Comments received (status: 11 December 12:00)

1 (page 2) 20171124-180604 Mandy Cooper-Sarkar
2 (page 5) 20171124-192632 Achim Geiser
3 (page 6) 20171127-104013 Iris Abt
4 (page 7) 20171127-133650 Sergey Levonian
5 (page 8) 20171130-184207 Joerg Gayler
6 (page 13) 20171201-133658 Misha Lisovyi (plus attachment)
7 (page 14) 20171202-170022 Brian Foster
8 (page 24) 20171203-174808 Dietrich Wegener
9 (page 25) 20171204-135338 Stefan Schmitt
10 (page 28) 20171205-121157 Ewald Paul
11 (page 29) 20171207-214529 Boris Levchenko.txt
12 (page 30) 20171208-004918 Katja Krueger
13 (page 32) 20171208-055147 Dieter Haidt (plus attachment)
14 (page 33) 20171208-173515 Jan Olsson and Nelly Gogitidze
15 (page 37) 20171208-184645 Achim Geiser: answer to Jan and Nelly
16 (page 38) 20171210-115907 Max Klein
17 (page 41) 20171211-000318 Oleg Kuprash
18 (page 43) 20171211-131140 Peter Truoel (plus attachment)
19 (page 44) 20171211-141058 Daniel Britzger
Attached at the end of the pdf file:
$=================================$
page 51: Attachment by Misha Lisovyi (pdf draft with comments in yellow)
page 106: Attachment by Dieter Haidt (word document)
page 111: Attachment by Peter Truoel (pdf document)

From Amanda.Cooper-Sarkar@physics.ox.ac.uk Fri Nov 24 19:06:18 2017
Date: Fri, 24 Nov 2017 18:06:04 +0000
From: Amanda Cooper-Sarkar [Amanda.Cooper-Sarkar@physics.ox.ac.uk](mailto:Amanda.Cooper-Sarkar@physics.ox.ac.uk)
To: Karin Daum [karin.daum@desy.de](mailto:karin.daum@desy.de), "h1zeus-eb17bc@desy.de" <h1zeus-eb17bc@desy .de>
Subject: RE: [h1zeus-eb17bc] Draft 1.0 _Native English speaker check
This is mostly about English style-- but there are just a few comments and quest ions inside it!

Line 28 no comma after 'momenta'
Line 29 'scale' should be 'scales'
Line 34 say either 'different' or 'various' but not both
Line 46 don't need the word 'further' and it sounds clumsy
Line 52 'given above' sounds better than 'given before'
Line 55/57 re-arrange sentence ' Therefore the charm and beauty contributions ca n be disentangled by using observables directly sensitive to th lifetime of the decaying heavy flavoured hadrons' then it reads smoothly
Line 58 suggest a colon after 'heavy flavoured hadron'
Line 60 suggest a semi colon before'the number of tracks' and again before 'the invariant mass;
Otherwise this last sentence of the paragraph is difficult to follow.
Line 68 'B mesons'
Line 73 suggest that we don't need the words 'under consideration'
Line 76 'and the large statistical correlations'
Line 82 something looks odd in the spacing of 'data set'
Line 98 the flow of the sentence does not need the word 'predictions' before 'us ing ABMP16'
Line 102 I think this says the FONLL-C scheme extended by low-x resummation is $u$ sed, but FONLL-C without low-x resummation is also used, so after the bracket it needs to say 'both with and without low-x resummation'.
Line 105/6 I think it would flow better as 'from HERA to make and NLO determinat ion of the running charm and beauty quark masses, as defined in the QCD Lagrangi an in the modified minimum-subtraction scheme.'
Line 111 'frameworks for'
Line 113/4/5 to improve the flow 'in section 4 and in section 5 they are compare d.....and VFNS.'

Line 117 'dependence'
Line 118 'Finally, the conclusions are given in Section 7'
Section 2
' combined analyses'
Line 126 cut the word 'only'
Line 147 'in[26] using scale dependent (running) heavy quark masses'
Line 158 the phrase 'in the MSbar scheme' is redundant
Line 168 don't need the word 'also' since you began the sentence with 'In additi on'

Section 3
Line 213/4'The different forms of the convolution integral for \sigma** and \si gma** necessitate the consideration of different sets of theory parameters'
Line 216 '....limits to estimate the....'
Line 231 'kept fixed'
Just one physics question here - the conditions for the HERAPDF1.0 are mostly th e same as those for the present analysis, which seems appropriate- but this is $n$ ot so for the renormalisation and factorisation scales... you don't comment on $t$ he extent to which this matters/or doesn't matter? I don't quite see how a corre sponding PDf can be used if the settings are not quite the same?

Line 237 comma after 'assumptions' and
Line 239 comma after 'tagging' makes the sentence easier to read.
Line 242 'in addition to those needed for \sigma^\{th\}_\{red\}'
Line 273 'to any order provided these calculations include uncertainties for pot enetial deviations from the 'true' result'

Section 4
Line 321 , are reduced significantly - by up to factors of two or more'
Line 329 ' and these reductions are independent of $\mathrm{X} \_\{\mathrm{Bj}\}$ and $\mathrm{Q}^{\wedge} 2^{\prime}$
Line 337 ' and reaches about $15 \%$ at small $x \_\{B j\} . .^{\prime}$
Line 342 'in precision of about $20 \%$..'
Line 343 'reaches about $30 \%$ in the range..'
Table 2 caption: typo in 'uncertainties' in the last line
Table 4 caption: 'in units of $\backslash$ sigma after the first iteration of the combinatio n'
A question the Tables give total systematic uncertainties. Shouldn't we tell the reader that the full correlations are also available, and where to get them at this point in the text?- at least in a footnote, or refer to where we do tell th em?
Figures 4 and 5 captions: I think the phrase 'For better visibility' is better t han 'For presentation purposes'

Section 5
Line 362/3- don't we also compare to the NNPDF calculation without the low-x res ummation? The text reads as if we only compare to the low-s resummation version.

Line $372-I$ suppose you may be asked to justify why the uncertainties of the HERA PDF FF that you consider are only the experimental and not also the ALL model/pa ram and why you include scale uncertainty but not say alphas uncertainty??

Fig 14 caption: 'similar size as those shown for the FONLL calculations' (no nee d for the word 'plain'
Line 429 'agree well with the previous measurement given that the theoretical ca lculations show tension in describing the underlying process'
Table 5 caption: says the \chi^2 and d.o.f are given, but the d.o.f is not given what is given is the number of data points and the p-values.
Also in the final sentence of the caption after the comma say 'reducing the numb er of data points to 47'

Section 6
Line 439 'The theory description of the $x_{-}\{B j\}$ dependence of the reduced cross section of charm production is also investigated'
Line 473 no need for the word 'independently'
Line 486 'noticeably' is misspelt
Line 499 'to the inclusive data only fixing...' leave out the word 'with'
Line 515 'typically a few MeV'
Line 517 'from all other variations of the parameters'
Line $534{ }^{\prime}$ The sensitivity to..'
Line 535 'demonstrate that' (there are two parts to the subject so the verb must be the plural)
Line 536 'from inclusive HERA data alone' -the word 'data ' is missing...and I t hik you mean 'are not sensitive in this framework' athough it is also tru that $t$ hey are not sensible!
Line 540 'by this fit' rather than 'by the fit' makes it clearer
Line 544 'especially that of the gluon and because the description of the data i s similar...'
Fig 15: caption 'obtained from the present QCD fit'
Fig 16: caption 'determined by the present fit'
Fig 17: same as for Fig 16
Line 551 'which are more precise than the heavy flavour measurements in the kine matic region of overlap'

Section 6.3
Just a comment while reading paragraph 566/579- it seems to me we have now refer red to 'the present fit' or 'the fit of this analysis' many times and it is gett ing a bit clumsy. Why don't we give 'the fit of this analysis' a name we can ref er to it by? Like HERAPDF FFmemb.

Line 581 drop the word 'with' and 'reqiting different values of the minimum $x$ _ $\{B$ j' don't repeat the word 'values'
Line 582 comma before 'in the range'

Line 589/590 put commas around the phrase 'shown in....at the scale..GeV^2' Fig 20 caption: 'from the present fit'-unless we give it a name Fig 22 caption: 'from the present QCD fits..' the caption mentions full lines $f$ or the yellow fit and says nothing about the graphics of the blue fit-doesn't th is need to be clarified for black and white?

Paragraph beginning Line 593 at this point the reader is getting really confused
as to which fit is which. If we don't name our fits then we will need to say so mething like (note the re-arranged order) :
Line 594/6 'obtained from the present fit to the heavy flavour data and the full inclusive data set and from the alternative fit in which inclusive data are sub ject to the cut $x_{-}\{B j\}>0.01$, to the reference cross-sections of HERAPDf2.0 FF3
$A^{\prime}$ Yes I think you need to specify what the reference cross section is again sin ce the reader has so MANY fits to keep in mind.
Line 597 'imposed on'...'are rising more strongly' (adverb not adjective)
Line 599 'an $x_{-}\{B j\}$ cut'
Line 602 ' three PDF sets discussed in the last paragraph'
Line 603 drop 'also'...Figure 24 rather than Figures
Line 608 'predict' since calculations is plural
Line 612 'it does not seem possible to resolve the $\sim 3 \backslash$ sigma tension in describin g..' (no need to say 'in theory' when the sentence starts with 'In the theoretic al framework..'
Line 613 'from HERA using this simple approach of changing..'
Line 616 'than that observed at NLO'
Line 617/8'some tensions in the theoretical description of the inclusive DIS da ta'
Line 620 'However, a dedicated investigation shows 'r' you may be asked to say a bit more about this? Like what did you actually do?
Fig 24 caption: 'from the present fits'
Section 7
Line 647 cut the word 'mainly' - it is said in the text, it sounds clumsy in the conclusion
Line 648 I would also cut 'especially the' here, it reads better, detail can be in the text.

That's it
Mandy
-----Original Message-----
From: h1zeus-eb17bc-request@desy.de [mailto:h1zeus-eb17bc-request@desy.de] On Be half Of Karin Daum
Sent: 23 November 2017 09:37
To: h1zeus-eb17bc@desy.de
Subject: [h1zeus-eb17.bc] Draft 1.0
Dear EB members,
please find attached the version 1.0 of the draft ready for circulation within $t$ he collaborations

Best regards
Karin

From geiser@mail.desy.de Fri Nov 24 19:26:37 2017
Date: Fri, 24 Nov 2017 19:26:32 +0100 (CET)
From: Achim Geiser [geiser@mail.desy.de](mailto:geiser@mail.desy.de)
To: "h1zeus-eb17bc@desy.de" [h1zeus-eb17bc@desy.de](mailto:h1zeus-eb17bc@desy.de)
Subject: RE: [h1zeus-eb17bc] Draft 1.0
Dear all,
Since I had the privilege to contribute a bit to the editing, most of my comments were accounted for already.

One additional one which we discussed during the presentation:
I suggest to add a table with the breakdown of the uncertainties for
the mass measurements, e.g. similar to the one given in Table 20 of DESY-14-083 (beauty mass measurement).

And a minor textual suggestion after rerereading:
Line 105/6: Please add a comma after "Lagrangian", since the scheme refers to the mass running derived from the Lagrangian and not to the Lagrangian itself, which doesn't have a scheme.

Best regards, Achim

From isa@mppmu.mpg.de Mon Nov 27 10:40:23 2017
Date: Mon, 27 Nov 2017 10:40:13 +0100
From: Iris Abt [isa@mppmu.mpg.de](mailto:isa@mppmu.mpg.de)
To: "h1zeus-eb17bc@desy.de" [h1zeus-eb17bc@desy.de](mailto:h1zeus-eb17bc@desy.de)
Subject: [h1zeus-eb17bc] Suggestions
Dear Karin,
I strongly support Mandy's suggestion to name the fits.
As for all the parameters of the fits and the question what is what. How about a table with all the settings?

Best,
Iris

From levonian@mail.desy.de Mon Nov 27 13:37:01 2017
Date: Mon, 27 Nov 2017 13:36:50 +0100 (CET)
From: Sergey Levonian [levonian@mail.desy.de](mailto:levonian@mail.desy.de)
To: Karin Daum [karin.daum@desy.de](mailto:karin.daum@desy.de)
Cc: h1zeus-eb17bc@desy.de
Subject: Re: [h1zeus-eb17bc] Draft 1.0

Dear Karin,
I have only few minor remarks to the new draft.

1. Now we use three different "jargon names" for the same theory calculation:
-- low-x resummation (e.g. in Intro, line 102)
-- log $x$ resummation (l. 432)
-- $\log 1 / x$ resummation (l. 634, Conclusions)
Showld it be unified?
2. Figures 2,3 look redundant to me.

All necessary information is present in Fig.4-6 anyway.
3. Cosmetics:

- I suggest to move column "N_b" immediately after the column "N_c" in table 1.
- l. 100: O(as)^3 -> O(as^3) (move power 3 inside the brackets)
- l. 163: here you use \cal\{O\} for the "order of", while before
that (lines 95-108) and after (l. 102) - plain O is used.
- Figures displaying PDFs (fig. 15, 22) do not have global "H1 and ZEUS" title. Is that by purpose? To show that we have nothing to do with that? ))
- Concerning Fig. 20 we already discussed this during T0 meeting: (label ccbar, bbar for sigmas; less number of x-grid lines; etc.)

Sergey

From gayler@mail.desy.de Thu Nov 30 18:42:13 2017
Date: Thu, 30 Nov 2017 18:42:07 +0100 (CET)
From: Joerg Gayler [gayler@mail.desy.de](mailto:gayler@mail.desy.de)
To: h1zeus-eb17bc@desy.de
Cc: joerg.gayler@desy.de
Subject: [h1zeus-eb17bc] Combination and QCD analysis of beauty and charm

Dear editors of the paper on combined charm and bottom,
congratulations that you could finish this complex analysis. I appreciate very much that this final combination was possible leading to improved HERA results.

I wonder why the parameters of the main fit are not given.
The discussion with Fig. 24 I find a bit overdoing. That a quite different gluon distribution does not describe the NC inclusive any more can be expected.

Several points are unclear to me and I hope they can be phrased more clearly. I have many questions and suggestions.

At several places $I$ find some confusion of the content of figures, main text and captions.

Going through the draft in sequence:
1 Introduction
------------------

29 scale --> scales
54, 63 not very clear what the "fragmentation fractions" mean in this context.

84 meaning a bit unclear due to the length: "suitable for comparison" may be intended to refer to the "consistent dataset", but it reads at first like referring to the cross-correlations, in spite of the comma.

95-103 The acronyms RTOPT, FONLL-C have some meaning, different from acronyms of names like ABKM09. The meaning should be given as you do e.g. for FFNS, VFNS, NLO, NNLO.

311 suggest frameworks are ---> framework is
2 Open heavy flavor production in DIS
$119+2$ suggest: analyses combined ---> combined analyses
(more easy to understand)
equation 1: I find alpha^2 (Q^2) difficult to read, at first sight somewhat misleading. Suggest (alpha(Q^2))^2

126 Ref[42] gives a calculation for $\mathrm{F}_{\mathrm{H}} \mathrm{L}$. The way it is quoted here, one expects a solid experimental result.
Suggest: In charm production it is expected to reach
Or to make it weaker in some other way.
158 there is an extra "in the MS bar scheme." in this line.
163 O(m_Q^2) has unusual O.
168 NLL: "next-to-leading log" somehow clear, especially with next line,
but better explained already together with FONLL in line 101.
3 Combination
188 may be somewhat confusing. Not very clear whether data set 8 of present table 1 is superseded, but still listed?
Data set 9 refers to present table 1, data set 8 not?
204 HVQDIS is a name not a scheme, but I still suggest to write The program for heavy quark production in DIS HVQDIS [43]

250 fro ---> from
253, 254 the difference to $244-252$ should be more obvious.
It took me quite some time to notice, that now we talk about fractions.
May be bring only fractions in bolt or include these lines in the upper bullet.

270-274 somewhat difficult to read. Maybe it helps if in the sentence the theoretical ratio of eq. (3) appears, to make clear that the mentioned cross sections have nothing to do with experiment.
Suggest e.g.: While the central values of
sigma_red^th (x_Bj, Q^2)/sigma^th_vis,bin (see eq. 3) are obtained
Suggest also to write "deviations from the unknown "true" QCD
result" to help understanding.
287 which compreiseS ---> which comprise
eq. 4 the delta_i,e,statmu^i,e and delta_i,e, uncorrm^i look like deltas. I wondered some time, why the deltas are the relative uncertainties and not the sigmas. Suggest to write
(mu^i,e \cdot delta_i,e,stat)^2 and (m^i \cdot delta_i,e, uncorr)^2. I find this much more easy to grasp.

292,293 I have the impression that this sentence is not correct or difficult to understand. What refers "they change" to? What is changing? The uncertainties? The cross sections? Why changing? To say "they are assumed to be proportional to the expected central values" seems more clear to me. Is this what is meant?

4 Combined cross sections
319 I have problems to understand.
I take the second row in table 4. I guess these are mu_r, mu_f variations.
Nominal 1 sigma corresponds to factor 2 variation.
Following the description of the caption of Table 4:
0.82 sigma $=0.82 * 2=1.64$. If I shift by that: $2-1.64=0.36$

Thus the effective (fitted) upward varied scale is
mu_r,nominal (1 + 0.36).
Looking to the downward variation:
$0.5 * 0.82=0.41$. A shift by that: $0.5-0.41=0.09$
Then the new variation would be from
(1-0.09) mu_r, nominal to (1 + 0.36) mu_r, nominal.
I am sure this is not what you do.
Is it rather that you use as the new uncertainty
(1-0.41) mu_r,nominal to (1.64) mu_r,nominal?
But then 0.82 is not a shift, rather a scale factor.
And what is the reduction factor? The reduction of the uncertainty range? In the last case $I$ get 0.8 , not 0.45 .

So what is exactly done? For the time being I assume that the text should be more clear or explicit (here or in table 4 caption).

321 "reduced to factors 2 or larger": suggest "reduced by factors $1 / 2$ or smaller"

5 Comparison with theory predictions
397 see points to Fig. 10 (11), 12 (13) below.
412 prefer here: which aims FOR
420 To the data?
421 "p-values" is a bit jargon like. fit probabilities?
429 "if"? Logic? Meant like "as"?
6 QCD analysis
I wonder why the PDF parameters of the central fit are not given.
The paper is long and explicit and presents the detailed equation (6),
but not the results.
457 The chi^2 definition is a central issue of the analysis and should be given fully within the present paper explaining the difference with respect to eq. (4).

466 Naive question: where are the non-valence, non-anti $u(x), d(x), s(x) ?$
473,474 Procedure is unclear, because you include the parameters
"independently" and only "one at a time". How can the procedure then arrive at more than one of these parameters?

479 what is the basis of this alpha_s variation? A reference?
514 suggest: effects on the model uncertaintieS --->
effects on the model uncertainty
(the somewhat difficult sentence more easily to understand)
527 a bit disappointing that we just state that we have done something and claim agreement but give not the result of the study.

536-578 I do not understand: The uncertainties are covered?
What $I$ see is that the result $1.8+0.14-0.13$ is quite far away from 1.29 (even more from PDG value). This indicates some inconsistency in the fit of inclusive data only. So the reader regrets not to get other uncertainties beyond "fit".

545 one could also add that the fitted quark masses are not far from the ones previously used.

549-552 A bit difficult to understand: "which are more precise"? More precise than charm and beauty data or more precise in this kinematic region?

You mention then the "few per cent" differences, but in spite of "dominance of inclusive" data there are sizable differences in the region of large $x$ in Fig. 19, where there are no bottom (and charm) data. A comment?

573 Unclear what this average for a given $x, Q 2$ point means. You average over the acceptance of the detector contributing to the cross section? Fig. 20 and caption tell nothing about averaging.

577,578 it may still be consistent with being independent of Q2, but Q2 > 200 does not support this and looks different.

Footnote 7 "the data have been used in the fit": this is always the case.

Suggest "the data have been used in the fit to adjust theoretical uncertainties. Therefore the theory"

587 if this shallow minimum of $c+b$ only is worth mentioning, then we can not say that $D I S+c+b$ is increasing in this region, it is rather falling again.

588 it would be convenient to have the degrees of freedom together with the 91 here available.

602-608 I find Fig. 24 a bit overdoing. It is fairly obvious that a much steeper gluon distribution as shown in Fig. 22 has also influence on the inclusive NC cross section. I suggest to skip this plot.

613 it reads a bit naive that one may expect to describe the inclusive NC data using a gluon distribution which is clearly outside the uncertainty band of NC gluon fits.

620, 621 Some indication what in this "dedicated investigation" is done? Now it reads a bit strange. The mass measurement is considered by the authors as important, they show some problems with theory and then just tell that these are unimportant for the main result (at least the abstract gives the impression that this is the main result).

647 reads a bit naive, see 613.

Table 1: why N_b not put directly right of Nc?
caption: Tagging not mentioned. It should also get some explanations, especially VTX.

Tables 2,3: caption: $I$ am afraid, also $Q 2$ and $x$ must be mentioned
Table 4: second row: theory, scales. I guess these are mu_r, mu_f. Should be indicated for clarity.
Caption: reduction of what? Suggest: reduction of correlated uncertainties.
Also main text is short here. See also comment to line 319.

Table 5 : Suggest to add in first column to "HERA 2012 c" the reference [36] (to make understanding more easy). But the publication is 2013.

Prefer in first column "Present" instead of "New".
caption: fit probability should be mentioned. ("p-value" as in text is a bit jargon like.
at the end: reduces --> reduced (or no "is")

Fig.2: the data points are rather large, so the inner error bars and even the outer ones are mostly invisible. Suggest either smaller dots or different symbols e.g. x.

Fig. 10 and 12: legend and colours consistant? Comparing the figures, I think that in Fig. 12 the colour choice for appr. should be the same as in Fig. 10 .
Is the labeling correct?. I expected to see the drop of appr. NNLO at large $x$ also in Fig. 12(?).
Similar for Fig. 11 and 13.
Fig. 12 caption: last word: calculationS ---> calculation Only one is presented with uncertainties.

Fig. 13 caption: similar size AS those
last word: calculations ---> calculation

Fig. 15:
The legend DIS +c+b with yellow fork suggests that also the yellow curve is a band. Should be removed.
Legend DIS only is too close to mu_f^2
This x Fitter Logo needed for license reasons? Can it be removed?
caption: I see no broken lines.
The blue error band is given according the figure for DIS only, but the caption tells that inclusive DIS has no error band.
"experimental/fit" looks like either or, or respectively. What is actually meant?

Figs. 16, 17 solid and dashed the other way round than in caption. Also for error band plot, legend and caption not consistent.

Figs. 18,19 Which uncertainty do we want to show? From the new fit or of the reference?

Fig. 20: the caption tells nothing on averaging $x$ in contrast to main text
Fig. 22: "full lines"? Both cases have full lines.
Fig. 24: Better: Combined reduced NC cross sections ..
but the abbreviation $N C$ is not yet introduced.
The dashed dotted line within the red band is hardly visible. See also comment at 602-608

Have a nice time in advent season, Joerg

From mikhaylo.lisovyi@desy.de Fri Dec 1 13:37:08 2017
Date: Fri, 1 Dec 2017 13:36:58 +0100 (CET)
From: "Lisovyi, Mikhaylo" [mikhaylo.lisovyi@desy.de](mailto:mikhaylo.lisovyi@desy.de)
To: "Zenaiev, Oleksandr" [oleksandr.zenaiev@desy.de](mailto:oleksandr.zenaiev@desy.de)
Cc: h1zeus-eb17bc@desy.de
Subject: [h1zeus-eb17.bc] Comments to the c+b combination paper
Dear Sasha, Karin and EB member,
I'm happy to see this important result coming out. Both results and text and in a very good shape.
I have mostly small comments, that are added as in-text notes in the draft itsel f: https://cernbox.cern.ch/index.php/s/KynFt4Hqrp0eA5B.

The only proposal for addition, that i have, is to add a figure of charm-to-incl usive and beauty-to-inclusive ratios of reduced cross sections, similar to what H1 has published in the past and what was also extracted for the previous combin ed data in Fig 49 on page 74 of arXiv:1506.07519. This plot is not difficult to produce and it is a very nice textbook plot showing the charge^2 asymptotic when HF mass does not play a role in kinematics (moderate Q 2 and low x.)

Most likely a web browser will not display them, but it works in most pdf viewer $s$, when you download the file. The comments are on pages (starting from 0 as the title page):
$0,1,5,8,15,16,31,54$.
Best regards, misha.

From Brian.Foster@physics.ox.ac.uk Sat Dec 2 18:00:44 2017
Date: Sat, 2 Dec 2017 17:00:22 +0000
From: Brian Foster [Brian.Foster@physics.ox.ac.uk](mailto:Brian.Foster@physics.ox.ac.uk)
To: "h1zeus-eb17bc@desy.de" [h1zeus-eb17bc@desy.de](mailto:h1zeus-eb17bc@desy.de)
Subject: [h1zeus-eb17bc] Comments on the draft
General comment. I find this paper very difficult to understand. It has too much detail and the important results are obscured.
I have not attempted to make issues of hyphenation and punctuation consistent throughout but have tried to correct the most obvious inconsistencies where they cannot be ignored.

The introduction is far too long and detailed. Much of the text belongs in the experimental method
not the introduction.
line 21: no hyphen in deep inelastic
28: no comma after momenta
29: hard scales not hard scale
32,33: no hyphen before scheme
34: either different or various but not both
36: longevity is the wrong word in this context. lifetime is probably clear enough.

47: that not which
48: is small, so that the mesons
54: I dont understand what is meant by fragmentation fractions here - should it be fragmentation functions?

55: "Therefore the charm and beauty contributions can be disentangled by using observables directly sensitive to the lifetime of the decaying heavy flavoured hadrons

58/59: "of the particle with lifetime information w.r.t. the flight direction doesnt make sense

60: neither does " the number of tracks with lifetime information - is what is being tried here to give pairs of variables that can be used? If so, then why does
the last one, the invariant mass, not have a second variable?
62: no comma
65: B mesons
66: comma after mass
67: delete being
72: comma after sample; phase space limitations is a vey strange concept replace with losses or inefficiency"

74: Fully inclusive or semi-inclusive lepton analyses, which are sensitive to both charm and beauty production, profit from larger.polar angle.
They are however affected by a worse signal-to-background ratio and.
78: comma after paper - a simultaneous determination - a simultaneous combination is meaningless

80: comma after [36]

81: comma after result
84: delete suitable for.
86: The procedure use is based on that described in
90: ..this procedure leads to a significant reduction of systematic uncertainties.

98-100: respectively cannot be used like this. Replace with:
"In addition, QCD calculations in the RTOPT VFNS at NLO [32] and NNLO are compared with the data. The
NLO calculations are at $O\left(\wedge 2 \_s\right)$ for PDFs and massive parts of the coefficient functions, $O\left(\_s\right)$ for massless parts of the coefficient functions; the NNLO calculations split identically but are one order of a_s higher. Why is there no reference for the NNLO calculation? If it is part of [32], move the reference to the end of the sentence.

110: comma after 2
111: framework not frameworks
112: Section 3, not 3.1
115: comma after 6"
116: charm and beauty quarks
118: Section 7 contains the conclusions.
120: I dont understand what Open is supposed to imply in this context - it is just confusing. We dont mention quarkonia, for obvious reasons. Delete.
119.1 deep inelastic
119.2: the analyses combined makes no sense. Replace by the combined analysis
119.3: .exchange dominates

124: comma after addressed
129: comma after QCD
131: are realised is not correct. Probably can be used is what is meant
136: a correct theoretical treatment is always mandatory. Replace with a careful theoretical

139: comma after paper
140: the before full- delete the comma before and. Comma after scheme"
147: comma after [26]
148: comma after paper
150: delete respectively - I have no idea what it means in this context.
157: comma before heavy
158: delete in the MSbar scheme
161: what is meant here by approximate NNLO ?

The sentence starting at 160 is completely impenetrable. Rewrite.
163: comma after schemes
164: comma before with
165: comma after calculations
178: dont use different and individual - it sounds like we want to mean different things - use either different twice of individual twice.

180: "from both not both from
186: datasets
187: are included for the first time in this analysis
187: no comma after note"
188: dataset - and throughout decide whether it is data set or dataset and use whichever consistently! I prefer dataset

192: comma after measurements - this sentence is however unintelligible. Rewrite.

195: In the case of inclusive $D$ meson cross sections, small
197: delete the removal of - and in 199
203.1: .. in the full phase space

205: replace in terms of by as a function of
208: in PQCD, \sigma etc etc
211: comma after \sigma^th_vis
212: In the case ofprogramme, non-perturbative...
223: comma after constant and after chosen"
226: No heavy flavour measurements were included in the determination of these PDF sets

230: comma after PDFs
231: ..was kept fixed at
233: I dont understand what this sentence means
235: [40]; the differences are found to be smaller than the cross-section uncertainties

247: comma after system"
249: what does have been transported mean?
250: "Transverse fragmentation is modelled by assigning to the charmed hadron a transverse momentum kT with respect to the direction of the charmed quark

259: no hyphen
263: PYTHIA
263: Why is this bullet in past tense while the others are present? In my view

ALL these bullets and indeed all the text in this section should be past tense -
they describe what we did.
272: I have no idea what the sentence starting The resulting reduced. is supposed to mean.

278: comma after [36]
284. This sentence is impossibly complicated. Here is an attempt to split it up, but I am not sure I understand precisely what
it is trying to say: "The three sums run over the input data sets e listed in table 1. The (xBj, Q^2) grid points i for which the measured cross sections $\mu^{\wedge} i, e$ are combined
with the cross sections $m^{\wedge}$ i. The sources $j$ of the shifts b_j are in units of standard deviations of the correlated uncertainties, which are obtained from the correlated
systematic uncertainties and the statistical correlation between the charm and beauty cross section measurements.

290: "The components of the vector m are the combined cross sections m_i, while those of the vector $b$ are the shifts b_j.

292/3: In the present analysis, the correlated and uncorrelated systematic uncertainties vary proportionally to the expected central values."

296: comma after table 1
297: comma after necessary
305/6: commas before together" and after uncertainties
308: delete respectively
309: comma after combination"
1st paragraph of section 4- dont repeat information - delete either the first mention of conservative estimates of the uncertainties of delete the
last sentence of the paragraph.
318: comma after 4 and after listed"
321: .reduced by factors of 2 or more.
323: comma after the close bracket
326: combination; some are further significantly reduced due to the inclusion of new precise data [19-21].

328: comma after observed
329: what is independent?
334: comma after 6
347: comma after sections
348: Predictions using not predictions of
351: In the case of VNFS, recent calculations.
357: comma after 9
372: comma after set
377: comma after precise
Tuesday January 09, 2018

378: comma after considered
380: For beauty production (figure 9), the predictions
385: comma after $\mathrm{GeV}^{\wedge} 2$
386: comma after region
387: comma after x_bj"
388: comma after second $\mathrm{GeV}^{\wedge} 2$
389: comma after second x_bj
391: comma after 11
392: comma after HERA
393: comma after uncertainties
395: comma after 12
397: comma after 10
399: delete within their uncertainties
400: differences, about 10\% smaller than the reference.
405: In figure 13, the same ratios discussed in the preceding paragraph are shown for beauty production."

406: comma after HERA
Paragraph beginning at line 408:
What is actually being used here, and what is shown on Fig. 14? The key to Fig. 14 shows "NNPDF31sx_nnlo_as_0118 FONLL-C
and "NNPDF31sx_nnlonllx_as_0118 FONLL-C but the text says "(NLLO+NLLsx) and without (NNLO) low-x resummation This
doesnt seem consistent. Should it perhaps say NLLO and (NNLO+NLLsx) without (NNLO) low-x resummation? That at least
is consistent with the figure caption and matches my recollection of what Ball et al. actually did. Anyway, I dont understand what we say about this figure. However, the predictions lie significantly below the data in most of the phase space. That is true for the dashed purple line, presumably the leading log one, but not for the solid purple band, which is generally speaking a better fit to the data than either the dashed or the HERAPDF band - at least up to $32 \mathrm{GeV}^{\wedge} 2$, after which there is little to choose between them. So
a) I dont know what models the text is actually commenting on and
b) irrespective of that, what we have written is wrong.

413: The charm data from the previous combination have already been used for the determination of the NNPDF3.1sx PDFs.
417. I dont understand the sentence Overall, the description is not improved It surely is from Fig. 14 alone - does Overall mean looking at other variables that we dont discuss?

425: comma after combination
426: comma after 4
428: I cant understand the sentence starting "The observed changes
430: comma after considered

432: comma after sections
434: comma after cases
439: The theory description of .production is also investigated."
442: comma after program
452: usually rather than commonly
453: comma after applicable
454: comma after data
Why are we showing Figs. 18 \& 19 anyway? As far as $I$ can see, the data in there and the HERAPDF fit is identical to Figs 13 and 14 . The only addition is Figs 18 and 19
is the purple dashed line labelled NLO fit in the legend, but this is referred to neither in this paragraph nor in the figure caption. What in fact is the chisq between the HERAPDF fit and this
data - it looks pretty terrible - yet we make no comment about it - nor did we for Figs 13,14? And finally, why is
the order of Figs 18, 19 reversed compared to 13, 14?
459.2: comma before the density

460: "where $x$ is the momentum fraction transferred to the struck parton in the infinite momentum frame of the incoming proton.

475: delete the comma
476: comma after [40]
486: insert that before with.
486: ..parameter; changing this parameter noticeably affects the mass determination.

487: furthermore doesnt make sense here - perhaps you mean In addition?
488: /... uncertainty is determined at each x_Bj value from the maximum differences

489: delete the sentence "This uncertainty. - this is obvious and adding this statement just makes the reader wonder why it is there.

494: comma after data
498: comma after 15
498: I am having to guess what is meant here - I think it should be Also shown are the PDFs whose experimental uncertainties arise from a fit to the inclusive data only,
with the heavy quark masses fixed to their PDG values [51]. No significant differences between the two PDFs are observed.

501: functions doesnt make sense. Do you mean regions?
502: density, a slight enhancement compared to that determined from the inclusive data only can be observed around $x \sim \sim 10^{\wedge}-3$ when including the heavy flavour data in the fit.

Actually, just delete from 501: "When comparing to 505: uncertainties. If it isnt significant we shouldnt be commenting on it.

506: comma after the first data
508: comma after analysis
Footnote 5 - this has to be a proper sentence : This did not include scale.
513: comma after mass - also yields not yields also - comma after
contribution
515: delete of"
524: delete given - comma after uncertainties
529: comma after case
530: comma after parameters
531: comma after the second GeV
533: comma after 0
534: to not on
536: from inclusive HERA data alone
537: unclear what covered means here
539: what PDF set?
539: This sentence is too complicated - there are 3 separate combineds involved.

543: can be observed. This is to be expected because of the similarities of the PDFs, especially of the gluon.
The description of the data.
546: Another impossibly complicated sentence!
Figure 18 show the ratio of data and predictions from the results of this analysis as well as the ratio of data to predictions based on the fixed HERAPDF2.0 FF3A PDF set, for charm quarks. Figure 19 shows the same ratios for beauty.

549: delete the rest of this paragraph after calculations.
556: This section seems to me to have the wrong title - almost all of its content is about various fits to all the DIS data and c+b as a function of $x$ _Bj. I dont understand the thrust of the discussion either. It seems to me that Fig 14 indicates
that the purple band of FONLL-C is an improved description giving a reasonable overall chisq for the charm data - and presumably
from the inclusive data, from which it was mostly derived. It is fine to examine why our HERAPDF formalism doesnt work well, but we leave the
impression that QCD fits are failing here - while it seems as if at least one approach, FONLL-C, gives a reasonable description.

559: delete "with the fitted parameters and the PDF parametrisation chosen.
560: comma after [36]
563: the font seems to have changed briefly here
563: no hyphen
564: paper. All calculations/
567: "The contribution to charm production at HERA arising from light flavours
amounts to five to eight per cent '
570: accessible by" is wrong but $I$ dont know if you mean accessible to or accessible from

575: ...the beauty data is limited to a higher x range, 0.0040 .1 because of."'

576: comma after data
In fact, $I$ cant tell anything of the sort from Fig. 20 - what "steeper slope is referred to in line 577?? Figure 20 is very difficult to interpret.

577: comma after evident and before consistent
578: " Due to the larger experimental uncertainties, no conclusion can be drawn for the beauty data.

Footnote 7: ...p-value given here do not.
580: comma after function"
581: delete with
587: delete the
592: Delete within experimental uncertainties.
593: "In figure 23, a comparison is presented of the ratios of the combined reduced charm cross section, cc red and the cross-section predictions obtained
from the fit to the heavy-flavour data and the inclusive data fulfilling xBj 0.01 to the reference cross sections. The predictions from the fit to the heavy flavour data and
the full inclusive data set are also shown. As expected, the charm cross sections inclusive data rise more strongly towards "

598: In general, the ..
599: "A similar study for beauty was also made but no significant differences were observed. Delete the rest of the paragraph.

603: In figure 24, these predictions are compared to the inclusive reduced cross sections

605/6: ... obtained in this analysis by the fit to the combined heavy flavour and inclusive data agree with the inclusive measurement. What? Is there some distinction
here between inclusive data and inclusive measurement? Is this too different sets of inclusive data? Please clarify!

608: predict not predicts. Larger - than what?
609: Delete comma
610: "within the framework for PDFs applied by excluding the low-xBj inclusive data in the fit. What does this mean? What framework for PDFs? And what is being applied?

612: 3 sigma tension in theory Does this mean that the tension is theoretical not real? I can have a guess at what this is supposed to mean:
" In the theoretical framework used in this analysis, it seems impossible using only variation in the gluon density to resolve the 3 tension between the fits to the inclusive and charm data. However, it does seem possible for FONNL!
614. Delete the rest of the paragraph starting from However. and replace with As shown in section 5 , this tension between the charm and inclusive data is unlikely to be resolved at NNLO, which gives a worse fit to the charm data. However, the quark mass measurements are not significantly affected by . By what? The current text implies that we have investigated the tensions but it is
unclear how or what we have investigated. Please clarify.
623: no hyphen
624: Now that we are actually summarising what we HAVE written about what we HAVE done, to use the present tense is incomprehensible.
experiments have been combined. The beauty cross sections have been combined.

629: ..combined data have been compared Hyphen between leading and order
630: the charm data. The beauty data, which have larger experimental uncertainties, are well described by the QCD predictions.

632: The next-to-leading-order calculations in the fixed-flavour-number scheme

We cannot end this very important paper with an inconclusive discussion about disagreements between inclusive and charm QCD fits at NLO and NNLO.
I serious doubt whether this whole subject should be included at all since our discussion of it in the main body of the paper
seems to me to be confusing and inconclusive. It detracts from the important results of the paper. If it remains, then the order of the final two paragraphs must be swapped so that we end with the determination of the running masses.

636: HERA are analysed in next-to-leading-order QCD in the fixed-flavour-number scheme

641: "The QCD analysis reveals some tensions in describing both the inclusive and the charm HERA DIS data in the same fit.

643: delete the theoretical framework of - A study in which inclusive data with x_bj < 0.01 were excluded from the fit was carried out.

645: could be achieved in this way. However, the resulting PDFs fail to describe the inclusive data in the excluded xBj region, a situation that is not improved at higher orders in QCD. Delete the rest of the paragraph from This points."

Reference [32] is missing authors' names
Reference [44] - there should be an and: before the third authors name in both references

Reference [59] Sjoestrands name is incorrectly spelled.
Caption to Table 1: ..For each dataset, the Q^2
Caption to Table 4: add comma after For each source and after
simultaneously. Delete to the data set number
Caption to Table 5: is reduced not is reduces
Caption to Fig. 4, 5. Shifted by what amount in x_bj?
Caption to Fig. 6. Shifted by what in x_bj? Towards larger values makes no
sense - delete.
The captions until Fig 5 had commas around the $\backslash$ sigmas. Make the captions of Fig

7 and the rest consistent and add commas.
Captions to Figs. 12, 13: They are of similar size to those presented for
Caption to Fig. 14 - the text is completely unintelligible unless one is also looking at the main text. This is not how figure captions should be
written. The models shown should be spelled out, not using acronyms at least the first time they are used, and if necessary a reference should be given.

Caption to Fig. 15 - there arent any dashed lines - just diagonally shaded.I dont understand the last sentence - so what uncertainties are shown? And how?

Captions to Figs. 18 \& 19 - what is the purple dashed line labelled NLO fit and why is it not referred to either in the caption or the main text?

Figure 20 is not a proper figure - it should be labelled a) and b) and it should be stated in the axis labels that the top one is charm and the bottom one beauty.
The right hand labels should be indicated to be $Q^{\wedge} 2$. The caption should be changed accordingly.

Caption to Fig. 21 - again, the caption can only be understood in conjunction with the main text - we need to spell out the details, such as what the fit is.

Figure 22. Please label on the figure that there are u_v, d_v etc - otherwise they are useless for people giving talks.

Figure 24 is too small. We make ourselves look
foolish by pretending we believe people can see the difference between dashed and dashed-dotted lines when in fact they cant see the lines at all. Splitting it into (at least) two parts is essential. We could leave the current figure with just the data and red and blue features as it is quite a useful visualisation - then have additional split plots containing these dot-dashed etc lines.

Cheers,
Brian

Brian Foster
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Gruppe FLA
Notkestrasse 85
22607 Hamburg
Germany
Tel: +49 4089983201

From wegener@physik.tu-dortmund.de Sun Dec 3 17:48:18 2017
Date: Sun, 3 Dec 2017 17:48:08 +0100
From: Dietrich Wegener [wegener@physik.tu-dortmund.de](mailto:wegener@physik.tu-dortmund.de)
To: h1zeus-eb17bc@desy.de
Subject: [h1zeus-eb17bc] comments draft 2 combination and qcd analysis of bwaty and charm production

Dera collegues,
its really an interesting paper, well presented and should be published soon.
Here a few commenst and corrections
1.line 17/ 18: ..(fit) ...(mod) ...(par): puzzling -> eliminate,
explanation in formula (7) on page 14 sufficient
2.line 100 : O(alpha**3)
3.line 144 massive coefficient function - slang
4. lines 223/224 reference for values
5. line 250 fro -> from
6. line 252: reference for the value of $k \_T$
7. line 268: corresponding clustering algorithm: not clear which
8. line 348: FFNS [24-31] and the VFNS [32-35]
9. line 372.. set theory uncertainties are given .. not visible in fig8 and fig 9
10. line 634: .. do not improve the overall dscription, with and without the inclusion of $\log (1 / x)$ resummation:
Where was this shown?
11. ref 18: Measurement of .... missing
12. ref [25\}: The 3,4,5 - flavor NNLO ...
13. ref [26]: Running Heavy Quark Masses in DIS
14. ref[28]: Phys. Rev. D96(2017)014011

15 ref[29]:On the value of heavy flavor distributins at high energy colliders
16 ref [32] R.S. Thorne ... Phys. Rev. D86(2012) 074017
17. ref [34] Impact of Heavy Quark Masses on parton distributions at

LHC phenomenology
Nucl. Phys. b855... unbiased global....
18. ref [36] Aaron -> H. Abramovicz
19. ref [41] A. Behring et al Phys. Rev D92 (2015)11405
20. ref $[46,48,49,50,52]$ titles of publications missing (be consistent!)
21. ref [59] High energy event generation with PYTHIA 6.1 hep-ph/0010017

22 ref [60] BELLE title and hep-ex
BABAR Phys. Rev. D67(2003) 031101 hep-ex/0208018
23 fig 7: fig caption "HERA 2012" -> [36]
24 fig 15: shading for DIS+c+b hardly visible; true?
25 fig 20: " brown colors" hardly separable
2 fig 24:dashed lines-> not visible
Regards Dietrich

From sschmitt@mail.desy.de Mon Dec 4 13:53:45 2017
Date: Mon, 4 Dec 2017 13:53:38 +0100 (CET)
From: Stefan Schmitt [sschmitt@mail.desy.de](mailto:sschmitt@mail.desy.de)
To: h1zeus-eb17bc@desy.de
Subject: [h1zeus-eb17bc] Comments to the combined c+b paper
Dear Sasha and Karin,
congratulations for releasing this nice paper draft. I only have minor comments to draft V1 as listed below. I am looking forward to a timely publication.

Best regards,
Stefan

General:
========
I would suggest to reduce the number of figures. My proposal is as
follows:

- remove figure 2 and 3. In the text, remove the second part of line 330 (after "are") and the first part of line 331 (up to "are").
- remove figure 16 and 17. In the text, remove end of line 541 (" in figures ...") and the rest of this paragraph.
[this may require some small further adjustments in the new paragraph]
Of couse we should keep the removed figures as extra material
Lines 593-601: as others mentioned as well, for this paragraph it may enhance clarity to have name tags for the different fits which are compared.
The tags will have to be used consistently in the figures.
Figures:
$=$ = $=$ = $=$ =

Figure 1: remove the "statistics" box. Add a legend which says something like this:

Gaussian fit:
mu=0.03+/-0.05
sigma $=0.77+/-0.03$
Figure 15: change blue hatched style to filled area (or reverse hatching direction) Color: can we use a color code for the DIS+c+b fit which is consistent with figure 18/19 Consistent colors with fig 21?

Figure 20:
remove the grid. I like the horizontal line at unity but the other dotted lines are too much for my taste For the x-axis, I would prefer the label: <x> (maybe:) try to use the same y-axis range for charm and beauty

Figure 21:
choose Colors consistent with figure 15,22
The horizontal axis is labelled x_min but in the text (line 587) it is named $x \_B j, m i n$ (same comment holds for figure caption)

Figure 22:
choose consistent colors with fig 21
choose consistent color of DIS+c+b with fig 18/19
small text comments:
92-93: too many times "reduced" "reduces"
proposal: fullstop after measurements and remove: "and thereby
... further"
140-141: "... at all scales..." I do not understand,
should this read "at all orders"???
188-189: I find it hard to read without confusing dataset 8 of [36]
with the present dataset 8. Proposal: put the details to the end, all
in a bracket:
... data set 9 supercedes a data set of the previos combination
(data set 8 in table 1 of [36]).
212: ... "the corresponding fragmentation functions"
250: (typo): "... originating from ..."
346-347: I think it is not recessary to explain that we do this
"Before" we do someting else.
-> remove line 346, start with:
"The combined heavy flavor data are compared ..."
348: replace: "pre-existing" by: "various"
371: remove the statement "The theory predictions are obtained without
fitting the data." (this is obvious in this section)
448: "if not" -> "unless"
455: ... are above $3.5 \mathrm{GeV}^{\wedge} 2$ for all these measurements.
467: remove "the": ... determined by QCD sum rules".
(otherwsie we will have to be more specific about which sum rules we
use, but it is too much for this paper)
468-469: The parameter $C^{\prime}$ _g=25 is fixed [64].
(this avoids the odd sequence of number and [ref]: ... 25 [64].)
486: "... parameter while the change ..."
502: ... around x~3x10^-3 ... (This is what I get from figure 20)
514/515: ".. typically of a few MeV" -> "typically a few MeV in size"
page 14, footnote 5: (use a complete sentence)
The previous charm mass result did not ...
556: maybe try this to make x_Bj bold:
begin \{boldmath \}
section\{ ... \}
label\{...\}
end\{boldmath \}
563: "equivalent to" has a different font?
563,589,612: please unify
2.9sigma without space
1.8 sigma with space
3 sigma with space
(I prefer to have no space or a small space only)

609: remove comma: ... study shows that..."
634: ... description, neither with nor without ...

## 647-650:

... by changing only the PDFs of the proton.
The alternative next-to-leading order or next-to-next-to leading order QCD calculations considered are not able to provide a better description of the combined heavy flavour data either.

From paul@physik.uni-bonn.de Tue Dec 5 12:12:02 2017
Date: Tue, 5 Dec 2017 12:11:57 +0100
(CET)
From: paul [paul@physik.uni-bonn.de](mailto:paul@physik.uni-bonn.de)
To: h1zeus-eb17bc@desy.de
Cc: paul [paul@physik.uni-bonn.de](mailto:paul@physik.uni-bonn.de)
Subject: [h1zeus-eb17bc] comments
> Dear all,
> The current analysis is based on considerably more charm data than in [36]
$>$ and, because of the beauty decays to charm, also on more beauty data.
$>$ Published charm and beauty cross sections are combined separately. Are the
> possible correlations between charm and beauty cross sections somewhere
$>$ quantified and taken into account in the systematics considered in this
> paper? How large are those? How do our published charm cross sections in [36]
$>$ compare to the current charm cross sections?
$>$
$>$ Table5 and caption do not fit together: the values of "d.o.f." mentioned in
> the caption, are missing in the table! And for "HERA 2012 c" the proper
> reference should be included.
$>$ I have spend some time to study figure 18 and the discussion of it in section
$>6.3$ and in the conclusions. In figure 18, we see that the
$>$ description by NLO FFNS is rather poor in some Q^2 bins, either in shape
$>$ or in magnitude or in both. The corresponding
$>$ b-results in Fig. 19 look somewhat better. This might be partially explained
> by the larger uncertainty of the data points, however I think we cannot rule
> out that this is not only a question of the difference event
$>$ statistics. I suggest to discuss this observation in more detail in the
$>$ text!
$>$ In lines $557 / 558$ is noted that the dependence on $x$ _JB is steeper than the
> predicted flat distributions. I think this is not the whole truth. We see
> that the magnitude value is also significantly different in at
$>$ least two Q^2 bins. In line 561, a "partial chi^2-value" is quoted which is
> not explaind at all. In case that this number is an overall estimate for all
$>Q^{\wedge} 2$-bins together, it makes no sense. Moreover I cannot see that to quantify
$>$ an overall deviation of 2.9 sigma without ideas about the origin delivers
$>$ any meaningful information. Would it make sense to give chi^2 for each $Q^{\wedge} 2$
> range sepaparately?
$>$ There is another observation which worries me. Looking into [36] at figures 6
$>$ to 8, the data are well described by various predictions. In
> lines 559/560 is written that in [36] we see a similar behaviour, whereas
$>$ have the impression that the new data are changed significantly, e.g. for
$>Q^{\wedge} 2=2.5 \mathrm{GeV}^{\wedge} 2$. In lines 609/610 and 642-645 is explained that the inclusive
$>$ data with x_BJ<0.01 are causing the disagreements. I wonder if it makes sense
> to present $\bar{a} l$ so results without the inclusive data.
> In lines 565/566 is said "show some tensions desribing the combined data".
$>$ "Some tensions" is also mentioned as a conclusion (in line 641)
> The "tension" should be somewhat quantified and the relation to
> the inclusive data should be discussed in the text.
$>$ Not having been close to this analysis, from going through the paper
> draft only, I have the general impression that important problems with
$>$ the problems with the current results, e.g. the difference to previously
> published charm data, are not discussed in a convincing manner.

Best regards,
Ewald

From levtchen@mail.desy.de Thu Dec 7 21:45:39 2017
Date: Thu, 7 Dec 2017 21:45:29 +0100 (CET)
From: Boris Levchenko [levtchen@mail.desy.de](mailto:levtchen@mail.desy.de)
To: h1zeus-eb17bc@desy.de
Subject: [h1zeus-eb17.bc] remarks

Dear All,
Here is a collection of detected misprints:
1.100, replace )^3 by ^3)
l.481, replace 2.5 by $2.5 \mathrm{GeV}^{\wedge} 2$
l. befor 512 , Eq. $7,2-$ nd line, -0.033 is not the same as in Abstract and Conclusions
l. 531, Is it really the fit gives mb(mb) $=8.45 \mathrm{GeV}$ ?
l.665, 668, 671: replace ," by ",
l.673, the article title is not complete
l. 678, 680, 683, 685, 690, 693, 696, 699: replace ," by ",
l.701, the article title is not complete
1.717, 720, 722, 724, 731: replace ," by ",
l.793, replace ". by ",
l.807, replace .," by ",
l. 827, replace HERA, by HERA",
l.832, replace ," by ",
l.834, replace," by ",
p. 45, in Fig 15 insert: more space between lines with an inserted text to avoid the text overlap
p. 52, Fig 22, the same as p. 45
p. 54, Fig 24, second line from top. One need to avoid overlap 'FF3A' and ' ----NLO fit'. Add a shift to left

With best regards,
Boris

Dr. B.B. Levchenko
DESY, ZEUS
/
Experimental High Energy Physics Department
Institute of Nuclear Physics,
Moscow State University Tel:+007 4959395881
RU-119991 Moscow, Russia Fax:+007 4959393064

From katja.krueger@desy.de Fri Dec 8 00:49:26 2017
Date: Fri, 8 Dec 2017 00:49:18 +0100
From: Katja Krüger [katja.krueger@desy.de](mailto:katja.krueger@desy.de)
To: h1zeus-eb17bc@desy.de
Subject: Re: [h1zeus-eb17bc] Draft 1.0
Dear All,
I only have very few comments to the new draft, most of my comments have been an swered before.

## Cheers,

Katja

- general: will we provide a table with the detailed breakdown of the uncertaint ies as a data file? if yes, should we mention that somewhere?
- l. 41: mesons samples -> meson samples
- l. 54: I think the on average fits better earlier in the sentence, e.g. The pr oper lifetime of $B$ mesons is on average
- l. 101: I think you cannot compare to a scheme, but only to predictions or cal culations in a scheme
- 1. 112: section 3.1 -> section 3
- 1. 116: measurementS
- l. 218,232: use the same order of mu_r and mu_f in all the equations, so swap them in "mu_f=mu_r=sqrt ( $\mathrm{Q}^{\wedge} 2+4 \mathrm{~m} \_\mathrm{Q}^{\wedge} 2$ )
- eq. 4, text below it, and tables 2 and 3: to me the treatment of the correlate d stat. unc. is not fully clear. In l. 267 it says that the correlated unc. comp rise systematic and statistical components. But then in the next sentence $I$ read that the gamma are correlated syst., the delta_stat statistical and the delta_u ncorr the uncor. syst. unc. Which one of the three contains the statistical corr elations? And in tables 2 and 3 we use a slightly different notation with delta_ stat, delta_uncor and delta_cor
- l. 328: Im a bit surprised that this effect of cross-calibration between charm and beauty is so small (or maybe I misunderstand the sentence). I assumed that the significantly smaller uncertainties on the charm cross section due to the co mbination would lead to a sizeable (more than $10 \%$ ) reduction of the beauty uncer tainties since the VTX measurement is one of the most precise beauty measurement $s$, and for this the (anti-) correlation is large.
- l. 360: remove program
- l. 401ff: many calculations -> use prediction instead for one or two
- l. 480: the _variation of the_ strangeness fraction
- 1. 496: The ratio \chi2 -> The ratio \chi^2 (exponent!)
- l. 498: as discussed in the last meeting: I think the differences between the new fit to the inclusive data only, fixing the heavy quark masses, to HERAPDF2.0 FF3A are rather small (running mass vs. pole mass, ), and we use HERAPDF2.0 FF3 A in the previous section as a reference, so I think it makes sense to point out these differences here.
- section 6.2: I think it would make some parts easier to read if we give names/ abbreviations to the variants of the fit, instead of having to write a long stat ement describing the fit every time
- l. 563: equivalent to is in a different font
- table 5: abm11_3n_nlo is the only PDF not written with capital letters
- caption of table 5: remove one the in line 4; reduces -> reduced in line 5

```
> On 23 Nov 2017, at 10:36, Karin Daum <karin.daum@desy.de> wrote:
> Dear EB members,
>
> please find attached the version 1.0 of the draft ready for circulation within
    the collaborations
>
> Best regards
>
> Karin
```

> <cbcomb.pdf>

From dieter.haidt@desy.de Fri Dec 8 05:51:50 2017
Date: Fri, 8 Dec 2017 05:51:47 +0100
From: "Haidt, Dieter" [dieter.haidt@desy.de](mailto:dieter.haidt@desy.de)
To: Oleksandr Zenaiev [oleksandr.zenaiev@desy.de](mailto:oleksandr.zenaiev@desy.de), Karin Daum <karin.daum@desy.d e>
Cc: Stefan Schmitt [sschmitt@mail.desy.de](mailto:sschmitt@mail.desy.de), Erich Lohrmann <erich.lohrmann@desy. de>, Dieter Haidt [haidt@mail.desy.de](mailto:haidt@mail.desy.de)
Subject: Comments to cb-draft
Dear Authors,
thank you very much for the draft on the analysis of the charm and
beauty data. I have attached some comments. $t$ may be most convenient to discuss the various issues orally. Please feel free to distribute my comments to those interested.

Best whishes
Dieter
[ Part 2, ]
[ Application/VND.OPENXMLFORMATS-OFFICEDOCUMENT.WORDPROCESSINGML.DOCUM ]
[ ENT (Name: "Comments to cb-analysis draft Nov 2016.docx") 28 KB. ]
[ Unable to print this part. ]

From olsson@desy.de Fri Dec 8 17:35:21 2017
Date: Fri, 8 Dec 2017 17:35:15 +0100 (CET)
From: Jan-Erik Olsson [olsson@desy.de](mailto:olsson@desy.de)
To: h1zeus-eb17bc@desy.de
Cc: Jan-Erik Olsson [olsson@desy.de](mailto:olsson@desy.de)
Subject: [h1zeus-eb17bc] Comments to c,b paper

Dear Editors and Referees of the paper
"Combination and QCD Analysis of Beauty and Charm Production Cross Section Measurements in Deep-Inelastic ep Scattering at HERA"

We are happy to congratulate you to this paper, and to the completion of this analysis! We have read the present draft from 23.11.2017 and find that you have given a very consise description of the analysis and its results, as presented in several meetings in the last couple of years.

We have no questions to the first part of the paper, the cross section measurements. However, in the second part of the paper, the QCD analysis with the expressed focus on the determination of the running quark masses, a question arises: We are aware that the ZEUS collaboration already published a "prerunner" of this paper, namely DESY 14-083 (also quoted as ref. 21 in the present paper), arXiv.1405.6915
and we are also aware that the result on the charm mass presented there was subsequently critized by Richard Ball, in

$$
\operatorname{arXiv} .1612 .03790
$$

If we understand correctly, the criticism points to an incorrect or incomplete treatment of pole quark mass vs. running quark mass, leading to unjustified small errors on the obtained running mass.

We now wonder if this criticism is valid also for the present paper. Indeed, we do miss a introduction/discussion in the present paper, in which the roles of pole and running quark masses in the used Monte Carlo simulations and in the used fit procedures are detailed. We think that such an introduction/discussion would be very valuable for the general reader and that such an introduction/discussion is also appropriate in a paper which focuses on the $c$ and $b$ running masses determination.

Maybe this would also give increased weight to the physics message of this paper, beyond the cross section measurements and agreement or disagreement with selected models, which now constitute the sole physics message?

Independent of this criticism, we also think that the message of the paper would gain from the addition in the Conclusions, of a paragraph which points to the possible improvements in the LHC cross section predictions, due to these new $c$ and $b$ running mass measurements.

A small point, which was heavily discussed in the last H1/ZEUS meeting: We support the suggestion to quantify the deviation, which is remarked on in lines 642-3, Conclusions.

The paper is very well written, and we found only very few mistakes in spelling, or choice of words. One general remark is of course that the paper has a tendency to formulate very long sentences, using
"German grammar, verb at the very far end", which makes the reading sometimes very tedious. This tendency is worsened by the lack of commas in many sentences, leading to ambivalence in the actual meaning of the text.
Thus, we hope that an English native will have a serious go with the paper text, making it more smooth and fluent, before the final publication! We make a few suggestions in this direction below.

We wish all success in the further publication procedure, and congratulate again to the paper and the tremendous work behind it!

Our detailed text comments follow below.
Best greetings,
Jan and Nelly
Line
5: in the Title is used "deep inelastic", as is also written in two places in the Abstract (lines 9,15). In the paper body text however, "deep-inelastic" is used everywhere.
--> Consistent usage?
14: Perturbative QCD predictions are compared to the combined data. The latter are used together with the combined inclusive deepinelastic scattering...
(We reverse the order in the comparison: Theory is always compared to the data, never the other way around)

26: "the mass of the heavy quark involved"
28: "momenta, of the" --> "momenta of the"
29: "several hard scales" (plural !)
35: Why no charge indication on "D", but indicating charges on $D^{*}$ ?
45: "significantly suppressed further" -->
"further significantly suppressed"
51-52: "Although the first two reasons given above for the suppression of beauty production relative to charm production also hold in this case,"

54: "is on average about a factor of 2 to 3 larger than that of D mesons, when taking..."

55-57: "Therefore, using... flavoured hadrons, the charm..."
67: "are on average harder" --> "have on average higher momenta"
68: "relative to the production cross section" -->
"relative to the observed \$c\$-induced fraction"
73: "limitations because" --> "limitations, because"
74: "inclusive or lepton" --> "inclusive and lepton"
94: "by the data" "by the new results"
108: "have not yet been fully" --> "are not fully"
113: Better: "... charm cross sections are given in section 4.
In section 4 theoretical calculations based... VFNS
are compared to these cross sections."
116: "charm and beauty quark" --> "charm and beauty quarks"
118: "are presented" --> "is presented"
"dependance" --> "dependence"
119+2: "analyses combined" --> "combined analyses"
125: "reaches up to" --> "reaches"

```
129: "occur for" --> "are involved in"
131: "are realised" --> "are used", or "are applied"
139: "used for" --> "used in"
168: "NLL" --> "next-to-leading-log (NLL)"
169: "next-to-leading-log" --> "NLL"
175: "interaction point" --> (better) "interaction region"
208: "in general sigma can be" --> "sigma can in general be"
237-239: "assumptions on ... tagging have to be made." -->
    "assumptions have to be made on ... tagging.
264: "programme" --> "program" (cf. computer program,
                                    government programme)
281: "directly taken" --> "taken directly"
299: "(dataset 1) using" --> "(dataset 1), using"
336: "and below" --> "and lower"
340: "data of" --> "results of"
342: "previous measurement." --> "previous measurements."
377: "show a somewhat steeper xBj dependence than" -->
    "show an xBj dependence somewhat steeper than"
408: "show some tension in general" -->
    "in general show some tension"
486: "noticabely" --> "noticeably
491: "fit, model, and" --> "fit, model and "
517: "parameterisation" --> "parametrisation"
        Note: You are almost everywhere using the spelling
                    "parametrisation". This is OK, as would also be
                    the spelling "parameterisation", which now occurs
                        in a few places. We suggest to make a search in the
                latex-source, and use only one of these spellings.
535: "demonstrates" --> "demonstrate" (plural actor!)
544: "PDFs especially of the gluon and the" -->
    "PDFs, in particular the gluon PDF. The"
570: "to see the ranges of x accessible by" -->
    "to determine which ranges of x are accessible"
575: "0.1 the" --> "0.1, the"
577: "evident showing" --> "evident, showing"
footnote 7: "does not" --> "do not"
"chi2 or p-value" --> "chi2 and p-value"
```

581: "performed with requiring different values of the minimum xBj values" --> "performed, varying the values of the minimum xBj"

```
584: "as function" --> "as a function"
586: "0.04 while" --> "0.04, while"
597: "imposed to" --> "imposed on"
597: "rising stronger towards small xBj" -->
    "rising stronger for smaller xBj-values"
602: "calculated for inclusive DIS also." -->
    "calculated also for inclusive DIS."
603: "figures 24" --> "figure 24"
609: "shows, that" --> "shows that"
610: "excluding ... in" --> "excluding ... from"
Ref. 44: "S.Alekhin, J.Bluemlein and S.Moch"
            "I.Bierenbaum, J.Bluemlein and S.Klein"
Ref. 59: "Sj\"ostrand et al."
Ref. 61: "G.Curci, W.Furmanski and R.Petronzio"
    "S.Moch, J.A.M.Vermaseren and A.Vogt"
    "A.Vogt, S.Moch and J.A.M.Vermaseren"
Tables 2,3 captions: "obtained by" --> "obtained through"
                                    "uncetrainties" --> "uncertainties"
Table 4: "luminosity" --> "integrated luminosity" 6 times
Table 4, caption: "extracted...sections simultaneously a" -->
    "simultaneously extracted ... sections a"
Table 5, caption: "the the" --> "the"
    "is reduces" --> "is reduced" or "reduces"
Figure 12, caption "uncertianties" --> "uncertainties"
Figure 13, caption" "They are of similar size than those" -->
                                "These are in size similar to those"
Figure 14, caption: "They are of similar size as those" -->
                "These are in size similar to those"
Figure 20, caption: "data as" --> "data, as"
                        "for the different" --> "for different"
                            We also suggest to use labels a) and b) in this figure,
                instead of "upper and lower panel". Who knows what
                orientation this figure will have in the final
                publication?
Figure 21, caption: "data (triangles) only" -->
                            "data only (triangles)"
Figure 23, caption: "shaded band" --> "shaded bands" (like in fig.22)
Figure 23,24 captions: "(full line)" --> "(full lines)"
```

From geiser@mail.desy.de Fri Dec 8 18:46:52 2017
Date: Fri, 8 Dec 2017 18:46:45 +0100 (CET)
From: Achim Geiser [geiser@mail.desy.de](mailto:geiser@mail.desy.de)
To: Jan-Erik Olsson [olsson@desy.de](mailto:olsson@desy.de)
Cc: h1zeus-eb17bc@desy.de
Subject: Re: [h1zeus-eb17bc] Comments to c,b paper
Dear Jan and Nelly,
Thanks a lot for your comments and in particular for making me aware of the conference report by Richard Ball, which was not known to me. I must say that $I$ do not agree with his arguments in several places, of which I want to discuss only one here. I think everyone agrees that the conversion between running mass and pole mass is not converging well (as he points out), but the conclusion known and plausible to me is just the opposite: so-called renormalons affect the pole mass (and not the running mass), and this is one of the reasons why (proven by many), FFNS heavy flavour cross section predictions converge better in the running mass than in the pole mass scheme. VFNS approaches are then yet another story, yet see arXiv:1605.01946 of which several of Richard Balls usual collaborators happily are authors.

I don't think that we should have ourselves lured into this dispute by an unpublished conference report contradicting several published papers most of which we already cite, but we will of course discuss this at the EB.

Best regards, Achim
On Fri, 8 Dec 2017, Jan-Erik Olsson wrote:
$>$
> Dear Editors and Referees of the paper
"Combination and QCD Analysis of Beauty and Charm Production Cross Section Measurements in Deep-Inelastic ep Scattering at HERA"

From mklein@hep.ph.liv.ac.uk Sun Dec 10 12:59:21 2017
Date: Sun, 10 Dec 2017 11:59:07 +0000 (GMT)
From: Max Klein [mklein@hep.ph.liv.ac.uk](mailto:mklein@hep.ph.liv.ac.uk)
To: h1zeus-eb17bc@desy.de
Cc: Max Klein [mklein@hep.ph.liv.ac.uk](mailto:mklein@hep.ph.liv.ac.uk)
Subject: [h1zeus-eb17bc] Comments on Fcc,bb - Max Klein

Dear Colleagues
congratulations to this combination and analysis, a monument to the strong c,b and QCD efforts of our Collaborations, many thanks.

Below please find some comments on the paper draft 1.0 (Nov23) with apologies for being somewhat late.

Best regards, Max
p0 112 is the photon virtuality $Q$ or $Q 2$ or -Q2? I would term Q2 once as what it is the negative 4 -momentum transfer^2. at high 22 it may also be the $Z$ virtuality, and we measure to 2000 GeV2
l17/18 I would term the 'fit' error source 'exp' because there sits all the experimental uncertainty, even if we use a fit to determine these for mc , mb
p1 I would move 4 paragraphs, $138-177$ from the introduction to data samples 3.1. it is a long, qualitative discussion, important, but in my view not for the general introduction.
p2 199-102 I would delete the statements in parentheses (O) alphas2.. respectively) and (O(alphas3... functions) as they are very technical and not important at this point. you cite [32] for NLO but then talk abut NNLO, is there no/a ref to that?

## p3 1108 delete 'ongoing'

eq 1 and elsewhere, probably too late, I know, but I would use 'x' instead of 'x_Bj' as in DIS x_Bj is known to be $x$, there is no $x$ _f to confuse the notation
eq 1 and eq 2 I would write alpha, not alpha(Q2) but in 1123 then write elm coupling alpha=alpha(Q). the finestructure constant running is less dramatic than the one of alphas as we all know.
p4 1158 delete .in the MSbar scheme
1163 there is a Landau O before in a different style, l99ff also I would write $\mathrm{Q} 2=O(\mathrm{mQ} 2)$ because the $O$ already implies $\backslash$ simeq
p5 1188 data set --> dataset
p10 1366 and figures: I would delete ABM09, it is past and brings nothing to the discussion, even if by any reason it is higher at $Q 22.5$ for cc, to have ABM11 and 16 should suffice. the fact that it has not used HERA HQ data is fine, but it neither used DIS data or LHC, so a comparison cannot be leading to any real conclusion.
p12 eq 6 and discussion: do we know what happens to the $x$ dependence if $A^{\prime}$ _g is set to zero, and the gluon is not allowed to 'disappear'? the steepness of $\operatorname{Fcc}, \mathrm{bb}$ will have to do with xg and this negative term we obtained from RT has improved the chi2 but it puts also epWZ16 to an extreme $x g$ prediction and should perhaps be questioned in the context of the discussion that is following in our paper, see also remarks below

```
p13 l471 i guess fs = s/(dbar+ubar) .??
```

p14 1529-538 I would delete that exercise because if you only fit Fcc and Fbb (which is what I conclude is described here) you must fail, and you do.
p15+16 6.3
I find there is not enough motivation, from fig 21 (chi^2 vs xmin) to go on for a study with $x>0.01$ for the DIS data. that is really extreme. it is obvious that such a fit cannot describe the data at $x=10^{\wedge}-4$, it has no handle. Moreover, you even see that the full DIS+c,b chi2 is improving! when going to lower xmin. I thus would stop the paper at fig21 and state that one observes a trend that the $x$ dependence is somewhat! steeper in Fcc than the fit wants it to be. give it more freedom and it follows, but do not cut all the HERA NC+CC data out. of course, if the only constraint at small $x$ is $\operatorname{Fcc}$ (and bb) then you reproduce them better, fig 23, but that is almost a trivial statement: you let the inclusive cross sections go to whereever and fit the low $x$ HQ data. this in my view is not illustrating or telling us anything really. we observe certain tension between Fcc,bb and inclusive DIS, which we could not resolve in our framework, that is an interesting result, not the $x<0.01$ toy fit study.
p17 perhasp call it a Summary rather than Conclusions
1630 i would list the QCD predictions here which were used
1640 perhaps one needs here a line to say what HERAPDF2.0 FF3A is
1641 'some tension' needs to be quantified here, is it a valid fit or is not? if indeed you accepted a bit of my reservation against the high weight given to the $x>0.01$ approach, the conclusion would basically be that in a joint $N C+C C+c, b$ fit the $x$ dependence of $F c c$ cannot be reproduced well, neither in NLO nor NNLO.

Acknowledgements: perhaps one thanks SA, JB, SM?
p19 [28] is: Published in Phys.Rev. D96 (2017) no.1, 014011
p22 [59] Sjoestrand
p30 are the nr of points 52,47 really the same in the 2012 set and now?
what about stating that the inclusion of the PDF uncertainties has a negligible effect on the chi2 and delete the rather repetetive last column?
p52 the gluon is too sensitive to the radical 0.01 cut. it does one good thing, it indicates that Fcc wants it to rise. since the inclusive DIS data seem not to want that, Fcc^thy is less steep than the data. therefore i would think that $A^{\prime} \quad$ g=0 may be an interesting case study. one may argue that xg has to be positive and exclude the negative term, perhaps this helps and it is a less radical cure or case study than the 0.01 thing.
a very nice result and very important paper, thank you!

Prof. Max Klein
University of Liverpool Department of Physics L69 7ZE UK
tel: +44 (0) 1517943353
CERN, 1211 Geneva 23 Switzerland
tel: +41 (0) 227671319

From oleg.kuprash@cern.ch Mon Dec 11 01:03:34 2017
Date: Mon, 11 Dec 2017 00:03:18 +0000
From: Oleg Kuprash [oleg.kuprash@cern.ch](mailto:oleg.kuprash@cern.ch)
To: "h1zeus-eb17bc@desy.de" [h1zeus-eb17bc@desy.de](mailto:h1zeus-eb17bc@desy.de)
Subject: [h1zeus-eb17bc] Comments for the paper draft
Dear Analysis Team,
Congratulations with performing the cross section combination and QCD analysis. The paper draft is nicely detailed.
Please find my proposed textual comments below.
Sorry for sending them after the deadline. I hope they could still be
considered.
With best regards,
Oleg
L28: Probably, "\$p_\{T\}\$, " was meant to be inserted between "momenta, " and "of the outgoing quarks"? Otherwise the comma seems to be not needed.
L29: "several hard scale" -> "several hard scales".
L34: It seems "different various" needs to be replaced with only one of the
words.
L100: "O(\alpha_s)^3" -> "O(\alpha_s^3)".
L102: "low-x" -> "low-\$x\$".
L112: "section 3.1" -> "section 3".
L117: "dependance" -> "dependence" (?)
L118: Is it a study of the $x \_B j-d e p e n d e n c e$ of the measurement, or of the x_Bj-dependence of the cross section?
L150: ", respectively" seems to be redundant.
L156: Is it "programme" or "program"? Different versions are used throughout the draft.
L158: Please remove ". in the MS-bar scheme".
L178: I was for a moment confused reading the sentence. Could the number of data sets entering the combination be also quoted? E.g. "The 13 data sets included in the combination are listed in table 1 and correspond to 209 individual charm and 57 beauty cross section measurements."
L192: For consistency, it is better to use Roman "red" in the notation for the reduced cross section.
L193: Same as above for "vis,bin".
L206: Capital "E" in "Eq" is used for denoting the equation in this line, but small "e" is used in the rest of the draft. It would be good to consistently use the same style (HERAPDF 2.0 paper was using "Eq.", "Tab.", "Fig.", "Section"). L231: "kept fix" -> "kept fixed".
L231: There seem to be a whitespace between "GeV" and "." Please remove it.
L231: "factorisation were" -> "factorisation scales were".
L250: "fro" -> "from".
L263: Since the \{\sc \} environment seems to be used for xFitter and HERAFitter, would it make sense also to use it for other programs, like Pythia, OPENQCDRAD, QCDNUM, and HVQDIS?
L264: Move "MC" closer to "Monte Carlo".
L344: "( data" -> "(data".
L359: It might sound better (and more fair to our theory colleagues) to replace "with" -> "within".
L360: Probably "program" can be safely removed. It is already called framework.
The two sentences in L358-360 and L361-363 might be combined into one sentence, and kept in the place of the first one. E.g.: "The theory predictions are obtained within the open-source QCD fit framework for PDF determination $\{$ \sc xFitter\} [45] (version 1.2.0), which uses the program \{\sc OPENQCDRAD\} [44] for the calculation of reduced cross sections."
L453: The comma between "applicable" and "the" would make the reading a bit easier.
L496: The "(d.o.f.)" has already been introduced earlier in the text.
L507: Is the word "form" used in the meaning of "shape"?
L511: Full stop is missing in the end of the sentence. The text of the footnote should probably start from a capital letter.
L519: Is "both masses" denoting the values of charm quark mass determined in
this and previous [36] papers? Or is it about charm and beauty masses? If it's the latter, $I^{\prime} d$ suggest to replace "both" $->$ "both charm and beauty".
Footnote 7: "does not" -> "do not".
L563: The text "equivalent to" is displayed in a different font on my system (Adobe Acrobat Reader on Windows 10), compared to the rest of the text.
L585,586,587: Please use Roman font for "min", as in L582.
L603: "figures" -> "figure".
L660: Can the symbols "0" and "+-" go as superscripts for "D"?
L661: The whitespace in "e p" could be removed.
L808: To be consistent, finish with a semicolon instead of full stop. Fig. 15 caption: "Q_0" -> "Q^\{2\}_\{0\}".
Fig. 15: Maybe it's my screen only, but both PDF sets look as continuous lines, while the caption says that one of them is dashed.
Fig. 15: Caption says that the uncertainties for the fit to inclusive data only are not shown. But they are shown on the plot (and the opposite is for DIS $+\mathrm{c}+\mathrm{b}$ data). Is the legend correct?
Fig. 20: Are the data points in this figure distinguishable from each other when printed in grayscale?
Fig. 22: In the caption, "(full lines)" could be removed (all lines look full). Fig. 23,24: In the caption, use Roman font for "min" in the notation for \$x_\{ $\left.\left.\mathrm{mathrm}_{\mathrm{m}}^{\mathrm{B}} \mathrm{j}, \mathrm{min}\right\}\right\}$.

Thanks a lot!

From truoel@physik.uzh.ch Mon Dec 11 13:12:08 2017
Date: Mon, 11 Dec 2017 13:11:40 +0100
From: Peter Truöl [truoel@physik.uzh.ch](mailto:truoel@physik.uzh.ch)
To: h1zeus-eb17bc@desy.de
Cc: Peter Truöl [truoel@physik.uzh.ch](mailto:truoel@physik.uzh.ch)
Subject: [h1zeus-eb17bc] Comments to Charm/Beauty-Kombination
Dear colleagues,
with a bit of delay caused by internet-breakdown in our mountain house
here are my minor comments to
the report on your apparently rather elaborate work.
Thanks and kind regards
Peter Truöl

```
[ Part 2, ]
[ Application/VND.OPENXMLFORMATS-OFFICEDOCUMENT.WORDPROCESSINGML.DOCUM ]
[ ENT (Name: "Comments_H1_ZEUS_11_2017.docx") 135 KB. ]
[ Unable to print this part. ]
[ Part 3, Application/PDF (Name: "Comments_H1_ZEUS_11_2017.pdf") 53 ]
[ KB. ]
[ Unable to print this part. ]
[ Part 4, Text/X-VCARD (Name: "truoel.vcf") 14 lines. ]
[ Unable to print this part. ]
```

From daniel.britzger@desy.de Mon Dec 11 14:08:21 2017
Date: Mon, 11 Dec 2017 14:10:58 +0100
From: Daniel Britzger [daniel.britzger@desy.de](mailto:daniel.britzger@desy.de)
To: h1zeus-eb17bc@desy.de
Subject: [h1zeus-eb17bc] Comments to the paper. Please ignore if too late...
Hi Sasha, Karin, et al.
please excuse the late sending of the comments.
Please ignore my comments, if those are considered as 'too late'.
The analysis is in a very well shape and the results are also very well presented. Congratulations !

I have only minor general remarks, but a number of smaller corrections and improvements of consistency.

Thanks for this impressive work. It is really a great paper.
Cheers, Daniel

General comments
The introduction appears to be too long. It may be appropriate to have a subsection "3.0 discussion of data sets", or "measurement techniques" and mention in the introdction only, that there have been different measurement techniques employed, which is then benefitial for the combination (138-177).
-> E.g. 171: This paragraph has no references and cannot be understood as it is.
-> E.g. 195-l103. All these details have not to be discussed in the introduction
\times -> \cdot or ' '

Often, the subscript or superscripts are not in roman fonts $x$ _ $\{\{\backslash$ rm Bj\},min\} -> $x_{\_}\{$\rm Bj,min\} etc...

The sections $6.2+6.3$ appear like, that you were lazy to write the paper in a more compact form.

Section 6.2 should be shortened. It is difficult to get the relevant information.

Section 6.3 should be sharpened as well. A table of the tests performed would maybe help.

All appearances of \alpha, have a larger font. Very strange. datasets -> data sets (multiple times)

## Title

Maybe add the mass determination, as this is also more prominent in the abstract than the QCD analysis:
Combination of ... and determination of charm and beuaty quark masses

L9 _all_ measurements ?
L9 'neutral current' is missing
113 \times -> 'r or \times -> '\cdot'
L14 vice-versa: Perturbative QCD corrections are compared to data
115. New sentence for 'together with combined inclusive...'

116 It should be mentioned, how these masses are obtained
116 beauty and charm -> Order consistently throughout the paper 121 ^^^

L17,18 (fit) is not a reasonable uncertainty. I suppose, this is just a linear error propagation of exp. uncertainties. Thus, it should be called (exp). Only in case, it is NOT just an error propagation of exp. uncertainties, one may consider other terminologies.
-> In case, it is (fit), it would be interesting to have a split-up into exp and 'fit' uncertainties
-> In l500, and 501 it is correctly named, 'experimental' uncertainties.
L17,18 (fit) (mod) (par) are not defined
L21. electron-proton -> lepton-proton ??
L21. It may be appropriate to add a footnote, that "beauty" denotes the "bottom" quark.

L24. Did the measurements showed that boson-gluon fusion is dominating, or rather the calculations ??

L21-137. I propose to have one paragraph for the data, and one paragraph for the theory introduction

L42. is suppressed by about a factor of $1 / 4$ (this does not hold for NNLO)
L46. "... is significantly suppressed further". ->
"... is significicantly suppressed further for the accessible kinematic ranges at HERA."

L49 "often escape detection": a bit colloquial: maybe somethine along: "are outside the acceptance of the HERA detectors" ??

154 of the D mesons~\cite\{add reference\}
L58 $\mathrm{P} \_\mathrm{T}^{\wedge}\{$ rel $\}->\mathrm{P}_{\mathrm{l}}\{\text { \rm } \mathrm{T}\}^{\wedge}\{$ \rm rel $\}$
-> large or small 'p' ??
159 w.r.t. -> with respect to
166. that of the D meson~\cite\{add reference\}

182 in DIS -> in NC DIS
184 cross-correlations -> colloquial
-> including the resulting correlations of systemtic uncertainties
184. remove 'suitable for comparison..." (this is obvious)

186-194. This paragraph appears not to belong to the introduction, or can be significantly shortened.

L95-1103. For the introduction it is sufficient, that different PDFs are studied in NLO and aNNLO, and different heavy-flavor schemes are studied.
1107. An NNLO ... -> this sentence does not belong to the introduction
111. What doe mean: 'briefly introduced' as there is already a 1.5-page introduction of these data in the introduction

1117 \xbj-dependance -> \xbj dependence ('hyphen' and typo)
L118. Finally, ... (remove this sentence), or add. "this introduction ends herewith."

L110-L118, It would be convenient to have click-able cross-references.

Section 2

```
L119+1 'neutural-current deep-inelastic ep scattering' -> NC DIS (as
already introdcued earlier (if NC is introduced in l21))
```

L119+2 'the virtuality of the exchanged boson is small' -> \Qsq is small
1119. is dominating?! dominating over what?! -> Is the data probably corrected for $g Z$ and $Z Z$ exchange?
l119+4, 125. particles are mainly abrevieated with small letters: u,d,c,b,t,g,gamma, etc... (but weak bosons Z,W,H)
$1119+6$ : F_L -> F_\{ Irm L$\}$
eq1. There, the nomenclature of using ' $Q^{\prime}$ for heavy-quarks is ambigious with ' $Q^{\prime}$ ' ( $\backslash$ alpha ( $Q^{\wedge} 2$ ) * $F^{\prime} 2^{\wedge}\{Q Q\}$ )

1121 'heavy QQbar pair' -> of a heavy quark pair (qqbar)
1122 electro-weak -> electroweak
eq1. I think, there is some problem with this definition: alpha_em(Q) can be taken out of the structure functions only

+ if gZ and ZZ exhange are excluded, or
+ alpha_em(0) if data are corrected for running alpha_em(Q)
This is because the $g Z$ and $Z Z$ terms are proportional to kappa_z, with kappa_z~alpha_em(0), but not to alpha_em(Q), because alpha_em(Q) is a purely QED correction.
It is sufficient, to mention explicitly that not gammaZ and ZZ
contributions are not considered in this paper. (see also l199)
l 132 At photon virtualities not $->$ At $\backslash 2 s q$ not
1138 'a correct theoretical treatment of the $h \mathrm{f}$ masses is mandatory' ->
'a correct treatment of the $h \mathrm{f}$ masses in the calculations is mandatory'

1139. FFNS~\cite\{reference needed to review or original article\}, or it has to be introduced first, and then abbreviated.
1140. $\backslash m u r^{\wedge} 2=\backslash m u f^{\wedge} 2=\backslash Q s q+4 \backslash m q s q \quad($ avoids the sqrt)
1141. corrupted sentence.
1142. this wass already said.

L163 \mathcal\{O\} or \mathscr\{O\} or 'O'
-> elsewhere (O(asmz)) is used
1163 it should be muf^2 here, right? What is actaully done?!
1164 \Qsq -> \mufsq (right?)
1173 past or present?
1180. In HERAPDF2.0, I think we have used 'HERA I' and 'HERA II', i.e. withough '-'.

1180 (1992--2000) -> (in the years: 1992-2000)
l180. A remark on 1176 should be made, that CST and MVD were only available at HERA II

1189 30~<br>% -> 30<br>, <br>%
1189. I think, one wants to say here, that these data are statistically correlated. (which is said now only indirectly)
1191. Use consistently: 'of reference~\cite\{xyx\}' or 'of~\cite\{xyz\}' The latter was used before rather often.
192. _\{red\} -> _\{ \rm red\}

1193 _\{ \rm vis,bin\} (as in eq3)
1194 eta not defined
1195 In case of inclusive $D$ meson cross sections~\cite\{add papers\}
1205. Well. for a single point, there is no 'normalisation' or 'shape'. What is meant here?

1206 Eq -> eq
l207. 'Uncertainties are correspondingly reduced.' -> Uncertainties are scaled accrodingly.

1209-1212. This should to be reformulated. (what does 'however' refer to?, why the convolutions is mentione two-times, although it is
irrelevant for the purpose of this paragraph, etc...)
1213. Why there are different forms of convolution integrals?

1220 use consistently 'c quark' ('b quark') or 'charm and beauty quark'
1220,223. References would be good to have here
1225. These variants are strictly-speaking no variants of HERAPDF1.0, since they are published within ref [39]. It should be said, that PDFs are determined following the HERAPDF1.0 approach, using HERA I data. Or simpler: add ref[36] after HERAPDF1.0 [36,39]

1229, 231. replace renorm. and fact... simply by mu_r and mu_f
L232. Q denotes the heavy quark, right?
L 245 c-quark -> charm quark (or c quark)
eq4,1288 stat -> \rm stat, uncorr -> \rm uncorr

L294. I don't understand this sentence.
L303 (1305). It would be interesting to quote the formula, how the error breakdown into stat. and uncorr. uncertaitnies of the results are obtained.
1310. remove 'and a conservative estimate...' (repetitive to 1314)

1323 a unique (a 'junique')
1336,338,3881481,1487, (and often elsewhere) consistently:
$' x y<Q 2<123$ GeV2' (as in abstract) or
'xy GeV2 < Q2< 1234 GeV2'
1354 HERAPDF 2.0 -> HERAPDF2.0
1357 remove MSbar running mass
1359-1375. Wasn't this already said?
1410 Replace footnote 4 by reference [A. BCD, private communication] or [Calculation provided by...]

1421 p-values: \$p\$-values or $\$ p \$$ values.

1446: Shorten: mur2 and muf2 are set to $Q 2+4 m Q 2$ (or remove this sentence, as already said).

1455 above 3.5 GeV2 -> above Q2min.
Or: "..data since always \muf2 > Q2min'.

L458. The question arises, why the log-term is not included in the combination.

L459+2 mu_\{ \rm f, 0\} $\rightarrow$ mu_\{f,0\} (as muf elsewhere)
eq5. This 'generic form' misses the prime part, right?
1477. What are 'fit' uncertainties. A reference should be added, if there is some specific definition. If this is only a linear error propagation of stat. and syst. uncertainties as defined above, then this should be denoted as 'exp'.
1478. Model uncertainties (mod)

1482 'parameterisation uncertianty' (par)
l485. '... is only 5 units worse...' this should maybe go to the results section.

L490. The total PDF uncertainty -> Isn't it just 'the total uncertainty' ??
l 496 'd.o.f.' -> n_\{d.o.f.\} In natural science one commonly uses a single letter for numerical numbers.

1496 \chi2 -> \chi^2
1496 is similar -> is of similar size
1499. What are 'experimental' uncertainties here !?
1505. 'experimental uncertainties' :)
1517. ' The running charm quark mass determined here agrees' -> The
value of mc (mc) agrees...'
1521. A cross check of what?

1521-528. This can be shortened or droped. E.g.:
"The uncertainties (which ones?) are found to be consistent with an alternative error propagation using a Monte Carlo method~\cite\{xFitter\}."
1529. Improve these sentences.

L529-538. This can be shortened. What is the message of this paragraph? Everybody expects, that when using data, which is not sensitive to those parameters, then the results are not good.

L543. Split sentence into two. "... . This is to be expected..."
1546-1555. These lines can be shortened, and discussed together with the previous paragraph (as no additional information is added, but only differently displayed).

1556 \boldmath \xbj
1561. 'partial chi2 is not defined.'
$->$ It maybe good, to calculate the chisq as a 'full' chisq for these data here.

1563 'equivalent to' is written in a different font. Very strange...
1572. Fig 20 does not show $\langle x\rangle$ but ' $x$ '

1572 <x> needs to be defined here. How <x> is calculated ?
$\rightarrow>$ The values $\langle x>$ are defined as..., and calculated using HVQDIS.
(reference is not needed here)
1575. This sentence is kind of obvious.

1566-1579. Does this study mean, that the PDF fit does not correctly
'shape' the gluon??
1627. 'have significantly reduced uncertainties', compared to what?

Compared to individal data sets, of course. Compared to all data sets? well, this is what is presented.
1629. Split sentence into two: "...predictions. The charm data..."
1633. 'provide the best description..... do not improve'. Kind of obvious.
1634. In the text, it is logx resummation, not logl/x $->$ consistency.

Table 2. Drop the column 'bin' or rename: 'data point number'
Table 2. header x-> x_Bj
Table 2. The precision of sigma_red should meet the precision of the uncertainties (4 digits?)

Table 4. How the 'reduction factor' is defined?
Table 4 caption: datasets - data sets
Table 5. Dataset -> Data set

Table 5. The data set names are not defined.
Table 5 does not fit the text-width
-> Maybe the last column can be droped, and just mentioned that the PDF uncertainty does not significantly reduce chi2. (although, I really admire this study!!!)
Table 5 stat, uncorr, cor tot -> $\{$ \rm ...\}

Fig 1 'H1 and ZEUS' labels are missing
Fig 1. The numerical values in the box need explanation, or a dedicated selection should be presented.
Fig 1. The pull distribution of what ?
Fig 1. It appears, that there are too many uncertainties with pull=0. Are those understood and included in table 4?

Fig 4 and others:
having $Q 2$ pads going from top-to-bottom, would allow to zoom into the relvant $x$-region (as it is done for fig6)

Fig 15. 'H1 and ZEUS' labels are missing
Fig 20. ' $Q^{\wedge} 2^{\prime}$ and 'data' sould be added to the legend 'charm' and 'beauty' as well
Fig 20 should be labelled 'H1 and ZEUS'
Fig 20. Is it $x$ or $\langle x>$ at the $x$-axis?
Fig21. x-axis: x2/N_dat, but in caption: x2/d.o.f.
Fig21. 'when including in the fit only' mayb better: -> 'when using only for the fit'
Fig21. $x \_m i n ~->~ x \_\{\backslash m ~ m i n\}$

# Combination and QCD analysis of beauty and charm production cross section measurements in deep inelastic ep scattering at HERA 

## The H1 and ZEUS Collaborations


#### Abstract

Measurements of open beauty and charm production cross sections in deep inelastic ep scattering at HERA from the H1 and ZEUS Collaborations are combined. Reduced cross sections for beauty and charm production are obtained in the kinematic range of photon virtuality $2.5 \leq Q^{2} \leq 2000 \mathrm{GeV}^{2}$ and Bjorken scaling variable $3 \times 10^{-5} \leq x_{\mathrm{Bj}} \leq 5 \times 10^{-2}$. The combination method accounts for the correlations of the statistical and systematic uncertainties among the different data sets. The combined data are compared to perturbative QCD predictions and used together with the combined inclusive deep inelastic scattering cross sections from HERA in a next-to-leading order QCD analysis. The running charm and beauty quark masses are determined as $m_{c}\left(m_{c}\right)=1.290_{-0.041}^{+0.046}(\mathrm{fit})_{-0.014}^{+0.062}(\mathrm{mod})_{-0.031}^{+0.007}(\mathrm{par})$ GeV and $m_{b}\left(m_{b}\right)=4.049_{-0.109}^{+0.104}(\mathrm{fit})_{-0.032}^{+0.090}(\mathrm{mod})_{-0.031}^{+0.001}(\mathrm{par}) \mathrm{GeV}$.


## 1 Introduction

Measurements of open charm and beauty production in deep-inelastic electron ${ }^{1}$-proton scattering (DIS) at HERA provide important input for stringent tests of the theory of strong interactions, quantum chromodynamics (QCD). Measurements at HERA [1-23] have shown that heavy flavour production in DIS proceeds predominantly via the boson-gluon-fusion process, $\gamma g \rightarrow \mathrm{Q} \overline{\mathrm{Q}}$, where Q is the heavy quark. The heavy flavour production cross section depends strongly on the gluon distribution in the proton and the mass of the heavy quarks involved. This mass provides a sufficiently high scale for the applicability of perturbative QCD (pQCD). However, other hard scales are also present in this process: the transverse momenta, of the outgoing quarks and the virtuality, $Q^{2}$, of the exchanged photon. The presence of several hard scale complicates the calculation of heavy flavour production in pQCD. Different approaches have been developed to cope with the multiple scale problem inherent in this process. In this paper, the massive fixed-flavour-number-scheme (FFNS) [24-31] and different implementations of the variable-flavour-number-scheme (VFNS) [32-35] are considered.

At HERA different various flavour tagging methods are applied for beauty and charm cross section measurements. The full reconstruction of $D$ or $D^{* \pm}$ mesons $[1,2,4-6,10-12,15,16$, 18-20], the longevity of heavy flavoured hadrons [7-9,14,21] and their semi-leptonic decays [13,22,23] are exploited. Usage of / Use of?

Using fully reconstructed $D$ or $D^{* \pm}$ mesons gives the best signal-to-background ratio for measurements of the charm production process. Although the branching ratios of beauty hadrons to $D$ and $D^{* \pm}$ mesons are large, the contribution from beauty production to the observed $D$ or $D^{* \pm}$ mesons samples is small for several reasons. Firstly, beauty production is suppressed relative to charm production by a factor $1 / 4$ because of the quark's electric charge coupling to the photon. Secondly, the boson-gluon-fusion cross section depends on the invariant mass of the outgoing partons, $\hat{s}$, which has a threshold value of $4 m_{\mathrm{Q}}^{2}$. Because the beauty quark mass, $m_{b}$, is about three times the charm quark mass, $m_{c}$, beauty production is significantly suppressed further. Thirdly, in beauty production $D$ and $D^{* \pm}$ mesons originate from the fragmentation of charm quarks which are produced by the weak decay of the beauty quark. Therefore the momentum fraction of the beauty quark carried by the $D$ or $D^{* \pm}$ meson is small such that the mesons often escape detection.

Fully inclusive analyses based on the lifetime of the heavy flavoured mesons are sensitive to both beauty and charm production. Although the first two reasons for the suppression of beauty production relative to charm production given before also hold in this case, sensitivity to beauty production can be enhanced by several means. The proper lifetime of $B$ mesons is about a factor of 2 to 3 that of $D$ mesons on average when taking into account the fragmentation fractions of the corresponding quarks. Therefore using observables directly sensitive to the lifetime of the decaying heavy flavoured hadrons the charm and beauty contributions can be disentangled.||The separation can be further improved by the simultaneous use of observables sensitive to the mass of the heavy flavoured hadron, e.g. relative transverse momentum, $p_{T}^{\text {rel }}$, of the particle with lifetime information w.r.t. the flight direction of the decaying heavy flavoured hadron, the number of tracks with lifetime information or the invariant mass obtained from charged particles attached to a secondary vertex candidate.\|

[^0]How about b fragmentation and B meson decays into D mesons? Current phrasing confusing or wron

Therefore,

This part is a mess. lifetime information wrt flight direction or momentum wrt flight direction? Additional commas would heln

The analysis of lepton production is sensitive to semi-leptonic decays of both, beauty and charmed hadrons. When taking into account the fragmentation fractions of the heavy quarks as well as the fact that in beauty production leptons may originate both from the $b \rightarrow c$ and the $c \rightarrow s$ transition, the semi-leptonic branching fraction of $B$ meson is about twice that of $D$ mesons. Because of the large $B$ meson mass leptons originating directly from the $B$ decay are on average harder than those being produced in $D$ meson decays. Therefore the experimentally observed fraction of $b$-induced leptons is enhanced relative to the production cross section. Similar methods as outlined in the previous paragraph are then used to further facilitate the separation of the beauty and charm contribution on a statistical basis.

While the measurement of fully reconstructed $D$ or $D^{* \pm}$ mesons yields the cleanest charm production sample it suffers from small branching fractions and significant phase space limitations because all particles from the $D$ or $D^{* \pm}$ meson decay under consideration have to be measured. Fully inclusive or lepton production analyses are sensitive to both beauty and charm production. Such analyses profit from larger branching fractions and better coverage in polar angle at the cost of a worse signal to background ratio and large statistical correlations between beauty and charm measurements inherent to these methods.

In this paper a simultaneous combination of beauty and charm production cross section measurements is presented. This analysis is an extension of the previous H1 and ZEUS combination of charm measurements in DIS [36] including new charm and beauty data [13,14,19-23] and extracting combined beauty cross sections for the first time. As a result a single consistent dataset from HERA of reduced cross sections for beauty and charm production in DIS in the kinematic range of photon virtuality $2.5 \leq Q^{2} \leq 2000 \mathrm{GeV}^{2}$ and Bjorken scaling variable $3 \times 10^{-5} \leq x_{\mathrm{Bj}} \leq 5 \times 10^{-2}$ is obtained, including all cross-correlations, suitable for comparison with theoretical predictions.

The combination is based on the procedure described in [36-39]. The correlated systematic uncertainties and the normalisation of the different measurements are accounted for such that one consistent data set is obtained. Since different experimental techniques of beauty and charm tagging have been employed using different detectors and methods of kinematic reconstruction, this combination leads to a significant reduction of statistical and systematic uncertainties. The simultaneous combination of charm and beauty cross section measurements reduces the correlations between beauty and charm measurements and thereby the uncertainties on the combined beauty and charm cross sections are reduced further. The combined reduced charm cross sections of the previous analysis [36] are superseded by the data presented in this paper.

The combined data are compared to theoretical predictions obtained in the FFNS at next-toleading order (NLO, $O\left(\alpha_{s}^{2}\right)$ ) QCD using HERAPDF2.0 [40], ABKM09 [25,26] and ABM11 [27] parton distribution functions (PDFs), and to approximate next-to-next-to-leading order (NNLO, $O\left(\alpha_{s}^{3}\right)$ ) predictions using ABMP16 [28] PDFs. In addition QCD calculations in the RTOPT VFNS at NLO [32] ( $O\left(\alpha_{s}^{2}\right)$ for PDFs and massive parts of the coefficient functions, $O\left(\alpha_{s}\right)$ for massless parts of the coefficient functions) and NNLO $\left(O\left(\alpha_{s}\right)^{3}\right.$ and $O\left(\alpha_{s}^{2}\right)$, respectively) are confronted with the data. A comparison is also made to the FONLL-C scheme [33,34] $\left(O\left(\alpha_{s}^{3}\right)\right.$ (NNLO) in the PDF evolution, $O\left(\alpha_{s}^{2}\right)$ in all coefficient functions) extended by low-x resummation [35].

A QCD analysis is performed of the new combined heavy flavour data together with the combined inclusive DIS cross section data from HERA [40] and the running charm and beauty
quark masses, as defined in the QCD Lagrangian in the modified minimum-subtraction ( $\overline{\mathrm{MS}}$ ) scheme, are determined at NLO. An NNLO mass determination is not attempted since the ongoing calculations of the corresponding $O\left(\alpha_{s}^{3}\right)$ massive terms [41] have not yet been fully completed.

The paper is organised as follows. In section 2 the reduced heavy flavour cross section is defined and the theoretical frameworks of heavy flavour production are briefly introduced. The data samples to be combined and the combination method are presented in section 3.1. The resulting combined reduced beauty and charm cross sections are presented in section 4 and compared with theoretical calculations based on existing PDF sets and with existing predictions at NLO and at NNLO in the FFNS and VFNS in section 5. In section 6 the NLO QCD analysis is described and the measurement of the running masses of the charm and beauty quark in the $\overline{\mathrm{MS}}$ scheme at NLO are presented. This section also contains a study of the $x_{\mathrm{Bj}}$-dependance of the heavy flavour cross section measurement. Finally, the paper is concluded in section 7.

## 2 Open heavy flavour production in DIS

In this paper, beauty and charm production via neutral-current deep-inelastic ep scattering are considered. In the kinematic range explored by the analyses combined, the virtuality of the exchanged boson is small, i.e. $Q^{2} \ll M_{Z}^{2}$, such that the virtual photon exchange is dominating. The cross section for the production of a heavy flavour of type Q , with Q being either beauty, $b$, or charm, $c$, may then be written in terms of the heavy flavour contributions to the structure functions $F_{2}$ and $F_{L}, F_{2}^{\mathrm{Q} \overline{\mathrm{Q}}}\left(x_{\mathrm{Bj}}, Q^{2}\right)$ and $F_{L}^{\mathrm{Q} \overline{\mathrm{Q}}}\left(x_{\mathrm{Bj}}, Q^{2}\right)$, as

$$
\begin{equation*}
\frac{\mathrm{d}^{2} \sigma^{\mathrm{Q} \overline{\mathrm{Q}}}}{\mathrm{~d} x_{\mathrm{Bj}} \mathrm{~d} Q^{2}}=\frac{2 \pi \alpha^{2}\left(Q^{2}\right)}{x_{\mathrm{Bj}} Q^{4}}\left(\left[1+(1-y)^{2}\right] F_{2}^{\mathrm{Q} \overline{\mathrm{Q}}}\left(x_{\mathrm{Bj}}, Q^{2}\right)-y^{2} F_{L}^{\mathrm{QQ}}\left(x_{\mathrm{Bj}}, Q^{2}\right)\right), \tag{1}
\end{equation*}
$$

where $x_{\mathrm{Bj}}$ and $y$ denote the Bjorken scaling variable and the lepton inelasticity, respectively. The superscripts $\mathrm{Q} \overline{\mathrm{Q}}$ indicate the presence of a heavy $\mathrm{Q} \overline{\mathrm{Q}}$ pair in the final state. The cross section $\mathrm{d}^{2} \sigma^{\mathrm{Q}} / \mathrm{d} x_{\mathrm{Bj}} \mathrm{d} Q^{2}$ is given at the Born level without QED and electro-weak radiative corrections, except for the running electromagnetic coupling, $\alpha\left(Q^{2}\right)$.

In this paper, the results are presented in terms of reduced cross sections, defined as follows:

$$
\begin{align*}
\sigma_{\mathrm{red}}^{\mathrm{Q} \overline{\mathrm{Q}}} & =\frac{\mathrm{d}^{2} \sigma^{\mathrm{Q} \overline{\mathrm{Q}}}}{\mathrm{~d} x_{\mathrm{Bj}} \mathrm{~d} Q^{2}} \cdot \frac{x_{\mathrm{Bj}} Q^{4}}{2 \pi \alpha^{2}\left(Q^{2}\right)\left(1+(1-y)^{2}\right)} \\
& =F_{2}^{\mathrm{Q} \overline{\mathrm{Q}}}-\frac{y^{2}}{1+(1-y)^{2}} F_{L}^{\mathrm{Q} \overline{\mathrm{Q}}} \tag{2}
\end{align*}
$$

In the kinematic range addressed the expected contribution from the exchange of longitudinally polarised photons, $F_{L}^{\mathrm{Q} \overline{\mathrm{Q}}}$, is small. In charm production it reaches up to a few per cent at high $y$ only [42]. The structure functions $F_{2}^{\mathrm{QQ}}$ and $F_{L}^{\mathrm{QQ}}$ are always calculated to the same order (mostly $O\left(\alpha_{s}^{2}\right)$ ) throughout this paper.

### 2.1 Theory of heavy flavour production

In the framework of pQCD several scales occur for heavy flavour production in DIS: the mass $m_{\mathrm{Q}}$ of the heavy quark, the photon virtuality $Q^{2}$ and the transverse momenta $p_{T}$ of the emerging heavy quarks. Therefore, several theoretical approaches are realised for describing this process. At photon virtualities not very much larger than the heavy quark mass, heavy flavours are produced dynamically by the photon-gluon-fusion process. The creation of a $\bar{Q} \overline{\mathrm{Q}}$ pair sets a lower limit of $2 m_{\mathrm{Q}}$ to the mass of the hadronic final state. This low mass cutoff affects the kinematics and the higher order corrections in the phase space accessible at HERA. Therefore, a correct theoretical treatment of the heavy flavour masses is mandatory for the pQCD analysis of heavy flavour production as well as for the determination of the PDFs of the proton from data including heavy flavours.

In this paper the FFNS is used for pQCD calculations for the corrections of measurements to full phase space, and in the QCD fits. In this scheme heavy quarks are treated as massive at all scales and are not considered as partons in the proton. The number of (light) active flavours in the PDFs, $n_{f}$, is set to three and heavy quarks are produced only in the hard scattering process. The leading order (LO) contribution to heavy flavour production $\left(O\left(\alpha_{s}\right)\right.$ in the coefficient functions) is the boson-gluon-fusion process. The NLO massive coefficient functions using on-shell mass renormalisation (pole masses) were calculated in [24] and adopted by many global QCD analysis groups [27,29-31], providing PDFs in this scheme. They were extended to the $\overline{\mathrm{MS}}$ scheme in [26] in which heavy quark masses are scale dependent (running). In all FFNS heavy flavour calculations presented in this paper the default renormalisation scale $\mu_{r}$ and factorisation scale $\mu_{f}$ are set to $\mu_{r}=\mu_{f}=\sqrt{Q^{2}+4 m_{\mathrm{Q}}^{2}}$, where $m_{\mathrm{Q}}$ is the appropriate pole or running mass, respectively.

For the extraction of the combined reduced cross sections of beauty and charm production presented in this paper, the FFNS at NLO is used to calculate inclusive [24] and exclusive [43] quantities in the pole mass scheme. This is currently the only scheme for which exclusive calculations are available.

The QCD analysis at NLO including the extraction of the heavy quark running masses is performed in the FFNS with the OPENQCDRAD programme [44] in the XFItTER (former HERAFITTER) framework [45]. In OPENQCDRAD heavy quark production is calculated either using the $\overline{\mathrm{MS}}$ or the pole mass treatment of heavy quark masses. in the $\overline{\mathrm{MS}}$ scheme. In this paper the $\overline{\mathrm{MS}}$ scheme is adopted.

A comparison is also made of the RTOPT [32] implementation of the VFNS at NLO and approximate NNLO, as implemented for the default VFNS variants of HERAPDF2.0 [40], with the charm and beauty data, and of the FONLL-C implementation [34,35] with the charm data only. In VFNS schemes heavy quarks are treated as massive at small $Q^{2}$ up to $Q^{2} \approx \mathscr{O}\left(m_{Q}^{2}\right)$ and as massless at $Q^{2} \gg m_{Q}^{2}$ with interpolation prescriptions between the two regimes which avoid double counting of common terms. In the FONLL-C calculations the massive part of the charm coefficient functions is treated at $\mathrm{NLO}\left(O\left(\alpha_{s}^{2}\right)\right)$ while the massless part and the PDFs are treated at NNLO $\left(O\left(\alpha_{s}^{2}\right)\right.$ and $O\left(\alpha_{s}^{3}\right)$, respectively). In addition to the default FONLL scheme, which already includes NLL resummation of quasi-collinear final state gluon radiation, also a variant is considered which includes next-to-leading-log small- $x$ resummation in the PDFs (NLLsx) [35].

## 3 Combination of H1 and ZEUS measurements

### 3.1 Data samples

The H1 [46] and ZEUS [47] detectors were general purpose instruments which consisted of tracking systems surrounded by electromagnetic and hadronic calorimeters and muon detectors, ensuring close to $4 \pi$ coverage of the $e p$ interaction point. Both detectors were equipped with high-resolution silicon vertex detectors: the Central Silicon Tracker [48] for H1 and the Micro Vertex Detector [49] for ZEUS.

The data sets included in the combination are listed in table 1 and correspond to 209 individual charm and 57 different beauty cross section measurements. The data have been obtained both from the HERA-I (1992-2000) and HERA-II (2003-2007) data-taking periods. The combination includes measurements of charm and beauty production performed using different tagging techniques: the reconstruction of particular decays of $D$ mesons $[4,6,10,12,15,18-20]$ (datasets $2-7,9,10$ ), the inclusive analysis of tracks exploiting lifetime information [14,21] (datasets 1,11 ) and the reconstruction of electrons and muons from heavy-flavour semi-leptonic decays [13,22,23] (datasets $8,12,13$ ).

The data sets 1 to 8 have already been used in the previous combination [36] of charm cross section measurements, while the data sets 9 to 13 are newly included. It is important to note, that data set 9 supersedes data set 8 (Table 1 of reference [36]) of the previous charm combination [36], because the earlier analysis was based only on about $30 \%$ of the final statistics collected during the HERA-II running period.

For the inclusive lifetime analysis of reference [14] (data set 1) the reduced cross sections $\sigma_{\text {red }}^{c \bar{c}}$ and $\sigma_{\text {red }}^{\mathrm{b} \bar{b}}$ are taken directly from the publication. For all other measurements the combination starts from the measurement of visible cross sections $\sigma_{v i s, b i n}$ defined as the $D$-meson, lepton or jet production cross section in a particular $p_{T}$ and $\eta$ range, given in the corresponding publications, in bins of $Q^{2}$ and $x_{\mathrm{Bj}}$ or $y$. In case of inclusive $D$ meson cross sections small measurements? beauty contributions as estimated in the corresponding papers are subtracted. All published visible cross section measurements include corrections for the removal of radiation of real photons from the incoming and outgoing lepton using the HERACLES programme [50]. Some also include corresponding corrections for the removal of virtual electroweak effects, except for the running of the electromagnetic coupling $\alpha$. QED corrections to the incoming and outgoing quarks are not considered. All cross sections are updated using the most recent hadron decay branching ratios [51].

### 3.2 Extrapolation of visible cross sections to $\sigma_{\text {red }}^{Q \bar{Q}}$

Except for data set 1 of table 1, for which only measurements expressed in full phase space are available, the visible cross sections $\sigma_{\mathrm{vis}, \text { bin }}$ measured in a limited phase space are converted to reduced cross sections $\sigma_{\text {red }}^{Q \bar{Q}}$ using a common theory. The reduced cross section of a heavy flavour Q at a reference $\left(x_{\mathrm{Bj}}, Q^{2}\right)$ point is extracted according to

$$
\begin{equation*}
\sigma_{\text {red }}^{\mathrm{Q} \overline{\mathrm{Q}}}\left(x_{\mathrm{Bj}}, Q^{2}\right)=\sigma_{\text {vis }, \text { bin }} \frac{\sigma_{\text {red }}^{\mathrm{Q} \overline{\mathrm{Q}}, \mathrm{th}}\left(x_{\mathrm{Bj}}, Q^{2}\right)}{\sigma_{\text {vis }, \text { bin }}^{\mathrm{th}}} . \tag{3}
\end{equation*}
$$

The HVQDIS programme [43] is used to calculate the theory predictions for $\sigma_{\text {red }}^{Q \bar{Q}, t h}\left(x_{\mathrm{Bj}}, Q^{2}\right)$ and $\sigma_{\mathrm{vis}, \mathrm{bin}}^{\mathrm{th}}$ in the NLO FFNS. Only the shape of these theory predictions in terms of kinematic variables is relevant for the corrections, while their normalisation cancels in Eq. (3). Uncertainties are correspondingly reduced.

In pQCD in general $\sigma_{\text {red }}^{\text {th }}$ can be written as the convolution integral of the proton PDFs with the hard matrix elements. For the identification of heavy flavour production, however, specific particles used for tagging have to be measured in the hadronic final state. This requires that in the calculation of $\sigma_{\mathrm{vis}}^{\mathrm{th}}$ the convolution includes the proton PDFs, the hard matrix elements and the fragmentation functions. In case of the HVQDIS programme non-perturbative fragmentation functions are used. The different forms of the convolution integrals for $\sigma_{\text {red }}^{\text {th }}$ and $\sigma_{\text {vis }}^{\text {th }}$ lead to different sets of theory parameters to be considered.

The following parameters are used consistently in these NLO calculations and are varied within the quoted limits for estimating the uncertainties in the predictions introduced by these parameters:

- The renormalisation and factorisation scales are taken as $\mu_{r}=\mu_{f}=\sqrt{Q^{2}+4 m_{\mathrm{Q}}^{2}}$. The scales are varied simultaneously up or down by a factor of two.
- The pole masses of the $\boldsymbol{c}$ and $\boldsymbol{b}$ quarks are set to $m_{c}=1.50 \pm 0.15 \mathrm{GeV}, m_{b}=4.50 \pm$ 0.25 GeV , respectively. These variations also affect the values of the renormalisation and factorisation scales.
- For the strong coupling constant the value $\alpha_{s}^{n_{f}=3}\left(M_{Z}\right)=0.105 \pm 0.002$ is chosen which corresponds to $\alpha_{s}^{n_{f}=5}\left(M_{Z}\right)=0.116 \pm 0.002$.
- The proton PDFs are described by a series of FFNS variants of the HERAPDF1.0 set [39] at NLO determined within the XFITTER framework. In the determination of these PDF sets no heavy flavour measurements were included. These PDF sets are those used in the previous combination [36] which were calculated for $m_{c}=1.5 \pm 0.15 \mathrm{GeV}, \alpha_{s}^{n_{f}=3}\left(M_{Z}\right)=$ $0.105 \pm 0.002$ and a simultaneous variation of the renormalisation and factorisations scales up and down by a factor two. For the determination of the PDFs the beauty mass was kept fix at $m_{b}=4.50 \mathrm{GeV}$. The renormalisation and factorisation were set to $\mu_{r}=\mu_{f}=Q$ for the light flavours and to $\mu_{f}=\mu_{r}=\sqrt{Q^{2}+4 m_{\mathrm{Q}}^{2}}$ for the heavy flavours. For all parameter settings used here, the corresponding PDF set is used. As a cross check of the extrapolation procedure, the cross sections are also evaluated with the 3-flavour NLO versions of the HERAPDF2.0 set (FF3A) [40] and the differences are found to be well within uncertainties.

For the calculation of $\sigma_{\mathrm{vis}}^{\mathrm{th}}$, assumptions on the fragmentation of the heavy quarks into particular hadrons and, when necessary, on the subsequent decays of the heavy flavoured hadrons into the particles used for tagging have to be made. The fragmentation model for $c$ quarks is based on the measurements by H1 [53] and ZEUS [54] and is used as described in detail in the previous charm combination [36]. It is only briefly summarised below.

In the calculation of $\sigma_{\mathrm{vis}}^{\mathrm{th}}$ the following settings and parameters are used in addition and are varied within the quoted limits:

- The charm fragmentation function is described by the Kartvilishvili function [52] controlled by a single parameter $\alpha_{K}$ to describe the longitudinal fraction of the $c$-quark momentum transferred to the $D$ or $D^{* \pm}$ meson. Depending on the invariant mass $\hat{s}$ of the outgoing parton system different values of $\alpha_{K}$ and their uncertainties as measured at HERA $[53,54]$ are used. The variation of $\alpha_{K}$ as a function of $\hat{s}$ observed in $D^{* \pm}$ measurements has been transported to the longitudinal fragmentation function of ground state $D$ mesons not originating fro $D^{* \pm}$ decays. Transverse fragmentation is modelled by assigning a transverse momentum $k_{T}$ to the charmed hadron with respect to the direction of the charmed quark with an average value of $\left\langle k_{T}\right\rangle=0.35 \pm 0.15 \mathrm{GeV}$.
- The charm fragmentation fractions of a charm quark into a specific charmed hadron and their uncertainties are taken from [57].
- The beauty fragmentation function is parameterised according to Peterson et al. [55] with $\varepsilon_{b}=0.0035 \pm 0.0020[56]$.
- The branching ratios of $D$ and $D^{* \pm}$ mesons into the specific decay channels analysed and their uncertainties are taken from [51].
- The branching fractions of semi-leptonic decays of heavy-quarks to a muon or electron and their uncertainties are taken from [51].
- The decay spectra of leptons originating from charmed hadrons are modelled according to [58].
- The decay spectrum for beauty hadrons into leptons was taken from the Pythia [59] Monte Carlo programme (MC), mixing direct semi-leptonic decays and cascade decays through charm according to the measured branching ratios [51]. It was checked that the MC describes BELLE and BABAR data [60] well.
- When necessary for the extrapolation procedure, parton-level jets are reconstructed using the corresponding clustering algorithms, and the cross sections are corrected for jet hadronisation effects using corrections derived in the original papers [21,23]. ${ }^{2}$

While the central values for these extrapolations are obtained in the FFNS pole mass scheme at NLO, their uncertainties are calculated such that they should cover potential deviations from the 'true' QCD result. The resulting reduced cross sections, which include these uncertainties, are thus comparable to calculations in any QCD scheme to any order which include uncertainties for potential deviations from the 'true' result.

### 3.3 Combination method

The quantities to be combined are the reduced charm and beauty cross sections, $\sigma_{\text {red }}^{c \bar{c}}$ and $\sigma_{\text {red }}^{b \bar{b}}$, respectively. The combined cross sections are determined at common $\left(x_{\mathrm{Bj}}, Q^{2}\right)$ grid points. For $\sigma_{\text {red }}^{c \bar{c}}$ the grid is chosen to be the same as in [36] leading to $52\left(x_{\mathrm{Bj}}, Q^{2}\right)$ points, while for $\sigma_{\text {red }}^{b \bar{b}}$ a

[^1]subset of 27 of these points is used. The combined reduced cross sections are provided at the centre-of-mass energy $\sqrt{s}=318 \mathrm{GeV}$. The results of the H 1 inclusive lifetime analysis (dataset 1) are directly taken from the original measurement in the form of $\sigma_{\text {red }}^{c \bar{c}}$ and $\sigma_{\text {red }}^{b \bar{b}}$. When needed, these measurements are transformed to the common grid $\left(x_{\mathrm{Bj}}, Q^{2}\right)$ points using the NLO FFNS calculations [24]. The uncertainties on the resulting scaling factors are found to be negligible.

The combination of the reduced cross sections is based on the $\chi^{2}$ minimisation procedure [37] also used in the previous HERA combinations [36,38-40]. The total $\chi^{2}$ is defined as

$$
\begin{equation*}
\chi_{\exp }^{2}(\boldsymbol{m}, \boldsymbol{b})=\sum_{e}\left[\sum_{i} \frac{\left(m^{i}-\sum_{j} \gamma_{j}^{j, e} m^{i} b_{j}-\mu^{i, e}\right)^{2}}{\left(\delta_{i, e, s t a t} \mu^{i, e}\right)^{2}+\left(\delta_{i, e, \text { uncorr }} m^{i}\right)^{2}}\right]+\sum_{j} b_{j}{ }^{2} . \tag{4}
\end{equation*}
$$

The three sums are running over the different input data sets $e$ listed in table 1 , the $\left(x_{\mathrm{Bj}}, Q^{2}\right)$ grid points $i$ for which the measured cross sections $\mu^{i, e}$ are combined to the cross sections $m^{i}$ and the sources $j$ of the shifts $b_{j}$ in units of standard deviations of the correlated uncertainties which comprises the correlated systematic uncertainties and the statistical correlation between the charm and beauty cross section measurements. The quantities $\gamma_{j}^{i, e}, \delta_{i, e, \text { stat }}$ and $\delta_{i, e, \text { uncorr }}$ denote the relative correlated systematic, relative statistical and relative uncorrelated systematic uncertainties, respectively. The components of the vectors $\boldsymbol{m}$ and $\boldsymbol{b}$ are the combined cross sections $m^{i}$ and the shifts $b_{j}$, respectively.

In the present analysis, the correlated and uncorrelated systematic uncertainties are predominantly of multiplicative nature, i.e. they change proportionally to the expected central values. The statistical uncertainties are mainly background dominated and thus are treated as constant. All experimental systematic uncertainties are treated as independent between H1 and ZEUS. For the datasets 1,8 and 11 of table 1 statistical correlations between charm and beauty cross sections are accounted for as reported in the original papers. Where necessary the statistical correlation factors are corrected to take into account differences in the kinematic region of the charm and beauty measurements (dataset 11) or binning schemes (dataset 1 ) using theoretical predictions calculated with the HVQDIS programme. The consistent treatment of the correlations of statistical and systematic uncertainties, including the correlations between the charm and beauty data sets where relevant, yields a significant reduction of the overall uncertainties of the combined data.

## 4 Combined cross sections

The values of the combined cross sections $\sigma_{\text {red }}^{c \bar{c}}$ and $\sigma_{\text {red }}^{b \bar{b}}$ together with the statistical, the uncorrelated and correlated systematic and the total uncertainties are listed in tables 2 and 3. A total of 209 charm and 57 beauty data points are combined simultaneously to obtain 52 reduced charm and 27 reduced beauty cross-section measurements, respectively. A $\chi^{2}$ value of 149 for 187 degrees of freedom (d.o.f.) is obtained in the combination indicating good consistency of the input data sets and a conservative estimate of the uncertainties of the individual measurements. The distribution of pulls of the 266 input data points with respect to the combined cross sections is presented in figure 1. It is consistent with a Gaussian around zero without any significant outliers. The observed width of the pull distribution is smaller than unity which indicates a conservative estimate of the systematic uncertainties.

There are 167 sources of correlated uncertainties in total. These are 71 experimental systematic sources, 16 sources due to the extrapolation procedure (including the uncertainties on the fragmentation fractions and branching ratios) and 80 statistical charm and beauty correlations. In table 4 the sources of correlated systematic and extrapolation uncertainties are listed together with the shifts and reductions obtained as a result of the combination. All shifts of the systematic sources with respect to their nominal values are smaller than $1.4 \sigma$. Several systematic uncertainties are significantly reduced up to factors of two or larger. The reductions are due to the different heavy flavour tagging methods applied and to the fact that for a given process (beauty or charm production) an unique cross section is probed by the different measurements at a given $\left(x_{\mathrm{Bj}}, Q^{2}\right)$ point. Those uncertainties for which large reductions have been observed already in the previous analysis [36] are reduced to at least the same level in the current combination, some of them are further reduced significantly due to new precise data [19-21] included. The shifts and reductions obtained for 80 statistical correlations between beauty and charm cross sections are not shown. Only small reductions in the range of $10 \%$ are observed consistent with being independent of $x_{\mathrm{Bj}}$ and $Q^{2}$.

The combined reduced cross sections $\sigma_{\text {red }}^{c \bar{c}}$ and $\sigma_{\text {red }}^{b \bar{b}}$ are shown as a function of $x_{\mathrm{Bj}}$ in bins of $Q^{2}$ in figures 2 and 3, respectively. These cross sections are compared to the input H 1 and ZEUS data in figures 4 and 5 . The combined cross sections are significantly more precise than any of the individual input data sets for charm as well as for beauty production. This is illustrated in figure 6 where the charm and beauty measurements for $Q^{2}=32 \mathrm{GeV}^{2}$ are shown. The uncertainty of the combined reduced charm cross section is $9 \%$ on average and reaches values of about $5 \%$ and below in the region $12 \mathrm{GeV}^{2} \leq Q^{2} \leq 60 \mathrm{GeV}^{2}$. The uncertainty of the combined reduced beauty cross section is about $25 \%$ on average and reaches on average $15 \%$ at small $x_{\mathrm{Bj}}$ and $12 \mathrm{GeV}^{2} \leq Q^{2}<200 \mathrm{GeV}^{2}$.

In figure 7 the combined reduced charm cross sections of this analysis are compared to the data of the previously published combination [36]. Good consistency between the different combinations can be observed. The detailed analysis of the cross section measurements reveals a relative improvement in precision of $20 \%$ on average with respect to the previous measurement. The improvement reaches about $30 \%$ on average in the range $7 \mathrm{GeV}^{2} \leq Q^{2} \leq 60 \mathrm{GeV}^{2}$, where the newly added data sets ( data sets $9-11$ in table 1) contribute with high precision.

## 5 Comparison with theory predictions

Before performing a dedicated QCD analysis of the combined charm, beauty and inclusive reduced cross sections the combined heavy flavour data are compared with calculations using pre-existing PDF sets. Predictions of the FFNS and the VFNS are considered. The main focus is on calculations using HERAPDF2.0 PDF sets. The data are also compared to FFNS predictions based on different variants of PDF sets at NLO and approximate NNLO provided by the ABM group [25,27,28]. In case of the VFNS recent calculations of the NNPDF group based on the NNPDF3.1sx PDF set [35] at NNLO, which specifically aim for a better description of the DIS structure functions at small $x_{\mathrm{Bj}}$ and $Q^{2}$, are also confronted with the combined heavy flavour measurements. Calculations in the FFNS based on the HERAPDF 2.0 FF3A PDF set will be considered as reference calculations in the subsequent parts of the paper.

### 5.1 FFNS predictions

In figures 8 and 9 theoretical predictions of the FFNS in the $\overline{\mathrm{MS}}$ running mass scheme are compared to the combined reduced cross sections $\sigma_{\text {red }}^{c \bar{c}}$ and $\sigma_{\text {red }}^{b \bar{b}}$, respectively. The theory predictions are obtained with the open-source QCD fit framework for PDF determination XFITTER [45] program (version 1.2.0). The running heavy flavour masses are set to the world average values [51] of $m_{c}\left(m_{c}\right)=1.27 \pm 0.03 \mathrm{GeV}$ and $m_{b}\left(m_{b}\right)=4.18 \pm 0.03 \mathrm{GeV}$. The cross section predictions are obtained using the OPENQCDRAD program [44] interfaced to the XFITTER framework. The predicted reduced cross sections are calculated using the HERAPDF2.0 FF3A [40] and ABM11 [27] NLO PDF sets using NLO $\left(O\left(\alpha_{s}^{2}\right)\right)$ coefficient functions and the ABMP16 [28] NNLO PDF set using approximate NNLO coefficient functions. The charm data are also compared to NLO predictions based on the ABKM09 [25] NLO PDF set already used in the previous analysis [36] of combined charm data. This PDF set was determined for a charm quark mass of $m_{c}\left(m_{c}\right)=1.18 \mathrm{GeV}$. The PDF sets considered were extracted without explicitly using heavy flavour data from HERA with the exception of the ABMP16 set, in which the HERA charm data from the previous combination [36] and some of the beauty data [14,21] have been included. The theory predictions are obtained without fitting the data. For the predictions based on the HERAPDF2.0 FF3A set theory uncertainties are given which are calculated by adding in quadrature the uncertainties from the PDF set ${ }^{3}$, the simultaneous variation of $\mu_{r}$ and $\mu_{f}$ by a factor of two up and down and the variation of the quark masses within the quoted uncertainties.

The FFNS calculations reasonably describe the charm data (figure 8) although in the kinematic range where the data are very precise the data show a somewhat steeper $x_{\mathrm{Bj}}$ dependence than predicted by the calculations. For the different PDF sets and QCD orders considered the predictions are quite similar at larger $Q^{2}$ while some differences can be observed at smaller $Q^{2}$ or $x_{\mathrm{Bj}}$. In case of beauty production (figure 9 ) the predictions are in good agreement with the data within the considerably larger experimental uncertainties.

The description of the charm production data is illustrated further in figure 10 , which shows the ratios of the reduced cross sections for data, ABKM09, ABM11 and ABMP16 with respect to the NLO reduced cross sections predicted in the FFNS using the HERAPDF2.0 FF3A set. For $Q^{2} \geq 18 \mathrm{GeV}^{2}$ the theory predictions are similar in the kinematic region accessible at HERA. In this region the predictions based on the different PDF sets are well within the theoretical uncertainties obtained for the HERAPDF2.0 FF3A set. Towards smaller $Q^{2}$ and $x_{\mathrm{Bj}}$ some differences in the predictions become evident. In the region of $7 \mathrm{GeV}^{2} \leq Q^{2} \leq 120 \mathrm{GeV}^{2}$ the theory tends to be below the data at small $x_{\mathrm{Bj}}$ and above the data at large $x_{\mathrm{Bj}}$ independent of the PDF set used.

In figure 11 the corresponding ratios are shown for the reduced beauty cross sections. In the kinematic region accessible at HERA the predictions based on the different PDF sets are very similar. Within the experimental uncertainties the data are well described by all calculations.

### 5.2 VFNS predictions

In figure 12 predictions of the RTOPT [32] NLO and approximate NNLO VFNS using the corresponding NLO and NNLO HERAPDF2.0 PDF sets are compared to the charm measurements.

[^2]As in figure 10 the ratio of data and theory predictions to the reference calculations are shown. While the NLO VFNS predictions are in general consistent with both the data cross sections and the reference calculations within their uncertainties, the approximate NNLO cross sections show somewhat larger differences predicting about $10 \%$ smaller cross sections than the reference calculations in the region $12 \mathrm{GeV}^{2} \leq Q^{2} \leq 120 \mathrm{GeV}^{2}$. On the other hand, at $Q^{2} \leq 7 \mathrm{GeV}^{2}$ the $x_{\mathrm{Bj}}$-slopes of the NNLO VFNS predictions tend to describe the data somewhat better than the reference calculations. Overall, the NLO and approximate NNLO VFNS calculations describe the data about equally well, but not better than the reference FFNS calculations.

In figure 13 the corresponding ratios are presented for beauty production. In the kinematic region accessible in DIS beauty production at HERA the differences between the different calculations are small in comparison to the experimental uncertainties of the measurements.

The calculations considered so far show some tension in general in describing the $x_{\mathrm{Bj}}{ }^{-}$ slopes of the measured reduced charm cross sections over a large range in $Q^{2}$. Therefore the charm data are compared in figure 14 to recent calculations [35] ${ }^{4}$ in the FONNL-C scheme with (NLLO+NLLsx) and without (NNLO) low- $x$ resummation in both $O\left(\alpha_{s}^{2}\right)$ matrix elements and $O\left(\alpha_{s}^{3}\right)$ PDF evolution, using the NNPDF3.1sx framework, which aims at a better description of the structure functions at low $x_{\mathrm{Bj}}$ and $Q^{2}$. For the determination of the NNPDF3.1sx PDFs the charm data from the previous combination have already been used. Both calculations provide a better description of the $x_{\mathrm{Bj}}$-shape of the measured charm cross sections for $Q^{2} \leq 32 \mathrm{GeV}^{2}$. However, the predictions lie significantly below the data in most of the phase space. This is especially the case for the NNLO+NLLsx calculations. Overall, the description is not improved with respect to the FFNS reference calculations.

### 5.3 Summary of theory comparison

The comparison of the different predictions considered to the data is summarised in table 5 in which the agreement with data is expressed in terms of $\chi^{2}$ and the corresponding p -values. The table also includes a comparison to the previous combined reduced charm cross section measurement [36]. The agreement of the various predictions with the combined charm cross section measurements of the current analysis is poorer than with the results of the previous combination for which consistency between theory and data within the experimental uncertainties was observed for most of the calculations. As shown in section 4 the combined charm cross sections of the current analysis agree well with the previous measurement but have considerably smaller uncertainties due to the high precision data added. The observed changes in the $\chi^{2}$-values are consistent with the improvement in data precision if the calculations show tensions in describing the underlying process. Among the calculations considered the NLO FFNS calculations provide the best description of the charm data. The observed $\sim 3 \sigma$ tensions are not resolved by VFNS calculations either with or without additional $\log x$ resummation. For the beauty cross sections good agreement of theory and data is observed within the large experimental uncertainties. In all cases the effect of the PDF uncertainties on the $\chi^{2}$ values is negligible.

[^3]
## 6 QCD analysis

The combined beauty and charm data are used together with the combined HERA inclusive DIS data [40] to perform a QCD analysis in the FFNS at NLO. The main focus of this analysis is the simultaneous determination of the running heavy quark masses $m_{c}\left(m_{c}\right)$ and $m_{b}\left(m_{b}\right)$. Furthermore the theory description of the $x_{\mathrm{Bj}}$-dependence of the reduced cross section of charm production is investigated.

### 6.1 Theoretical formalism and settings

The analysis is performed with the XFITTER [45] program in which the scale evolution of partons is calculated through DGLAP equations [61] at NLO, as implemented in the QCDNUM program [62]. The theoretical FFNS predictions for the HERA data are obtained using the OPENQCDRAD program [44] interfaced in the XFITTER framework. The number of active flavours is set to $n_{f}=3$ at all scales. The renormalisation and factorisation scales for heavyflavour production are set to $\mu_{r}=\mu_{f}=\sqrt{Q^{2}+4 m_{\mathrm{Q}}^{2}}$, where $m_{\mathrm{Q}}$ denotes the running mass of $c$ or $b$ quarks. The heavy-quark masses are left free in the fit if not stated otherwise. For the lightflavour contributions to the inclusive DIS cross sections, the pQCD scales are set to $\mu_{r}=\mu_{f}=$ $Q$. The massless contribution to the longitudinal structure function $F_{L}$ is calculated to $O\left(\alpha_{s}\right)$. The strong coupling strength is set to $\alpha_{s}^{n_{f}=3}\left(M_{Z}\right)=0.106$, corresponding to $\alpha_{s}^{n_{f}=5}\left(M_{Z}\right)=$ 0.118. In order to perform the analysis in the kinematic region where pQCD is commonly assumed to be applicable the $Q^{2}$ range of the inclusive HERA data is restricted to $Q^{2} \geq Q_{\mathrm{min}}^{2}=$ $3.5 \mathrm{GeV}^{2}$. No such cut is applied to the charm and beauty data since the relevant scales $\mu_{r}^{2}=$ $\mu_{f}^{2}=Q^{2}+4 m_{\mathrm{Q}}^{2}$ are always above $3.5 \mathrm{GeV}^{2}$.

The $\chi^{2}$ definition used for the HERA DIS data follows that of equation (32) in reference [40]. It includes an additional logarithmic term that is relevant when the estimated statistical and uncorrelated systematic uncertainties in the data are rescaled during the fit [63]. The correlated systematic uncertainties are treated through nuisance parameters.

The procedure for the determination of the PDFs follows the approach of HERAPDF2.0 [40]. At the starting scale $\mu_{\mathrm{f}, 0}$ the density functions of a parton $f$ of the proton are parametrised using the generic form:

$$
\begin{equation*}
x f(x)=A x^{B}(1-x)^{C}\left(1+D x+E x^{2}\right), \tag{5}
\end{equation*}
$$

where $x$ is the momentum fraction in the infinite momentum frame of the incoming proton transferred to the struck parton. The parametrised PDFs are the gluon distribution $\operatorname{xg}(x)$, the valence quark distributions $x u_{v}(x)$ and $x d_{v}(x)$, and the $u$ - and $d$-type antiquark distributions $x \bar{U}(x)$ and $x \bar{D}(x)$.

At the initial QCD evolution scale $\mu_{\mathrm{f}, 0}^{2}=1.9 \mathrm{GeV}^{2}$, the default parametrisation of the PDFs has the form:

$$
\begin{align*}
x g(x) & =A_{g} x^{B_{g}}(1-x)^{C_{g}}-A_{g}^{\prime} x^{B_{g}^{\prime}}(1-x)^{C_{g}^{\prime}}, \\
x u_{v}(x) & =A_{u_{v}} x^{B_{u_{v}}}(1-x)^{C_{u_{v}}}\left(1+E_{u_{v}} x^{2}\right), \\
x d_{v}(x) & =A_{d_{v}} x^{B_{d_{v}}}(1-x)^{C_{d_{v}}},  \tag{6}\\
x \bar{U}(x) & =A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}\left(1+D_{\bar{U}} x\right), \\
x \bar{D}(x) & =A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}},
\end{align*}
$$

assuming the relations $x \bar{U}(x)=x \bar{u}(x)$ and $x \bar{D}(x)=x \bar{d}(x)+x \bar{s}(x)$. Here, $x \bar{u}(x), x \bar{d}(x)$, and $x \bar{s}(x)$ are the up, down, and strange antiquark distributions, respectively. The sea quark distribution is defined as $\mathrm{x} \Sigma(x)=x \bar{u}(x)+x \bar{d}(x)+x \bar{s}(x)$. The normalisation parameters $A_{u_{v}}, A_{d_{v}}$, and $A_{g}$ are determined by the QCD sum rules. The $B$ and $B^{\prime}$ parameters determine the PDFs at small $x$, and the $C$ parameters describe the shape of the distributions as $x \rightarrow 1$. The parameter $C_{g}^{\prime}$ is fixed to 25 [64]. Additional constraints $B_{\bar{U}}=B_{\bar{D}}$ and $A_{\bar{U}}=A_{\bar{D}}\left(1-f_{s}\right)$ are imposed to ensure the same normalisation for the $x u$ and $x d$ distributions as $x \rightarrow 0$. The strangeness fraction $f_{s}=x s /(x d+x s)$ is fixed to $f_{s}=0.4$ as in the HERAPDF2.0 analysis [40].

The parameters in equation (5) are selected by first fitting with all $D$ and $E$ parameters set to zero, and then including them independently one at a time in the fit. The improvement in the $\chi^{2}$ of the fit is monitored and the procedure is stopped when no further improvement is observed. This leads to the same 14 free PDF parameters, as in the inclusive HERAPDF2.0 analysis [40].

The PDF uncertainties are estimated according to the general approach of HERAPDF2.0 [40] in which the fit, model, and parametrisation uncertainties are taken into account. Fit uncertainties are determined using the tolerance criterion of $\Delta \chi^{2}=1$. Model uncertainties arise from the variations of the strong coupling constant $\alpha_{s}^{n_{f}=3}\left(M_{Z}\right)=0.106 \pm 0.0015$, the simultaneous variation of the factorisation and renormalisation scales up and down by a factor of two, the strangeness fraction $0.3 \leq f_{s} \leq 0.5$, and the value of $2.5 \leq Q_{\min }^{2} \leq 5.0 \mathrm{GeV}^{2}$ imposed on the inclusive HERA data. The parametrisation uncertainty is estimated by extending the functional form in equation (6) of all parton density functions with additional parameters $D$ and $E$ added one at a time. An additional parametrisation uncertainty is considered by using the functional form in equation (6) with $E_{u_{v}}=0$. The $\chi^{2}$ in this variant of the fit is only 5 units worse than with the released $E_{u_{v}}$ parameter and the change of this parameter noticabely affects the mass determination. Furthermore, $\mu_{\mathrm{f}, 0}^{2}$ is varied within $1.6 \mathrm{GeV}^{2}<\mu_{\mathrm{f}, 0}^{2}<2.2 \mathrm{GeV}^{2}$. The parametrisation uncertainty is constructed at each $x_{\mathrm{Bj}}$ value, built from the maximal differences between the PDFs resulting from the central fit and all parametrisation variations. This uncertainty is valid in the $x$ range covered by the PDF fit to the data. The total PDF uncertainty is obtained by adding the fit, model, and parametrisation uncertainties in quadrature. In the following, the quoted uncertainties correspond to $68 \%$ CL.

### 6.2 QCD fit and determination of the running heavy quark masses

In the QCD fit to the HERA combined inclusive and combined heavy flavour data the running heavy quark masses are fitted simultaneously together with the PDF parameters. The fit yields a total $\chi^{2}=1435$ for 1208 degrees of freedom (d.o.f.). The ratio $\chi 2 /$ d.o.f. $=1.19$ is similar to the values obtained in the analysis of the HERA combined inclusive data [40].

In figure 15 the PDFs at the scale $\mu_{\mathrm{f}, 0}=1.9 \mathrm{GeV}^{2}$ are presented. Also shown are the PDFs with experimental uncertainties from a fit to the inclusive data only with fixing the heavy quark masses to their PDG values [51]. Only marginal differences, well within the experimental uncertainties, between these two PDF sets are visible. When comparing the central functions of the gluon density a slight enhancement of the gluon density around $x \approx 10^{-3}$ can be observed when including the heavy flavour data in the fit compared to the gluon density determined from the inclusive data only. This is the region in $x$ where the charm data are most precise. However, the difference observed is within the experimental uncertainties. When used together
with the full sets of inclusive HERA data the heavy flavour data have only little influence on the form of the PDFs determined with quark masses fixed to their expected values. Despite the more precise heavy flavour data available in the current analysis this finding does not alter the conclusion made on this point in the HERAPDF2.0 analysis [40]. However, the smaller uncertainties of the new combination reduce the uncertainty of the charm mass determination with respect to the previous result ${ }^{5}$

The running heavy quark masses are determined as:

$$
\begin{align*}
& m_{c}\left(m_{c}\right)=1.290_{-0.041}^{+0.046}(\mathrm{fit})_{-0.014}^{+0.062}(\mathrm{mod})_{-0.031}^{+0.007}(\text { par }) \mathrm{GeV} \\
& m_{b}\left(m_{b}\right)=4.049_{-0.109}^{+0.104}(\text { fit })_{-0.032}^{+0.009}(\mathrm{mod})_{-0.033}^{+0.001}(\text { par }) \mathrm{GeV} \tag{7}
\end{align*}
$$

The model uncertainties are dominated by theoretical uncertainties arising from the scale variations. In case of the charm quark mass the variation in $\alpha_{s}$ yields also a sizeable contribution while the effects on the model uncertainties of all other model variations are small, typically of a few MeV , both for $m_{c}\left(m_{c}\right)$ and $m_{b}\left(m_{b}\right)$. The main contribution to the parametrisation uncertainties comes from the fit variant in which the term $E_{u_{v}}$ is set to zero. The parametrisation uncertainties from all other variations of the parameterisation are negligible. The running charm quark mass determined here agrees well with result from the previous analysis of HERA combined charm cross sections [36] and both masses are in agreement with the corresponding PDG values [51].

A cross check is performed using the Monte Carlo method [65,66]. It is based on analysing a large number of pseudo data sets called replicas. For this cross check, 500 replicas are created by taking the combined data and fluctuating the values of the reduced cross sections randomly within their given statistical and systematic uncertainties taking into account correlations. All uncertainties are assumed to follow a Gaussian distribution. The central values for the fitted parameters and their uncertainties are estimated using the mean and RMS values over the replicas. The obtained heavy-quark masses and their fit uncertainties are in agreement with those quoted in equation (7).

Fits to the combined inclusive data only are also tried. In this case the fit results are very sensitive to the choice of the PDF parametrisation. When using the default 14 parameters the masses are determined to be $m_{c}\left(m_{c}\right)=1.80_{-0.13}^{+0.14}$ (fit) $\mathrm{GeV}, m_{b}\left(m_{b}\right)=8.45_{-1.81}^{+2.28}($ fit $) \mathrm{GeV}$ where only the fit uncertainties are quoted. In the variant of the fit using the inclusive data only and the reduced parametrisation with $E_{u_{v}}=0$ the central fitted values for the heavy-quark masses are: $m_{c}\left(m_{c}\right)=1.45 \mathrm{GeV}, m_{b}\left(m_{b}\right)=4.00 \mathrm{GeV}$. The sensitivity on the PDF parameterisation and the large fit uncertainties for a given parameterisation demonstrates that attempts to extract heavy quark masses from inclusive HERA alone are not sensible in this framework. The uncertainties of the mass determinations induced by this behaviour of the inclusive data are covered by the extra $E_{u_{v}}$ variation.

The NLO FFNS predictions based on the PDF set and the running beauty and charm quark masses determined by the fit to the combined inclusive and combined heavy flavour data are compared to the combined charm and beauty cross sections in figures 16 and 17 , respectively. The predictions based on the HERAPDF2.0 set are also included in the figures. Only minor differences between the different predictions can be observed which is to be expected, because

[^4]of the similarities of the PDFs especially of the gluon and the description of the data is similar to that observed for the predictions based on the HERAPDF2.0 FF3A set.

In order to better visualise the differences the ratios of data and predictions based on the PDF set and running beauty and charm masses obtained in this analysis to the predictions based on the fixed HERAPDF2.0 FF3A PDF set are shown in figures 18 and 19 for charm and beauty, respectively. The description of the data is almost identical for both calculations. There are only marginal differences between the two calculations because of the dominance of the inclusive cross section measurements which are more precise in the kinematic region accessible with the heavy flavour measurements. In general the predictions based on HERAPDF2.0 FF3A are a few per cent above those obtained with the parameters fitted in this analysis for charm and below in case of beauty. This can be explained by the fact that the fitted masses $m_{c}\left(m_{c}\right)\left(m_{b}\left(m_{b}\right)\right)$ are slightly larger (smaller) than those used for the predictions based on HERAPDF2.0 FF3A.

### 6.3 Study of the $x_{\mathrm{Bj}}$ dependence of the reduced charm cross section

The comparison of the measured charm cross sections and the calculations in figure 18 indicates a steeper $x_{\mathrm{Bj}}$ dependence of the measured charm cross section than expected in NLO FFNS with the fitted parameters and the PDF parametrisation chosen. A similar behaviour can be observed already for the charm cross sections from the previous combination [36] albeit at lower significance due to the larger uncertainties. The partial $\chi^{2}$ value of 116 for the heavy flavour data ${ }^{6}$ (d.o.f. $=79$ ) in the fit presented is somewhat large and corresponds to a p-value ${ }^{7}$ of 0.004 , equivalent to $2.9 \sigma$. A similar $x_{\mathrm{Bj}}$-behaviour can also be observed in figures 10 and 12 for most of the other calculations presented in this paper and all calculations discussed show some tensions in describing the combined charm data (table 5) using fits to the inclusive data.

Heavy flavour production is dominated by the boson-gluon-fusion process which is directly related to the gluon density function. The light flavour initiated contribution to charm production at HERA amounts to five to eight per cent varying only slightly with $x_{\mathrm{Bj}}$ or $Q^{2}$ [42]. To study the behaviour of the heavy flavour cross sections as a function of the partonic $x$ and to see the ranges of $x$ accessible by charm and beauty production in DIS at HERA, the ratio of the measured reduced cross sections to the NLO FFNS predictions based on the PDFs and masses determined in the fit in this paper is presented in figure 20 as a function of $\langle x\rangle$ instead of $x_{\mathrm{Bj}}$. The average $x$-values for the $\left(x_{\mathrm{Bj}}, Q^{2}\right)$ points at which the cross sections are measured are calculated at NLO using HVQDIS [43]. While the charm measurements cover the range

What is mean $x$ ? Averaged over what? was it defined? $0.0005 \lesssim\langle x\rangle \lesssim 0.1$ the beauty data can access only a higher $x$-range, $0.004 \lesssim\langle x\rangle \lesssim 0.1$, because of the large beauty quark mass. For the charm data a clear deviation from the reference calculation is evident showing a steeper slope in $x$ in the range $0.0005 \lesssim\langle x\rangle \lesssim 0.01$ consistent with being independent of $Q^{2}$. Due to the larger experimental uncertainties no conclusion can be made for the beauty data. Was this quantified by doing a straight-line fit and looking at the significance of te slope value?

[^5]To reduce the impact of the inclusive data a series of fits is performed with requiring
applied to the heavy flavour data. The $\chi^{2} /$ d.
and the partial $\chi^{2} /$ d.o.f. for the heavy flavour data only are presented in figure 21 as function of $x_{\mathrm{Bj}, \text { min }}$. The partial $\chi^{2} /$ d.o.f. for the heavy flavour data improves significantly with rising $x_{\mathrm{Bj}, \text { min }}$-cut reaching a minimum at $x_{\mathrm{Bj}, \text { min }} \approx 0.04$ while the $\chi^{2} /$ d.o.f. for the inclusive plus heavy flavour data sample slightly increases. For the further studies $x_{\mathrm{Bj}, \text { min }}=0.01$ is chosen. The total $\chi^{2}$ is 822 for 651 degrees of freedom. The partial $\chi^{2}$ of the heavy flavour data improves to 98 (corresponding to a $p$-value of 0.07 or $1.8 \sigma$ ). What chi2 is it? Is that the stat component only or does it include the HF figure 22 at the scale $\mu_{\mathrm{f}, 0}^{2}=1.9 \mathrm{GeV}^{2}$ is signif systematics components? determined when including all inclusive meas functions are consistent with the result of the deratrin witriricipermititarurtitantits.

In figure 23 a comparison is presented of the ratios of the combined reduced charm cross section, $\sigma_{\text {red }}^{c \bar{c}}$, the cross section predictions obtained from the fit to the heavy flavour data and the inclusive data fulfilling $x_{\mathrm{Bj}} \geq 0.01$ and the predictions from the fit to the heavy flavour data and the full inclusive data set to the reference cross sections. As expected the charm cross sections fitted with the $x_{\mathrm{Bj}}$-cut imposed to the inclusive data are rising stronger towards small $x_{\mathrm{Bj}}$ and describe the data better than the other predictions. In general the predictions from the fit with $x_{\mathrm{Bj}}$-cut follow nicely the charm data. A similar study is also made for the beauty measurements (not shown). Here also differences are visible but they are small compared to the experimental uncertainties.

Cross section predictions based on the three PDF sets discussed are calculated for inclusive DIS also. In figures 24 these predictions are compared to the measured combined inclusive reduced cross sections [40] for neutral current positron-proton scattering, $e^{+} p \rightarrow e^{+} X$. The predictions based on HERAPDF2.0 FF3A and on the PDF set obtained in this analysis by the fit to the combined heavy flavour and inclusive data agree with the inclusive measurement. The calculations based on the PDF set determined by requiring $x_{\mathrm{Bj}} \geq 0.01$ for the inclusive data predicts significantly larger inclusive reduced cross sections at small $x_{\mathrm{Bj}}$.

This study shows, that a better description of Where does 3 sigma estimate come from? so far there was no explicit within the framework for PDFs applied by exclu ${ }^{\text {t }}$ ever, the calculations then fail to describe the inc ension quantificaton or did i overlook it?
work considered it seems not possible to resolve the $\sim 3 \sigma$ tension in theory in describing both the inclusive and charm measurements from HERA with this simple approach of changing mainly the gluon density. However, the comparison of various theory predictions to the charm data in section 5 suggests that the situation is unlikely to improve at NNLO because the NNLO predictions presented provide a poorer description of the charm data than observed at NLO. The combined inclusive analysis [40] already revealed some tensions of the theoretical calculations in describing the inclusive DIS data. The current analysis reveals some additional tensions in describing both the combined charm data and the combined inclusive data simultaneously. A dedicated investig 1 would suggest to add something like "tensions of the *NLO and NNLO* measurements beyond the quqtheoretical predictions". At the moment the text is to general and gives an impression that it is true in QCD in general. i,e, to all orders. However, we know that order of various components (F2light, FLlight, F2hf) does matter for the

## 7 Conclusions

Measurements of beauty and charm production cross sections in deep-inelastic ep scattering by the H1 and ZEUS experiments are combined at the level of reduced cross sections, accounting for their statistical and systematic correlations. The beauty cross sections are combined for the first time. The data sets are found to be consistent and the combined data have significantly reduced uncertainties. The combined reduced charm cross sections presented in this paper are significantly more precise than the previously published combined charm measurements. The combined data are compared to next-to-leading and approximate next-to-next-to-leading order QCD predictions, which are found to be in fair agreement with the charm data, whereas the beauty data are described well given the larger experimental uncertainties of the beauty measurements. The next-to-leading order calculations in the fixed-flavour-number-scheme provide the best description of the heavy flavour data. Variable-flavour-number-scheme calculations do not improve the overall description, with or without the inclusion of $\log \frac{1}{x}$ resummation.

The combined heavy flavour data together with the published combined inclusive data from HERA are subjected to a next-to-leading order QCD analysis in the fixed-flavour-numberscheme using the $\overline{\mathrm{MS}}$ running mass definition. The running heavy quark masses are determined as $m_{c}\left(m_{c}\right)=1.290_{-0.041}^{+0.046}(\mathrm{fit})_{-0.014}^{+0.062}(\bmod )_{-0.031}^{+0.007}(\mathrm{par}) \mathrm{GeV}$ for the charm quark and $m_{b}\left(m_{b}\right)=$ $4.049_{-0.109}^{+0.104}(\mathrm{fit})_{-0.032}^{+0.090}(\mathrm{mod})_{-0.031}^{+0.001}$ (par) GeV for the beauty quark. The simultaneously determined parton density functions are found to agree well with HERAPDF2.0 FF3A.

The QCD analysis reveals some tensions in describing at the same time both the inclusive and the charm HERA DIS data. The measured reduced charm cross sections show a stronger $x_{\mathrm{Bj}}$ dependence than calculated in the theoretical framework of the QCD analysis. A study is performed in which inclusive data with $x_{\mathrm{Bj}}<0.01$ are excluded from the fit. A much better description of the charm data can be achieved this way, however, the resulting PDFs fail to describe the inclusive data in the excluded $x_{\mathrm{Bj}}$ region. This points to difficulties in resolving the observed tensions in the theoretical calculations by changing mainly the gluon density distribution in the proton. However, the other next-to-leading order and especially the next-to-next-leading order QCD calculations considered do not provide a better agreement with the combined heavy flavour data.

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| Data set | Tagging | $\begin{gathered} \hline Q^{2} \text { range } \\ {\left[\mathrm{GeV}^{2}\right]} \end{gathered}$ | $N_{c}$ | $\begin{array}{r} \mathscr{L} \\ {\left[\mathrm{pb}^{-1}\right]} \end{array}$ | $\begin{array}{r} \sqrt{s} \\ {[\mathrm{GeV}]} \end{array}$ | $N_{b}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 H1 VTX [14] | VTX | $5-2000$ | 29 | 245 | 318 | 12 |
| 2 H1 $D^{*+}$ HERA-I [10] | $D^{*+}$ | $2-100$ | 17 | 47 | 318 |  |
| 3 H1 $D^{*+}$ HERA-II (medium $Q^{2}$ ) [18] | $D^{*+}$ | 5 - 100 | 25 | 348 | 318 |  |
| 4 H1 $D^{*+}$ HERA-II (high $Q^{2}$ ) [15] | $D^{*+}$ | $100-1000$ | 6 | 351 | 318 |  |
| 5 ZEUS $D^{*+} 96-97$ [4] | $D^{*+}$ | 1 - 200 | 21 | 37 | 300 |  |
| 6 ZEUS $D^{*+} 98-00$ [6] | $D^{*+}$ | 1.5 - 1000 | 31 | 82 | 318 |  |
| 7 ZEUS $D^{0} 2005$ [12] | $D^{0}$ | $5-1000$ | 9 | 134 | 318 |  |
| 8 ZEUS $\mu 2005$ [13] | $\mu$ | $20-10000$ | 8 | 126 | 318 | 8 |
| 9 ZEUS $D^{+}$HERA-II [19] | $D^{+}$ | $5-1000$ | 14 | 354 | 318 |  |
| 10 ZEUS $D^{*+}$ HERA-II [20] | $D^{*+}$ | $5-1000$ | 31 | 363 | 318 |  |
| 11 ZEUS VTX HERA-II [21] | VTX | $5-1000$ | 18 | 354 | 318 | 17 |
| 12 ZEUS e HERA-II [22] | $e$ | $10-1000$ |  | 363 | 318 | 9 |
| 13 ZEUS $\mu+$ jet HERA-I [23] | $\mu$ | $2-3000$ |  | 114 | 318 | 11 |

Table 1: Data sets used in the combination. For each data set the $Q^{2}$ range, integrated luminosity $(\mathscr{L})$, centre-of-mass energy $(\sqrt{s})$ and the numbers of charm $\left(N_{c}\right)$ and beauty $\left(N_{b}\right)$ measurements are given.

| bin | $Q^{2}\left[\mathrm{GeV}^{2}\right]$ | $x$ | $\sigma_{\text {red }}^{c \bar{c}}$ | $\delta_{\text {stat }}[\%]$ | $\delta_{\text {uncor }}[\%]$ | $\delta_{\text {cor }}[\%]$ | $\delta_{\text {tot }}[\%]$ |
| :---: | :---: | :---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 2.5 | 0.00003 | 0.11423 | 8.9 | 10.7 | 9.4 | 16.9 |
| 2 | 2.5 | 0.00007 | 0.11054 | 5.8 | 6.7 | 8.2 | 12.1 |
| 3 | 2.5 | 0.00013 | 0.09111 | 7.1 | 6.2 | 7.9 | 12.3 |
| 4 | 2.5 | 0.00018 | 0.09170 | 4.8 | 9.6 | 7.2 | 12.9 |
| 5 | 2.5 | 0.00035 | 0.05437 | 5.3 | 8.2 | 6.9 | 12.0 |
| 6 | 5.0 | 0.00007 | 0.15321 | 11.6 | 9.6 | 8.2 | 17.1 |
| 7 | 5.0 | 0.00018 | 0.15385 | 5.3 | 3.4 | 7.8 | 10.0 |
| 8 | 5.0 | 0.00035 | 0.11642 | 5.2 | 5.3 | 5.7 | 9.3 |
| 9 | 5.0 | 0.00100 | 0.07763 | 4.8 | 8.7 | 5.6 | 11.4 |
| 10 | 7.0 | 0.00013 | 0.22486 | 4.3 | 3.3 | 6.7 | 8.6 |
| 11 | 7.0 | 0.00018 | 0.20231 | 6.8 | 5.7 | 7.2 | 11.4 |
| 12 | 7.0 | 0.00030 | 0.17669 | 2.3 | 2.4 | 5.4 | 6.4 |
| 13 | 7.0 | 0.00050 | 0.16158 | 2.5 | 1.8 | 5.2 | 6.0 |
| 14 | 7.0 | 0.00080 | 0.11994 | 4.6 | 4.0 | 4.9 | 7.8 |
| 15 | 7.0 | 0.00160 | 0.09023 | 4.1 | 3.9 | 5.2 | 7.7 |
| 16 | 12.0 | 0.00022 | 0.31613 | 4.9 | 2.9 | 5.7 | 8.0 |
| 17 | 12.0 | 0.00032 | 0.29041 | 2.9 | 1.5 | 6.3 | 7.1 |
| 18 | 12.0 | 0.00050 | 0.24098 | 2.4 | 1.3 | 4.6 | 5.3 |
| 19 | 12.0 | 0.00080 | 0.18134 | 2.1 | 1.4 | 4.5 | 5.1 |
| 20 | 12.0 | 0.00150 | 0.14761 | 3.2 | 1.5 | 5.1 | 6.2 |
| 21 | 12.0 | 0.00300 | 0.10103 | 4.4 | 4.0 | 5.1 | 7.8 |
| 22 | 18.0 | 0.00035 | 0.31977 | 5.2 | 3.3 | 5.2 | 8.1 |
| 23 | 18.0 | 0.00050 | 0.29049 | 2.6 | 1.4 | 6.4 | 7.0 |
| 24 | 18.0 | 0.00080 | 0.25539 | 2.2 | 1.2 | 4.2 | 4.9 |
| 25 | 18.0 | 0.00135 | 0.20163 | 2.0 | 1.1 | 4.1 | 4.7 |
| 26 | 18.0 | 0.00250 | 0.16300 | 1.9 | 1.3 | 4.2 | 4.7 |
| 27 | 18.0 | 0.00450 | 0.11367 | 5.5 | 4.1 | 5.4 | 8.7 |

Table 2: The averaged reduced cross section for charm production, $\sigma_{\text {red }}^{c \bar{c}}$, obtained by the combination of H 1 and ZEUS measurements. The cross section values are given together with the statistical $\left(\delta_{\text {stat }}\right)$ and the uncorrelated ( $\delta_{\text {uncor }}$ ) and correlated $\left(\delta_{\text {cor }}\right)$ systematic uncertainties. The total uncertainties ( $\delta_{t o t}$ ) are obtained by adding the statistical, uncorrelated and correlated systematic uncetrainties in quadrature. All uncertainties are quoted in per cent.

| bin | $Q^{2}\left[\mathrm{GeV}^{2}\right]$ | $x$ | $\sigma_{\text {red }}^{c \bar{c}}$ | $\delta_{\text {stat }}[\%]$ | $\delta_{\text {uncor }}[\%]$ | $\delta_{\text {cor }}[\%]$ | $\delta_{\text {tot }}[\%]$ |
| :---: | :---: | :---: | ---: | ---: | ---: | ---: | ---: |
| 28 | 32.0 | 0.00060 | 0.38846 | 8.5 | 9.3 | 5.8 | 13.9 |
| 29 | 32.0 | 0.00080 | 0.37557 | 2.3 | 1.4 | 4.4 | 5.2 |
| 30 | 32.0 | 0.00140 | 0.28070 | 2.0 | 1.1 | 3.4 | 4.1 |
| 31 | 32.0 | 0.00240 | 0.21897 | 2.3 | 1.4 | 3.9 | 4.7 |
| 32 | 32.0 | 0.00320 | 0.20149 | 3.6 | 1.6 | 5.4 | 6.6 |
| 33 | 32.0 | 0.00550 | 0.15534 | 4.2 | 3.0 | 4.1 | 6.6 |
| 34 | 32.0 | 0.00800 | 0.09403 | 8.7 | 5.4 | 6.0 | 11.9 |
| 35 | 60.0 | 0.00140 | 0.32542 | 3.2 | 1.4 | 4.8 | 5.9 |
| 36 | 60.0 | 0.00200 | 0.32893 | 2.3 | 1.2 | 4.1 | 4.9 |
| 37 | 60.0 | 0.00320 | 0.25762 | 2.2 | 1.2 | 3.6 | 4.4 |
| 38 | 60.0 | 0.00500 | 0.19250 | 2.3 | 1.6 | 4.1 | 5.0 |
| 39 | 60.0 | 0.00800 | 0.15960 | 4.8 | 3.1 | 3.4 | 6.7 |
| 40 | 60.0 | 0.01500 | 0.09458 | 8.1 | 6.5 | 4.9 | 11.5 |
| 41 | 120.0 | 0.00200 | 0.37661 | 3.3 | 2.6 | 5.0 | 6.5 |
| 42 | 120.0 | 0.00320 | 0.22743 | 14.6 | 13.7 | 2.7 | 20.2 |
| 43 | 120.0 | 0.00550 | 0.21729 | 3.3 | 1.6 | 5.4 | 6.5 |
| 44 | 120.0 | 0.01000 | 0.15186 | 3.9 | 2.3 | 5.2 | 6.9 |
| 45 | 120.0 | 0.02500 | 0.07022 | 13.6 | 12.6 | 4.4 | 19.1 |
| 46 | 200.0 | 0.00500 | 0.23889 | 3.1 | 2.4 | 4.5 | 6.0 |
| 47 | 200.0 | 0.01300 | 0.17035 | 3.4 | 2.3 | 5.0 | 6.5 |
| 48 | 350.0 | 0.01000 | 0.22300 | 5.1 | 3.0 | 6.4 | 8.7 |
| 49 | 350.0 | 0.02500 | 0.10646 | 6.1 | 2.9 | 7.4 | 10.0 |
| 50 | 650.0 | 0.01300 | 0.20260 | 5.4 | 3.7 | 9.1 | 11.2 |
| 51 | 650.0 | 0.03200 | 0.08846 | 7.8 | 3.8 | 12.8 | 15.4 |
| 52 | 2000.0 | 0.05000 | 0.06026 | 16.0 | 6.7 | 26.4 | 31.6 |

Table 2: continued

| bin | $Q^{2}\left[\mathrm{GeV}^{2}\right]$ | $x$ | $\sigma_{\text {red }}^{b \bar{b}}$ | $\delta_{\text {stat }}$ | $\delta_{\text {uncor }}$ | $\delta_{\text {cor }}$ | $\delta_{\text {tot }}$ |
| :---: | :---: | :---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 2.5 | 0.00013 | 0.00184 | 28.4 | 22.4 | 11.4 | 37.9 |
| 2 | 5.0 | 0.00018 | 0.00476 | 10.5 | 7.1 | 19.8 | 23.5 |
| 3 | 7.0 | 0.00013 | 0.00593 | 8.8 | 11.2 | 12.7 | 19.1 |
| 4 | 7.0 | 0.00030 | 0.00398 | 8.5 | 10.3 | 15.2 | 20.2 |
| 5 | 12.0 | 0.00032 | 0.00715 | 4.9 | 5.8 | 10.5 | 13.0 |
| 6 | 12.0 | 0.00080 | 0.00409 | 4.6 | 6.9 | 11.1 | 13.9 |
| 7 | 12.0 | 0.00150 | 0.00145 | 32.2 | 26.9 | 3.6 | 42.1 |
| 8 | 18.0 | 0.00080 | 0.00817 | 4.8 | 5.0 | 12.8 | 14.5 |
| 9 | 32.0 | 0.00060 | 0.02074 | 8.9 | 7.8 | 8.9 | 14.8 |
| 10 | 32.0 | 0.00080 | 0.01516 | 5.8 | 6.1 | 10.0 | 13.1 |
| 11 | 32.0 | 0.00140 | 0.01135 | 3.9 | 5.3 | 9.0 | 11.2 |
| 12 | 32.0 | 0.00240 | 0.00824 | 9.0 | 9.5 | 12.9 | 18.4 |
| 13 | 32.0 | 0.00320 | 0.00464 | 32.2 | 41.9 | 3.0 | 52.9 |
| 14 | 32.0 | 0.00550 | 0.00579 | 39.8 | 20.4 | 57.4 | 72.8 |
| 15 | 60.0 | 0.00140 | 0.02599 | 4.8 | 6.9 | 8.8 | 12.2 |
| 16 | 60.0 | 0.00200 | 0.01672 | 7.5 | 6.5 | 10.5 | 14.4 |
| 17 | 60.0 | 0.00320 | 0.00975 | 10.7 | 7.7 | 14.4 | 19.5 |
| 18 | 60.0 | 0.00500 | 0.01287 | 5.4 | 4.2 | 14.7 | 16.2 |
| 19 | 120.0 | 0.00200 | 0.02876 | 6.3 | 5.4 | 9.0 | 12.2 |
| 20 | 120.0 | 0.00550 | 0.01268 | 21.2 | 14.9 | 10.9 | 28.1 |
| 21 | 120.0 | 0.01000 | 0.01485 | 20.5 | 20.6 | 23.6 | 37.5 |
| 22 | 200.0 | 0.00500 | 0.02737 | 3.8 | 3.7 | 6.9 | 8.7 |
| 23 | 200.0 | 0.01300 | 0.01231 | 9.5 | 4.8 | 19.5 | 22.2 |
| 24 | 350.0 | 0.02500 | 0.01381 | 20.4 | 26.2 | 35.0 | 48.2 |
| 25 | 650.0 | 0.01300 | 0.01641 | 8.1 | 7.5 | 13.1 | 17.1 |
| 26 | 650.0 | 0.03200 | 0.01027 | 8.1 | 8.7 | 14.6 | 18.8 |
| 27 | 2000.0 | 0.05000 | 0.00522 | 30.6 | 15.2 | 47.6 | 58.6 |

Table 3: The averaged reduced cross section for beauty production, $\sigma_{\text {red }}^{b \bar{b}}$, obtained by the combination of H1 and ZEUS measurements. The cross section values are given together with the statistical $\left(\delta_{\text {stat }}\right)$ and the uncorrelated ( $\delta_{\text {uncor }}$ ) and correlated $\left(\delta_{\text {cor }}\right)$ systematic uncertainties. The total uncertainties $\left(\delta_{t o t}\right)$ are obtained by adding the statistical, uncorrelated and correlated systematic uncetrainties in quadrature. All uncertainties are quoted in per cent.

| Data set | Name | shift $[\sigma]$ | reduction factor |
| :--- | :--- | :---: | :---: |
| $2-7,8 \mathrm{c}, 9,10,11 \mathrm{c}$, | theory, $m_{c}$ | 0.29 | 0.65 |
| $2-13$ | theory, scales | -0.82 | 0.45 |
| $2-13$ | theory, $\alpha_{S}\left(M_{Z}\right)$ | 0.17 | 0.95 |
| $1-7,8 \mathrm{c}, 9,10$ | theory, $c$ fragmentation $\alpha_{K}$ | -0.82 | 0.80 |
| $2-7,8 \mathrm{c}, 9,10$ | theory, $c$ fragmentation $\hat{s}$ | -1.44 | 0.83 |
| $2-7,8 \mathrm{c}, 9,10$ | theory, $c$ transverse fragmentation | -0.10 | 0.90 |
| $2-7,10$ | $f\left(c \rightarrow D^{*+}\right)$ | 0.43 | 0.92 |
| $2-6,10$ | $B R\left(D^{*+} \rightarrow D^{0} \pi^{+}\right)$ | 0.14 | 0.99 |
| $2-7,10$ | $B R\left(D^{0} \rightarrow K^{-} \pi^{+}\right)$ | 0.47 | 0.98 |
| $1-4$ | H1 CJC efficiency | 0.29 | 0.78 |
| 2 | H1 luminosity (1998-2000) | -0.05 | 0.97 |
| 2 | H1 trigger efficiency (HERA-I) | -0.07 | 0.94 |
| $2-4$ | H1 electron energy | 0.29 | 0.67 |
| $2-4$ | H1 electron polar angle | 0.23 | 0.74 |
| 2 | H1 MC alternative fragmentation | -0.09 | 0.68 |
| 3,4 | H1 primary vertex fit | 0.31 | 0.98 |
| $1,3,4$ | H1 hadronic energy scale | -0.06 | 0.81 |
| 3,4 | H1 luminosity (HERA-II) | -0.19 | 0.77 |
| 3,4 | H1 trigger efficiency (HERA-II) | -0.06 | 0.98 |
| 3,4 | H1 fragmentation model in MC | -0.17 | 0.87 |
| $1,3,4$ | H1 photoproduction background | 0.31 | 0.91 |
| 3,4 | H1 efficiency using alternative MC model | 0.30 | 0.71 |
| 1 | H1 vertex resolution | -0.53 | 0.88 |
| 1 | H1 CST efficiency | -0.34 | 0.89 |
| 1 | H1 B multiplicity | 0.26 | 0.79 |
| 1 | H1 $D^{+}$multiplicity | -0.30 | 0.94 |
| 1 | H1 $D^{*+}$ multiplicity | -0.02 | 0.98 |
| 1 | H1 $D_{s}^{+}$multiplicity | 0.09 | 0.97 |
|  |  |  |  |

Table 4: Sources of bin-to-bin correlated systematic uncertainties considered in the combination. For each source the affected datasets are given, together with the shift and reduction factor in units of $\sigma$ in the combination obtained after the first iteration. For those measurements which have extracted beauty and charm cross sections simultaneously a suffix $b$ or $c$ to the data set number indicates that the given systematic source applies only to the beauty or charm measurements, respectively.

| Data set | Name | shift [ $\sigma$ ] | reduction factor |
| :---: | :---: | :---: | :---: |
| 1 | H1 $b$ fragmentation | -0.05 | 0.96 |
| 1 | H1 VTX model: $x$ reweighting | -0.20 | 0.92 |
| 1 | H1 VTX model: $p_{T}$ reweighting | -0.31 | 0.68 |
| 1 | H1 VTX model: $\eta(c)$ reweighting | -0.36 | 0.80 |
| 1 | H1 VTX uds background | -0.14 | 0.43 |
| 1 | H1 VTX $\phi$ of $c$ quark | 0.05 | 0.84 |
| 1 | H1 VTX $F_{2}$ normalisation | -0.05 | 0.93 |
| 9,10,11 | ZEUS luminosity (HERA-II) | -1.24 | 0.88 |
| 9,10,11 | ZEUS tracking efficiency | 0.03 | 0.88 |
| 11 | ZEUS VTX decay length smearing (tail) | -0.23 | 0.96 |
| 9,10,11 | ZEUS hadronic energy scale | 0.08 | 0.54 |
| 9,10,11 | ZEUS electron energy scale | 0.24 | 0.55 |
| 11 | ZEUS VTX $Q^{2}$ reweighting in charm MC | -0.10 | 1.00 |
| 11 | ZEUS VTX $Q^{2}$ reweighting in beauty MC | 0.04 | 1.00 |
| 11 | ZEUS VTX $\eta$ (jet) reweighting in charm MC | -0.57 | 0.97 |
| 11 | ZEUS VTX $\eta$ (jet) reweighting in beauty MC | 0.10 | 0.99 |
| 11 | ZEUS VTX $E_{T}(\mathrm{jet})$ reweighting in charm MC | 0.48 | 0.96 |
| 11 | ZEUS VTX $E_{T}$ (jet) reweighting in beauty MC | -0.43 | 0.92 |
| 11 | ZEUS VTX light-flavour background | 0.48 | 0.85 |
| 11 | ZEUS VTX charm fragmentation fucntion | -0.91 | 0.87 |
| 11 | ZEUS VTX beauty fragmentation fucntion | -0.17 | 0.95 |
| 9 | $f\left(c \rightarrow D^{+}\right)$ | -0.11 | 0.94 |
| 9 | $B R\left(D^{+} \rightarrow K^{-} \pi^{+} \pi^{+}\right)$ | -0.10 | 0.95 |
| 9 | ZEUS $D^{+}$decay length smearing | 0.05 | 0.99 |
| 9,10 | ZEUS beauty MC normalisation | 0.67 | 0.85 |
| 9 | ZEUS $D^{+} \eta$ MC reweighting | 0.23 | 0.85 |
| 9 | ZEUS $D^{+} p_{T}, Q^{2} \mathrm{MC}$ reweighting | 0.92 | 0.66 |
| 9 | ZEUS $D^{+}$MVD hit efficiency | -0.04 | 0.99 |
| 9 | ZEUS $D^{+}$secondary vertex description | -0.08 | 0.97 |
| 5,13 | ZEUS luminosity (1996-1997) | 0.57 | 0.95 |

Table 4: continued

| Data set | Name | shift [ $\sigma$ ] | reduction factor |
| :---: | :---: | :---: | :---: |
| 6,13 | ZEUS luminosity (1998-2000) | 0.42 | 0.87 |
| 10 | ZEUS $D^{*+} p_{T}\left(\pi_{s}\right)$ description | 0.84 | 0.92 |
| 10 | ZEUS $D^{*+}$ beauty MC efficiency | -0.17 | 0.97 |
| 10 | ZEUS $D^{*+}$ photoproduction background | 0.39 | 0.96 |
| 10 | ZEUS $D^{*+}$ diffractive background | -0.35 | 0.92 |
| 10 | ZEUS $D^{*+} p_{T}, Q^{2} \mathrm{MC}$ reweighting | -0.45 | 0.91 |
| 10 | ZEUS $D^{*+} \eta$ MC reweighting | 0.34 | 0.77 |
| 10 | ZEUS $D^{*+} \Delta(M)$ window efficiency | -0.77 | 0.92 |
| 7 | $f\left(c \rightarrow D^{0}\right)$ | 0.32 | 0.99 |
| 7,8,12 | ZEUS luminosity (2005) | 0.66 | 0.91 |
| 8c | $B R(c \rightarrow l)$ | -0.10 | 0.97 |
| 8 | ZEUS $\mu$ : B/RMUON efficiency | 0.54 | 0.90 |
| 8 | ZEUS $\mu$ : FMUON efficiency | 0.15 | 0.95 |
| 8 | ZEUS $\mu$ : energy scale | -0.01 | 0.67 |
| 8 | ZEUS $\mu$ : $p_{T}^{\text {miss }}$ calibration | 0.13 | 0.66 |
| 8 | ZEUS $\mu$ : hadronic resolution | 0.62 | 0.58 |
| 8 | ZEUS $\mu$ : IP resolution | -0.70 | 0.83 |
| 8 | ZEUS $\mu$ : MC model | -0.08 | 0.75 |
| 1b | H1 VTX beauty: $Q^{2}$ charm reweighting | -0.02 | 1.00 |
| 1b | H1 VTX beauty: $Q^{2}$ beauty reweighting | -0.02 | 0.99 |
| 1b | H1 VTX beauty: $x$ reweighting | 0.09 | 0.89 |
| 1b | H1 VTX beauty: $p_{T}$ reweighting | -1.06 | 0.82 |
| 1b | H1 VTX beauty: $\eta$ reweighting | 0.01 | 0.91 |
| 1b | H1 VTX beauty: $\operatorname{BR}\left(D^{+}\right)$ | -0.21 | 0.99 |
| 1b | H1 VTX beauty: $\operatorname{BR}\left(D^{0}\right)$ | 0.16 | 1.00 |
| 8b,11b,12,13 | theory, $m_{b}$ | 0.60 | 0.93 |
| 8b,12,13 | theory, $b$ fragmentation | -0.71 | 0.97 |
| 8b,12,13, | $B R(b \rightarrow l)$ | -0.60 | 0.97 |
| 13 | ZEUS muon efficiency (HERA-I) | -1.02 | 0.91 |

Table 4: continued

| Dataset | PDF (scheme) | $\chi^{2}$ [ $p$-value] | $\chi^{2}$ with PDF unc. |
| :---: | :---: | :---: | :---: |
| HERA 2012 c | HERAPDF20_NLO_FF3A (FFNS) | 59 [0.23] | 59 |
|  | ABKM09 (FFNS) | 59 [0.23] | - |
|  | abm11_3n_nlo (FFNS) | 62 [0.16] | 62 |
|  | ABMP16_3_nnlo (FFNS) | 70 [0.05] | 69 |
| $\left(\mathrm{N}_{\text {dat }}=52\right)$ | HERAPDF20_NLO_EIG (RT OPT) | 71 [0.04] | 70 |
|  | HERAPDF20_NNLO_EIG (RT OPT) | 66 [0.09] | 65 |
|  | NNPDF31sx_nnlo_as_0118 (FONLL-C) | $106\left[1.5 \cdot 10^{-6}\right]$ | - |
| $\left(\mathrm{N}_{\mathrm{dat}}=47\right)$ | NNPDF31sx_nnlonllx_as_0118 (FONLL-C) | 71 [0.013] | - |
| New combined $c$ | HERAPDF20_NLO_FF3A (FFNS) | 86 [0.002] | 85 |
|  | ABKM09 (FFNS) | 82 [0.005] | - |
|  | abm11_3n_nlo (FFNS) | 92 [0.0005] | 91 |
| $\left(\mathrm{N}_{\text {dat }}=52\right)$ | ABMP16_3_nnlo (FFNS) | $109\left[6 \cdot 10^{-6}\right]$ | 106 |
|  | HERAPDF20_NLO_EIG (RT OPT) | $99\left[9 \cdot 10^{-5}\right]$ | 98 |
|  | HERAPDF20_NNLO_EIG (RT OPT) | $102\left[4 \cdot 10^{-5}\right]$ | 99 |
|  | NNPDF31sx_nnlo_as_0118 (FONLL-C) | $140\left[1.5 \cdot 10^{-11}\right]$ | - |
| $\left(\mathrm{N}_{\mathrm{dat}}=47\right)$ | NNPDF31sx_nnlonllx_as_0118 (FONLL-C) | $114\left[5 \cdot 10^{-7}\right]$ | - |
| New combined $b$$\left(\mathrm{N}_{\mathrm{dat}}=27\right)$ | HERAPDF20_NLO_FF3A (FFNS) | 33 [0.20] | 33 |
|  | abm11_3n_nlo (FFNS) | 34 [0.17] | 34 |
|  | ABMP16_3_nnlo (FFNS) | 41 [0.04] | 41 |
|  | HERAPDF20_NLO_EIG (RT OPT) | 33 [0.20] | 33 |
|  | HERAPDF20_NNLO_EIG (RT OPT) | 45 [0.016] | 45 |

Table 5: The $\chi^{2}$ values and d.o.f. of the charm and beauty data with respect to the NLO and approximate NNLO calculations using various PDFs as described in the text. The $\chi^{2}$ values that include PDF uncertainties are shown separately. The measurements at $Q^{2}=2.5 \mathrm{GeV}^{2}$ are excluded in the calculations of the $\chi^{2}$ values for the the NNPDF3.1sx predictions, by which the number of data points is reduces to 47 . (See caption of figure 14 for further explantions.)


Figure 1: The pull distribution for the combination of the charm and beauty reduced cross sections.

[^6]

Figure 2: Combined measurements of the reduced charm production cross sections, $\sigma_{\text {red }}^{c \bar{c}}$, as a function of $x_{\mathrm{Bj}}$ for different values of $Q^{2}$. The inner error bars indicate the uncorrelated part of the uncertainties and the outer error bars represent the total uncertainties.


Figure 3: Combined measurements of the reduced beauty production cross sections, $\sigma_{\text {red }}^{b \bar{b}}$, as a function of $x_{\mathrm{Bj}}$ for different values of $Q^{2}$. The inner error bars indicate the uncorrelated part of the uncertainties and the outer error bars represent the total uncertainties.

```
■ H1 VTX
o ZEUS \mu 2005
ZEUS D}\mp@subsup{}{}{0
| H1 D* HERA-II
v H1 D* HERA-I
\square ZEUS D* 98-00
# ZEUS D+
~}\mathrm{ ZEUS VTX
- HERA
```

v H1 D* HERA-I
$\triangle$ ZEUS D* 96-97

* ZEUS D* HERA-II


Figure 4: Combined measurements of the reduced charm production cross sections, $\sigma_{\text {red }}^{c \bar{c}}$, (full circles) as a function of $x_{\mathrm{Bj}}$ for different values of $Q^{2}$. The inner error bars indicate the uncorrelated part of the uncertainties and the outer error bars represent the total uncertainties. The input measurements are also shown by the different markers. For presentation purposes each individual measurement is shifted in $x_{\mathrm{Bj}}$.


Figure 5: Combined measurements of the reduced beauty production cross sections, $\sigma_{\text {red }}^{b \bar{b}}$, (full circles) as a function of $x_{\mathrm{Bj}}$ for different values of $Q^{2}$. The inner error bars indicate the uncorrelated part of the uncertainties and the outer error bars represent the total uncertainties. The input measurements are also shown by the different markers. For presentation purposes each individual measurement is shifted in $x_{\mathrm{Bj}}$.


Figure 6: Reduced cross sections as a function of $x_{\mathrm{Bj}}$ at $Q^{2}=32 \mathrm{GeV}^{2}$ for charm (upper panel) and beauty production (lower panel). The combined cross sections (full black circles) are compared to the input measurements shown by the different markers. The inner error bars indicate the uncorrelated part of the uncertainties and the outer error bars represent the total uncertainties. For better visibility the individual input data are displaced in $x_{\mathrm{Bj}}$ towards larger values.


Figure 7: Combined reduced cross sections $\sigma_{\text {red }}^{c \bar{c}}$ (full circles) as a function of $x_{\mathrm{Bj}}$ for given values of $Q^{2}$, compared to the results of the previous combination, denoted as 'HERA 2012' (open circles).

- HERA
----- NLO ABKM09
-     -         -             - appr. NNLO ABMP16

NLO HERAPDF2.0 FF3A
NLO ABM11


Figure 8: Combined reduced charm cross sections $\sigma_{\text {red }}^{c \bar{c}}$ (full circles) as a function of $x_{\mathrm{Bj}}$ for given values of $Q^{2}$, compared to the NLO QCD FFNS predictions based on the HERAPDF2.0 FF3A (solid lines), ABKM09 (dashed lines) and ABM11 (dotted lines) PDF sets. Also shown is the approximate NNLO prediction using ABMP16 (dashed-dotted lines). The shaded bands on the HERAPDF2.0 FF3A predictions show the theory uncertainties obtained by adding PDF, scale and charm quark mass uncertainties in quadrature.


Figure 9: Combined reduced beauty cross sections $\sigma_{\mathrm{red}}^{b \bar{b}}$ (full circles) as a function of $x_{\mathrm{Bj}}$ for given values of $Q^{2}$, compared to the NLO QCD FFNS predictions based on the HERAPDF2.0 FF3A (solid lines), ABKM09 (dashed lines) and ABM11 (dotted lines) PDF sets. Also shown is the prediction in approximate NNLO using ABMP16 (dashed-dotted lines). The shaded bands on the HERAPDF2.0 FF3A predictions show the theory uncertainties obtained by adding PDF, scale and beauty quark mass uncertainties in quadrature.

- HERA
----- NLO ABKM09
appr. NNLO ABMP16

NLO HERAPDF2.0 FF3A
NLO ABM11

H1 and ZEUS


Figure 10: Combined reduced charm cross sections $\sigma_{\text {red }}^{c \bar{c}}$ (full circles) as a function of $x_{\mathrm{Bj}}$ for given values of $Q^{2}$, compared to the NLO and approximate NNLO QCD theoretical FFNS predictions obtained using various PDFs, as in Fig. 8, normalised to the predictions obtained using HERAPDF2.0 FF3A.


Figure 11: Combined reduced beauty cross sections $\sigma_{\text {red }}^{b \bar{b}}$ (full circles) as a function of $x_{\mathrm{Bj}}$ for given values of $Q^{2}$, compared to the NLO and approximate NNLO QCD theoretical FFNS predictions obtained using various PDFs, as in Fig. 9, normalised to the predictions obtained using HERAPDF2.0 FF3A.


Figure 12: Combined reduced charm cross sections $\sigma_{\text {red }}^{c \bar{c}}$ (full circles) as a function of $x_{\mathrm{Bj}}$ for given values of $Q^{2}$, compared to NLO (dashed-dotted lines) and approximate NNLO (dashed lines) VFNS predictions based on HERAPDF2.0 using corresponding NLO and NNLO HERAPDF2.0 PDF sets, normalised to the FFNS predictions obtained using HERAPDF2.0 FF3A. The uncertianties for the VFNS predictions are of similar size as those presented for the FFNS calculations.


Figure 13: Combined reduced beauty cross sections $\sigma_{\text {red }}^{b \bar{b}}$ (full circles) as a function of $x_{\mathrm{Bj}}$ for given values of $Q^{2}$, compared to the NLO (dashed-dotted lines) and approximate NNLO (dashed lines) VFNS predictions based on HERAPDF2.0 using corresponding NLO and NNLO HERAPDF2.0 PDF sets, normalised to the FFNS predictions obtained using HERAPDF2.0 FF3A. For the VFNS predictions no uncertainties are given. They are of similar size than those presented for the FFNS calculations.


Figure 14: Combined reduced charm cross sections $\sigma_{\text {red }}^{c \bar{c}}$ (full circles) as a function of $x_{\mathrm{Bj}}$ for given values of $Q^{2}$, compared to FONNL-C (dotted lines with uncertainty bands) and FONNLC+NLLsx (dashed lines) VFNS predictions based on NNPDF3.1sx PDF sets without and with $\log \frac{1}{x}$ resummation, normalised to the FFNS predictions obtained using HERAPDF2.0 FF3A. For better clarity of the presentation the uncertainties of the FONNL+NLLsx calculations are not shown. They are of similar size as those shown for the plain FONLL calculations. No FONNL predictions based on NNPDF3.1sx are shown at $Q^{2}=2.5 \mathrm{GeV}^{2}$ because this value lies below the starting scale of the QCD evolution in the calculation $\left(2.6 \mathrm{GeV}^{2}\right)$.


Figure 15: Parton density functions $x \cdot f\left(x, Q^{2}\right)$ at the starting scale $Q_{0}=1.9 \mathrm{GeV}^{2}$ with $f=$ $u_{v}, d_{v}, g, \Sigma$ for the valence up quark (a), the valence down quark (b), the gluon (c) and the sea quarks (d) obtained from the QCD fit to the combined inclusive and heavy flavour data (full lines) and to the combined inclusive data only (dashed lines). The experimental/fit uncertainties obtained from the fit to the combined inclusive and heavy flavour data are indicated by the shaded bands. For better visibility the uncertainties from the fit to the inclusive data only are not shown.


Figure 16: Combined reduced charm cross sections $\sigma_{\text {red }}^{c \bar{c}}$ (full circles) as a function of $x_{\mathrm{Bj}}$ for given values of $Q^{2}$, compared to the NLO QCD FFNS predictions based on the PDF set determined by the fit to the inclusive and heavy flavour data (solid lines) and on the HERAPDF2.0 FF3A (dashed lines) set. The shaded bands on the predictions using the reference calculation show the theory uncertainties obtained by adding PDF, scale and charm quark mass uncertainties in quadrature.


Figure 17: Combined reduced beauty cross sections $\sigma_{\text {red }}^{b \bar{b}}$ (full circles) as a function of $x_{\mathrm{Bj}}$ for given values of $Q^{2}$, compared to the NLO QCD FFNS predictions based on the PDF set determined by the fit to the inclusive and heavy flavour data (solid lines) and on the HERAPDF2.0 FF3A (dashed lines) set. The shaded bands on the predictions using the fitted PDF set show the theory uncertainties obtained by adding PDF, scale and charm quark mass uncertainties in quadrature.


Figure 18: Combined reduced charm cross sections $\sigma_{\text {red }}^{c \bar{c}}$ (full circles) as a function of $x_{\mathrm{Bj}}$ for given values of $Q^{2}$, compared to the NLO FFNS predictions resulting from the fit of this analysis, normalised to the reference cross sections using HERAPDF2.0 FF3A.


Figure 19: Combined reduced beauty cross sections $\sigma_{\text {red }}^{b \bar{b}}$ (full circles) as a function of $x_{\mathrm{Bj}}$ for given values of $Q^{2}$, compared to the NLO FFNS predictions resulting from the fit of this analysis, normalised to the reference cross sections using HERAPDF2.0 FF3A.


Figure 20: Ratio of the combined reduced cross sections $\sigma_{\text {red }}^{c \bar{c}}$ (upper panel) and $\sigma_{\text {red }}^{b \bar{b}}$ (lower panel) to the NLO FFNS cross section predictions obtained from the fit to the inclusive and heavy flavour data as a function of the partonic $x$ for the different values of $Q^{2}$.

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Figure 21: The $\chi^{2} /$ d.o.f. values determined for the heavy flavour data (triangles) only and for the inclusive plus heavy flavour data (dots) when including in the fit only inclusive data with $x_{\mathrm{Bj}} \geq x_{\text {min }}$.


Figure 22: Parton density functions $x \cdot f\left(x, Q^{2}\right)$ at the starting scale $Q_{0}^{2}=1.9 \mathrm{GeV}^{2}$ with $f=$ $u_{v}, d_{v}, g, \Sigma$ for the valence up quark (a), the valence down quark (b), the gluon (c) and the sea quarks (d) obtained from the QCD fit to the combined inclusive and heavy flavour data without requiring a minimum $x_{\mathrm{Bj}}$ for the inclusive data included in the fit (full lines) and with a minimum cut of $x_{\mathrm{Bj}} \geq 0.01$ for the inclusive data included in the fit. The experimental/fit uncertainties are shown by the hatched bands.


Figure 23: Combined reduced charm cross sections $\sigma_{\text {red }}^{c \bar{c}}$ (full circles) as a function of $x_{\mathrm{Bj}}$ for given values of $Q^{2}$, compared to the NLO FFNS predictions resulting from the fit to heavy flavour data and the inclusive data with $x_{\mathrm{Bj}, \min } \geq 0.01$ (dashed dotted lines) and without $x_{\mathrm{Bj}, \min }{ }^{-}$ cut (dashed lines) normalised to the reference cross sections using HERAPDF2.0 FF3A (full line). The experimental/fit uncertainties of the reference cross sections are indicated by the shaded band.


Figure 24: Combined reduced cross sections $\sigma_{\mathrm{red}}\left(f\right.$ full circles) as a function of $x_{\mathrm{Bj}}$ for given values of $Q^{2}$, compared to the NLO FFNS predictions resulting from the fit to heavy flavour data and the inclusive data with $x_{\mathrm{Bj}, \text { min }} \geq 0.01$ (dashed dotted lines) and without $x_{\mathrm{Bj}, \text { min }}$-cut (dashed lines) normalised to the reference cross sections using HERAPDF2.0 FF3A (full line).

## Comments to draft 1.0 (Nov 23, 2017) of Combination and QCD analysis of beauty and charm production cross section measurements in deep inelastic ep scattering at HERA

## Dieter Haidt

## General

The analysis of HERA data has reached its mature and final form. The present publication finalizes the analysis of the inclusive charm and beauty production cross sections and complements the already published measurements of the fully inclusive production cross sections. The authors and the two Collaborations H1 and ZEUS can claim with pride that to the community are given unique and fundamental measurements which have value in themselves, serve as input to other experimental analyses and as testing ground for QCD.

The publication raises in addition several important and critical remarks on the interpretation of the new data within QCD in its present status and points the theoretical groups to areas which require further development. An important aspect is our evaluation to what degree there is agreement between data and theory. To this end it is inappropriate to use the term tension. Our task is to make up our mind and state whether there is either agreement or failure and quantify our judgement by quoting the confidence level either pro or contra. In the case
of a discrepancy we should spell it out clearly and put the finger on the elements leading to this decision.

Another qualifying statement concerns the term stringent, which in my opinion is not justified. Our data has indeed reached high quality and high precision. On the contrary, the status of the theory at present does not make clearcut predictions. Chapter 6 mentions some of the drawbacks related to the use of QCD at first order, the uncertainties in dealing with massive quarks etc. For the time being the theory allows merely for a test of consistency. Given my critical remarks below I am not sure whether the observed disagreement in Figure 8 ( $x$ distribution) will remain, since I would blame the gluon for it. However, if the disagreement persists, then it is worth to note it as an important result.

## Detailed comments

Drop the word tension which appears in the lines 408429431 565612617619620641647

Line 22 : drop stringent (see comment above)
Line 803 : Sjøstrand (spelling)
Conclusions : fair versus discrepancy - make a decision
439 QCD does not predict the x-distribution of the parton distributions

497 Quote table with fit parameters and correlations, including the fit of the mass parameters and correlations which represents one of the important results

Comment after eq. 7 (line 512) : the experimental and theoretical uncertainty of the fitted masses are comparable.

529 tried -> investigated

533 understand correlation between Euv and other parameters. The block (sea,gluon) should reveal a strong correlation. Why is that not so ?

559 can -> could
Fig 20 add average $x$ in caption
578 discrepancy for $\mathrm{Q}^{2}<18$ and $\mathrm{x}<0.01$
The review of Oleksandr Zenaiev in EPJC should be quoted. It provides detailed insight in the treatment of fully inclusive data and inclusive heavy quark data. In addition it shows the benefit of combining HERA data with LHCb data which has led to a significant improvement in the knowledge of the gluon distribution.

## DGLAP

There are two types of analysis.
The DGLAP analysis applied to fully inclusive data. It assumes
a. the shape of all partons in $\mathrm{X}_{\text {bjorken }}$ at $\mathrm{Q}_{0}{ }^{2}=1.9 \mathrm{GeV}^{2}$ with 14 free parameters and in addition the $c$ and $b$ masses either fixed or free
b. the predicted $\mathrm{Q}^{2}$ evolution of the parton distributions (which are not observables) require additional assumptions to predict the observable cross sections
c. the treatment of the massive c-and b-quarks
d. the region of applicability

The resulting fit for the free parameters can be bad for several reasons
a. inadequate shape of pdf at starting scale
b. bias in data
c. inadequacy of theory

It is intrinsic to any DGLAP analysis that the gluon is only indirectly determined. Furthermore, there are significant
correlations between the fit parameters caused by the fact that there is no flavor separation.

The prediction of the c - and b -cross sections proceeds in two steps

1. DGLAP analysis of light quarks
2. Gamma-gluon fusion for c- and b-production with explicit dependence upon the gluon distribution.

Application : use pdf from inclusive data + PDG masses of c and $b$ to predict $c$ - and b-cross sections. The important point is that the input required for QCD to predict the heavy quark cross sections is obtained from independent data. This is possible and we can claim a genuine test of QCD and can conclude whether there is agreement or not. Further applications would be possible by relaxing the mass constraints.

As I understand this clearcut test is not persued in the present draft because of the apparent sensitivity to the c- and bmasses. The origin of this apparent sensitivity is, in my opinion, to be found in the unsatisfactory knowledge of the gluon distribution. The correlation between Euv and the second term in describing the gluon distribution reveals the problem (see below). Whether or not you agree with my criticism below, the statement that the fully inclusive data do not allow for a determination of the heavy quark masses should be elaborated. In particular it should be elucidated to what extent the uncertainty in the gluon distribution affects the sensitivity to the masses.

The simultaneous fit to the inclusive and heavy quark data, as described in chapter 6, provides a determination of the pdf and the masses, but no genuine test of the theory. The important point is clearly stated : the masses are obtained thanks to the heavy quark data and constitute a substantial result.
a. the two blue regions in uv and dv reflect the fact that there is the number constraint of 2 and 1 . Why isn't there a strong correlation between the sea and gluon parameters ?
b. the big uncertainty in the blue curve is expected, but why is the yellow curve not equally uncertain?
c. the gluon distribution vanishes at low $x$, while subfigure (d) shows that there are many qqbar pairs. It is counterintuitive to admit that there are really no gluons at low $x$ at the starting scale. I am aware that parton distributions are not observables and even distributions running below 0 are acceptable as long as their use to predict observables leads to finite and positive numbers. It is assumed that $\mathrm{Q}_{0}{ }^{2}=1.9 \mathrm{GeV}^{2}$ belongs to the perturbative region. Then there are processes gamma+q -> q+gluon (similarly antiquark) and both q and g contribute to the low-x sea in equal amount. Is the effect of higher orders such that gluons get suppressed, but quark-antiquark pairs get favoured at low x ? If, on the contrary, the starting scale is deemed to be nonperturbative, then the gluon and sea distributions towards low $x$ with approximate averages of their distributions <xsea>=0.15 and <gluon>=0.5 will have tails towards low $x$. Is it reasonable that the tail of the large gluon component vanishes at low $x$, while the antiquark component increases ? The adhoc ansatz à la MRS for the gluon (which I dislike) is prone to suppress gluons at low $x$ as long as the evolution has not yet had a significant impact. I give further arguments below why I am doubtful about the fitted shape of the gluon.
d. the subfigure (d) is not informative, better show deviation with respect to a standard curve. In comparison to the gluon distribution the qqbar distribution pairs increases steadily with decreasing $x$.

Question : comparison with masses from spectroscopy ? Is already answered in draft

## The Euv problem and the role of the momentum sumrule

One of the DGLAP equations governs the evolution of the nonsinglet with the splitting function Pqq, while the other two DGLAP equations are a coupled system describing the evolution of singlet-gluon, where also Pqq appears albeit affected by a small $1^{\text {st }}$ order correction (easy to handle). The evolution of the valence is fully determined by the $1^{\text {st }}$ DGLAP equation. Being a nonsinglet the distribution has the characteristic behavior getting slowly (with $\mathrm{Q}^{2}$ ) degraded, i.e. $<x>$ moves from about 0.4 to 0.3 over the $\mathrm{Q}^{2}$ range of the data. The known running of <xvalence> has to be compensated by a sharing between the sea and the gluon distribution in order to satisfy the momentum sumrule. No correlation is expected between the parameters of the valence on the one side and the sea and gluon parameters on the other side, because the $1^{\text {st }}$ DGLAP equation is independent of the gluon. This is perhaps not fully correct, since the valence contributes a small amount to the gluon derivative through Pgq*singlet (where the singlet contains also uv and dv).
The momentum sumrule does not depend upon $\mathrm{Q}^{2}$. In the MRST para-metrization the sum expected to be 1 decreases with $\mathrm{Q}^{2}$ and deviates at large $\mathrm{Q}^{2}$ by more than $3 \%$. I don't know how this sumrule is build in the program. In any case, it is an integral running from $\mathbf{0}$ to 1 , so care has to be taken given our triangular shape of the phase space in ( $1 / \mathrm{x}, \mathrm{Q}^{2}$ ).

The observed correlation (Euv, Apri) and (Euv, ) is, I think, an artefact. I noticed that the $u$-valence distribution can be well approximated by setting Euv=0 and lowering the c-parameter from 4.9 to 2.9. This power for (1-x), by the way, is in good agreement with neutrino data. The present large power in (1-x) is large because of the presence of the adhoc ( $1+$ Euv $\mathrm{x}^{2}$ ) term and the actual treatment of the gluon. At present uv and dv are not treated on the same footing. A more appropriate method
may be to consider $u$ and $d / u$ both for the valence and th sea. Data from W-production and decay may be useful.

I stop at this point and wait for your answer and perhaps for your disagreement with my remarks. I would be happy to discuss the relevant issues personally with you rather than bothering you with formulating a written answer. Eventhough I have raised some critical points, I appreciate very much the effort you have devoted to this publication.

Comments to „Combination and QCD Analysis of Beauty and Charm Production Cross Section Measurements in Deep Inelastic ep Scattering at HERA"
Peter Truöl

General: Within the heavy flavour working group active during the pre-data phase of H1, which I was asked to coordinate, we started off with the notion that HERA was among other things a „charm factory". This illusion quickly disappeared when we realized that in lack of suitable triggers the heavy quarks would escape nearly unnoticed. It is therefore gratifying that in the end, 25 years later, the final analysis of the relevant cross sections measured by both collaborations has been finished and is ready for publication. Many thanks to all involved in the preparation of this final section of the long journey, among them some of members of my group in the early phases. My minor comments only concern the text.

It seems that the different chapters have been written by different persons with the consequence that there appear some repetitions which need to be weeded out.
The title of the paper could be shortened to
„QCD Analysis of Beauty and Charm Production Data from Deep Inelastic ep Scattering at HERA"

Abstract (shorten somewhat, e.g. like): Open beauty and charm production cross sections in deep inelastic ep scattering measured at HERA by the H 1 and ZEUS collaborations are combined. The data cover a kinematic range of photon virtuality ....... The combination method accounts for correlations of the statistical and systematic uncertainties among the different data sets. The data are compared to perturbative QCD predictions and also used together with inclusive deep inelastic scattering data from HERA in a next-to-leading order QCD analysis. ....

Throughout the text: It is not necessary to repeat „heavy quark", „charm and beauty" within a section several times, if it is clear from the beginning that nothing else is being discussed, first examples line 25 and 34 below.

Semi-leptonic -> semileptonic
Both „program" and „programme" are used in the text, decide on one them

Introduction:
L 25: The cross section therefore depends strongly on the gluon distribution in the proton and the mass of the heavy quarks involved.

L 28: ... the transverse momenta of the outgoing quarks and the virtuality, $\mathrm{Q}^{2}$, of the exchanged photon. The presence of several hard scales ...

L 34: At HERA different various flavour tagging methods have been applied.

L 53: The proper lifetime of $B$ mesons is about a factor of two to three larger than that of $D$ mesons on average

L 91: The simultaneous combination of charm and beauty cross section measurements reduces the correlations between them and hence also the uncertainties.

L 98: In addition QCD calculations in the RTOPT VFNS <- what does this mean
L 104: The new data are subjected to a QCD analysis together with inclusive DIS cross section data from HERA [40] allowing for running charm and beauty quark masses in NLO, as defined in the QCD Lagrangian in the modified minimum subtraction (MS) scheme.

L 110: The paper is organized as follows. In section 2 the reduced heavy flavour cross section is defined and the theoretical framework is briefly introduced. The data samples and the combination method are presented in section 3.1. The resulting reduced cross sections are presented in section 4 and compared with theoretical calculations based on existing PDF sets at NLO and at NNLO in the FFNS and VFNS in section 5 . In section 6 the NLO QCD analysis
is described and the measurement of the running masses of the charm and beauty quark in the MS scheme at NLO is presented. This section also contains a study of the xBj-dependance of the cross section. Finally, the paper is concluded in section 7.

## Section 2:

Electro-weak -> electroweak, per cent -> percent
L 119: In the kinematic range explored by the analysis of the data presented here the virtuality of the exchanged boson is small, i.e. $Q^{2} \ll M^{2}$, such that virtual photon exchange dominates.

L 120: ... where y denotes the lepton inelasticity. (Bjorken has been defined before)
L129-131: cut, already in introduction

L 158: drop "in the MS scheme."
L 160-162 ff: RTOPT ? (authors of ref. 32 ?), FONLL_C ? This paragraph can only be understood by specialists.

## Section 3:

L 170 - 171: ... high-resolution vertex detectors [48,49]. (the references to the vertex detectors suffice, the names are irrelevant)

L 179: drop "and correspond to 209 individual charm and 57 different beauty cross section measurements", appears again in L 306-307

L 181: ... includes measurements using different tagging methods: (we know by that we deal with charm and beauty)

L 184: .... muons from semileptonic decays
L 204-206: .... theoretical predictions for ... and .... in the NLO FFNS scheme. Only their shape in function of the kinematic variables is relevant for the corrections, while their normalisation cancels in Eq. (3).

L 231: ... was fixed at ..

L 233: For all parameter sets the corresponding PDF set is used.
P 7 footnote 2: While ... -> Since ...
L 279: The results are converted to a centre-of-mass energy $\mathrm{V} s=318 \mathrm{GeV}$.

L 281: The combination is based on the ..... procedure [37] used previously [36,38-40]
L 302: .... yields a significant reduction of the overall uncertainties of the combined data, as detailed in the next section.

Section 4:
L 310: drop "and a conservative estimate of the uncertainties of the individual measurements." comes again in L 314

Section 5:
L 346 ff : Shorten to "Before performing a dedicated QCD analysis of the data they are compared with calculations using pre-existing PDF sets. Predictions in the FFN and the VFN schemes are considered focusing on results using HERAPDF2.0 PDF sets."

In the following "combined" could be cut everywhere no uncombined data are considered anyway, maybe even "combined reduced charm (beauty) cross section" could be replaced by "charm (beauty) data" or "results" to make the chapters shorter and more readable.
I guess for combinations such as "theory predictions" and the likes "theoretical" would be better.

Footnote 4: The calculated cross sections ... were provided by the authors.

Section 5.3 Summary of the comparison to theoretical predictions
L 421-430 a rather clumsy explanation of a simple fact, why not just write:
The table also includes a comparison to the combined charm data published previously [36]. The apparent poorer agreement of the new data compared to the previous results can be traced to the increased precision of the new data.

Section 6:
L 494: In the QCD fit the running heavy quark masses are fitted simultaneously with the PDF parameters. The fit yields a total $X^{2}=1435$ for 1208 degrees of freedom (d.o.f.). The ratio $X^{2} /$ d.o.f. $=1.19$ is similar to the values obtained in the analysis of the HERA inclusive data [40].

L512: The model uncertainties are dominated by those arising from the scale variations.
L 514: ... while the other sources lead to uncertainties of typically a few MeV ...
L515: .... Is set to zero, the other contributions are negligible.
L 539 ff : The NLO FFNS predictions based on the PDF set and the running beauty and charm quark masses determined by the fit are compared to the data in figures 16 and 17 , respectively.

L 546 ff : In order to better visualize the differences of the present to the latter analysis the ratios of data to predictions are shown in figures 18 and 19 for charm and beauty, respectively.

## Section 7:

L 527: The charm cross sections presented in this paper are significantly more precise than those previously published. The data are compared ....

References: A few inconsistencies
1-23: reorganize in chronological order ?
$18,26,28,32,35,66$ details are missing
61 - 65 the information following doi: should be scratched

Figures:
Except for the theoretical curves 8 and 9 are identical to 2 and 3 , hence one could omit the latter; if captions contain identical sentences a reference to the first occurrence may suffice.


[^0]:    ${ }^{1}$ In this paper the term 'electron' denotes both electron and positron if not stated otherwise.

[^1]:    ${ }^{2}$ While no such corrections are provided in [23], an uncertainty of 5\% is assigned to cover the untreated hadronisation effects [23].

[^2]:    ${ }^{3}$ Only experimental uncertainties ('EIG') of HERAPDF2.0 are considered.

[^3]:    ${ }^{4}$ The cross section predictions and their uncertainties were provided by the authors.

[^4]:    ${ }^{5}$ which did not yet include scale variations and had a less flexible PDF parametrisation [36]. The beauty mass determination improves the previous result based on a single data set [21].

[^5]:    ${ }^{6}$ It is not possible to quote the charm and the beauty contribution to this $\chi^{2}$ value separately because of the correlations between the combined charm and beauty measurements.
    ${ }^{7}$ The $\chi^{2}$ and the p -value given here does not correspond exactly to the statistical definition of $\chi^{2}$ or p -value because the data have been used in the fit and therefore the theory is somewhat shifted towards the measurements. However this bias is expected to be small because the predictions are mainly constrained by the much larger and more precise inclusive data sample.

[^6]:    What is shown with the solid-line Gaussian? A fit or a reference with std=1? A comment in the caption would be handy

