

# **Next great challenge beyond SLAC E144: probing a fully non-perturbative QED with electron-laser interaction**

Probing strong-field QED in electron-photon interactions

21-24 August 2018 / DESY, Hamburg, Germany

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# Acknowledgements

1. **Thanks to the organizers for invitation!**
2. Content of the talk was extensively discussed with/contributed by:
  - **N.B. Narozhny**, **A.A. Mironov**  
(National Research Nuclear University MEPhI, Moscow, Russia)
  - **S. Meuren**  
(Department of Astrophysical Sciences, Princeton University, USA)
  - **V. Yakimenko, F. Fiuza, M.J. Hogan, G. White**  
(SLAC National Accelerator Laboratory, Menlo Park, CA USA)
  - **F. Del Gaudio, T. Grismayer, L.O. Silva**  
(GoLP/Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico, Universidade de Lisboa, Portugal)
  - **A.M. Pukhov, C. Baumann**  
(Heinrich-Heine-Universität University, Düsseldorf, Germany)
  - **I.Yu. Kostyukov, E.N. Nerush**  
(Institute of Applied Physics RAS, Nijniy Novgorod, Russia)
3. Support from **RFBR** and the **Foundation for the Advancement of Theoretical Physics and Mathematics “BASIS”**
4. Details: **V. Yakimenko et al., arXiv:1807.09271**

# Introduction

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# Intense Field QED (IFQED)

- In QED electron motion in **external EM field** is described by


$$\Rightarrow = \rightarrow + \rightarrow \cdot \rightarrow + \rightarrow \cdot \rightarrow \cdot \rightarrow + \dots$$

- Expansion parameter:  $a_0 \simeq \frac{e}{m} \mathcal{A}$  can be  $\gtrsim 1 \implies$  **IFQED!**
- IFQED effects reveal for  $a_0 \gg 1$ , for laser intensity  $I \gtrsim 10^{18} \text{W/cm}^2$
- Dressed propagator**

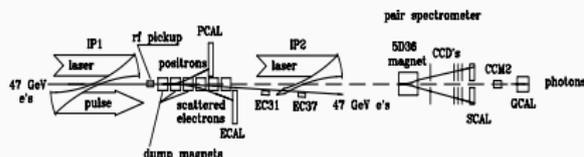

$$\Rightarrow = \rightarrow + \rightarrow \cdot \rightarrow \Rightarrow$$

can be solved *exactly* for certain field configurations: *plane wave, constant field (including CCF), Coulomb field, etc.*

- Physical quantities (probability rates, cross sections) are computed as in ordinary QED, but replacing *bare* propagators with **dressed** ones
- Such **IFQED dressing takes into account exactly only interaction with an external field, but not radiative corrections** (see below)...

# The landmark SLAC E144 experiment (mid 1990's)

<https://www.slac.stanford.edu/exp/e144/e144.html>

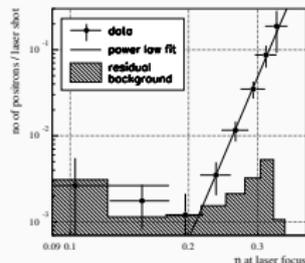


Main goals:

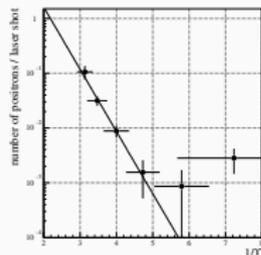
- First experimental study of IFQED (by **optical high-power laser**  $I \sim 10^{18} \text{W/cm}^2$  + linac)

- Direct observation of:

- **nonlinear Compton scattering**  $e^- + n\gamma_L \rightarrow e^- + \gamma'$



- **multiphoton Breit-Wheeler process**  $\gamma' + n\gamma_L \rightarrow e^+e^-$



# Parameters of laser-matter interaction at extreme intensities

Classical non-linearity parameter:

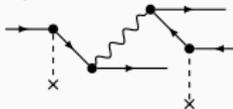
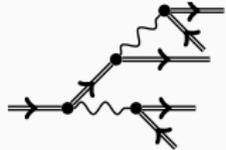
$$a_0 = \frac{e\sqrt{-\langle A^\mu A_\mu \rangle}}{mc} \sim \frac{eE_0}{mc\omega}$$

(non-)perturbativity w.r.t. ext. field

Dynamical quantum parameter:

$$\chi = \frac{e\hbar\sqrt{-(F^{\mu\nu}p_\nu)^2}}{m^3c^4} \sim \gamma \frac{E_\perp}{E_S}$$

importance of quantum recoil

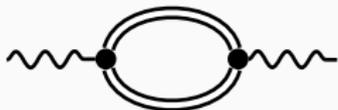
Regime	$a_0 \ll 1$	$a_0 \gtrsim 1$
$\chi \ll 1$	classical non-relativistic 	classical relativistic 
$\chi \gtrsim 1$	fully perturbative QED 	IFQED (SLAC E144; cascades) 
<b>Next challenge:</b> $\chi \gtrsim 1600$	—	Radiative corrections (loops) become also important: <b>perturbation theory breakdown!</b>

**Radiative corrections in CCF  
(universal model for a single  
particle in subcritical field**

**$F \ll m^2/e$  if  $a_0 \gg 1$  and  $\chi \gtrsim 1$ )**

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# One-loop radiative corrections in CCF



Polarization operator

$$\mathcal{P}_{\mu\nu}(x, x') = e^2 \text{Tr} [\gamma_\mu S^e(x, x') \gamma_\nu S^e(x', x)]$$

Physical effect: **photon mass**:

$$m_{\gamma \parallel, \perp}^2(\chi) = \frac{\alpha m^2}{6\sqrt{\pi}} \int_1^\infty du \frac{8u + 1 \mp 3}{zu\sqrt{u(u-1)}} f'(z)$$
$$\simeq \frac{5 \mp 1}{28\pi^2} 3^{7/6} \Gamma^4\left(\frac{2}{3}\right) (1 - i\sqrt{3}) \alpha m^2 \chi^{2/3}, \quad \chi \gg 1,$$
$$z = \left(\frac{4u}{\chi}\right)^{2/3}, \quad f(z) = \frac{i}{\sqrt{\pi}} \int_0^\infty dt e^{-i(z t + t^3/3)}.$$

N.B. Narozhnyi, JETP **28**, 371 (1969); V.I. Ritus, JETP **30**, 1181 (1970).

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Similarly, **mass operator** modifies electron mass  $m$  by  $\sim \alpha m \chi^{2/3}$

V.I. Ritus, JETP **30**, 1181 (1970); Ann. Phys. **69**, 555 (1972).

# Implication: perturbative theory breakdown at $\alpha\chi^{2/3} \gtrsim 1$

- If  $\alpha\chi^{2/3} \simeq 1$  then **radiative corrections cease being small**:

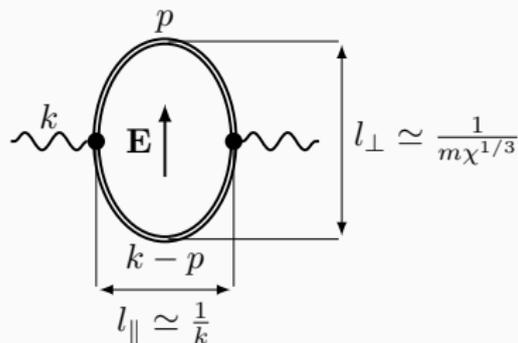
$$m_\gamma^2 \simeq \alpha m^2 \chi^{2/3} \simeq m^2, \quad \Delta m_e \simeq \alpha m \chi^{2/3} \simeq m$$

Moreover, the free path in a rest frame  $(\alpha m \chi^{2/3})^{-1}$  is sub-Compton!

Indicates **IFQED perturbation theory breakdown** for  $\alpha\chi^{2/3} \gtrsim 1$   
(explicitly: for  $\chi \gtrsim 1600$ )!

- First realized by N. Narozhny and V. Ritus  $\sim 50$  years ago. Hence the name **Ritus-Narozhny conjecture** (themselves they called it **enhancement of radiative corrections in external field**).
- Recall that in ordinary QED the radiative corrections use to scale like  $\propto \alpha \log \gamma$  and can never compare to bare quantities at any reasonable energy scale (the PT breakdown corresponds to a Landau pole).  
**Physical reasons for enhancement in CCF?!**

## Intuitive insight: loop scale



- In a transverse field  $\mathbf{p}_{\perp} \sim e\mathbf{E}t$  and:

$$\varepsilon = \sqrt{p_{\parallel}^2 + e^2 E^2 t^2 + m^2} \approx p_{\parallel} + \frac{e^2 E^2 t^2}{2p_{\parallel}}$$

- The uncertainty principle ( $p_{\parallel} \sim k$ ):

$$\Delta\varepsilon \cdot t \simeq \frac{e^2 E^2 t^3}{k} \simeq 1$$

$$\Rightarrow t_{loop} \simeq \left( \frac{k}{e^2 E^2} \right)^{1/3} \simeq \frac{k}{m^2 \chi^{2/3}}$$

- Corresponding transverse momentum gain and transverse size of the loop (for further details see arXiv:1807.09271):

$$p_{\perp} \simeq eEt_{loop} \simeq m\chi^{1/3}, \quad \boxed{l_{\perp} \simeq \frac{1}{p_{\perp}} \simeq \frac{1}{m\chi^{1/3}}} \quad \text{and} \quad \boxed{l_{\parallel} \simeq \frac{1}{p_{\parallel}} \sim \frac{1}{k}}$$

(indeed  $m \ll p_{\perp} \ll k$ , justifying the initial expansion).

## Intuitive insight: loop value

- In a strong transverse field the loop becomes squeezed to sub-Compton scale ( $l_{\parallel}, l_{\perp} \ll 1/m$ ) – an example of a general duality:

**STRONG FIELD  $\leftrightarrow$  SMALL DISTANCE**

- Having estimated the loop dimensions, we can estimate the loop volume:

$$V_{loop} \simeq \pi l_{\perp}^2 l_{\parallel} \simeq \frac{1}{m\chi^{1/3}} \times \frac{1}{m\chi^{1/3}} \times \frac{1}{k} \simeq \frac{1}{km^2\chi^{2/3}}$$

and the photon mass as the **plasma frequency of a 'relativistic plasma of virtual pairs'**:

$$m_{\gamma}^2 \simeq \omega_p^2 \equiv \frac{8\pi e^2}{m\gamma} n_{e^+e^-} \simeq \frac{\alpha}{k} \frac{1}{V_{loop}} \simeq \alpha m^2 \chi^{2/3}$$

# Direct calculation of higher-order corrections

- Some higher (up to 3-loop) order diagrams were also computed:

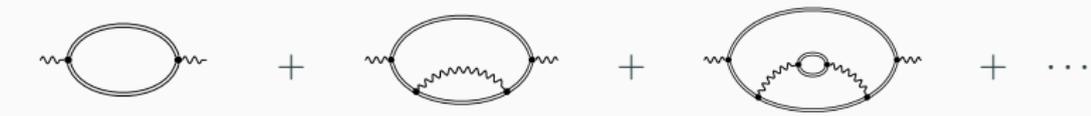


$\sim \alpha \chi^{2/3}$   
(Ritus, 1970)

$\sim \alpha^2 \chi \log \chi$   
(Ritus, 1972)

$\sim \alpha^3 \chi^{5/3}$   
(Narozhny, 1980)

+ ...



$\sim \alpha \chi^{2/3}$   
(Narozhny, 1968)

$\sim \alpha^2 \chi^{2/3} \log \chi$   
(Morozov, 1977)

$\sim \alpha^3 \chi \log^2 \chi$   
(Narozhny, 1980)

+ ...

[only dominant contributions shown, for review see A.M.F., J. Phys.: Conf. Ser. 826, 012027 (2017)]

- This is in agreement with the **Ritus-Narozhny conjecture** that the expansion parameter is  $\alpha \chi^{2/3}$ .

## **Towards experiments**

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# How to probe experimentally? Option 1

- Option 1: laser-beam collisions:



- Assuming  $\omega_L = 1\text{eV}$  ( $\lambda = 1.24\mu\text{m}$ ) and (sub-)period laser pulse:

$\gamma$ ( $\epsilon_e$ [GeV])	$10^7$ (5000)	$10^6$ (500)	$10^5$ (50)	$10^4$ (5)	$10^3$ (0.5)
$a_0$	50	500	5000	$5 \times 10^4$	$5 \times 10^5$
$I_L$ [W/cm <sup>2</sup> ]	$2.5 \times 10^{21}$	$2.5 \times 10^{23}$	$2.5 \times 10^{25}$	$2.5 \times 10^{27}$	$2.5 \times 10^{29}$
$\chi \sim 2 \frac{\omega}{m} a_0 \gamma$	2000	2000	2000	2000	2000
$\alpha \chi^{2/3}$	1.16	1.16	1.16	1.16	1.16
Radiation loss $\delta \sim \frac{\alpha m}{\gamma} \chi^{2/3} \cdot \frac{\pi}{\omega}$	0.182	1.82	18.2	182	1820

- Some options look prospective, but would require **multi-TeV** class electron accelerator...

## How to probe experimentally? Option 2

- Option 2: same, but using pulse compression or XFEL

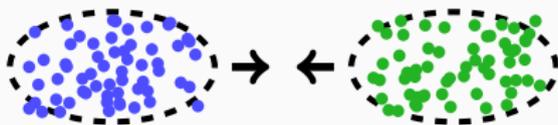


- Assuming  $\omega_L = 10\text{keV}$  ( $\lambda = 0.12\text{nm}$ ) and (sub-)period laser pulse:

$\gamma$ ( $\epsilon_e$ [GeV])	$10^7$ (5000)	$10^6$ (500)	$10^5$ (50)	$10^4$ (5)	$10^3$ (0.5)
$a_0$	$5 \times 10^{-3}$	$5 \times 10^{-2}$	0.5	5	50
$I_L$ [W/cm <sup>2</sup> ]	$2.5 \times 10^{21}$	$2.5 \times 10^{23}$	$2.5 \times 10^{25}$	$2.5 \times 10^{27}$	$2.5 \times 10^{29}$
$\chi \sim 2 \frac{\omega}{m} a_0 \gamma$	2000	2000	2000	2000	2000
$\alpha \chi^{2/3}$	1.16	1.16	1.16	1.16	1.16
Radiation loss $\delta \sim \frac{\alpha m}{\gamma} \chi^{2/3} \cdot \frac{\pi}{\omega}$	$1.82 \times 10^{-5}$	$1.82 \times 10^{-4}$	$1.82 \times 10^{-3}$	$1.82 \times 10^{-2}$	0.182

- All options are prospective and requirements for electron accelerator maybe relaxed, but at cost of further progress in X-ray intensity...Sub-period X-rays also a challenge...

## Option 3: beam-beam collisions (arXiv:1807.09271)



- Beams: 100GeV,  $I = 10^6$  A,  $\sigma_z = 100$ nm ( $N_e \sim 10^9$ ),  $\sigma_r = 10$ nm
- Probe beam sees beam density

$$\tilde{n}_e \sim \frac{N_e}{\pi\sigma_r^2(\sigma_z/\gamma)} \sim 10^{31} \text{cm}^{-3} \simeq \lambda_C^{-3}$$

- Beam disruption parameter:

$$D = \frac{2N_e r_e \sigma_z}{\gamma \sigma_r^2} \simeq 0.1$$

- Average energy loss in quantum regime:

$$\delta \simeq W_{rad}(\chi \sim 1) \cdot \sigma_z \simeq \frac{\alpha \sigma_z}{\gamma \lambda_C} \sim 0.1$$

- Nonlinearity and beamstrahlung parameter:

$$a_0 \simeq \frac{r_e N_e}{\sigma_r} \simeq 10^3,$$

$$\chi \equiv \Upsilon \simeq \frac{r_e^2 N_e \gamma}{\alpha \sigma_r \sigma_z} \simeq 10^3$$

# Proposal (arXiv:1807.09271)

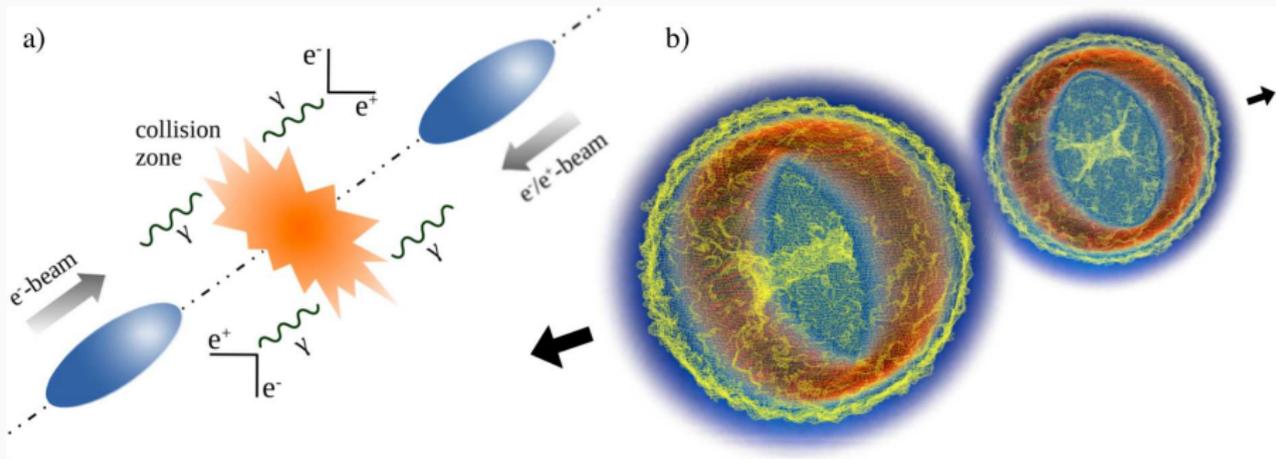
Parameter	[Unit]	NpQED Collider	FACET-II	ILC	CLIC
Machine Length	[km]	5	1	31	48
Beam Energy	[GeV]	125	10	250	1500
Bunch Charge	[nC]	1.4	1.2	3.2	0.6
Peak Current	[kA]	1700	300	1.3	12.1
rms Energy Spread	[%]	0.1	0.85	0.12	0.34
rms Bunch Length	[ $\mu\text{m}$ ]	0.01	0.48	300	44
rms Bunch Size	$\sigma_x$ [ $\mu\text{m}$ ]	0.01	3	0.47	0.045
	$\sigma_y$ [ $\mu\text{m}$ ]	0.01	2	0.006	0.001
Beamstrahlung Parameter	$\chi_{\text{av}}$ $\chi_{\text{max}}$	969 1721	–	0.06 0.15	5 12
Disruption Parameters	$D_x$ $D_y$	0.001 0.001	–	0.3 24.4	0.15 6.8
Peak electric field	[TV/m]	4500	3.2	0.2	2.7
Beam Power	[MW]	$10^{-3}$	$10^{-4}$	5	14
Luminosity	[ $\text{cm}^{-2}\text{s}^{-1}$ ]	$10^{30}$	–	$10^{34}$	$10^{34}$

# 3D simulation with OSIRIS (see arXiv:1807.09271)

electrons

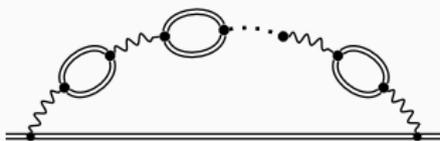
electrons with  $\alpha\chi^{2/3} > 1$

photons with  $\hbar\omega > 2mc^2$



# Theoretical challenges and working program

- Main challenge: **missing theory**
- Preliminary steps [currently we are **here**]:
  - Simplified building blocks (**dressed propagators**) for  $\chi \gg 1$
  - Calculation of all-order bubble correction



(conjectured as leading) to confirm/reject the RN conjecture

- Explicit and accurate reconsideration of **renormalization** in IFQED, especially of its **finite part** (with the possible insight into the **IFQED renormgroup flow**)
- Genuine self-consistent non-perturbative solution of IFQED SDEs

$$\text{---} = \text{===} + \text{---} \text{---} \text{---} \text{---} \text{---}$$

The diagram shows an equation for a dressed propagator. On the left is a single horizontal line. On the right is the sum of a double horizontal line and a diagram consisting of a double horizontal line connected to a shaded circle, which is then connected to a wavy line.

$$\text{---} = \text{---} + \text{---} \text{---} \text{---}$$

The diagram shows an equation for a dressed wavy propagator. On the left is a single wavy line. On the right is the sum of another wavy line and a diagram consisting of a wavy line connected to a circle, which is then connected to a shaded circle, which is finally connected to another wavy line.

- **Vertex problem:**

$$\text{Diagram} = \text{Diagram}_1 + \text{Diagram}_2 + \text{Diagram}_3 + \text{Diagram}_4 + \dots$$

Disagreement in the literature:

$$= \begin{cases} \mathcal{O}(\alpha\chi^{2/3}), & \text{Morozov et al, JETP 53, 1103 (1981) } \text{☹} \\ \exists \text{ gauge : } \mathcal{O}(\alpha), & \text{Gusynin et al, NPB 563, 361 (1999) } \text{☺} \end{cases}$$

**Refinement needed!**

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- Prediction of novel **physical effects/signatures** for the non-perturbative regime

**Questions?**