Aspects of SUSY Breaking in String Theory

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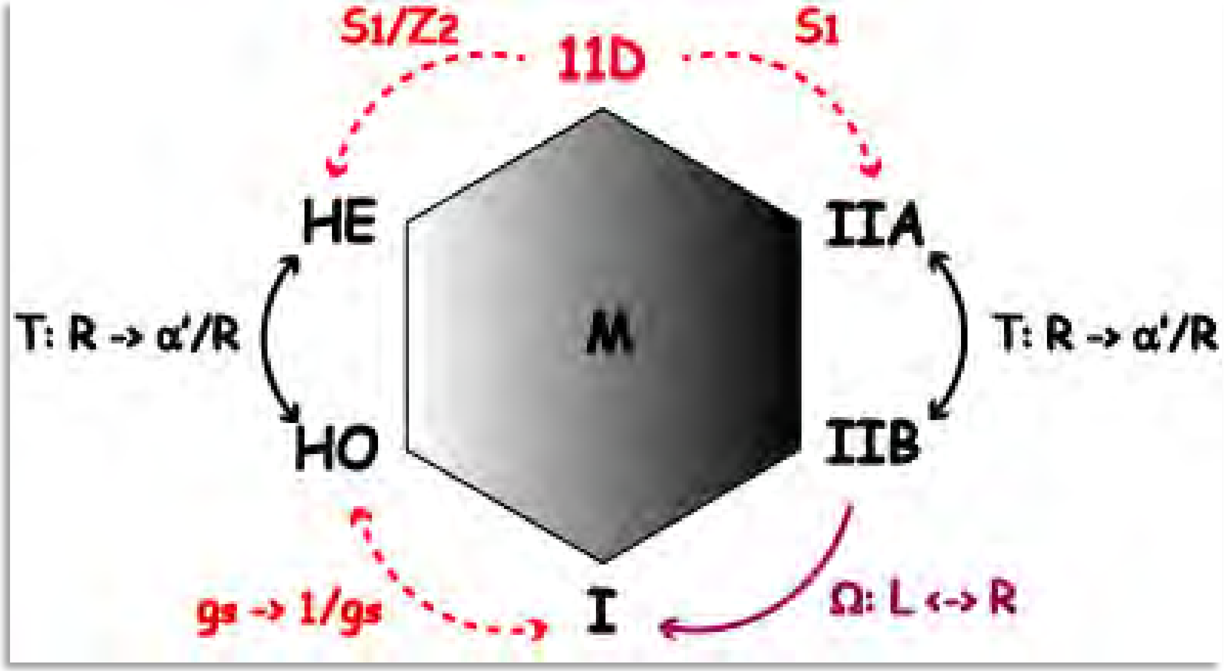
10D Superstrings

And

(Brane) SUSY Breaking

The 10-11 Duality Hexagon



- **Highest point** reached by (SUSY) String Theory
- Exhibits **dramatically** the **narrow limits** of our understanding
- **Solid arrows:** perturbative links
- **Dashed arrows:** non-perturbative links suggested by 10&11D supergravity
- **HERE:** SUSY eliminates tachyons and stabilizes the 10D Minkowski vacua

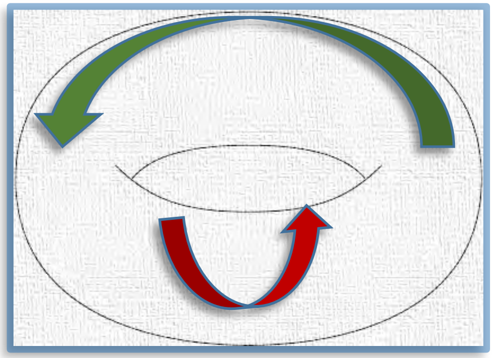
BROKEN SUSY?

Ten-Dimensional (Closed) Superstrings

❖ Building principles of (closed) string spectra and the vacuum energy:

- spin-statistics *(GSO projections)*
- modular invariance

(Gliozzi, Scherk, Olive, 1977)



$$\begin{pmatrix} O_8 \\ V_8 \\ S_8 \\ C_8 \end{pmatrix} :$$

$$\tau \to -\frac{1}{\tau} \ : \ S = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \qquad \tau \to \tau+1 \ : \ T = e^{-i\frac{\pi}{3}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\tau \to -\frac{1}{\tau} \ : \ \eta(-1/\tau) = (-i\tau)^{\frac{1}{2}}\eta(\tau) \qquad \tau \to \tau+1 \ : \eta(\tau+1) = e^{i\frac{\pi}{12}}\,\eta(\tau)$$

- **IIA, IIB:**

$$\mathcal{T}_{IIA} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\frac{(V_8 - S_8)(\bar{V}_8 - \bar{C}_8)}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8} \qquad \mathcal{T}_{IIB} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\frac{|V_8 - S_8|^2}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8}$$

$IIA: \ (e_\mu^a, B_{\mu\nu}, \phi, \psi_{\mu\,L}, \psi_{\mu\,R}, \psi_L, \psi_R, A_\mu, C_{\mu\nu\rho})$

$IIB: \ (e_\mu^a, B_{\mu\nu}^{1,2}, \phi^{1,2}, \psi_{\mu\,L}^{1,2}, \psi_R^{1,2}, D_{\mu\nu\rho\sigma}^{+})$

- **HE, HO:**

$$\mathcal{T}_{HE} = \int_{\mathcal{F}} \frac{d^2\tau}{Im\tau^2}\frac{(V_8 - S_8)(\bar{O}_{16} + \bar{S}_{16})^2}{Im\tau^4\,\eta^8\,\bar{\eta}^8} \qquad \mathcal{T}_{HO} = \int_{\mathcal{F}} \frac{d^2\tau}{Im\tau^2}\frac{(V_8 - S_8)(\bar{O}_{32} + \bar{S}_{32})}{Im\tau^4\,\eta^8\,\bar{\eta}^8}$$

$HE: \ (e_\mu^a, B_{\mu\nu}, \phi, \psi_{\mu\,L}, \psi_R) \oplus (E_8 \times E_8 \text{ super YM})$

$HO: \ (e_\mu^a, B_{\mu\nu}, \phi, \psi_{\mu\,L}, \psi_R) \oplus (SO(32) \text{ super YM})$

$$\boxed{\text{SUSY:} \ V_8 = S_8 = C_8}$$

- **0A, 0B:**

$$\mathcal{T}_{0A} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\frac{|O_8|^2 + |V_8|^2 + S_8\bar{C}_8 + C_8\bar{S}_8}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8} \qquad \mathcal{T}_{0B} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\frac{|O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8}$$

$0A: \ (e_\mu^a, B_{\mu\nu}, \phi, T, A_\mu^{1,2}, C_{\mu\nu\rho}^{1,2})$

$0B: \ (e_\mu^a, B_{\mu\nu}^{1,2,3}, \phi^{1,2,3}, T, D_{\mu\nu\rho\sigma})$

- **H_{16x16}:**

$$\mathcal{T}_{SO(16)\times SO(16)} = \int_{\mathcal{F}} \frac{d^2\tau}{Im\tau^2}\frac{1}{Im\tau^4\,\eta^8\,\bar{\eta}^8}\left[O_8(\bar{V}_{16}\bar{C}_{16} + \bar{C}_{16}\bar{V}_{16}) + V_8(\bar{O}_{16}\bar{O}_{16} + \bar{S}_{16}\bar{S}_{16}) - S_8(\bar{O}_{16}\bar{S}_{16} + \bar{S}_{16}\bar{O}_{16}) - C_8(\bar{V}_{16}\bar{V}_{16} + \bar{C}_{16}\bar{C}_{16})\right]$$

$H_{16\times16}: \ (e_\mu^a, B_{\mu\nu}, \phi) \oplus A_\mu^{(120,1)\oplus(1,120)} \oplus \psi_L^{(128,1)\oplus(1,128)} \oplus \psi_R^{(16,16)}$

(Dixon, Harvey, 1986)
(Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1986)

IIA & IIB on a Circle

- COMPACTIFY IIA and IIB on a circle of radius R:

$$\mathcal{T}_{IIA} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{(V_8 - S_8)(\bar{V}_8 - \bar{C}_8)}{(Im\tau)^{\frac{7}{2}} \eta^8 \bar{\eta}^8} \times \sum_{m,n} q^{\frac{\alpha'}{4}\left(\frac{m}{R}+\frac{nR}{\alpha'}\right)^2} \bar{q}^{\frac{\alpha'}{4}\left(\frac{m}{R}-\frac{nR}{\alpha'}\right)^2}$$

$$\mathcal{T}_{IIB} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{|V_8 - S_8|^2}{(Im\tau)^{\frac{7}{2}} \eta^8 \bar{\eta}^8} \times \sum_{m,n} q^{\frac{\alpha'}{4}\left(\frac{m}{R}+\frac{nR}{\alpha'}\right)^2} \bar{q}^{\frac{\alpha'}{4}\left(\frac{m}{R}-\frac{nR}{\alpha'}\right)^2}$$

- CORRECTION factor:

$$\sum_{m,n} \Lambda_{m,n} \equiv \sum_{m,n} q^{\frac{\alpha'}{4}\left(\frac{m}{R}+\frac{nR}{\alpha'}\right)^2} \bar{q}^{\frac{\alpha'}{4}\left(\frac{m}{R}-\frac{nR}{\alpha'}\right)^2}$$

- NOTICE the invariance of the (stringy) sum in the Kaluza-Klein factor under $R \rightarrow \alpha'/R$
- BUT: this combines into a "R-moving parity": $X_L(\tau+\sigma) \rightarrow X_L(\tau+\sigma)$, $X_R(\tau-\sigma) \rightarrow -X_R(\tau-\sigma)$, … which flips the relative chirality of the two Ramond vacua: IIA ↔ IIB as the dual $R \rightarrow \infty$

SUSY BREAKING?

Scherk-Schwarz reduction: (Bose periodic & Fermi antiperiodic)

(Circle) Scherk-Schwarz

(Sherk, Schwarz, 1979)
(Rohm, 1984)
(Ferrara, Kounnas, Porrati, Zwirner, 1987)
....

❖ Scherk-Schwarz (R') = "shift-orbifold (2R'=R) [X → X + π R combined with $(-1)^F$ x I]

$$\mathcal{T}_{IIA} \to \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{1}{(Im\tau)^{\frac{7}{2}}\ \eta^8\,\bar\eta^8} \times \Big[(V_8\,\bar V_8 + S_8\,\bar C_8) \sum_{m,n} \frac{1+(-1)^m}{2}\,\Lambda_{m,n} - (V_8\,\bar C_8 + S_8\,\bar V_8) \sum_{m,n} \frac{1-(-1)^m}{2}\,\Lambda_{m,n}$$
$$(O_8\,\bar O_8 + C_8\,\bar S_8) \sum_{m,n} \frac{1+(-1)^m}{2}\,\Lambda_{m,n+\frac{1}{2}} - (O_8\,\bar S_8 + C_8\,\bar O_8) \sum_{m,n} \frac{1-(-1)^m}{2}\,\Lambda_{m,n+\frac{1}{2}} \Big]$$
$$\mathcal{T}_{IIB} \to \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{1}{(Im\tau)^{\frac{7}{2}}\ \eta^8\,\bar\eta^8} \times \Big[(V_8\,\bar V_8 + S_8\,\bar S_8) \sum_{m,n} \frac{1+(-1)^m}{2}\,\Lambda_{m,n} - (V_8\,\bar S_8 + S_8\,\bar V_8) \sum_{m,n} \frac{1-(-1)^m}{2}\,\Lambda_{m,n}$$
$$(O_8\,\bar O_8 + C_8\,\bar C_8) \sum_{m,n} \frac{1+(-1)^m}{2}\,\Lambda_{m,n+\frac{1}{2}} - (O_8\,\bar C_8 + C_8\,\bar O_8) \sum_{m,n} \frac{1-(-1)^m}{2}\,\Lambda_{m,n+\frac{1}{2}} \Big]$$

- ❖ NOW: "twisted" sectors where as R → 0 TACHYONS appear
- ❖ A VACUUM ENERGY is generated
- ❖ AFTER a T-duality R → 0 translates into R → ∞, and

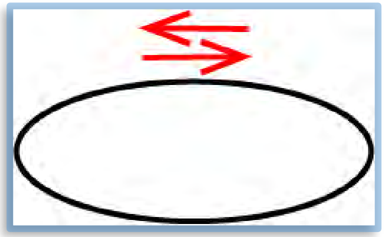
$$\mathcal{T}_{IIB} \to \mathcal{T}_{0A} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{|O_8|^2 + |V_8|^2 + S_8\,\bar C_8 + C_8\,\bar S_8}{(Im\tau)^4\ \eta^8\,\bar\eta^8}$$
$$\mathcal{T}_{IIA} \to \mathcal{T}_{0B} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{|O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2}{(Im\tau)^4\ \eta^8\,\bar\eta^8}$$

(Blum, Dienes, 1998)

Open Descendants, or "Orientifolds"

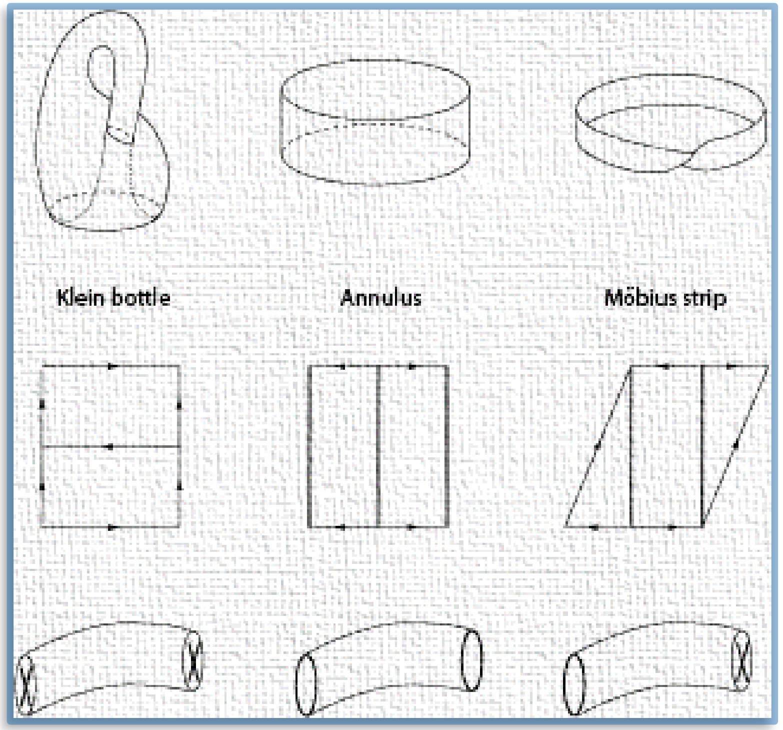
(AS, 1987)

- Models with OPEN strings DESCEND from closed ones via orientifold projections. In particular: the SO(32) type-I superstring descends from type-IIB

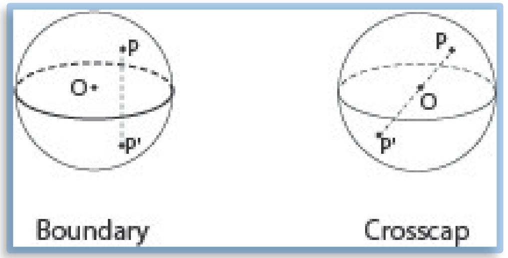


→ MIX L and R string MODES

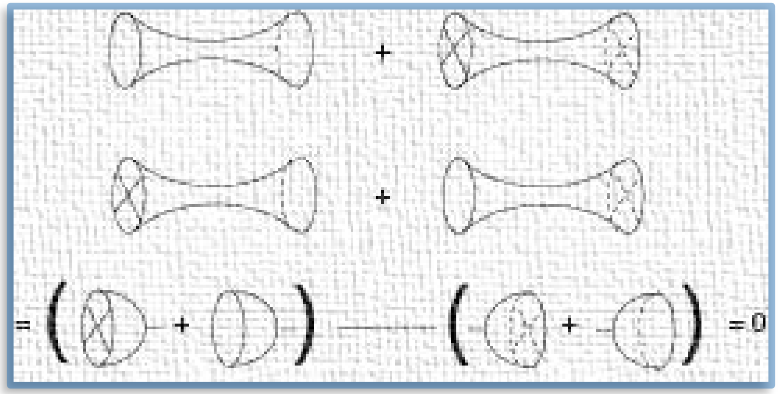
(+ M. Bianchi, G. Pradisi, 1988- ; D. Fioravanti, 1993; Y. Stanev, 1994-)



- *New 2D ingredients:*



- RR tadpole(s): neutrality conditions



- The procedure fills vacua with D-branes and O-planes

(Polchinski, 1995)

10D Tachyon-Free Orientifolds

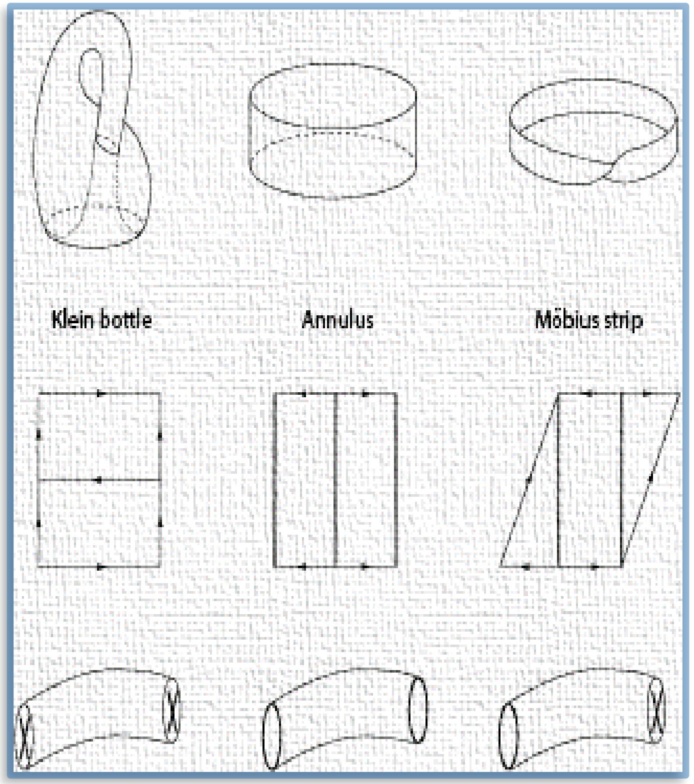
1) **SO(32) type-I:** in vacuum **BPS** combination **(O_ orientifold** (T<0,Q<0) and **D-branes** (T>0,Q>0)**)**. Massless Weyl fermions in the adjoint of SO(32)

$$\mathcal{T}_{SO(32)} = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\,\frac{|V_8 - S_8|^2}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8}\,[\tau,\bar{\tau}] \qquad \mathcal{K}_{SO(32)} = \frac{1}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\,\frac{V_8 - S_8}{(\tau_2)^4\,\eta^8}\,[2i\tau_2]$$

$$\mathcal{A}_{SO(32)} = \frac{\mathcal{N}^2}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\,\frac{V_8 - S_8}{(\tau_2)^4\,\eta^8}\,[i\tau_2/2] \qquad \mathcal{M}_{SO(32)} = -\,\frac{\mathcal{N}}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\,\frac{(\hat{V}_8 - \hat{S}_8)}{(\tau_2)^4\,\hat{\eta}^8}\,[i\tau_2/2 + 1/2]$$

$$\mathcal{N} = 32$$

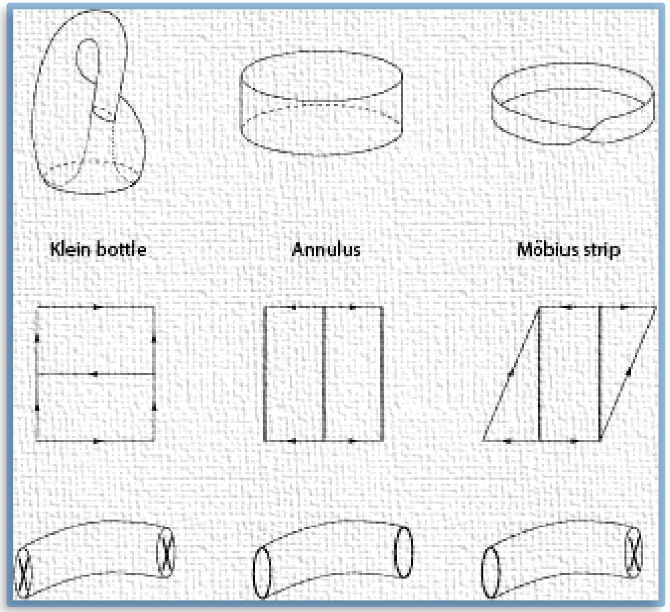
$$(e_\mu^a, B_{\mu\nu}, \phi, \psi_\mu, \psi) \oplus (A_\mu^{[ab]}, \lambda^{[ab]}) \;, \quad \mathrm{SO(32)\ gauge\ group}$$



D-branes
T>0, Q>0

~~O-~~
~~T<0, Q<0~~

10D Tachyon-Free Orientifolds



1. U(32) type-0'b: $(e_\mu^a, B_{\mu\nu}, D^+_{\mu\nu\rho\sigma}, \phi, \phi') \oplus (A_{\mu\, a}{}^{b}, \lambda^{[ab]}, \lambda_{[ab]})$, U(32) gauge group

   [NO SUSY, T > 0 , NO TACHYONS after orientifold projection] *(AS, 1995)*

$$\mathcal{T}_{0'B} = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\, \frac{|O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2}{(Im\tau)^4\, \eta^8\, \bar\eta^8}[\tau,\bar\tau] \qquad \mathcal{K}_{0'B} = \frac{1}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{-O_8 + V_8 + S_8 - C_8}{(\tau_2)^4\, \eta^8}[2i\tau_2]$$

$$\mathcal{A}_{0'B} = \int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{\mathcal{N}\bar{\mathcal{N}}\, V_8 - \frac{\mathcal{N}^2+\bar{\mathcal{N}}^2}{2}\, C_8}{(\tau_2)^4\, \eta^8}[i\tau_2/2] \qquad \mathcal{M}_{0'B} = -\,\frac{\mathcal{N}+\bar{\mathcal{N}}}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{\hat C_8}{(\tau_2)^4\, \hat\eta^8}[i\tau_2/2+1/2]$$

$$\mathcal{N} = \bar{\mathcal{N}} = 32$$

2. USp(32) type-I: $(e_\mu^a, B_{\mu\nu}, \phi, \psi_\mu, \psi) \oplus (A_\mu^{(ab)}, \lambda^{[ab]})$, USp(32) gauge group

   [non-linear SUSY, T > 0] *(Sugimoto, 1999)*

D-antibranes
T>0, Q<0

~~Q+~~
~~T>0, Q>0~~

$$\mathcal{T}_{Usp(32)} = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\, \frac{|V_8 - S_8|^2}{(Im\tau)^4\, \eta^8\, \bar\eta^8}[\tau,\bar\tau] \qquad \mathcal{K}_{USp(32)} = \frac{1}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{V_8 - S_8}{(\tau_2)^4\, \eta^8}[2i\tau_2]$$

$$\mathcal{A}_{Usp(32)} = \frac{\mathcal{N}^2}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{V_8 - S_8}{(\tau_2)^4\, \eta^8}[i\tau_2/2] \qquad \mathcal{M}_{USp(32)} = -\,\frac{\mathcal{N}}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\, \frac{(\text{“}-\text{”}\hat V_8 - \hat S_8)}{(\tau_2)^4\, \hat\eta^8}[i\tau_2/2+1/2]$$

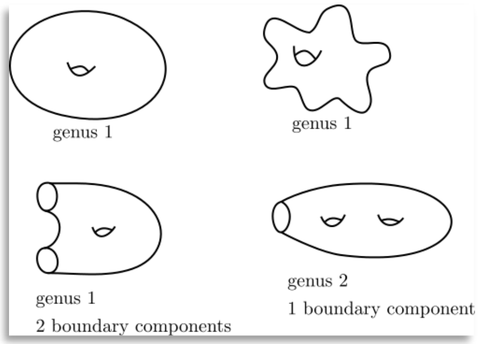
$$\mathcal{N} = 32$$

Only sign of V flipped w.r.t. type-I SO(32)!

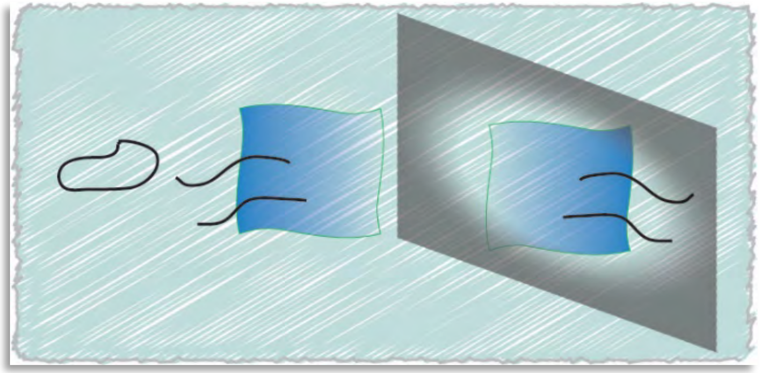
Orientifolds and Brane SUSY Breaking

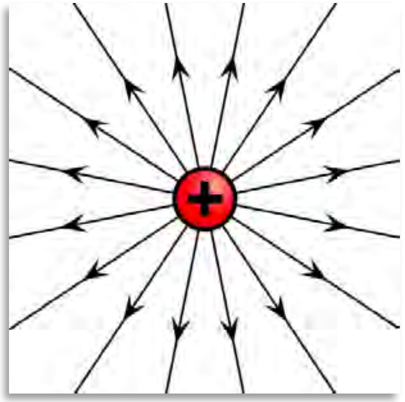
❖ String spectra: CLOSED *or* OPEN+CLOSED

- [Unified → "orientifold" construction] *(AS, 1987)*
  [+M. Bianchi, G. Pradisi, 1988-96 ; Y. Stanev, 1994-96]
- [Vacuum filled with D-branes and Orientifolds (mirrors)] *(Polchinski, 1995)*



❖ Different options to fill the vacuum :





- SUSY collections of D-branes and Orientifolds → Superstrings

❖ (Tachyon-free) Non-SUSY → BSB

(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Angelantonj, 1999)
(Aldazabal, Uranga, 1999)

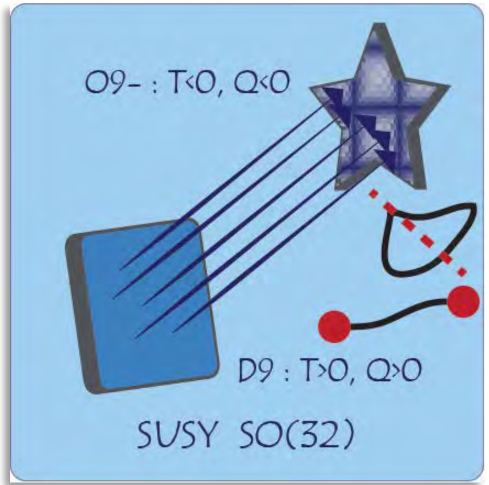
❖ BSB : D+O Tensions → "critical" exponential potential $V = V_0 \, e^{2\varphi}$

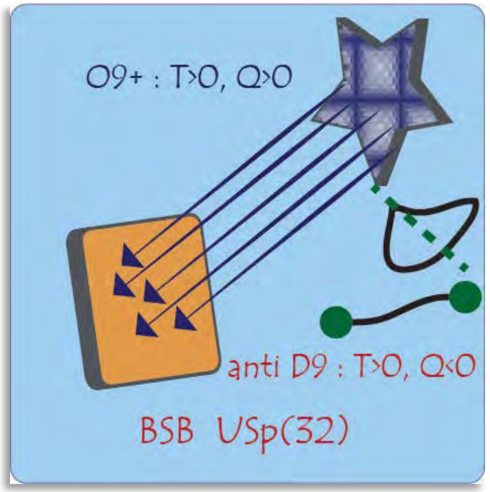
Brane SUSY Breaking (BSB)

(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Angelantonj, 1999)
(Aldazabal, Uranga, 1999)

BSB

- SUSY **BROKEN** at string scale in **open** sector, **EXACT** in closed **sector**
- Stable vacuum (classically)





Tension unbalance → "critical" exponential (runaway) potential

$$S_{10} = \frac{1}{2k_{10}^2} \int d^{10}x\sqrt{-g}\,\{e^{-2\phi}\left(-R + 4(\partial\phi)^2\right) - Te^{-\phi} + \ldots\}$$

[SAME exponential (from D – anti D): *KKLT uplift (2003)*] (Kachru, Kallosh, Linde, Trivedi, 2003)

NON-LINEAR SUSY:

- COSMOLOGY: hints of a pre-inflationary phase
- STATIC Solutions: puzzles related to (in)stability

Pre-Inflation

From

(Brane) SUSY Breaking

Cosmological Potentials

- What potentials lead to slow-roll, and where ?

$$ds^2 = -dt^2 + e^{2A(t)}\, d\mathbf{x}\cdot d\mathbf{x}$$

$$\ddot{\phi} + 3\dot{\phi}\sqrt{\frac{1}{3}\dot{\phi}^2 + \frac{2}{3}V(\phi)} + V' = 0$$

Driving force from V′ *vs* friction from V

- If V does not vanish : convenient gauge "makes the damping term neater"

$$ds^2 = e^{2\mathcal{B}(t)}\, dt^2 - e^{\frac{2\mathcal{A}(t)}{d-1}}\, d\mathbf{x}\cdot d\mathbf{x}$$

$$V e^{2\mathcal{B}} = V_0$$

$$\tau = t\sqrt{\frac{d-1}{d-2}}\,, \qquad \varphi = \phi\sqrt{\frac{(d-1)}{2(d-2)}}$$

$$\dot{\mathcal{A}}^2 - \dot{\varphi}^2 = 1$$

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1+\dot{\varphi}^2} + \frac{V_\varphi}{2V}\left(1+\dot{\varphi}^2\right) = 0$$

- Now driving from logV *vs* O(1) damping

$$V = \varphi^n \longrightarrow \frac{V'}{2V} = \frac{n}{2\varphi}$$

  - ❖ Quadratic potential? Far away from origin *(Linde, 1983)*
  - ❖ Exponential potential? YES or NO

$$V(\varphi) = V_0\, e^{2\gamma\varphi} \longrightarrow \frac{V'}{2V} = \gamma$$

$V = e^{2\gamma\varphi}$ & Climbing Scalars

- **$\gamma < 1$?** Both signs of speed

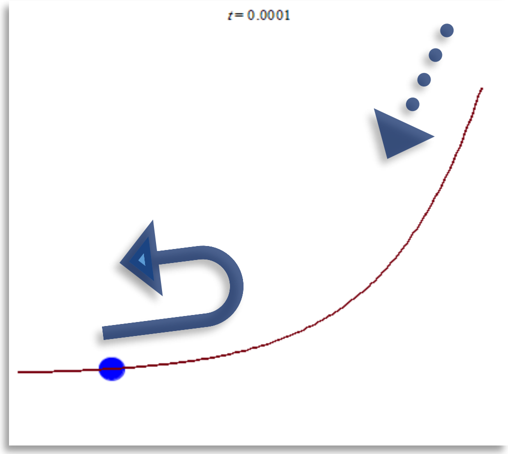
(Halliwell, 1987;…, Dudas and Mourad, 1999; Russo, 2004; Dudas, Kitazawa, AS, 2010)

a. **"Climbing" solution** (φ climbs, then descends):

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\ \coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\ \tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

b. **"Descending" solution** (φ only descends):

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\ \tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\ \coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$



Limiting τ- speed (LM attractor):

(Lucchin and Matarrese, 1985)

$$v_{lim} = -\frac{\gamma}{\sqrt{1-\gamma^2}}$$

$\gamma = 1$ is "critical": LM attractor & descending solution disappear there and beyond !

CLIMBING : in ALL asymptotically exponential potentials with $\gamma \geq 1$!

BSB in STRING THEORY HAS PRECISELY $\gamma = 1$ → WEAK coupling ($g_s = e^{\varphi}$)

- **$\gamma = 1$:**

$$\varphi(\tau) = \varphi_0 + \frac{1}{2}\left[\log|\tau-\tau_0| - \frac{1}{2}(\tau-\tau_0)^2\right]$$

$$\mathcal{A}(\tau) = \mathcal{A}_0 + \frac{1}{2}\left[\log|\tau-\tau_0| + \frac{1}{2}(\tau-\tau_0)^2\right]$$

Critical Exponentials and BSB

❖ BSB in STRING THEORY PREDICTS the exponent in $V = V_0 \, e^{2\,\varphi}$

(Dudas, Kitazawa, AS, 2010)
(AS, 2013)
(Fré, AS, Sorin, 2013)

- D=10 : Polyakov expansion and dilaton tadpole

$$\mathcal{S} \,=\, \frac{1}{2\,k_N^2}\int d^{10}x\,\sqrt{-\det g}\left[R \,+\, \frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\,\phi\,\partial_\nu\,\phi \,-\, T\,e^{\frac{3}{2}\,\phi} \,+\, \ldots\right] \qquad \longrightarrow \qquad \gamma = 1 \;(\text{for} \;\; \varphi)$$

- D < 10 : two combinations of ϕ and "breathing mode" σ → $(\Phi_s\,,\,\Phi_t\,)$
- Φ_t → "critical" potential (γ = 1) & CLIMBING, IF Φ_s is stabilized

$$S_d \;=\; \frac{1}{2\kappa_d^2}\int d^dx\sqrt{-g}\,\left[R \,+\, \frac{1}{2}\,(\partial\Phi_s)^2 \,+\, \frac{1}{2}\,(\partial\Phi_t)^2 \,-\, T_9\,e^{\sqrt{\frac{2(d-1)}{d-2}}\;\Phi_t} \,+\, \ldots\right]$$

- If Φ_s is stabilized: a SOLITON (p-brane) that couples via $(g_s)^{-\,\alpha}$ yields :

[D9-brane of 10D BSB: p=9, α=1]

∀ Broken SUSY in STRING THEORY: γ ≥ 1 !

$$\gamma \;=\; \frac{1}{12}\,(p \;+\; 9 \;-\; 6\,\alpha)$$

[**NOTE:** all multiples of $\frac{1}{12} \simeq 0.08$ (→ n_s ≅ 0.96)]

BSB: Pre-Inflation with a bounce?

(Dudas, Kitazawa, AS, 2010)

$$\mathcal{S} = \frac{1}{2\,k_{10}^2}\int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R+4(\partial\phi)^2\right]-\frac{1}{12}\,\mathcal{H}_3^2-\frac{1}{4}\,e^{-\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{-\phi}+\ldots\right\}$$

1. **"Critical" tadpole exponent**: precisely at onset of the **"climbing phenomenon"**.

$$ds^2 = |\alpha\,t^2|^{\frac{2}{9}}\,e^{\frac{1}{2}\phi_0}\,e^{-\frac{1}{4}\alpha\,t^2}\,\delta_{ij}\,dx^i\,dx^j - |\alpha\,t^2|^{-\frac{1}{3}}\,e^{-\phi_0}\,e^{\frac{3}{4}\alpha\,t^2}\,dt^2$$
$$e^{\phi} = e^{\phi_0}\,|\alpha\,t^2|^{\frac{1}{3}}\,e^{-\frac{3}{4}\alpha\,t^2}$$

(Lucchin, Matarrese, 1985)
(Halliwell , 1987)
…………
(Dudas, Mourad, 2000)
(Russo, 2004)
(Dudas, Kitazawa, AS, 2010)
…………

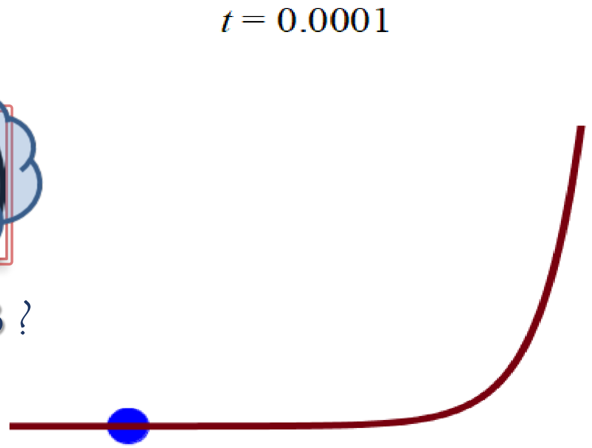
2. **Scalar** → emerges from initial singularity **"climbing up"** **ANY potential V(φ)** **that ADDS to BSB SOFTER TERMS**

Now: bounded string loop corrections [**NOT SO** curvature corrections, however]

$$V(\varphi) = V_0\left(e^{2\varphi}+e^{2\gamma\varphi}+\widetilde{V}(\varphi)\right)$$

3. **Slow-roll after bounce and deceleration:** Last stages of deceleration **imprinted in CMB** ?

→ **Low-ℓ lack of power** [& **Low-ℓ enhancement of tensor-to-scalar ratio r**].



Fast-Roll and the Mukhanov-Sasaki Equation

- MS equation :

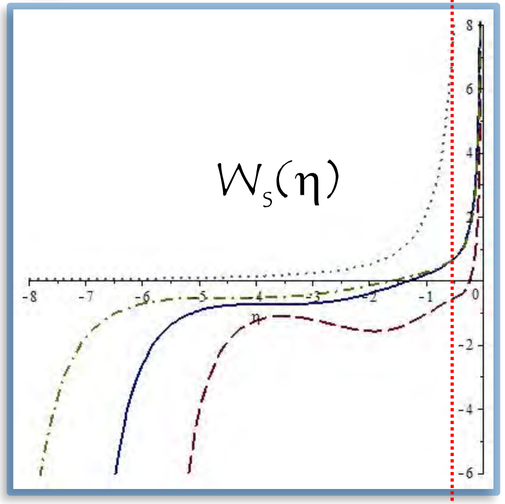
$$\left(\frac{d^2}{d\eta^2} \,+\, k^2 \,-\, W_s(\eta) \right) v_k(\eta) \;=\; 0$$

- Limiting W_s :

$$W_s \underset{\eta \to -\eta_0}{\sim} -\,\frac{1}{4}\,\frac{1}{(\eta+\eta_0)^2}\;, \qquad W_s \underset{\eta \to -0}{\sim} \frac{\nu^2 - \frac{1}{4}}{\eta^2} \quad \left(\nu \,=\, \frac{3}{2}\,\frac{1-\gamma^2}{1-3\,\gamma^2}\right)$$

- Power :

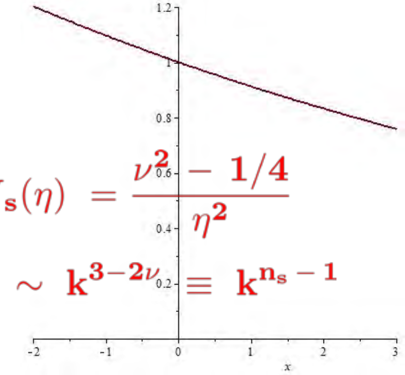
$$P(k) \;=\; \frac{k^3}{2\,\pi^2}\left|\frac{v_k(-\epsilon)}{z(-\epsilon)}\right|^2$$



❖ Pre – inflationary fast roll : $P(k) \sim k^3$

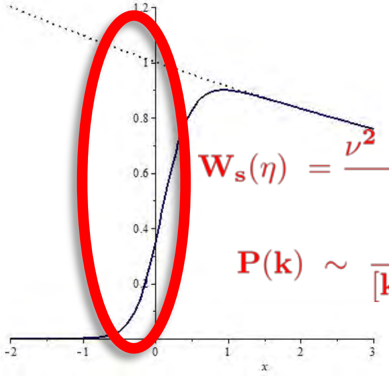
WKB : $$v_k(-\epsilon) \;\sim\; \frac{1}{\sqrt[4]{|W_s(-\epsilon) \,-\, k^2|}}\,\exp\left(\int_{-\eta^\star}^{-\epsilon}\sqrt{|W_s(y) \,-\, k^2|}\,dy\right)$$

(Chibisov, Mukhanov, 1981)



$$\mathbf{W_s}(\eta) \;=\; \frac{\nu^2 \,-\, 1/4}{\eta^2}$$

$$\mathbf{P(k)} \;\sim\; \mathbf{k}^{3-2\nu} \;\equiv\; \mathbf{k}^{\mathbf{n_s}\,-\,1}$$



$$\mathbf{W_s}(\eta) \;=\; \frac{\nu^2 \,-\, 1/4}{\eta^2} \;-\; \boldsymbol{\Delta}^2$$

$$\mathbf{P(k)} \;\sim\; \frac{\mathbf{k}^3}{[\mathbf{k}^2 \,+\, \boldsymbol{\Delta}^2]^\nu}$$

Fast-Roll and the Mukhanov-Sasaki Equation

- MS equation :

$$\left(\frac{d^2}{d\eta^2} + k^2 - W_s(\eta) \right) v_k(\eta) = 0$$

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$$W_s \underset{\eta \to -\eta_0}{\sim} -\frac{1}{4}\,\frac{1}{(\eta+\eta_0)^2}\,, \qquad W_s \underset{\eta \to -0}{\sim} \frac{\nu^2 - \frac{1}{4}}{\eta^2} \quad \left(\nu = \frac{3}{2}\,\frac{1-\gamma^2}{1-3\gamma^2}\right)$$

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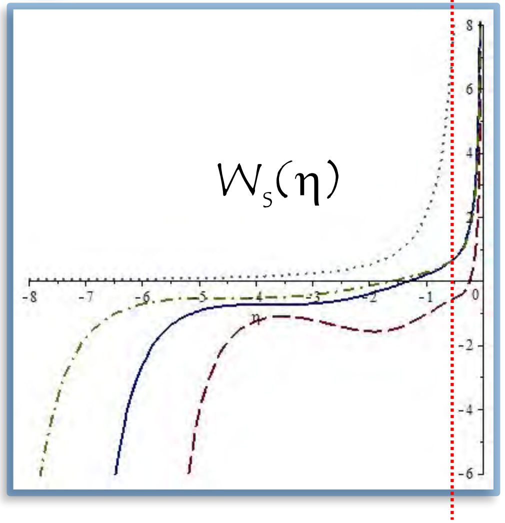
❖ Pre – inflationary fast roll : P(k) ~ k^3

WKB : $$v_k(-\epsilon) \sim \frac{1}{\sqrt[4]{|W_s(-\epsilon) - k^2|}} \exp\left(\int_{-\eta^\star}^{-\epsilon} \sqrt{|W_s(y) - k^2|}\, dy \right)$$

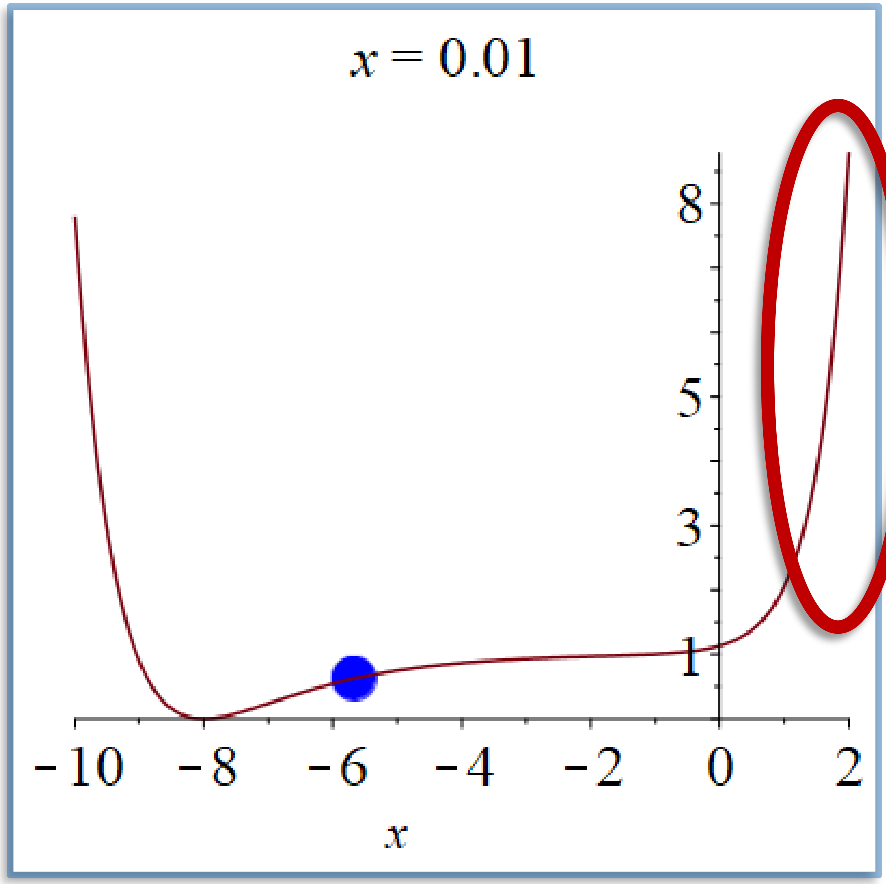


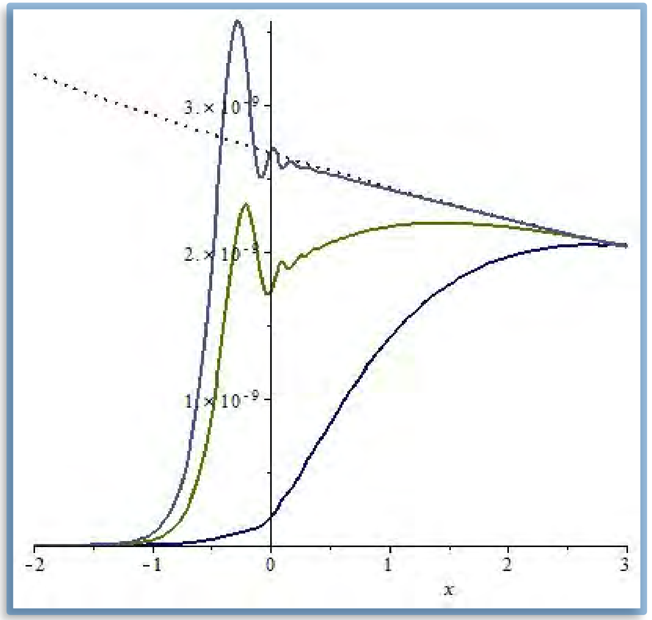
LOW CMB QUADRUPOLE FROM THIS PHENOMENON ?

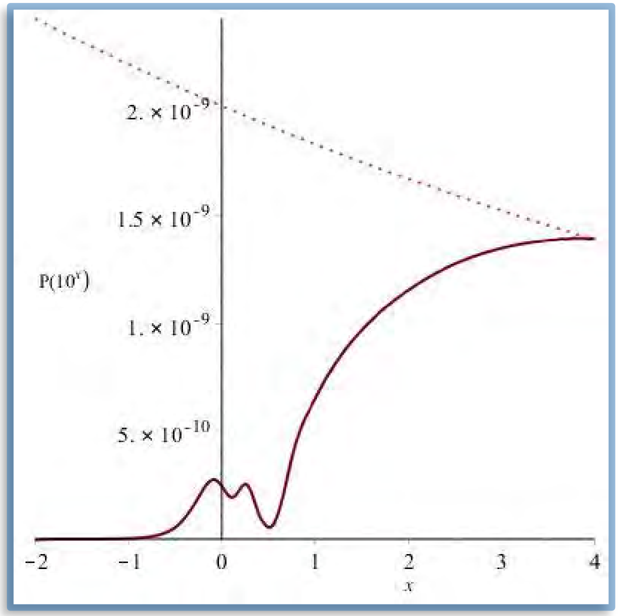
Additional signature → pre-inflationary peak !

Pre-Inflation with a Bounce

(Dudas, Kitazawa, AS, 2010)
(Dudas, Kitazawa, Patil, AS, 2012)





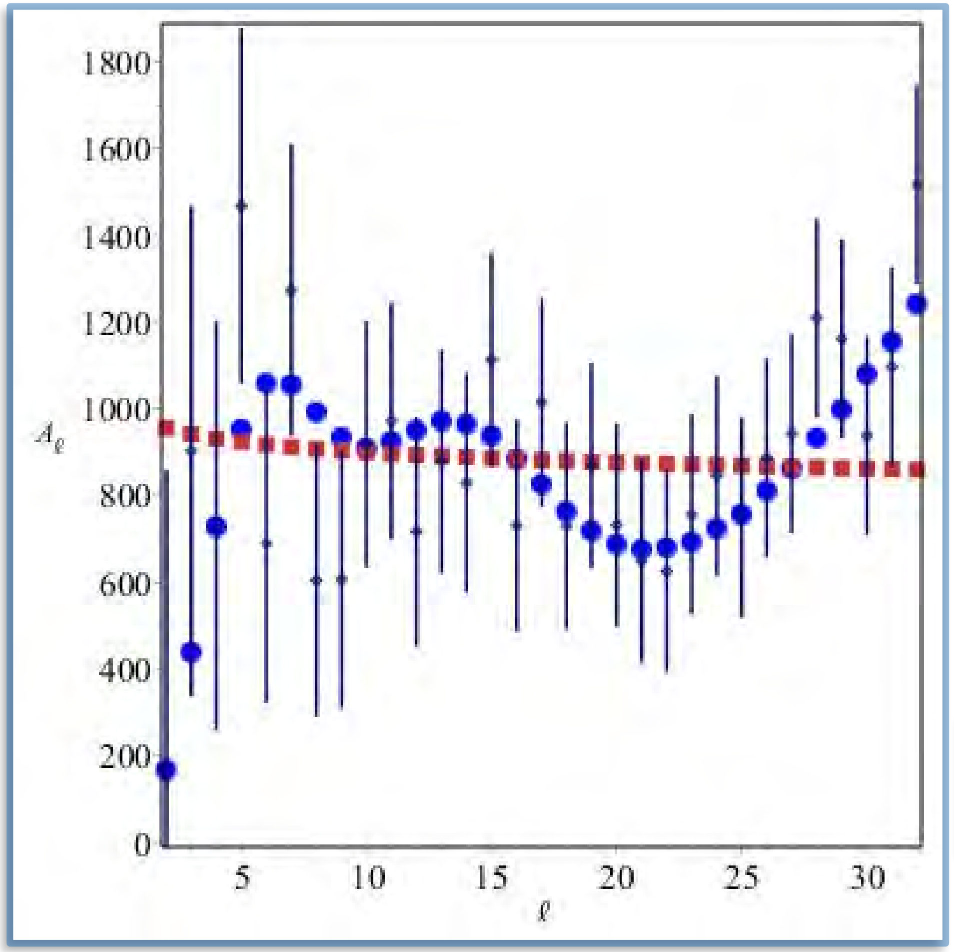


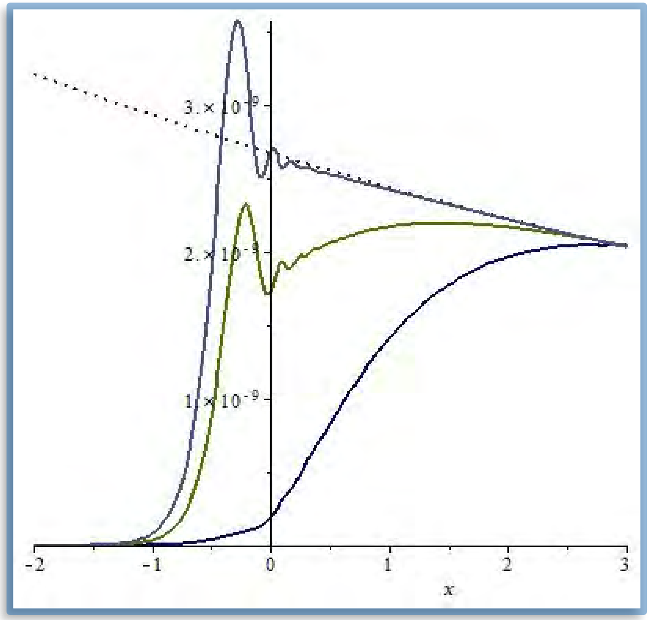
$$V(\varphi) = V_0 \left\{ e^{2\varphi} + \frac{1}{2}\, e^{2\gamma\varphi} + a_1\, e^{-a_2(\varphi+a_3)^2} + \left[1 - e^{-\frac{2}{3}(\varphi+\Delta)}\right]^2 \right\} - v_0$$

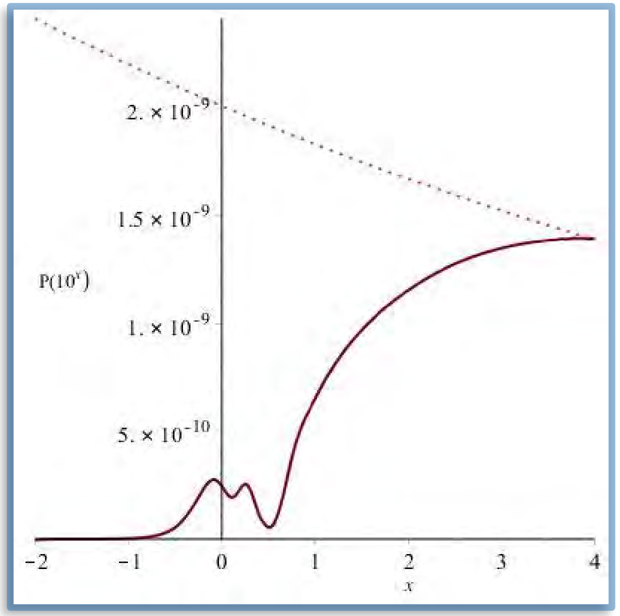
Low-l lack of power in CMB from a decelerating inflaton ?

Pre-Inflation with a Bounce

(Dudas, Kitazawa, AS, 2010)
(Dudas, Kitazawa, Patil, AS, 2012)







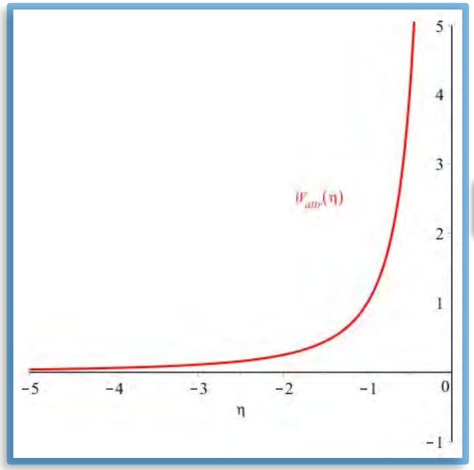
$$V(\varphi) = V_0 \left\{ e^{2\varphi} + \frac{1}{2}\, e^{2\gamma\varphi} + a_1\, e^{-a_2(\varphi+a_3)^2} + \left[1 - e^{-\frac{2}{3}(\varphi+\Delta)}\right]^2 \right\} - v_0$$

Low-l lack of power in CMB from a decelerating inflaton ?

Pre-Inflationary Relics

In the CMB?

Analytic Power Spectra



$$\frac{d^2 v_k(\eta)}{d\eta^2} + \left[k^2 - W_s(\eta)\right] v_k(\eta) = 0$$

$$W_s = \frac{\nu^2 - \frac{1}{4}}{\eta^2} \longrightarrow P(k) \sim k^3 \times k^{-2\nu}$$

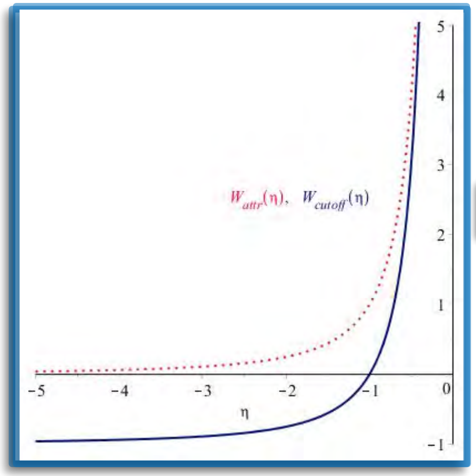
- **IF** W_s crosses the real axis → **POWER CUTOFF**
- One can also produce a **"caricature" PRE-INFLATIONARY PEAK**
- ❖ **Tensor-to-scalar ratio r:** typically **grows by about one order of magnitude** in region of power cut

$$W_S = \frac{\nu^2 - \frac{1}{4}}{\eta^2}\left[c\left(1+\frac{\eta}{\eta_0}\right) + (1-c)\left(1+\frac{\eta}{\eta_0}\right)^2\right]$$

(Dudas, Kitazawa, Patil, AS, 2012)

$$P_{\mathcal{R}}(k) \sim \frac{(k\,\eta_0)^3 \exp\left(\frac{\pi\left(\frac{c}{2}-1\right)\left(\nu^2-\frac{1}{4}\right)}{\sqrt{(k\,\eta_0)^2 + (c-1)\left(\nu^2-\frac{1}{4}\right)}}\right)}{\left|\Gamma\left(\nu+\frac{1}{2}+\frac{i\left(\frac{c}{2}-1\right)\left(\nu^2-\frac{1}{4}\right)}{\sqrt{(k\,\eta_0)^2 + (c-1)\left(\nu^2-\frac{1}{4}\right)}}\right)\right|^2 \left[(k\,\eta_0)^2 + (c-1)\left(\nu^2-\frac{1}{4}\right)\right]^{\nu}}$$

Analytic Power Spectra



$$\frac{d^2 v_k(\eta)}{d\eta^2} + \left[k^2 + \Delta^2 - W_s(\eta)\right] v_k(\eta) = 0$$

$$W_s' = \frac{\nu^2 - \frac{1}{4}}{\eta^2} - \Delta^2 \longrightarrow P(k) \sim \frac{k^3}{\left[k^2 + \Delta^2\right]^{\nu}}$$

- **IF** W_s crosses the real axis → **POWER CUTOFF**
- One can also produce a **"caricature" PRE-INFLATIONARY PEAK**
- **Tensor-to-scalar ratio r:** typically **grows by about one order of magnitude** in region of power cut

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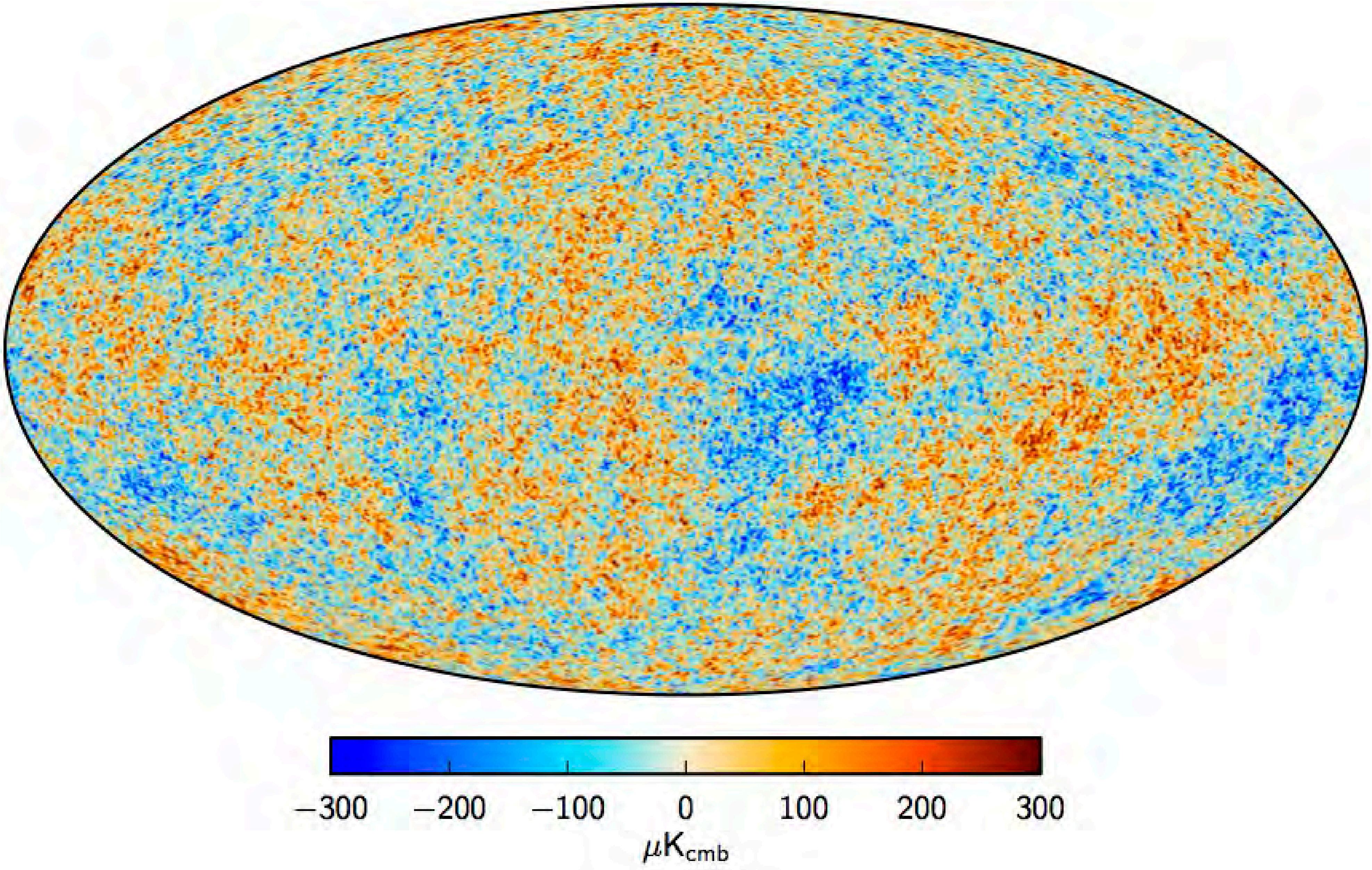
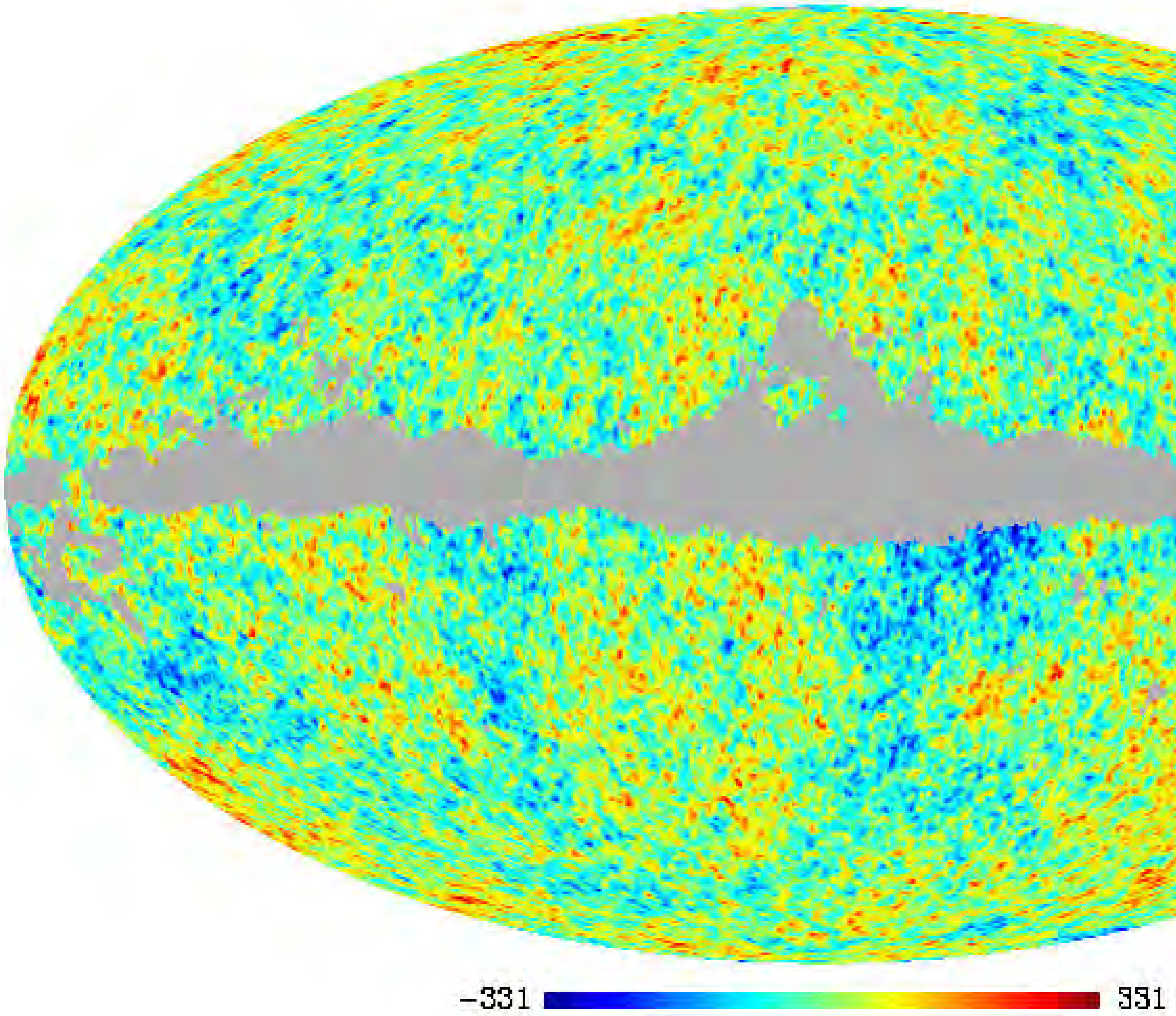

−300
−200
−100
0
100
200
300
μK_cmb

Planck CMB

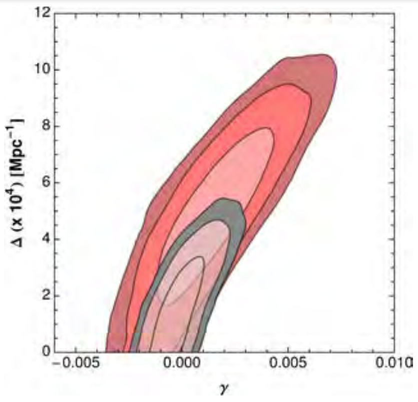


Pre-Inflationary Relics in the CMB?

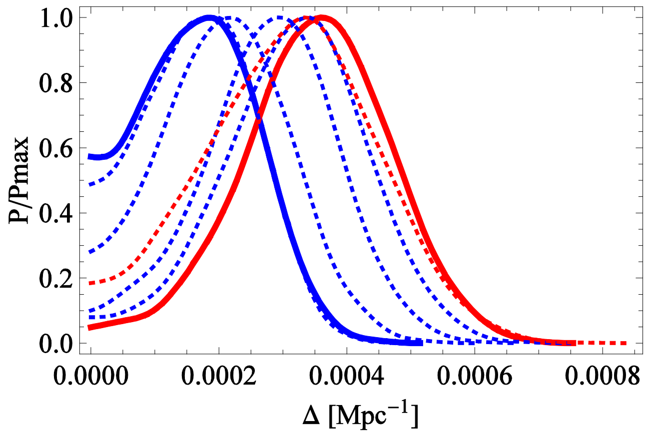
(Gruppuso, Mandolesi, Natoli, Kitazawa, AS, 2015)

- Extend ΛCDM to allow for low-ℓ suppression:

$$\mathcal{P}(k) \;=\; A\;(k/k_0)^{n_s-1} \;\rightarrow\; \frac{A\;(k/k_0)^3}{\left[(k/k_0)^2+(\Delta/k_0)^2\right]^{\nu}}$$



- ❖ NO effects on standard ΛCDM parameters (6+16 nuisance)
- ❖ A new scale Δ. Preferred value? Depends on GALACTIC MASK



$$\Delta = (0.351 \pm 0.114) \times 10^{-3}\,\mathrm{Mpc}^{-1}$$

RED : + 30−degree extended mask

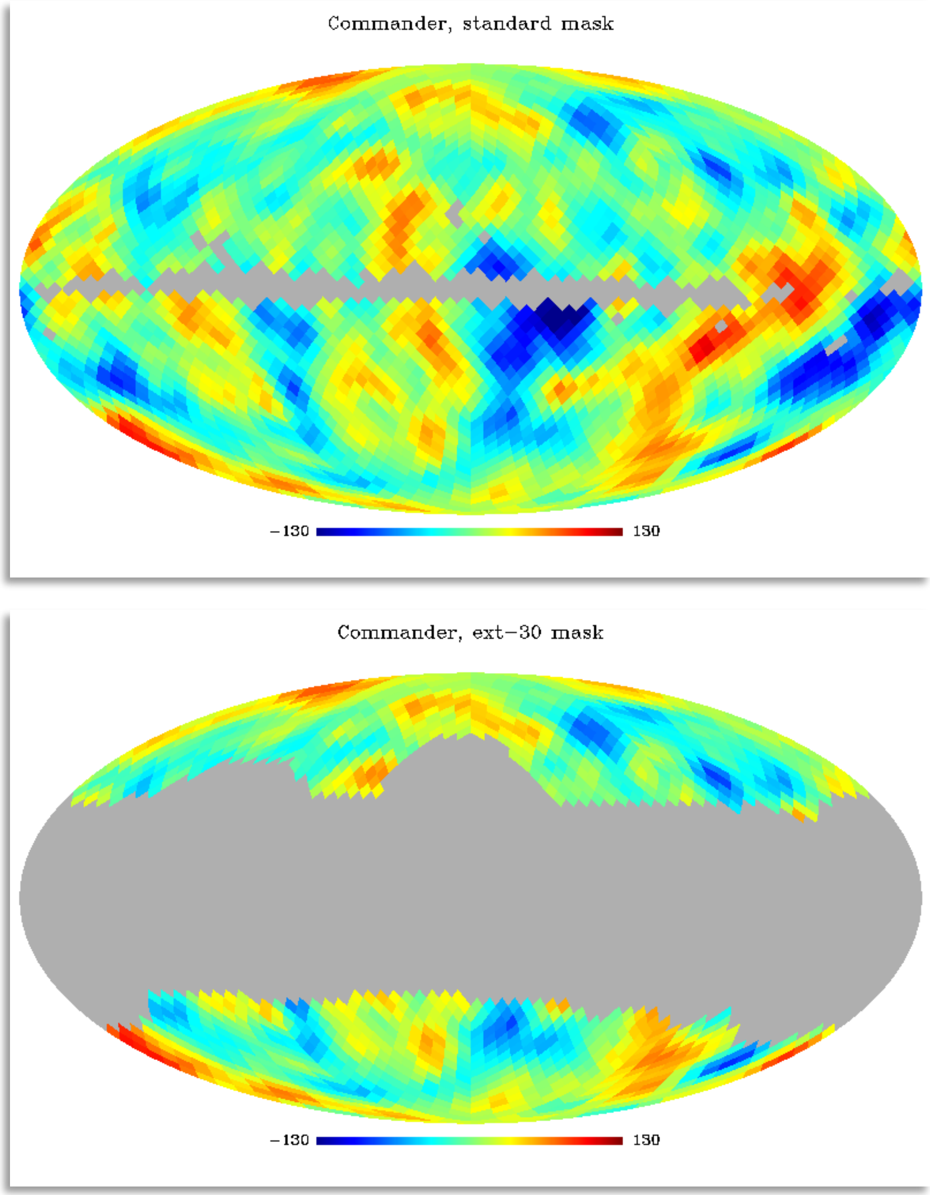
$> 99\%$ confidence level

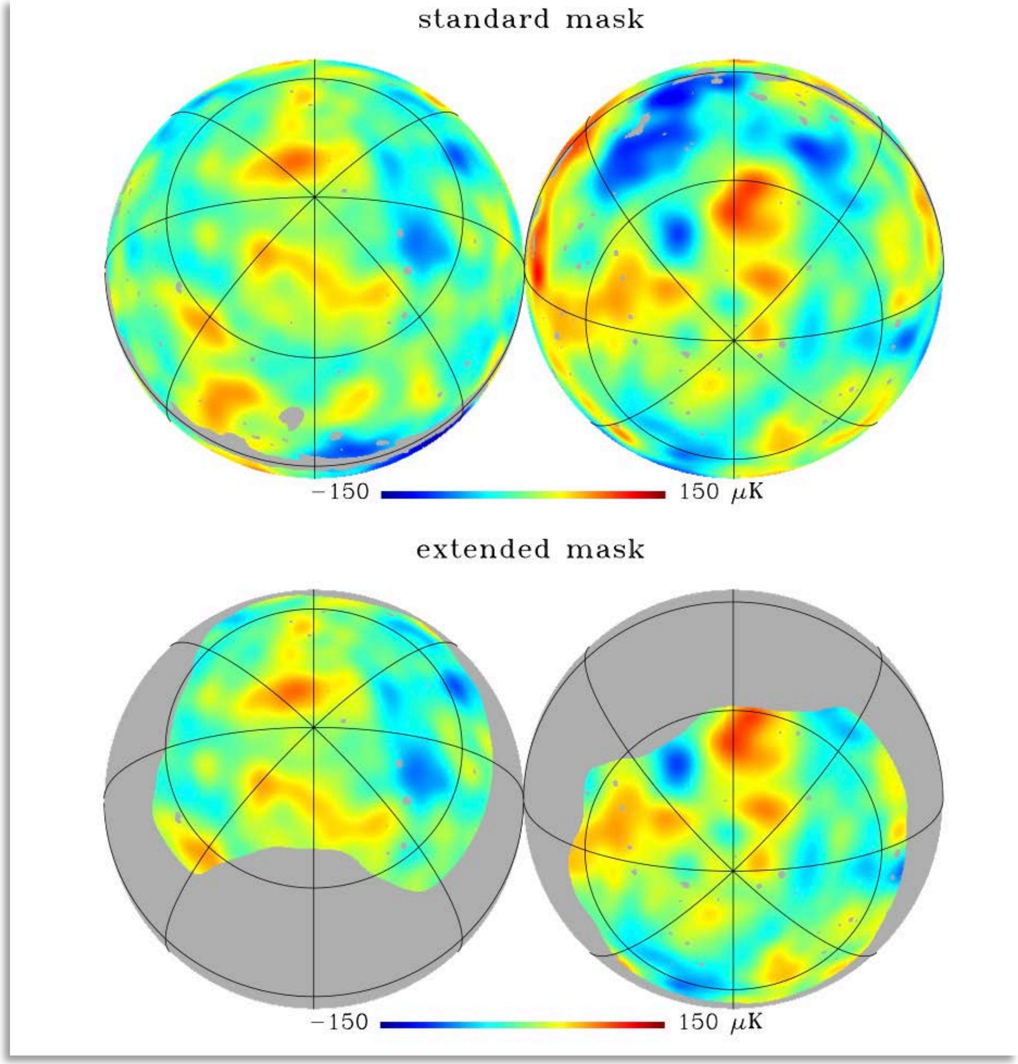
- What is the corresponding energy scale at onset of inflation?

$$\Delta^{Infl} \;\sim\; 2.4 \times 10^{12}\; e^{N-60}\;\mathrm{GeV} \;\sim\; 10^{12} - 10^{14}\mathrm{GeV} \;\; \text{for} \;\; \mathrm{N} \;\sim\; 60-65$$

Pre-Inflationary Relics in the CMB?

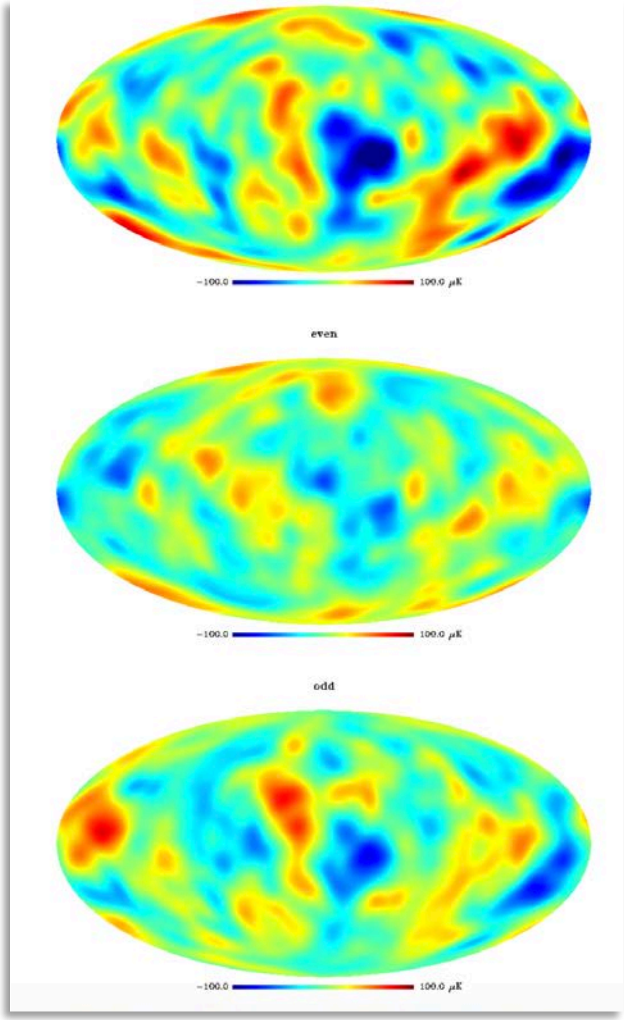
(Gruppuso, Kitazawa, Lattanzi, Mandolesi, Natoli, AS, 2017)

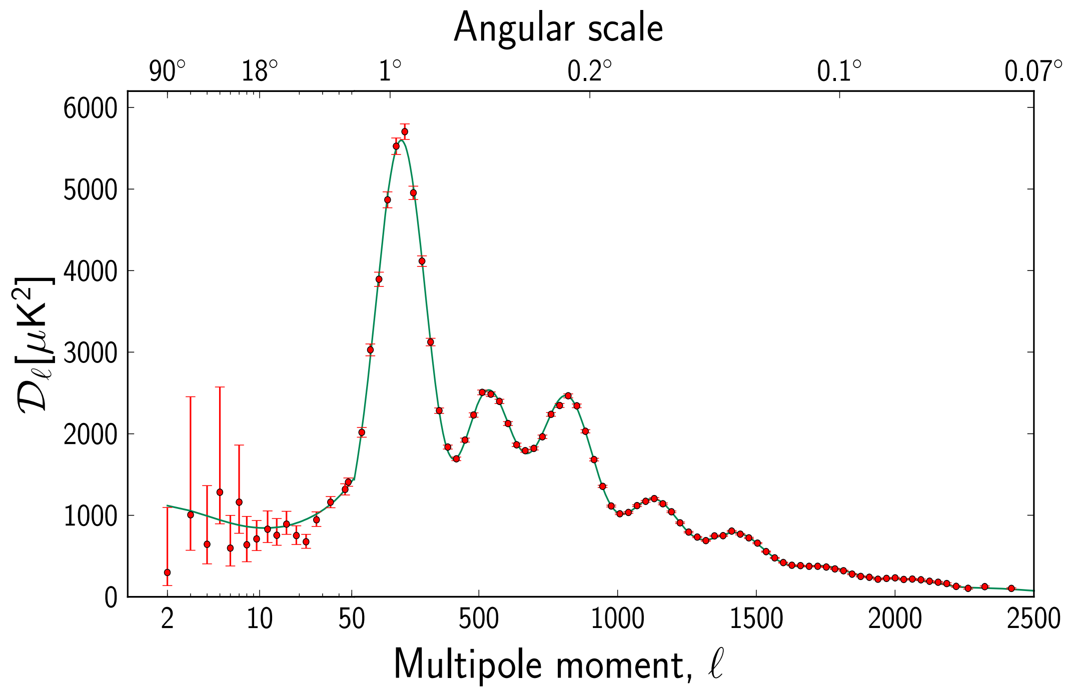


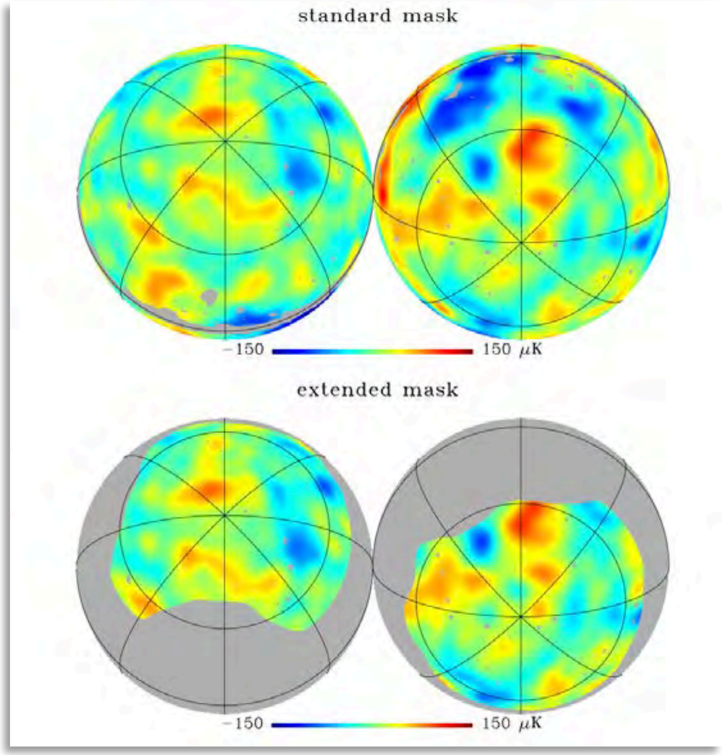


Even-Odd Asymmetry

(Gruppuso, Kitazawa, Lattanzi, Mandolesi, Natoli, AS, 2017)





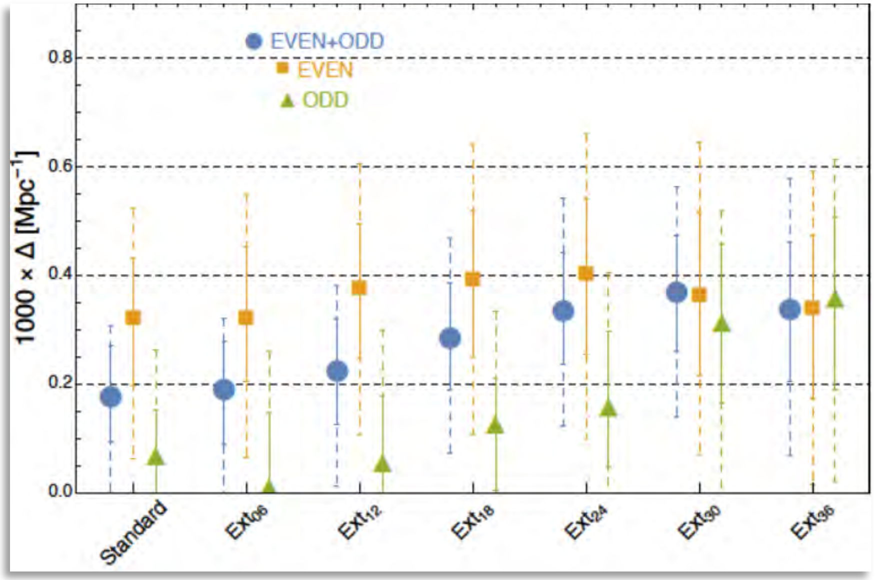


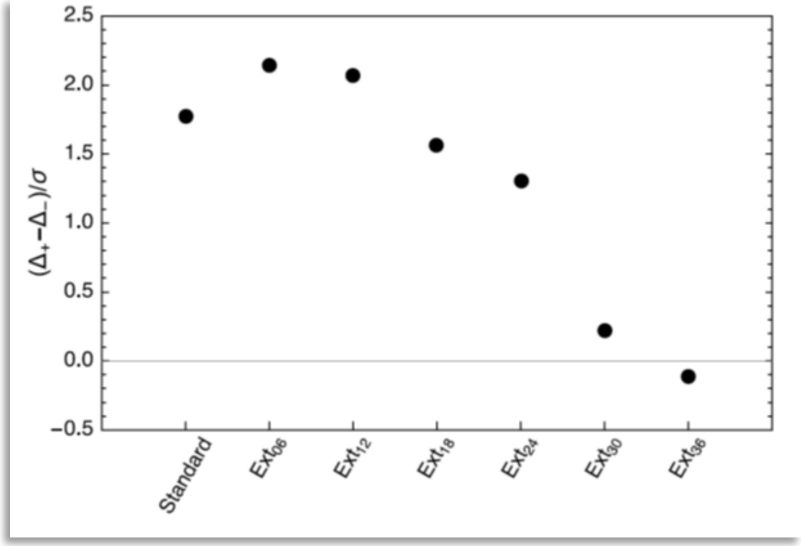
- **As we have seen:** Δ better detected **away from the GALACTIC PLANE**
- Where does the lack-of-power come from? **EVEN MULTIPOLES**
- **NOTE:** "even" map (middle left) far smoother than all (up) or odd (down) !

❖ **[EVEN vs ODD:** signature of oscillations near the transition? **]**

(Even & Odd) Detections of Δ

(Gruppuso, Kitazawa, Lattanzi, Mandolesi, Natoli, AS, 2017)

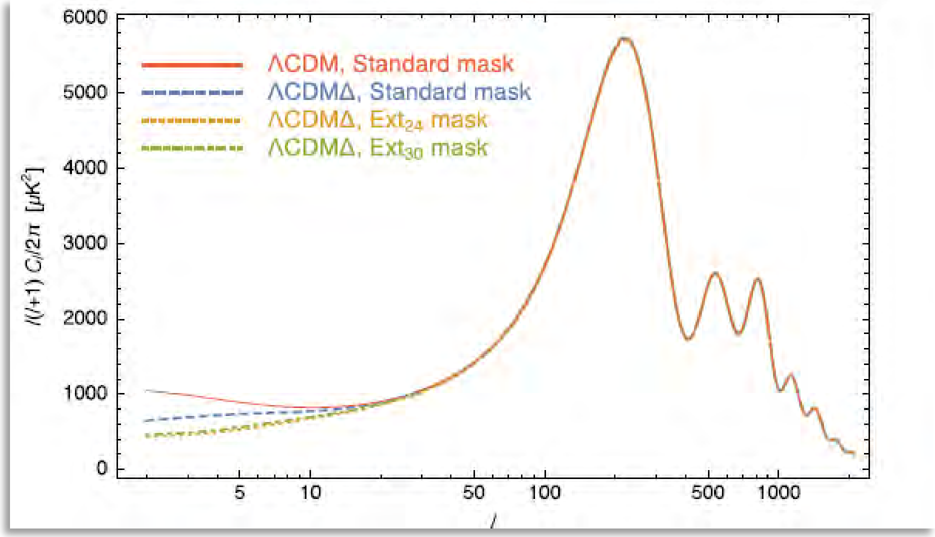


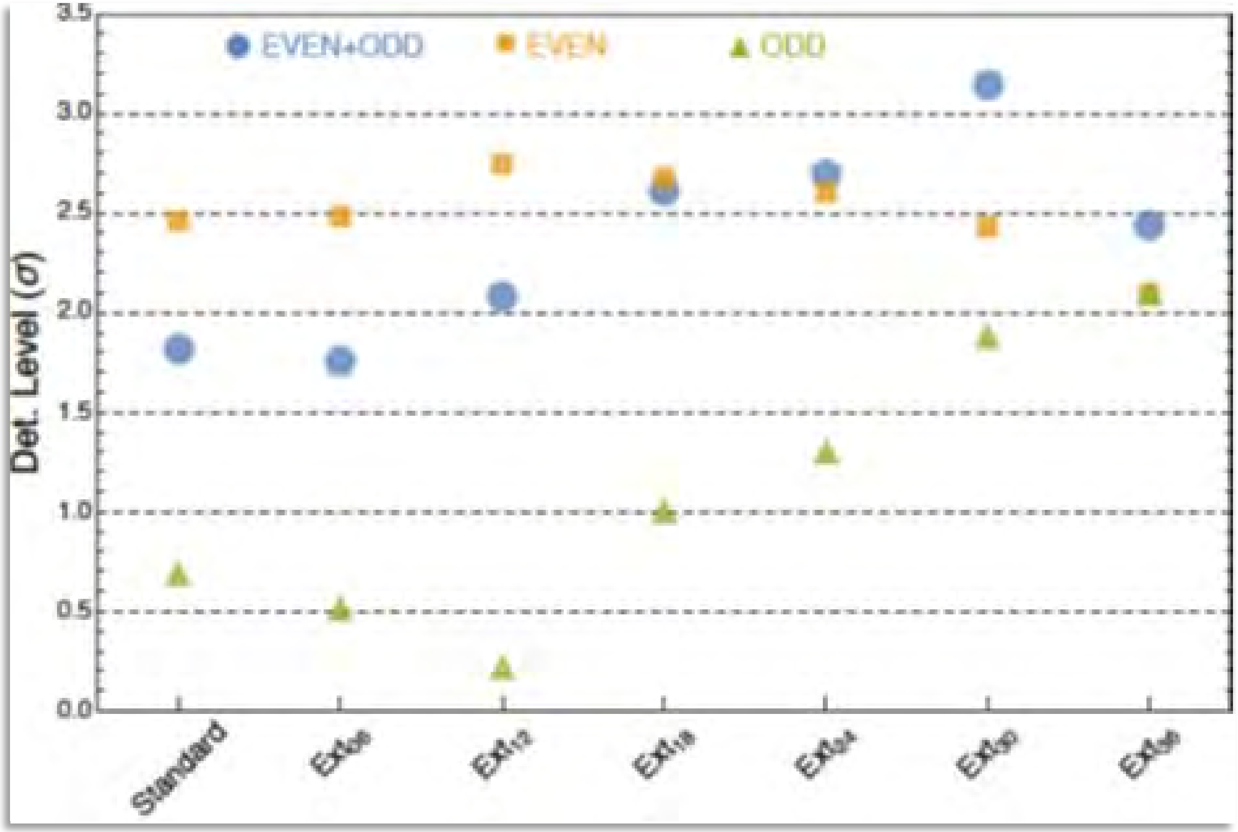


Case	Label	dataset	Detection level (%)	Detection level (σ)
a	Standard	full	93.26	1.83
a	Standard	even	98.59	2.46
a	Standard	odd	52.52	0.72
b	Ext_{06}	full	92.30	1.77
b	Ext_{06}	even	98.65	2.47
b	Ext_{06}	odd	41.03	0.54
c	Ext_{12}	full	96.41	2.10
c	Ext_{12}	even	99.39	2.74
c	Ext_{12}	odd	18.93	0.24
d	Ext_{18}	full	99.15	2.63
d	Ext_{18}	even	99.23	2.67
d	Ext_{18}	odd	69.80	1.03
e	Ext_{24}	full	99.32	2.71
e	Ext_{24}	even	99.05	2.59
e	Ext_{24}	odd	81.57	1.33
f	Ext_{30}	full	99.84	3.16
f	Ext_{30}	even	98.47	2.43
f	Ext_{30}	odd	94.37	1.91
g	Ext_{36}	full	98.60	2.46
g	Ext_{36}	even	96.27	2.08
g	Ext_{36}	odd	96.60	2.12

Future Prospects, I

(Gruppuso, Kitazawa, Lattanzi, Mandolesi, Natoli, AS, 2017)





- Δ does not affect standard ΛCMB parameters

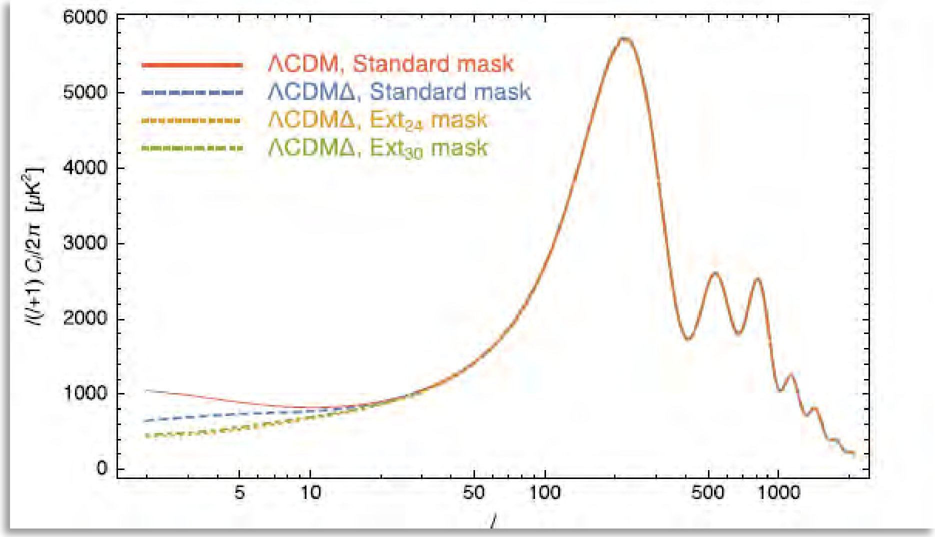
WHAT NEXT?

POLARIZATION

- cosmic-variance limited E-mode could lead to a 5-6 σ detection of Δ (or could rule it out)

Future Prospects, I

(Gruppuso, Kitazawa, Lattanzi, Mandolesi, Natoli, AS, 2017)



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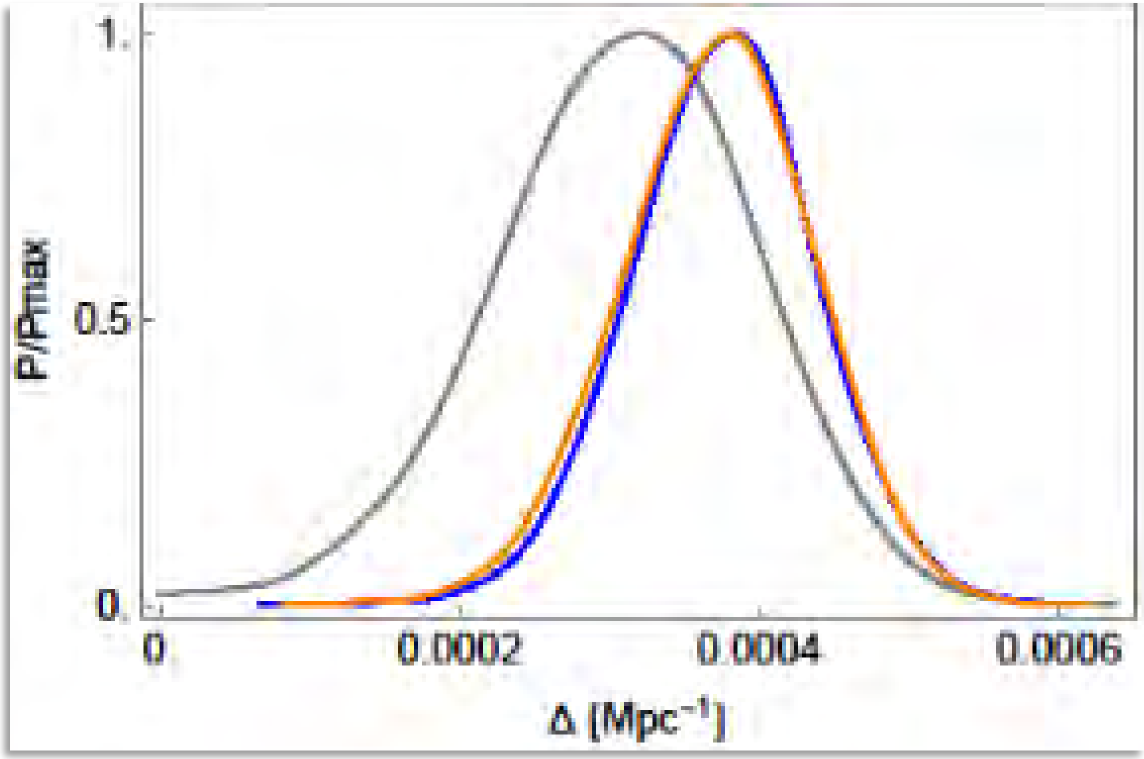
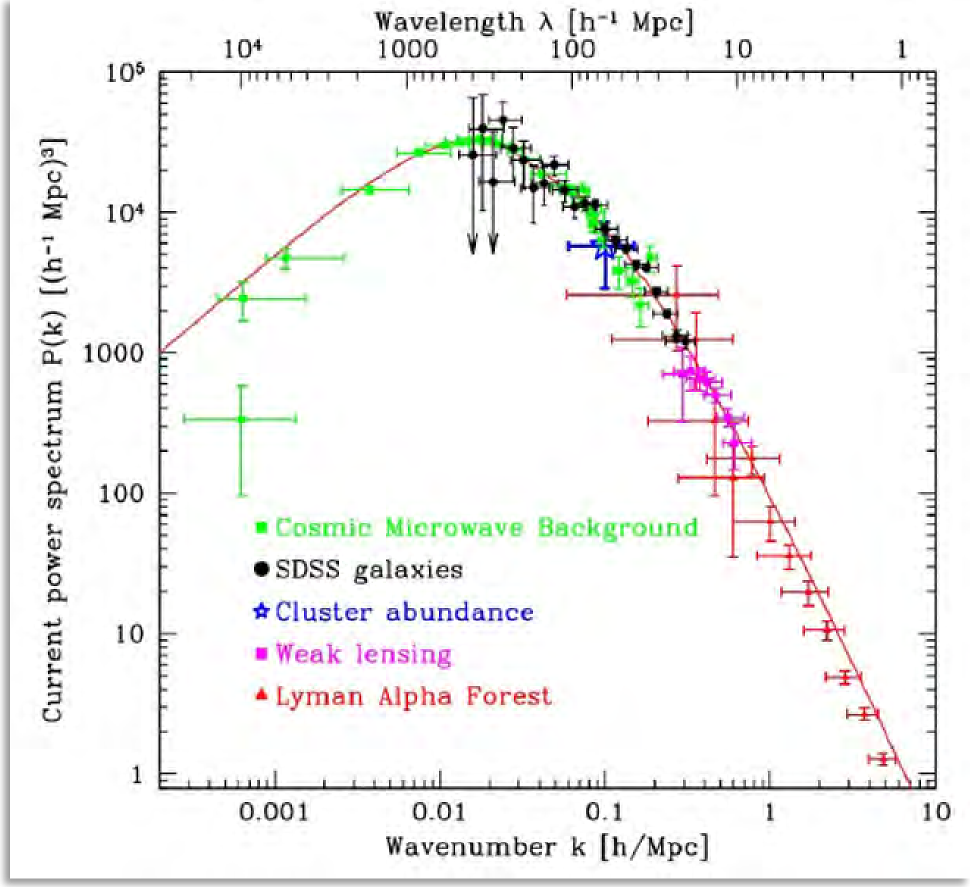
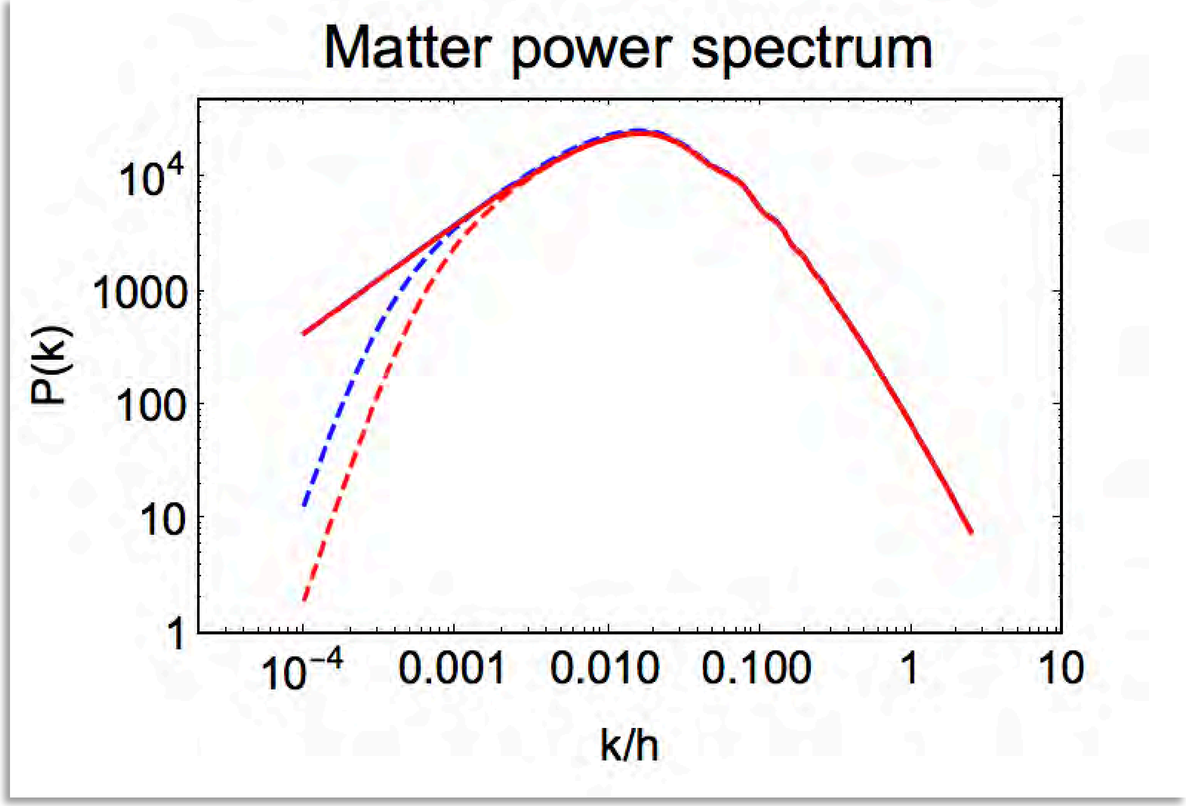


Future Prospects, II

(Gruppuso, Kitazawa, Lattanzi, Mandolesi, Natoli, AS, in progress)

OTHER IMPRINTS OF Δ? LARGEST - SCALE STRUCTURES





Constrained Superfields and 4D Models (SUSY branes & BSB)

BSB: 10D non-linear supergravity

(Dudas, Mourad, 2001)
(Pradisi, Riccioni, 2001)

❖ PROBLEM: couple a Volkov-Akulov-like goldstino to 10D Supergravity (+YM), ADAPTING (Akulov, Volkov, 1973)

- (Off-shell) 4D SUSY transformations on the goldstino can be induced, by analogy, from superspace shifts:

$$\delta\,\lambda_\alpha \;=\; \frac{1}{k}\,\xi_\alpha \;-\; i\,k\,\left(\theta^\alpha\,\sigma^\mu_{\alpha\dot\alpha}\,\overline{\xi}^{\dot\alpha} \;-\; \xi^\alpha\,\sigma^\mu_{\alpha\dot\alpha}\,\overline{\theta}^{\dot\alpha}\right)\partial_\mu\,\lambda_\alpha$$
$$\delta\,\overline{\lambda}_{\dot\alpha} \;=\; \frac{1}{k}\,\overline{\xi}_{\dot\alpha} \;-\; i\,k\,\left(\theta^\alpha\,\sigma^\mu_{\alpha\dot\alpha}\,\overline{\xi}^{\dot\alpha} \;-\; \xi^\alpha\,\sigma^\mu_{\alpha\dot\alpha}\,\overline{\theta}^{\dot\alpha}\right)\partial_\mu\,\overline{\lambda}_{\dot\alpha}$$
$$[\delta_2\,,\,\delta_1]\,\lambda_\alpha \;=\; 2\,i\left(\xi_1^\alpha\,\sigma^\mu_{\alpha\dot\alpha}\,\overline{\xi}_2^{\dot\alpha} \;-\; \xi_2^\alpha\,\sigma^\mu_{\alpha\dot\alpha}\,\overline{\xi}_1^{\dot\alpha}\right)\partial_\mu\,\lambda_\alpha$$

- SUSY transformations: act as special diffeomorphisms on $\hat{e}_\mu^a \;=\; \delta_\mu^a \;-\; i\,k^2\,\partial_\mu\,\lambda^\alpha\,\sigma^a_{\alpha\dot\alpha}\,\overline{\lambda}^{\dot\alpha} \;+\; i\,k^2\,\lambda^\alpha\,\sigma^a_{\alpha\dot\alpha}\,\partial_\mu\,\overline{\lambda}^{\dot\alpha}$
- Invariant (flat) goldstino action: $\mathcal{S} \;=\; -\,\frac{1}{2\,k^2}\int d^4x\;\det\left(\delta_\mu^a \;+\; i\,k^2\,\lambda^\alpha\,\sigma^\mu_{\alpha\dot\alpha}\,\partial_\mu\,\overline{\lambda}^{\dot\alpha} \;+\; \text{h.c.}\right) \;+\; \ldots$
- Can extend the methods for coupling to 4D supergravity, extending the work in (Samuel, Wess, 1973) and then, in 10D:

$$\mathcal{S} \;=\; \mathcal{S}_0 \;-\; \int d^{10}x\,\sqrt{-g}\,e^{-\phi}\left[1 \;+\; \frac{1}{4}\,\overline{\lambda}\Gamma^M D_M\,\lambda \;-\; \frac{1}{4}\,\overline{\lambda}\Gamma^M\psi_M \;+\; \frac{1}{\sqrt{2}}\,\overline{\lambda}\,\chi\right] \;+\; \ldots$$

- All orders in fermions ? Complicated since transformations are redefined …

(Bonnefoy, Dudas, Mourad, in progress)

Constrained Superfields in D=4

- Consider an O(N) invariant scalar model:
  - For $\lambda \to \infty$: a σ-model obtains, which describes the dynamics in the valley of minima

$$\mathcal{S} \;=\; \int d^D x \left[-\,\frac{1}{2}\,\partial_\mu \phi^T\,\partial^\mu \phi \;-\; \lambda \left(\phi^T\,\phi \;-\; \rho^2\right)^2 \right]$$

$$\mathcal{S} \;=\; -\int d^D x\,\frac{1}{2}\,\partial_\mu \phi^T\,\partial^\mu \phi\;, \qquad \phi^T\,\phi \;=\; \rho^2$$

- In the Wess-Zumino multiplet of 4D SUSY, when the scalar becomes very massive
  - Only a fermion with broken SUSY ($\lambda \to \infty$ below)

    Volkov-Akulov (1973) model (up to field redefinitions)

(Rocek, 1978)
(Casalbuoni, De Curtis, Dominici, Feruglio, Gatto, 1989)
(Komargodski, Seiberg, 2009)

$$\mathcal{S} \;=\; \int d^4x\; d^2\,\theta\, d^2\,\overline{\theta}\left[\,\overline{\Phi}\,\Phi \;+\; \lambda\,\left(\overline{\Phi}\,\Phi\right)^2\,\right] \;+\; \int d^4x\; d^2\,\theta\, f\,\Phi \;+\; \text{h.c.}$$

$$\lambda \;\to \infty \;\; : \quad \Phi \;=\; \frac{\psi^\alpha\,\psi_\alpha}{2\,F} \;+\theta^\alpha\,\psi_\alpha \;+\frac{\theta^2}{2}\;F\;, \qquad \Phi^2 \;=\; 0 \quad [F \neq 0]$$

  - Several N=1 → N=0 constraints were investigated in recent years.

4D Supergravity with Constrained Superfields

- Constrained superfields: a powerful and instructive tool in Supergravity
  - they allow to resort directly to the general N=1 expression for the potential

$$\mathcal{V} \;=\; e^{\mathcal{G}} \left[\left(\mathcal{G}^{-1}\right)^{i\bar{j}} \mathcal{G}_i \, \mathcal{G}_{\bar{j}} \;-\; 3 \right] \qquad \text{with} \qquad \mathcal{G} \;=\; \mathcal{K} \;+\; \log |\mathcal{W}|^2$$

while enforcing in it the constraints. Simple to build interesting examples: NO extra fields

  - Example 1 (Volkov-Akulov supergravity): (X^2=0)

$$\begin{aligned} \mathcal{K} &= -\,3 \log\left(1 \,-\, X\,\overline{X}\right) \\ \mathcal{W} &= f\,X \,+\, \mathcal{W}_0 \\ \mathcal{V} &= \frac{1}{3}\,|f|^2 \,-\, 3\,|\mathcal{W}_0|^2 \end{aligned}$$

(Antoniadis, Dudas, Ferrara, AS, 2014)
(Dudas, Ferrara, Kehagias, AS, 2015)
(Bergshoeff, Freedman, Kallosh, Van Proeyen, 2015)
(Antoniadis, Markou, 2015)

  - Example 2 (Volkov-Akulov-Starobinsky supergravity): (X^2=0, with some redefinitions)

$$\begin{aligned} \mathcal{K} &= -\,3 \log\left(T \,+\, \overline{T} \,-\, X\,\overline{X}\right) \qquad T \,=\, e^{\phi\sqrt{\frac{2}{3}}} \,+\, i\,a\sqrt{\frac{2}{3}} \\ \mathcal{W} &= f\,X \,+\, M\,X\,T \,+\, \mathcal{W}_0 \\ \mathcal{L} &= \frac{R}{2} \,-\, \frac{1}{2}\,(\partial\phi)^2 \,-\, \frac{1}{2}\,e^{-2\phi\sqrt{\frac{2}{3}}}\,(\partial a)^2 \,-\, \frac{M^2}{12}\left(1 \,-\, e^{-\sqrt{\frac{2}{3}}\,\phi}\right)^2 \,-\, \frac{M^2}{18}\,e^{-2\phi\sqrt{\frac{2}{3}}}\,a^2 \end{aligned}$$

  - Important additions by many authors: exit from inflation, other cosmological models,…
  - [BSB:

$$\mathcal{K} \;=\; -\,3 \log\left(T \,+\, \overline{T}\right) \,+\, \frac{X\,\overline{X}}{1 \,+\, \left(\frac{T+\overline{T}}{2}\right)^2 v\left(\frac{T+\overline{T}}{2}\right)} \,, \qquad \mathcal{W} \;=\; f\,X \,+\, \mathcal{W}_0$$

$$\mathcal{V} \;=\; e^{-\sqrt{6}\,\phi} \,+\, v\,(\phi)$$

]

(Ferrara, Kallosh, Linde, 2014)
(Dall'Agata, Zwirner, 2014)
(Bergshoeff, Freedman, Kallosh, Van Proeyen, 2015)
(Antoniadis, Markou, 2015)
……….

4D Toy Models with Constrained Superfields
(BPS-like: 2 → 1 breaking)

(Bagger, Galperin, 1997)
(Rocek, Tseytlin, 1999)
…………….

- **N=2 vector multiplet (1 vector + 1 chiral multiplet in N=1 language):**
  - one can consider the **decoupling limit** for a very massive chiral multiplet
  - **partial breaking N=2 → N=1** occurs, with "magnetic" coupling m
  - non-linear constraint: $$W^\alpha W_\alpha \;-\; 2\,\Phi\left(\frac{1}{4}\,\overline{D}^{\,2}\,\overline{\Phi}\;-\;m\right)\;=\;0\;\longrightarrow\;\Phi^2\;=\;0\;\;\Phi\,W_\alpha\;=\;0$$
  - N=2 language: $$\mathcal{X}\;=\;\Phi\;+\;i\sqrt{2}\,\tilde{\theta}^\alpha\,W_\alpha\;-\;\tilde{\theta}^2\left(\frac{1}{4}\,\bar{D}^2\,\bar{\Phi}\;-\;m\right)\;\longrightarrow\;\mathcal{X}^2\;=\;0$$
  - highest component yields **SUSY Born-Infeld theory**: $$D^2+2\,F(\overline{F}+m)-\frac{1}{2}\,F_{\mu\nu}\,F^{\mu\nu}-\frac{i}{2}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}\;=\;0$$
  - can extend to a number n of **abelian** vector multiplets: $$d_{ABC}\left[W^{\alpha\,B}\,W^C_\alpha\;-\;2\,\Phi^B\left(\frac{1}{4}\,\overline{D}^{\,2}\,\overline{\Phi}^C\;-\;m^C\right)\right]\;=0$$

(Hughes, (Liu), Polchinski, 1986)
(Antoniadis, Partouche, Taylor, 1996)
(Ferrara, Girardello, Porrati, 1996)

(Ferrara, Porrati, AS, 2014)
(Ferrara, Porrati, AS, Stora, Yeranyan, 2014)

- **MICROSCOPICALLY:** toy models for separated D-branes. **Non-Abelian lessons?**

4D Models with Constrained Superfields
(BSB-like: 2 → 0 breaking)

- **N=2 vector multiplet (1 vector + 1 chiral multiplet in N=1 language):** (Dudas, Ferrara, AS, 2017)
  - one can consider a different **decoupling limit**, leaving behind a vector and two goldstini
  - **Non-linear constraint:**
    $$\Phi\, W^\alpha W_\alpha \,-\, \Phi^2 \left(\frac{1}{4}\, \overline{D}^{\,2}\, \overline{\Phi} \,-\, m \right) \,=\, 0 \,\longrightarrow\, \Phi^3 \,=\, 0 \quad \Phi^2\, W_\alpha \,=\, 0$$
  - **In N=2 language:**
    $$\mathcal{X} \,=\, \Phi \,+\, i\sqrt{2}\, \tilde{\theta}^\alpha W_\alpha \,-\, \tilde{\theta}^{\,2} \left(\frac{1}{4}\, \bar{D}^2\, \bar{\Phi} \,-\, m \right) \,\longrightarrow\, \mathcal{X}^3 \,=\, 0$$
  - **Away from a singular 2→1 corner:** complex scalar in terms of the vector and gaugini

$$\begin{aligned}
Z &= \mathcal{O} \,+\, u\,\Box\,(\overline{u}\,\Box\,\mathcal{O}) \\
\mathcal{O} &= \frac{b}{a} \,-\, u\,\Box\left(\frac{\bar{b}}{\bar{a}}\right) \\
u &= \frac{(\psi\,\psi)\,(\lambda\,\lambda)}{a_0^2} \\
a &= D^2 + 2\,F(\overline{F}+m) - \frac{1}{2}\,F_{\mu\nu}\left(F^{\mu\nu} + i\,\widetilde{F}^{\mu\nu}\right) - 2\,i\,\psi\,\sigma^\mu\,\partial_\mu\bar{\psi} - 2\,i\,\lambda\,\sigma^\mu\,\partial_\mu\bar{\lambda} \\
a_0 &= D^2 \,+\, 2\,F(\overline{F}+m) \,-\, \frac{1}{2}\,F_{\mu\nu}\left(F^{\mu\nu} + i\,\widetilde{F}^{\mu\nu}\right) \\
b &= (\overline{F} \,+\, m)\,\psi\,\psi \,-\, i\sqrt{2}\,\psi\,(D \,+\, i\,\sigma^{\mu\nu}F_{\mu\nu})\,\lambda \,+\, F\,\lambda\,\lambda
\end{aligned}$$

Thank You