NNLO contributions to Bhabha scattering

Tord Riemann, DESY, Zeuthen
30 June 2009, FCAL Meeting, DESY, Zeuthen

based on work with:
S. Actis (RWTH Aachen)
M. Czakon (U. Würzburg, now RWTH Aachen)
J. Gluza (Silesian U. Katowice)

- Bhabha scattering – Born cross-section and experimental aspects
- Electroweak one-loop contributions
- QED two-loop contributions to Bhabha Scattering
  ACGR: Phys. Rev. Letters 100 (2008) [arXiv:0711..3847]
  ACGR: Phys. Rev. D78 (2008) 085019 [arXiv:0807.4691]
- Summary and outlook

Born cross-section and experimental aspects

H. Bhabha,

"The Scattering of Positrons by Electrons with Exchange on Dirac's Theory of the Positron",

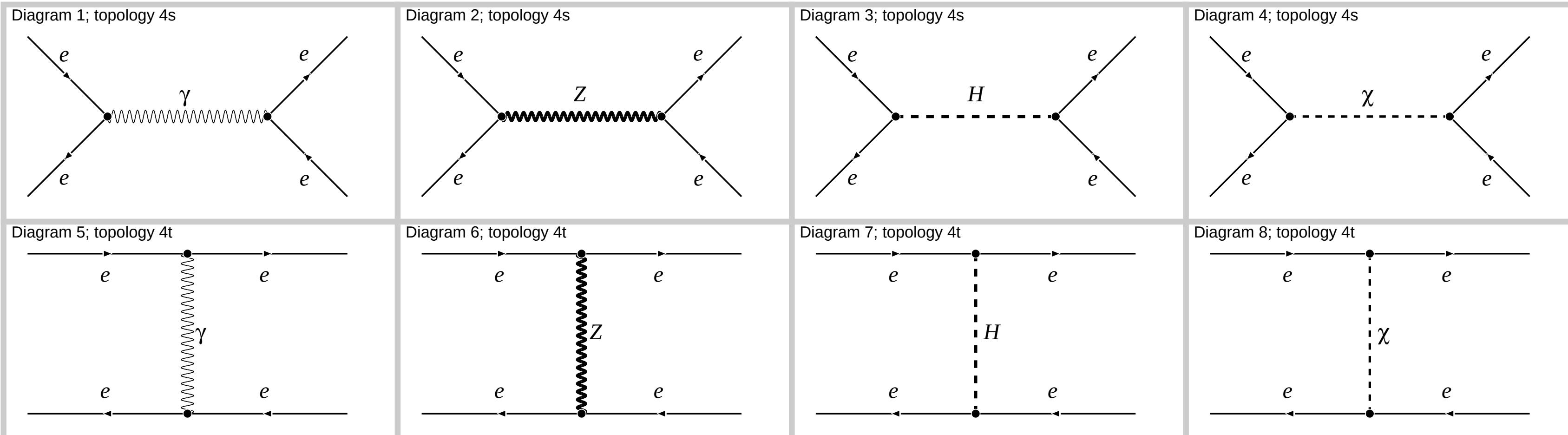
Proc. Roy. Soc. A154 (1936) 195

$$\frac{d\sigma_0}{d\Omega} = \frac{\alpha^2}{s}\left(\frac{s}{t}+1+\frac{t}{s}\right)^2$$

where the relations between beam energy, scattering angle and s, t are:

$$\begin{aligned} s &= 4E^2 \\ t &= \frac{s}{2}(1-\cos\vartheta) \end{aligned}$$

- $|\mathcal{M}_s + \mathcal{M}_t|^2$
- simple process with zero [one] mass scales
- strong forward peak, huge statistics
- there: QED dominating [if no new physics]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Electroweak Born and 1-loop contributions

The Born cross-section is:

$$\frac{d\sigma_{ew}}{d\Omega} = \frac{\alpha^2}{4s}\left(T_s + T_{st} + T_t\right),$$

with

$$T_s = (1+\cos^2\theta)\left[1 + 2\textbf{Re}\chi(s)\left(v^2\right) + |\chi(s)|^2\left(1+v^2\right)^2\right] + 2\cos\theta\left[2\textbf{Re}\chi(s) + |\chi(s)|^2\left(4v^2\right)\right],$$

$$T_{st} = -2\frac{(1+\cos\theta)^2}{(1-\cos\theta)}\left\{1 + \left[\chi(t) + \textbf{Re}\chi(s)\right]\left(1+v^2\right) + \chi(t)\textbf{Re}\chi(s)\left[(1+v^2)^2 + 4v^2\right]\right\},$$

$$T_t = 2\,\frac{(1+\cos\theta)^2}{(1-\cos\theta)^2}\left\{1 + 2\chi(t)\left(1+v^2\right) + \chi(t)^2\left[(1+v^2)^2 + 4v^2\right]\right\} + \frac{8}{(1-\cos\theta)^2}\left[1 - \chi(t)\left(1-v^2\right)\right]^2.$$

We choose the following conventions:

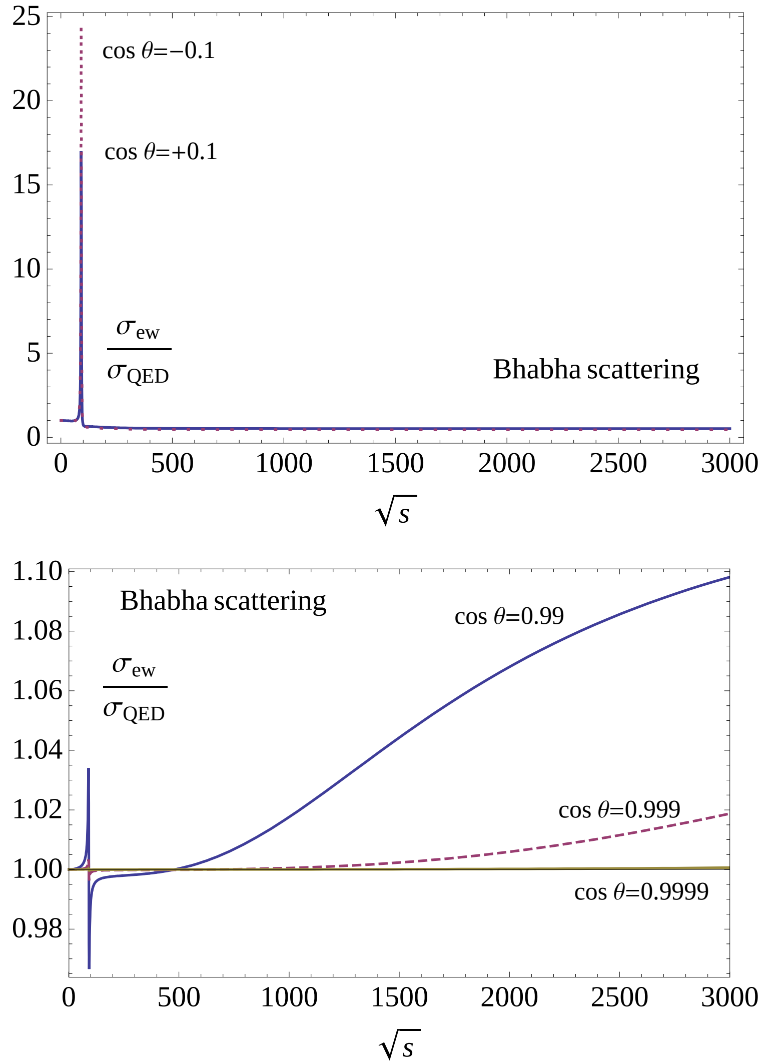
$$v = 1 - 4s_w^2,$$

$$\chi(s) = \frac{G_F}{\sqrt{2}}\frac{M_Z^2}{8\pi\alpha}\frac{s}{s - M_Z^2 + iM_Z\Gamma_Z},$$

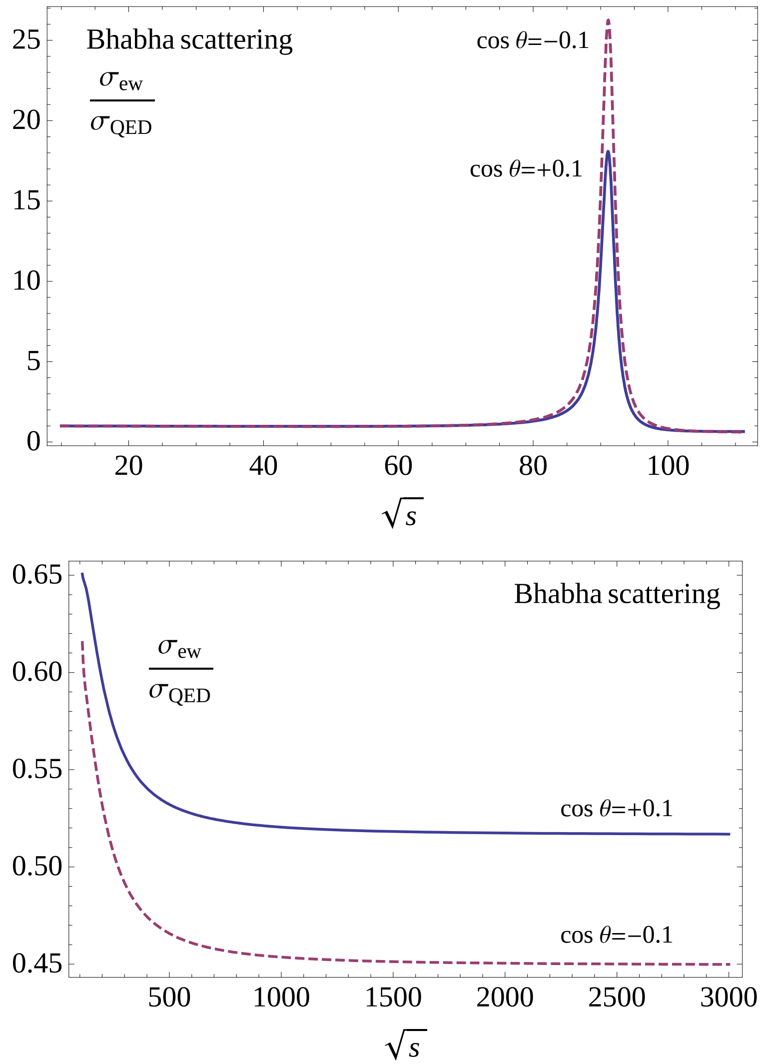
$$\chi(t) = \frac{G_F}{\sqrt{2}}\frac{M_Z^2}{8\pi\alpha}\frac{t}{t - M_Z^2}.$$

Among the quantities α, G_F, s_w^2, M_Z there are only three independent, and Γ_Z is predicted by the theory as well.
The phrasing *effective Born cross-section* **means here that we use, besides α, the following input variables:**

$$\begin{aligned} s_w^2 &= 0.23, \\ M_Z &= 91.1876 \pm 0.0021 \text{ GeV}, \\ \Gamma_Z &= 2.4952 \pm 0.0023 \text{ GeV}, \\ G_F &= (1.16637 \pm 0.00001) \times 10^{-5} \text{ GeV}^{-2}. \end{aligned}$$

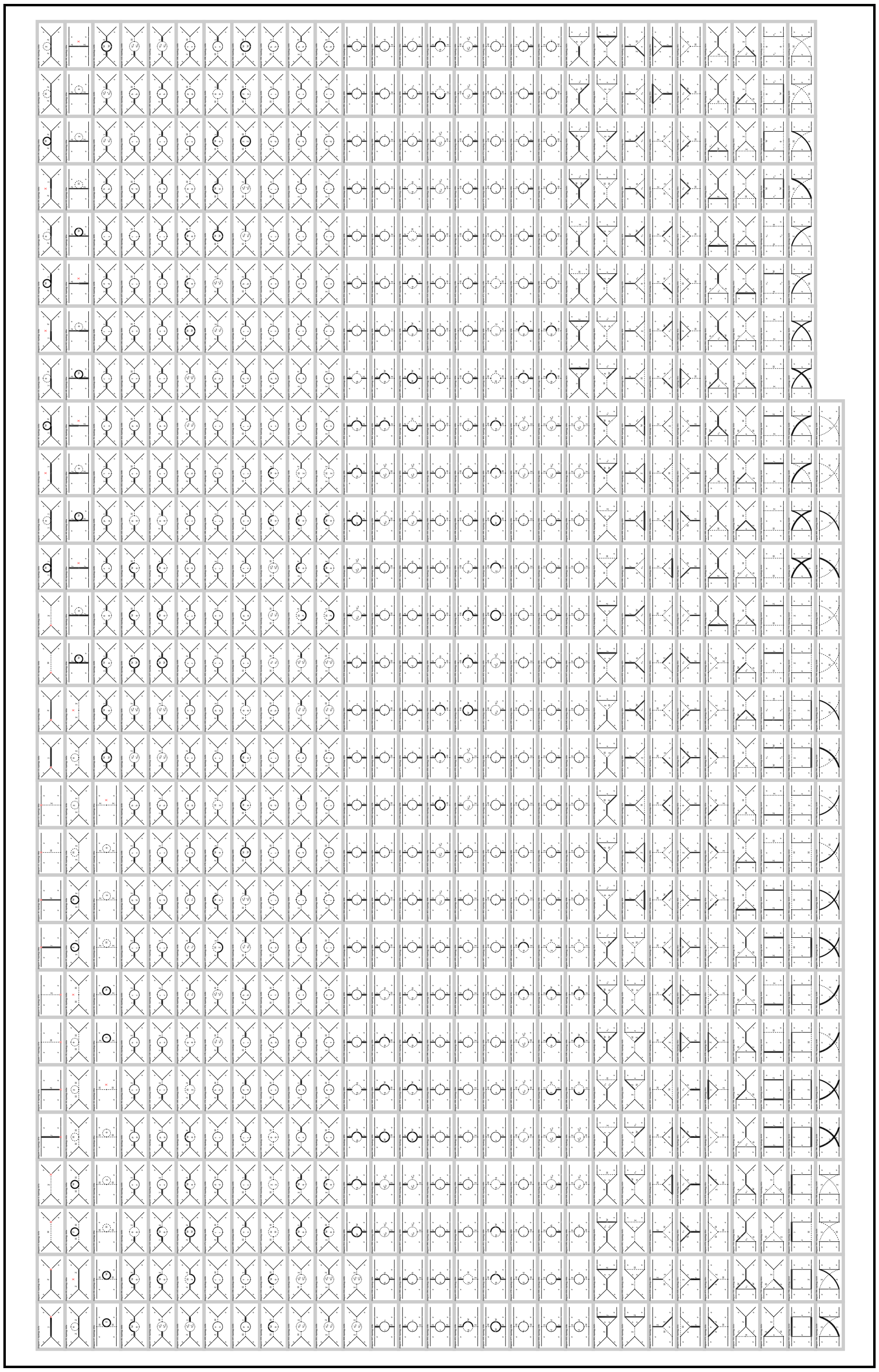


Ratio of electroweak to QED Bhabha scattering cross-section at large angles (up) and small angles (down) as a function of $\sqrt{s}$.



Ratio of electroweak to QED Bhabha scattering cross-section at large angles in the energy ranges of LEP1/GigaZ (up) and ILC (down).

DIANA OneLoop_page5.ps shows the many one-loop diagrams



Weak corrections to Bhabha scattering

Bardin,Hollik,T.R., Z.PhysikC49(1991)485

Table 2:
The differential Bhabha cross section in nbarn as function of the scattering angle and the cms-energy.
$M_Z = 91.16\ GeV$, $m_t = 150\ GeV$, $M_H = 100\ GeV$.
Upper rows: DZ, lower rows: H.
δ_m: *largest relative deviation in per mille.*

$\sqrt{s}$ (GeV) / θ	60	89	91.16	93	200
15°	129.6	65.11	57.93	49.00	11.82
	129.6	65.11	57.93	49.00	11.82
45°	1.451	1.376	1.755	.4833	11.67
	1.451	1.377	1.756	.4837	11.68
60°	.4303	.6124	1.125	.2697	.03075
	.4305	.6129	1.126	.2699	.03077
75°	.1717	.3627	.8718	.2232	.01072
	.1718	.3630	.8720	.2233	.01072
90°	.08873	.2768	.7790	.2088	.004862
	.08876	.2769	.7787	.2087	.004855
105°	.05917	.2690	.8082	.2157	.002858
	.05918	.2690	.8074	.2157	.002853
120°	.04906	.3053	.9323	.2429	.002077
	.04906	.3051	.9309	.2426	.002074
135°	.04671	.3626	1.111	.2838	.001743
	.04672	.3624	1.109	.2833	.001742
165°	.04839	.4638	1.425	.3590	.001539
	.04839	.4635	1.422	.3584	.001540
δ_m	0.6	0.8	1.8	2.0	1.7

The 1991 result is state of the art in e.g. ZFITTER and BHWIDE.

Results: Numerical comparison in all $f\bar{f}$

Bhabha $e^-e^+ \to e^-e^+\ (\gamma)$ **at LC:** $\sqrt{s} = 500$ GeV, $E_{\rm max}(\gamma_{\rm soft}) = \frac{\sqrt{s}}{10}$

$\cos\theta$	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\rm Born}$ (pb)	$\left[\frac{d\sigma}{d\cos\theta}\right]_{\mathcal{O}(\alpha^3)=\text{Born+QED+weak+soft}}$	Group
−0.9999	0.21482 70434 05632 5	0.14889 12125 78083 7	aITALC
−0.9999	0.21482 70434 05632 6	0.14889 12189 28404 0	*FeynArts*
−0.9	0.21699 88288 10920 5	0.19344 50785 26863 6	aITALC
−0.9	0.21699 88288 10920 0	0.19344 50785 26862 2	*FeynArts*
−0.9	0.21699 88288 41513 1	0.19344 50785 62637 9	$m_e = 0$
+0.0	0.59814 23072 50330 3	0.54667 71794 69423 1	aITALC
+0.0	0.59814 23072 50329 4	0.54667 71794 69421 8	*FeynArts*
+0.0	0.59814 23072 88584 4	0.54667 71794 99961 4	$m_e = 0$
+0.9	$0.18916\,03223\,32270\,6 \cdot 10^3$	$0.17292\,83490\,66507\,2 \cdot 10^3$	aITALC
+0.9	$0.18916\,03223\,32270\,6 \cdot 10^3$	$0.17292\,83490\,66508\,0 \cdot 10^3$	*FeynArts*
+0.9	$0.18916\,03223\,31848\,5 \cdot 10^3$	$0.17292\,83490\,61347\,4 \cdot 10^3$	$m_e = 0$
+0.9999	$0.20842\,90676\,46142\,9 \cdot 10^9$	$0.19140\,17861\,11341\,6 \cdot 10^9$	aITALC
+0.9999	$0.20842\,90676\,46436\,4 \cdot 10^9$	$0.19140\,17861\,11979\,0 \cdot 10^9$	*FeynArts*

Great independent agreement up to 14 digits! : limit in double precision

Previous agreement with *FeynArts*: 11 digits hep-ph/0307132**, SANC: 10 digits** hep-ph/0207156

Thanks to T. Hahn, numbers supplied with *FeynArts* + *FormCalc* + *LoopTools*

Really precise predictions . . . include 2-loop QED corrections

for both

- **Small angle Bhabha scattering**
- **Large angle Bhabha scattering**

aim is at 10^{-4} accuracy

2-loop Bhabha scattering: What to be done?

- **Calculate:**

$$\sigma = (2 \to 2) + (2 \to 3) + (2 \to 4)$$

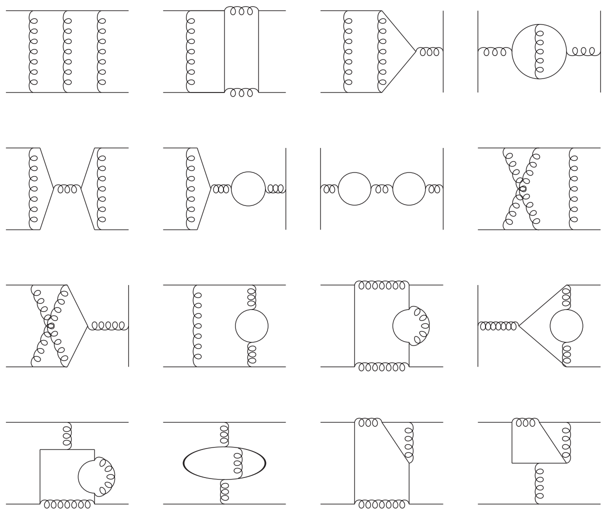
$$\begin{aligned}\sigma = & \left|\textbf{Born} + (\textbf{1-loop}) + (\textbf{2-loop})\right|^2 \\ & + \left|(\textbf{Born} + \textbf{1-}\gamma) + (\textbf{1-loop} + \textbf{1-}\gamma)\right|^2 \\ & + \left|(\textbf{Born} + \textbf{2-}\gamma)\right|^2\end{aligned}$$

- **Do not include: (1-loop) $\times$ (2-loop) and $|$2-loop$|^2$**
  $|$(1-loop $+$ 1-γ)$|^2$
- **Difficult: $|$2-loop$|^2$ – is done in 2007**
- **Difficult: (Born $+$ 1-γ) $\times$ (1-loop $+$ 1-γ) – is being done**
- **Easier: Real pair production corrections – just done, to be published (Czyz, Gluza, Gunia, Riemann, Worek)**

Two Loop Bhabha Scattering

To calculate Bhabha scattering it is best to first compute $e^+e^- \to \mu^+\mu^-$, since it's closely related but has less diagrams.

There are 47 QED diagrams contributing to $e^+e^- \to \mu^+\mu^-$.

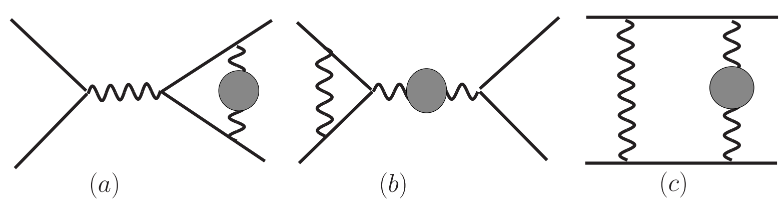


The Bhabha scattering amplitude can be obtained from $e^+e^- \to \mu^+\mu^-$ simply by summing it with the crossed amplitude (including fermi minus sign).

Adapted from Z.Bern, LoopFest 2002

The diagrams with electrons and photons define an $n_f = 1$ problem.

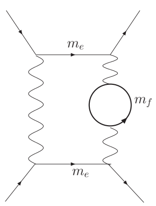
But there are additional ones with heavier fermions.
So we have to investigate an $n_f = 2$ problem
For self-energies starting with 1-loops, and for vertices and boxes starting with 2-loops:



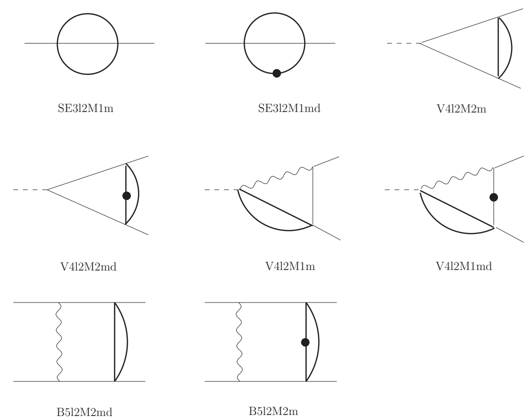
The $n_f = 2$ contributions have been determined in 2007

- Self-energies are not a two-masses-problem
- 2-vertices are known (for $m_e^2 = m_f^2$ and $m_e^2 << m_f^2$): G. Burgers PLB 164 (1885), Kniehl, Krawczyk, Kühn, Stuart PLB 209 (1988)
- What is really new: the 2-boxes with two different fermions involved

Box-master integrals: Actis, Czakon, Gluza, TR (ACGR), PRD 71 (2005)



$\longrightarrow$



- $m_e^2 << m_f^2 << s, t$: Becher,Melnikov JHEP 6 (2007) and ACGR NPB 786 (2007)
- $m_e^2 << m_f^2, s, t$: ACGR 0710.5111 $->$ APP B38 (2007)
  and Bonciani,Ferroglia,Penin 0710.4775 (2007)
- $m_e^2 << m_{hadrons}^2, s, t$: ACGR 0711.3847 $->$ PRL 100 (2008)
  and Kuehn et al. 0807.1284 (2008)

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How to evaluate the $N_f = 2$ diagrams?

We did it in 2 ways

- Decompose the 2-loop integrals to master integrals, solve them.
  Here: In the limit $m_e^2 << m_f^2 << s, t, u$
  This was done in hep-ph/07042400v2 $\longrightarrow$ ACGR, NPB 786 (2007)
- Alternatively, rewrite the 2-loop integrals as dispersion integrals.
  Decompose the loop integrals afterwards into master integrals
  The master integrals are simpler, of one-loop type, but the numerical dispersion integration remains then.

Advantages of the dispersion integrals:

- get easily the range $m_e^2 << m_f^2, s, t, u$
- method applies also to hadronic insertions

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Dispersion Integrals

$$\frac{g_{\mu\nu}}{q^2 + i\,\delta} \to \frac{g_{\mu\alpha}}{q^2 + i\,\delta} \left(q^2\, g^{\alpha\beta} - q^\alpha\, q^\beta\right) \Pi_{\rm had}(q^2)\, \frac{g_{\beta\nu}}{q^2 + i\,\delta},$$

the once-subtracted dispersion integral

$$\Pi_{\rm had}(q^2) = -\frac{q^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{dz}{z} \frac{\textbf{Im}\,\Pi_{\rm had}(z)}{q^2 - z + i\,\delta}.$$

Finally, one relates Im $\Pi_{\rm had}$ **to the hadronic cross-section ratio** $R_{\rm had}$**,**

$$\textbf{Im}\Pi_{\rm had}(z) = -\frac{\alpha}{3} R_{\rm had}(z) = -\frac{\alpha}{3} \frac{\sigma_{e^+e^-\to {\rm hadrons}}(z)}{(4\pi\alpha^2)/(3z)},$$

For heavy fermion insertions, we have instead of $R_{\rm had}(z)$**:**

$$R_f(z) = Q_f^2 C_f (1 + 2m_f^2/z)\sqrt{1 - 4m_f^2/z},$$

Replacing the $\Pi_{\rm had}(q^2)$ **in a vertex or in box diagram by the** z**-dispersion integral and exchanging the** $\int d^4k$ **with the** $\int dz$ **creates one-loop diagrams with a subsequent** z**-integration.**

.

The kernel functions for the dispersion integrals

$$\Delta\alpha(x) = \Delta\alpha^{(5)}_{\rm had}(x) + \Pi_e(x) + \sum_{f=\mu,\tau,t} \Pi_f(x)$$

$$\Delta\alpha^{(5)}_{\rm had}(x) = \frac{\alpha}{\pi}\,\frac{x}{3}\int_{4m_\pi^2}^{\infty} dz\,\frac{R^{(5)}_{\rm had}(z)}{z}\,\frac{1}{x-z+i\,\delta}$$

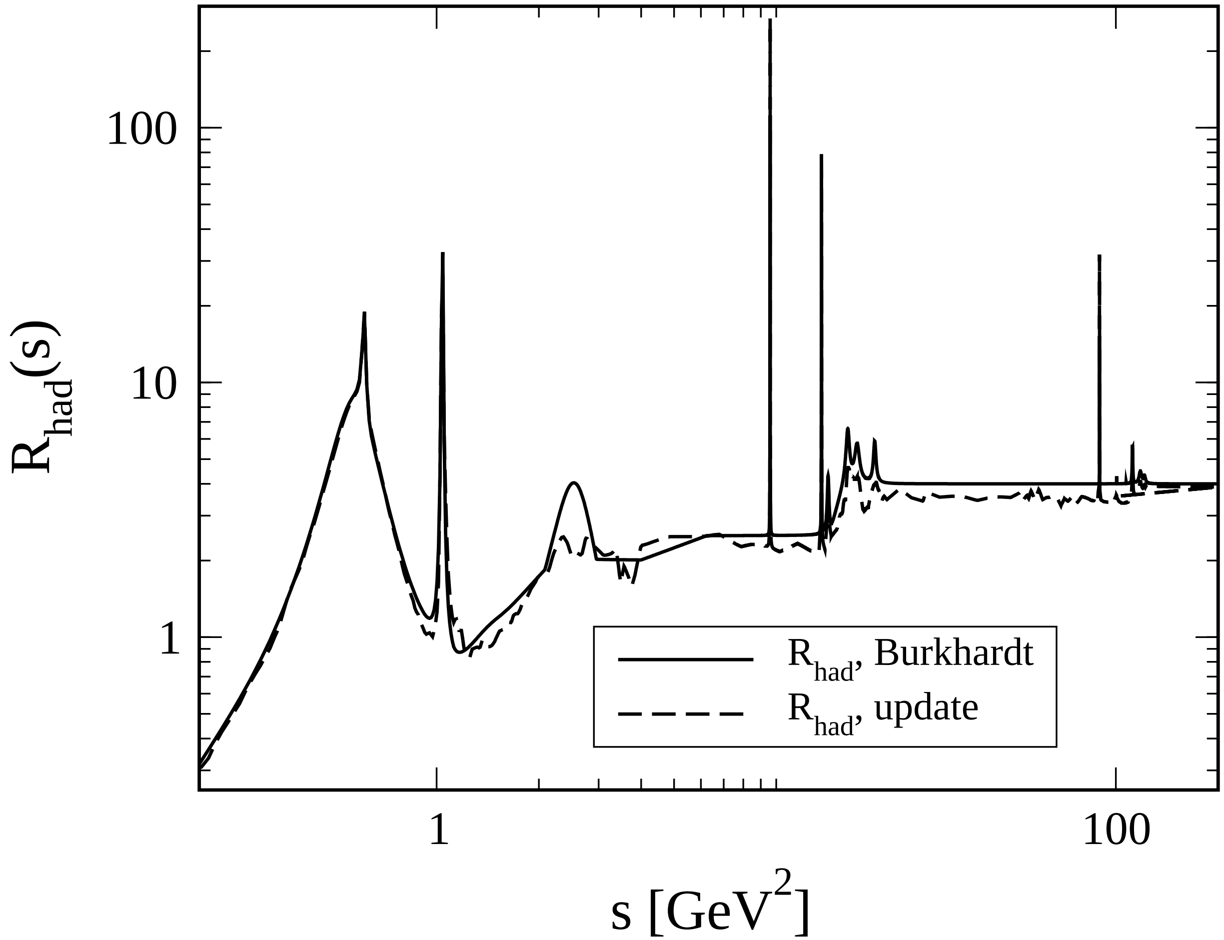
$$V_2(x) = V_{2e}(x) + V_{2\rm rest}(x)$$

$$V_{2\rm rest}(x) = \int_{4M^2}^{\infty} dz\,\frac{R(z)}{z}\,K_V(x+i\delta;z)$$

$$K_V(x;z) = \frac{1}{3}\left\{-\frac{7}{8} - \frac{z}{2\,x} + \left(\frac{3}{4}+\frac{z}{2\,x}\right)\ln\left(-\frac{x}{z}\right) - \frac{1}{2}\left(1+\frac{z}{x}\right)^2\left[\zeta_2 - \mathbf{Li}_2\left(1+\frac{x}{z}\right)\right]\right\}$$

$$B_i(x,y) = \int_{4M^2}^{\infty} dz\,\frac{R(z)}{z}\,K_{box,i}(x+i\delta, y+i\delta; z)$$

The $K_{box,i}(x,y;z)$ are determined as linear combinations of one-loop integrals with mass $z = M^2$.



A comparison of the parametrizations from [Burkhardt:1981jk]

and [rintpl:2008AA]

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Final formula and results

We distinguish 3 different categories of 2-loop contributions:

- **Running** α
- **the irreducible 2-loop vertices**
- **the 'rest': irreducible vertices and boxes plus 2-loop boxes**

$$
\begin{aligned}
\frac{d\overline{\sigma}}{d\Omega} &= c \int_{4M_\pi^2}^{\infty} dz \frac{R_{\rm had}(z)}{z} \frac{1}{t-z} F_1(z) \qquad (1) \\
&+ \quad c \int_{4M_\pi^2}^{\infty} \frac{dz}{z\,(s-z)} \Big\{ R_{\rm had}(z) \Big[F_2(z) + F_3(z) \ln\Big|1-\frac{z}{s}\Big| \Big] \\
&- \quad R_{\rm h}(s) \Big[F_2(s) + F_3(s) \ln\Big|1-\frac{z}{s}\Big| \Big] \Big\} \\
&+ \quad c\, \frac{R_{\rm h}(s)}{s} \Big\{ F_2(s) \ln\Big(\frac{s}{4M_\pi^2}-1\Big) - 6\zeta_2 F_a(s) \\
&+ \quad F_3(s) \Big[2\zeta_2 + \frac{1}{2}\ln^2\Big(\frac{s}{4M_\pi^2}-1\Big) + \mathbf{Li}_2\Big(1-\frac{s}{4M_\pi^2}\Big) \Big] \Big\},
\end{aligned}
$$

with $c = \alpha^4/(\pi^2 s)$ **and** $R_{\rm h}(s) = \theta(s-4M_\pi^2)\ R_{\rm had}(s)$**.**

$$
\begin{aligned}
F_1(z) \quad = \quad & \frac{1}{3}\Big\{ 9\,\bar{c}(s,t)\ln\Big(\frac{s}{m_e^2}\Big) + \Big[-z^2\Big(\frac{1}{s}+\frac{2}{t}+2\,\frac{s}{t^2}\Big) + z\,\Big(4+4\,\frac{s}{t}+2\,\frac{t}{s}\Big) + \frac{1}{2}\,\frac{t^2}{s} + 6\,\frac{s^2}{t} \\
+ \quad & 5\,s+4\,t\Big]\,\ln\Big(-\frac{t}{s}\Big) + s\,\Big(-\frac{z}{t}+\frac{3}{2}\Big)\,\ln\Big(1+\frac{t}{s}\Big) + \Big[\frac{1}{2}\,\frac{z^2}{s} + 2\,z\,\Big(1+\frac{s}{t}\Big) - \frac{11}{4}\,s - 2\,t\Big]\,\ln^2\Big(-\frac{t}{s}\Big) \\
- \quad & \Big[\frac{1}{2}\,\frac{z^2}{t} - z\,\Big(1+\frac{s}{t}\Big) + \frac{t^2}{s} + 2\,\frac{s^2}{t} + \frac{9}{2}\,s + \frac{15}{4}\,t\Big]\,\ln^2\Big(1+\frac{t}{s}\Big) + \Big[\frac{z^2}{t} - 2\,z\,\Big(1+\frac{s}{t}\Big) + 2\,\frac{s^2}{t} + 5\,s + \frac{5}{2}\,t\Big] \\
\times \quad & \ln\Big(-\frac{t}{s}\Big)\,\ln\Big(1+\frac{t}{s}\Big) - 4\,\Big[\frac{t^2}{s} + 2\,\frac{s^2}{t} + 3\,\Big(s+t\Big)\Big]\,\Big[1+\mathrm{Li}_2\Big(-\frac{t}{s}\Big)\Big] - \Big[\frac{t^2}{s} + 2\,\frac{s^2}{t} + 3\,\Big(s+t\Big)\Big]\,\ln\Big(\frac{z}{s}\Big)\,\ln\Big(1+ \\
- \quad & \Big[2\,\frac{z^2}{t} - 4\,z\,\Big(1+\frac{s}{t}\Big) - 4\,\frac{t^2}{s} - 2\,\frac{s^2}{t} + s - \frac{11}{2}\,t\Big]\,\zeta_2 + \Big[z^2\,\Big(\frac{1}{s} + 2\,\frac{s}{t^2} + \frac{2}{t}\Big) - z\,\Big(\frac{t}{s} + 2\frac{s}{t} + 2\Big)\Big]\,\ln\Big(\frac{z}{s}\Big) \\
- \quad & \Big[z^2\,\Big(\frac{1}{s} + \frac{1}{t}\Big) + 2\,z\,\Big(1+\frac{s}{t}\Big) + s + 2\,\frac{s^2}{t}\Big]\,\ln\Big(\frac{z}{s}\Big)\,\ln\Big(1+\frac{z}{s}\Big) + \Big[\frac{z^2}{s} + 4\,z\,\Big(1+\frac{s}{t}\Big) - \frac{t^2}{s} - 4\,\Big(s+t\Big)\Big] \\
\times \quad & \ln\Big(\frac{z}{s}\Big)\ln\Big(1-\frac{z}{t}\Big) - \Big[z^2\,\Big(\frac{1}{s} + 2\frac{s}{t^2} + \frac{2}{t}\Big) - 2\,z\,\Big(\frac{t}{s} + 2\,\frac{s}{t} + 2\Big) + \frac{t^2}{s} + 2\,\Big(s+t\Big)\Big]\,\ln\Big(1-\frac{z}{t}\Big) \\
+ \quad & \Big[\frac{z^2}{t} - 2\,z\,\Big(1+\frac{s}{t}\Big) + 2\,\frac{t^2}{s} + 8\,s + 4\,\frac{s^2}{t} + 7\,t\Big]\,\ln\Big(1-\frac{z}{t}\Big)\,\ln\Big(1+\frac{t}{s}\Big) + \Big[\frac{z^2}{s} + 4\,z\,\Big(1+\frac{s}{t}\Big) - \frac{t^2}{s} - 4\,\Big(s+t\Big) \\
\times \quad & \mathrm{Li}_2\,\Big(\frac{z}{t}\Big) - \Big[z^2\,\Big(\frac{1}{s}+\frac{1}{t}\Big) + 2\,z\,\Big(1+\frac{s}{t}\Big) + s + 2\,\frac{s^2}{t}\Big]\,\mathrm{Li}_2\,\Big(-\frac{z}{s}\Big) - \Big[\,\frac{z^2}{t} - 2\,z\,\Big(1+\frac{s}{t}\Big) + \frac{t^2}{s} + 5\,s + 2\,\frac{s^2}{t} + 4\,t \\
\times \quad & \mathrm{Li}_2\,\Big(1+\frac{z}{u}\Big)\Big\} + 4\,\bar{c}(s,t)\ln\Big(\frac{2\,\omega}{\sqrt{s}}\Big)\,\Big[\ln\Big(\frac{s}{m_e^2}\Big) + \ln\Big(-\frac{t}{s}\Big) - \ln\Big(1+\frac{t}{s}\Big) - 1\Big],
\end{aligned}
$$

and similarly for $F_2(z)$ and $F_3(z)$.

The $\int_{4M^2} dz F_i(z)$ gives from the lower integration bound the logarithmically enhanced terms $\ln(= M^2)^n$, e.g. from terms like $A(z)\ \ln(z/s)$ or from $B(z)\mathrm{Li}_2\Big(\frac{z}{s}\Big)$.

Some numerical results

We will now discuss the numerical net effects arising from the $N_f = 2$ vertex plus box diagrams (i.e. excluding the pure running coupling effects):

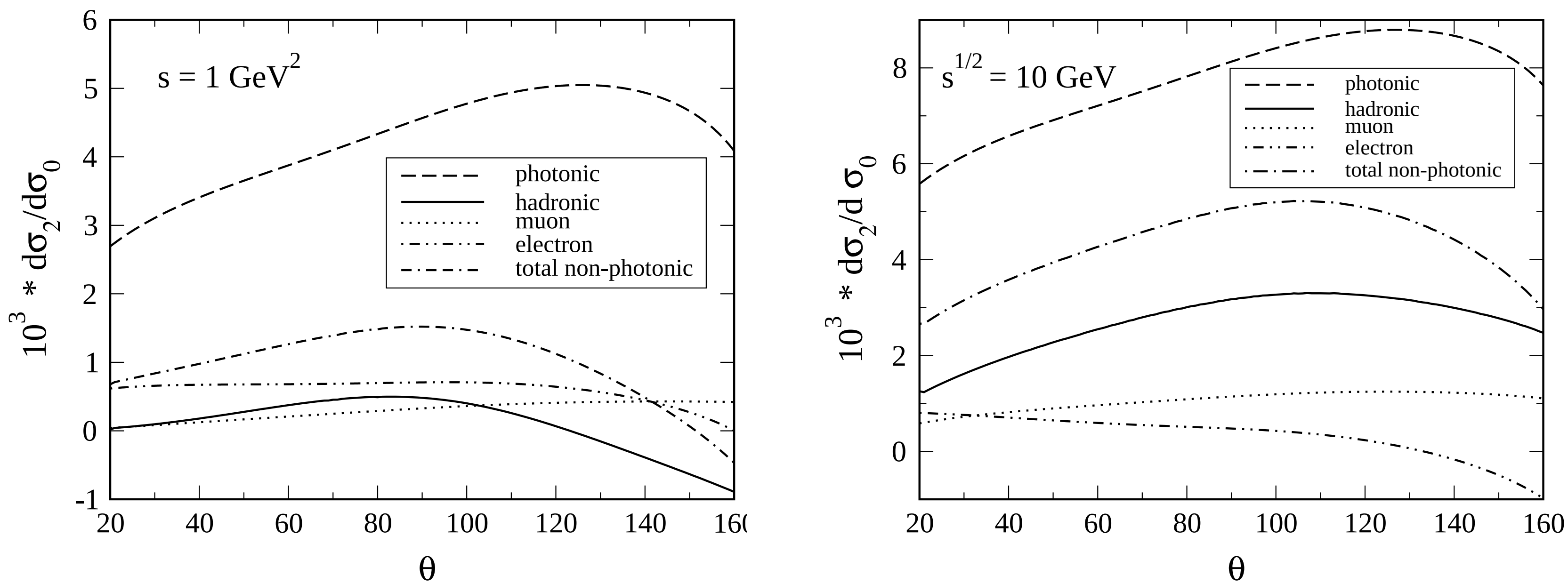
$$\frac{d\sigma_2}{d\Omega} = \frac{d\overline{\sigma}}{d\Omega} + \frac{d\sigma_v}{d\Omega},$$

with $d\overline{\sigma}/d\Omega$ from Eqn. (1). The expression for the irreducible vertex term $d\sigma_v/d\Omega$ derives directly from

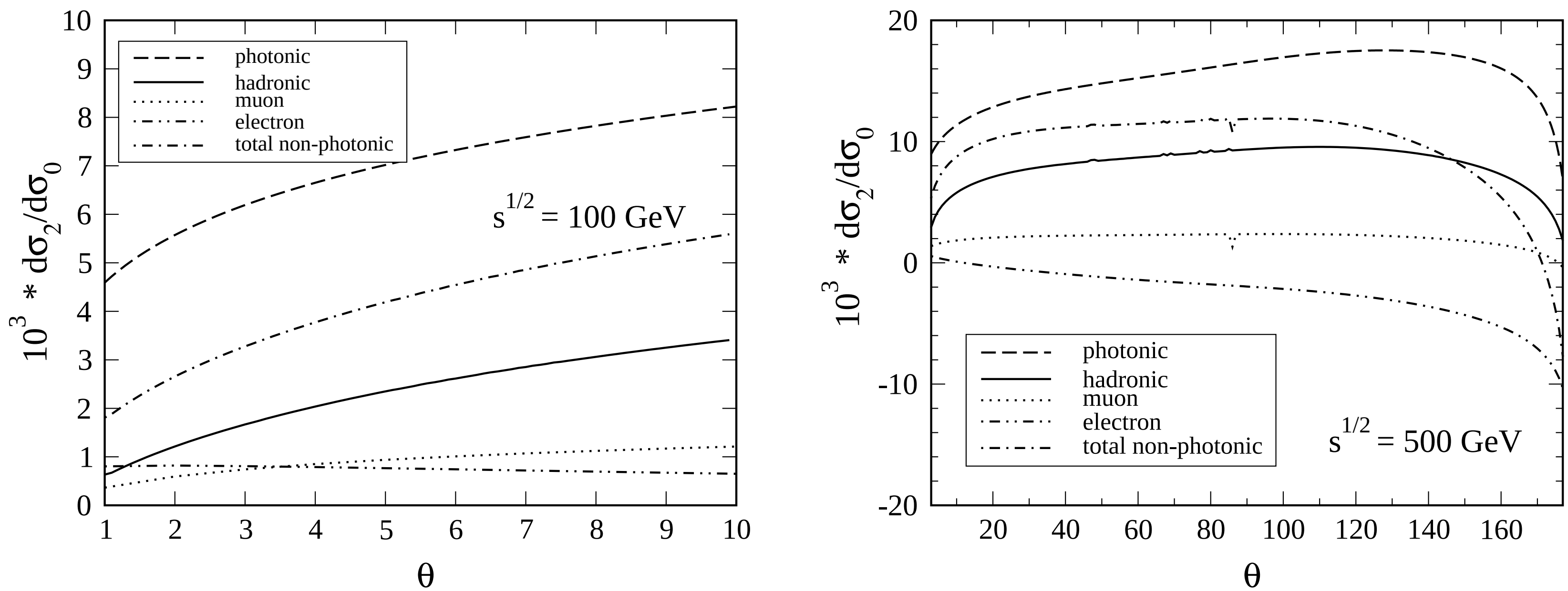
[Kniehl:1988id,webPage:2007x3]

. The $d\sigma_2/d\Omega$ is normalized to the pure photonic Bhabha Born cross section $d\sigma_0/d\Omega$:

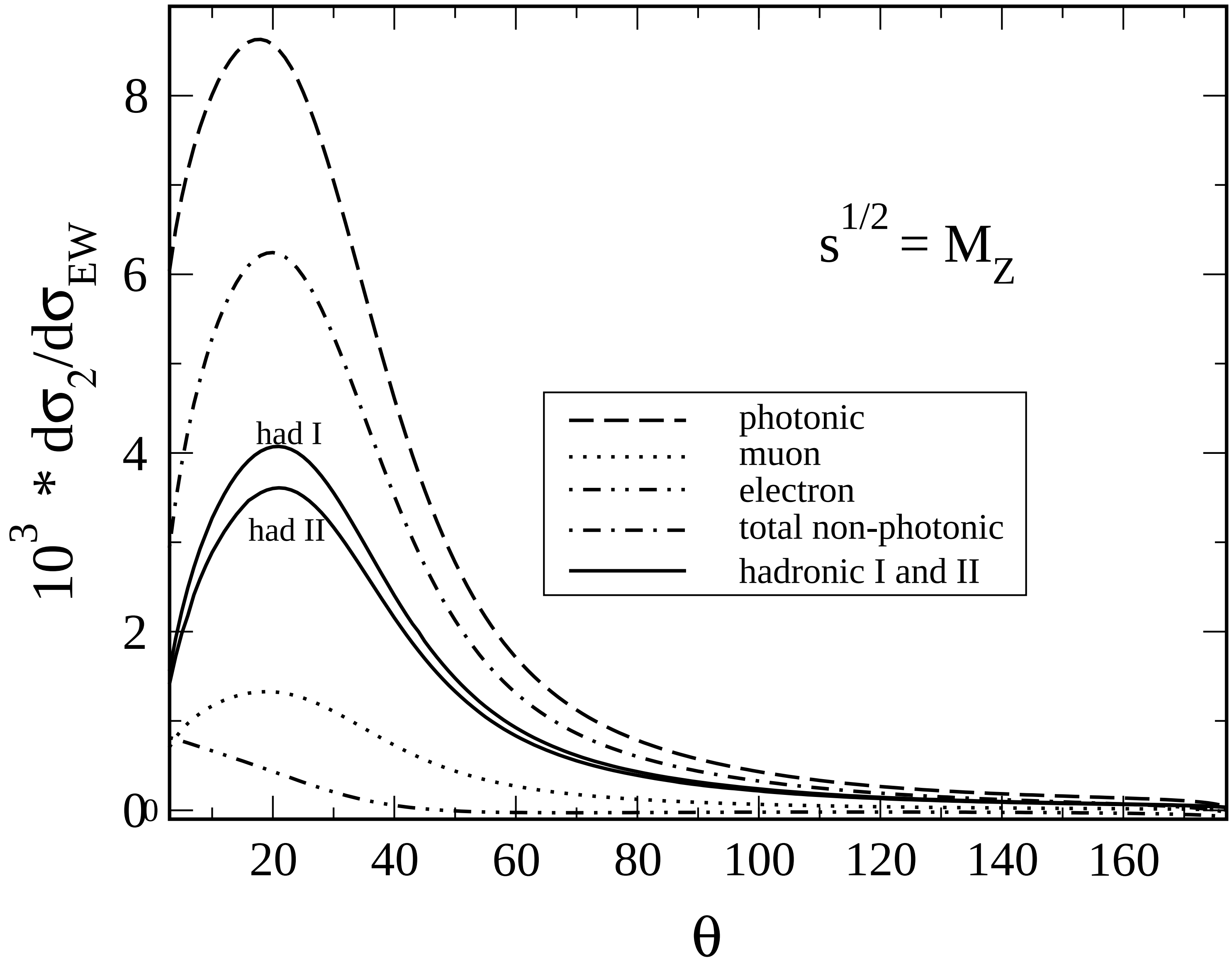
$$\frac{d\sigma_0}{d\Omega} = \frac{\alpha^2}{s}\left(\frac{s}{t} + 1 + \frac{t}{s}\right)^2.$$



Two-loop vertex and box corrections $d\sigma_2$ to Bhabha scattering in units of $10^{-3} d\sigma_0$ at meson factories, $\sqrt{s} = 1$ GeV (a) and $\sqrt{s} = 10$ GeV (b).



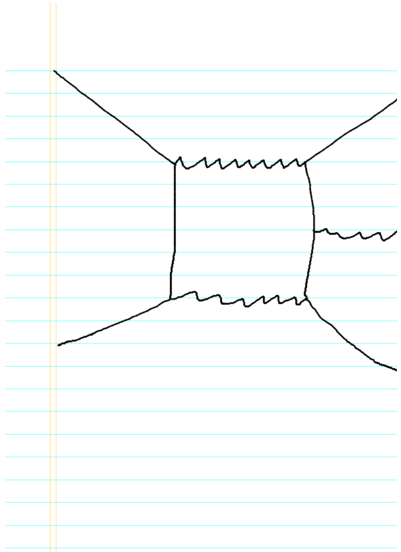
Two-loop vertex and box corrections $d\sigma_2$ to Bhabha scattering in units of $10^{-3} d\sigma_0$ at ILC energies of $\sqrt{s} = 100$ GeV (GigaZ option) and $\sqrt{s} = 500$ GeV.



Two-loop corrections to Bhabha scattering at $\sqrt{s} = M_Z$, normalized to the effective weak Born cross section.

Radiative loop corrections

Czyz, Gluza, Kajda, Sabonis, T.R.



Among the non-leading NNLO corrections are the so-called radiative loop corrections, interfering with lowest order bremsstrahlung.

The main problems arise from the pentagon diagrams.

Tools for tensor reduction of 5-point functions to scalar boxes, vertices, self-energies:

Czakon, Kajda, Gluza, Riemann, ambre.m, hexagon.m, MB.m

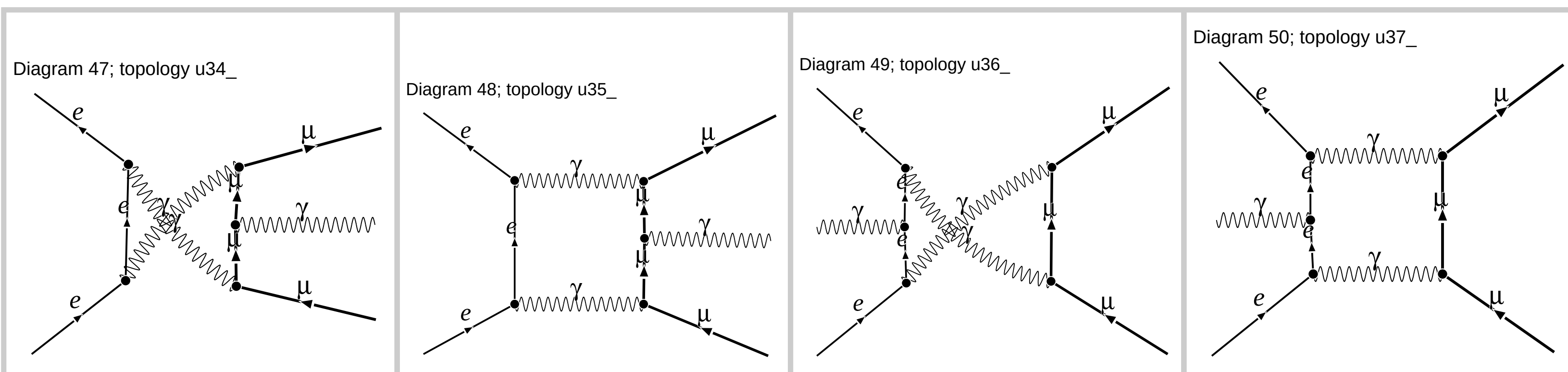
Status: We aim at automatic Fortran code generation for phase space integrations

- with DIANA, Fleischer/Tentyukov – creation of all diagrams
- with hexagon.m and LoopTools/FF, Hahn/vanOldenborgh and FORM, Vermaseren and Mathematica – treatment of the tensor loop integrals, and evaluation of the matrix elements with trace and helicity methods
- with PHOKHARA (H. Czyz et al.) – Monte Carlo phase space integration foreseen

We look first at the reaction

$$e^+e^- \to \mu^+\mu^-\gamma$$

with a resolved photon.
This has nothing to do with Bhabha scattering, but is a part of the Bhabha contributions and of physical interest by itself.



Four 5-point diagrams obtained using DIANA, Fleischer/Tentyukov.

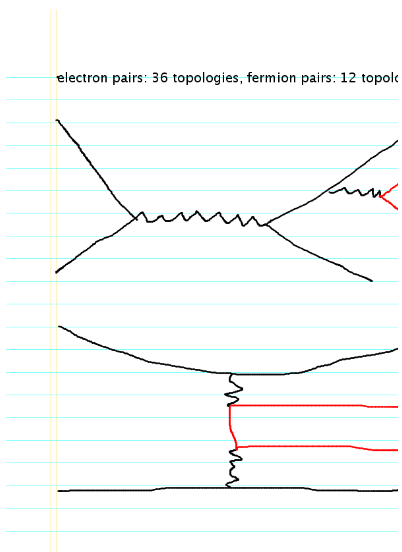
Pair corrections

M. Czakon, J. Gluza, T.R., M. Worek

Thanks to M. Worek's engagement, there are first results for event generation of Bhabha scattering with additional unresolved electron or muon pairs at $\sqrt{s} = 1.02, 10, 91$ GeV.

No cuts on the unresolved particles, but acceptance cuts on electron energy E_{min}, production angles $\theta_{\pm}$, acollinearity $\xi_{\max}$.

All particles are massive and observed, so there are no true singularities.



- **At low energies, logarithms are not enhanced at all**
- **There are diagrams with quite different kinematics**
- **then, realistic cuts play a crucial role**
- $\longrightarrow$ **use HELAC-PHEGAS, Kanaki/Papadopulos/Worek/Cafarella webpage** `http://helac-phegas.web.cern.ch/helac-phegas/`

Summary

- We know now the photonic, the $N_f = 1$, and the $N_f = 2$ contributions to 2-loop Bhabha scattering, including the hadronic corrections
- They are small, but non-negligible at the scale 10^{-3} ($\rightarrow$ No LEP influencing)
- To be evaluated yet:
  $\longrightarrow$ 1-loop diagrams with real photon emission, interfering with real (Born) radiation, including 5-point functions (massless case: Arbuzov et al.)
- To be evaluated yet:
  $\longrightarrow$ Real heavy pair emission corrections
- Both items are under study
- The Monte-Carlo codes then have to be tuned, correspondingly
  Some of the bigger effects are already included (leading logarithms, factorizing terms)