Axion predictions in Grand Unified Theories

Axion-WIMP workshop 2018
DESY Hamburg, 19/06/18 Anne Ernst

based on JHEP 02(2018)103, in collaboration with Andreas Ringwald and Carlos Tamarit

Motivation

* search for a **well-motivated model** solving fundamental **problems of the Standard Model**
* axion solves **Strong CP-problem** and is a good candidate for **Cold Dark Matter**
* can we use **GUT** to constrain the axion mass?
    * unification of gauge couplings
    * one gauge group instead of 3
* why **SO(10)**?
    * simplest SU(5) models: disfavoured
    * **neutrinos massive**: seesaw mechanism

Status

* $SO(10) \times U(1)_{PQ}$ models have been studied before [Lazarides, Kim, Bajc et al, Babu et al, Altarelli et al, ...]
* however, a few things were missing:
    * a systematic identification of axion field and decay constant in the presence of gauge symmetries
    * a systematic calculation of the couplings to other particles
    * a direct calculation of associated domain wall number
    * two-loop analysis of unification constraints including threshold corrections

GUT model building: non-SUSY SO(10)

$SO(10)$	$4_C 2_L 2_R$	$4_C 2_L 1_R$	$3_C 2_L 1_R 1_{B-L}$	$3_C 2_L 1_Y$	scale
16_F	$(4,2,1)$	$(4,2,0)$	$(3,2,0,\frac{1}{3})$	$(3,2,\frac{1}{6}) := Q$	M_Z
			$(1,2,0,-1)$	$(1,2,-\frac{1}{2}) := L$	M_Z
	$(\bar{4},1,2)$	$(\bar{4},1,\frac{1}{2})$	$(\bar{3},1,\frac{1}{2},-\frac{1}{3})$	$(\bar{3},1,\frac{1}{3}) := d$	M_Z
			$(1,1,\frac{1}{2},1)$	$(1,1,1) := e$	M_Z
		$(\bar{4},1,-\frac{1}{2})$	$(\bar{3},1,-\frac{1}{2},-\frac{1}{3})$	$(\bar{3},1,-\frac{2}{3}) := u$	M_Z
			$(1,1,-\frac{1}{2},1)$	$(1,1,0) := N$	$M_{\rm BL}$

- one generation of SM fermions + heavy right-handed neutrinos fits perfectly into one 16 representation of SO(10)

- most general Yukawa coupling:

$$\mathcal{L}_Y = 16_F \left(Y_{10} 10_H + Y_{120} 120_H + Y_{126} \overline{126}_H \right) 16_F + \text{h.c.}$$

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minimality

Role of PQ-symmetry in GUT model building

- complex 10_H representation necessary to reproduce realistic mass relations
- reduced **predictivity** in Yukawa sector

$$\mathcal{L}_Y = 16_F \left(Y_{10} 10_H + \tilde{Y}_{10} 10^*_H + Y_{126} \overline{126}_H \right) 16_F + \text{h.c.}$$

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* solution: impose global U(1) symmetry!

$$U(1)_{\rm PQ}: \quad 16_F \to 16_F e^{i\alpha}$$

$$10_H \to 10_H e^{-2i\alpha}$$

$$\overline{126}_H \to \overline{126}_H e^{-2i\alpha}$$

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* broken global symmetry → Goldstone boson!

Color-anomalous symmetry

→ solution to Strong CP -problem

Peccei-Quinn solution

* assume existence of anomalous global $U(1)_{\rm PQ}$ symmetry, spontaneously broken at a scale f_A
* Goldstone boson: $U(1)_{PQ}: \frac{A}{f_A} \to \frac{A}{f_A} + \epsilon \quad \frac{A}{f_A} \in [0, 2\pi)$
* $SU(3)_C - SU(3)_C - U(1)_{PQ}$ anomaly induces effective change in the Lagrangian :

$$\delta\mathcal{L} = -\frac{g^2}{32\pi^2}\frac{A}{f_A}G^{a\mu\nu}\tilde{G}^a_{\mu\nu}$$

* can rewrite CP violating term as

$$\mathcal{L}_{\rm eff} = -\frac{g^2}{32\pi^2}\underbrace{\left(\frac{A}{f_A} + \bar{\theta}\right)}_{\theta}G^{a\mu\nu}\tilde{G}^a_{\mu\nu}$$

Peccei-Quinn solution

* non-perturbative effects introduce potential for A !
* minimum at $\frac{A_0}{f_A} + \bar{\theta} = \theta = 0$

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Dynamical solution of strong CP problem !
Particle excitation of field A: the axion.

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Particle excitation of field A: the axion.

* Axion properties are described by axion decay constant f_A

$$m_A(T) f_A = \sqrt{\chi(T)}$$

Q: Can we use GUT to constrain f_A ?

Peccei-Quinn solution

* **non-perturbative effects** introduce potential for A !

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* Axion properties are described by **axion decay constant** f_A

$$m_A(T) f_A = \sqrt{\chi(T)}$$

Q: Can we use GUT to constrain f_A ?

A: It depends...

Higgs sector of SO(10): symmetry breaking chains

$SO(10)$	$4_C 2_L 2_R$	$4_C 2_L 1_R$	$3_C 2_L 1_R 1_{B-L}$	$3_C 2_L 1_Y$	$3_C 1_{\text{em}}$	scale
10_H	$(1,2,2)$	$(1,2,\frac{1}{2})$	$(1,2,\frac{1}{2},0)$	$(1,2,\frac{1}{2})$	$(1,0) =: H_d$	M_Z
		$(1,2,-\frac{1}{2})$	$(1,2,-\frac{1}{2},0)$	$(1,2,-\frac{1}{2})$	$(1,0) =: H_u$	M_Z
$\overline{126}_H$	$(10,1,3)$	$(10,1,1)$	$(1,1,1,-2)$	$(1,1,0) := \Delta_R$	$(1,0)$	M_{BL}
	$(15,2,2)$	$(15,2,\frac{1}{2})$	$(1,2,\frac{1}{2},0)$	$(1,2,\frac{1}{2})$	$(1,0) := \Sigma_d$	M_Z
		$(15,2,-\frac{1}{2})$	$(1,2,-\frac{1}{2},0)$	$(1,2,-\frac{1}{2})$	$(1,0) := \Sigma_u$	M_Z

* $\overline{126}_H$ cannot break SO(10) down to the Standard Model (as it leaves an SU(5) subgroup unbroken), we need at least one additional rep
* choosing 210_H, obtain the two-step symmetry breaking chain

$$SO(10) \xrightarrow{M_{\text{U}}-210_H} 4_C\, 2_L\, 2_R \xrightarrow{M_{\text{BL}}-\overline{126}_H} 3_C\, 2_L\, 1_Y \xrightarrow{M_Z-10_H} 3_C\, 1_{\text{em}}$$

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210_H	$(1,1,1) := \phi$	$(1,1,0)$	$(1,1,0,0)$	$(1,1,0)$	$(1,0)$	$M_{\rm U}$

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Physical PQ symmetry

$$U(1)_{\mathrm{PQ}}:$$

$$16_F \to 16_F e^{i\alpha}$$

$$10_H \to 10_H e^{-2i\alpha}$$

$$\overline{126}_H \to \overline{126}_H e^{-2i\alpha}$$

Physical PQ symmetry

* linear combination of gauge and global symmetries
    * axion is massless at the perturbative level
    * orthogonal to all gauge symmetries, in particular B-L
* lower decay constant!

$$U(1)_{\rm PQ}:$$
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$$10_H \to 10_H e^{-2i\alpha}$$
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$$f_A \sim M_Z$$

visible axion!

For an explicit construction of the physical axion, check out our paper!

Lifting the axion from the electroweak scale

	Model 1	Model 2	Model 3
extend PQ symmetry	X	X	X
extra scalar singlet			X
extra scalar multiplet		X	

M1: extended PQ symmetry

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M1: extended PQ symmetry

* minimal extension: include in the PQ symmetry
* axion construction similar as before, but now obtain

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M1: extended PQ symmetry

* minimal extension: include 210_H in the PQ symmetry
* axion construction similar as before, but now obtain

$$f_A \sim \frac{v_{\rm U}}{3} \sim \frac{\langle 210_H \rangle}{3}$$

fixed by the requirement of gauge coupling unification!

$U(1)_{\rm PQ}$:

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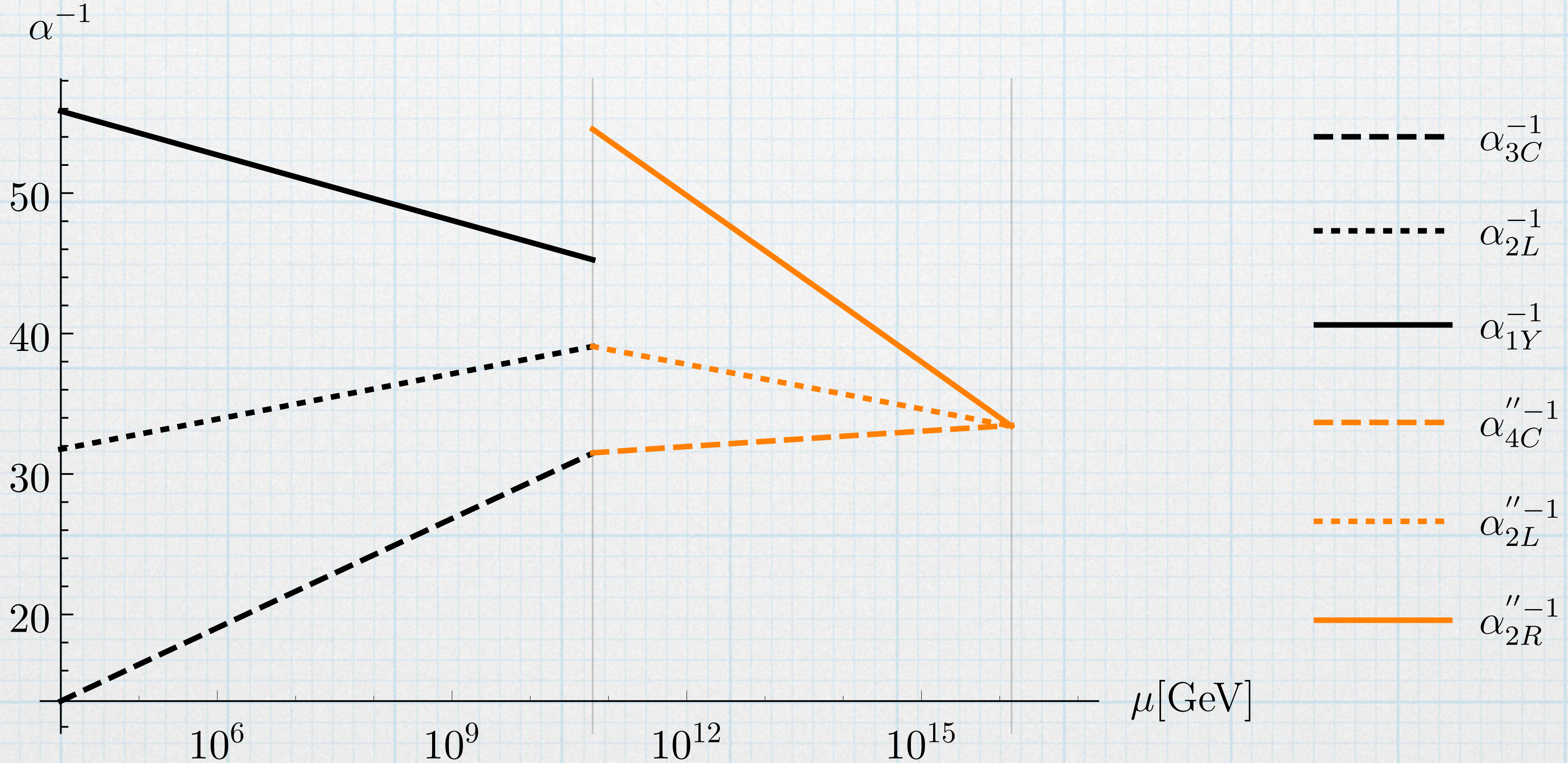
M1: RGE running

$$\frac{d\alpha_i^{-1}(\mu)}{d\ln\mu} = -\frac{a_i}{2\pi} - \sum_j \frac{b_{ij}}{8\pi^2\alpha_j^{-1}(\mu)}$$



(no threshold corrections)

M1: Matching conditions

* matching conditions depend on the group structure and the contained particles

* size of threshold corrections depends on the masses of heavy scalars (more specifically, the deviation from the threshold scale)

$$\alpha_{1Y}^{-1}(M_{\mathrm{BL}}) = \frac{3}{5}\alpha_{2R}^{''-1}(M_{\mathrm{BL}}) + \frac{2}{5}\alpha_{4C}^{''-1}(M_{\mathrm{BL}}) - \frac{\lambda_{1Y}}{12\pi}$$

$$\alpha_{2L}^{-1}(M_{\mathrm{BL}}) = \alpha_{2L}^{''-1}(M_{\mathrm{BL}}) - \frac{\lambda_{2L}}{12\pi}$$

$$\alpha_{3C}^{-1}(M_{\mathrm{BL}}) = \alpha_{4C}^{''-1}(M_{\mathrm{BL}}) - \frac{\lambda_{3C}}{12\pi}$$

$$\alpha_{2R}^{''-1}(M_U) = \alpha_G^{-1}(M_U) - \frac{\lambda_{2R}''}{12\pi}$$

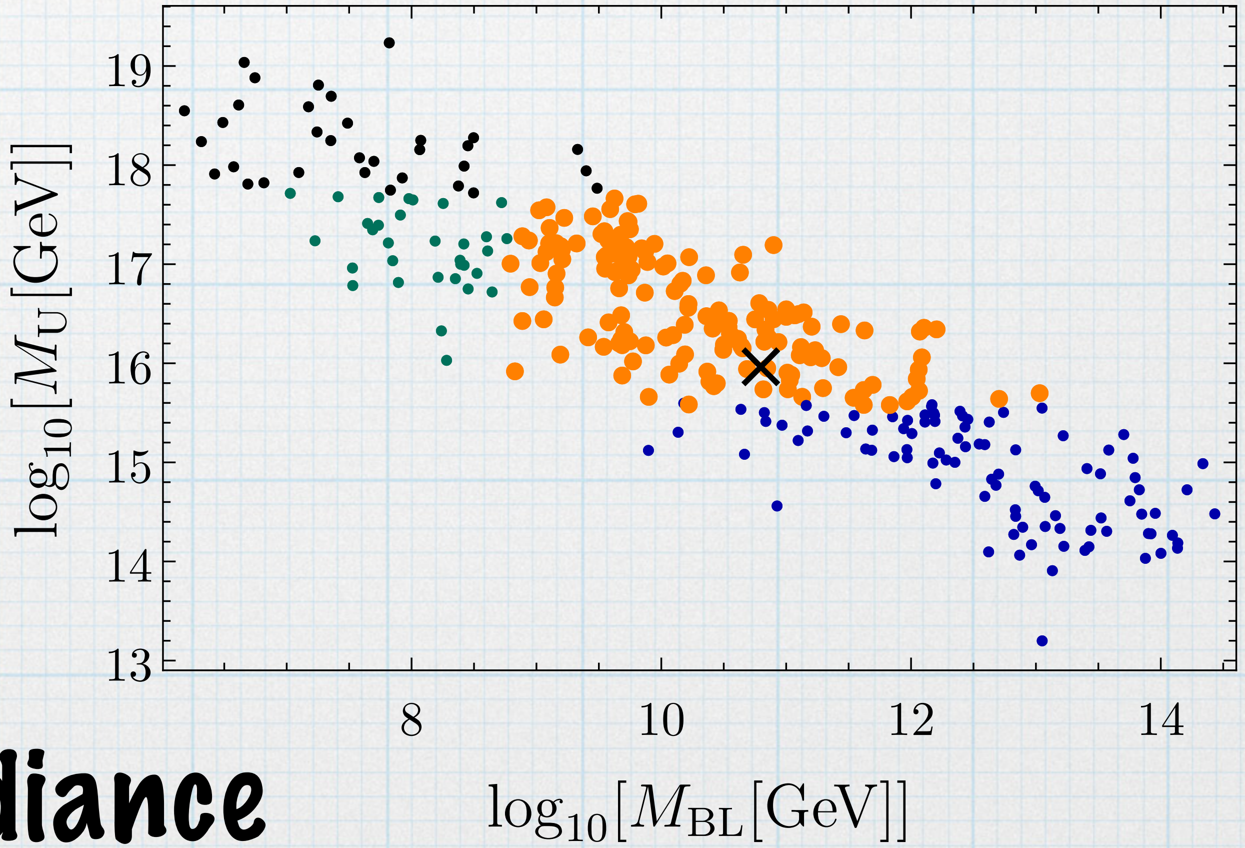
$$\alpha_{2L}^{''-1}(M_U) = \alpha_G^{-1}(M_U) - \frac{\lambda_{2L}''}{12\pi}$$

$$\alpha_{4C}^{''-1}(M_U) = \alpha_G^{-1}(M_U) - \frac{\lambda_{4C}''}{12\pi}$$

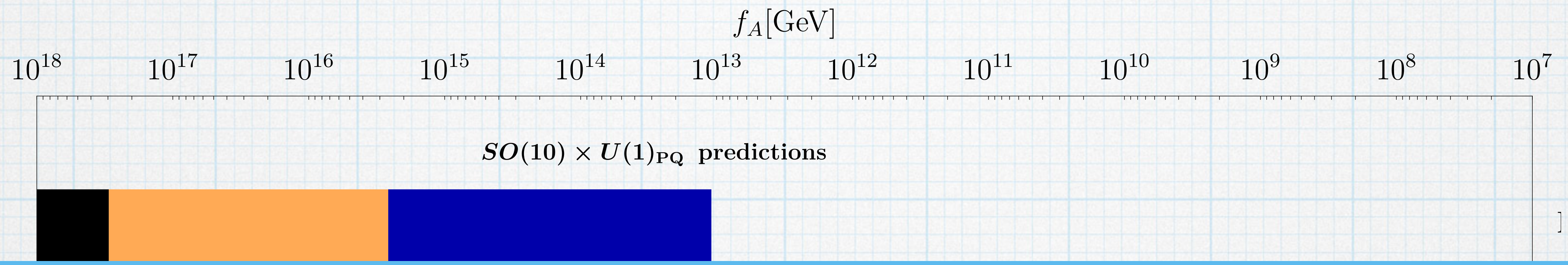
M1: gauge coupling unification

* in lack of detailed knowledge of the scalar sector, scalar masses have been randomized in the interval $[\frac{1}{10}M_T, 10M_T]$

* imposed limits:
  * proton stability
  * B-L scale
  * black hole superradiance



M1: predictions



- relatively sharp prediction of axion mass
- axion decay constant at the GUT scale

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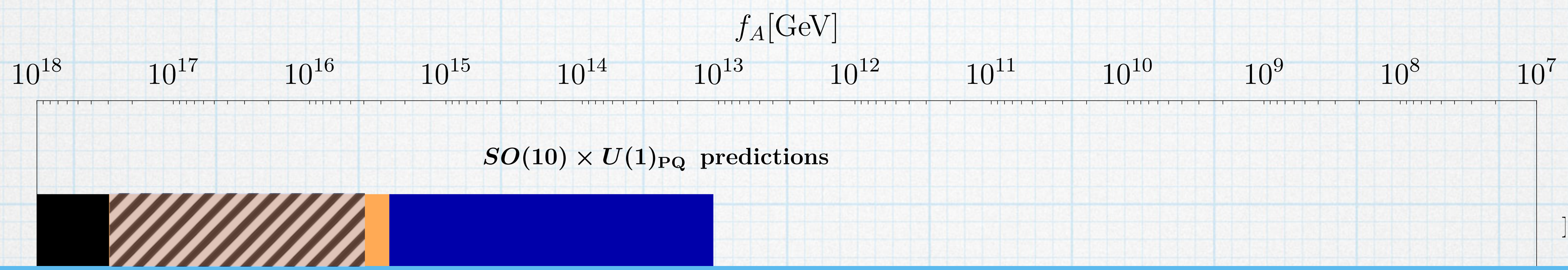

allowed region if HyperK discovers proton decay

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- axion decay constant at the GUT scale

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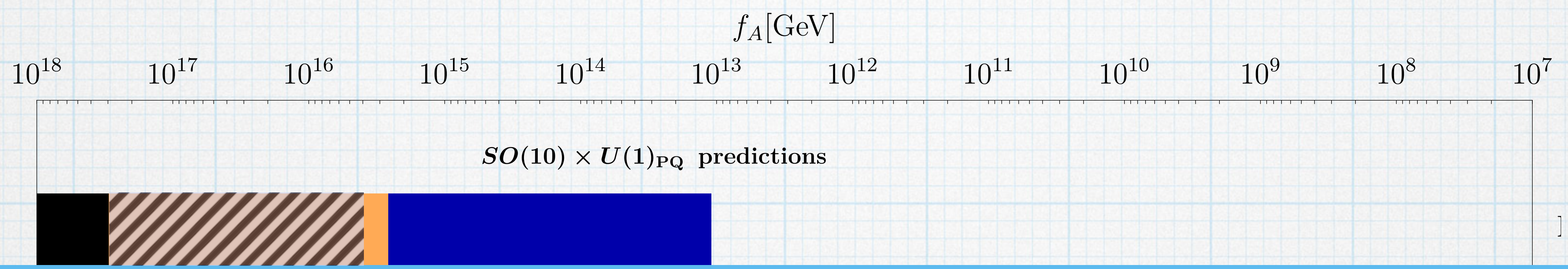

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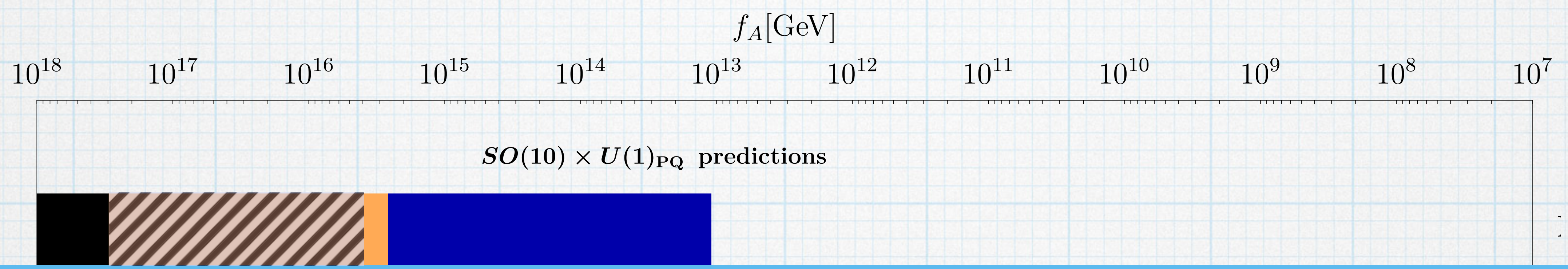
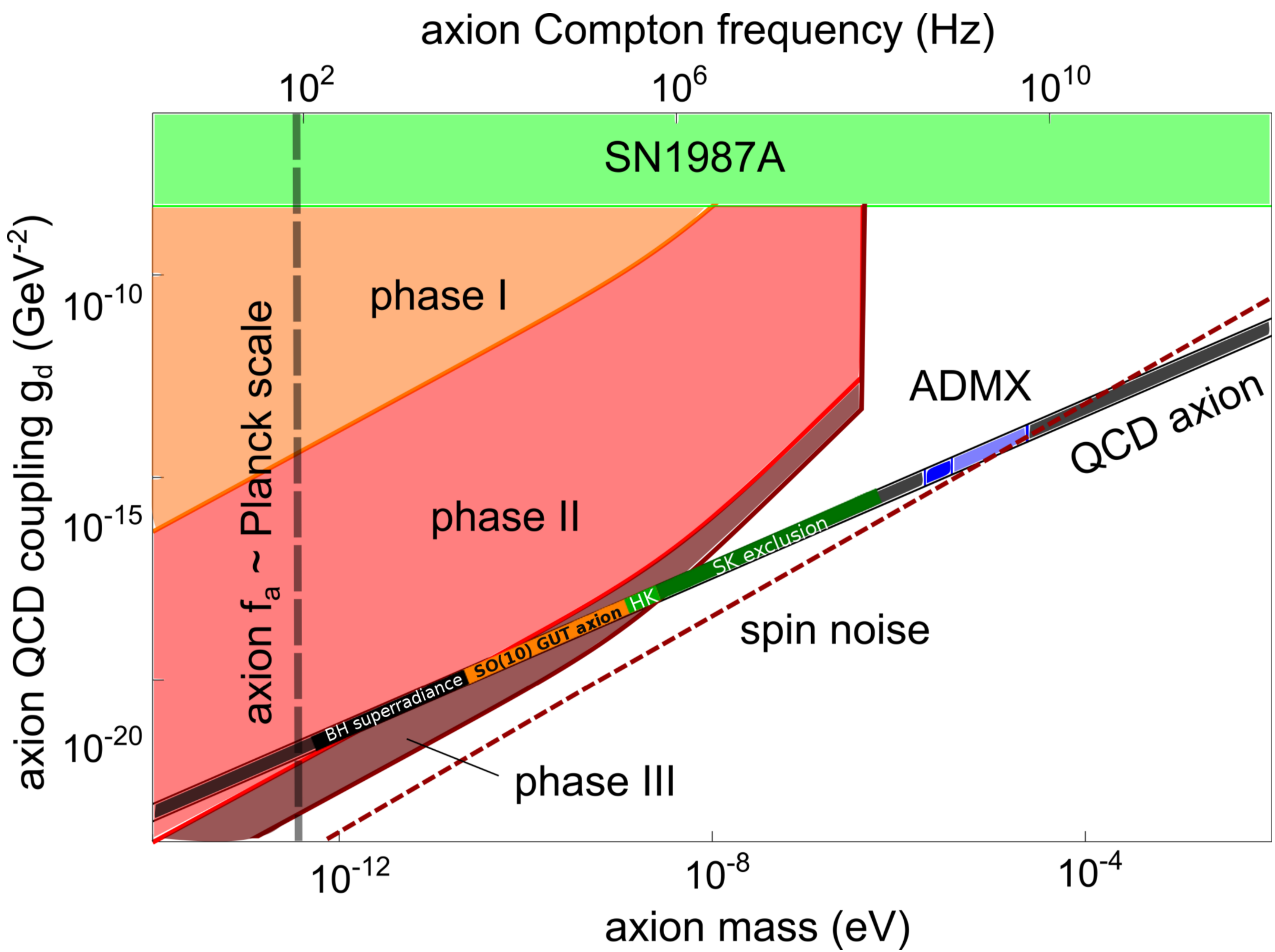


allowed region if HyperK excludes proton decay

- relatively sharp prediction of axion mass
- axion decay constant at the GUT scale

CASPEr electric

[arXiv: 1711.08999]



M2: additional multiplet

* include PQ charged multiplet 45_H
* axion decay constant

$$f_A = \frac{v_{\rm PQ}}{3} = \frac{\langle 45_H \rangle}{3}$$

$U(1)_{\rm PQ}$:

$$16_F \to 16_F e^{i\alpha}$$
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can we constrain this using gauge coupling unification?

M2: three-step symmetry breaking

$M_{\rm PQ} > M_{\rm BL}$:

$$SO(10) \xrightarrow{M_{\rm U}-210_H} 4_C\, 2_L\, 2_R \xrightarrow{M_{\rm PQ}-45_H} 4_C\, 2_L\, 1_R \xrightarrow{M_{\rm BL}-126_H} 3_C\, 2_L\, 1_Y \xrightarrow{M_Z-10_H} 3_C\, 1_{\rm em}$$

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45_H	$(1,1,3)$	$(1,1,0) := \sigma$	$(1,1,0,0)$	$(1,1,0)$	$(1,0)$	$M_{\rm PQ}$
$\overline{126}_H$	$(10,1,3)$	$(10,1,1)$	$(1,1,1,-2)$	$(1,1,0) := \Delta_R$	$(1,0)$	$M_{\rm BL}$
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M2: three-step symmetry breaking

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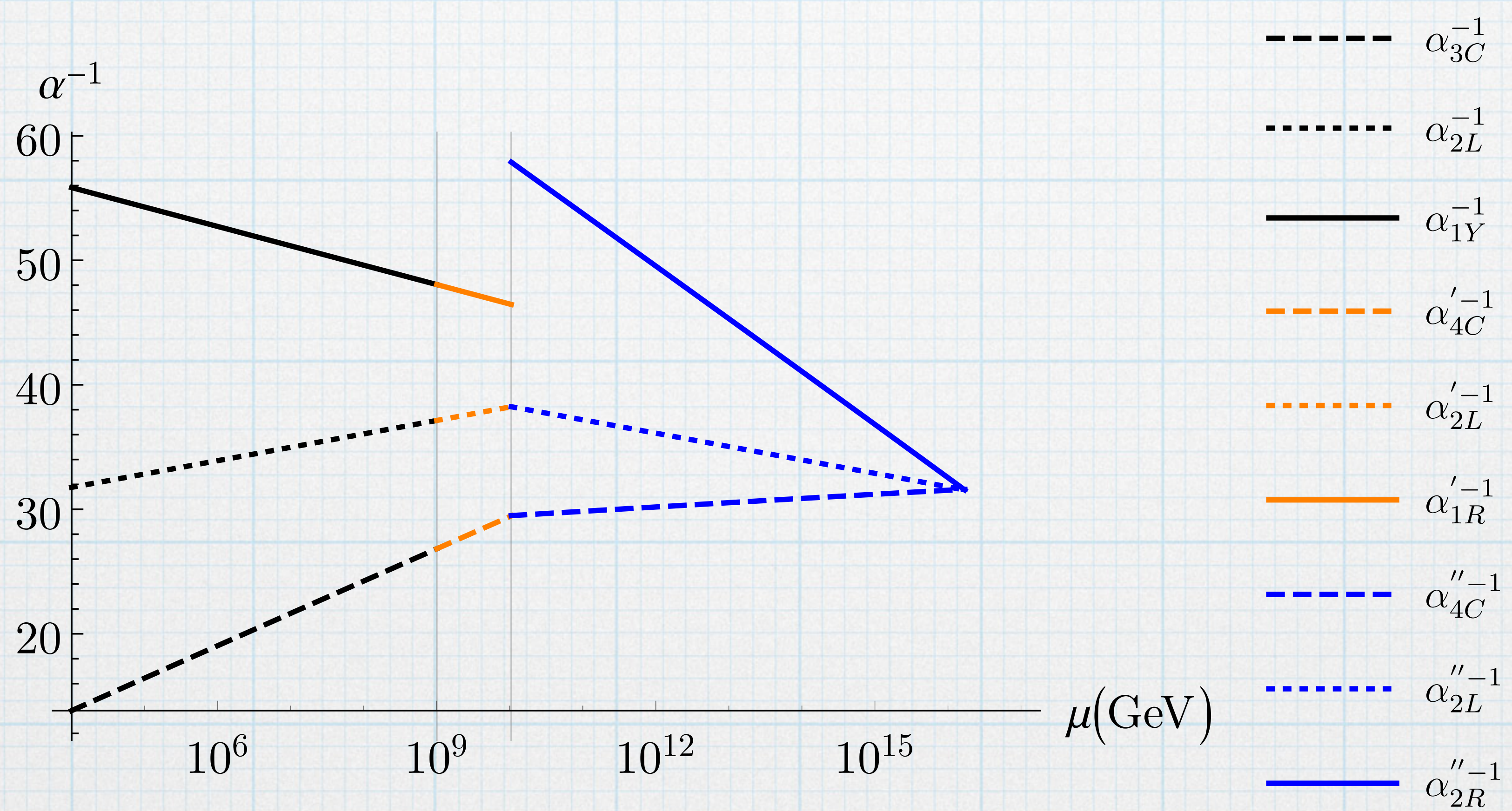
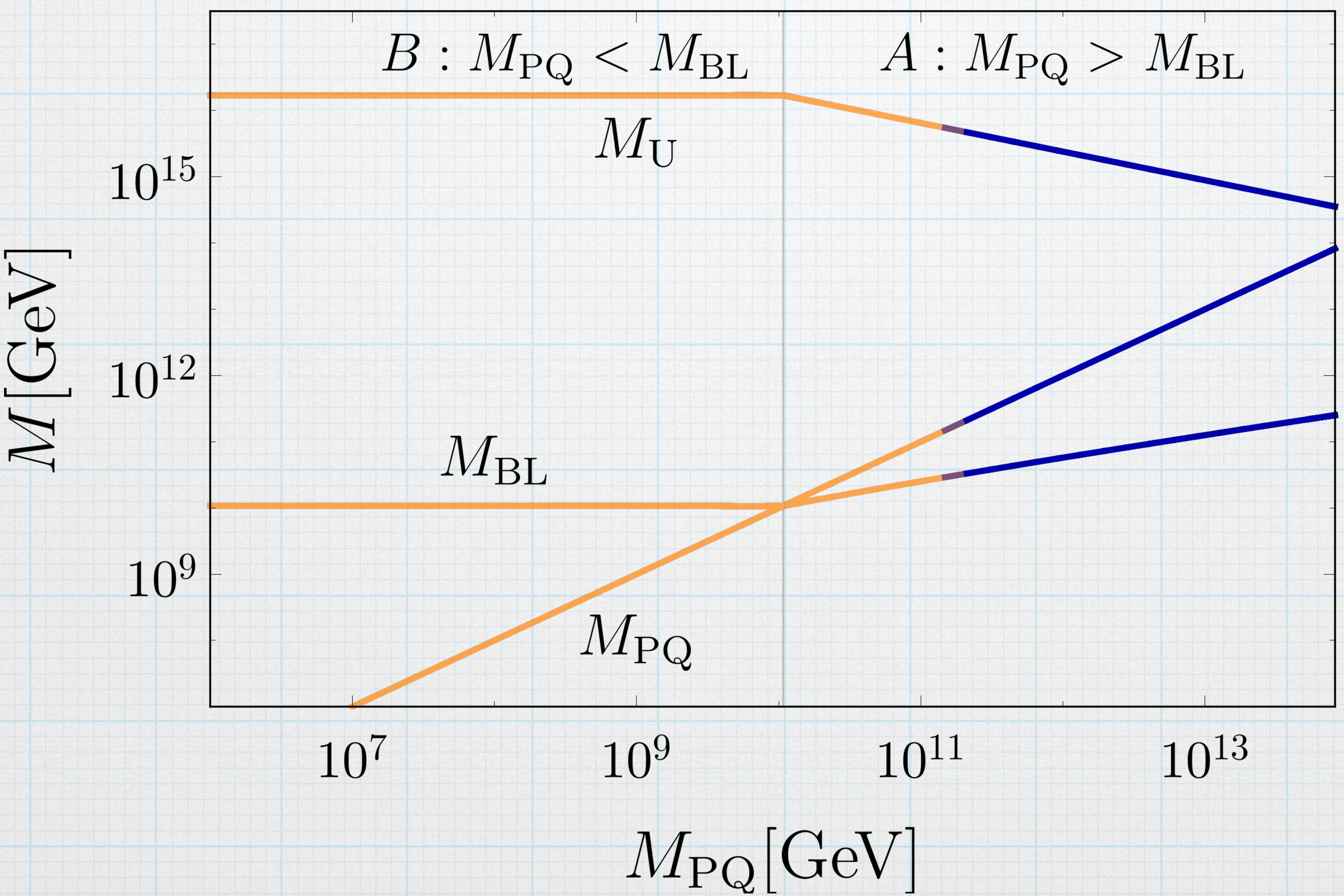


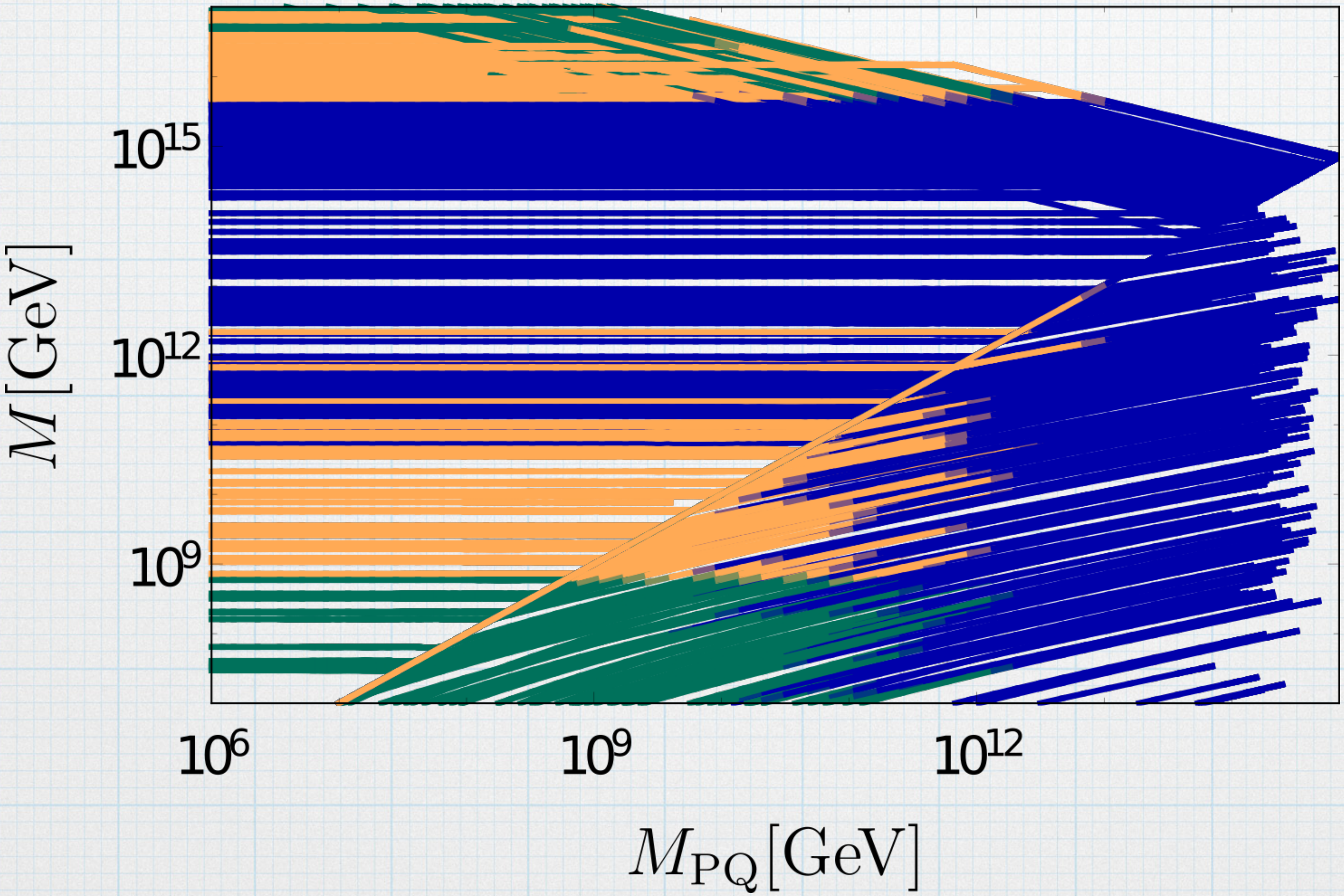
B: $M_{\rm PQ} < M_{\rm BL}$:

$$SO(10) \xrightarrow{M_{\rm U}-210_H} 4_C\, 2_L\, 2_R \xrightarrow{M_{\rm BL}-\overline{126}_H} 3_C\, 2_L\, 1_Y \xrightarrow{M_Z-10_H} 3_C\, 1_{\rm em}$$

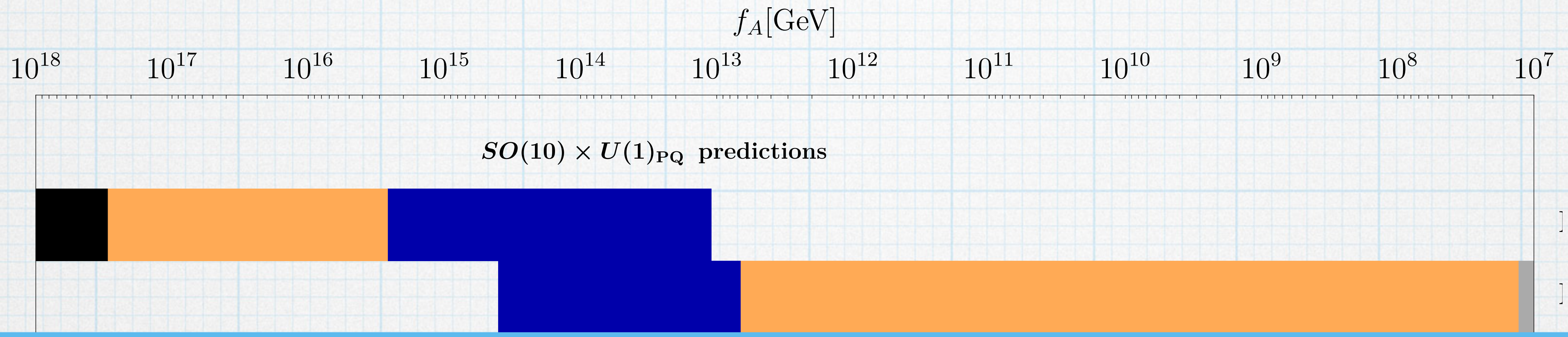
M2: three-step symmetry breaking



M2: three-step symmetry breaking



M2: predictions



- axion mass largely unconstrained
- accommodates a natural DM candidate axion

M3: additional singlet

- include PQ charged singlet
- axion decay constant given by vev of S
- as S is not charged under the gauge symmetry, **no constraints** can be placed on axion mass in this model

$U(1)_{\rm PQ}:$

$$16_F \to 16_F e^{i\alpha}$$

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$$S \to S e^{4i\alpha}$$

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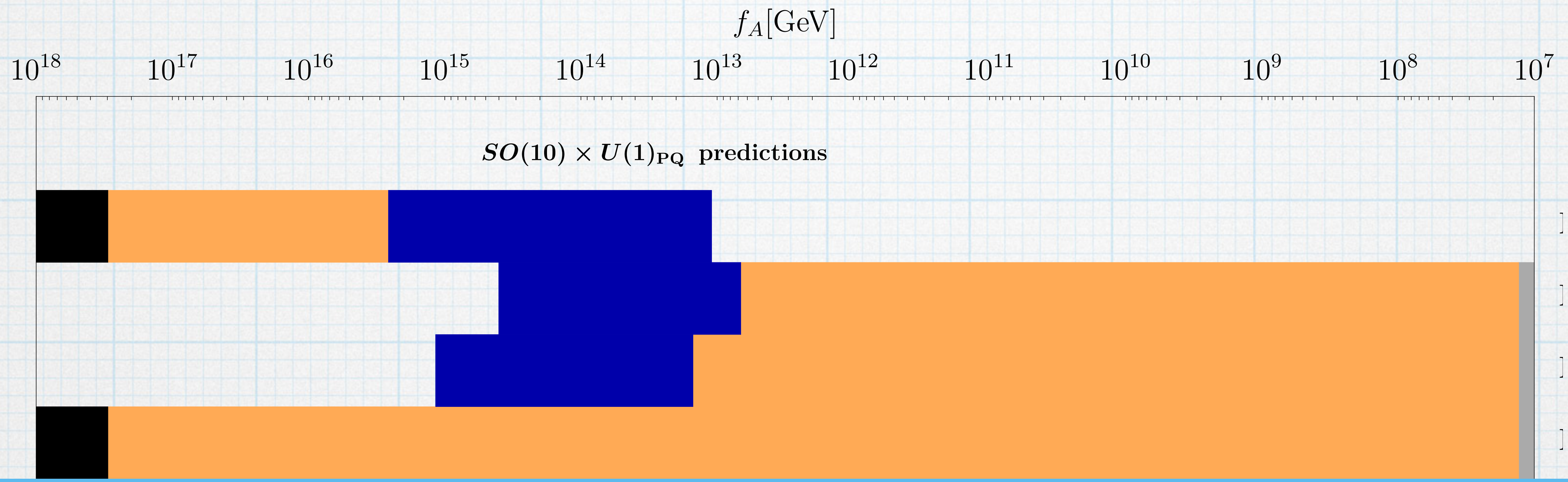
* similar to M2, M3 in its simplest version has domain wall problem → define M3.2, a model with an extra singlet and two generations of extra fermions

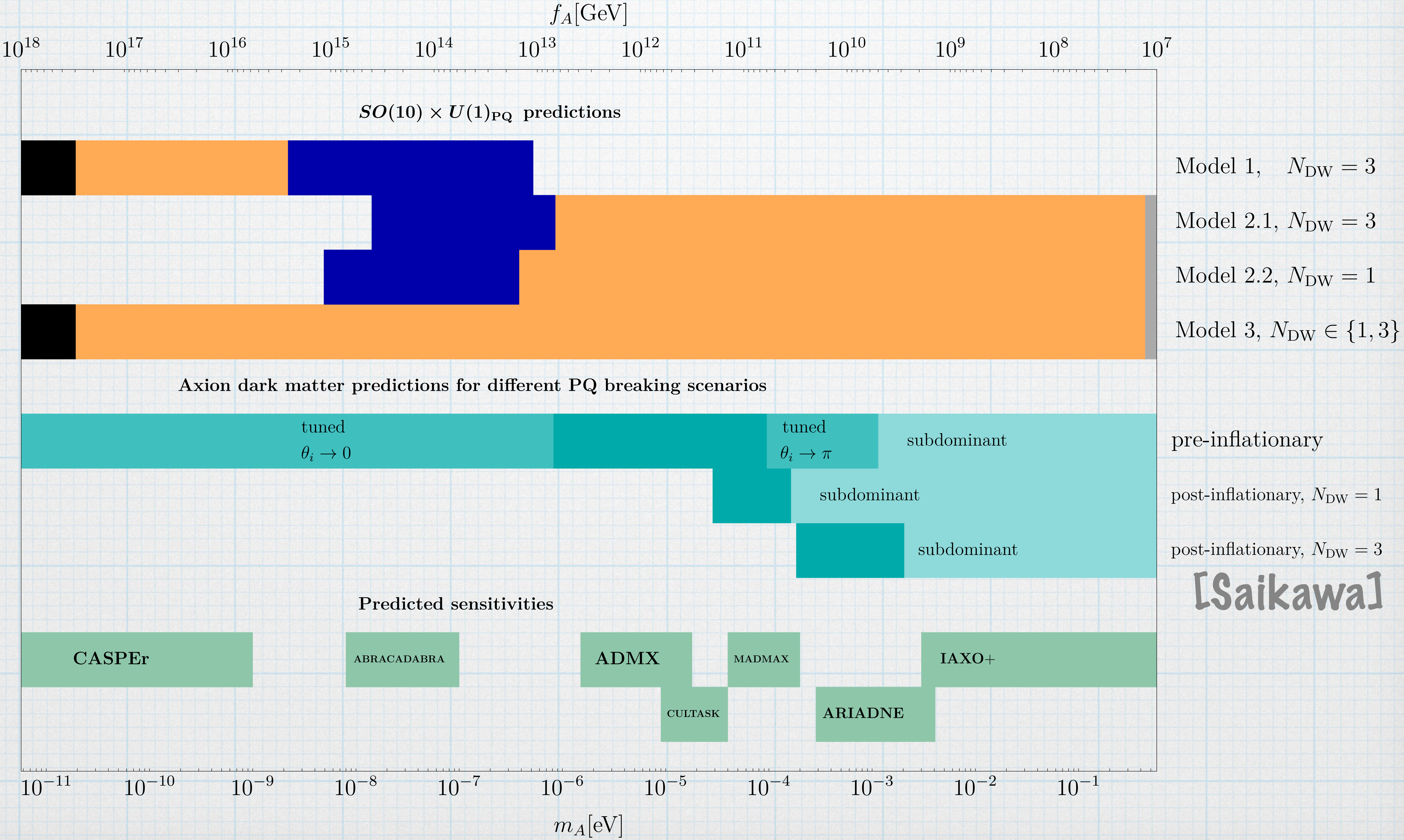
M3: (no) predictions



- axion mass unconstrained
- accommodates a natural DM candidate axion

Summary

	16_F	$\overline{126}_H$	10_H	210_H	45_H	S	10_F
Model 1	1	-2	-2	4	–	–	–
Model 2.1	1	-2	-2	0	4	–	–
Model 2.2	1	-2	-2	0	4	–	-2
Model 3	1	-2	-2	0	–	4	–

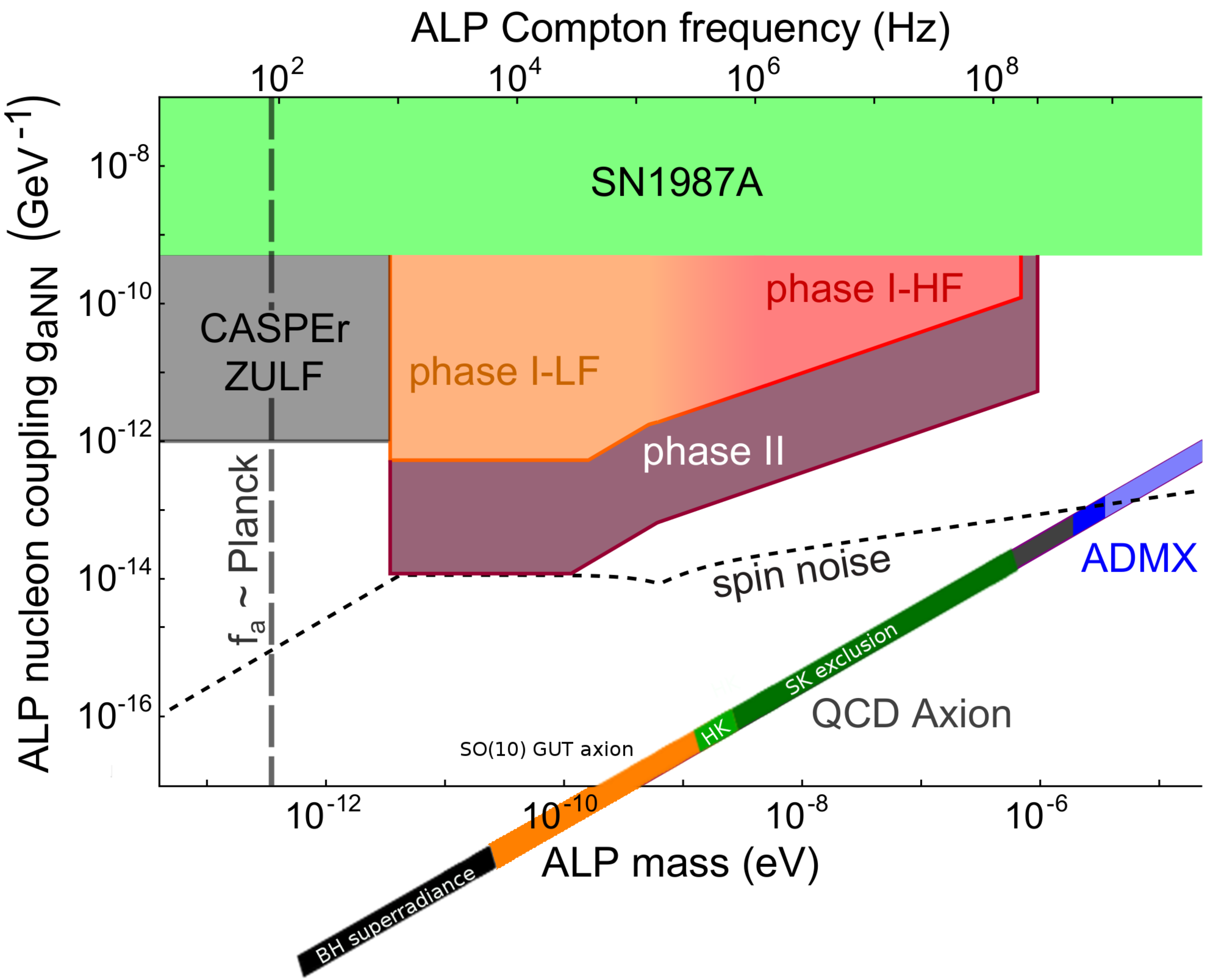


[Saikawa]

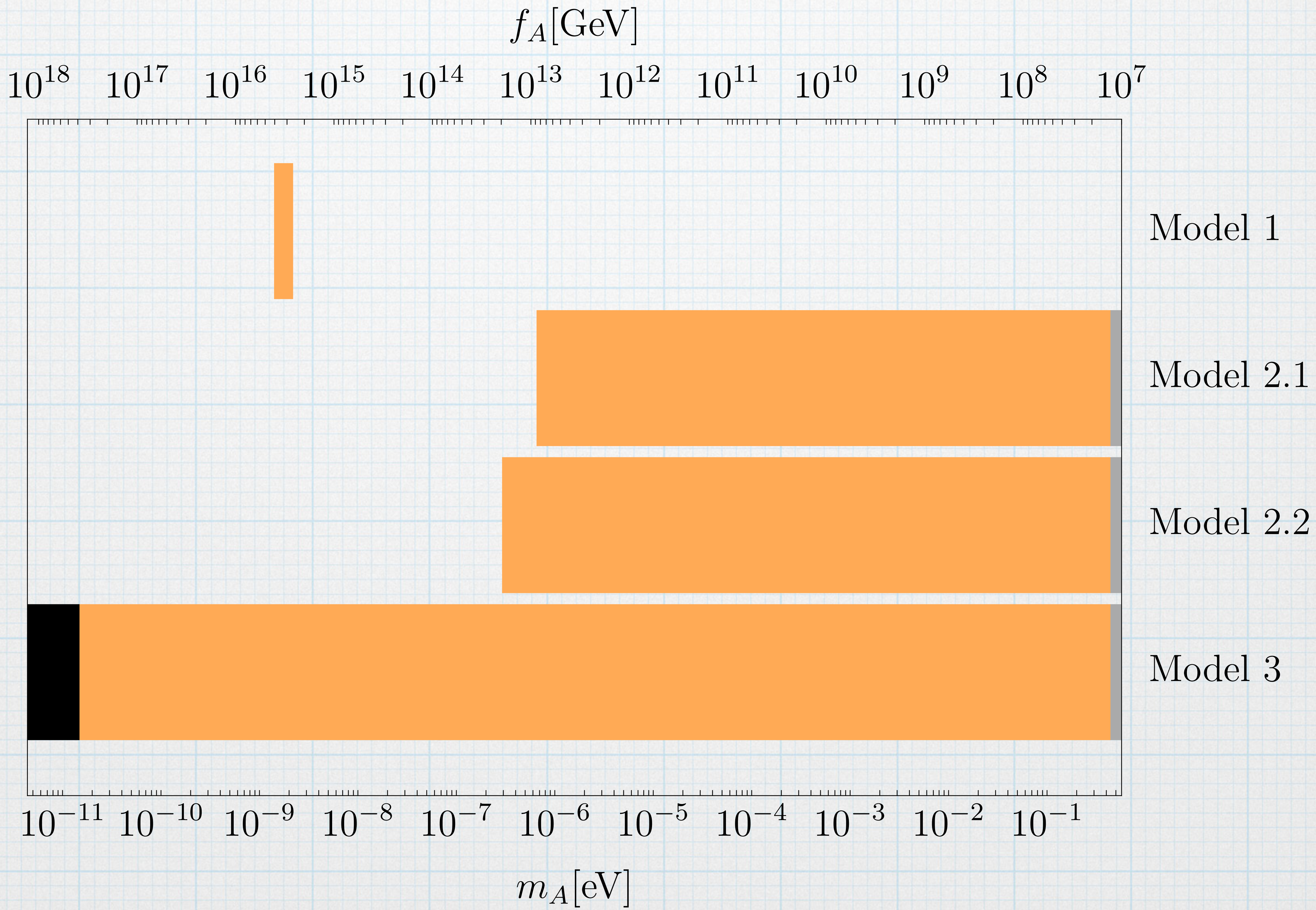
Backup

CASPEr wind

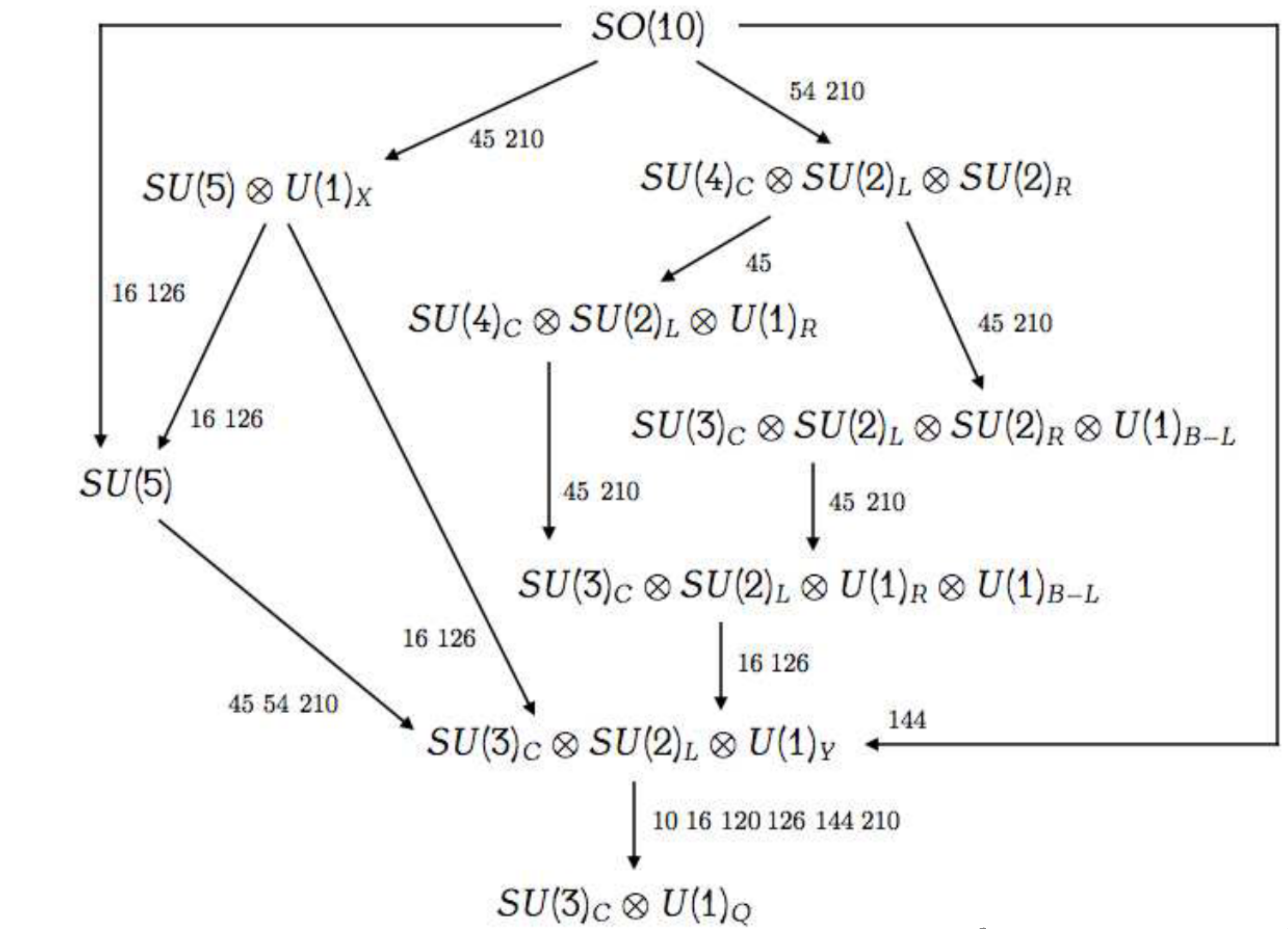
[arXiv: 1711.08999]



If Hyper-Kamiokande were to discover proton decay in the next decade



Symmetry breaking chains



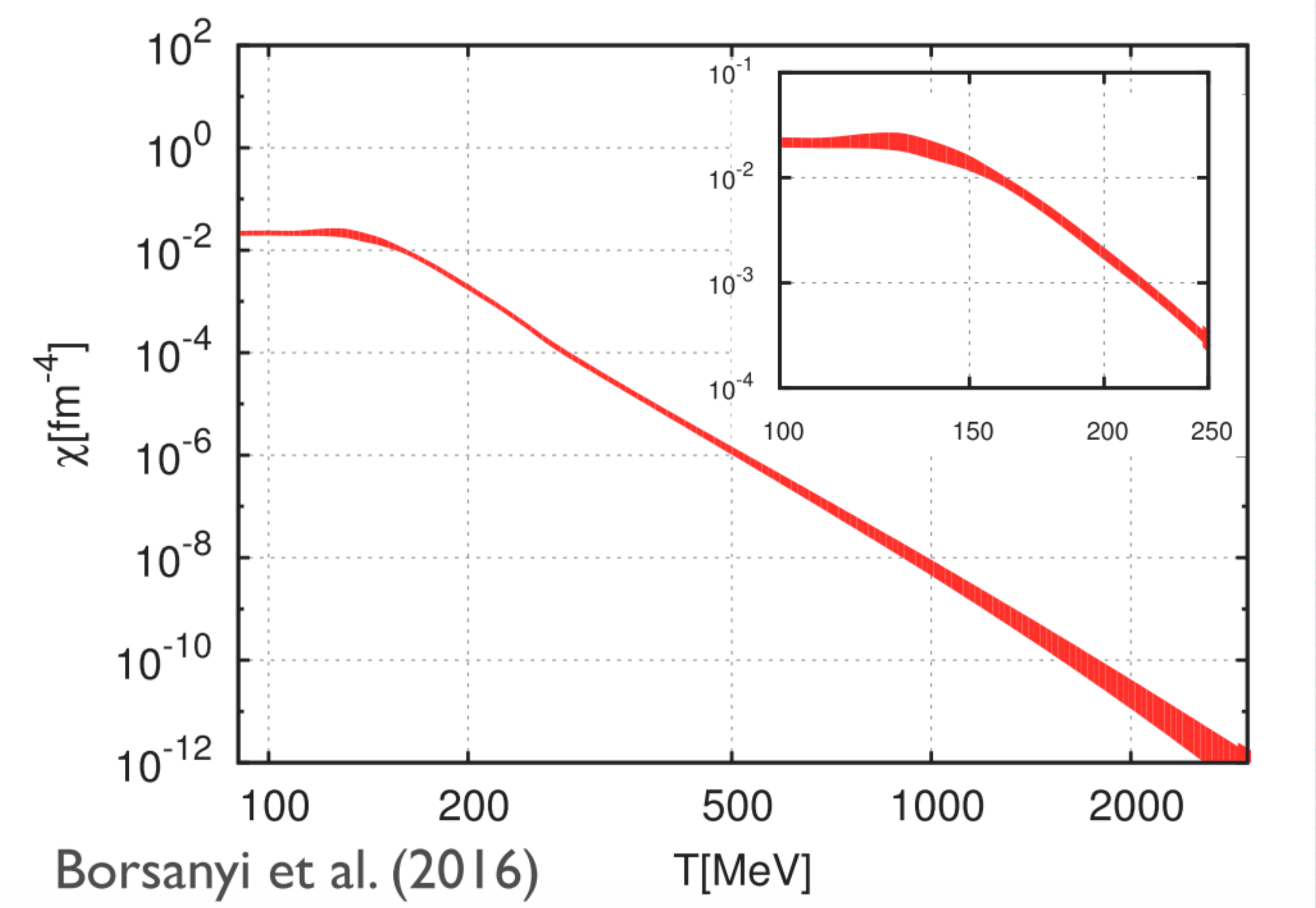
[Di Luzio 2011]

Axion properties

* (model dependent) couplings to gluons, photons and fermions, suppressed by $1/f_A$

* temperature-dependent mass

$$m_A(T) f_A = \sqrt{\chi(T)}$$



Axion properties

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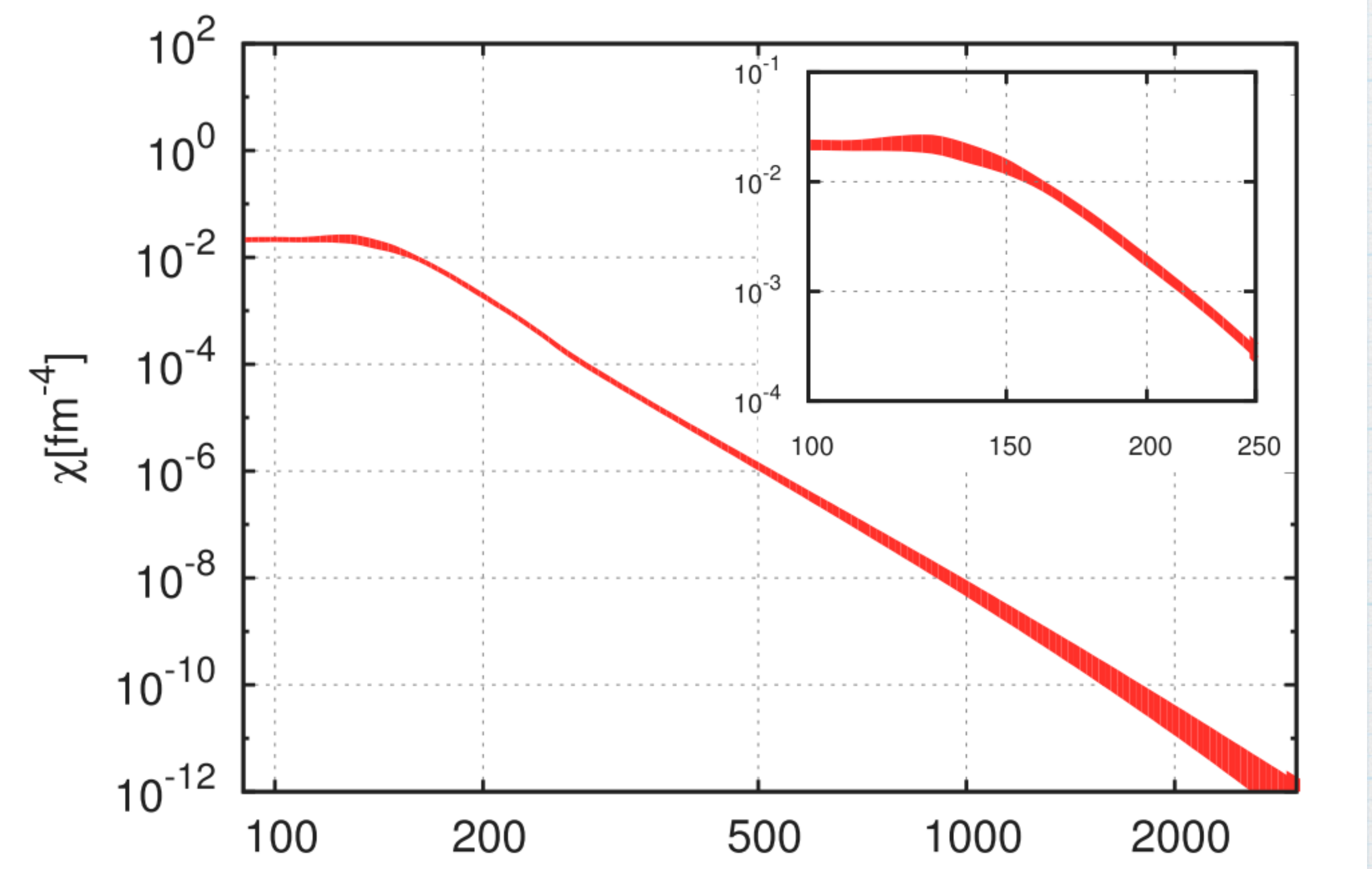
„axion decay constant“



Borsanyi et al. (2016)

Axion production: Misalignment mechanism

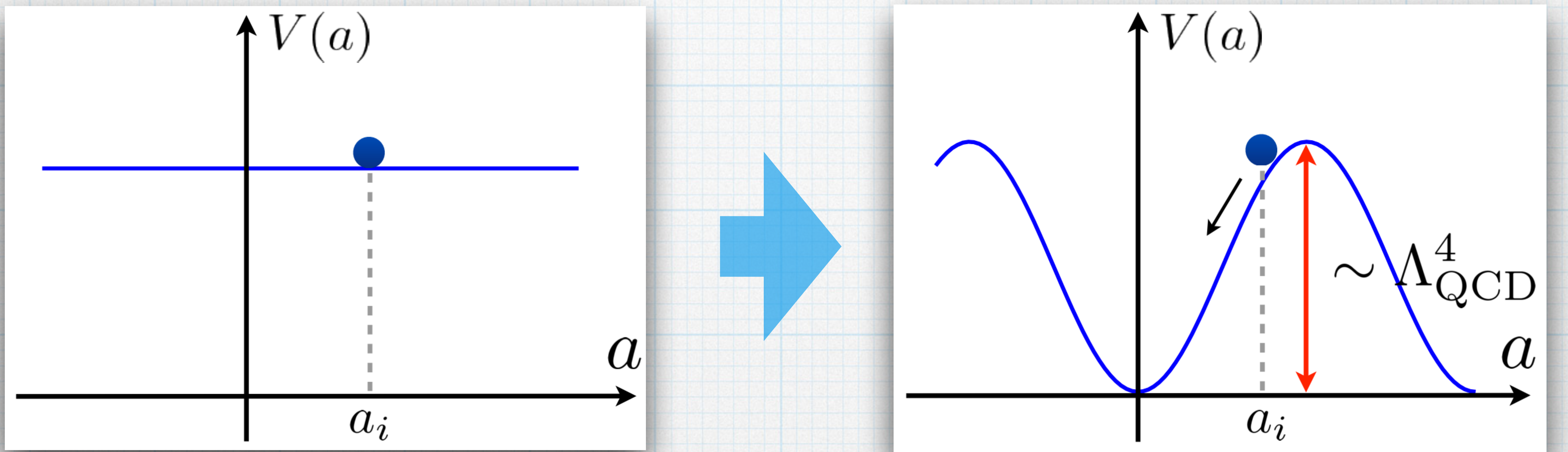
* scalar field in expanding FRW universe

$$\ddot{\phi} + 3H\dot{\phi} + m_a^2(T)\phi = 0$$

* at $m(T_{\rm osc}) \approx 3H(T_{\rm osc})$: field starts to oscillate

* oscillating field behaves as cold dark matter!

Axion production: Misalignment mechanism



[Saikawa]

$$\Omega_a h^2 \sim 2 \times 10^4 \left(\frac{f_A}{10^{16}\mathrm{GeV}} \right)^{7/6} \langle \theta_I^2 \rangle$$

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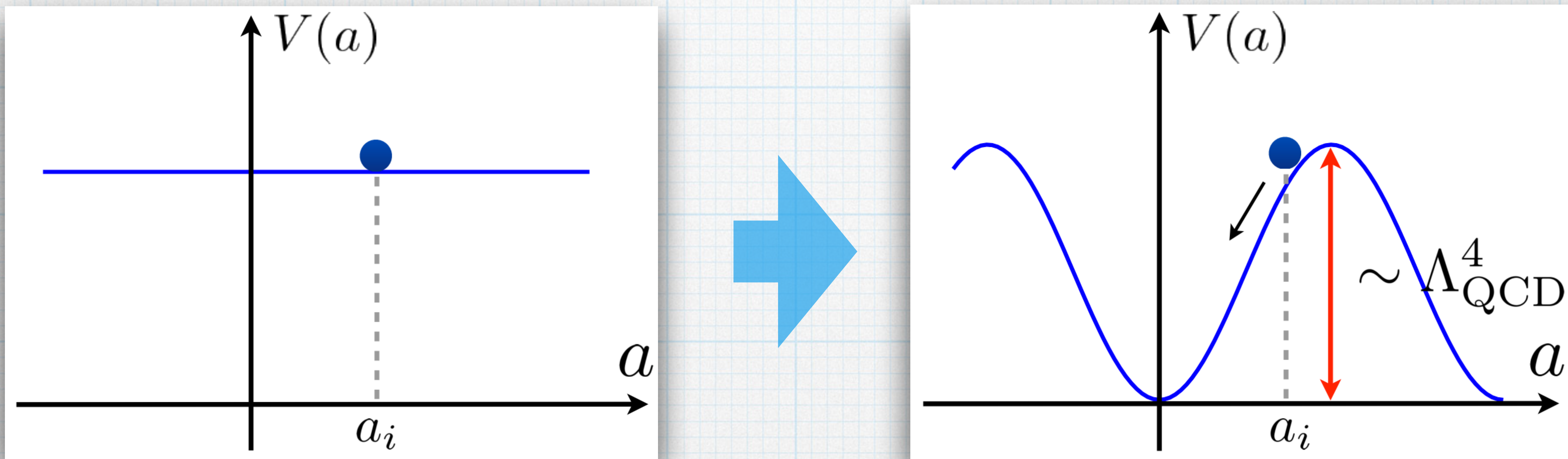

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„initial misalignment angle“

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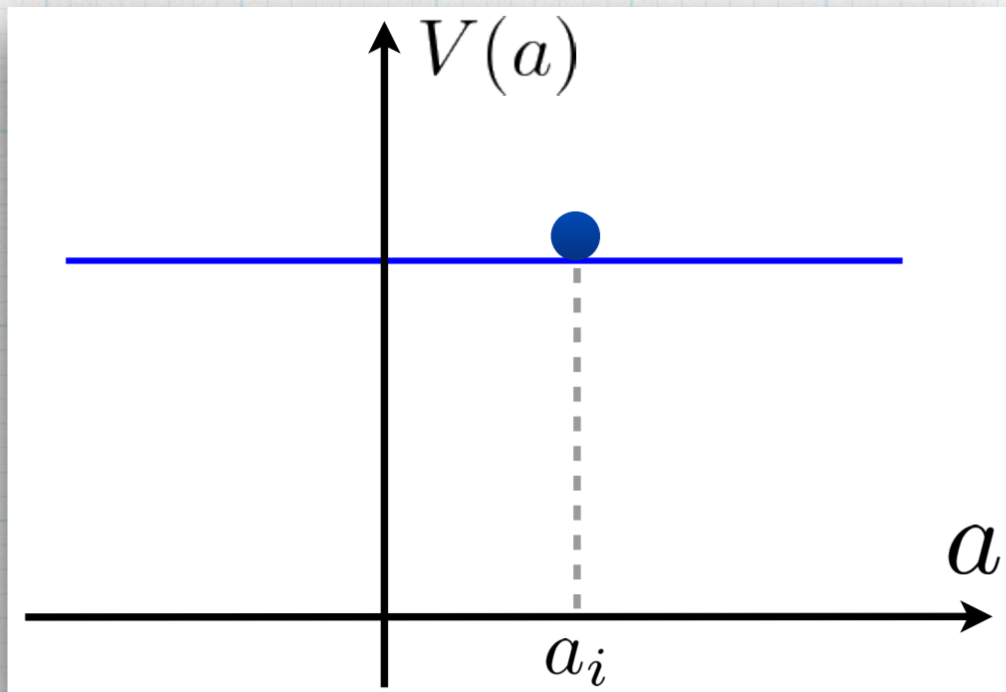

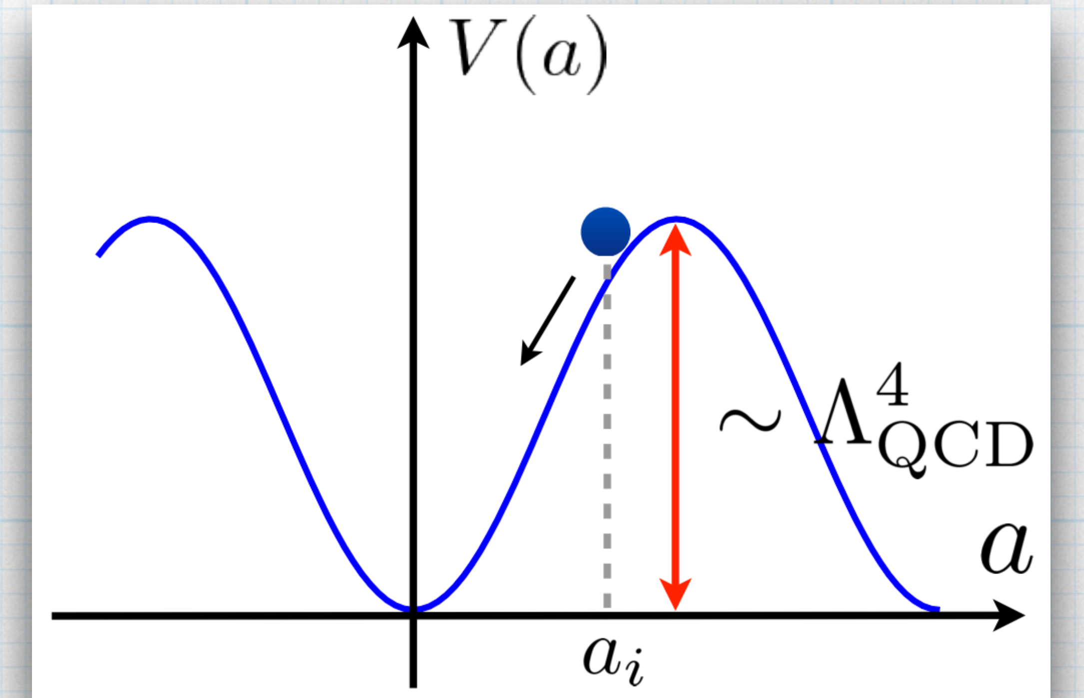

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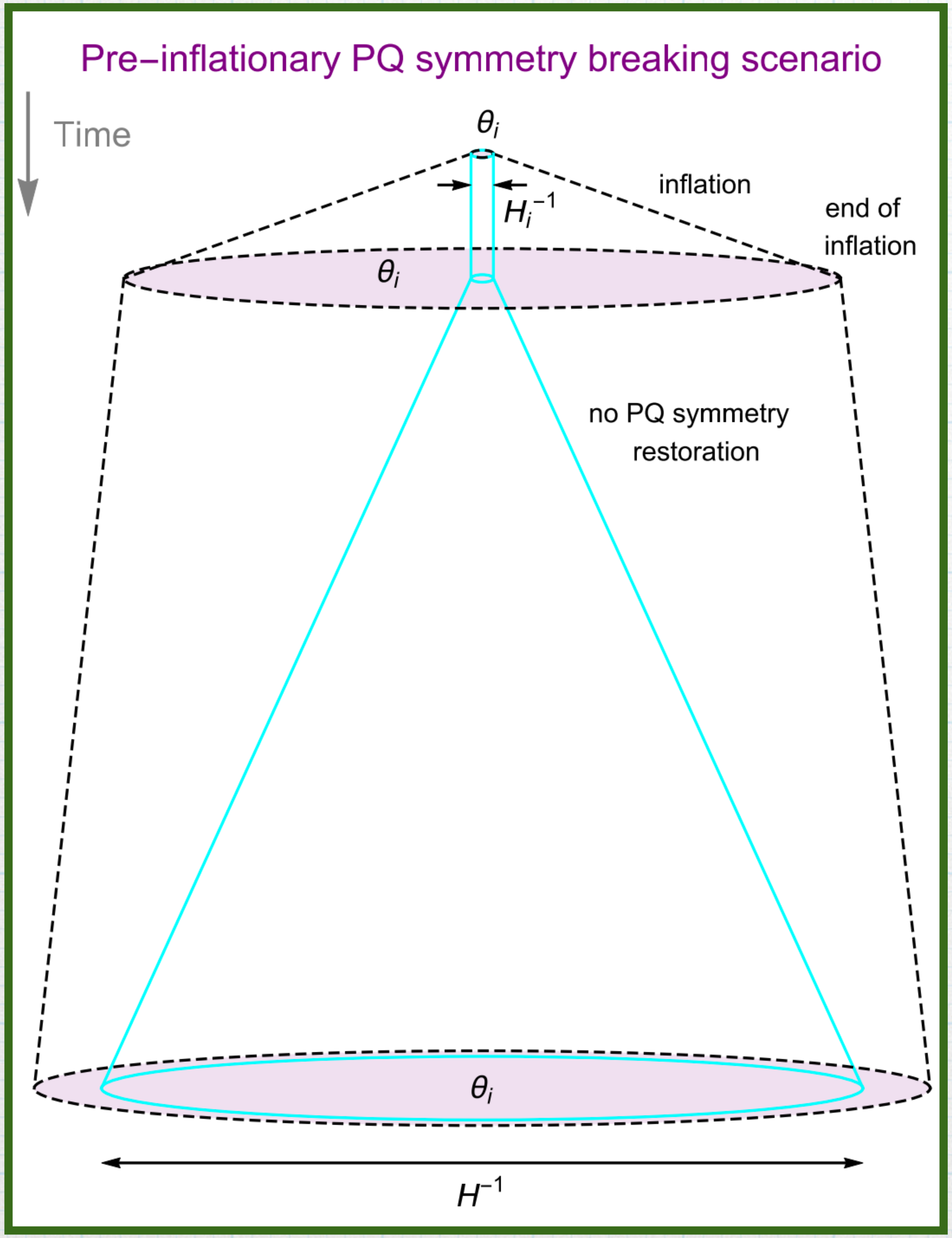
„initial misalignment angle"

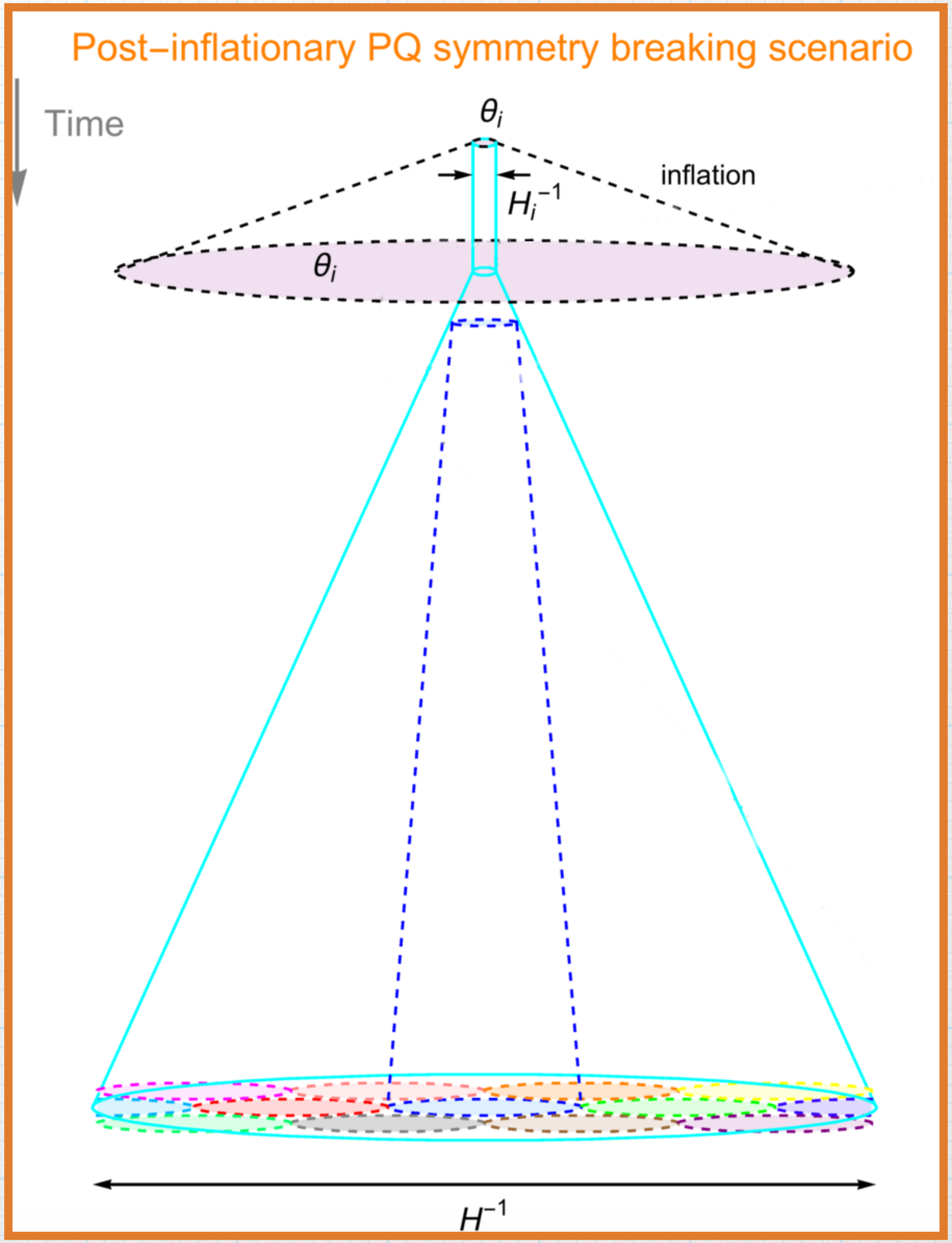
$$\Omega_{\mathrm{CDM}} h^2 \sim 0.11$$

[WMAP7]

PQ breaking: before or after inflation

[Saikawa]

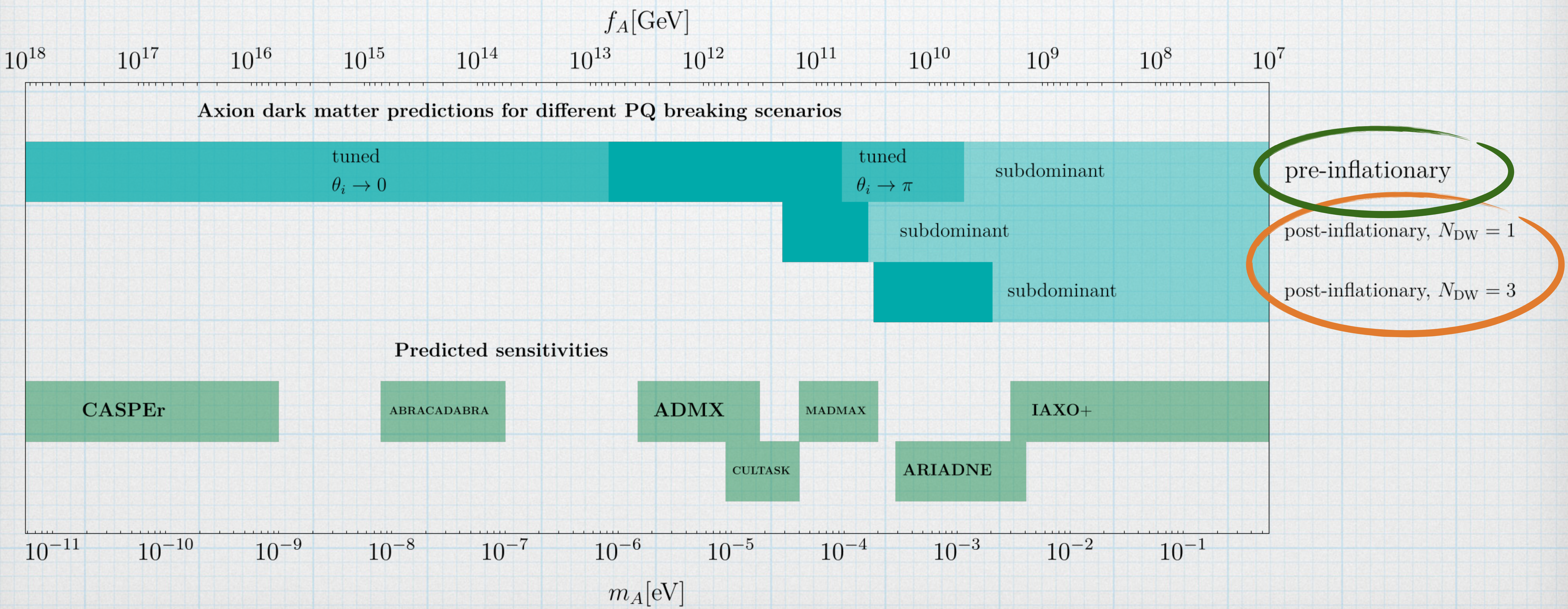




PQ breaking: before or after inflation

- θ_I is a free parameter - can be tuned
- anthropic constraints
- constraints from isocurvature perturbations
- „anthropic axion window"

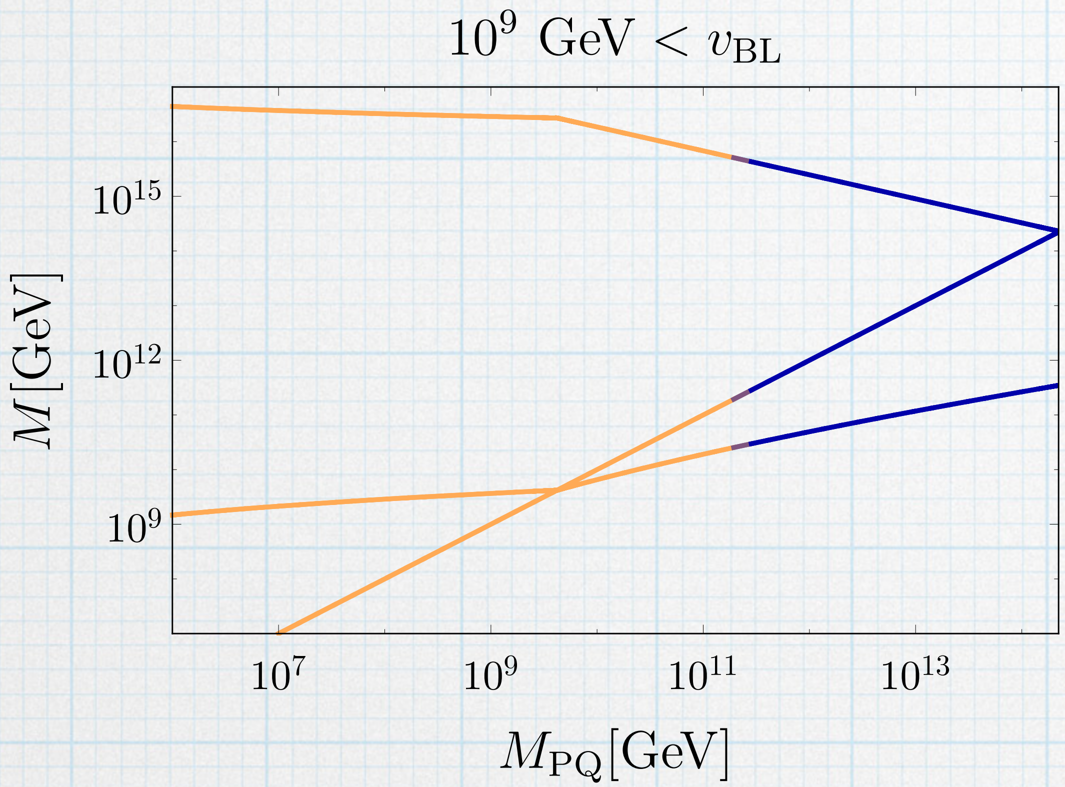
- $\langle \theta_I^2 \rangle = \frac{\pi^2}{3}$
- axion decay constant fixed
- topological defects
- „classic axion window“

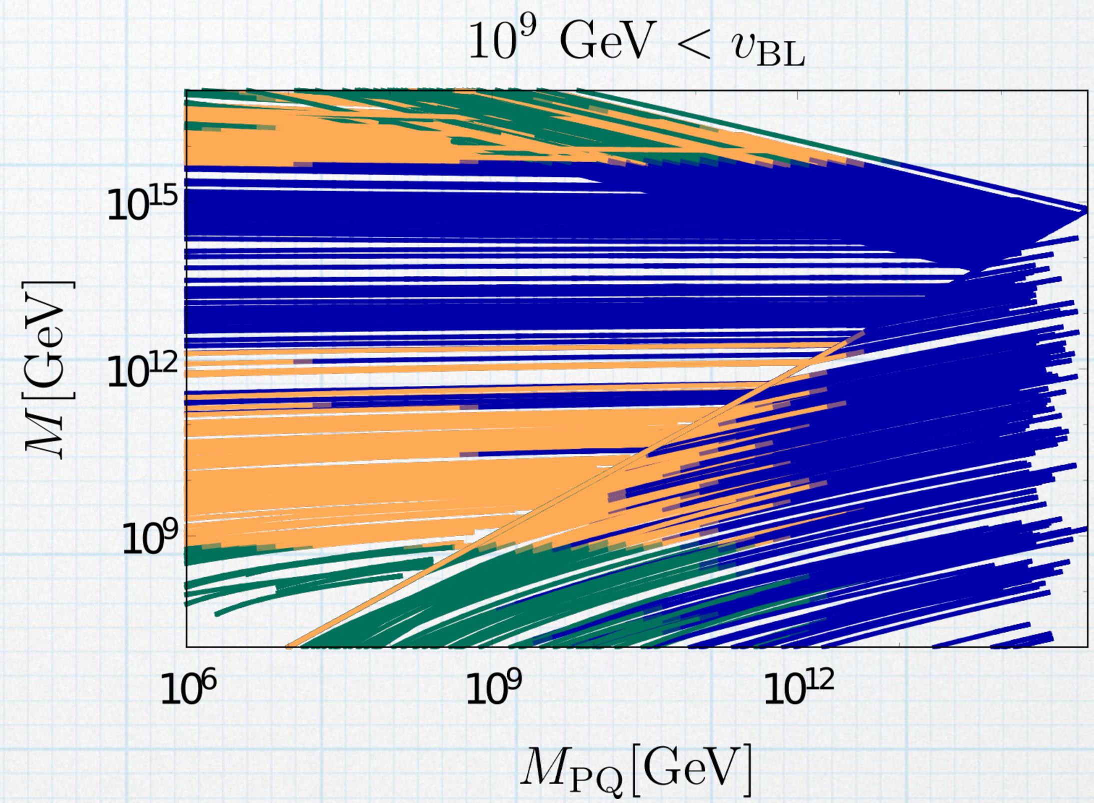


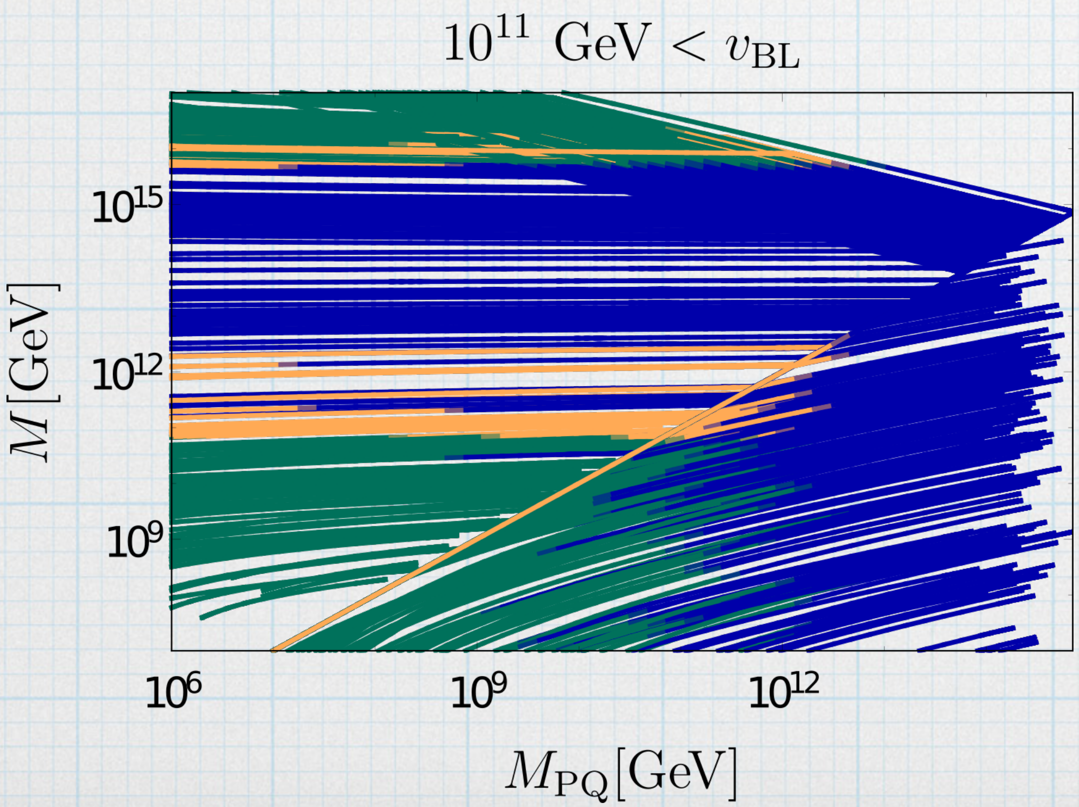
M2: domain wall number

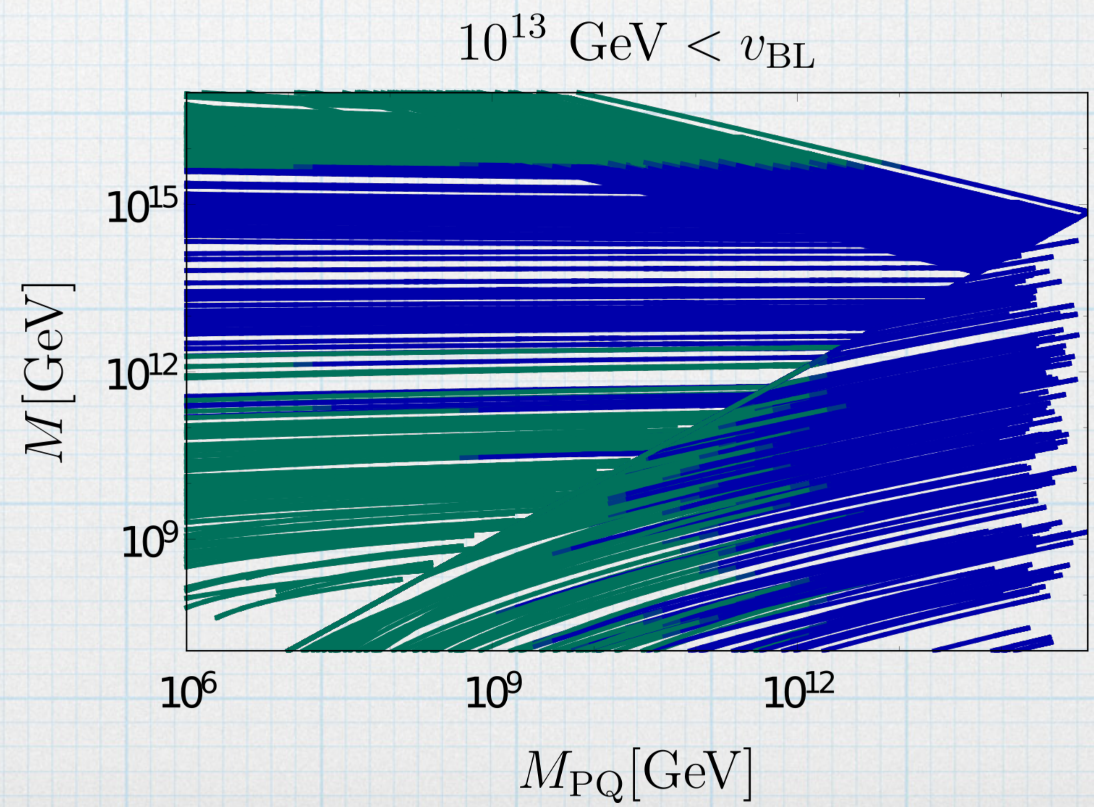
* this model has $N_{\rm DW} = 3$ which can cause cosmological problems if inflation happens before the PQ symmetry is broken
* Model 2.2: M2 + two additional generations of PQ charged fermions in the 10_F
* this lowers the domain wall number to 1 (Lazarides mechanism)
* additional particles change RGE running

M2.2: predictions









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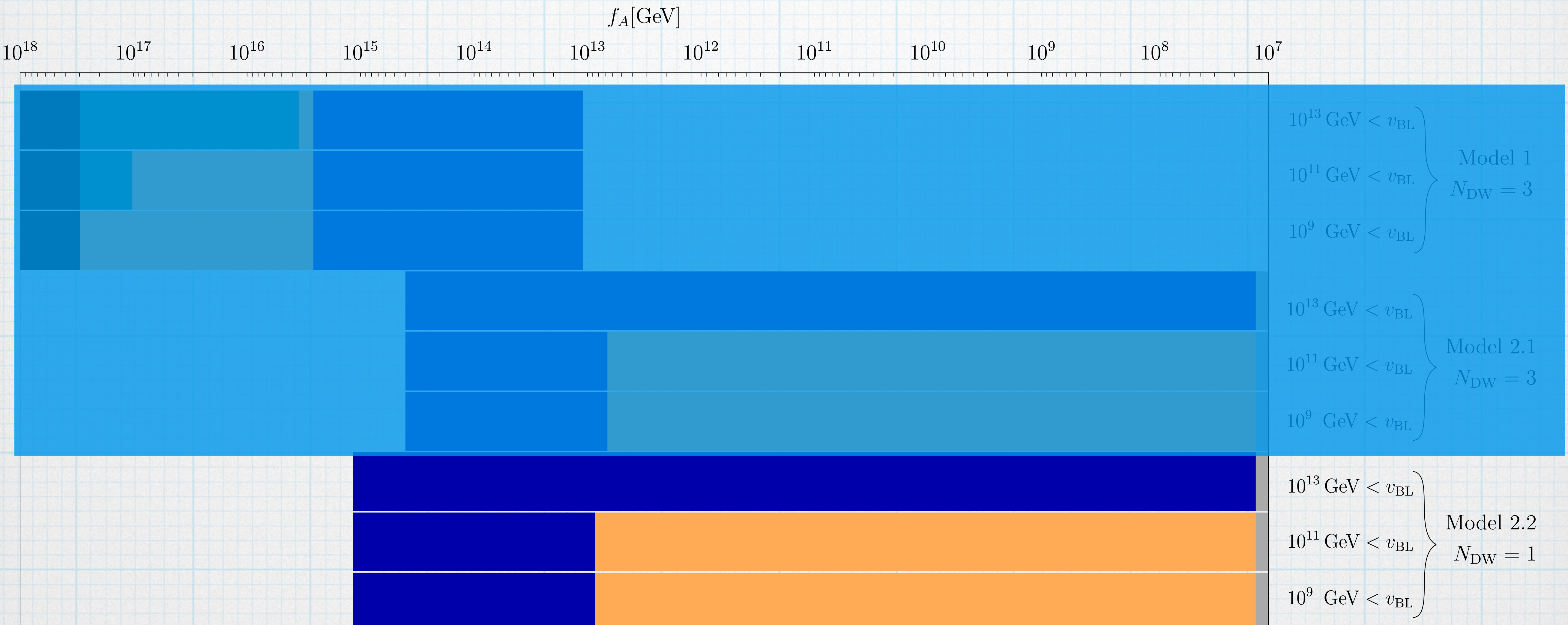

- axion mass largely unconstrained
- accommodates a natural DM candidate axion

Example: Axion construction in GUT theories

- axion is Goldstone boson of PQ symmetry breaking ➡ must be linear combination of phases $A = \sum_i c_i A_i$
- here we have defined $\phi_i = \frac{1}{\sqrt{2}}(v_i + \rho_i) e^{i \frac{A_i}{v_i}}$

field	vev	phase
$\phi_1 \equiv \Sigma_u$	v_1	A_1
$\phi_2 \equiv \Sigma_d$	v_2	A_2
$\phi_3 \equiv H_u$	v_3	A_3
$\phi_4 \equiv H_d$	v_4	A_4
$\phi_5 \equiv \Delta_R$	v_5	A_5
$\phi_6 \equiv \phi$	v_6	A_6

Example: Axion construction in GUT theories

* gauge invariance of the axion requires

$$c_1 v_1 - c_2 v_2 + c_3 v_3 - c_4 v_4 - 2c_5 v_5 = 0$$

$$c_5 v_5 = 0$$

field	vev	phase	$U(1)_{\rm BL}$	$U(1)_{\rm R}$	$U(1)_{\rm PQ}$
$\phi_1 \equiv \Sigma_u$	v_1	A_1	0	$-\frac{1}{2}$	-2
$\phi_2 \equiv \Sigma_d$	v_2	A_2	0	$\frac{1}{2}$	-2
$\phi_3 \equiv H_u$	v_3	A_3	0	$-\frac{1}{2}$	-2
$\phi_4 \equiv H_d$	v_4	A_4	0	$\frac{1}{2}$	-2
$\phi_5 \equiv \Delta_R$	v_5	A_5	-2	1	-2
$\phi_6 \equiv \phi$	v_6	A_6	0	0	0

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- imposed symmetries allow mass terms:

$$\begin{aligned} 10_H\, 10_H\, \overline{126}_H^\dagger\, \overline{126}_H^\dagger|_{\rm inv} + h.c. &\supset (1,2,2)(1,2,2,)(15,2,2)(15,2,2)|_{\rm inv} + h.c. \\ &\supset (H_u + H_d)(H_u + H_d)(\Sigma_u^\dagger + \Sigma_d^\dagger)(\Sigma_u^\dagger + \Sigma_d^\dagger)|_{\rm inv} + h.c. \\ &\supset -v_3^2 v_1^2 \left(\frac{A_3}{v_3} - \frac{A_1}{v_1}\right)^2 - v_4^2 v_2^2 \left(\frac{A_4}{v_4} - \frac{A_2}{v_2}\right)^2 . \end{aligned}$$

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- axion is perturbatively massless:

$$-\frac{c_1}{v_1} + \frac{c_3}{v_3} = 0 \qquad -\frac{c_2}{v_2} + \frac{c_4}{v_4} = 0 \qquad c_6 = 0$$

Example: Axion construction in GUT theories

* solving system of linear equations

$$A = -\frac{(A_4 v_4 + A_2 v_2)(v_3^2 + v_1^2) + (A_3 v_3 + A_1 v_1)(v_4^2 + v_2^2)}{\sqrt{v^2 (v_4^2 + v_2^2)(v_3^2 + v_1^2)}}, \quad v^2 \equiv \sum_{i=1}^{4} v_i^2$$

* after symmetry breaking, Axion appears in Yukawa couplings

$$\mathcal{L} \supset y^i_{ab} \phi_i \psi_a \psi_b + c.c. \supset \frac{y^i_{ab} v_i}{\sqrt{2}} e^{i q_i A / f_{\rm PQ}} \psi_a \psi_b + c.c.$$

* can be rotated away - but need to take into account Fujikawa's anomaly formula: [Dias et.al. 2014]

$$\mathscr{L}(q_{kR}) - \theta \frac{\alpha_s}{8\pi} G\tilde{G} \sim \mathscr{L}(e^{-i\alpha_k} q_{kR}) - \left(\theta + \sum_k \alpha_k\right) \frac{\alpha_s}{8\pi} G\tilde{G} - \left[3 \sum_k \alpha_k \left(C^{(k)}_{\rm em}\right)^2\right] \frac{\alpha_{\rm em}}{4\pi} F\tilde{F}$$

Example: Axion construction in GUT theories

$$\mathcal{L}_{\text{int, gauge}} = \frac{1}{2}\partial_\mu A \partial^\mu A + \frac{\alpha_s}{8\pi}\frac{A}{f_A} G^b_{\mu\nu}\tilde{G}^{b\mu\nu} + \frac{\alpha}{8\pi}\frac{8}{3}\frac{A}{f_A} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$f_A = \frac{1}{3}\sqrt{\frac{\left(v_1^2 + v_3^2\right)\left(v_2^2 + v_4^2\right)}{v^2}} \sim \frac{4}{3} M_Z$$

* obtain Axion effective Lagrangian
* note: even though B-L breaking vev is higher, Axion decay constant at the electroweak scale → experimentally excluded
* in general, if a vev breaks both PQ symmetry and a local U(1) symmetry, PQ symmetry survives to lower scales ('t Hooft mechanism)