Consistent Perturbative Fixed Point Calculations in Gauge Theories

DESY, Hamburg

March 20, 2018

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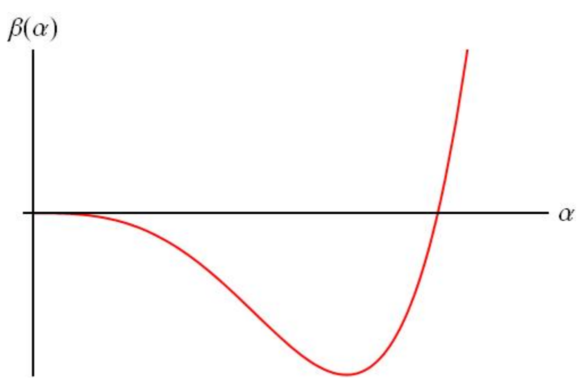
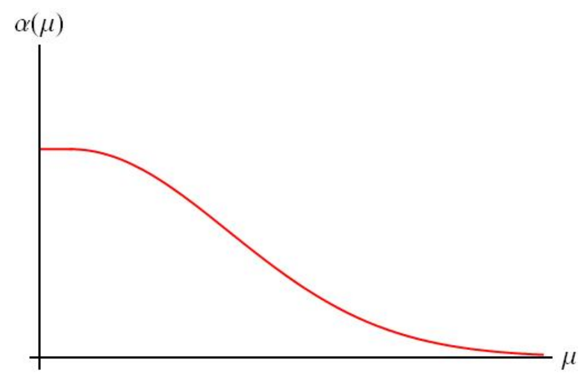
QCD in conformal window

- I will discuss QCD in a phase in which it is conformal in the deep infrared.
- In other words I will imagine taking the number of flavors to be sufficiently large such that the first two beta function coefficients have opposite signs.

W. E. Caswell, Phys. Rev. Lett. **33** (1974) 244
T. Banks & A. Zaks, Nucl. Phys. B **196**, 189 (1982)

	$[SU(N_c)]$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$
ψ	N_c	N_f	1	1
$\tilde{\psi}$	$\overline{N}_c$	1	$\overline{N}_f$	-1





For $N_c = 3$

$\Downarrow$

$8.05 < N_f < 16.5$

$\infty \leftarrow \alpha_{IR} \rightarrow 0$

SQCD in the conformal window

	$[SU(N_c)]$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
Φ	N_c	N_f	1	1	$\frac{N_f-N_c}{N_f}$
$\tilde{\Phi}$	$\overline{N}_c$	1	$\overline{N}_f$	-1	$\frac{N_f-N_c}{N_f}$

- In SQCD the extend of the conformal window is known exactly (duality checks, NSVZ beta function, etc)

N. Seiberg, Nucl. Phys. B **435**, 129 (1995)

$$\frac{3}{2}N_c < N_f < 3N_c$$

$$\infty \leftarrow \alpha_{IR} \rightarrow 0$$

Observables in the IR conformal phase

- At an IR conformal fixed point a set of natural physical quantities are the anomalous dimensions of various operators.

- The scaling equation of some (gauge invariant) operator *O* is

$$\frac{dO}{d\ln\mu} = -d_O O$$

- where $d_O = d_{classical} - \gamma_O$ and γ_O is the anomalous dimension.

- Two schemes S and S' :

$$\beta'(\alpha') = \frac{\partial \alpha'}{\partial \alpha}\beta(\alpha)$$

$$\gamma'_O(\alpha') = \gamma_O + \frac{\partial \ln F}{\partial \alpha}\beta(\alpha)$$

$$F = \frac{Z_{\bar{\psi}\psi}}{Z'_{\bar{\psi}\psi}}$$

- The existence of a fixed point is scheme independent.

- The anomalous dimension at the fixed point is scheme independent.

Observables in the IR conformal phase

- Initially we will focus on the anomalous dimension of the (mass) operator

$$O = \bar{\psi}\psi$$

Flavor non-singlet $\bar{\psi}T^a\psi$
and flavor singlet $\bar{\psi}\psi$
have equal anomalous dimensions

J. A. Gracey, Phys. Lett. B **488**, 175 (2000)

- Unitarity at the fixed point implies

$$d_{\bar{\psi}\psi} = 3 - \gamma_{\bar{\psi}\psi} > 1 \qquad \Rightarrow \qquad \gamma_{\bar{\psi}\psi} < 2$$

G. Mack, Commun. Math. Phys. **55**, 1 (1977)

- In the case of SQCD

$$O = \tilde{\Phi}\Phi \ , \qquad d_{\tilde{\Phi}\Phi} = 2 - \gamma_{\tilde{\Phi}\Phi} > 1 \qquad \Rightarrow \qquad \gamma_{\tilde{\Phi}\Phi} < 1$$

- Later we will also extend our analysis to include the anomalous dimension of several other operators….

Let us face it head on....

5-loop beta function and anomalous dimension

- 5-loop beta function and anomalous dimension are known in $\overline{\mathrm{MS}}$ scheme

F. Herzog et al., JHEP **1702**, 090 (2017)
P. A. Baikov et al., Phys. Rev. Lett. **118**, 082002 (2017)

$$\frac{2\pi}{\alpha^2}\beta(\alpha) = -\beta_1 - \beta_2\left(\frac{\alpha}{2\pi}\right) - \beta_3\left(\frac{\alpha}{2\pi}\right)^2 - \beta_4\left(\frac{\alpha}{2\pi}\right)^3 - \beta_5\left(\frac{\alpha}{2\pi}\right)^4 + O(\alpha^5)$$

$$\gamma_{\bar{\psi}\psi} = \gamma_1\left(\frac{\alpha}{2\pi}\right) + \gamma_2\left(\frac{\alpha}{2\pi}\right)^2 + \gamma_3\left(\frac{\alpha}{2\pi}\right)^3 + \gamma_4\left(\frac{\alpha}{2\pi}\right)^4 + \gamma_5\left(\frac{\alpha}{2\pi}\right)^5 + O(\alpha^6)$$

T. Luthe et al., JHEP **1701**, 081 (2017)
P. A. Baikov et al., JHEP **1410**, 076 (2014)

- Strategy:

Solve $\beta(\alpha_{IR}) = 0$

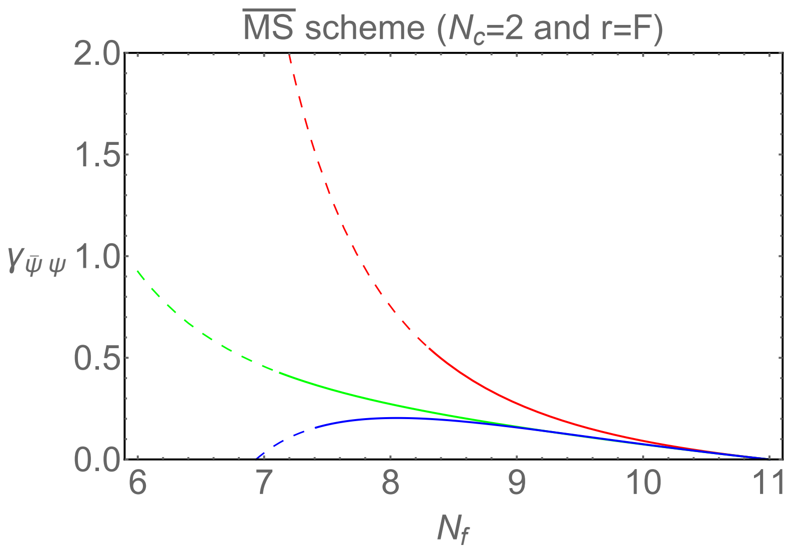
$\Downarrow$

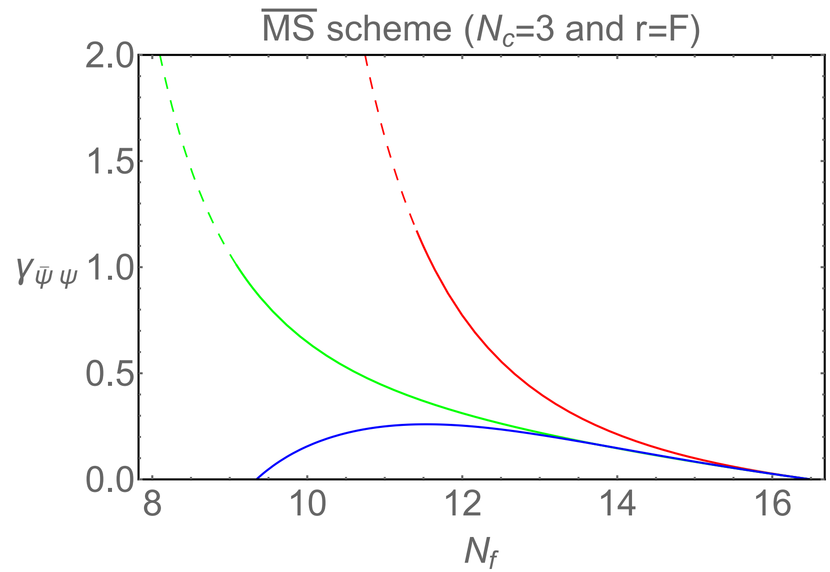
Take α_{IR} closest to the origin

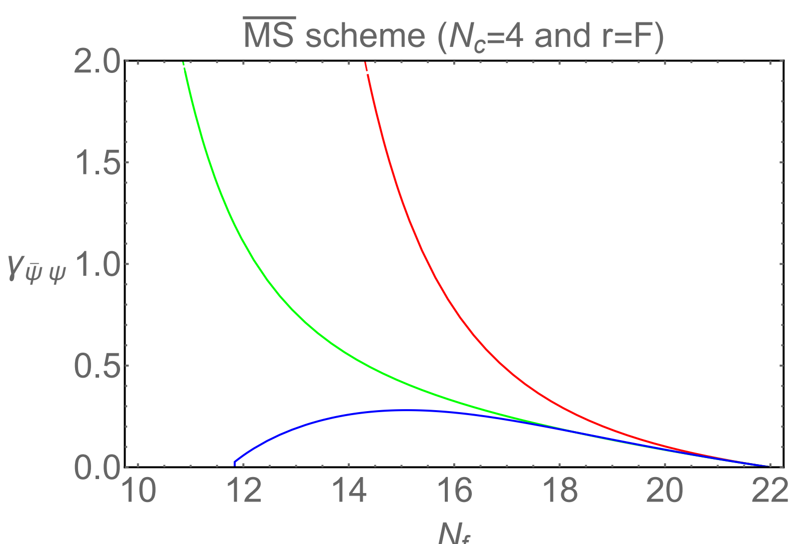
$\Downarrow$

Evaluate $\gamma_{\bar{\psi}\psi}(\alpha_{IR})$

2, 3 and 4 loops







Solid: $\alpha_{IR} < 1$

Dashed: $\alpha_{IR} > 1$

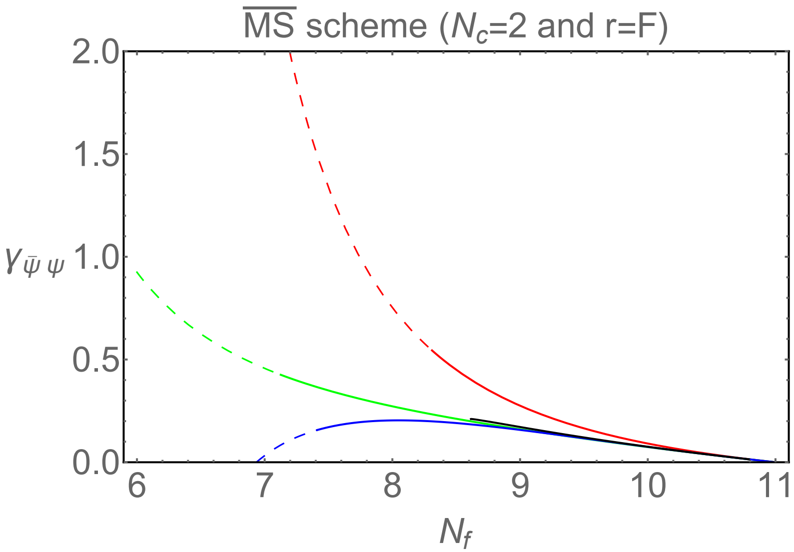
— : 2-loop

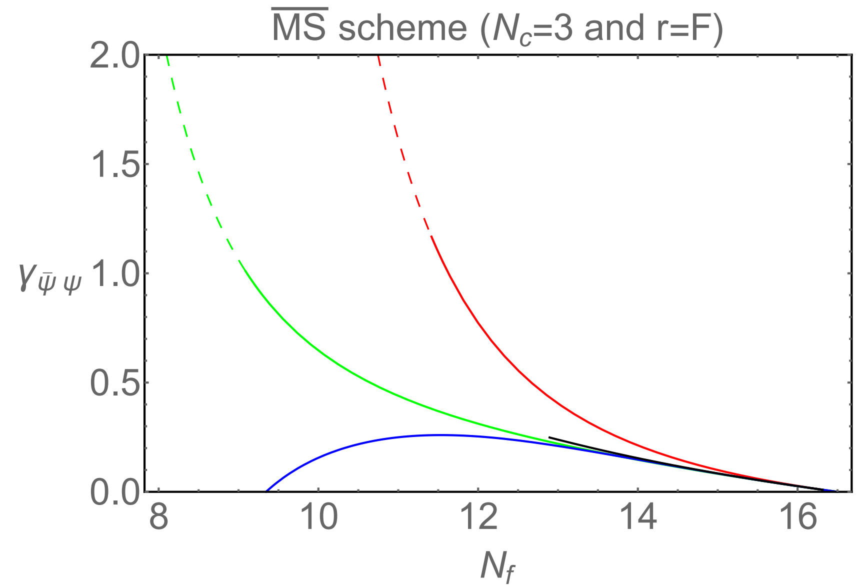
— : 3-loop

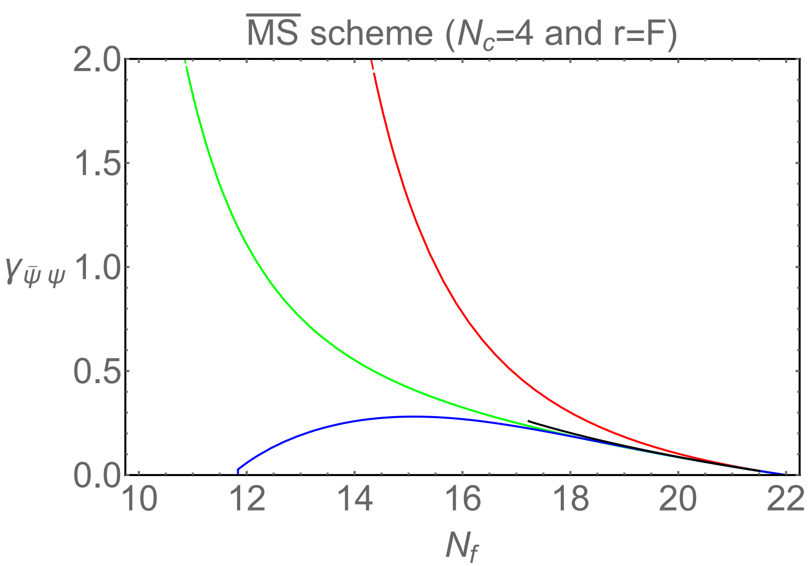
— : 4-loop

TAR & R. Shrock, Phys. Rev. D **83**, 056011 (2011)
C. Pica & F. Sannino, Phys. Rev. D **83**, 035013 (2011)

5-loops







Solid: $\alpha_{IR} < 1$
Dashed: $\alpha_{IR} > 1$

—: 2-loop
—: 3-loop
—: 4-loop
—: 5-loop

TAR & R. Shrock, Phys. Rev. D **94**, 105015 (2016)

Supersymmetric QCD (SQCD)

- In SQCD the anomalous dimension is known exactly (from the NSVZ beta function)

$$\gamma_{\Phi\tilde{\Phi}} = \frac{3C_2(G) - 2T(r)N_f}{2T(r)N_f}$$

V. A. Novikov et al., Phys. Lett. **B166**, 329 (1986)

- The 4-loop beta function and 3-loop anomalous dimension are known in $\overline{\mathrm{DR}}$ scheme

W. Siegel, Phys. Lett. B **84**, 193 (1979)
W. Siegel, Phys. Lett. B **94**, 37 (1980)
D. M. Capper, D. R. T. Jones & P. van Nieuwenhuizen, Nucl. Phys. B **167**, 479 (1980)

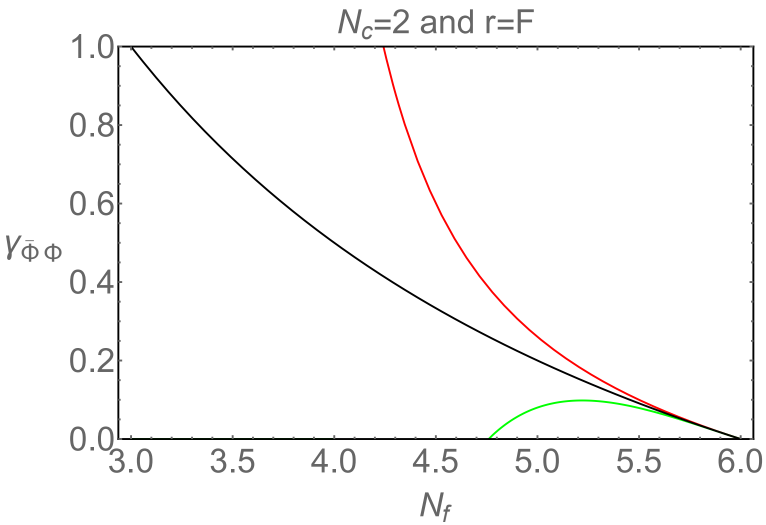
$$\frac{2\pi}{\alpha^2}\beta(\alpha) = -\beta_1 - \beta_2\left(\frac{\alpha}{2\pi}\right) - \beta_3\left(\frac{\alpha}{2\pi}\right)^2 - \beta_4\left(\frac{\alpha}{2\pi}\right)^3 + O(\alpha^4)$$

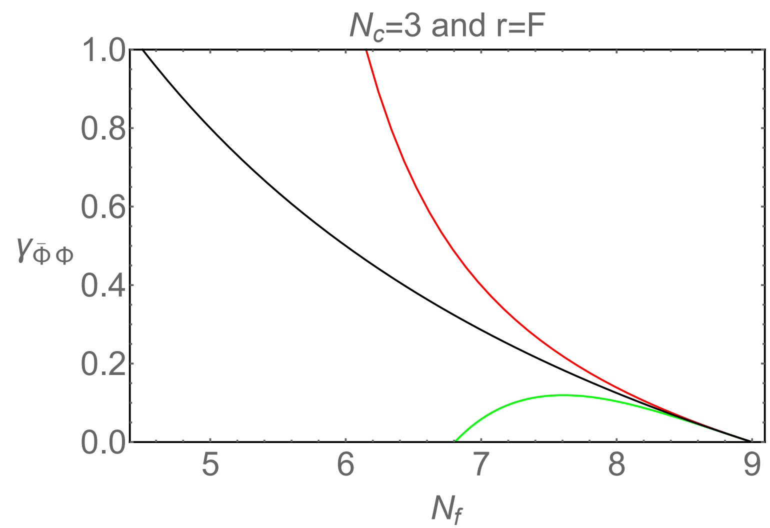
$$\gamma_{\Phi\tilde{\Phi}} = \gamma_1\left(\frac{\alpha}{2\pi}\right) + \gamma_2\left(\frac{\alpha}{2\pi}\right)^2 + \gamma_3\left(\frac{\alpha}{2\pi}\right)^3 + O(\alpha^4)$$

I. Jack et al., Phys. Lett. B **435**, 61 (1998)
I. Jack et al., Phys. Lett. B **386**, 138 (1996)
A. G. M. Pickering et al., Phys. Lett. B **510**, 347 (2001)
R. V. Harlander et al. Eur. Phys. J. C **63**, 383 (2009)

- Same procedure as before and then compare to the exact result…

2 and 3 loops vs exact result





—: 2-loop
—: 3-loop
—: exact

$$\gamma_{\bar{\Phi}\Phi} < 1 \qquad \Rightarrow \qquad \frac{3}{2}N_c < N_f < 3N_c$$

At 4-loops α_{IR} turns complex just below $3N_c$

TAR & R. Shrock, Phys. Rev. D **85**, 076009 (2012)

Usual statement:
Electric (magnetic) theory weakly (strongly) coupled for $2N_c < N_f < 3N_c$
Electric (magnetic) theory strongly (weakly) coupled for $\frac{3}{2}N_c < N_f < 2N_c$

Brief summary and outline

- Before we send perturbation theory to the grave we should remember that there are two sources of error in the calculation:

TAR & R. Shrock, Phys. Rev. D **86**, 085005 (2012)
TAR & R. Shrock, Phys. Rev. D **86**, 065032 (2012)
TAR, Phys. Rev. D **91**, 039906 (2015)

Truncation of perturbative expansion + scheme dependence

- Can we rinse the value obtained so far for scheme dependent contamination?

- There should be some scheme independent information hidden in the perturbative calculation. How can we extract it?

- At higher and higher loop order, more and more scheme independent information should be hidden in the perturbative calculation.

- Goal: Find a way to calculate which is scheme independent and exact order by order.

An advance

- We suggest to follow Banks and Zaks and calculate

T. Banks & A. Zaks, Nucl. Phys. B **196**, 189 (1982)
E. Gardi et al., JHEP **9807**, 007 (1998)
E. Gardi & G. Grunberg, JHEP **9903**, 024 (1999)
P. M. Stevenson, Mod. Phys. Lett. A **31**, 1650226 (2016)

$$\gamma = \sum_{i=1} c_i \Delta_f^i \ , \qquad \Delta_f = \bar{N}_f - N_f$$

- This expansion has several nice properties

  – The coefficients c_i are scheme independent since Δ_f is.

  – The coefficient c_i is exactly determined from the $i+1$ loop beta function and i loop anomalous dimension. It receives no corrections from higher orders!

- In this way the anomalous dimension can be calculated with only perturbation theory as an error to the finite order indicated.

An advance

- The usual procedure for finding α_{IR} is not very elegent and efficient.
- Instead: we are interested in

$$\frac{\alpha_{IR}}{2\pi} = \sum_{i=1} a_i \Delta_f^i$$

- Evaluate the beta function at α_{IR} and expand

$$0 = \beta(\alpha_{IR}) = \sum_{i=1} \beta_i \left(\sum_{j=1} a_j \Delta_f^j \right)^i = k_1 \Delta_f + k_2 \Delta_f^2 + k_3 \Delta_f^3 + O(\Delta_f^4)$$

- where

$$k_1 = a_1 \bar{\beta}_2^{(0)} - \bar{\beta}_1^{(1)}$$
$$k_2 = a_2 \bar{\beta}_2^{(0)} - a_1 \bar{\beta}_2^{(1)} + a_1^2 \bar{\beta}_3^{(0)}$$
$$k_3 = a_3 \bar{\beta}_2^{(0)} + 2a_1 a_2 \bar{\beta}_3^{(0)} - a_2 \bar{\beta}_2^{(1)} - a_1^2 \bar{\beta}_3^{(1)} + a_1^3 \bar{\beta}_4^{(0)}$$

$$\bar{\beta}_i^{(n)} = \frac{\partial^n \beta_i}{\partial N_f^n}|_{N_f = \bar{N}_f}$$

- Setting $k_1 = 0, \ k_2 = 0, \ k_3 = 0$ allows to solve for $a_1, \ a_2, \ a_3, ...$
- a_1 depends on the 2-loop beta function,
- a_2 depends on the 3-loop beta function,
- a_3 depends on the 4-loop beta function, etc….

An advance

- Now that we have solved for the fixed point value we can evaluate the anomalous dimension

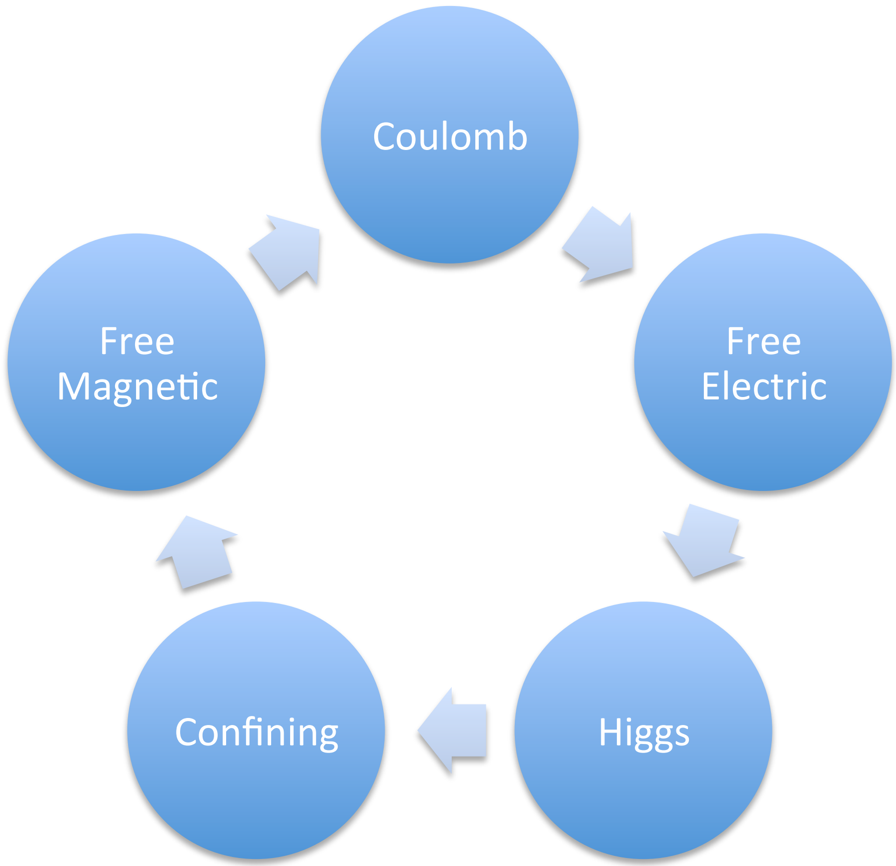
$$\gamma(\alpha_{IR}) = \sum_{i=1} \gamma_i \left(\sum_{j=1} a_j \Delta_f^j \right)^i = c_1 \Delta_f + c_2 \Delta_f^2 + c_3 \Delta_f^3 + O(\Delta_f^4)$$

$$c_1 = a_1 \overline{\gamma}_1^{(0)}$$
$$c_2 = a_2 \overline{\gamma}_1^{(0)} + a_1^2 \overline{\gamma}_2^{(0)} - a_1 \overline{\gamma}_1^{(1)}$$
$$c_3 = a_3 \overline{\gamma}_1^{(0)} + 2 a_1 a_2 \overline{\gamma}_2^{(0)} + a_1^3 \overline{\gamma}_3^{(0)} - a_1^2 \overline{\gamma}_2^{(1)}$$

$$\overline{\gamma}_i^{(n)} = \frac{\partial^n \gamma_i}{\partial N_f^n} \Big|_{N_f = \bar{N}_f}$$

- c_1 depends on the 2-loop beta function and 1-loop anomalous dimension,
  c_2 depends on the 3-loop beta function and 2-loop anomalous dimension,
  c_3 depends on the 4-loop beta function and 3-loop anomalous dimension.

Supersymmetric QCD (SQCD)



Supersymmetric QCD (SQCD)

- In SQCD we know the beta function to 4-loops and the anomalous dimension to 3-loops in the $\overline{\mathrm{DR}}$ scheme. So:

I. Jack et al., Phys. Lett. B **435**, 61 (1998)
I. Jack et al., Phys. Lett. B **386**, 138 (1996)
A. G. M. Pickering et al., Phys. Lett. B **510**, 3
R. V. Harlander et al. Eur. Phys. J. C **63**, 383

$$\gamma_{\Phi\tilde{\Phi}} = \frac{2T(r)}{3C_2(G)}\Delta_f + \left(\frac{2T(r)}{3C_2(G)}\right)^2 \Delta_f^2 + \left(\frac{2T(r)}{3C_2(G)}\right)^3 \Delta_f^3 + O(\Delta_f^4)$$

TAR, Phys. Rev. Lett. **117**, 071601 (2016)
TAR & R. Shrock, Phys. Rev. D **95**, 085012 (2017)

- The exact result (via NSVZ) is:

$$\gamma_{\Phi\tilde{\Phi}} = \frac{\frac{2T(r)}{3C_2(G)}\Delta_f}{1-\frac{2T(r)}{3C_2(G)}\Delta_f} = \frac{2T(r)}{3C_2(G)}\Delta_f + \left(\frac{2T(r)}{3C_2(G)}\right)^2 \Delta_f^2 + \ldots \left(\frac{2T(r)}{3C_2(G)}\right)^i \Delta_f^i + \ldots$$

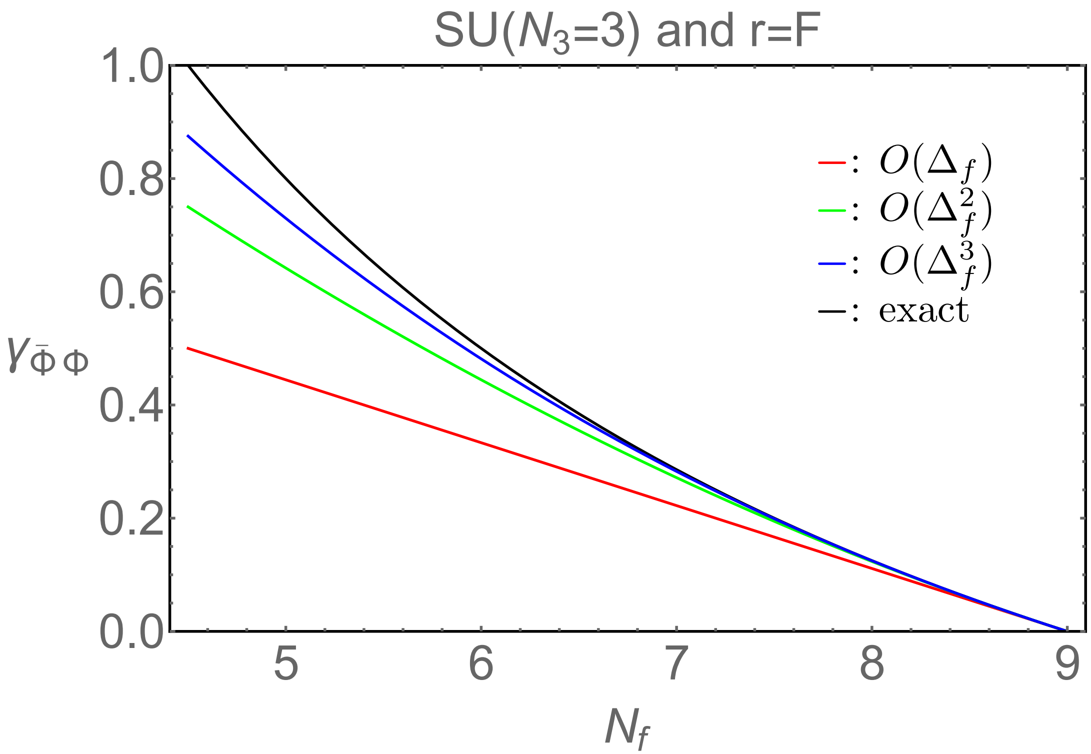
V. A. Novikov et al., Phys. Lett. **B166**, 329 (1986)

- Exact agreement to the order we can calculate.
- Confidence in $\overline{\mathrm{DR}}$ as a viable consistent scheme.
- How well does perturbation theory do and how fast does the series converge?

$$N_c = 3,\ \text{Fundamentals}: \qquad \gamma_{\tilde{\Phi}\Phi} = \frac{1}{9}\Delta_f + \left(\frac{1}{9}\right)^2 \Delta_f^2 + \left(\frac{1}{9}\right)^3 \Delta_f^3 + \ldots$$

- Note: We would have been able to guess the entire series (exact result) correctly….

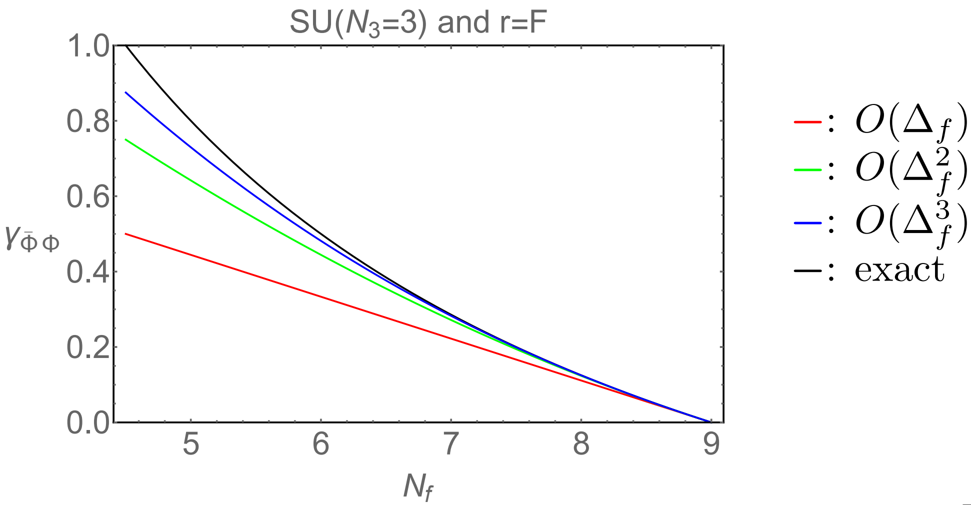
Supersymmetric QCD (SQCD)



TAR, Phys. Rev. Lett. **117**, 071601 (2016)
TAR & R. Shrock, Phys. Rev. D **95**, 085012 (2017)

- The perturbative result converges quickly to the exact result. High level of accuracy of perturbation theory throughout the entire conformal window.
- With perturbation theory + unitarity we would have nailed down the conformal window.

What is the relative error?



TAR, Phys. Rev. Lett. **117**, 071601 (2016)
TAR & R. Shrock, Phys. Rev. D **95**, 085012 (2017)

$$\sum_{j=1}^{s} x^j = \frac{x(x^s-1)}{x-1}$$

- Relative error at order s : $\epsilon_s = \dfrac{\gamma_{exact} - \gamma_s}{\gamma_{exact}} = \dfrac{\frac{\frac{\Delta_f}{3N_c}}{1-\frac{\Delta_f}{3N_c}} - \sum_{j=1}^{s}\left(\frac{\Delta_f}{3N_c}\right)^j}{\frac{\frac{\Delta_f}{3N_c}}{1-\frac{\Delta_f}{3N_c}}} = \left(\dfrac{\Delta_f}{3N_c}\right)^s$

TAR & R. Shrock, Accepted for pub. in PRD

- Example at bottom of window:

$$\Delta_f = 3N_c - \frac{3}{2}N_c = \frac{3}{2}N_c \qquad \Rightarrow \qquad \epsilon_s = \left(\frac{1}{2}\right)^s = e^{-(\ln 2)s}$$

- So: $O(\Delta_f^4):\ \epsilon_s = 6.25\%$, $O(\Delta_f^5):\ \epsilon_s = 3.125\%$, $O(\Delta_f^6):\ \epsilon_s = 1.56\%$

- If one is not happy with the relative error in the computation the full exact result can just be guessed.

Supersymmetric QCD (SQCD)

$$\gamma_{\Phi\tilde{\Phi}} = \frac{\frac{2T(r)}{3C_2(G)}\Delta_f}{1-\frac{2T(r)}{3C_2(G)}\Delta_f} = \frac{2T(r)}{3C_2(G)}\Delta_f + \left(\frac{2T(r)}{3C_2(G)}\right)^2 \Delta_f^2 + \ldots \left(\frac{2T(r)}{3C_2(G)}\right)^i \Delta_f^i + \ldots$$

- Having the exact result in hand we can also turn it around:

- If somebody, some day, calculates the $i+1$ loop beta function and i loop anomalous dimension then at a fixed point they *must* find

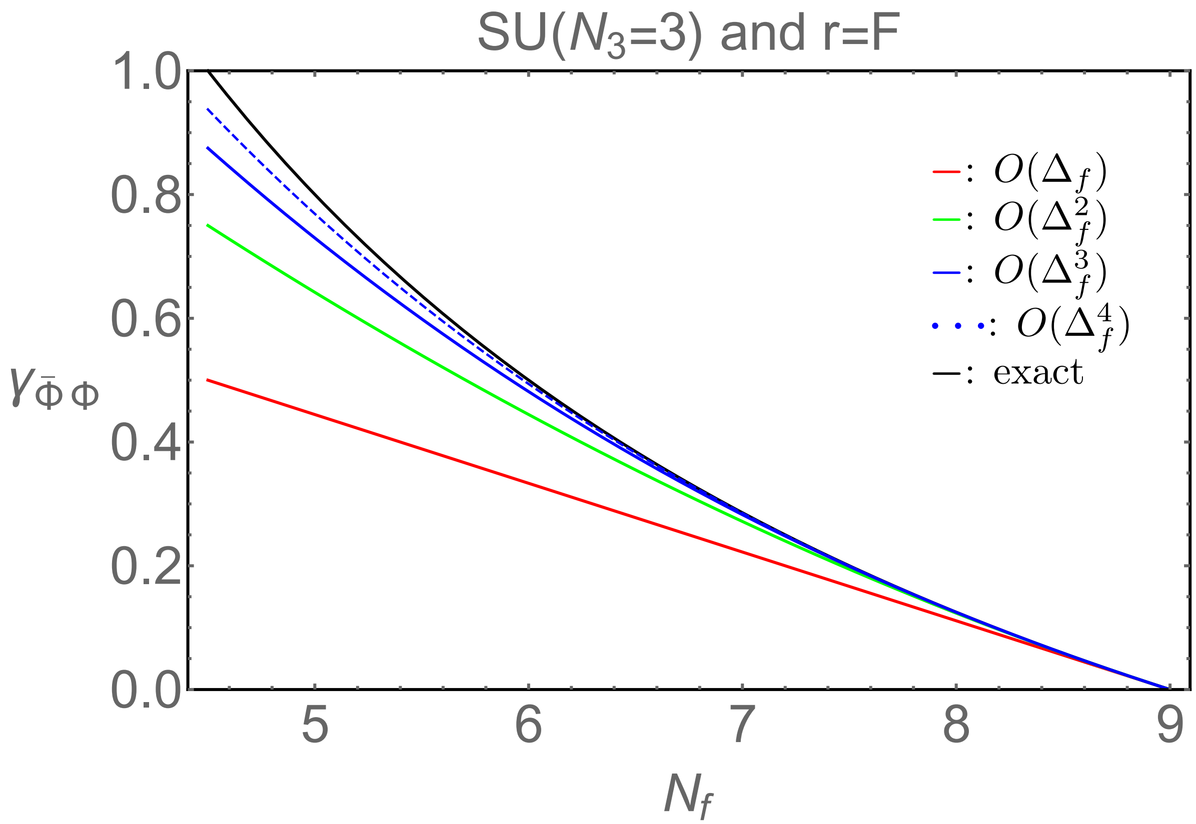
$$c_i = \left(\frac{2T(r)}{3C_2(G)}\right)^i$$

- This shows how the exact NSVZ result emerges order by order in perturbation theory.

- It also enables us to understand how well perturbation theory does order by order to *any* order without making a single loop calculation.

TAR, Phys. Rev. Lett. **117**, 071601 (2016)
TAR & R. Shrock, Phys. Rev. D **95**, 085012 (2017)

Supersymmetric QCD (SQCD)



- This is the order to which we can calculate in QCD

TAR, Phys. Rev. Lett. **117**, 071601 (2016)
TAR & R. Shrock, Phys. Rev. D **95**, 085012 (2017)

Supersymmetric QCD (SQCD)

- How about anomalous dimensions of other operators?

- In SQCD $$D_O = \frac{3}{2} R_O$$

- For baryons and antibaryons

$$B^{f_1 \ldots f_{N_c}} = \epsilon_{a_1 \ldots a_{N_c}} \Phi^{a_1, f_1} \cdots \Phi^{a_{N_c}, f_{N_c}}$$
$$\tilde{B}_{f_1 \ldots f_{N_c}} = \epsilon^{a_1 \ldots a_{N_c}} \tilde{\Phi}_{a_1, f_1} \cdots \tilde{\Phi}_{a_{N_c}, f_{N_c}}$$

- We find

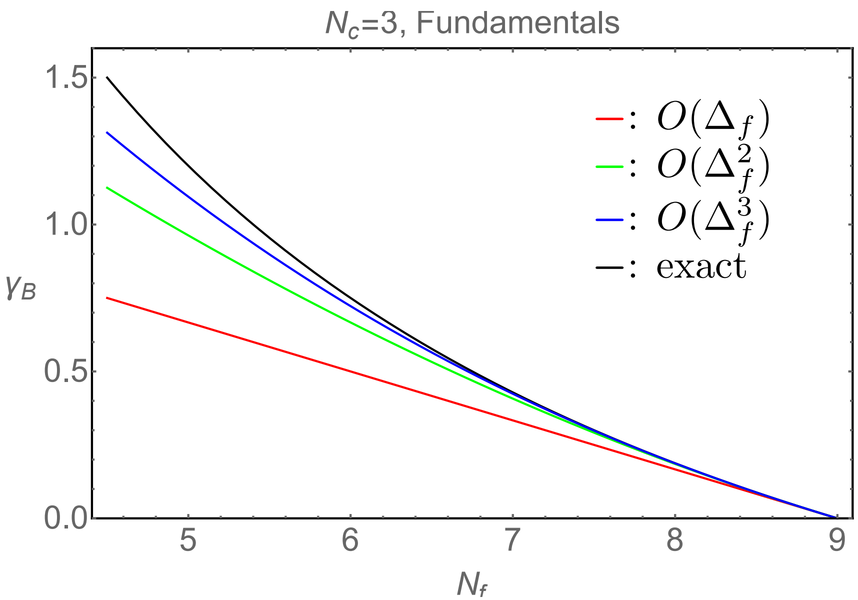
TAR & R. Shrock, arXiv:1706.06422, PRD in press

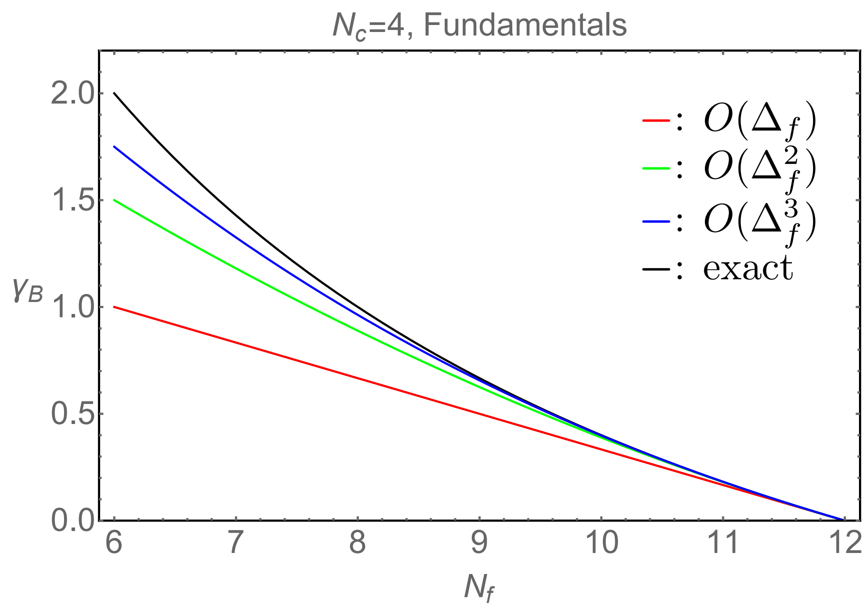
$$\gamma_B = \gamma_{\tilde{B}} = \frac{N_c}{2} \left(\frac{3N_c}{N_f} - 1 \right) = \frac{1}{6} \Delta_f + \frac{1}{18 N_c} \Delta_f^2 + \frac{1}{54 N_c^2} \Delta_f^3 + \ldots \qquad \Delta_f = 3N_c - N_f$$

- Or more generally

$$\Phi_{\rm prod} = M^{n_M} B^{n_B} \tilde{B}^{n_{\tilde{B}}} \qquad \gamma_{\Phi_{\rm prod}} = \left[n_M + \frac{n_B + n_{\tilde{B}}}{2} N_c \right] \left(\frac{3N_c}{N_f} - 1 \right)$$

Supersymmetric QCD (SQCD)





TAR & R. Shrock, Phys. Rev. D96 (2017), 105018

- In all cases is perturbation theory remarkably accurate.
- Note: all coefficients are always positive for all anomalous dimensions

Supersymmetric QCD (SQCD)

A$_2$

$$M_f^{f'} = \mathrm{Tr}\left(\tilde{\Phi}_f \Phi^{f'}\right)$$

$$B^{f_1\dots f_k} = \frac{1}{2^k k!}\epsilon_{a_1\dots a_{2k}}\Phi^{a_1 a_2 f_1}\dots\Phi^{a_{2k-1}a_{2k}f_k} \ , \qquad N_c = 2k$$

$$B^{f_1\dots f_{N_c}} = \frac{1}{N_c!}\epsilon_{a_1\dots a_{N_c}}\epsilon_{a'_1\dots a'_{N_c}}\Phi^{a_1 a'_1 f_1}\dots\Phi^{a_{N_c}a'_{N_c}f_{N_c}} \ , \qquad N_c = 2k+1$$

$$\gamma_B = \frac{N_c}{4}\left[\frac{3N_c}{N_f(N_c-2)} - 1\right] \ , \qquad N_c = 2k$$

$$\gamma_B = \frac{N_c}{2}\left[\frac{3N_c}{N_f(N_c-2)} - 1\right] \ , \qquad N_c = 2k+1$$

Adjoint

$$M_{ff'} = B_{ff'} = \tilde{B}_{ff'} = \mathrm{Tr}\left(\Phi_f \Phi_{f'}\right)$$

$$\gamma_M = \frac{3}{2N_f} - 1$$

- Again, in all cases perturbation theory is remarkably accurate!
- Note: all coefficients are always positive for all anomalous dimensions

S$_2$

$$M_f^{f'} = \mathrm{Tr}\left(\tilde{\Phi}_f \Phi^{f'}\right)$$

$$B^{f_1\dots f_{N_c}} = \frac{1}{N_c!}\epsilon_{a_1\dots a_{N_c}}\epsilon_{a'_1\dots a'_{N_c}}\Phi^{a_1 a'_1 f_1}\dots\Phi^{a_{N_c}a'_{N_c}f_{N_c}}$$

$$\tilde{B}_{f_1\dots f_{N_c}} = \frac{1}{N_c!}\epsilon^{a_1\dots a_{N_c}}\epsilon^{a'_1\dots a'_{N_c}}\Phi_{a_1 a'_1 f_1}\dots\Phi_{a_{N_c}a'_{N_c}f_{N_c}}$$

$$\gamma_M = \frac{3N_c}{N_f(N_c+2)} - 1$$

$$\gamma_B = \gamma_{\tilde{B}} = \frac{N_c}{2}\left[\frac{3N_c}{N_f(N_c+2)} - 1\right]$$

TAR & R. Shrock, Phys. Rev. D96 (2017), 105018

Central charges: a & c

- In the presence of a curved background the trace of the energy-momentum tensor is

$$T^{\mu}{}_{\mu} = \frac{1}{(4\pi)^2}\left(cW_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma} - aE_4\right)$$

- Where $W_{\mu\nu\rho\sigma}$ is the Weyl tensor and E_4 is the Euler density.

- Their values in the IR are known exactly

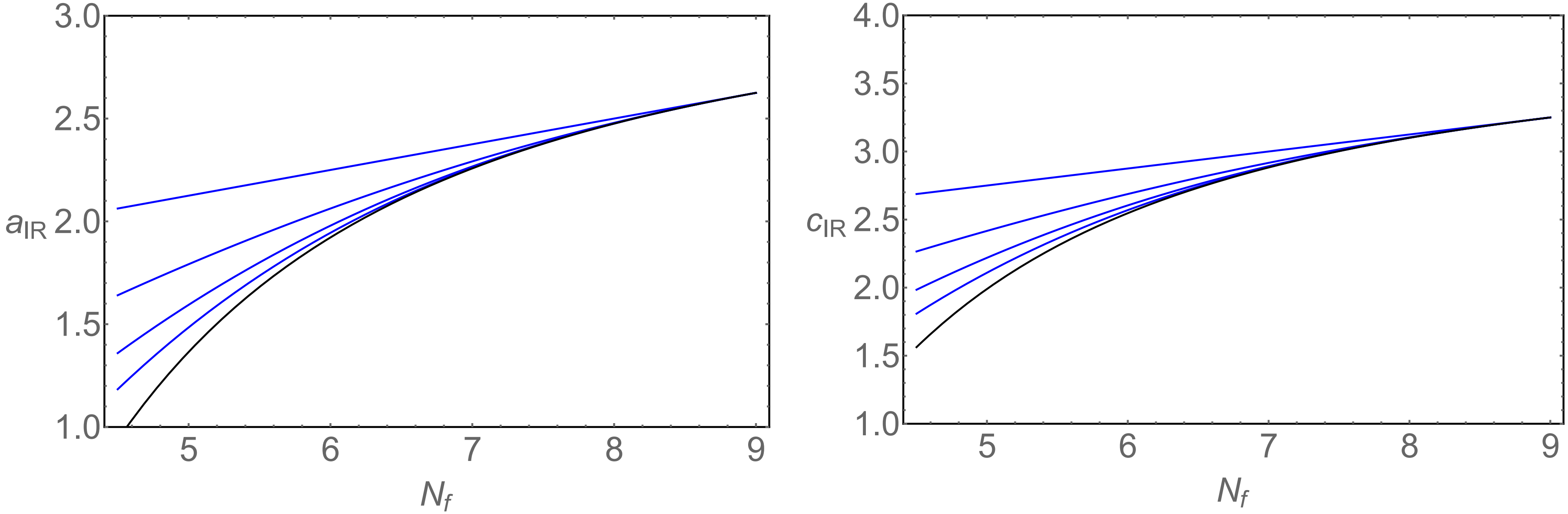
D. Anselmi et al., Nucl. Phys. B526 (1998) 543-571

$$a_{IR} = a_{UV} - \frac{1}{144}\sum_{j=2}^{\infty}\frac{j+1}{(3N_c)^{n-2}}\Delta_f^n\ , \qquad a_{UV} = \left(-\frac{3}{16} + \frac{5N_c^2}{16}\right) - \frac{N_c}{24}\Delta_f$$

$$c_{IR} = c_{UV} + \frac{N_c}{48}\left(2\Delta_f - \sum_{j=2}^{\infty}\frac{j+1}{(3N_c)^{j-1}}\Delta_f^j\right)\ , \qquad c_{UV} = \left(-\frac{1}{8} + \frac{3N_c^2}{8}\right) - \frac{N_c}{12}\Delta_f$$

TAR & R. Shrock, arXiv:1711.01116, Submitted for pub.

Central charges: a & c



TAR & R. Shrock, arXiv:1711.01116, Submitted for pub.

One last important quantity…

- The slope, $\beta'(\alpha_{IR})$, is a scheme independent quantity

D. Anselmi et al., Nucl. Phys. B **491**, 221 (1997)
E. Gardi et al., JHEP **9903**, 024 (1999)

$$\beta' = d_2\Delta_f^2 + d_3\Delta_f^3 + d_4\Delta_f^4 + \ldots$$

with

$$d_2 = \frac{2T(r)^2}{3C_2(G)C_2(r)} \qquad d_3 = \frac{2T(r)^3(C_2(G)+2C_2(r))}{(3C_2(G)C_2(r))^2}$$

$$d_4 = \frac{(N_c^4 - 2N_c^2 + 5) - 18N_c^2(N_c^2+1)\zeta_3}{108N_c^2(N_c^2-1)^3}$$

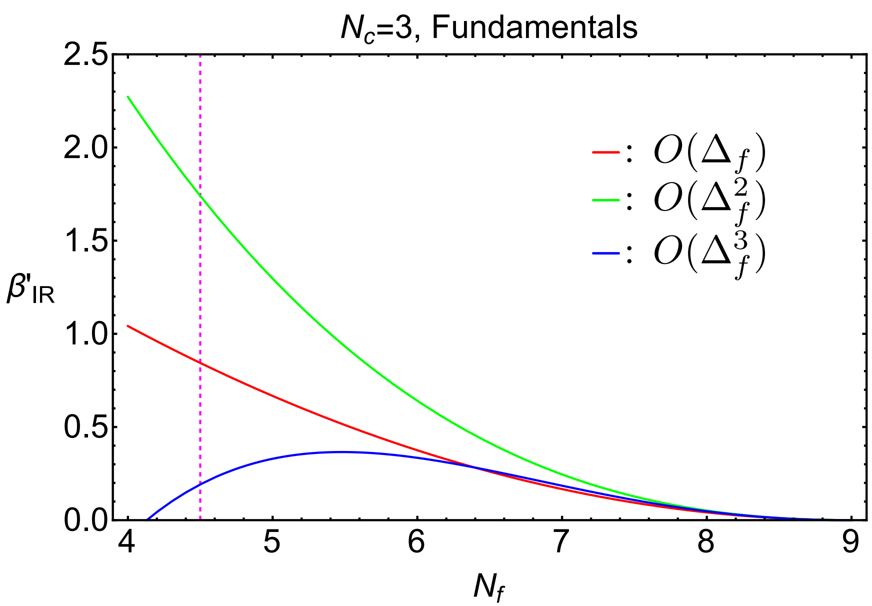
TAR & R. Shrock, arXiv:1706.06422, PRD in press

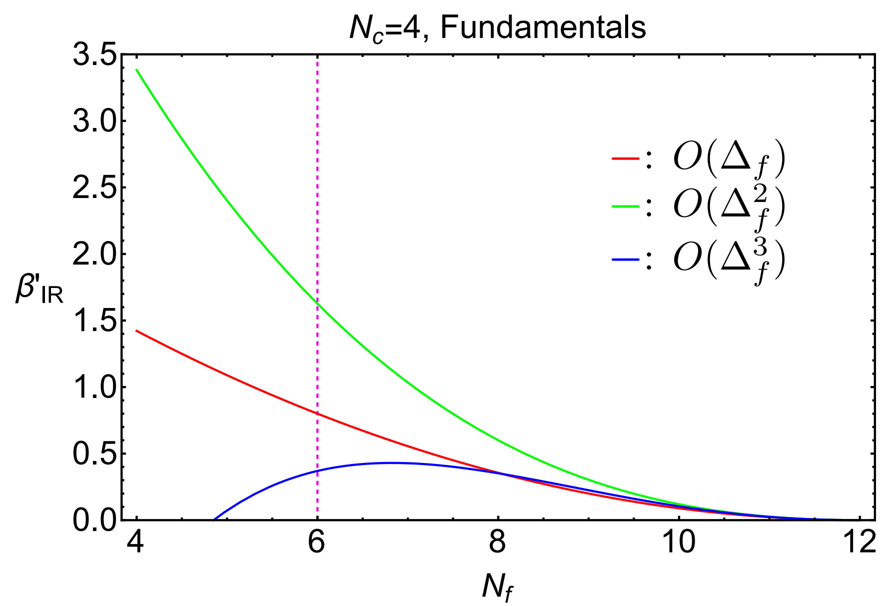
- It is not known exactly and guessing it would be difficult.

- An example:

$$N_c = 3: \qquad \beta' = 4.17\times 10^{-2}\Delta_f^2 + 9.83\times 10^{-3}\Delta_f^3 - 3.78\times 10^{-3}\Delta^4$$

One last important quantity…





- Duality dictates: $$\beta'\left(N_f = \frac{3}{2}N_c\right) = 0$$

D. Anselmi et al., Nucl. Phys. B **491**, 221 (1997)
E. Gardi et al., JHEP **9903**, 024 (1999)
TAR & R. Shrock, Phys. Rev. D96 (2017), 105018

- Interesting behavior despite seemingly slow convergence.

How about QCD.....?

QCD

- In QCD we know the 5-loop beta function and 5-loop (mass) anomalous dimension, so we can calculate

$$\gamma_{\bar{\psi}\psi} = c_1 \Delta_f + c_2 \Delta_f^2 + c_3 \Delta_f^3 + c_4 \Delta_f^4 + \ldots$$

$$\beta' = d_2 \Delta_f^2 + d_3 \Delta_f^3 + d_4 \Delta_f^4 + d_5 \Delta_f^5 + \ldots \qquad = -\gamma_{F^2}$$

TAR & R. Shrock, Phys. Rev. D **94**, 105014 (2016)
TAR & R. Shrock, Phys. Rev. D **94**, 125005 (2016)
TAR & R. Shrock, Phys. Rev. D **95**, 085012 (2017)
TAR & R. Shrock, Phys. Rev. D **95**, 105004 (2017)

$$c_1 = \frac{8T_rC_r}{C_A(7C_A + 11C_r)}\,, \qquad c_2 = \frac{4T_r^2C_r(35C_A^2 + 636C_AC_r + 352C_r^2)}{3C_A^2(7C_A + 11C_r)^3}$$

$$\begin{aligned}
c_3 = {} & \frac{4T_rC_r}{81C_A^4(7C_A + 11C_r)^5}\Bigg(-55419T_r^2C_A^5 + 432012T^2C_A^4C_r + 5632T_r^2C_r\frac{d_A^{abcd}d_A^{abcd}}{d_A}(-5 + 132\zeta_3) \\
& +16C_A^3\left(122043T_r^2C_r^2 + 6776\frac{d_F^{abcd}d_F^{abcd}}{d_A}(-11 + 24\zeta_3)\right) \\
& +704C_A^2\left(1521T_r^2C_r^3 + 112T_r\frac{d_F^{abcd}d_A^{abcd}}{d_A}(4 - 39\zeta_3) + 242C_r\frac{d_F^{abcd}d_F^{abcd}}{d_A}(-11 + 24\zeta_3)\right) \\
& +32T_rC_A\left(53361T_rC_r^4 - 3872C_r\frac{d_F^{abcd}d_A^{abcd}}{d_A}(-4 + 39\zeta_3) + 112T_r\frac{d_A^{abcd}d_A^{abcd}}{d_A}(-5 + 132\zeta_3)\right)\Bigg)
\end{aligned}$$

$$\begin{aligned}
c_4 = {} & \frac{T_f^2}{3^5C_A^5D^7}\Bigg[C_AC_fT_f^2\Big(19515671C_A^6 - 131455044C_A^5C_f + 1289299872C_A^4C_f^2 + 2660221312C_A^3C_f^3 \\
& + 1058481072C_A^2C_f^4 + 6953709312C_AC_f^5 + 1275715584C_f^6\Big) + 2^{10}C_fT_f^2D\Big(5789C_A^2 - 4168C_AC_f - 6820C_f^2\Big)\frac{d_A^{abcd}d_A^{abcd}}{d_A} \\
& - 2^{10}C_AC_fT_fD\Big(41671C_A^2 - 125477C_AC_f - 53240C_f^2\Big)\frac{d_R^{abcd}d_A^{abcd}}{d_A} \\
& - 2^8\cdot 11^2C_A^2C_fD(2569C_A^2 + 18604C_AC_f - 7964C_f^2\Big)\frac{d_R^{abcd}d_R^{abcd}}{d_A} \\
& - 2^{14}\cdot 3C_AT_f^2D^3\frac{d_R^{abcd}d_A^{abcd}}{d_R} + 2^{13}\cdot 33C_A^2T_fD^3\frac{d_R^{abcd}d_R^{abcd}}{d_R} \\
& + 2^8D\Big[-3C_AC_fT_f^2D\Big(4991C_A^4 - 17606C_A^3C_f + 33240C_A^2C_f^2 - 30672C_AC_f^3 + 9504C_f^4\Big) \\
& - 2^4C_fT_f^2\frac{d_A^{abcd}d_A^{abcd}}{d_A}\Big(17206C_A^2 - 60511C_AC_f - 45012C_f^2\Big) + 40C_AC_fT_f\frac{d_R^{abcd}d_A^{abcd}}{d_A}\Big(35168C_A^2 - 154253C_AC_f - 88572C_f^2\Big) \\
& - 88C_A^2C_f\frac{d_R^{abcd}d_R^{abcd}}{d_A}\Big(973C_A^2 - 93412C_AC_f - 56628C_f^2\Big) + 1440C_AT_f^2D^2\frac{d_R^{abcd}d_A^{abcd}}{d_R} - 7920C_A^2T_fD^2\frac{d_R^{abcd}d_R^{abcd}}{d_R}\Big]\zeta_3 \\
& + \frac{4505600C_AC_fD^2}{d_A}\Big[-4T_f^2d_A^{abcd}d_A^{abcd} + 2T_fd_R^{abcd}d_A^{abcd}(10C_A + 3C_f) + 11C_Ad_R^{abcd}d_R^{abcd}(C_A - 3C_f)\Big]\zeta_5\Bigg]\,. \qquad (3.5)
\end{aligned}$$

$$d_2 = \frac{2^5 T_f^2}{3^2 C_A D} \qquad d_3 = \frac{2^7 T_f^3 (5C_A + 3C_f)}{3^3 C_A^2 D^2}$$

$$\begin{aligned} d_4 \; = \; -\frac{2^3 T_f^2}{3^6 C_A^4 D^5} \Bigg[& -3C_A T_f^2 \left(137445 C_A^4 + 103600 C_A^3 C_f + 72616 C_A^2 C_f^2 + 951808 C_A C_f^3 - 63888 C_f^4\right) \\ & - 5120 T_f^2 D \frac{d_A^{abcd} d_A^{abcd}}{d_A} + 90112 C_A T_f D \frac{d_R^{abcd} d_A^{abcd}}{d_A} - 340736 C_A^2 D \frac{d_R^{abcd} d_R^{abcd}}{d_A} \\ & + 8448 D \left[C_A^2 T_f^2 \left(21 C_A^2 + 12 C_A C_f - 33 C_f^2\right) + 16 T_f^2 \frac{d_A^{abcd} d_A^{abcd}}{d_A} - 104 C_A T_f \frac{d_R^{abcd} d_A^{abcd}}{d_A} + 88 C_A^2 \frac{d_R^{abcd} d_R^{abcd}}{d_A} \right] \zeta_3 \Bigg] \end{aligned}$$

$$\begin{aligned} d_5 \; = \; \frac{2^4 T_f^3}{3^7 C_A^5 D^7} \Bigg[& - C_A T_f^2 \Big(39450145 C_A^6 + 235108272 C_A^5 C_f + 1043817726 C_A^4 C_f^2 + 765293216 C_A^3 C_f^3 \\ & - 737283360 C_A^2 C_f^4 + 730646400 C_A C_f^5 - 356750592 C_f^6 \Big) - 2^9 T_f^2 D \frac{d_A^{abcd} d_A^{abcd}}{d_A} (6139 C_A^2 + 2192 C_A C_f - 3300 C_f^2) \\ & + 2^9 C_A T_f D \frac{d_R^{abcd} d_A^{abcd}}{d_A} (43127 C_A^2 - 28325 C_A C_f - 2904 C_f^2) + 15488 C_A^2 D \frac{d_R^{abcd} d_R^{abcd}}{d_A} (2975 C_A^2 + 8308 C_A C_f - 12804 C_f^2) \\ & + 2^7 D \bigg[3 C_A T_f^2 D \left(6272 C_A^4 - 49823 C_A^3 C_f + 40656 C_A^2 C_f^2 + 13200 C_A C_f^3 + 2112 C_f^4\right) \\ & + 2^4 T_f^2 \frac{d_A^{abcd} d_A^{abcd}}{d_A} (19516 C_A^2 - 18535 C_A C_f - 21780 C_f^2) - 2^3 C_A T_f \frac{d_R^{abcd} d_A^{abcd}}{d_A} (182938 C_A^2 - 297649 C_A C_f - 197472 C_f^2) \\ & - 88 C_A^2 \frac{d_R^{abcd} d_R^{abcd}}{d_A} (245 C_A^2 + 62524 C_A C_f + 42108 C_f^2) \bigg] \zeta_3 \\ & + 2^{10} \cdot 55 C_A D^2 \bigg[9 C_A T_f^2 D (C_A + 2C_f)(C_A - C_f) + 160 T_f^2 \frac{d_A^{abcd} d_A^{abcd}}{d_A} \\ & - 80 T_f (10 C_A + 3 C_f) \frac{d_R^{abcd} d_A^{abcd}}{d_A} - 440 C_A (C_A - 3C_f) \frac{d_R^{abcd} d_R^{abcd}}{d_A} \bigg] \zeta_5 \Bigg] \, . \end{aligned} \qquad (4.9)$$

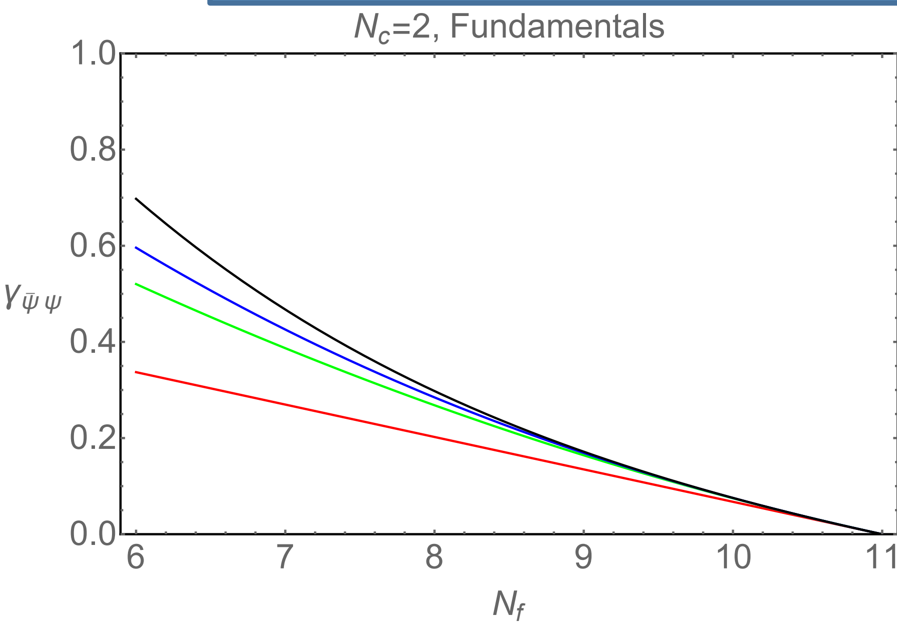
QCD

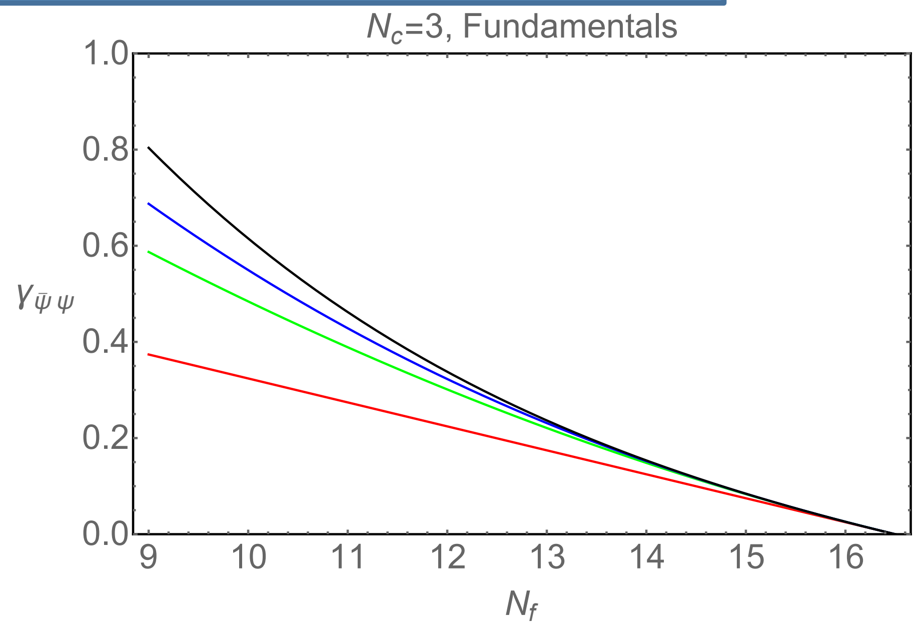
- Observations:
  - ζ_n appears. It is not a scheme artifact
  - Higher order group invariants $d_R^{abcd}d_R^{abcd},\ d_R^{abcd}d_A^{abcd},\ d_A^{abcd}d_A^{abcd}$ appear. They are not scheme artifacts
  - Very hard to guess the entire series (unfortunately)
- How about convergence? $(N_c = 3 \quad \text{and fundamentals})$

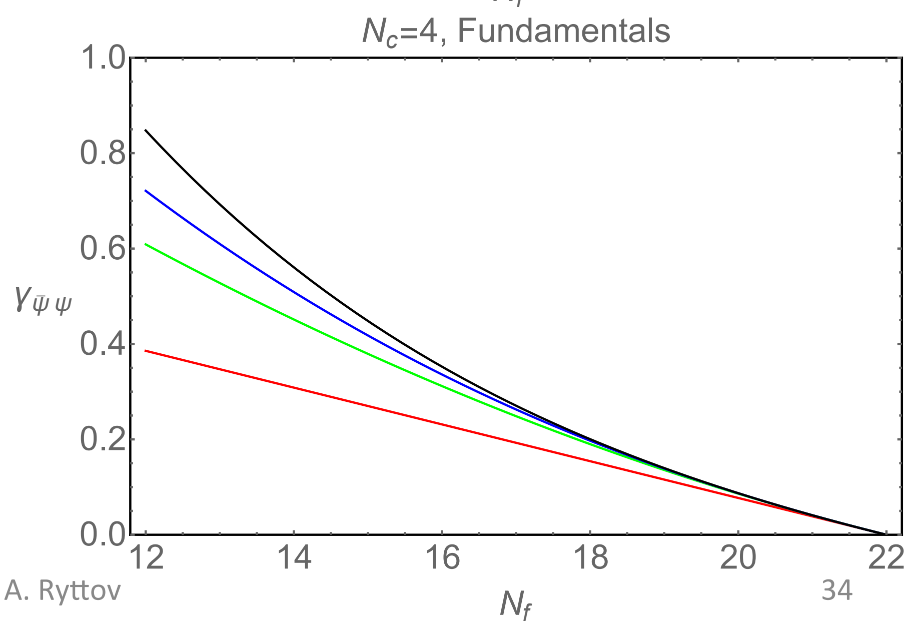
$$\gamma_{\bar{\psi}\psi} = 4.9\times 10^{-2}\Delta_f + 3.8\times 10^{-3}\Delta_f^2 + 2.4\times 10^{-4}\Delta_f^3 + 3.7\times 10^{-5}\Delta_f^4$$

$$\beta' = 8.3\times 10^{-3}\Delta_f^2 + 9.8\times 10^{-4}\Delta_f^2 - 4.6\times 10^{-5}\Delta_f^3 - 5.6\times 10^{-6}\Delta_f^4$$

QCD



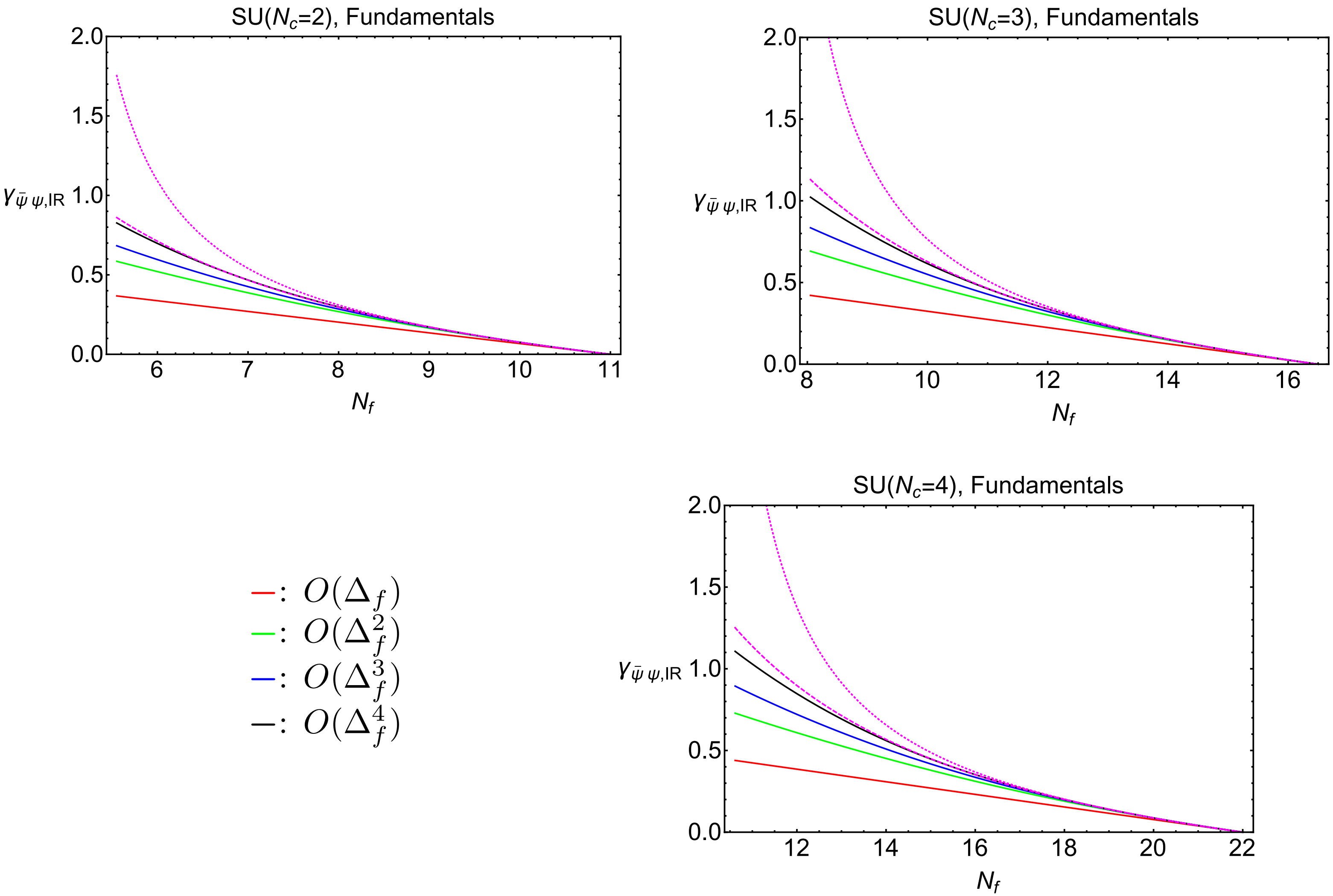




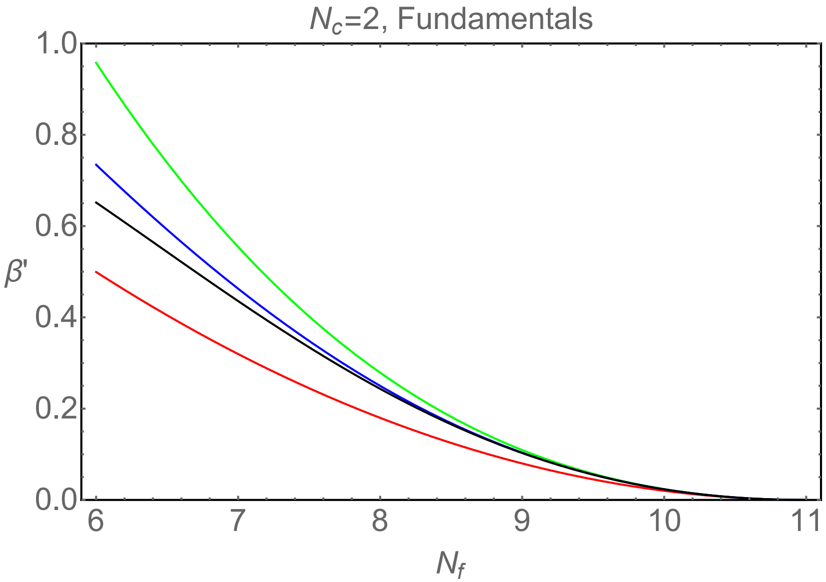
—: $O(\Delta_f)$
—: $O(\Delta_f^2)$
—: $O(\Delta_f^3)$
—: $O(\Delta_f^4)$

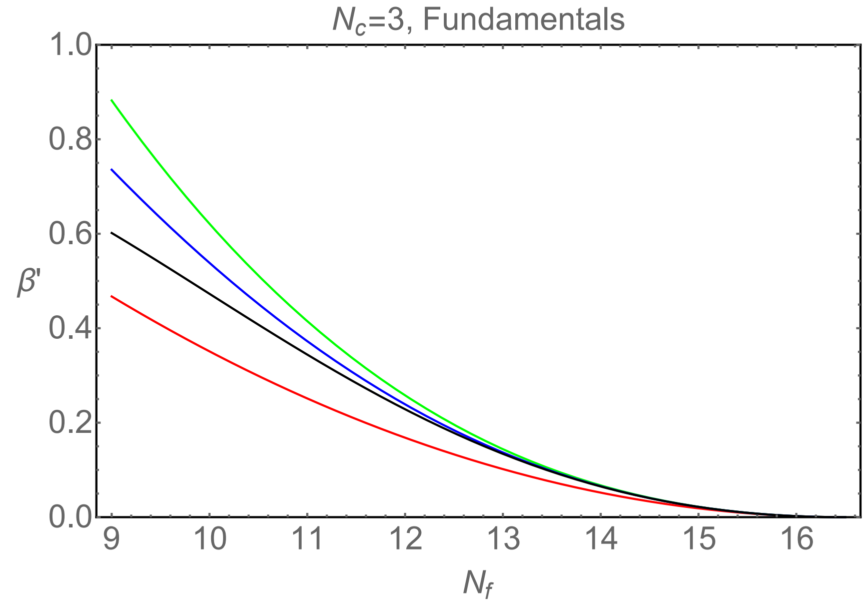
- Good convergence (similar to SQCD) up to $O(\Delta_f^4)$
- Coefficients are positive (also for SO(N), Sp(N))

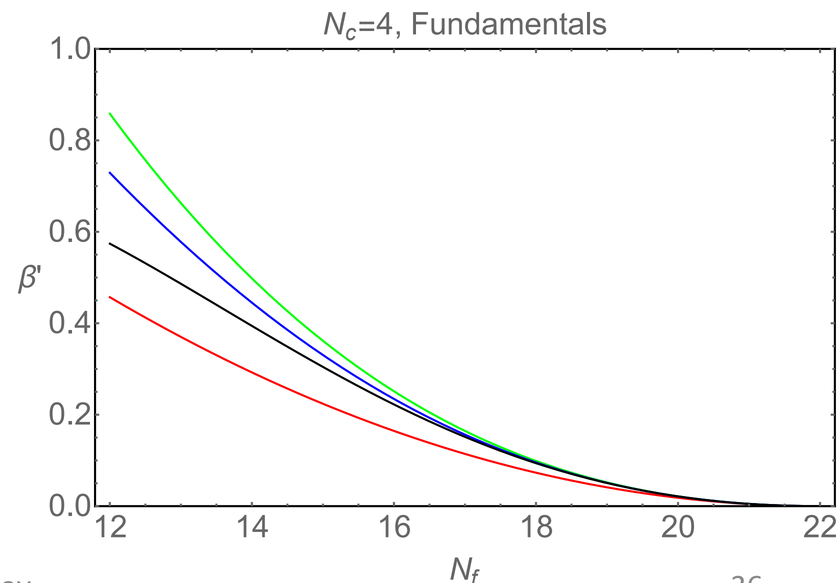
Pade approximation



QCD







—: $O(\Delta_f)$
—: $O(\Delta_f^2)$
—: $O(\Delta_f^3)$
—: $O(\Delta_f^4)$

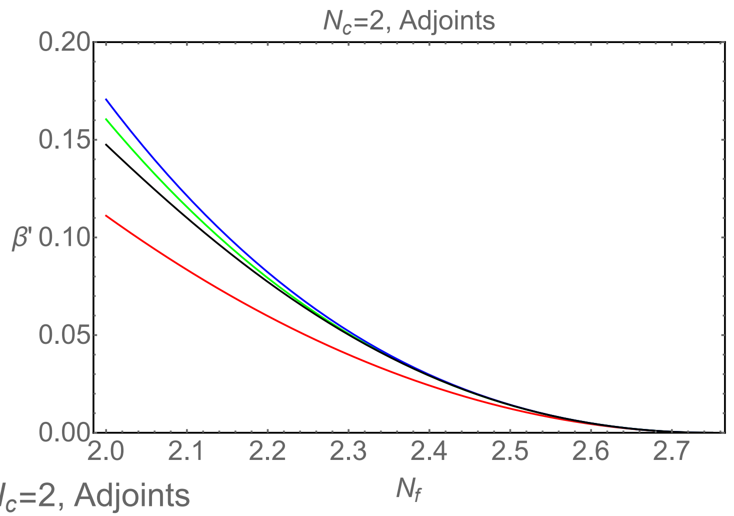
- Implications for (possible) dualities?
- Where does it cross 0?

Adjoint QCD

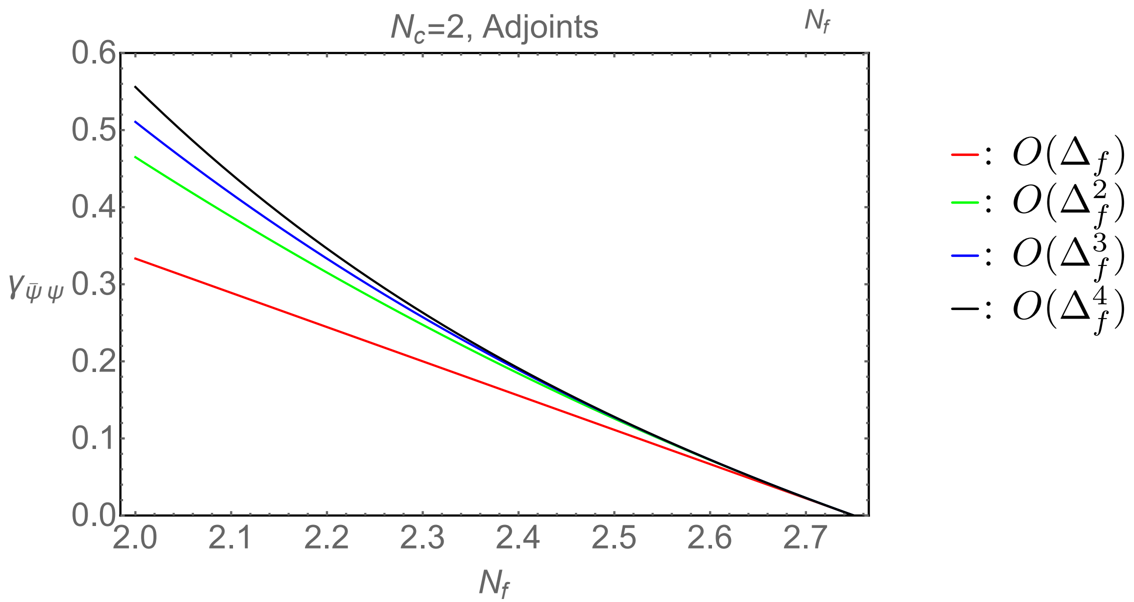
- Does it simplify for adjoint fermions? In SQCD the anomalous dimension is independent of the number of colors N_c.
- In adjoint QCD

$$c_1 = \frac{4}{9} \qquad c_2 = \frac{341}{1458} \qquad c_3 = \frac{61873}{472392} - \frac{592}{6561}\frac{1}{N_c^2}$$

$$c_4 = \frac{53389393}{612220032} + \frac{368}{59049}\zeta_3 + \left(-\frac{2170}{59049} + \frac{33952}{177147}\zeta_3\right)\frac{1}{N_c^2}$$

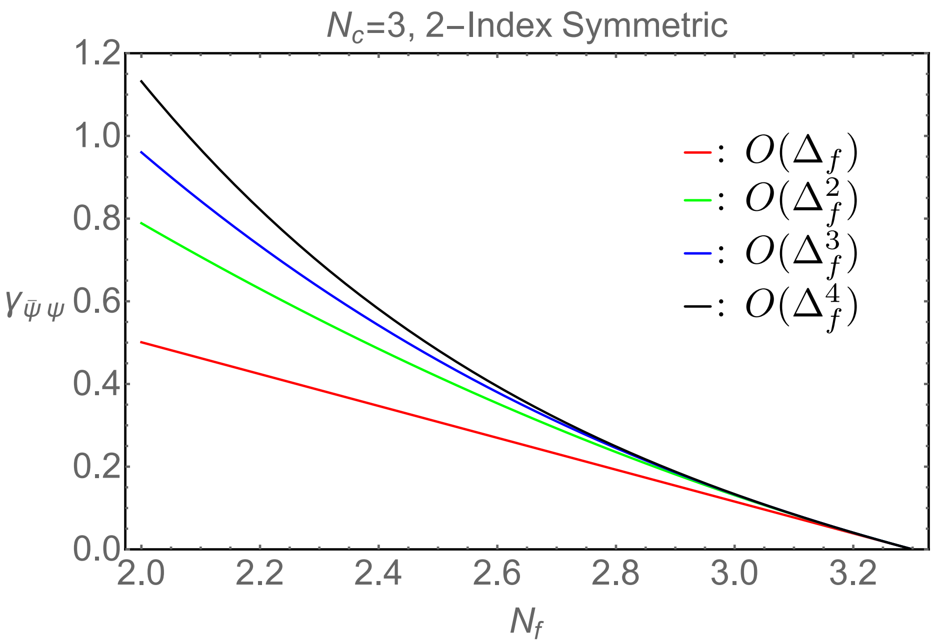


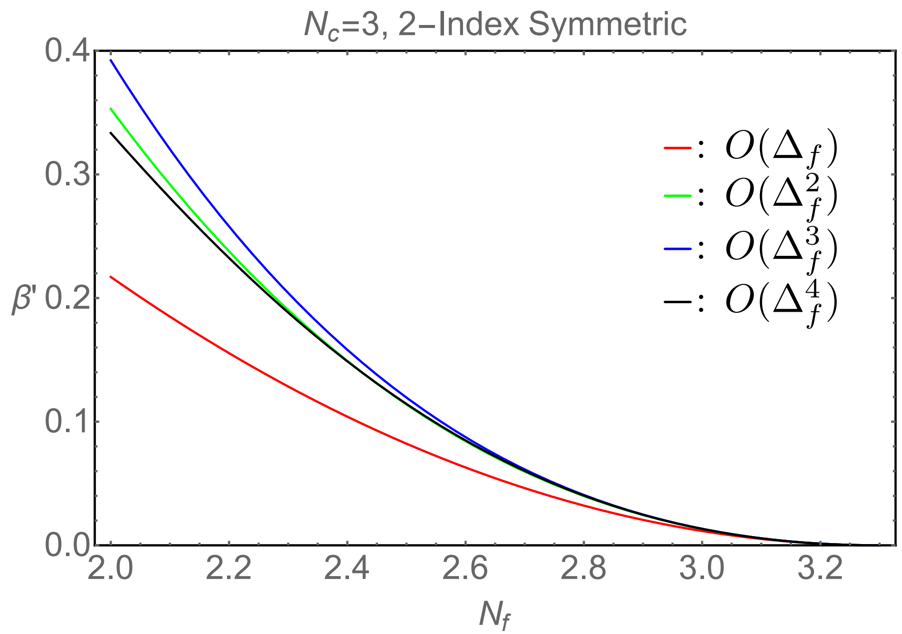
- There is a mild N_c dependence
- Also ζ_3 dependence



TAR, Phys. Rev. Lett. **117**, 071601 (2016)
TAR & R. Shrock, Phys. Rev. D **95**, 105004 (2017)

2-Index Symmetric Representation





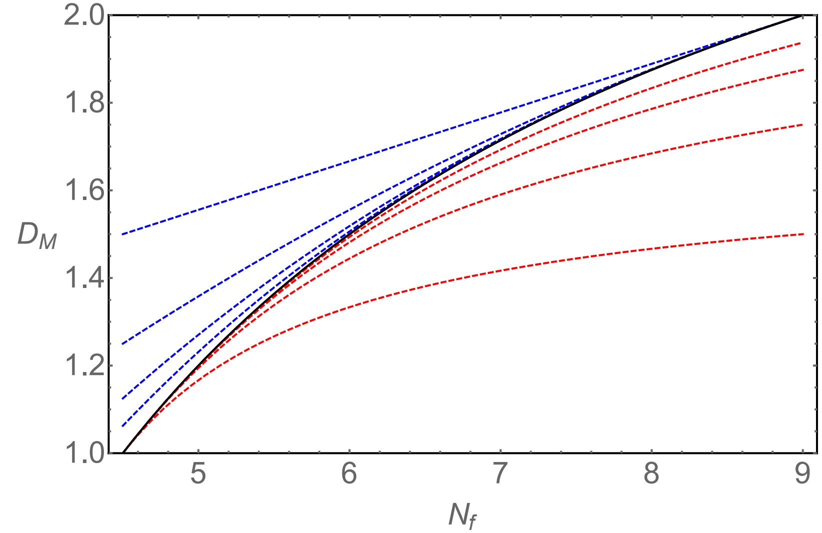
How about dualities…..?

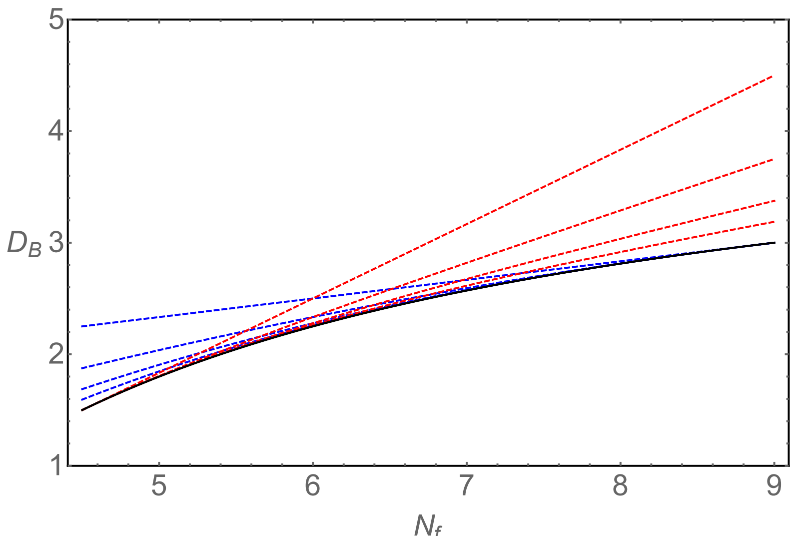
SUSY Duality

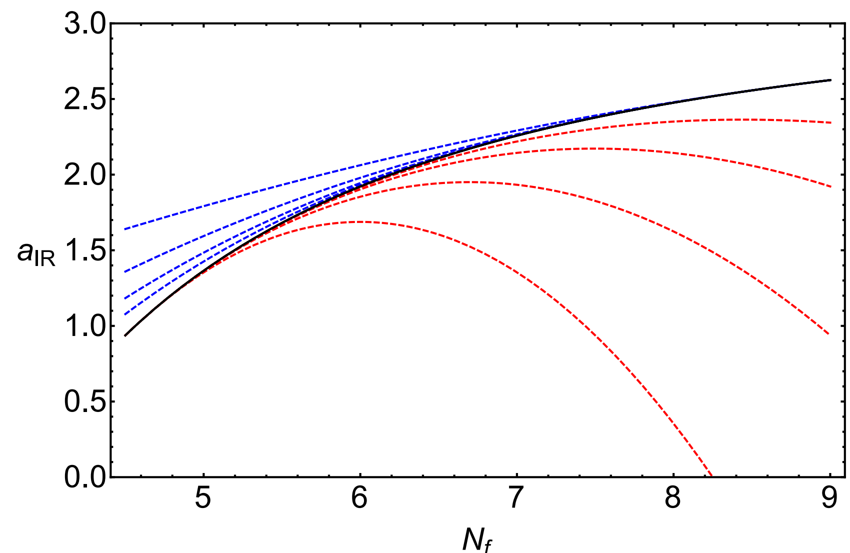
	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
Q	N_c	N_f	1	1	$\frac{N_f-N_c}{N_f}$
$\tilde{Q}$	$\overline{N}_c$	1	$\overline{N}_f$	-1	$\frac{N_f-N_c}{N_f}$

	$SU(N_f-N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
q	N_c	$\overline{N}_f$	1	$\frac{N_c}{N_f-N_c}$	$\frac{N_c}{N_f}$
$\tilde{q}$	$\overline{N}_c$	1	N_f	$-\frac{N_c}{N_f-N_c}$	$\frac{N_c}{N_f}$
ϕ	1	N_f	$\overline{N}_f$	0	$\frac{2(N_f-N_c)}{N_f}$

$$W = \lambda \phi q \tilde{q}$$







Non-SUSY Duality

Work in progress……….

Viewing things from a distance

- We have found a tool that is readily available in both QCD and SQCD. It works remarkably well in SQCD and there is evidence that it works equally well in QCD.

- In fact: in SQCD the physics is controlled by the first (handful) of orders throughout the entire conformal window.

- We now have a tool to ask about possible dual desctiptions of non-SUSY theories. To appear very soon..