# Status and (some) recent developments in Higgs physics in the SM and in the MSSM

#### Emanuele A. Bagnaschi (DESY Hamburg)



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# Introduction

## Experimental Higgs physics at the LHC

- Rich experimental program in Higgs Physics see the previous talks by M. Shevchenko and K. Brendlinger.
- Characterization of both total rates and differential cross-sections at an advanced stage.



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# BSM physics in the Higgs sector

# Characterization of the boson at 125 GeV

- Deviation of the Higgs couplings from the SM predictions.
- New states in the loops?

The differential measurements can probe NP differently from the inclusive results.





[ATLAS JHEP 01 (2018) 055]

#### New Higgs states

- Extended Higgs sector?
- Simple extension: Two Higgs Doublet Model (e.g. MSSM).

Precise predictions required to properly recast experimental results in NP models.

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# Higgs production channels at the LHC



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# **Gluon** fusion

- Inclusive cross section known in the SM @ N<sup>3</sup>LO QCD in the HQEFT  $(m_t \rightarrow \infty)$  [Anastasiou et al, ...] [1].
- Differential results for the inclusive process = available up to NNLO, pt resummation [Catani et al '03, ...]
- SM process known up to NLO; higher order . terms in the expansion  $1/m_t$  known [Marzani et al '08, Harlander et al '10, ...] [2].
- Soft-resummation available up to N<sup>3</sup>LL ([De . Florian et al '14, ..., [3]) in the HQEFT.
- H + j known up to NNLO in the . HQEFT [Boughezal et al, ...].
- Merged/matched MCs [NNLOPS, POWHEG, . MG5 aMC@NLO, SHERPA, ...]



- Result first published in [Anastasiou et al. '15], using a threshold expansion.
- Uncertainty estimation presented in [Anastasiou et al. '16].
- This year, the full computation was presented [Mistlberger '18], removing one of the item in the uncertainty.
- The code iHixs2 was also released [Dulat et al '18].

-1.15 pb +0.21%

-2.37%

 $\pm 0.37\%$ 



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 $\pm 1\%$ 

 $\pm 0.83\%$ 

 $\pm 1\%$ 

 $\pm 1.16\%$ 

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$\delta(scale)$	$\delta(trunc)$	$\delta$ (PDF-TH)	$\delta(EW)$	$\delta(t, b, c)$	$\delta(1/m_t)$
$^{+0.13} { m ~pb} \\ -1.20 { m ~pb}$	0	$\pm 0.56~{ m pb}$	$\pm 0.49~{ m pb}$	$\pm 0.41 \ \text{pb}$	$\pm 0.49~{ m pb}$
+0.28% -2.50%	0%	±1.16%	$\pm 1\%$	±0.85%	$\pm 1\%$

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- Presented in [Bison et al. '17], based on the formalism of [Monni et al. '16] (see also Ebert et al. '17 for another approach to p<sub>T</sub> space resummation).
- Nearly consistent N3LL resummation (missing four-loop cusp anomalous-dimension).

- Matched to NNLO result from [Caola et al.'16], N3LO normalization.
- N3LL is of order 10% 20% around the Sudakov region vs pure NNLL.
- NNLO+N3LL differs of several percent below the Sudakov peak, the rest is similar to NNLO+NNLL.



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- Very recent results presented in [Chen et al., '18].
- Resummation at N3LL using SCET.
- Uses the recent computation of the tree-loop rapidity anomalous dimension [Li et al. '16 and '16].
- Higher "resolution" with respect to the RADISH result.
- Perturbative uncertainties reduced to  $\leq 6\%$  for  $5 < p_T < 35$  GeV, then they rise to  $\pm 10\%$  and then decrease again.



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# Understanding the mass effects at NLO in H+jet: top quark

- Numerical approach based on SecDec by [Jones et al. '18].
- Expansion of the 2-loop integrals at  $\mathcal{O}(m_t^2/p_t^2)$ ,  $\mathcal{O}((m_H^2/p_t^2)^0)$  by [Kudashkin et al. '17, Lindert et al. '18].



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# Understanding the mass effects at NLO in H+jet: bottom quark

- Expansion at  $\mathcal{O}(m_b^2/p_t^2)$  presented in [Melnikov et al., '16, '17].
- Approach valid down to  $p_T \simeq 10$  GeV.
- Numerical results from [Lindert et al., '17].



Large QCD corrections, but rather flat (no change in the shape).

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# Understanding the mass effects at NNLL+NLO in H+jet: top+bottom quark

- Results valid in the range  $m_b < p_T < m_H$ , presented in [Caola et al., '18]; study focused on theory uncertainties.
- O(20%) uncertainty for the top+bottom interference.
- Accurate at NNLL+NLO; still open problem of the resummation of log(p<sub>T</sub>/m<sub>b</sub>),log(m<sub>H</sub>/m<sub>b</sub>).



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## Gluon fusion in the MSSM



#### SUSY

- Squark diagrams known up to NLO [4, 5, 6, 7].
- Complete NLO results for the squark-gluino contribution known only from semi-numeric computation or using Taylor/asymptotic expansions publicly available [Harlander et al '03, Degrassi et al '08,'10,'11,'12] [8].
- Need resummation of tan β enhanced contribution proportional to the bottom Yukawa [9]

$$\begin{split} \widetilde{Y_b}^h &=\; \frac{Y_b^h}{1+\Delta_b} \left(1-\Delta_b \, \frac{\cot \alpha}{\tan \beta}\right) \\ \widetilde{Y_b}^H &=\; \frac{Y_b^H}{1+\Delta_b} \left(1+\Delta_b \, \frac{\tan \alpha}{\tan \beta}\right) \\ \widetilde{Y_b}^A &=\; \frac{Y_b^A}{1+\Delta_b} \left(1-\Delta_b \, \cot^2 \beta\right) \end{split}$$

- Resummation of non-abelian log(m<sub>b</sub>/m<sub>h</sub>) still missing. Resummation of Abelian logs computed [Melnikov et al '16].
- Codes: SusHi, MoRe-SusHi, POWHEG-BOX/gg\_H\_MSSM, aMC\_SusHi

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### Scale uncertainty for gluon fusion



 Non trivial dependence on the parameters. For h up to 35%, for H up to 50% and for A up to 30%.

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# Validity of the SUSY expansions at NLO



- Contributions from diagrams with quark-squark-gluino are available only as an expansion (in the the limit m<sub>h</sub> → 0 for h and as an expansion in the inverse SUSY-particle masses for H and A).
- We compute a test factor  $T = A_{\tilde{q}}^{1l}/A_{\tilde{q}}^{1l,exp}$ , where  $q_i = \{\tilde{b}, \tilde{t}\}$  and  $A_{\tilde{q}}^{1l-exp}$  is the result by keeping just the leading  $\mathcal{O}(m_{\tilde{q}}^{-2})$ .
- We multiply the 2-loop stop and sbottom approximate contributions by T and take the resulting value for the cross section as a probe of the uncertainty in the expansion.

# The Higgs $p_T^H$ in the MSSM



 $m_b^{\text{mod}+}$  scenario, tan  $\beta = 17$ ,  $m_A = 500$  GeV,  $m_H = 499.9$  GeV

•  $\pm 40\%$  variation between the "normal" scale choices and the specific ones.

#### [EB et al. '11; EB, Vicini, '15; EB, Harlander, Mantler, Wiesemann, Vicini '15]

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# Higgs $p_T$ and MSSM $A/H \rightarrow \tau \tau$ searches in CMS

- CMS published (1803.06553) its updated  $H/A \rightarrow \tau \tau$  performed using a reweighting procedure to account for the different acceptance due to the BSM nature of the Higgs states.
- Work performed in the context of the Higgs XS working group, in collaboration with S. Liebler (KIT) and R. Wolf (CMS) and collaborators.
- POWHEG-BOX used for the shape and SusHi for the total inclusive cross-sections.



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### **Vector Boson Fusion**

#### The SM

- Know up to NNLO-QCD differentially using a structure function approach[Bolzoni et al '10, ...] [10].
- Non-factorizing contribution shown to be small [Harlander et al '08] [11, 10, 12].
- Codes: HAWK, POWHEG, MG5\_aMC@NLO, VBF@NNLO, proVBFH, VBFNLO.







 Computation at NNLO possible only in the DIS/VBF approximation.

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#### [Cruz-Martinez et al., '18]

 Independent result. Found error in the NLO H+3jet virtual corrections in the first computation.

### **Vector Boson Fusion**



#### The MSSM

- Known up to NLO in SUSY-QCD (also NLO-EW available) [Hollik et al 08', Figy et al 10']
- Small impact on the total cross section.
- In the decoupling limit, h is SM-like and the H couples only weakly with vector bosons. No AVV coupling at tree level.

- NLO-QCD corrections as DY (30% of the total cross section) [13].
- QCD corrections known up to NNLO [14].
- New channel, gg → ZH opens up NNLO and yields a large contribution (20% of the total cross section); now known at HQEFT-NNLO[Altenkamp et al '13] [15].
- Codes: HVNNLO, MCFM, VHNNLO, VH@NNLO, NNLOPS, POWHEG, MG5\_aMC@NLO



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[Spira '16]

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[Granata et al. '17]

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#### The MSSM

- As for VBF, in the decoupling limit only sensible production rate is for h.
- No pseudoscalar production tree level.
- Relative NLO-QCD corrections are the same as the SM.
- At NNLO, the gg → ZH contribution will be different due to the different top/bottom Yukawa.
- SUSY-QCD small [16].

#### The SM

- ttH: known up to NLO-QCD [Beenakker et al '01, ...] [17, 18, 19](O(20%)), SCET [Kulesza et al '16] [20], resummation, EW corrections.
- ttH: available @ NLO+PS in POWHEG-BOX, MG5\_aMC@NLO, SHERPA.
- bbH-4FS: up to NLO-QCD.
- bbH-5FS: up to NNLO-QCD (bbh@nnlo) [Harlander et al '03] [21].
- bbH, scheme matching: Santander, FONLL [Forte et al. '16], SCET [Bonvini et al '16].



dashed – ratio over LO; solid – ratio over NLO; crosses – LO+PS [Frederix et al 11]





#### The MSSM

- ttH suppressed for  $\tan \beta > 1$ .
- 4FS-ttA:NLO-QCD corrections for ttA know [19].
- 5FS-bbh: replace the bottom Yukawa with the resummed one.
- 4FS-(bb/tt)(A/H): Full NLO SUSY-QCD corrections recently computed [Dittmaier et al '14] [22, 23]
- Can be re-adsorbed with good approximations in the resummed bottom Yukawa coupling [Dittamaier et al '14].
- 5FS-code: SusHi (NNLO-QCD).

#### [YR4]



[Dittmaier et al '14]

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[Dittmaier et al '14]

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## Scale uncertainty for bottom annihilation



- We take the set  $C_{\mu} \equiv \{(\mu_R, \mu_F)\}$  of combinations of renormalization and factorization scales, where  $\mu_R = \{m_{\phi}/2, m_{\phi}, 2m_{\phi}\}$  and  $\mu_F = \{m_{\phi}/8, m_{\phi}/4, m_{\phi}/2\}$ , with the constraint  $2 \leq \mu_r/\mu_f \leq 8$
- Approximately independent of  $\tan \beta$
- *O*(20%) for h, from 20% to a few % for H and A

## Conclusions

- Precision physics can offer an insight on BSM extensions by looking at the deviation from SM predictions.
- For a meaningful precision physics program at the LHC, theoretical uncertainties should match the experimental precision of the results.
- The computation of radiative corrections for the SM Higgs boson is an ongoing effort.
- The MSSM, due to the lack of SUSY hints from the LHC and the higher complexity of the calculations, was less the focus of theorist attention in the past few years.

# **Backup slides**

# **Double Higgs production**

- Dominated by gluon fusion.
- Other channels (VBF, double Higgs strahlung, double quark-associated production) subdominant.
- Gluon fusion known up to NLO-QCD in SM via a numerical computation [Borowska et al ' 17].
- Known up to NNLO-QCD in the HQEFT [de Florian et al '13, ...] [24].
- NNLL soft and collinear resummation available [Shao et al '13, ..] [25].



# **Double Higgs production**



#### The MSSM

- Final states: *hh*, *hH*, *hA*, *HH*, *HA*, *AA*
- hA, HA dominated by DY like process
- The new SM results important, because bottom quark may dominate in the MSSM.
- Recent progress: gluon fusion SUSY corrections known at NLO-QCD in the limit of vanishing external momenta [Degrassi et al '16].
- Code: HPAIR

# **Higgs decays**

For the light SM-like Higgs, BSM physics can enter as

- Modified couplings.
- New intermediate state particles.
- New final states.



In an extended Higgs sector, we need to consider the decays of the new resonances

- Decay of CP-even neutral resonances.
- Decay of CP-odd neutral resonances.
- Decay of charged scalar resonances.



[YR3]

# Higgs decays to gluons

#### The SM

 Loop induced decay, known at NLO-QCD in the SM, N<sup>3</sup>LO in the HQEFT [J.R. Ellis et al '76, ...][26, 4, 27, 28, 29, 30].

#### The MSSM

- HQEFT not applicable, couplings to b-quark strongly enhanced in the large  $\tan\beta$  regime
- Squarks loop known up to NLO with full mass dependence [Bonciani et al '07, Mühlleitner et al '08] [6].
- Full QCD corrections computed either through expansions or numerically [Harlander et al '03, Degrassi et al '08, ...] [8].
- Sizable effect but can be decently described by the  $\Delta_b$  approximation



#### [Spira et al '16]



# Higgs decays to photons



#### The SM

- Full two-loop (NLO-QCD) corrections to quarks loop available[Spira et al '95] [4, 31, 5]
- In the HQEFT, QCD corrections known up to N<sup>3</sup>LO [26, 32, 4, 33].
- Perturbative behavior improved if expressed in terms of the running top mass @  $Q = M_H/2$
- Resummation of log(m<sub>b</sub>/m<sub>h</sub>) [34]

#### The MSSM

- NLO-QCD corrections for squark loops known [4, 6, 31, 5, 35].
- Pseudoscalar amplitudes show a Couloumb singularity at the tt
   threshold (regularized by the top and squark widths)
- Important phenomenological aspects, interplay of stops (and stau) to enhance/suppress  $pp \to H \to \gamma \gamma.$

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