Trident pair production

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Victor Dinu and GT, Phys. Rev. D **97**, no. 3, 036021 (2018)

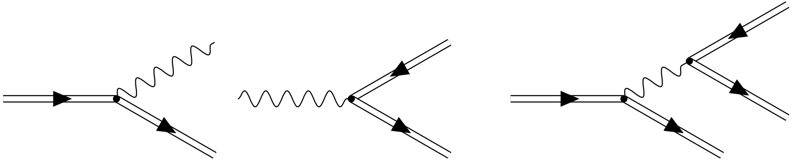
Outline

- Trident pair production

  Baier, Katkov, & Strakhovenko (1972); Ritus (1972);
  Hu, Müller & Keitel (2010); Ilderton (2011); King & Ruhl (2013);

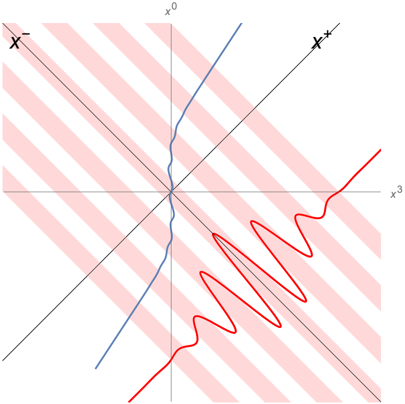
  V. Dinu & GT (2017); King & Fedotov (2018); Mackenroth & Di Piazza (2018)

- $a_0 = \frac{eE}{m\omega} \gtrsim 1$: trident more than Compton $\times$ Breit-Wheeler



Model high-intensity lasers with pulsed plane waves

- $a_0 = \frac{eE}{m\omega} > 1 \to$ treat field exactly
- $eF_{\mu\nu}(x^+) = k_\mu a'_\nu - k_\nu a'_\mu$
- Lightfront coordinates: $x^\pm = t \pm z$
- $k^2 = 0,\ k\cdot x = \omega x^+,\ k\cdot a(x^+) = 0$
- Pulsed plane waves: $a'(\pm\infty) = 0$
- Sol. to Lorentz force eq.



$$m\ddot{x}^\mu = eF^{\mu\nu}\dot{x}_\nu:\ \pi_\mu(x^+) := m\dot{x}_\mu = p_\mu - a_\mu + \frac{2ap - a^2}{2k\cdot p}k_\mu$$

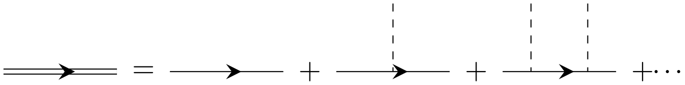
Volkov solution and Furry picture

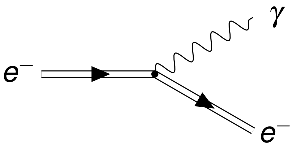
- Volkov: $(i\not{D} - m)\psi = 0$ $\qquad \psi(x) = \left(1 + \frac{\not{k}\not{a}}{2kp}\right) e^{-ipx - i\int^{x^+} \frac{2ap - a^2}{2kp}}$

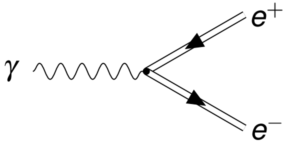
Volkov (1935)

- Furry picture: $H = H_{\text{“free”}}[a] + H_{\text{int}}[a]$

- Volkov solutions describe Lorentz force: $iD_\mu \varphi(x) = \pi_\mu(x^+)\varphi(x)$



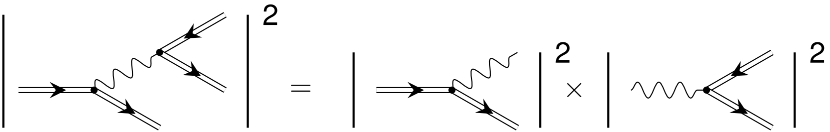




Trident - two-step and one-step

- Higher orders from sequence of first orders

  Baier, Katkov, & Strakhovenko (1972); Ritus (1972); King & Ruhl (2013)



$$+ \text{“one-step” terms} = \mathbb{P}_{\text{two}} + \mathbb{P}_{\text{one}}$$

- Particle-in-cell simulations at high intensity

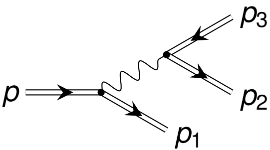
  Review: Gonoskov *et al.* PRE (2015)

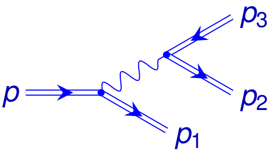
  - $a_0 = \frac{eE}{m\omega} \gg 1 \qquad \rightarrow \qquad |\mathbb{P}_{\text{one}}| \ll \mathbb{P}_{\text{two}}$

- Corrections from $\mathbb{P}_{\text{one}}$ for $a_0 > 1$

Direct and Exchange

- Exchange of identical particles in final state

$\Bigg|$  $- \quad (p_1 \leftrightarrow p_2) \Bigg|^2 \quad =$

$\Bigg|$  $\Bigg|^2 \quad + \quad (p_1 \leftrightarrow p_2) \quad + \quad$ "cross term" $\quad =$

"direct part" $+$ "exchange part" $= \mathbb{P}_{\text{dir}} + \mathbb{P}_{\text{ex}}$

- Notation: $\mathbb{P}_{\text{dir}} \neq \mathbb{P}_{\text{one}} = \mathbb{P}^{\text{dir}}_{\text{one}} + \mathbb{P}^{\text{ex}}_{\text{one}} \qquad \mathbb{P}_{\text{two}} = \mathbb{P}^{\text{dir}}_{\text{two}}$

Direct and Exchange

- $\mathbb{P}_{\text{ex}}$ more difficult than $\mathbb{P}_{\text{dir}}$
- $\mathbb{P}_{\text{ex}}$ neglected in some previous studies
- Expect $|\mathbb{P}_{\text{ex}}| \ll \mathbb{P}_{\text{dir}}$ for $\chi \gg 1$ where $\chi := a_0 b_0 = \frac{eE}{m\omega}\frac{kp}{m^2}$
- But for how large χ?
- And what about $\chi \lesssim 1$?
- Numerical and analytical calculation of $\mathbb{P}_{\text{ex}}$ Victor Dinu & GT (2017)
  - $\mathbb{P}_{\text{ex}} \sim \mathbb{P}_{\text{dir}}^{\text{one}}$ for $\chi \not\gg 1$

Lightfront quantisation

- $a_\mu(x^+), \pi_\mu(x^+), \varphi = e^{-ipx} f(x^+)$

- Use $x^+ = t + z$ instead of t

- Different forms of dynamics

  Dirac 1949

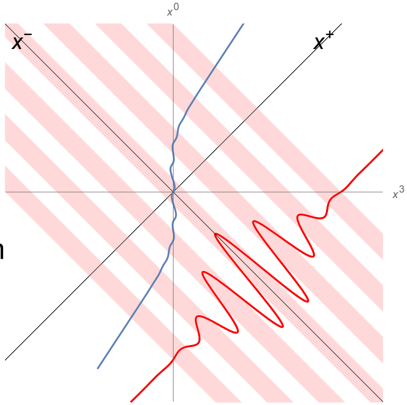
- x^+: front form, lightfront quantisation
  $[\Phi(x), \Phi^\dagger(y)]_{x^+=y^+}$

  For reviews see T. Heinzl and Brodsky, Pauli and Pinsky

- Used e.g. for non-perturbative QCD

- Lightfront quantisation + Furry picture for plane waves

  Neville & Rohrlich 1971

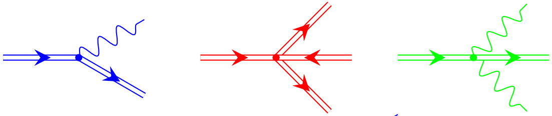


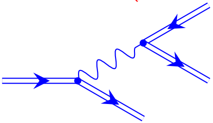
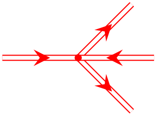
Lightfront Hamiltonian

- LF Hamiltonian $H = P_+ = (P_0 + P_3)/2$ for evolution in $x^+ = x^0 + x^3$: $|\,\psi; x^+\rangle = T_+ e^{-i\int^{x^+} H_{\rm int}}\,|\text{in}\rangle$

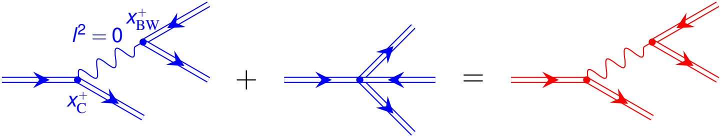
- "Instantaneous" terms in H. $p^2 = m^2$ in Ψ and $l^2 = 0$ in A_μ

$$H_{\rm int} = \frac{1}{2}\int \mathrm{d}\bar{x}\; ejA + \frac{e^2}{2} j_- \frac{1}{(i\partial_-)^2} j_- + e^2 \bar{\Psi} \not{A} \frac{\gamma^+}{4i\partial_-} \not{A} \Psi \qquad j^\mu = \bar{\Psi}\gamma^\mu\Psi\,,$$



trident amplitude =  + 

On-shell



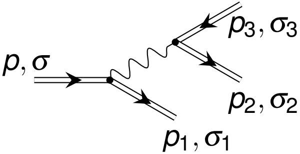
- LF Hamiltonian formalism
- On-shell: $l^2 = 0 \implies l_+ = \frac{l_\perp^2}{4 l_-}$
- Instantaneous terms
- x^+ ordered: $x^+_{\rm BW} > x^+_{\rm C}$
- cf. two-step & one-step

- standard covariant formalism
- Start: off-shell l_+-integral
- LF gauge: $\frac{g_{\mu\nu} - \frac{k_\mu l_\nu + l_\mu k_\nu}{kl}}{l^2 + i\varepsilon}$
- l_+ integral $\rightarrow$ terms with $\theta(x^+_{\rm BW} - x^+_{\rm C})$ and $\delta(x^+_{\rm BW} - x^+_{\rm C})$

Pair production probability

- Sum over momenta and spin

$$\mathbb{P} = \sum \Bigg|$$

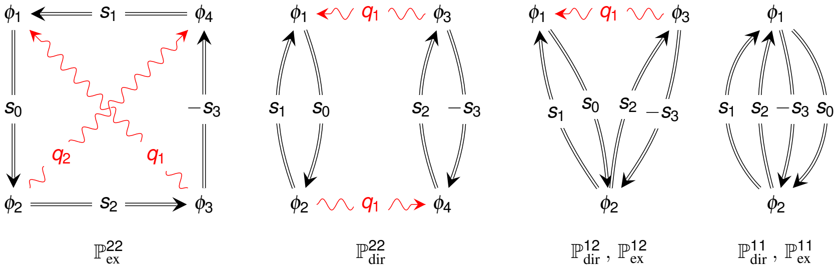


$$- \quad (p_1, \sigma_1 \leftrightarrow p_2, \sigma_2) \Bigg|^2$$

- $F_{\mu\nu}(x^+) \quad \rightarrow \quad \delta^3_{-,\perp}(p_1 + p_2 + p_3 - p)$
- Integrate over Gaussian $p_1^\perp, p_2^\perp$ integrals
- $P_- = P_0 - P_3 > 0 \quad \rightarrow \quad (p - p_1 - p_2)_- > 0$
- Prob. density: $\mathbb{P} = \int_0^1 \mathrm{d}s_1 \mathrm{d}s_2 \theta(1 - s_1 - s_2)\mathbb{P}(kp, s_1, s_2), \qquad s_i = \frac{p_{i-}}{p_-}$

Exact probability for arbitrary field shape

- LF formalism $\rightarrow$ 3 direct + 3 exchange terms



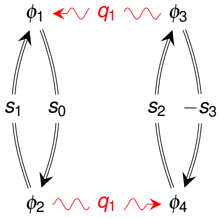
- Integrals over $\phi_i = kx_i = \omega x_i^+$. Long. momenta $s_i = \frac{kp_i}{kp}$, $q_i = 1 - s_i$

- Symmetries: $\mathbb{P}^{22}_{\text{ex}}$: $\phi_1 \to \phi_2 \to \phi_3 \to \phi_4, s_1 \to -s_0 \to s_2 \to s_3 \to s_1$

- Compact expressions for arbitrary plane waves $a_\perp(x^+)$

Victor Dinu & GT PRD (2018)

One-step two-step



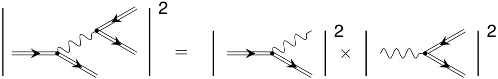
$$= \int \mathrm{d}^4\phi\, \theta(\theta_{42})\theta(\theta_{31})\ldots e^{i(\varphi_{21}+i\varphi_{43})}$$

$\mathbb{P}^{22}_{\mathrm{dir}}$

- Effective mass: $\varphi_{21} \propto M^2 = \langle\pi\rangle^2$ $\qquad \theta_{ij} = \phi_i - \phi_j$ $\qquad \sigma_{ij} = \frac{\phi_i+\phi_j}{2}$
- $\mathbb{P}_C = \int \mathrm{d}^2\phi \ldots e^{i\varphi_{21}}$ $\qquad \mathbb{P}_{BW} = \int \mathrm{d}^2\phi \ldots e^{i\varphi_{43}}$
- $\theta(\theta_{42})\theta(\theta_{31}) = \theta(\sigma_{43}-\sigma_{21})\left\{1-\theta\left(\frac{|\theta_{43}-\theta_{21}|}{2}-[\sigma_{43}-\sigma_{21}]\right)\right\}$
- $\mathbb{P}^{22}_{\mathrm{dir}} = \mathbb{P}_{\mathrm{two}} +$ contribution to $\mathbb{P}_{\mathrm{one}}$ $\qquad \mathbb{P}_{\mathrm{two}} = \sum_{\mathrm{pol.}} \mathbb{P}_C\mathbb{P}_{BW}$

Two-step and one-step

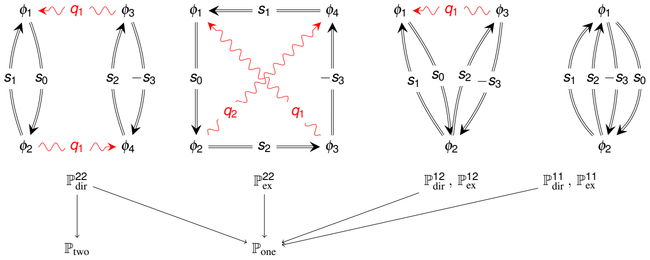
- Two-step and one-step separation:



$+$ "one-step" terms $= \mathbb{P}_{\text{two}} + \mathbb{P}_{\text{one}}$

- LF separation:

Victor Dinu & GT PRD (2018)



- From now on: $\mathbb{P}_{\text{two}}$, $\mathbb{P}^{\text{dir}}_{\text{one}}$ and $\mathbb{P}^{\text{ex}}_{\text{one}}$

$a_0 \gg 1$ and the locally constant field approximation

- Constant fields: $\mathbb{P}_{\text{two}} \sim (\Delta x^+)^2$ and $\mathbb{P}^{\text{dir}}_{\text{one}} \sim \Delta x^+$

  Baier, Katkov, and Strakhovenko (1972); Ritus (1972); King and Ruhl (2013)

- $a_0 = \frac{eE}{m\omega} \gg 1$: expand in $\frac{1}{a_0}$ $\qquad \mathbb{P} = a_0^2 P_2 + a_0 P_1 + P_0 + \mathcal{O}(\frac{1}{a_0})$

  Victor Dinu & GT PRD (2018)

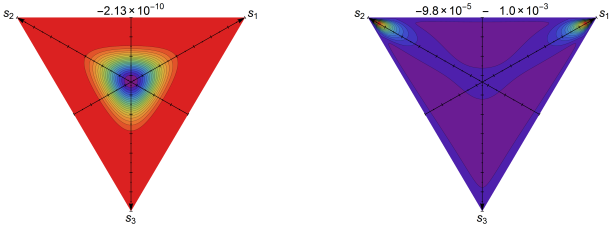
- $\mathbb{P}_{\text{two}} = a_0^2 P_2 + \mathcal{O}(a_0^0) \qquad \mathbb{P}_{\text{one}} = a_0 P_1 + \dots$

- Avoid large volume factors and include higher orders

- Both constant and non-constant fields

- Both $\mathbb{P}_{\text{dir}}$ and $\mathbb{P}_{\text{ex}}$

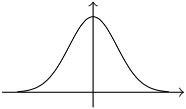
Constant field, $a_0 \gg 1$ and $\chi \ll 1$



- Longitudinal momenta $s_i = kp_i/kp$ ($s_1+s_2+s_3=1$) for $\chi=1/2$ and $\chi=16$
- Use saddle-point approx. for $\chi \ll 1$
- Constant field: $\mathbb{P}_{\rm two} \approx \alpha^2 \frac{(a_0\Delta\phi)^2}{64} e^{-\frac{16}{3\chi}}$ $\quad \mathbb{P}^{\rm dir}_{\rm one} \approx -\alpha^2 \frac{a_0\Delta\phi\sqrt{\chi}}{16\sqrt{6\pi}} e^{-\frac{16}{3\chi}}$ $\quad \mathbb{P}^{\rm ex}_{\rm one} \approx \frac{13}{18}\mathbb{P}^{\rm dir}_{\rm one}$
- $\mathbb{P}_{\rm two}$ and $\mathbb{P}^{\rm dir}_{\rm one}$ agree with literature. $\mathbb{P}^{\rm ex}_{\rm one}$ is new. V. Dinu & GT PRD (2018)
- $\mathbb{P}^{\rm ex}_{\rm one}$ as important as $\mathbb{P}^{\rm dir}_{\rm one}$

Pulsed Fields with $a_0 \gg 1$ and $\chi \ll 1$

- Pulsed plane wave $a(\phi) = a_0 f(\phi)$, $\phi = \omega x^+$, $f^{(3)}(0) = -\zeta$



- $\mathbb{P}_{\text{two}} = \alpha^2 \frac{\pi\sqrt{3}}{128} \frac{a_0^2 \chi}{\zeta} e^{-\frac{16}{3\chi}}$ $\quad \mathbb{P}^{\text{dir}}_{\text{one}} = -\alpha^2 \frac{a_0 \chi}{64\sqrt{\zeta}} e^{-\frac{16}{3\chi}}$ $\quad \mathbb{P}^{\text{ex}}_{\text{one}} = \frac{13}{18} \mathbb{P}^{\text{dir}}_{\text{one}}$

  V. Dinu & GT PRD (2018)

- $\mathbb{P}^{\text{ex}}_{\text{one}}$ on the same order as $\mathbb{P}^{\text{dir}}_{\text{one}}$ in general for $a_0 \gg 1$ and $\chi \ll 1$

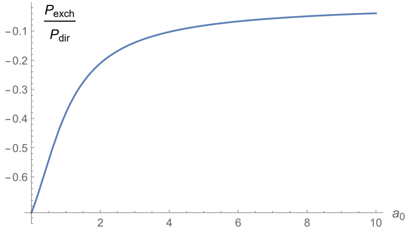
$a_0 \sim 1$ and $\chi \ll 1$

- Sauter pulse $a'(\phi) = a_0 \mathrm{sech}^2\phi$:

V. Dinu & GT PRD (2018)

$$\mathbb{P}^{\rm dir}_{\rm one} = -\frac{2}{\pi}\arctan\sqrt{1-\frac{a_0}{(1+a_0^2)\mathrm{arccot}a_0}}\,\mathbb{P}_{\rm two} \qquad \mathbb{P}^{\rm ex}_{\rm one} = \frac{13}{18}\mathbb{P}^{\rm dir}_{\rm one}$$

- $a_0 \sim 1$: $\mathbb{P}^{\rm dir}_{\rm one} \sim \mathbb{P}^{\rm ex}_{\rm one} \sim \mathbb{P}^{\rm dir}_{\rm two}$ $\qquad$ $a_0 \gg 1$: $\mathbb{P}^{\rm dir}_{\rm one} \sim \mathbb{P}^{\rm ex}_{\rm one} \ll \mathbb{P}^{\rm dir}_{\rm two}$



- $\mathbb{P} = \ldots e^{-\frac{8a_0}{\chi}[(1+a_0^2)\mathrm{arccot}\,a_0 - a_0]}$
- $a_0 \gg 1$: $\qquad \mathbb{P} = \ldots e^{-\frac{16}{3\chi}}$
- $a_0 \ll 1$: $\qquad \mathbb{P} \sim e^{-\frac{4\pi}{kp}} \sim |\tilde{a}(\frac{4\omega}{kp})|^2$

Monochromatic field, $a_0 \sim 1$ and $\chi \ll 1$

- $a'(\phi) = a_0 \cos\phi$: $\quad \mathbb{P}_{\rm two} \sim N\mathbb{P}^{\rm dir}_{\rm one} \sim N\mathbb{P}^{\rm ex}_{\rm one}$

- $\mathbb{P} = \text{prefactor} \exp\left\{-\frac{4a_0}{\chi}\left([2+a_0^2]\text{arcsinh}\frac{1}{a_0} - \sqrt{1+a_0^2}\right)\right\}$

  V. Dinu & GT PRD (2018)

- Compare with SLAC experiment:

  $\mathbb{P} \sim e^{-\frac{\sqrt{2}c}{\chi}} \qquad c_{\rm SLAC} = 2.4 \pm 0.1(\text{stat.})^{+0.2}_{-0.6}(\text{syst.}) \qquad c_{\rm we} \approx 2.46$

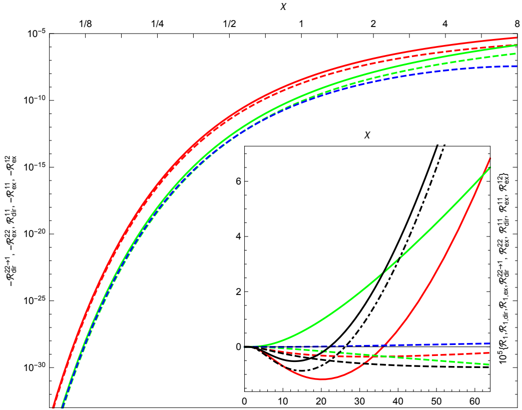
- Agreement

- However: too large error bars

- and too close to perturbative limit $\mathbb{P} \sim a_0^{8/kp}$

$a_0 \gg 1$ and general χ

- $\mathbb{P}^{\rm ex}_{\rm one} < 0$ for all χ
- $\mathbb{P}^{\rm dir}_{\rm one} < 0$ for $\chi \lesssim 20$
- $|\mathbb{P}^{\rm ex}_{\rm one}| > |\mathbb{P}^{\rm dir}_{\rm one}|$ for $17 \lesssim \chi \lesssim 26$
- $\mathbb{P}^{\rm dir}_{\rm one} \gg |\mathbb{P}^{\rm ex}_{\rm one}|$ for $\chi \gg 30$
- $\mathbb{P}^{\rm ex}_{\rm one}$ important up to quite large χ



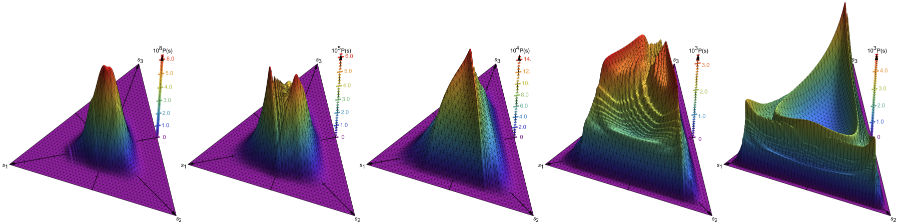
Victor Dinu & GT PRD (2018)

Momentum spectra at $a_0 = 1$

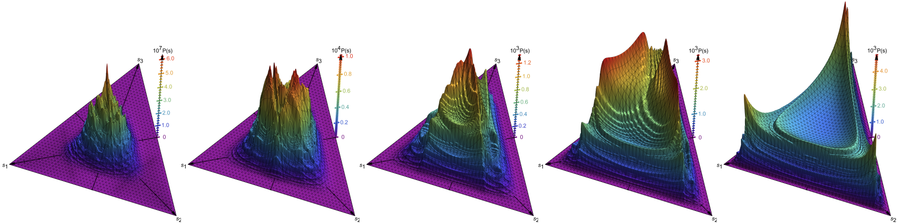
- $\mathbb{P}(s)$ for a long pulse

Victor Dinu & GT preliminary results

- circular polarization, $a_0 = 1$ and $b_0 = kp/m^2 = 0.5, 1, 2, 4, 8$:



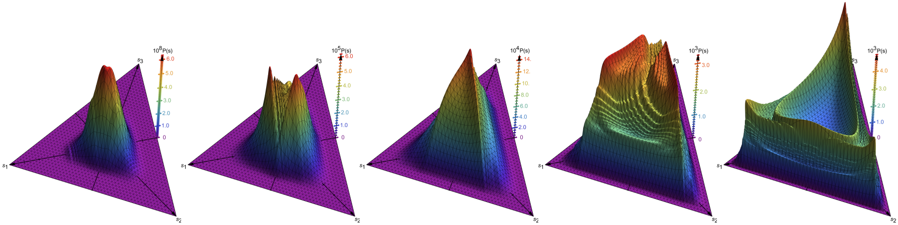
- linear polarization:



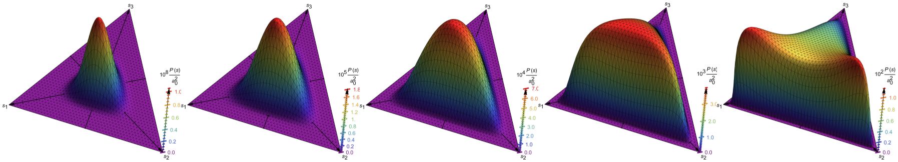
Far from LCF

- Exact for $a_0 = 1$

Victor Dinu & GT preliminary results



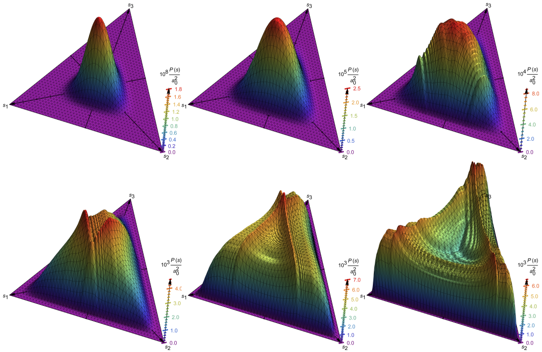
- Locally constant field approximation



$a_0 = 2$

- $a_0 = 2$, $b_0 = 0.25, 0.5, 1, 2, 4, 8$

Victor Dinu & GT preliminary results



Conclusions

- Strong fields $\rightarrow$ plane waves $\rightarrow$ lightfront formalism
- LF $\rightarrow$ compact $\mathbb{P}$ for arbitrary field shapes V. Dinu & GT PRD (2018)
- All terms, both $\mathbb{P}_{\rm dir}$ and $\mathbb{P}_{\rm ex}$
- $\mathbb{P}_{\rm ex} \sim \mathbb{P}_{\rm dir}^{\rm one}$
  - Analytically for $\chi \ll 1$ and $a_0 \gg 1$ or $a_0 \sim 1$
  - Numerically for $a_0 \gg 1$ and quite large χ
- Methods also useful for double nonlinear Compton scattering