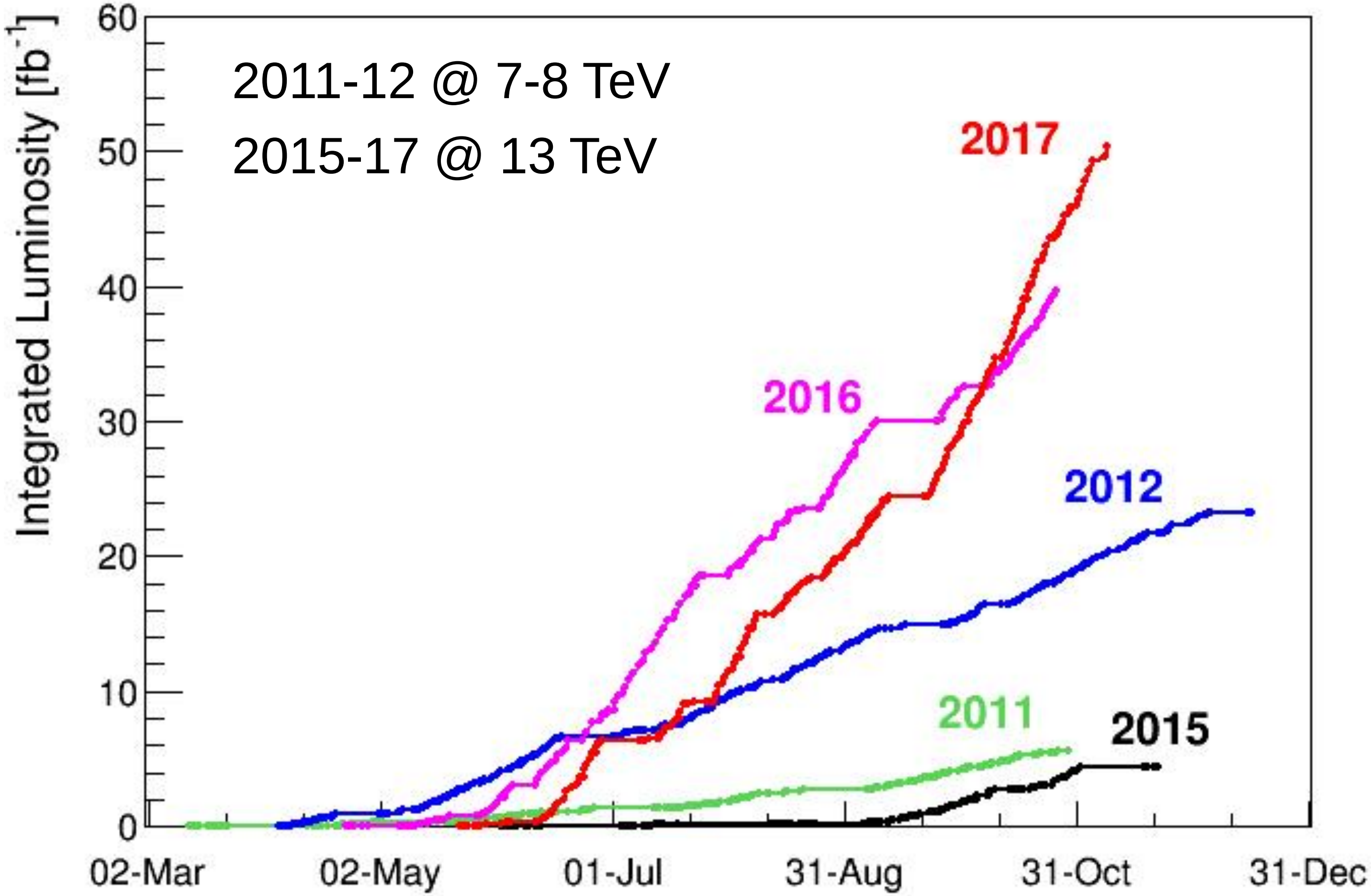
9 March 2018

Large Hadron Collider



- Proton-proton collisions at **13 TeV** center-of-mass energy, 6.5 times greater than previous (Tevatron, Fermilab, Illinois)
- Luminosity (collision rate) ~ 100 times greater
- New window into physics at shortest distances, since 2010.

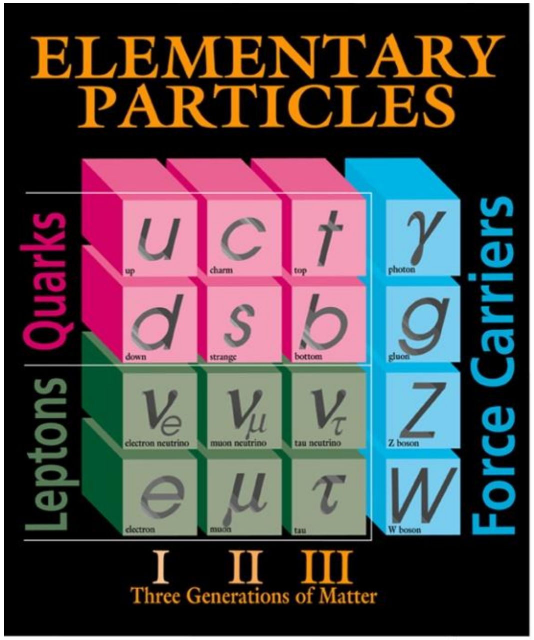
LHC performing exceedingly well



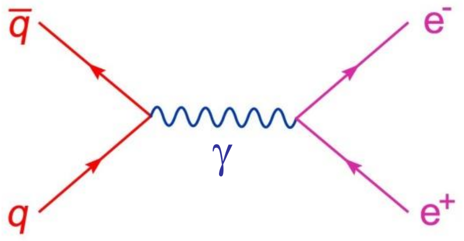
Standard Model

- All elementary forces except gravity in same basic framework
- Matter made of spin ½ fermions
- Forces carried by spin 1 vector bosons: γ $W^+ W^- Z^0$ g
- Add a spin 0 Higgs boson H to explain masses of $W^+ W^- Z^0$

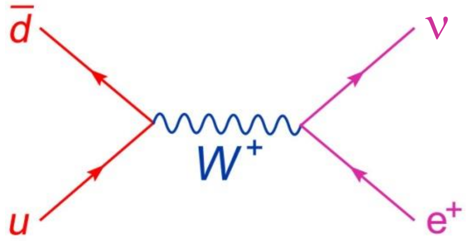
  → finite, testable predictions for all quantities



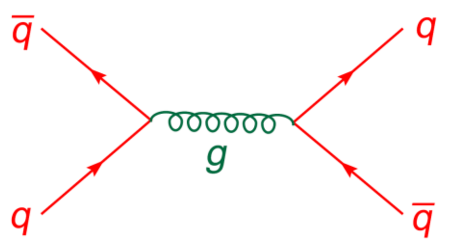
Fermilab 95-759



electromagnetism (QED)

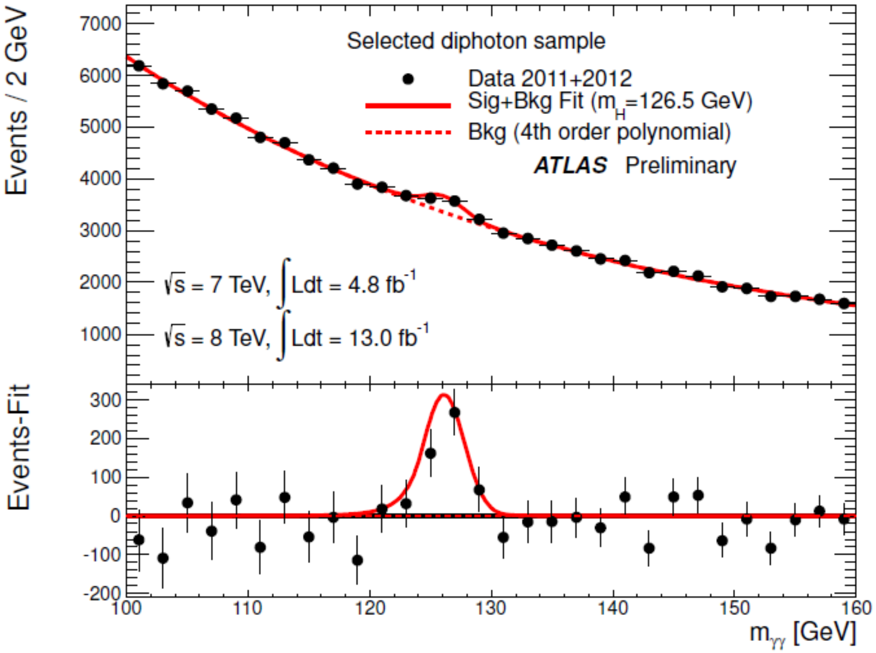


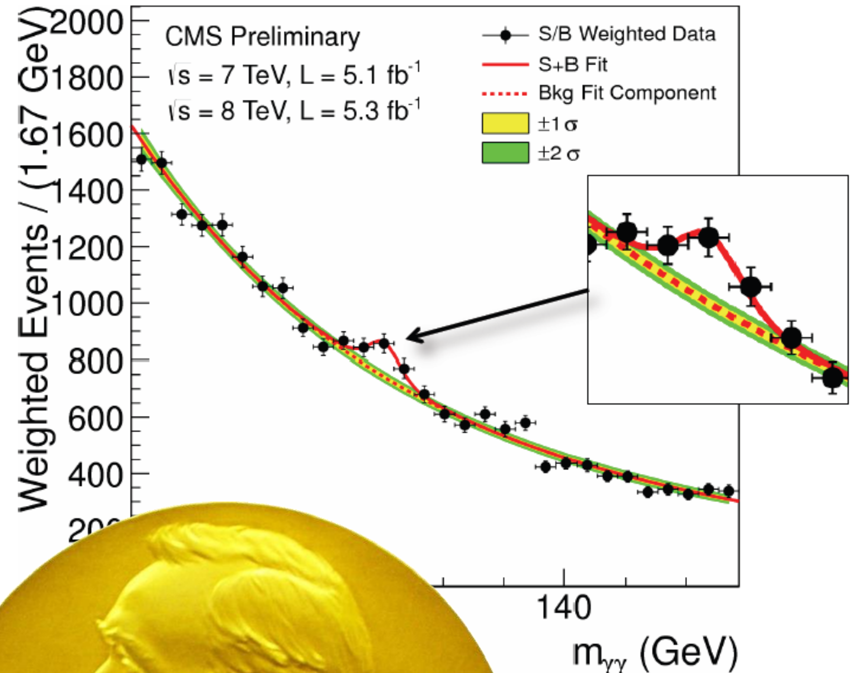
weak



strong (QCD)

Higgs-like particle discovered!

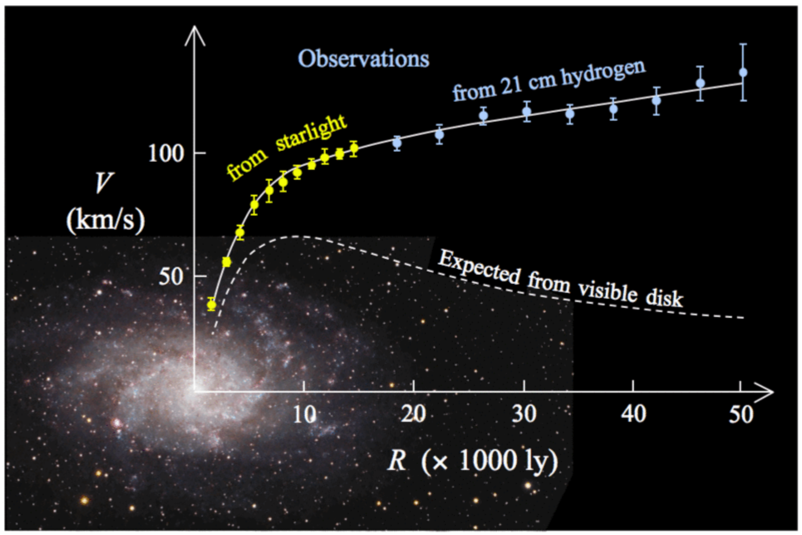


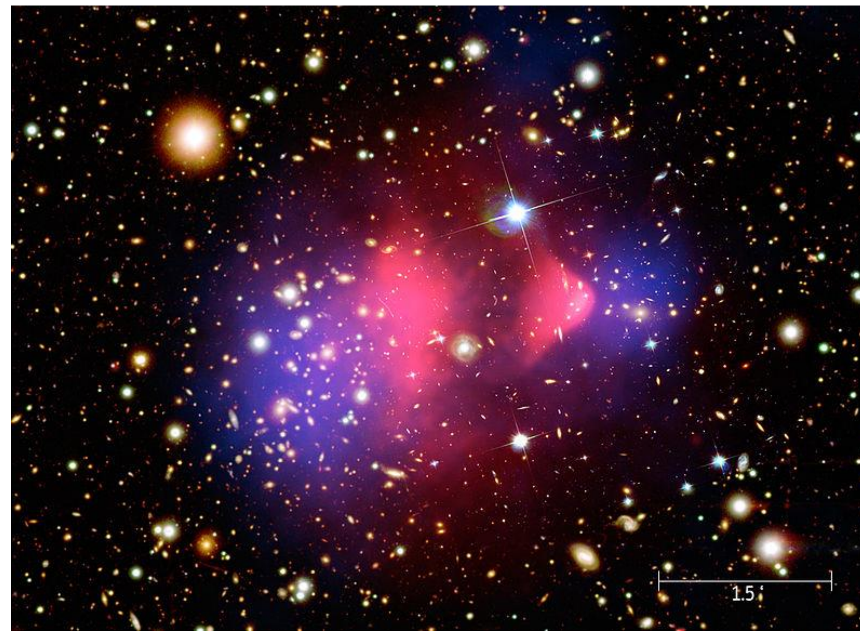


Is there Physics beyond the Standard Model at the LHC?

There is dark matter in the cosmos

talk by Moritz Hütten





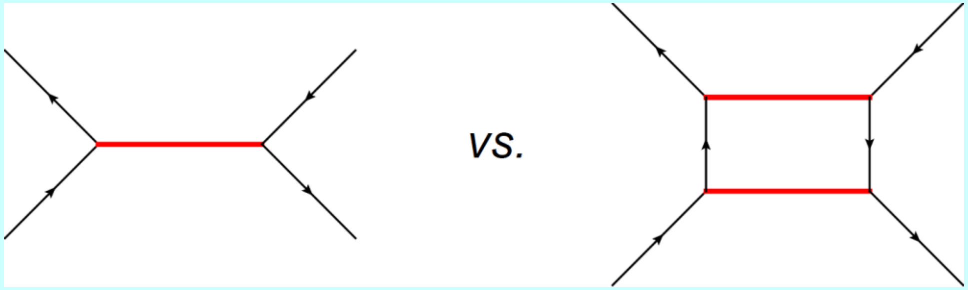
If it is an elementary particle, the LHC could produce it, or produce other particles that decay to it

Beyond the Standard Model

- **Hierarchy problem:**

In SM, electroweak scale m_W looks fine-tuned as soon as ultraviolet cutoff Λ is raised well above m_W.

- Many theories predict a host of new massive particles with masses ~ " m_W " i.e. within reach of the LHC.
- To prevent problems from precision electroweak physics,



such theories often have a discrete symmetry, for which the lightest odd particle is a **dark matter candidate**

Searching for BSM at LHC

- **Frameworks include:**
  - **supersymmetry**
  - **new dimensions of space-time**
  - **new forces**
  - etc.

- **Usually some colored particles which can be produced with large cross sections at the LHC.**

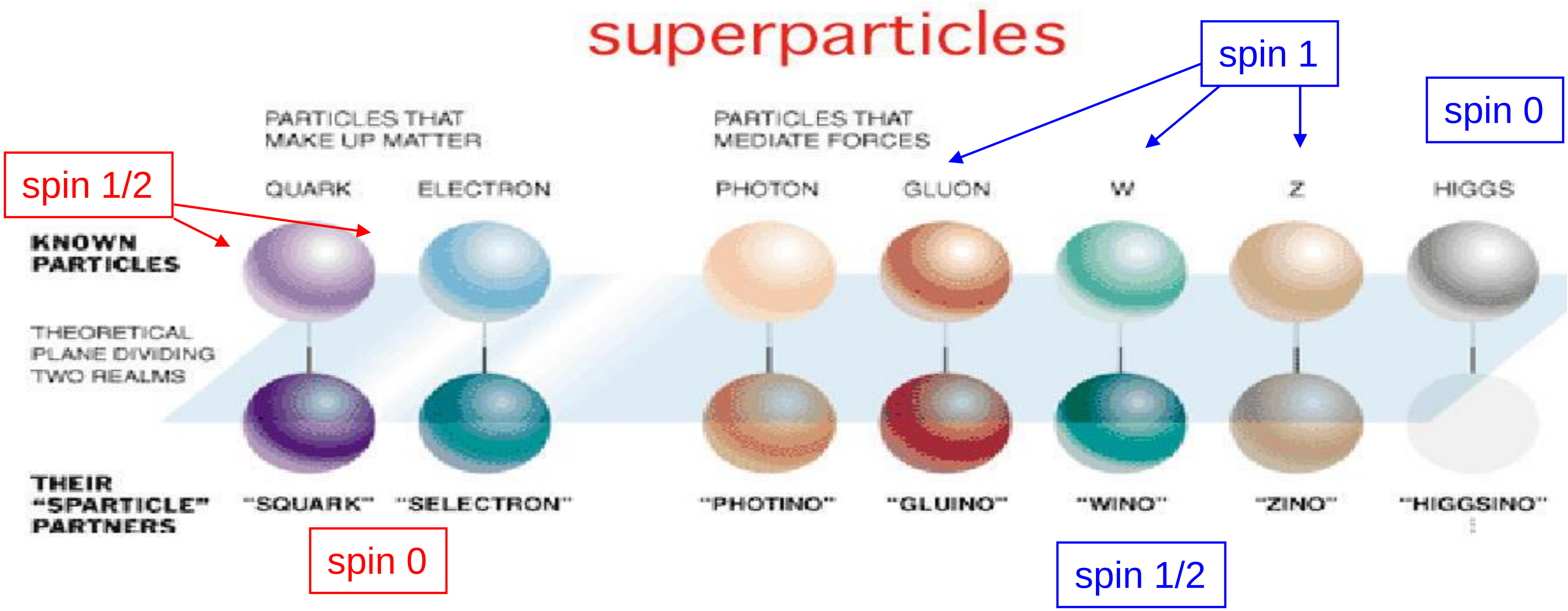
- Most new massive particles decay rapidly to old, ~massless particles:
  quarks, gluons, charged leptons, neutrinos, photons
  + **dark matter?**

- **How to distinguish new physics from old (Standard Model)?**
- **From other types of new physics?**

New Physics Example: Supersymmetry

talk by Philip Dießner

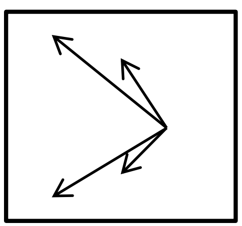
- Symmetry between fermions (matter) and bosons (forces)
- Very elegant, solves hierarchy problem
- Lightest supersymmetric particle (LSP) can be **dark matter**
- **Cornucopia** of new elementary particles at LHC.

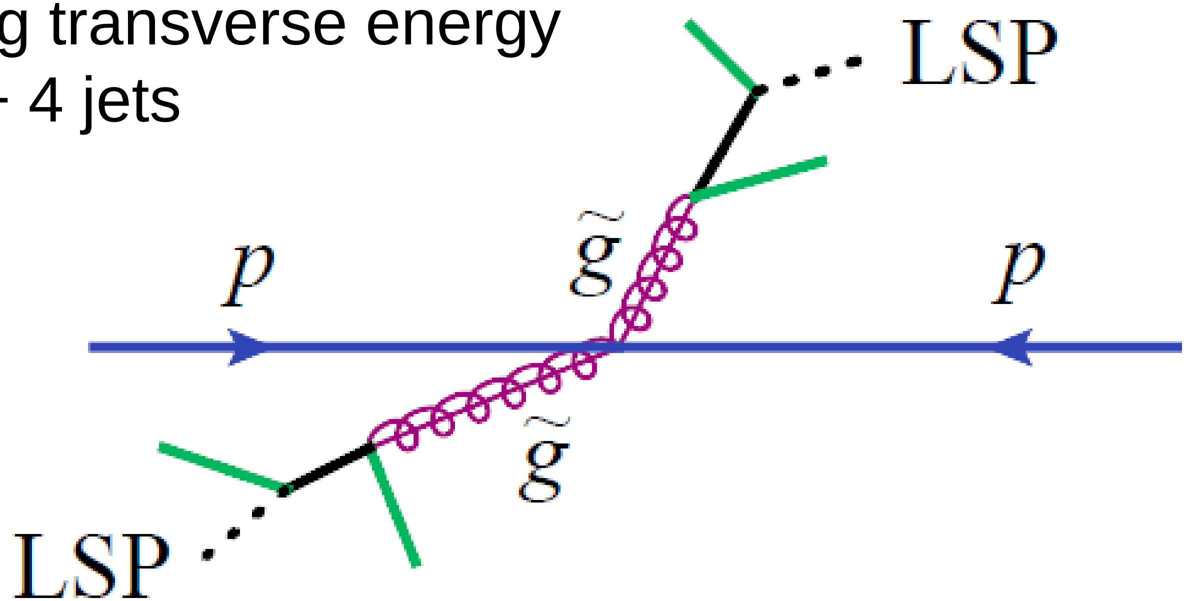


Classic SUSY dark matter signature

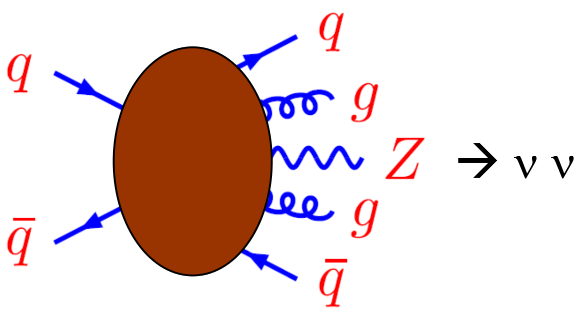
Heavy colored particles decay rapidly to stable Weakly Interacting Massive Particle (WIMP = LSP) plus jets

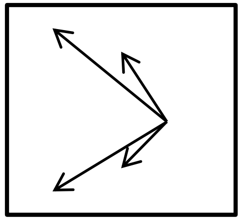
→ Missing transverse energy
MET + 4 jets





Not background free: happens in Standard Model too





MET + 4 jets from pp → *Z* + 4 jets,
Z → neutrinos

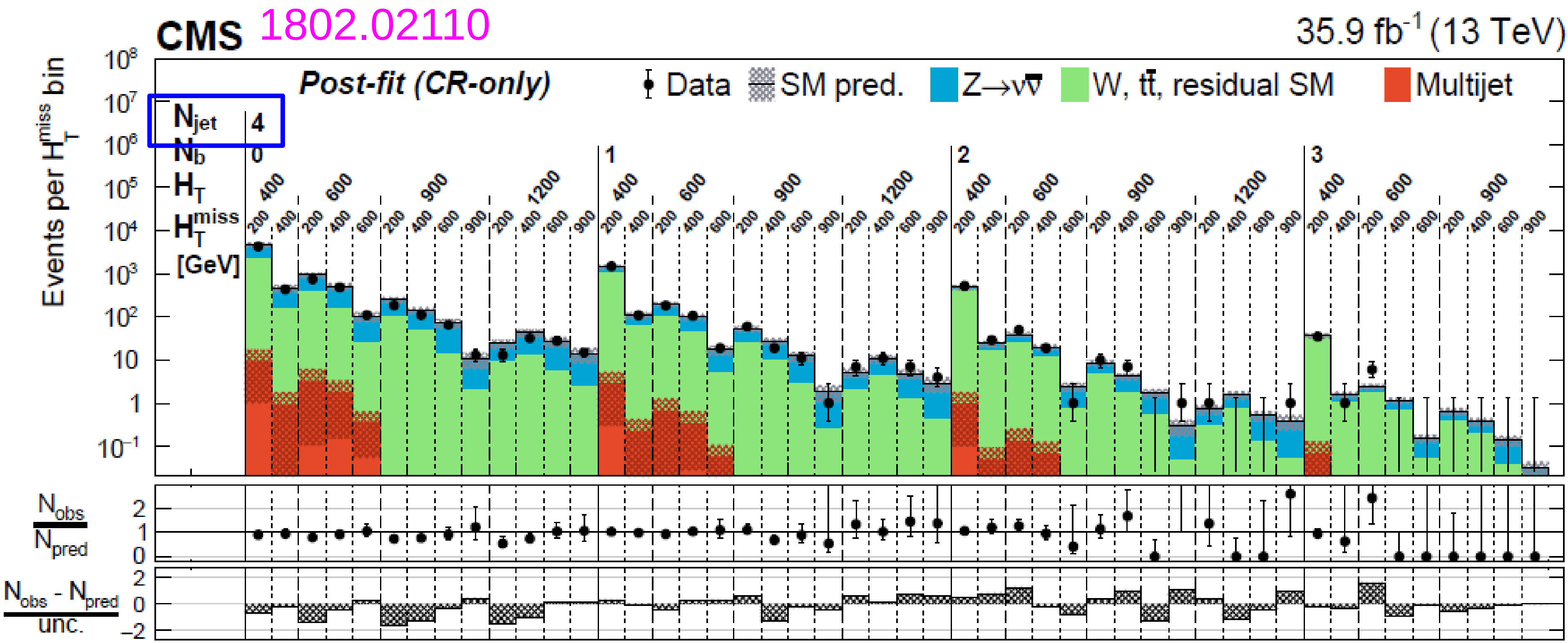
Neutrinos escape detector.

Irreducible background.

Plus there are many reducible backgrounds from W + jets, tt + jets, ...

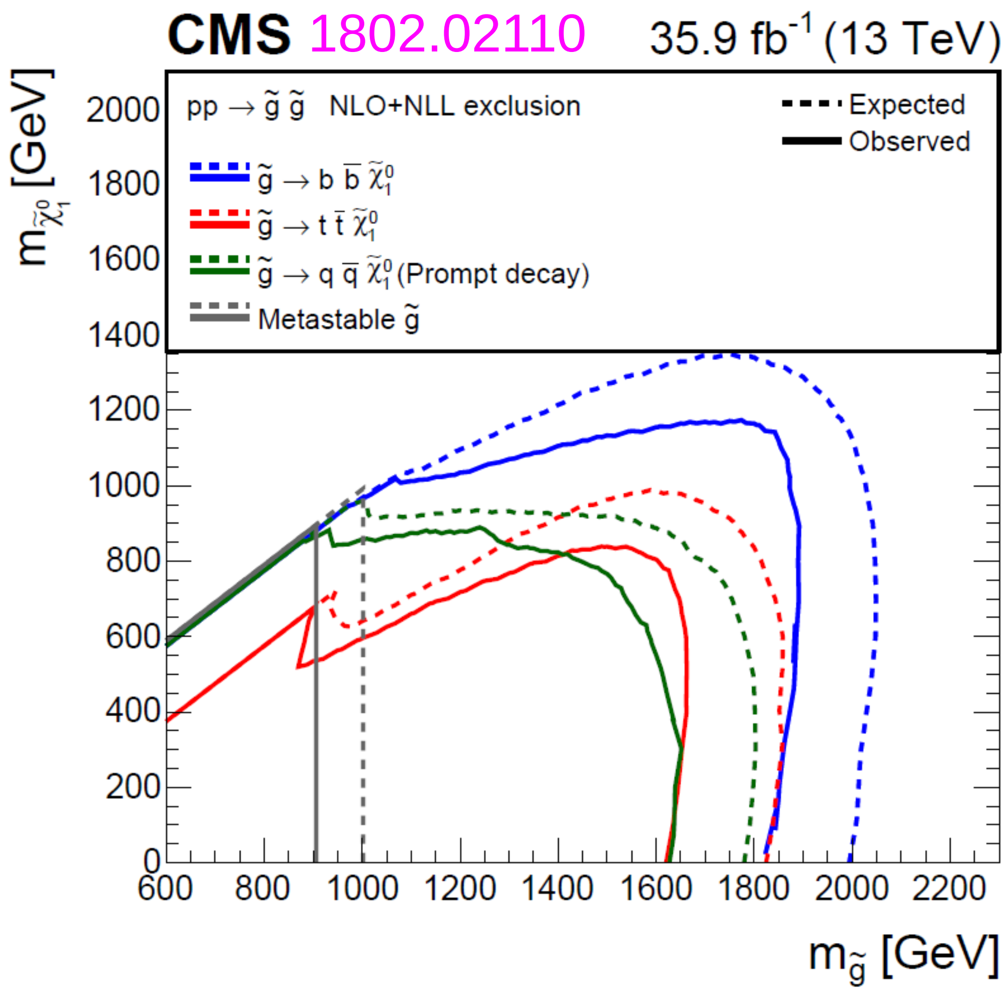
Precision theory (typically NLO) can help with this, usually when embedded in parton shower Monte Carlos

SUSY searches now very sophisticated

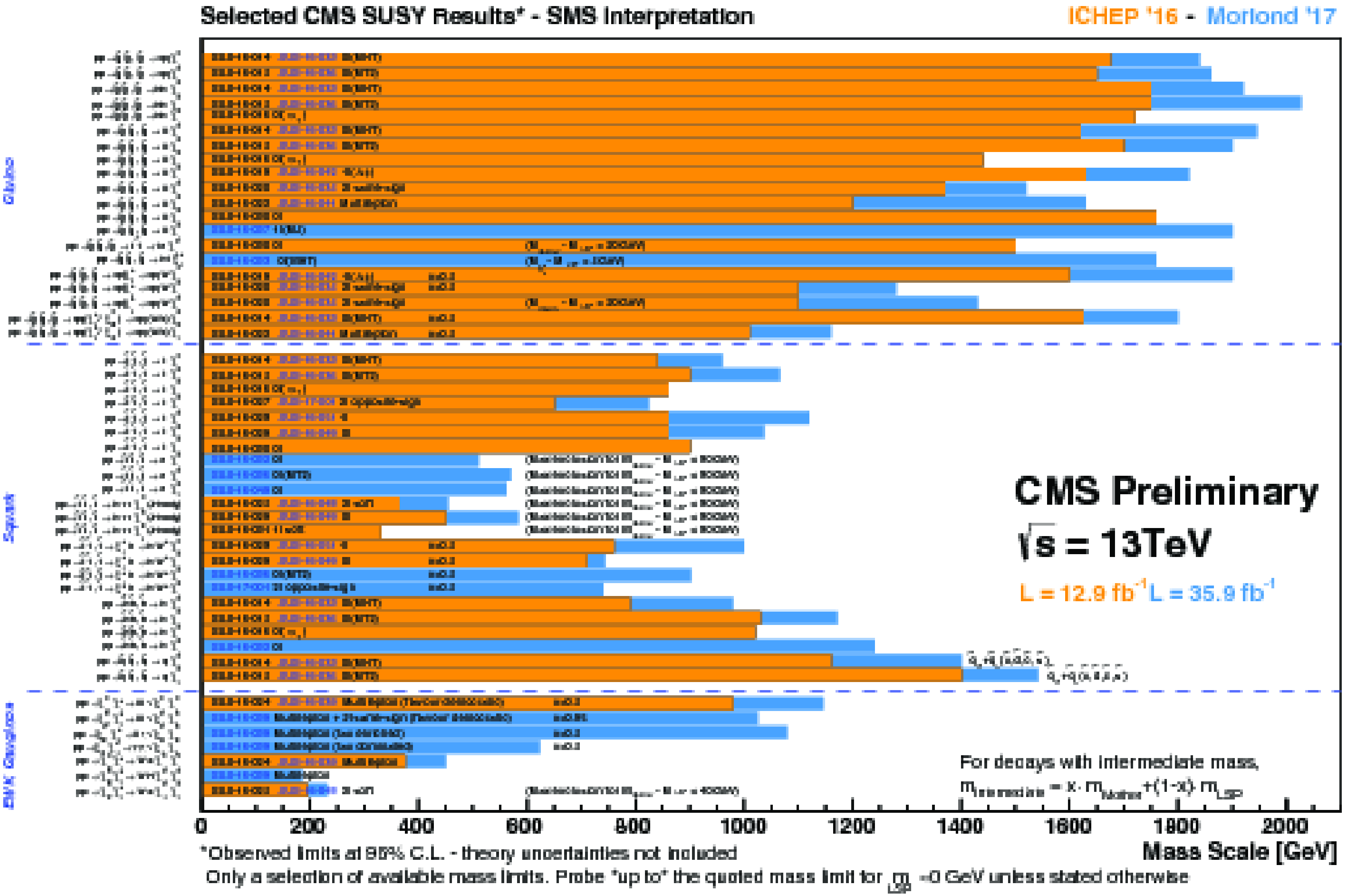


- Also N_{jet} = 1,2,3,5,6
- No significant excesses seen, so set lower limits on masses of superparticles.

Sample exclusion plot

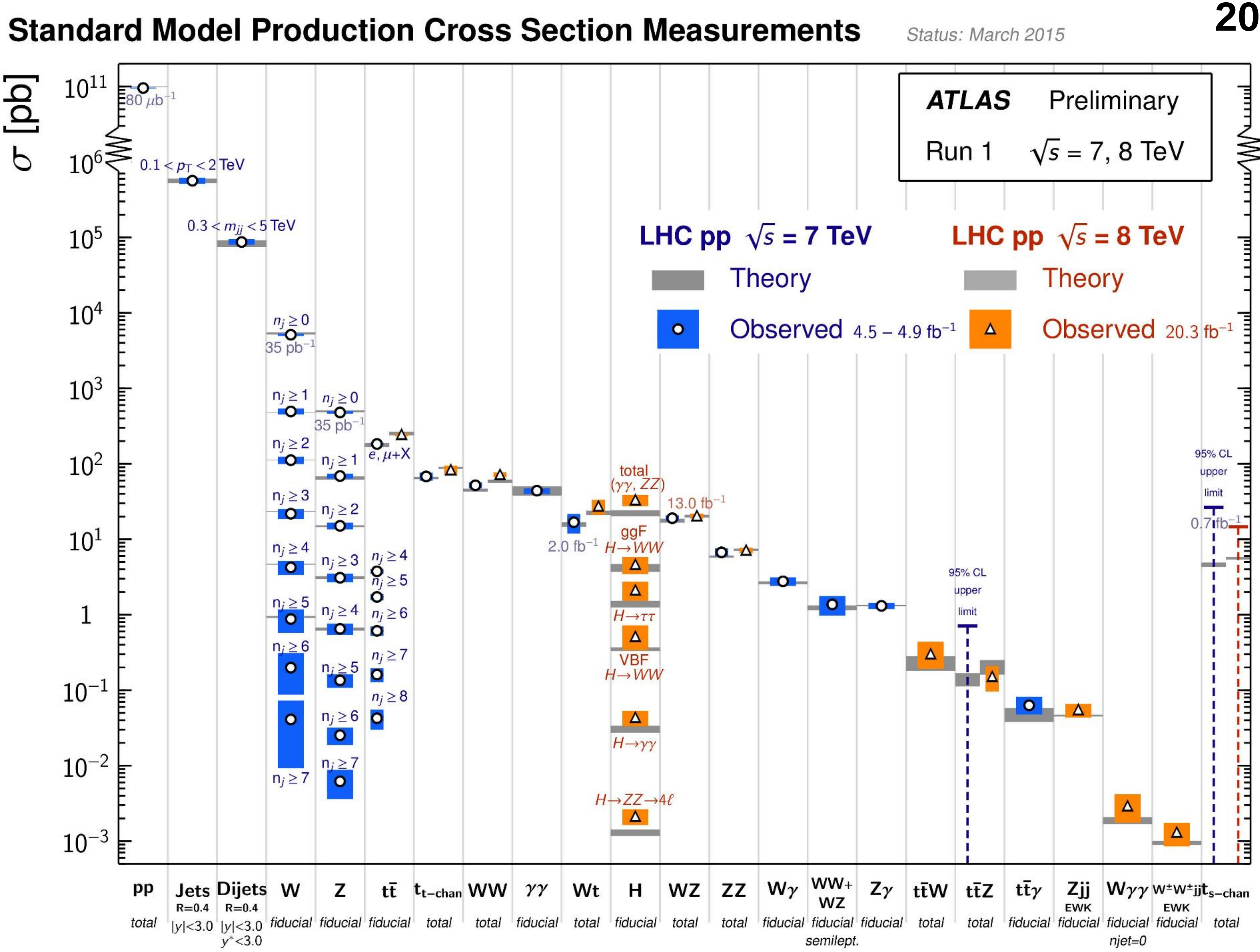


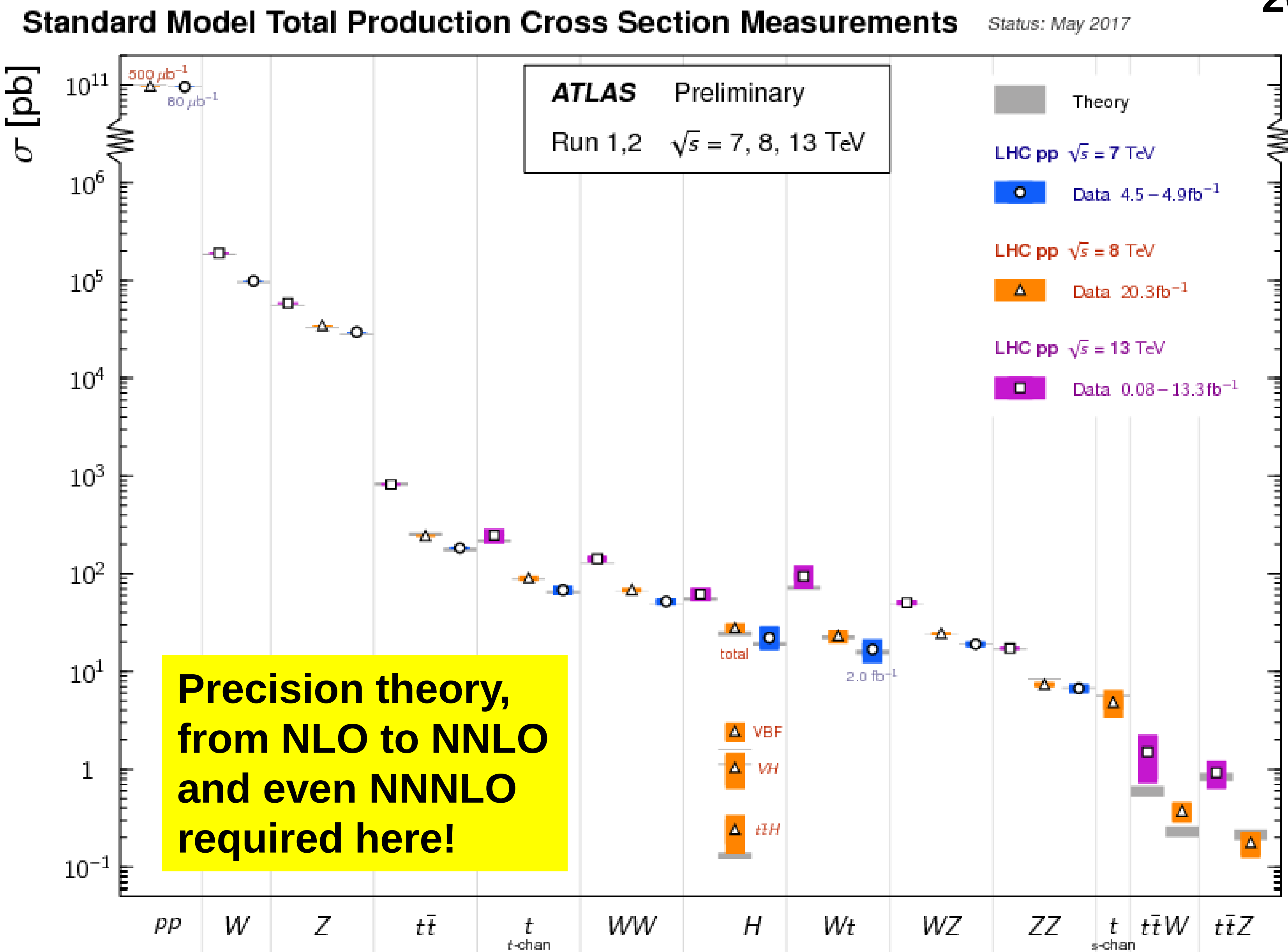
CMS SUSY Search Summary



From searches to measurements

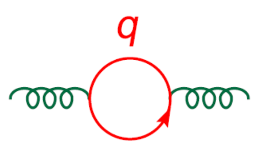
- No convincing evidence for SUSY, or any other direct production of new particles.
- Also look for deviations in rates for Standard Model processes, especially involving the brand-new Higgs boson.
- Measurements are hard, take a while to perform.
- More precise theory typically needed.


Standard Model Production Cross Section Measurements
Status: March 2015
ATLAS Preliminary
Run 1 √s = 7, 8 TeV
LHC pp √s = 7 TeV
Theory
Observed 4.5 – 4.9 fb⁻¹
LHC pp √s = 8 TeV
Theory
Observed 20.3 fb⁻¹
σ [pb]
pp total
Jets R=0.4 |y|<3.0
Dijets R=0.4 |y|<3.0 y*<3.0
W fiducial
Z fiducial
tt̄ fiducial
t t-chan total
WW total
γγ fiducial
Wt total
H fiducial
WZ total
ZZ total
Wγ fiducial
WW+WZ fiducial semilept.
Zγ fiducial
tt̄W total
tt̄Z total
tt̄γ fiducial
Zjj EWK fiducial
Wγγ fiducial njet=0
W±W±jj EWK fiducial
t s-chan total
95% CL upper limit



Precision theory, from NLO to NNLO and even NNNLO required here!

Asymptotic freedom and short-distance calculability



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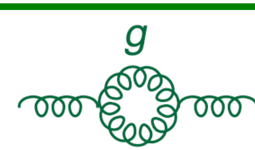
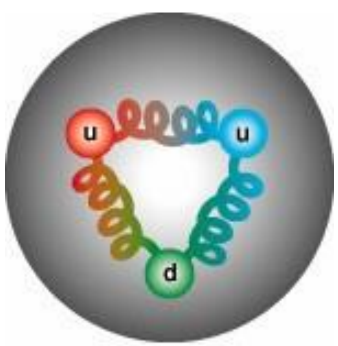


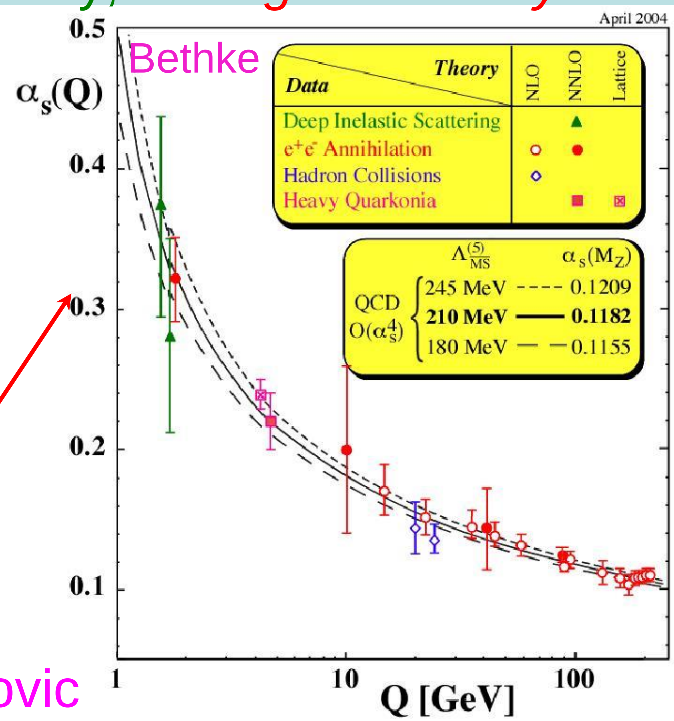
< 0

- Gluon self-interactions →

α_s → **0** asymptotically, but *logarithmically* at short distances (large Q)

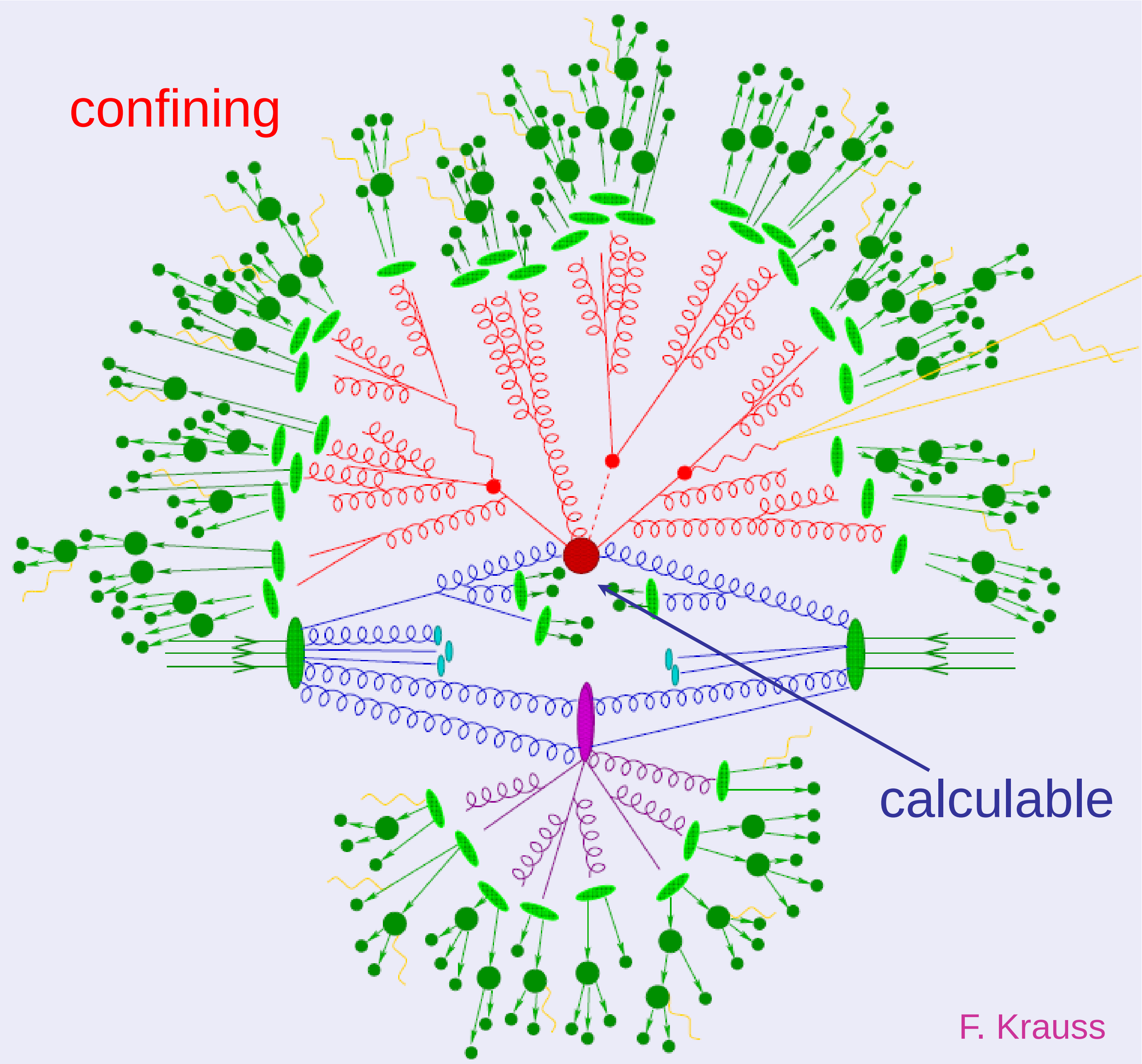


confining



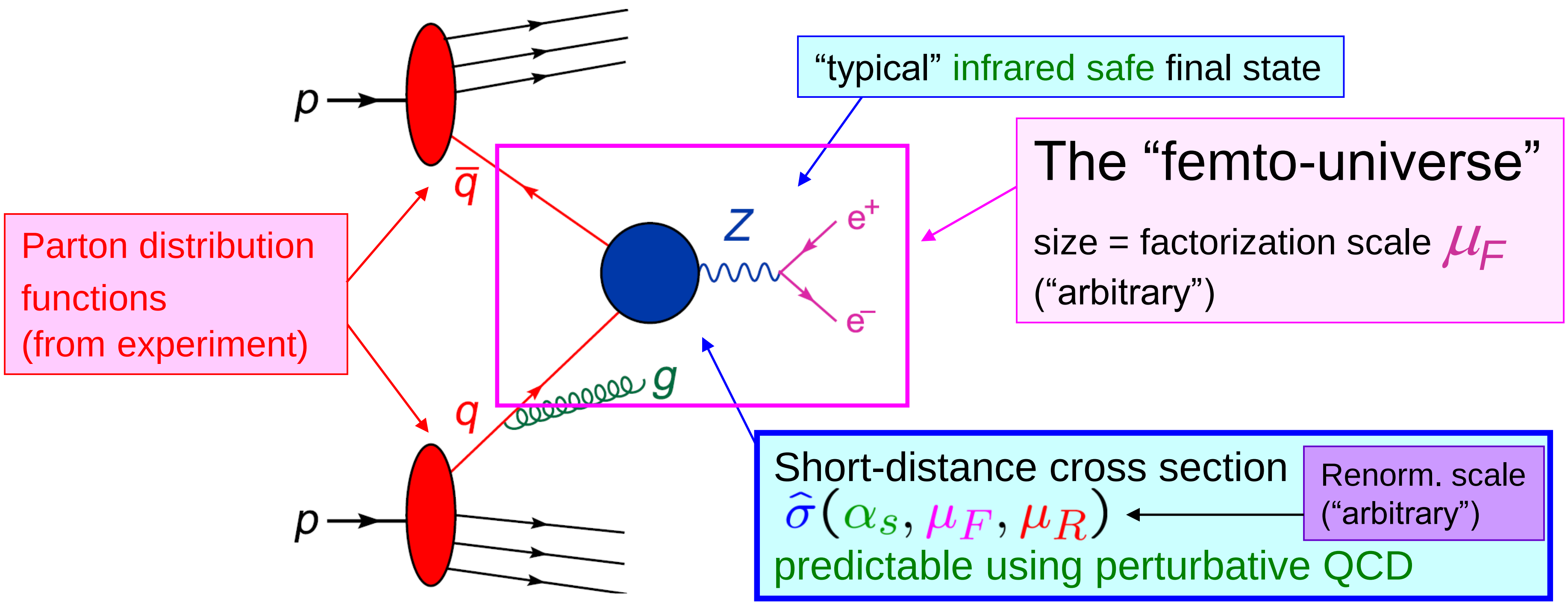
calculable

talk by Marina Marinkovic


confining
calculable
F. Krauss

QCD Factorization & Parton Model

At short distances, quarks and gluons (partons) in proton are almost free. Sampled “one at a time”



Perturbative Short-Distance Cross Section

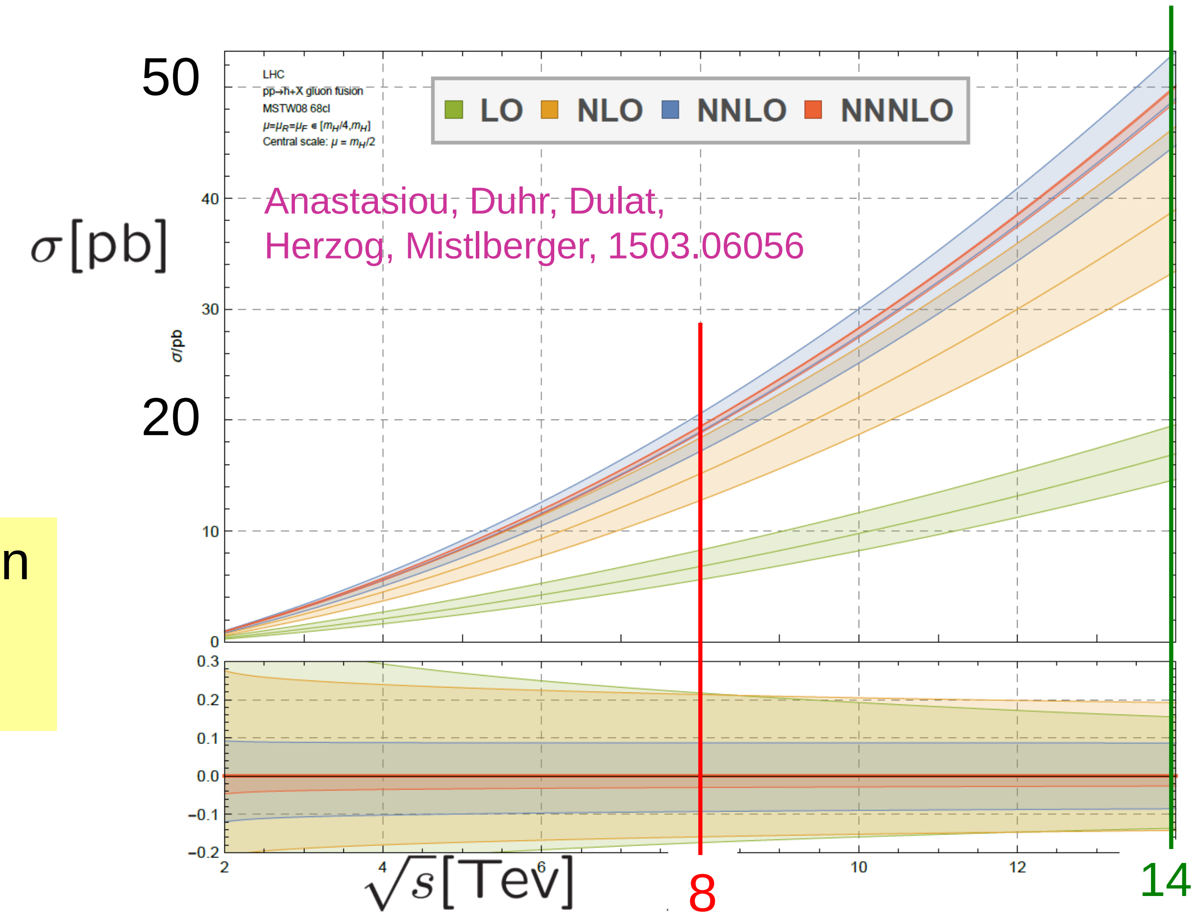
$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) \;=\; [\alpha_s(\mu_R)]^{n_\alpha} \left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi} \hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots \right]$$

LO NLO NNLO

Leading-order (LO) predictions qualitative:
poor convergence
of expansion in $\alpha_s(\mu)$
Uncertainty bands from varying $\mu_R = \mu_F = \mu$

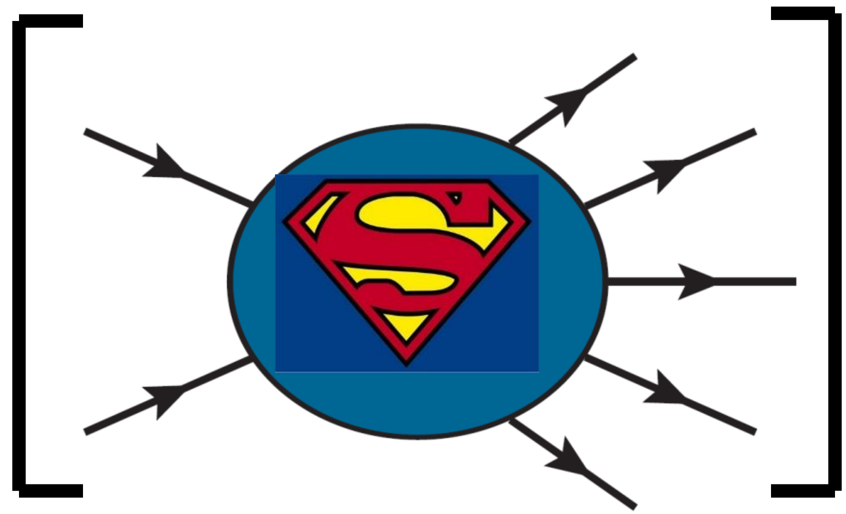
Example: Higgs gluon fusion cross section at LHC vs. CM energy $\sqrt{s}$

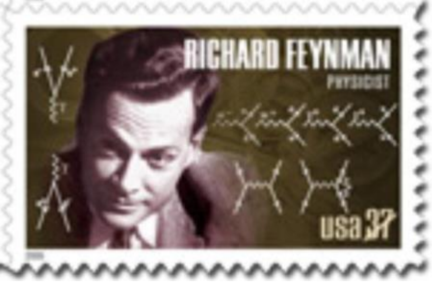
LO → NNNLO
→ factor of 2.7 increase!



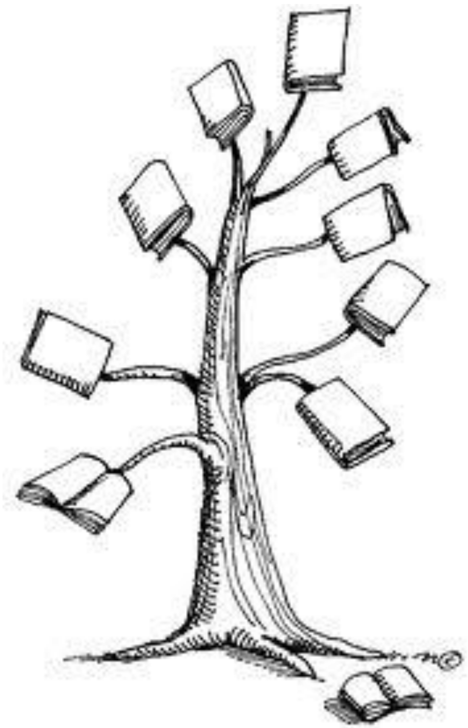
Short-distance cross sections built out of scattering amplitudes, ***S***-matrix elements

$$\sigma = \int$$





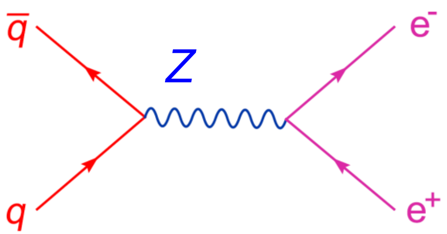
LO = Trees



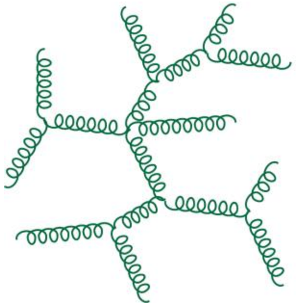
LO cross section uses only Feynman diagrams with no closed loops – tree diagrams.
Here's a very simple one:

$\hat{\sigma}^{(0)}$

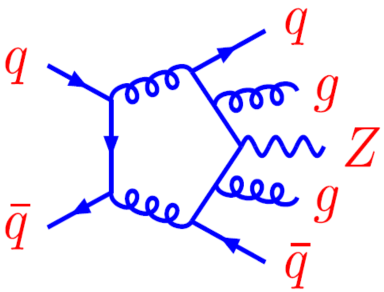
LO



Although there are many kinds of trees, some harder than others, "textbook" methods usually suffice



+ ...



NLO = Loops

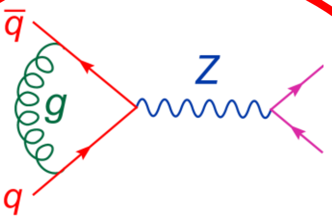
NLO cross section needs Feynman diagrams with exactly one closed loop

Where the fun really starts – textbook methods quickly fail, even with very powerful computers

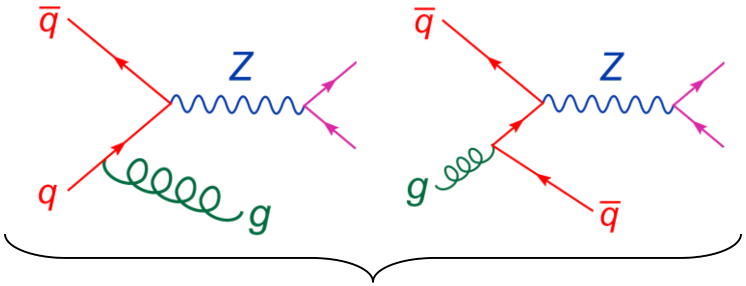
- NLO also needs tree-level amplitudes with one more parton
- Both terms infinite(!) – combine them to get a finite result

$\hat{\sigma}^{(1)}$

NLO



1 loop

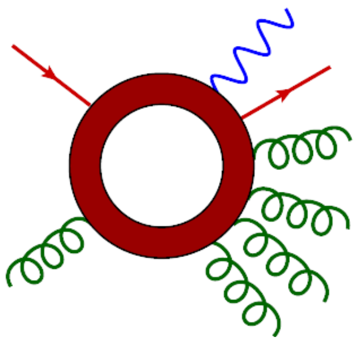


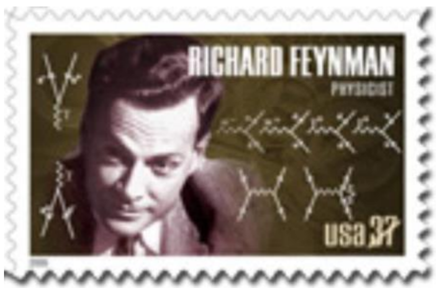
tree + 1 parton

- One-loop amplitudes were the bottleneck for a long time
– but now we know how to mass produce them!

Just one QCD loop can be a challenge

$q\bar{q} \rightarrow W + n$ gluons





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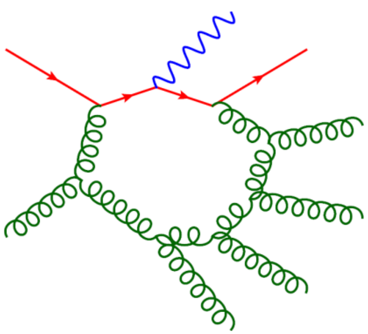
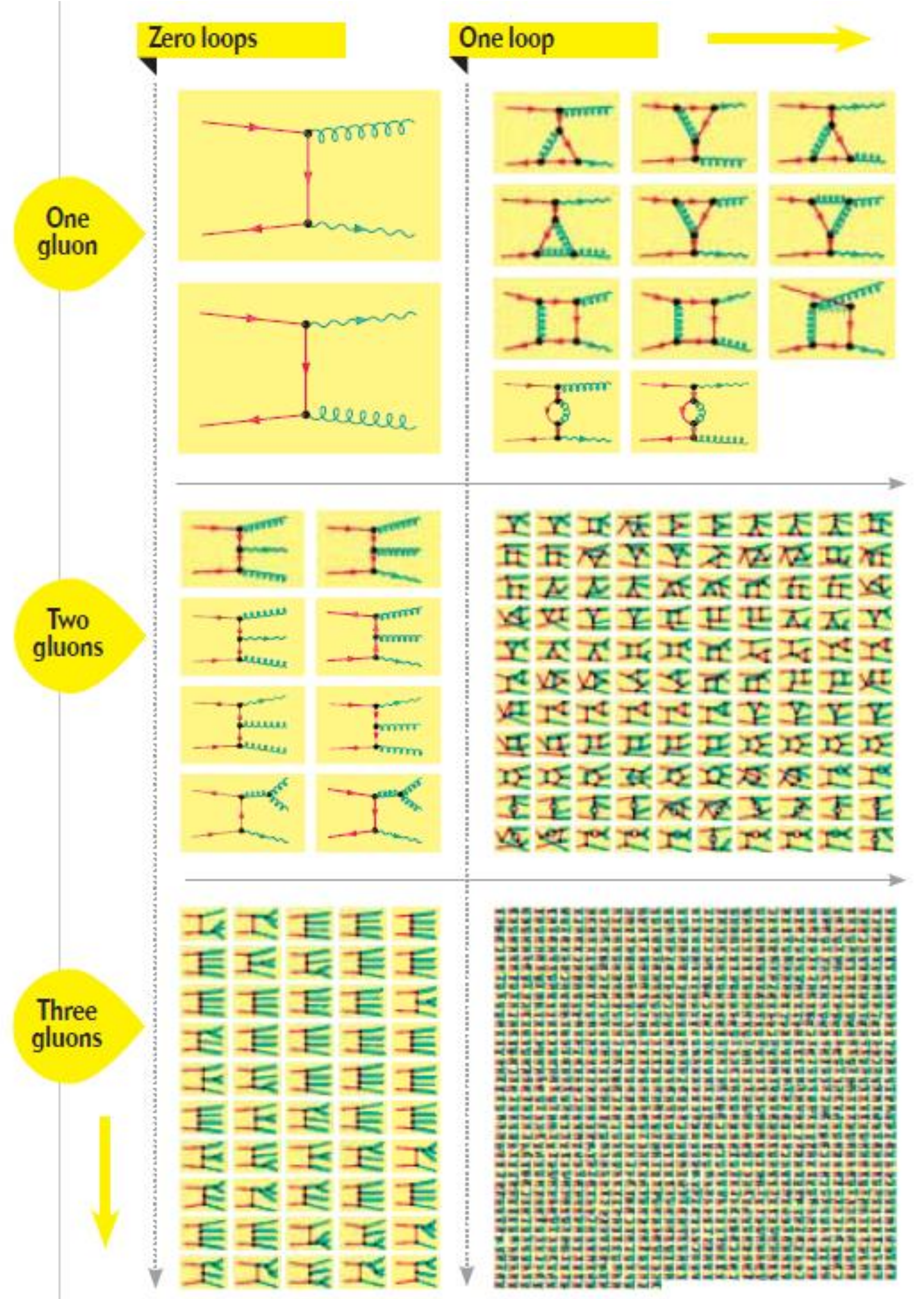
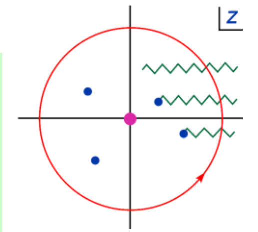


+ 256,264 more

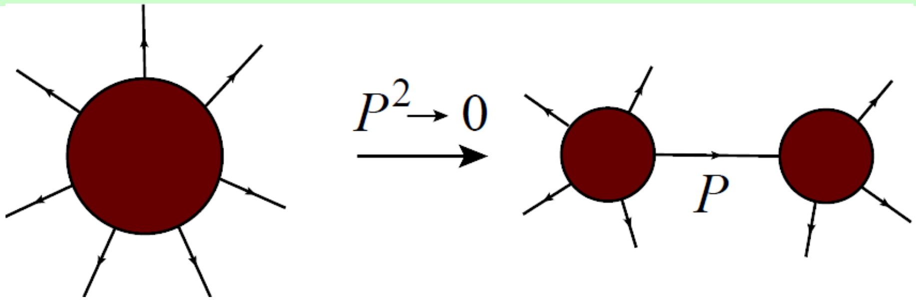


1960's Analytic *S*-Matrix

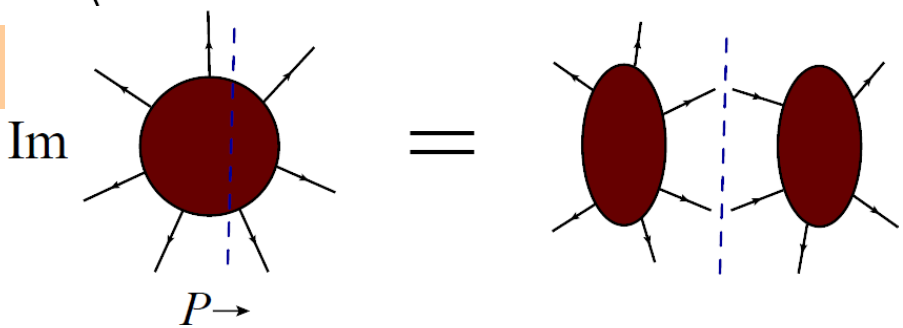


No QCD, no Lagrangian or Feynman rules for strong interactions.
Bootstrap program: Reconstruct scattering amplitudes **directly** from **analytic properties: "on-shell" information**

- Poles



- Branch cuts

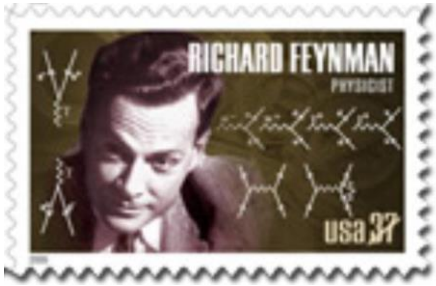


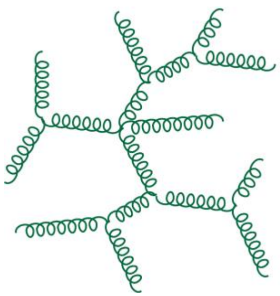
Landau; Cutkosky;
Chew, Mandelstam;
Frautschi;
Eden, Landshoff,
Olive, Polkinghorne;
Veneziano;
Virasoro, Shapiro;
… (1960s)

Analyticity fell out of favor in 1970s with the rise of QCD & Feynman rules

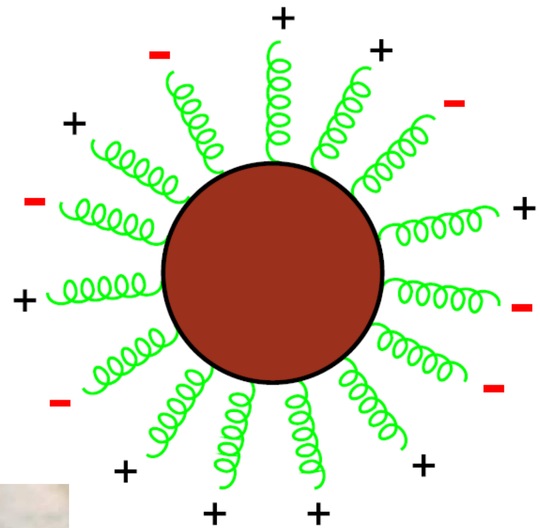
Resurrected for computing amplitudes in **perturbative** QCD
– as **alternative to Feynman diagrams!**
Perturbative information now assists analyticity.

Granularity vs. Fluidity



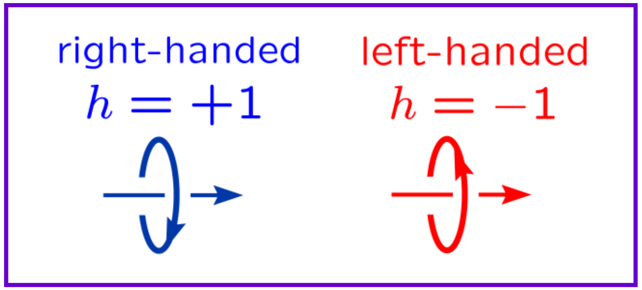


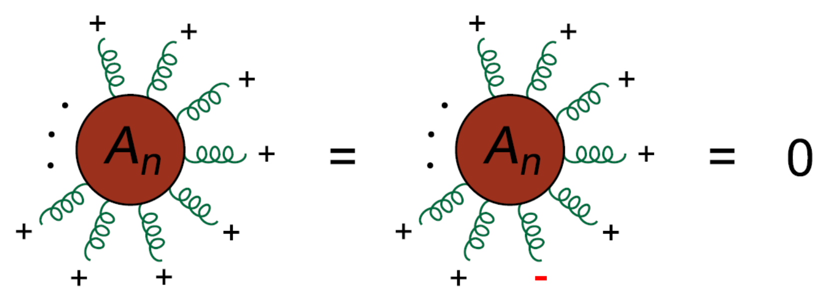
+ …



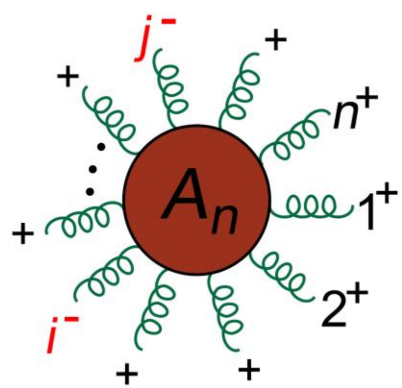
Helicity Formalism Exposes Tree-Level Simplicity in QCD

Many tree-level helicity amplitudes either vanish or are very short





Analyticity makes it possible to recycle this simplicity into loop amplitudes



$$= \frac{\langle i\, j\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\cdots\langle n\,1\rangle}$$

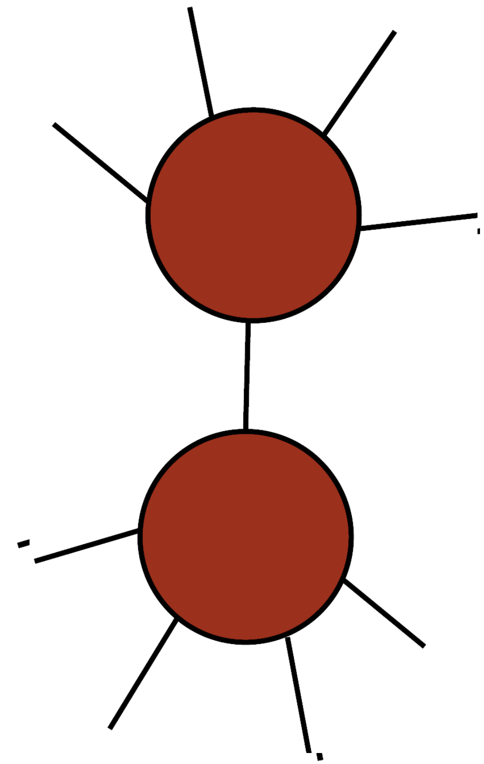
Parke-Taylor formula (1986)

Recycling “Plastic” Amplitudes

Amplitudes fall apart into simpler ones in special limits – pole information

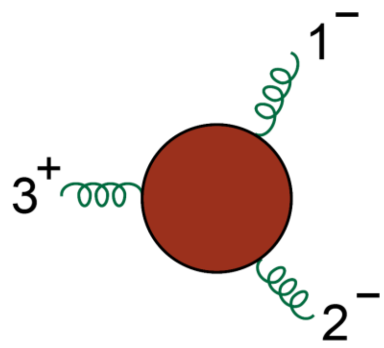
Picture leads directly to BCFW (on-shell) recursion relations: Reconstruct amplitude from poles in complex plane, where it factorizes into simpler amplitudes

Britto, Cachazo, Feng, Witten, hep-th/0501052



Trees recycled into trees

All Gluon Tree Amplitudes Built From:



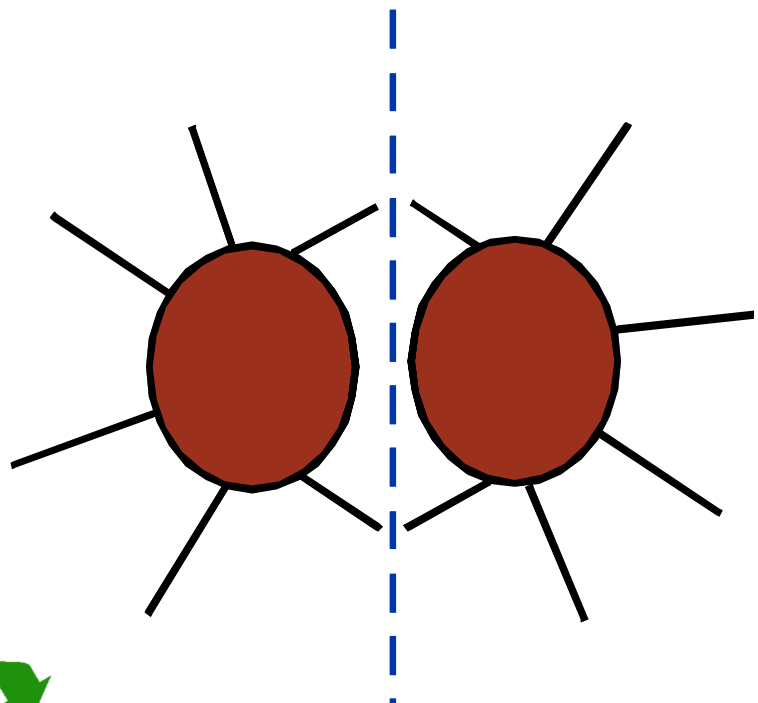
$$= \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle}$$

In contrast to Feynman vertices, it's on-shell, completely physical

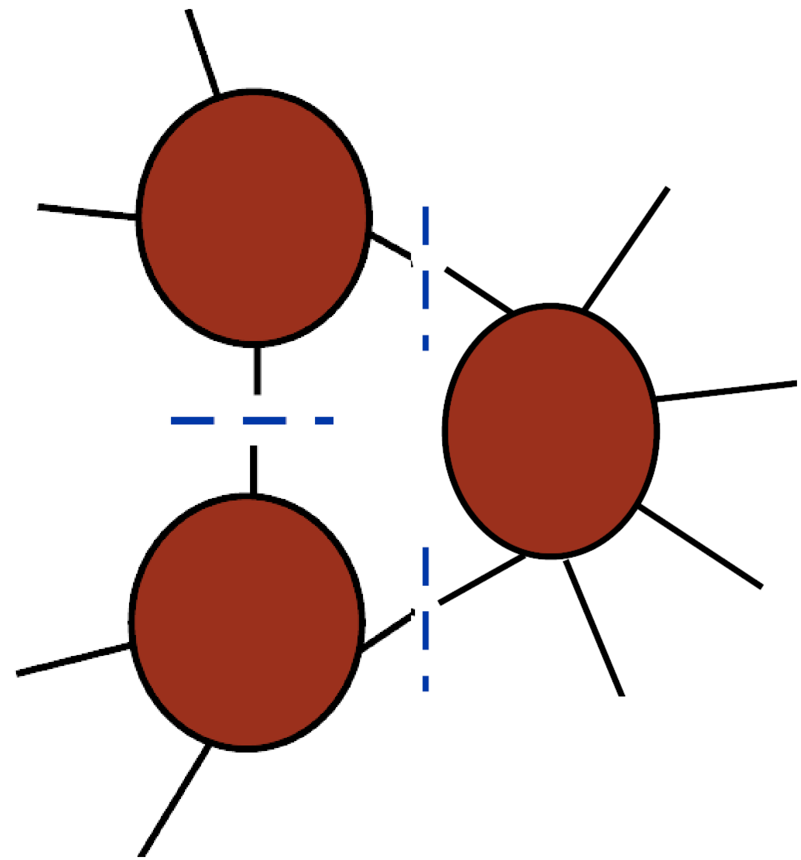
- On-shell recursion → very compact analytic formulae, fast numerical implementation.
- Can do same sort of thing at loop level.

Branch cut information → Generalized Unitarity (One-loop Plasticity)

Ordinary unitarity:
put 2 particles on shell



Generalized unitarity:
put 3 or 4 particles on shell



Trees recycled into loops!

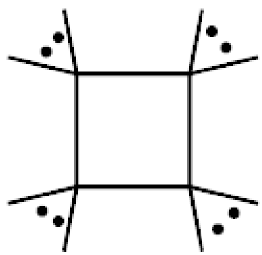
One-Loop Amplitude Decomposition

Bern, LD, Dunbar, Kosower (1994)

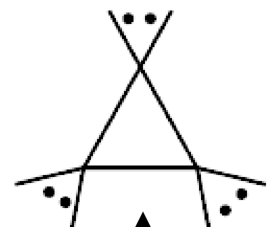
Missing from the old, nonperturbative analytic *S*-matrix

coefficients can be determined from products of trees using (generalized) unitarity

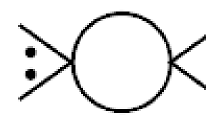
$$A^{1-\text{loop}} = \sum_i d_i$$



$$+ \sum_i c_i$$



$$+ \sum_i b_i$$



$$+ R + \mathcal{O}(\epsilon)$$

Known functions (integrals), same for all amplitudes

rational part; from D-dimensional trees, or recursively

Many Automated Programs for One-Loop QCD
→ Many New Processes at NLO

Blackhat: Berger, Bern, LD, Diana, Febres Cordero, Forde, Gleisberg, Höche, Ita, Kosower, Maître, Ozeren, 0803.4180, 0808.0941, 0907.1984, 1004.1659, 1009.2338…
+ **Sherpa** → NLO W,Z + 3,4,5 jets pure QCD 4 jets

CutTools: Ossola, Papadopolous, Pittau, 0711.3596
NLO WWW, WWZ, ... Binoth+OPP, 0804.0350
NLO $t\bar{t}b\bar{b}$, $t\bar{t}$ + 2 jets,…
Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, 0907.4723; 1002.4009
MadLoop: Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau 1103.0621
HELAC-NLO: Bevilacqua et al, 1110.1499

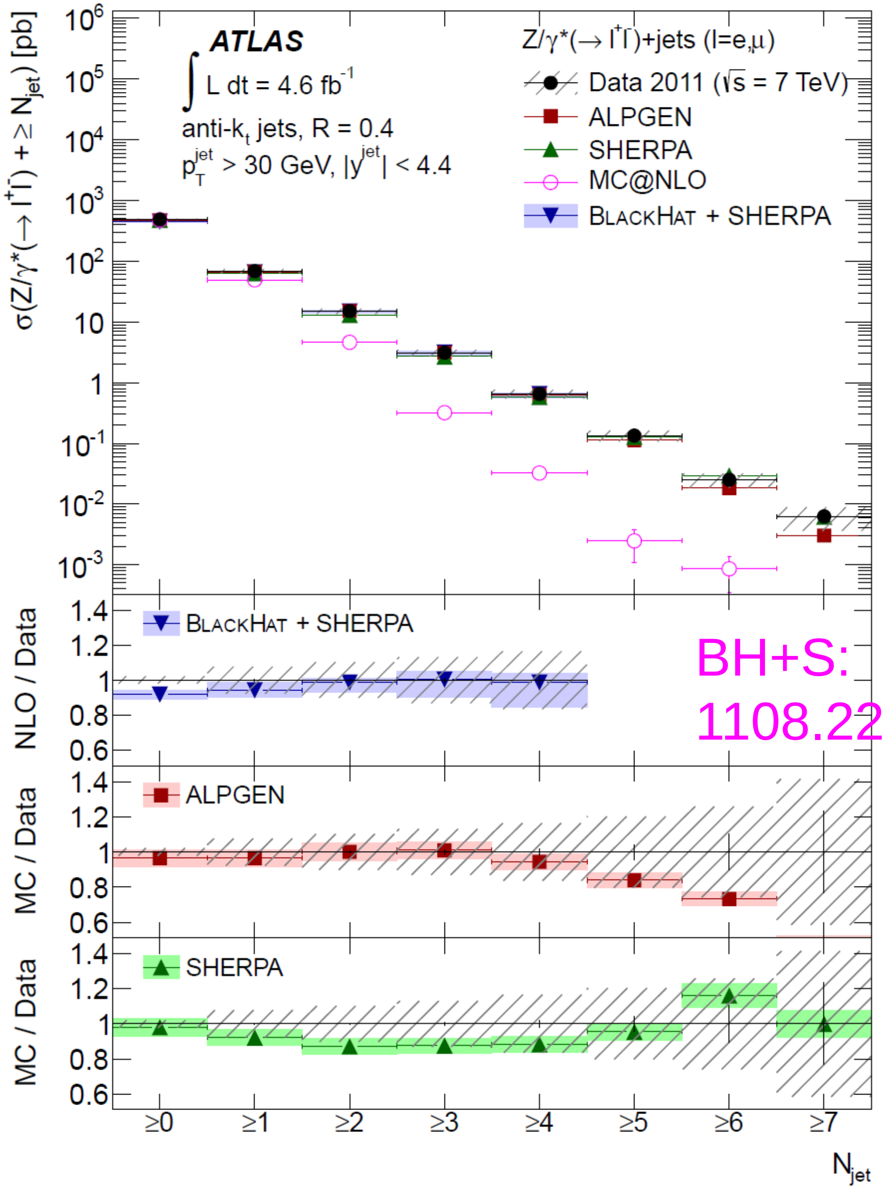
Rocket: Giele, Zanderighi, 0805.2152
Ellis, Giele, Kunszt, Melnikov, Zanderighi, 0810.2762
NLO W + 3 jets Ellis, Melnikov, Zanderighi, 0901.4101, 0906.1445
$W^+W^\pm$ + 2 jets Melia, Melnikov, Rontsch, Zanderighi, 1007.5313, 1104.2327

SAMURAI → GoSAM: Mastrolia, Ossola, Reiter, Tramontano, 1006.0710,…

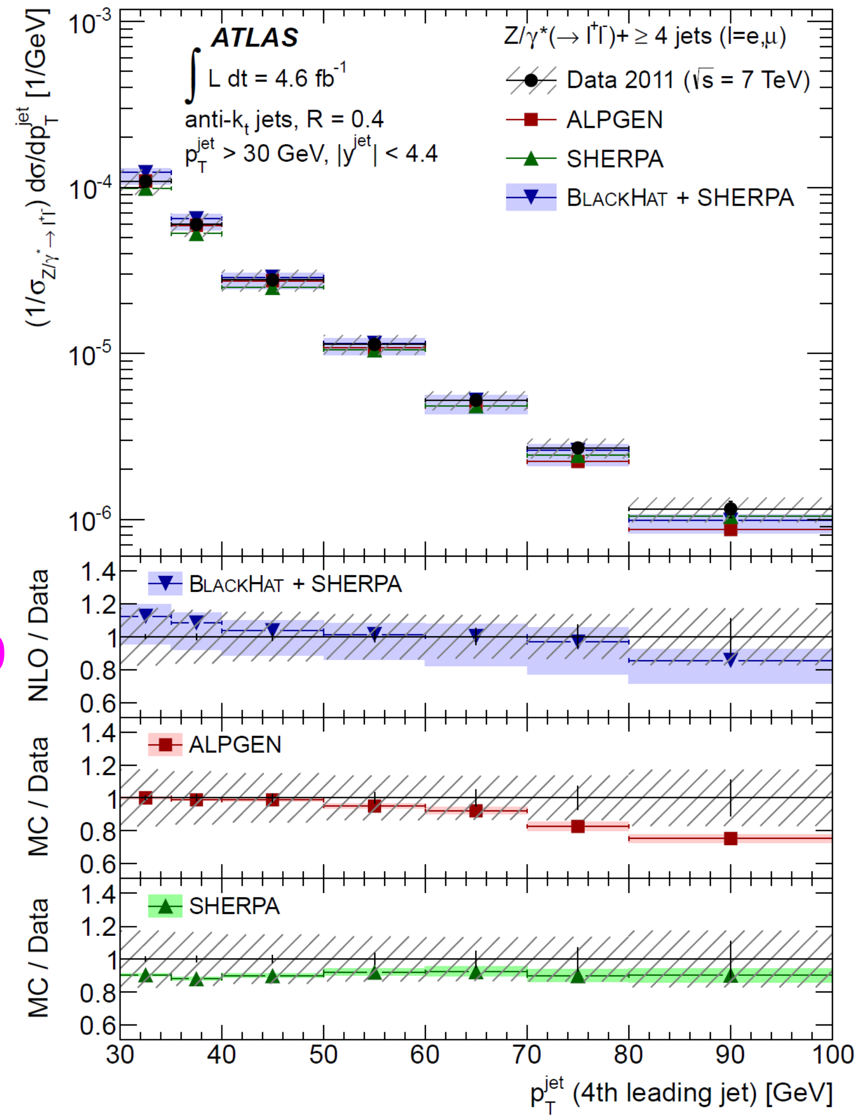
NGluon: Badger, Biedermann, Uwer, 1011.2900,…

NLO $pp \rightarrow Z + 1,2,3,4$ jets vs. ATLAS 2011 data

ATLAS 1304.7098

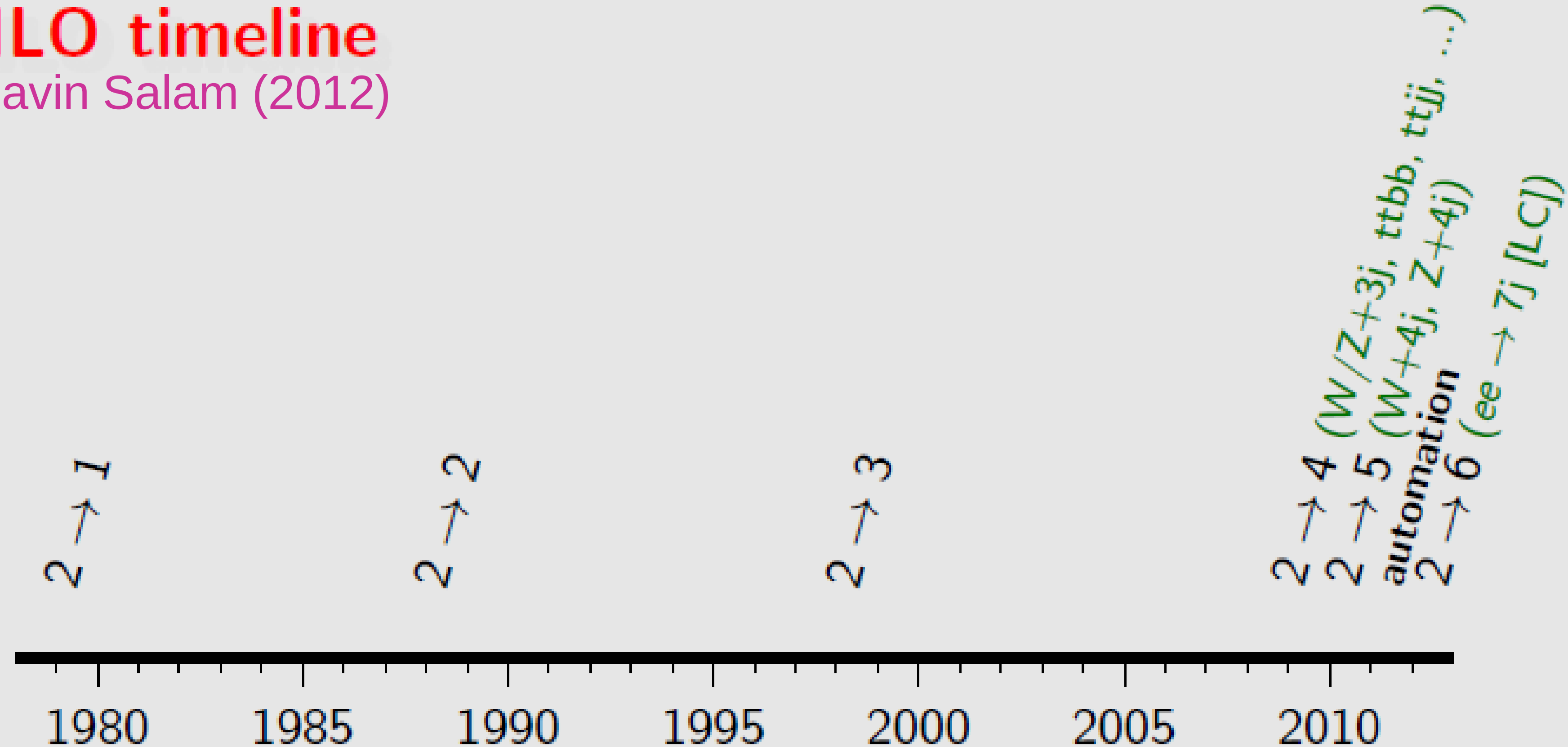


BH+S: 1108.2229



NLO timeline

Gavin Salam (2012)



2010: NLO $W+4j$ [BlackHat+Sherpa: Berger et al]	[unitarity]
2011: NLO $WWjj$ [Rocket: Melia et al]	[unitarity]
2011: NLO $Z+4j$ [BlackHat+Sherpa: Ita et al]	[unitarity]
2011: NLO $4j$ [BlackHat+Sherpa: Bern et al]	[unitarity]
2011: first automation [MadNLO: Hirschi et al]	[unitarity + feyn.diags]
2011: first automation [Helac NLO: Bevilacqua et al]	[unitarity]
2011: first automation [GoSam: Cullen et al]	[feyn.diags(+unitarity)]
2011: $e^+e^- \to 7j$ [Becker et al, leading colour]	[numerical loops]

On to two loops

- State-of-art currently stuck at 2 → 2

[First 2 -> 3 forays: Henn et al., 1511.05408, Badger et al., 1712.02229, Abreu et al., 1712.03946]

- Why? In part because 2 loop multiscale integrals are typically very hard
- All 1 loop integrals with external legs in D=4 are reducible to scalar box integrals + simpler

→ combinations of

$$\mathrm{Li}_2(x) = -\int_0^x \frac{dt}{t} \ln(1-t)$$

+ simpler

Brown-Feynman (1952), Melrose (1965), Passarino-Veltman (1979), van Neerven-Vermaseren (1984), Bern, LD, Kosower (1992)

Two loop integrals

- Become “elliptic” very quickly if there are internal particle masses, e.g. the London transport integral

  → sunset integral (sunrise integral?)

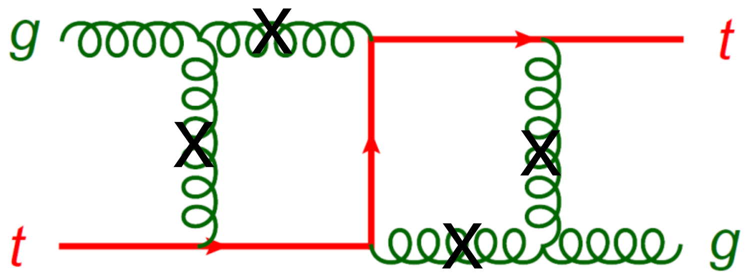
P →

=

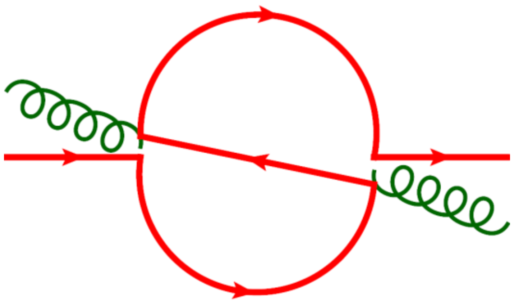
Broadhurst-Fleischer-Tarasov, 9304303, Berends-Böhm-Buza-Scharf (1994), Laporta-Remiddi, 0406160,
Adams-Bogner-Weinzierl, 1302.7004, Bloch-Vanhove, 1309.5865,…

Sunset inside top production

- At subleading color in $gg \rightarrow t\,\bar{t}$,



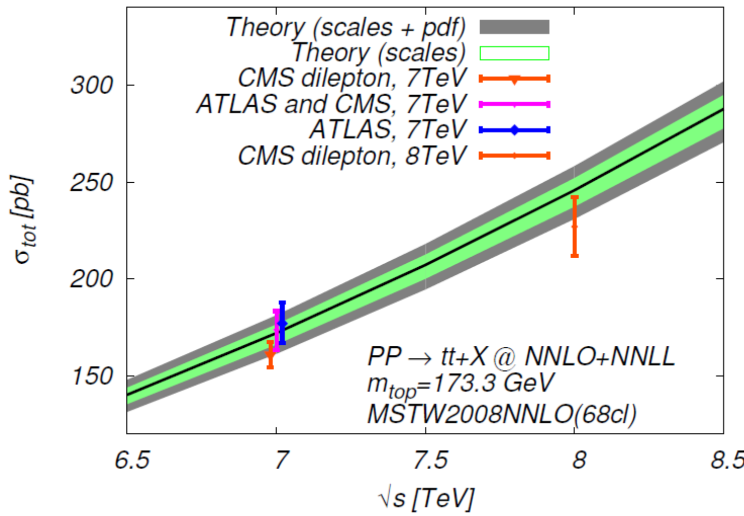
$\rightarrow$



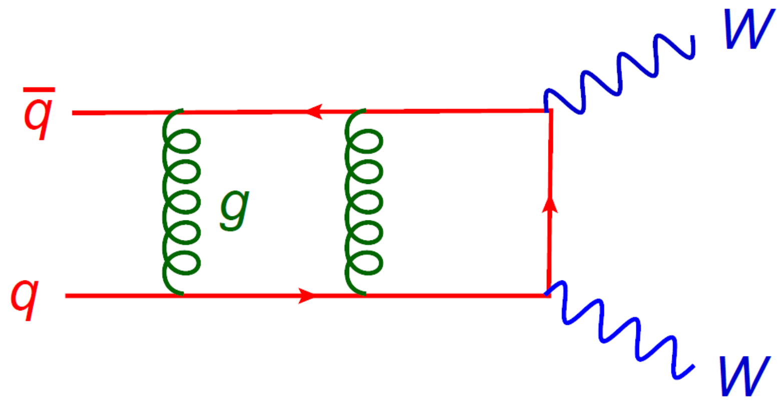
- Already done numerically

Czakon, Fiedler, Mitov, 1303.6254

- Better analytic understanding will aid in multiscale generalizations



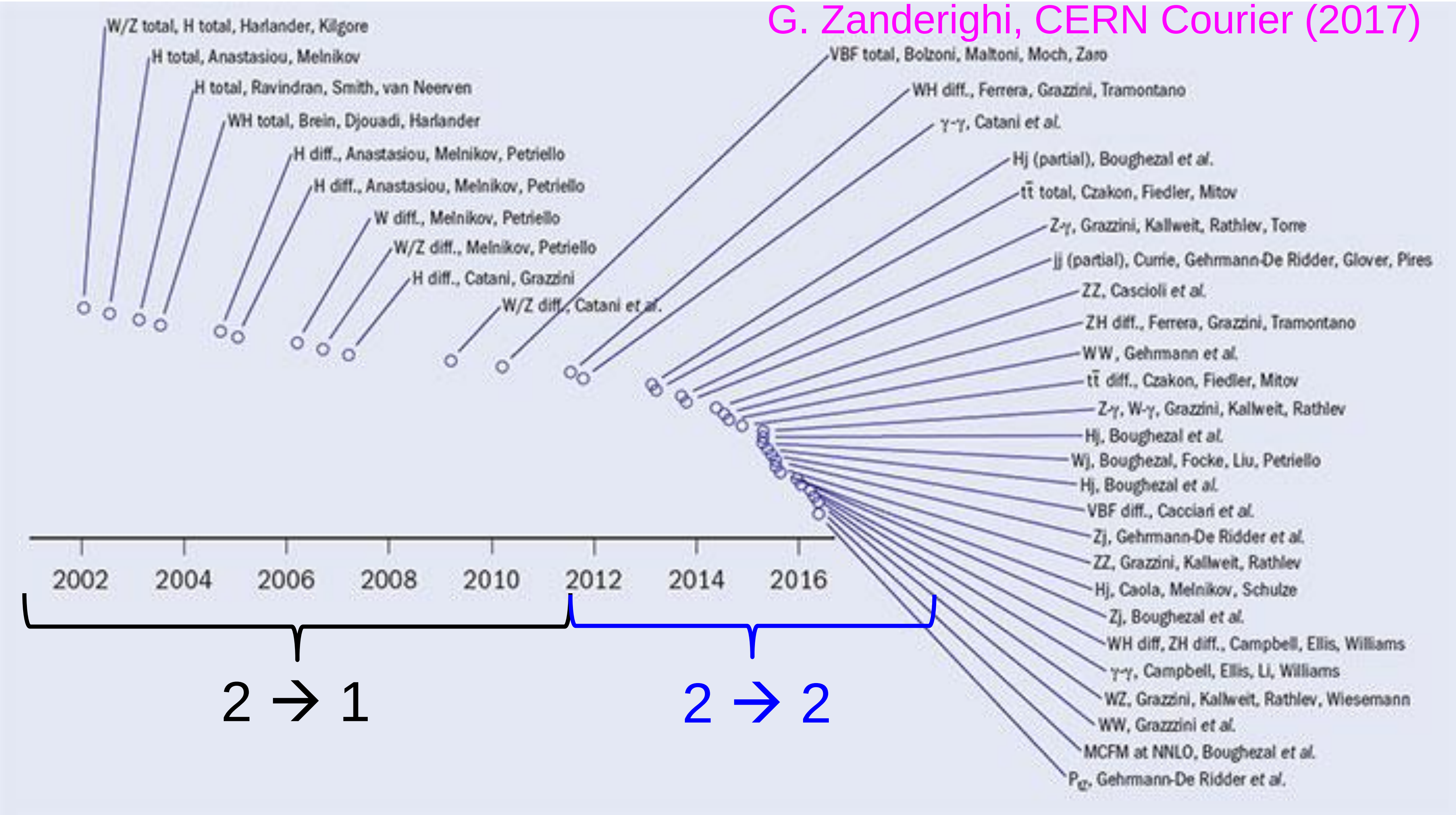
Massless (internal) 2 → 2



- Here, the 2 loop integrals typically belong to a simpler class of multiple polylogarithms, e.g. "G functions" Goncharov, 1105.2076
- Together with advances in handling real radiation, and stable one-loop 2 → 3 amplitudes, made possible a large class of 2 → 2 processes at NNLO

NNLO timeline

G. Zanderighi, CERN Courier (2017)



NNLO beyond 2 → 2

- Will require combining multi-loop generalized unitarity (lots of progress already but no time to review) with sophisticated methods for multi-scale multi-loop integration.

- But we know it is possible:

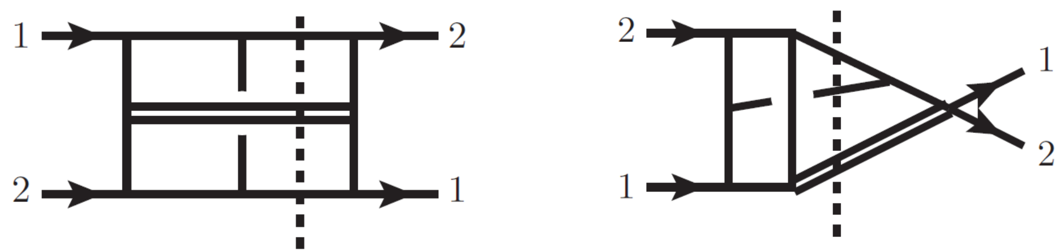
Trees can be recycled into multi loops!

$gg \rightarrow H$ at NNNLO

Anastasiou, Duhr, Dulat, Furlan, Herzog, Mistlberger, 1302.4379, 1311.1425, 1403.4616, 1411.3584, 1503.06056, 1505.04110

- First step, integrate out top quark loop, removes 1 scale and 1 loop:

- Last step, do some very complicated phase space integrals as loop integrals, using "reverse unitarity":



- Avoid the nastiest special functions by performing a very high order threshold expansion, removing 1 more scale.

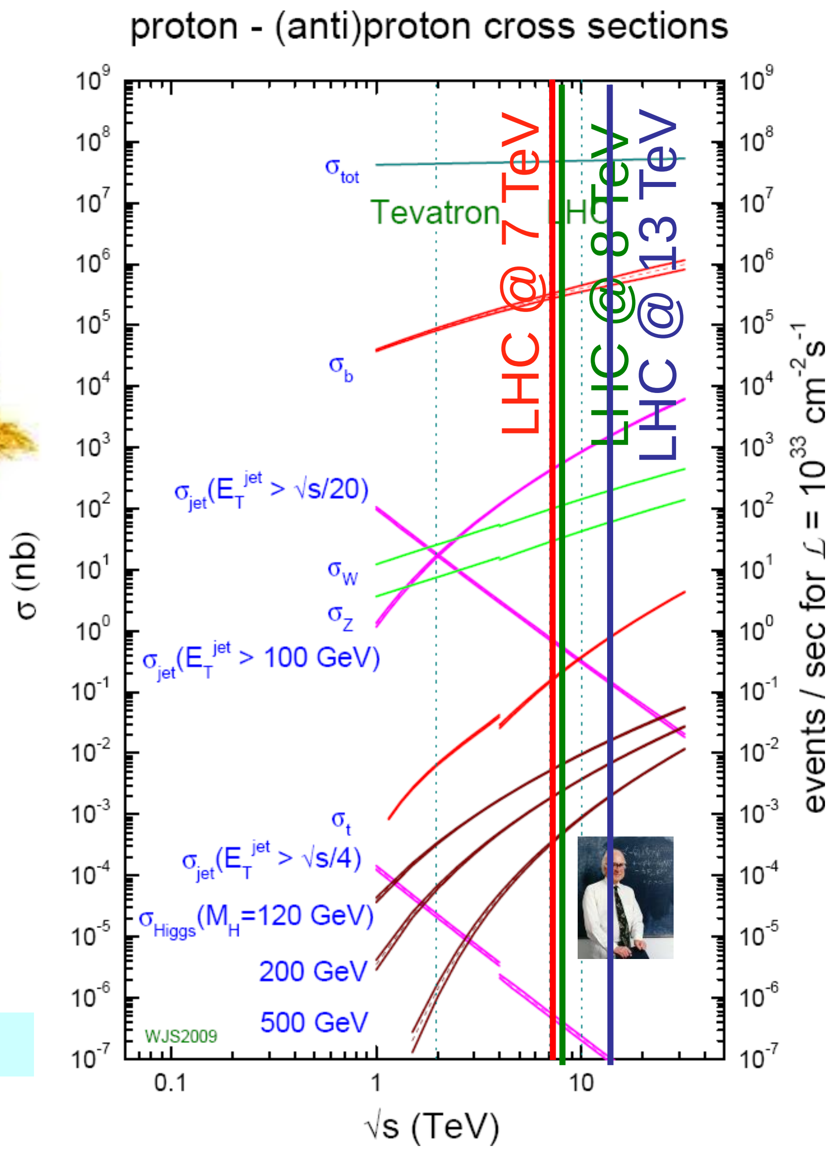
The "QCD for LHC" revolution

- Many important hadron collider processes have been computed at NLO and NNLO in the past decade (even one at NNNLO), well beyond what was previously thought possible
- Required a new understanding of scattering amplitudes, at a formal level, as well as efficient, stable implementation
- Many people contributed to this progress
- Parallel progress in understanding supersymmetric gauge & gravity theories
- Revolution is far from over; NNLO 2 → 3 awaits!

Extra Slides

LHC Data Dominated by Jets

new physics?? →



Jets from quarks and gluons.

- *q,g* from decay of new particles?
- Or from old QCD?

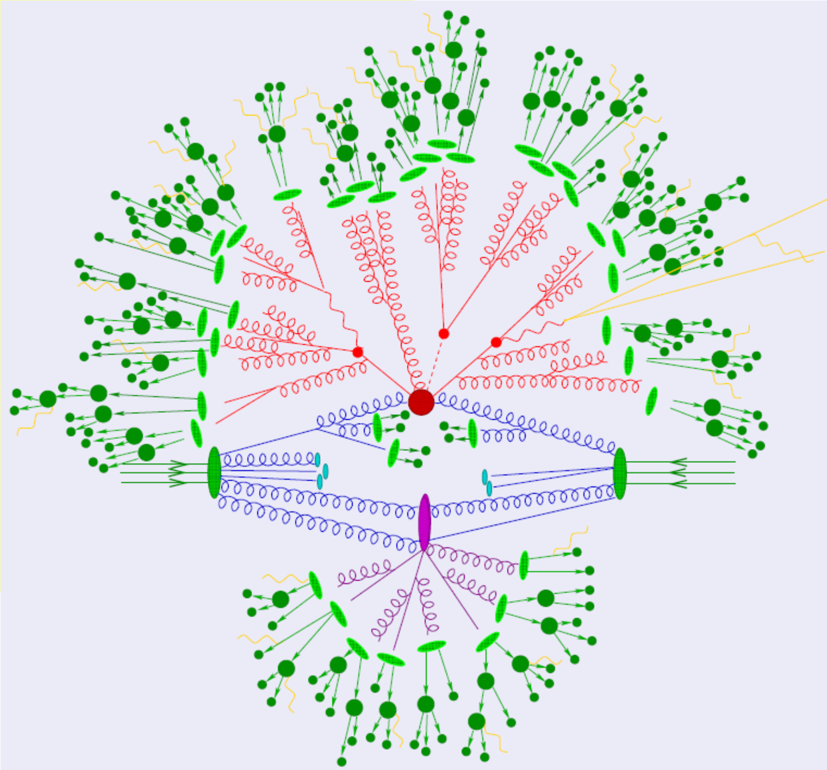
- Every process shown also with one more jet at ~ 1/5 the rate
- Need **accurate production rates** for

X *+ 1,2,3,… jets*

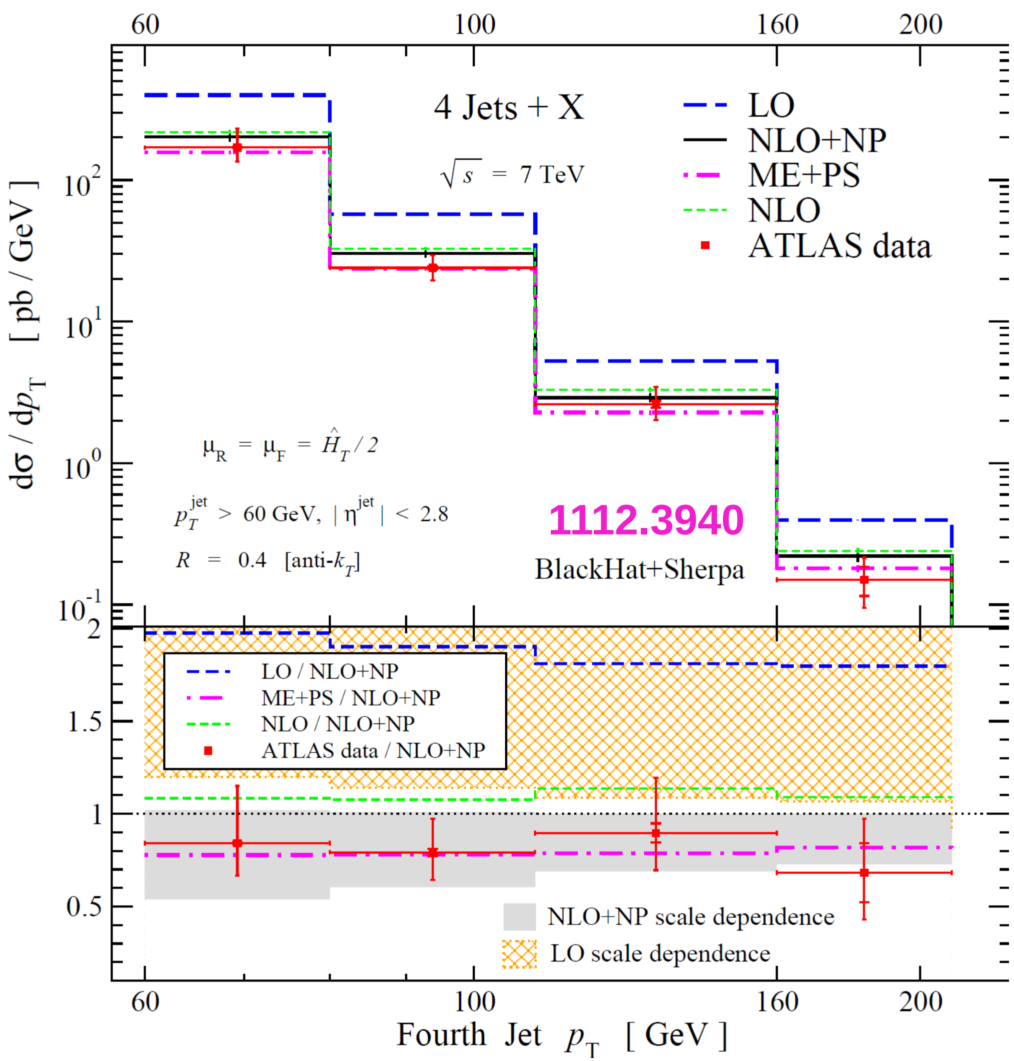
in Standard Model

Fixed order vs. Monte Carlo

- NLO fixed-order → few partons: no model of long-distance effects included;
  cannot pass through a detector simulation
- Methods available for matching NLO parton-level results to parton showers, accuracy:
  - **MC@NLO** Frixione, Webber (2002) ; …; SHERPA implementation
  - **POWHEG** Nason (2004); Frixione, Nason, Oleari (2007)
  - **SHERPA** Krauss et al. (2012,…)
- Recently implemented for increasingly complex final states



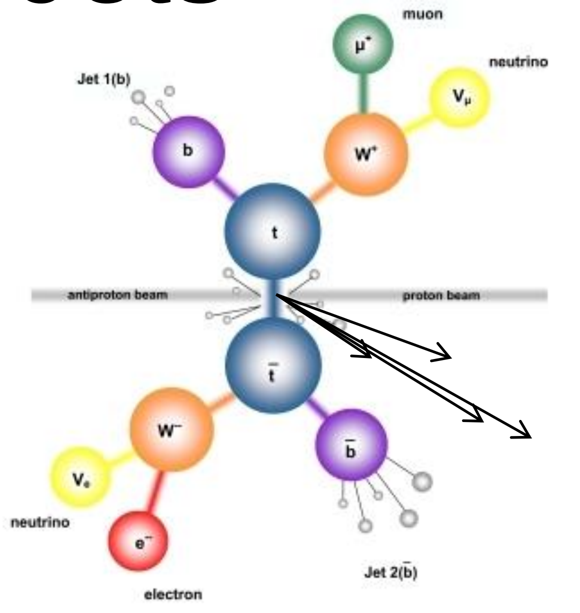
Pure QCD: *pp* → 4 jets vs. ATLAS data



4 jet events might hide pair production of 2 colored particles, each decaying to a pair of jets

Detailed study of multi-jet QCD dynamics may help understand other channels

Top Quark Pairs + Jets



- Like (W,Z) + jets, very important bkgd
- Cross sections large

– no electroweak couplings

- Jets boost $t\,\bar{t}$ system, increase MET,

provide jets to fake $t\,\bar{t}\,H,\ \ H \rightarrow b\bar{b}$.

- State of art circa 2010:
- **NLO *tt* + 1 jet:** Dittmaier, Uwer, Weinzierl, hep-ph/0703120,…
- **+ top decays:** Melnikov, Schulze, 1004.3284
- **+ NLO parton shower:** Kardos, Papadopoulos, Trócsányi, 1101.2672

- **NLO *tt* + *bb*:** Bredenstein, Denner, Dittmaier, Pozzorini, 0905.0110, 1001.4006; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, 0907.4723
- **NLO *tt* + 2 jets:** Bevilacqua, Czakon, Papadopoulos, Worek, 1002.4009

Processes currently known through NNLO

Kirill Melnikov (2016)

dijets	O(3%)	gluon-gluon, gluon-quark	PDFs, strong couplings, BSM
H+0 jet	O(3-5 %)	fully inclusive (N3LO)	Higgs couplings
H+1 jet	O(7%)	fully exclusive; Higgs decays, infinite mass tops	Higgs couplings, Higgs p_t, structure for the ggH vertex.
tT pair	O(4%)	fully exclusive, stable tops	top cross section, mass, p_t, FB asymmetry, PDFs, BSM
single top	O(1%)	fully exclusive, stable tops, t-channel	V_{tb}, width, PDFs
WBF	O(1%)	exclusive, VBF cuts	Higgs couplings
W+j	O(1%)	fully exclusive, decays	PDFs
Z+j	O(1-3%)	decays, off-shell effects	PDFs
ZH	O(3-5 %)	decays to bb at NLO	Higgs couplings (H-> bb)
ZZ	O(4%)	fully exclusive	Trilinear gauge couplings, BSM
WW	O(3%)	fully exclusive	Trilinear gauge couplings, BSM
top decay	O(1-2 %)	exclusive	Top couplings
H -> bb	O(1-2 %)	exclusive, massless	Higgs couplings, boosted

N3LO!