Vacuum Stability.

Complementary Constraints on SUSY Parameter Space

Jonas Wittbrodt

LHC Physics Discussion: ***SUSY***

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Based on: [W.G. Hollik, G. Weiglein, J. Wittbrodt; 1811.?????]

Vacuum Stability

The EW vacuum is a local minimum of the scalar potential. The universe has been in that state at least since BBN (so for $\sim T_H$).

- if it is the global minimum $\Rightarrow$ absolute stability
- if there are deeper minima
  - $\rightarrow$ lifetime of the EW vacuum $> T_H \Rightarrow$ long-lived metastability
  - $\rightarrow$ lifetime of the EW vacuum $< T_H \Rightarrow$ short-lived instability

Both absolute and metastability are fine with observations, but a short-lived EW vacuum is not.

This can be used to constrain particle physics models.

Overview

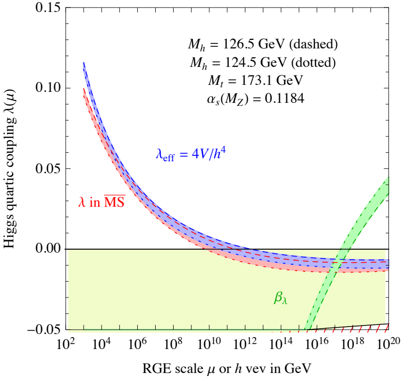
High Scale Vacuum Stability

- The SM

EW Scale Vacuum Stability

- Method
- Contraints on MSSM Benchmark Scenarios

High Scale Vacuum Stability in the SM

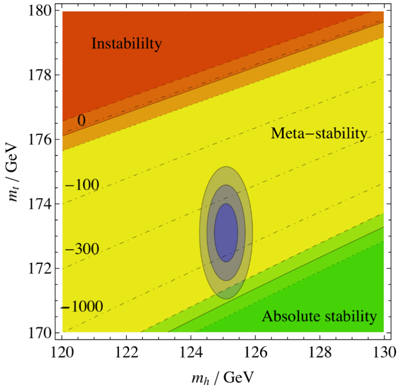


[Degrassi et.al.; 1205.6497]

> When extrapolating the SM to very high energies the quartic coupling λ turns negative.

⇒ A deeper minimum develops at very high field values.

High Scale Vacuum Stability in the SM



- Sensitive to the values of m_t and m_h

$$\beta_\lambda \approx 12\lambda^2 + 6y_t^2\lambda - 3y_t^4$$

- Current central values in a region of metastability

$$P_{\text{decayed}} = \frac{\Gamma}{V}(VT)_{\text{lc}} = 10^{-516^{-409}_{+202}}$$

[Chigusa, Moroi, Shoji; 1803.03902] [Andreassen, Frost, Schwartz; 1707.08124]

High Scale Vacuum Stability in the SM

The stability of the EW vacuum in the SM is in agreement with observations.

Ignoring:

- New physics below the Planck scale?
- Impact of Planck suppressed operators?

EW Scale Vacuum Stability

Additional scalar degrees of freedom can lead to vacuum stability constraints already at the EW scale.

- insensitive to high scale physics
- less sensitive to higher order effects

The most general renormalizable scalar potential at tree-level is

$$V(\phi_a) = \lambda_{abcd}\phi_a\phi_b\phi_c\phi_d + A_{abc}\phi_a\phi_b\phi_c + m^2_{ab}\phi_a\phi_b + t_a\phi_a + c$$

for $a, b, c, d \in \{1, \ldots, N\}$ for N real scalar degrees of freedom.

Directions in Fieldspace

$$V(\phi_a) = \lambda_{abcd}\phi_a\phi_b\phi_c\phi_d + A_{abc}\phi_a\phi_b\phi_c + m^2_{ab}\phi_a\phi_b + t_a\phi_a + c$$

Expand around EW vacuum $\vec{\phi} \to \vec{v} + \vec{\varphi}$:

$$V(\varphi_a) = \lambda(\vec{v})_{abcd}\varphi_a\varphi_b\varphi_c\varphi_d + A(\vec{v})_{abc}\varphi_a\varphi_b\varphi_c + m^2(\vec{v})_{ab}\varphi_a\varphi_b$$

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Introduce polar coordinates $\vec{\varphi} \to \varphi\hat{\varphi}$:

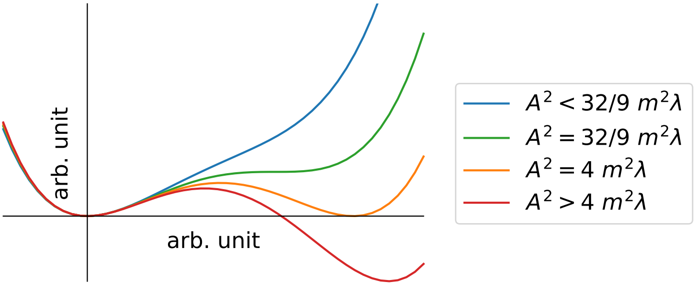
$$V(\varphi) = \lambda(\hat{\varphi})\varphi^4 - A(\hat{\varphi})\varphi^3 + m^2(\hat{\varphi})\varphi^2$$

> $\lambda > 0$ for physical potentials (bounded from below)

> $A > 0$ by choice ($\varphi \leftrightarrow -\varphi$)

> $m^2 > 0$ if the EW-vacuum is a local minimum

Stability of Fieldspace Directions



- at most one additional minimum for each $\hat{\varphi}$
- the additional minimum is deeper if $A(\hat{\varphi})^2 > 4m^2(\hat{\varphi})\lambda(\hat{\varphi})$

We use polynomial homotopy continuation to find all stationary points and identify deep minima from there.

Lifetime and Vacuum Decay

The vacuum tunneling decay width is given by

$$\frac{\Gamma}{V} = K e^{-B}$$

The bounce action B

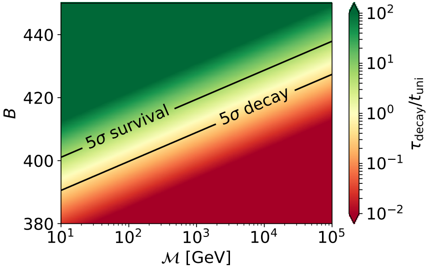
> analytic solution in straight path approximation

[Adams; hep-ph/9302321]

> associated uncertainty of $\mathcal{O}(10\%)$

[Masoumi, Olum, Wachter; 1702.00356]

The prefactor $K \sim \mathcal{M}^4$



$\Leftrightarrow B \in [390, 440] \sim 10\%$

The Scalar Sector of the MSSM

In SUSY theories every SM fermion gains a scalar superpartner.
In the MSSM the scalar potential including only the real, neutral Higgs and real $\widetilde{t}_r$, $\widetilde{t}_l$ fields reads

$$V((h_u)^{0,r},(h_d)^{0,r},(\widetilde{t}_r)^r,(\widetilde{t}_l)^r) =$$

$$\frac{g_1^2}{288}\left(3(h_u^2-h_d^2)+\widetilde{t}_l^{\,2}-4\widetilde{t}_r^{\,2}\right)^2+\frac{g_2^2}{32}\left(h_d^2-h_u^2+\widetilde{t}_l^{\,2}\right)^2+\frac{g_3^2}{24}\left(\widetilde{t}_l^{\,2}-\widetilde{t}_r^{\,2}\right)^2$$

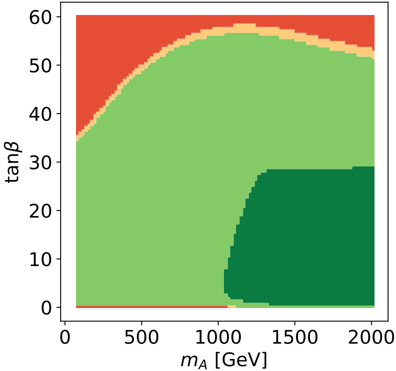
$$+\frac{y_t^2}{4}\left(h_u^2(\widetilde{t}_l^{\,2}+\widetilde{t}_r^{\,2})+\widetilde{t}_l^{\,2}\widetilde{t}_r^{\,2}\right)+\frac{y_t}{\sqrt{2}}(A_t h_u-\mu h_d)\widetilde{t}_l\widetilde{t}_r$$

$$+\frac{g_1^2+g_2^2}{16}\left(v_0^2 c_{2\beta}(h_u^2-h_d^2)\right)+\frac{m_A^2}{2}\left(c_\beta h_u-s_\beta h_d\right)^2+\frac{m_{Q_3}^2}{2}\widetilde{t}_l^{\,2}+\frac{m_{U_3}^2}{2}\widetilde{t}_r^{\,2}$$

Tree-level — the 1-loop effective potential has numerical and theoretical issues.

[Andreassen, Farhi, Frost, Schwartz; 1604.06090]

Vacuum Stability of the $M_h^{125}(\tilde{\tau})$ Scenario



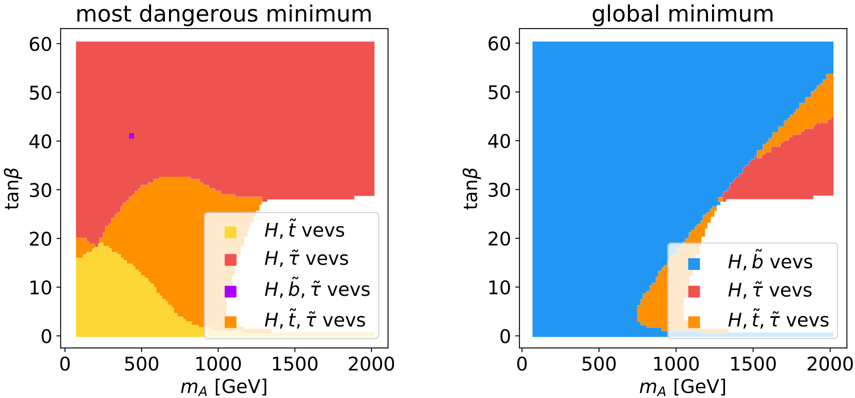
- stable, EW vacuum is the global minimum
- long-lived
- $390 < B < 440$
- short-lived

[Bahl, et.al.; 1808.07542]

$$X_t = A_t - \frac{\mu}{\tan\beta} = 2.8\,\text{TeV}\,, \quad A_b = A_t\,, \quad \mu = 1\,\text{TeV}\,,$$

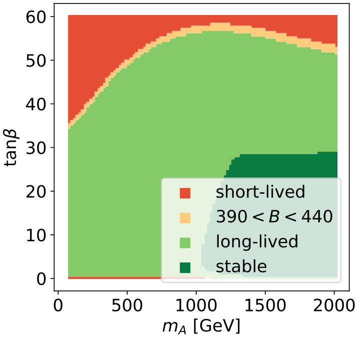
$$m_{Q_3,U_3,D_3} = 1.5\,\text{TeV}\,, \quad m_{L_3,E_3} = 350\,\text{GeV}\,, \quad A_\tau = 800\,\text{GeV}$$

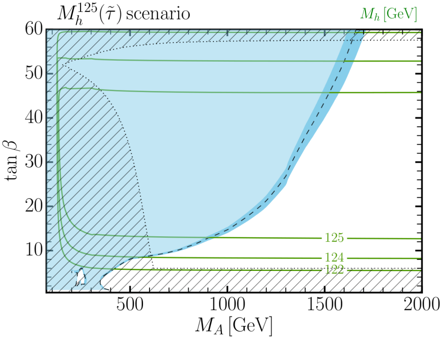
The Vacuum Structure



- fastest tunnelling in $\tilde{\tau}$ directions
- fastest tunneling in general not in the direction of the global minimum

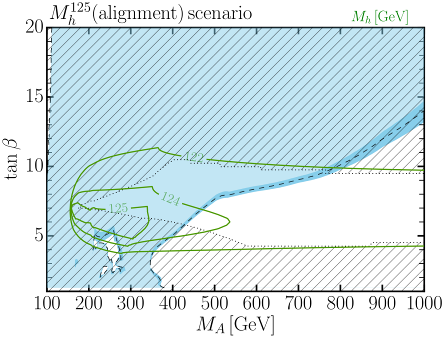
Complementarity to Experimental Constraints





- Vacuum stability constraints important at large $\tan\beta$
- complementary to $BR(h \to \gamma\gamma)$

The M_h^{125}(alignment) Scenario



alignment without decoupling

- tree-level and 1-loop cancel $\rightarrow$ SM-like h
- excluded at large $\tan\beta$ from $H \rightarrow \tau\tau$
- at small $\tan\beta$ requires $A_t, \mu \gg m_{\tilde{t}}$

$$A_t = A_b = A_\tau = 6.25\,\mathrm{TeV}\,, \quad \mu = 7.5\,\mathrm{TeV}\,,$$
$$m_{Q_3,U_3,D_3} = 2.5\,\mathrm{TeV}\,, \quad m_{L_3,E_3} = 2\,\mathrm{TeV}$$

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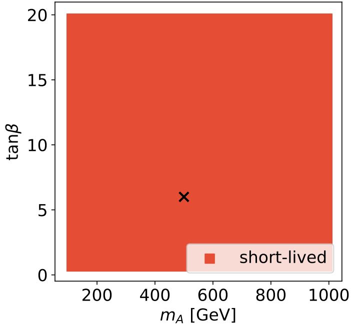

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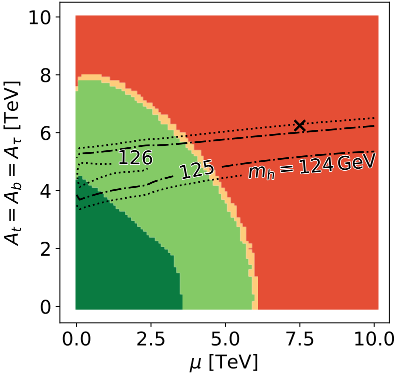
Vacuum Stability and Alignment Without Decoupling



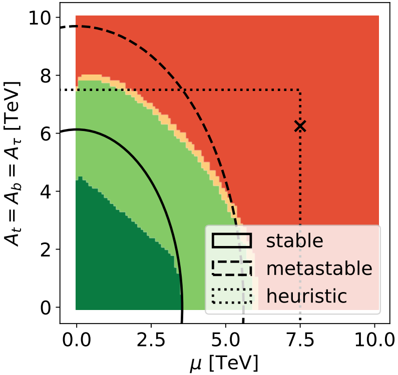
- stabilizing the vacuum would require, e.g.

$$A \to \sim 5\,\text{TeV} \quad \mu \to \sim 4\,\text{TeV}$$

- this reduces $BR(h \to \gamma\gamma)$ by $\sim 10\%$

The alignment without decoupling regime is strongly constrained by combining experimental searches with constraints from vacuum stability.

Vacuum Stability Constraints from the Literature



- analytical/empirical constraints for absolute stability and metastability
  [Kusenko et.al.; hep-ph/9602414]

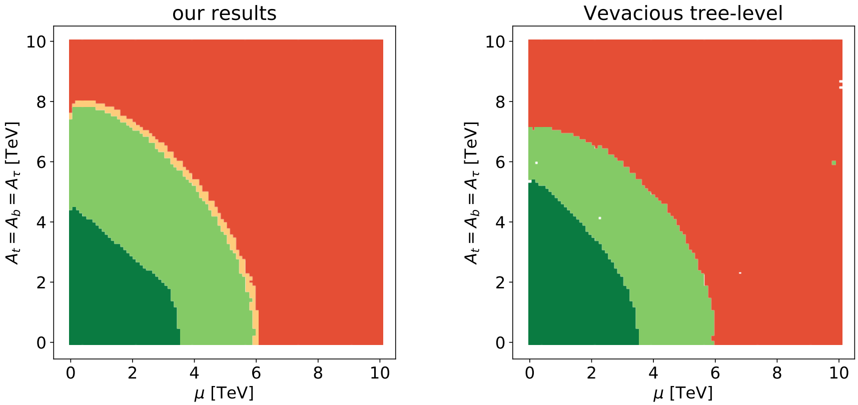
$$A_t^2 + 3\mu^2 < (m_{\tilde{t}_R}^2 + m_{\tilde{t}_L}^2) \cdot \begin{cases} 3 \\ 7.5 \end{cases}$$

- heuristic estimate
  $A/m, \mu/m \lesssim 3$

Analytic and empirical bounds provide only very rough estimates of the vacuum stability constraints.

Comparison with `Vevacious`

[Camargo-Molina, O'Leary, Porod, Staub; 1307.1477]



- good agreement in this plane, differences due to $\tilde{\tau}$-vevs not included in our `Vevacious` run
- our code is considerably faster ($\sim 5\times$) and more reliable

Summary

Vacuum stability provides important constraints on the parameter space of models with large scalar sectors and can provide complementary constraints to experimental searches.

- Vacuum stability constraints can help catch theorists as they push their models into corners of parameter space.
- Vacuum stability constraints on model parameter space are not very sensitivy to uncertainties in B.
- The most dangerous minimum for tunneling is not in general the global minimum of the theory.

We aim to provide efficient and reliable bounds from vacuum stability in any renormalizable model.

Upcoming paper [Hollik, Weiglein, JW; 1811.?????].

Considered Field Sets

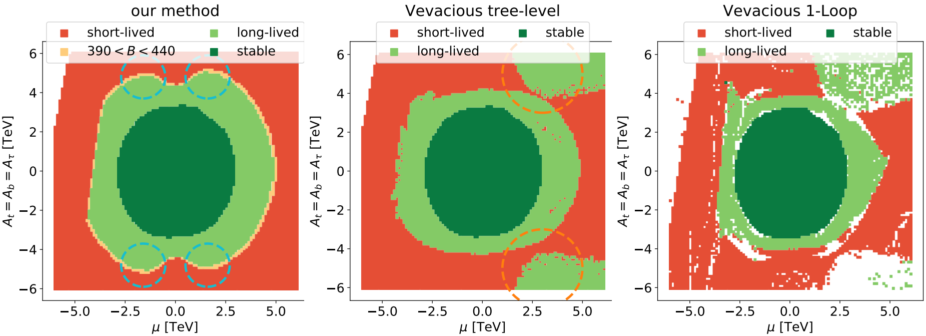
We look for stationary points using these three sets of fields:

$$\left\{\mathrm{Re}(h_u^0),\, \mathrm{Re}(h_d^0),\, \mathrm{Re}(\tilde{t}_L),\, \mathrm{Re}(\tilde{t}_R),\, \mathrm{Re}(\tilde{b}_L),\, \mathrm{Re}(\tilde{b}_R)\right\}$$

$$\left\{\mathrm{Re}(h_u^0),\, \mathrm{Re}(h_d^0),\, \mathrm{Re}(\tilde{t}_L),\, \mathrm{Re}(\tilde{t}_R),\, \mathrm{Re}(\tilde{\tau}_L),\, \mathrm{Re}(\tilde{\tau}_R)\right\}$$

$$\left\{\mathrm{Re}(h_u^0),\, \mathrm{Re}(h_d^0),\, \mathrm{Re}(\tilde{b}_L),\, \mathrm{Re}(\tilde{b}_R),\, \mathrm{Re}(\tilde{\tau}_L),\, \mathrm{Re}(\tilde{\tau}_R)\right\}$$

Detailed Comparison with `Vevacious`



Finding Deep Directions in Fieldspace

1. Solve $\vec{\nabla}_\phi V = 0$ to find all stationary points using polynomial homotopy continuation (PHC).
2. Compare the potential values at each stationary point to the value at the EW vacuum.
3. Get $\lambda(\hat{\varphi})$, $A(\hat{\varphi})$ and $m^2(\hat{\varphi})$ for $\hat{\varphi}$ pointing towards each deeper stationary point.

PHC in theory always finds all solutions. In practice it requires the system to be well conditioned to reduce coefficient variability.

Perturbative Expansion of the Bounce

$$V(\phi) = \lambda\phi^4 \quad \text{with } \lambda < 0$$

Analytic bounce solution

$$\phi_c(\rho) = \sqrt{-\frac{2}{\lambda}}\frac{R}{R^2+\rho^2} \;\Rightarrow\; B = -\frac{2\pi}{3\lambda}$$

The 1-loop effective action up to two derivatives is

$$S_{\text{eff}} = \underbrace{\lambda\phi^4}_{\text{LO}} + \underbrace{\frac{9\lambda^2}{4\pi^2}\phi^2\left(\ln\frac{12\lambda\phi^2}{\mu^2} - \frac{3}{2}\right)}_{\text{1L eff. potential}} + \underbrace{\frac{(\partial_\mu\phi)^2}{2}\left(1+\frac{\lambda}{4\pi^2}\right)}_{\text{1L } p^2} + \underbrace{\mathcal{O}(\partial^4)}_{\text{1L } p^4}$$

Leading to

$$B = -\frac{2\pi}{3\lambda} + 3\ln\frac{R\mu}{2\sqrt{6}} + \frac{19}{4} + \frac{1}{3} + \mathcal{O}(1)$$

[Andreassen, Farhi, Frost, Schwartz; 1604.06090]