Simulation studies for the MADMAX Axion direct detection experiment

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Universität Hamburg
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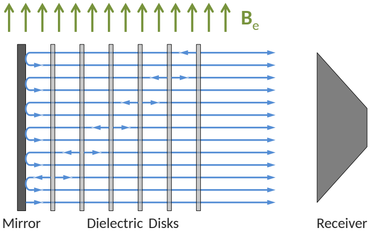
Particles, Strings, and the Early Universe
Collaborative Research Center SFB 676

Solving Axion-Maxwell equations for axion direct detection experiments

- Our software framework can solve the Maxwell-Axion equations for arbitrary geometries and materials

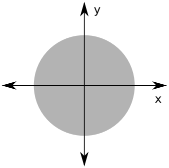
$$\nabla \times (\nabla \times \boldsymbol{E}^{(1)}) - \omega^2 \epsilon (1 + \frac{i\sigma}{\omega \epsilon}) \boldsymbol{E}^{(1)} = \omega \boldsymbol{B}^{(0)} a$$

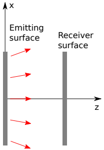
- Approach is based on the finite element method
- Applied to experimental setups related to the MADMAX experiment.



- MADMAX is a dielectric haloscope which utilizes the axion photon conversion at many dielectric discs [1] and can probe axion masses in the mass range $m_a = 40 - 400\ \mu\text{eV}$

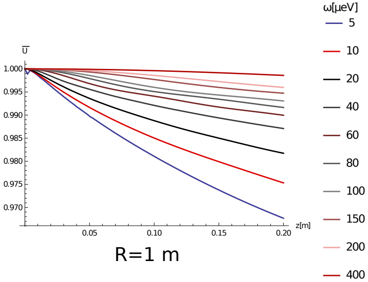
Circular conducting surface



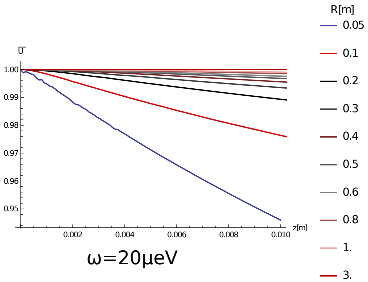


$$\boldsymbol{B}^{(0)} = \hat{\boldsymbol{e}}_y 10\mathrm{T}$$

$$\bar{U} := \frac{\text{received power on shifted circular surface with same radius}}{\text{emitted power}}$$



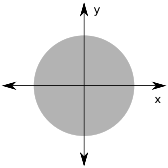
R=1 m

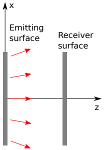


ω=20μeV

Losses increase for decreasing frequency and radius.

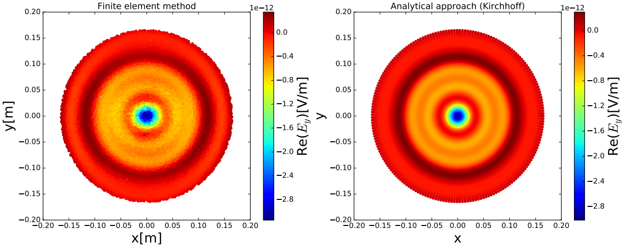
Circular conducting surface





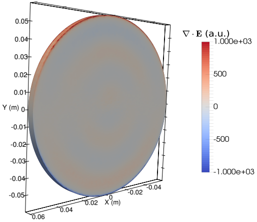
$$\boldsymbol{B}^{(0)} = \hat{\boldsymbol{e}}_y 10\text{T}$$

Kirchhoff diffraction theory

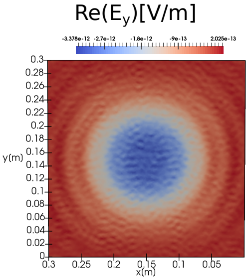


(cut at z=0.1 m away from emitting surface, $\lambda = 0.03$ m $\Leftrightarrow$ $m_a = 40$ µeV, Radius $= 0.06$ m)

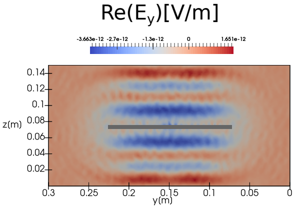
Circular dielectric disc



- $\boldsymbol{B}^{(0)} \sim 10T\hat{\boldsymbol{e}}_y$, $\lambda = 0.03$ m
- $\nabla \cdot \boldsymbol{E} = \rho_p$
- Diffraction/ interference effects
- Finite disc size leads to boundar charges which are absent in the case of infinite discs

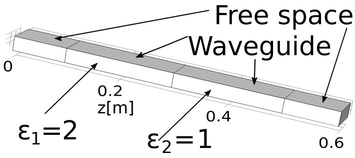


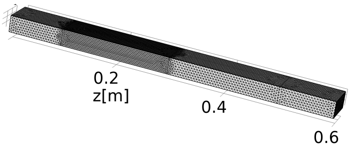
(cut at $z = 0.09$ m , Radius$= 0.075$ m)



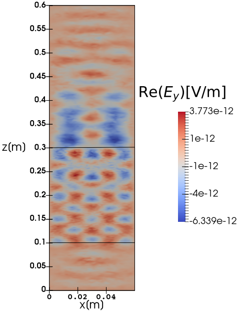
(cut at $y = 0.15$ m, Radius$= 0.075$ m)

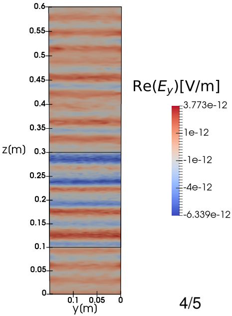
Waveguide with different dielectrics





$$\lambda = 0.03 \text{ m} , \quad \boldsymbol{B}^{(0)} = \hat{\boldsymbol{e}}_y 10T \sin(\frac{\pi(z-0.1)}{0.4})^2 \begin{cases} 1 & \text{inside waveguide} \\ 0 & \text{outside waveguide} \end{cases}$$





Summary and Outlook

- We can solve Axion-Maxwell equations for arbitrary geometries/materials
- Formalism applied to MADMAX related setups/ comparison to analytical calculations
- Next steps: Feasibility study for MADMAX and new experiments

Thank you for your attention!

Backup

Backup: Axion-Maxwell equations

Zero velocity ($a \sim e^{-i\omega t}$) limit and with linear media:

$$\nabla \cdot \epsilon \boldsymbol{E} = \rho_f \tag{1}$$

$$\nabla \times \mu^{-1} \boldsymbol{B} - \partial_t \epsilon \boldsymbol{E} = \boldsymbol{J}_f - i\omega g_{a\gamma} \boldsymbol{B} a \tag{2}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{3}$$

$$\nabla \times \boldsymbol{E} + \partial_t \boldsymbol{B} = 0 \tag{4}$$

$$(-\omega^2 + m_a^2) a = g_{a\gamma} \boldsymbol{E} \cdot \boldsymbol{B} \tag{5}$$

Perturbation Ansatz [2]

$$\boldsymbol{B}(\boldsymbol{x}, t) = \sum_{i=0}^{\infty} g_{a\gamma}^i \omega^i \boldsymbol{B}^{(i)}(\boldsymbol{x}, t) \tag{6}$$

$$\boldsymbol{E}(\boldsymbol{x}, t) = \sum_{i=0}^{\infty} g_{a\gamma}^i \omega^i \boldsymbol{E}^{(i)}(\boldsymbol{x}, t) \tag{7}$$

Similar expansion for $\boldsymbol{J}_f(\boldsymbol{x}, t)$ and $\rho_f(\boldsymbol{x}, t)$.

Axion-Maxwell equations

$\boldsymbol{J}_f^{(0)}(\boldsymbol{x})$ generates external B-field. $\rho_f^{(0)} = 0$.

Zeroth order is equivalent to solving the problem without axions.

First order:

$$\nabla \cdot \epsilon \boldsymbol{E}^{(1)} = \rho_f^{(1)} \tag{8}$$

$$\nabla \times \mu^{-1} \boldsymbol{B}^{(1)} - \partial_t \epsilon \boldsymbol{E}^{(1)} = \boldsymbol{J}_f^{(1)} + \sigma \boldsymbol{E}^{(1)} - i \boldsymbol{B}^{(0)} a \tag{9}$$

$$\nabla \cdot \boldsymbol{B}^{(1)} = 0 \tag{10}$$

$$\nabla \times \boldsymbol{E}^{(1)} + \partial_t \boldsymbol{B}^{(1)} = 0 \tag{11}$$

$$(-\omega^2 + m_a^2) a = g_{a\gamma} \boldsymbol{E}^{(0)} \cdot \boldsymbol{B}^{(0)} \tag{12}$$

$\boldsymbol{J}_f^{(1)} = 0 = \rho_f^{(1)}$ because conductors only at boundary of simulation domain.

We solve for the first order E-field:

$$\nabla \times (\nabla \times \boldsymbol{E}^{(1)}) - \omega^2 \epsilon (1 + \frac{i\sigma}{\omega\epsilon}) \boldsymbol{E}^{(1)} = \omega \boldsymbol{B}^{(0)} a \tag{13}$$

References

A.J. Millar, G. G. Raffelt, J. Redondo, F. Steffen, *Dielectric Haloscopes to Search for Axion Dark Matter: Theoretical Foundations*, JCAP **1701** (2017) 061

P. L. Hoang, J. Jeong, B. X. Cao, Y. Shin, B. Ko, Y. Semertzidis, *Theoretical models of axion electromagnetism for haloscope searches*, Physics of the Dark Universe, 2017