

Introduction to QCD

lecture one

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Helmholtz Alliance *Physics at the Terascale* School on Parton Distribution Functions, Oct 20, 2009, Hamburg

Introduction to QCD

- Introduction to QCD I *Tuesday, October 20, 2009*
- Introduction to QCD II *Wednesday, October 21, 2009*

Lecture 1

- The big questions
- Basics of perturbative QCD
- QCD factorization
- Parton distributions

Things we do know

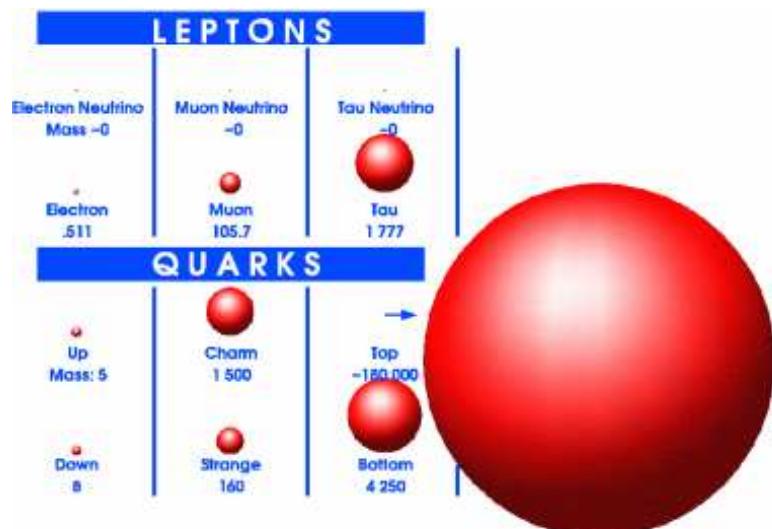
- Elementary particles
 - fermions (leptons, quarks)
(constituents of matter)
 - bosons
(carrier particles of forces)

Elementary Particles				Force Carriers
Quarks	u up	c charm	t top	
	d down	s strange	b bottom	γ photon
Leptons	ν_e e neutrino	ν_μ μ neutrino	ν_τ τ neutrino	W W boson
	e electron	μ	τ	Z Z boson

3 → I II III ← Generations

Things we do know

- Elementary particles
 - fermions (leptons, quarks)
(constituents of matter)
 - bosons
(carrier particles of forces)
- Masses of fermions



Elementary Particles				Force Carriers
Quarks	u up	c charm	t top	
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	ν_e e neutrino	ν_μ μ neutrino	ν_τ τ neutrino	W W boson
	e electron	μ muon	τ tau	Z Z boson

3 → I II III ← Generations

massive neutrinos first glimpse beyond SM



The Nobel Prize in Physics 2008

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"



Photo: SCANPIX



Photo: Kyodo/Reuters



Photo: Kyoto University

Yoichiro Nambu

1/2 of the prize

USA

Enrico Fermi Institute,
University of Chicago
Chicago, IL, USA

b. 1921

Makoto Kobayashi

1/4 of the prize

Japan

High Energy Accelerator Research Organization

Toshihide Maskawa

1/4 of the prize

Japan

b. 1940

Titles, data and places given above refer to the time of the award.

Nobel prize 2008 for spontaneous symmetry breaking and CP violation

Things we want to know

The Big Questions

- What is the origin of mass?
- What is 96% of the universe made of?
- Why is there no more antimatter?
- Do extra dimensions of space really exist?
- ...

Things we want to know

The Big Questions

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Highest energies at colliders until 201x

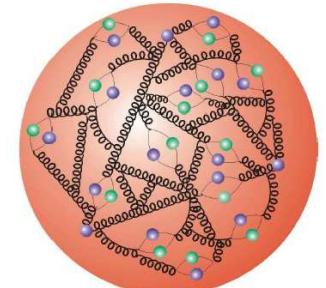
Energy frontier

- Search for Higgs boson, new massive particles at highest energies

$$E = m c^2$$

Hadron colliders

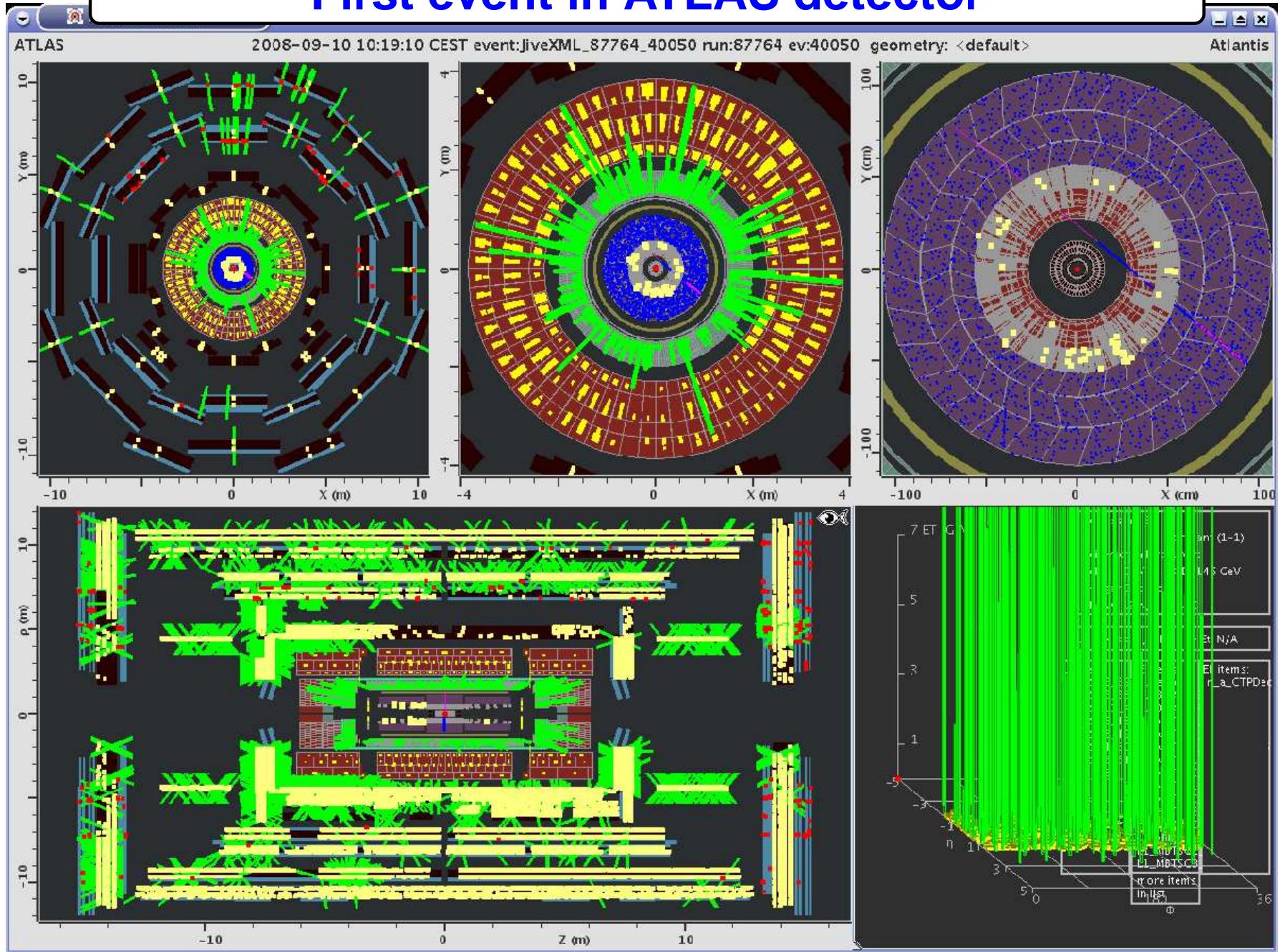
- Proton-(anti)-proton collisions reach TeV-scale
 - Tevatron $\sqrt{S} = 1.96 \text{ TeV}$ (until 2009), LHC with $\sqrt{S} = 7 \dots 14 \text{ TeV}$
- Proton: composite multi-particle bound state
 - collider: "wide-band beams" of quarks and gluons
- Protons are heavy
 - no significant synchrotron radiation $\sim \left(\frac{E}{m}\right)^4 / r$



Large Hadron Collider



First event in ATLAS detector



Theoretical predictions for the LHC

Challenge

- Solve master equation

new physics = data – Standard Model

- New physics searches require understanding of SM background
- LHC explores the energy frontier
 - theory has to match or exceed accuracy of LHC data

Theoretical predictions for the LHC

Challenge

- Solve master equation

new physics = data – Standard Model

- New physics searches require understanding of SM background
- LHC explores the energy frontier
 - theory has to match or exceed accuracy of LHC data

Tools

- LHC is a QCD machine
 - perturbative QCD is essential and established part of toolkit
(we no longer “test” QCD)
- Electroweak corrections important for precision predictions

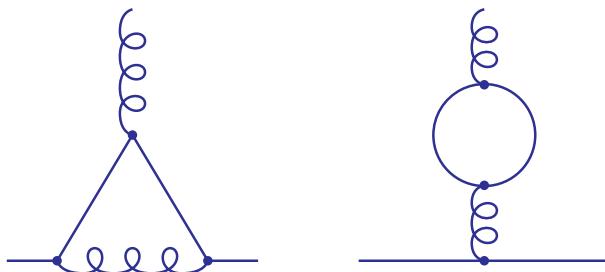
Plan

Rest of this lecture

- Asymptotic freedom
- Review of Feynman rules
- Colour ordering
- Spinor conventions
- Helicity amplitudes

Running coupling

- Effective coupling constant α_s
 - depends on resolution, momentum scale Q



- screening (like in QED)
- anti-screening (color charge of g)
- Scale dependence governed by β -function of QCD

$$\frac{d \alpha_s}{d \ln Q^2} = \beta(\alpha_s) = -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 - \dots$$

- perturbative expansion with coefficients $\beta_0, \beta_1, \beta_2, \dots$

$$\beta_0 = \frac{1}{4\pi} \left(\frac{11}{3} C_A - \frac{2}{3} n_f \right)$$

Asymptotic freedom in QCD



The Nobel Prize in Physics 2004

"for the discovery of asymptotic freedom in the theory of the strong interaction"



David J. Gross



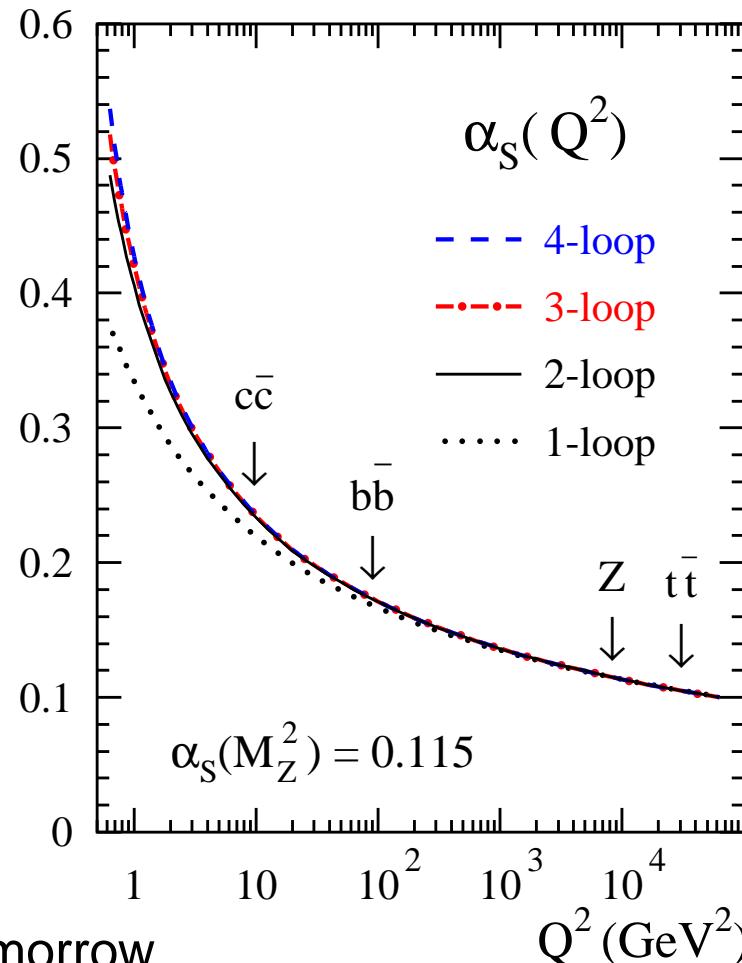
H. David Politzer



Frank Wilczek

Asymptotic freedom in QCD

- Solution of QCD β -function
 - perturbation theory applicable at large scales (but $\alpha_s \gg \alpha_{\text{QED}}$)



- Exercise session tomorrow
 - U. Langenfeld

Feynman rules (I)

- Propagators

- fermions, gluons, ghosts
- covariant gauge

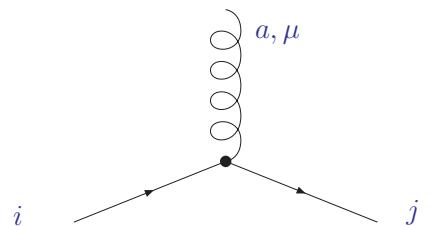
$$i \quad \xrightarrow{p} \quad j$$
$$\delta^{ij} \frac{i}{\not{p} - m}$$

$$a, \mu \quad \xrightarrow[p]{\text{wavy}} \quad b, \nu$$
$$\delta^{ab} i \left(\frac{-g^{\mu\nu}}{p^2} + (1 - \lambda) \frac{p^\mu p^\nu}{(p^2)^2} \right)$$

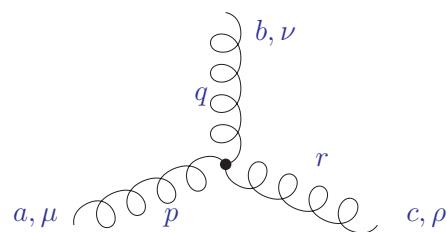
$$a \quad \xrightarrow[p]{\text{dashed}} \quad b$$
$$\delta^{ab} \frac{i}{\not{p}^2}$$

Feynman rules (II)

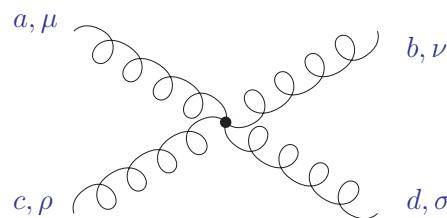
- Vertices



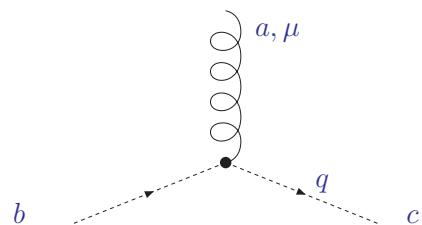
$$-i g (t^a)_{ji} \gamma^\mu$$



$$-g f^{abc} ((p - q)^\rho g^{\mu\nu} + (q - r)^\mu g^{\nu\rho} + (r - p)^\nu g^{\mu\rho})$$



$$\begin{aligned} & -i g^2 f^{xac} f^{xbd} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) \\ & -i g^2 f^{xad} f^{xbc} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}) \\ & -i g^2 f^{xab} f^{xcd} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) \end{aligned}$$



$$g f^{abc} q^\mu$$

Multiparticle production

- Number of Feynman diagrams for n -parton amplitudes grows quickly with n
 - example n -gluon amplitudes

n	diagrams
4	4
5	25
6	220
7	2485
8	34300
9	559405
10	10525900

- Feynman diagram evaluation very inefficient for many legs
 - too many diagrams, terms per diagram, kinematic variables

Quantum numbers

QCD amplitudes for n partons

- Complete amplitude \mathcal{A}
 - dependence on momenta k_i , helicities λ_i and colour a_i
- Keep track of quantum phases
 - computing transition amplitude rather than cross-section
- Use helicity/colour quantum-numbers of amplitude \mathcal{A}
 - decompose \mathcal{A} into simpler, gauge-invariant pieces
(so called partial amplitudes A)
- Transition to numerical evaluation at very end of calculation
 - square amplitudes, sum over helicities and colours and obtain unpolarized cross-sections
- NLO radiative corrections
 - combination of virtual and real corrections

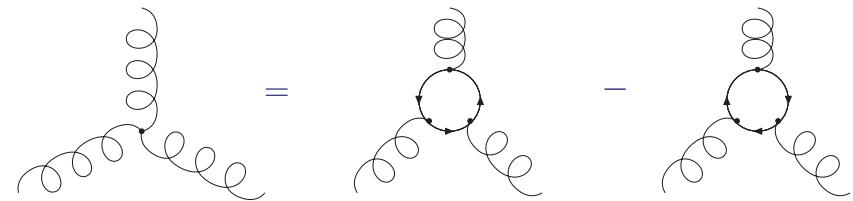
Colour ordering (I)

- $SU(N)$ -generators t^a from fundamental representation

$$\text{Tr} (t^a t^b) = \delta^{ab}$$

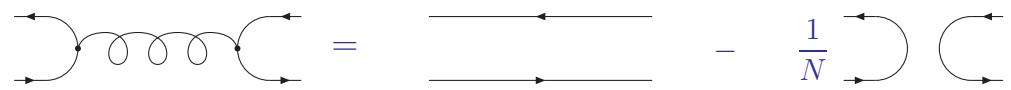
- $SU(N)$ -generators f^{abc} of adjoint representation

$$f^{abc} = -\frac{i}{\sqrt{2}} \text{Tr} ([t^a, t^b] t^c)$$



- Fierz identity

$$(t^a)_{i_1}^{j_1} (t^b)_{i_2}^{j_2} = \delta_{i_1}^{j_2} \delta_{i_2}^{j_1} - \frac{1}{N} \delta_{i_1}^{j_1} \delta_{i_2}^{j_2}$$



Partial amplitudes

- Color decomposition of n -parton amplitudes \mathcal{A}_n
 - colour ordered partial amplitudes A with kinematic information

Colour ordering (II)

- Tree level amplitude $\mathcal{A}^{\text{tree}}$ with n external gluons
 - sum over all non-cyclic permutations S_n/Z_n of external gluons

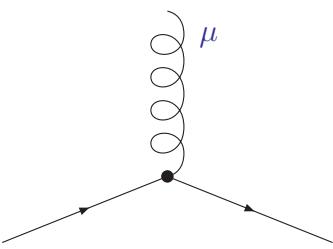
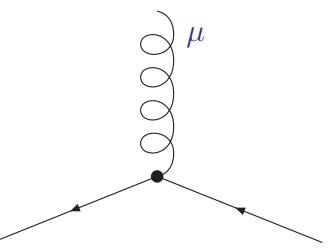
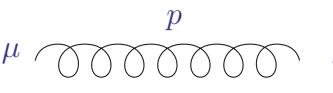
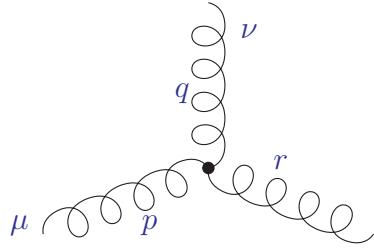
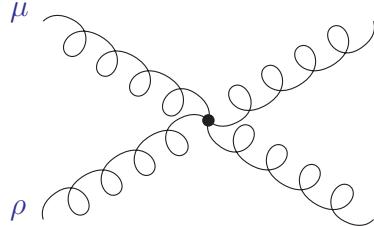
$$\mathcal{A}_n^{\text{tree}}(\{k_i, \lambda_i, a_i\}) = g^{n-2} \sum_{\sigma \in S_n/Z_n} \text{Tr}(t^{a_\sigma(1)} \dots t^{a_\sigma(n)}) A(\sigma(1^{\lambda_1}), \dots, \sigma(n^{\lambda_n}))$$

- Tree level amplitude $\mathcal{A}^{\text{tree}}$ with $q\bar{q}$ and $n-2$ external gluons

$$\mathcal{A}_n^{\text{tree}}(\{k_i, \lambda_i, a_i\}) = g^{n-2} \sum_{\sigma \in S_{n-2}} \left(t^{a_\sigma(3)} \dots t^{a_\sigma(n)} \right)_{i_2}^{j_1} A(1_{\bar{q}}^{\lambda_1}, 2_q^{\lambda_2}, \sigma(3^{\lambda_3}), \dots, \sigma(n^{\lambda_n}))$$

Colour ordered rules (III)

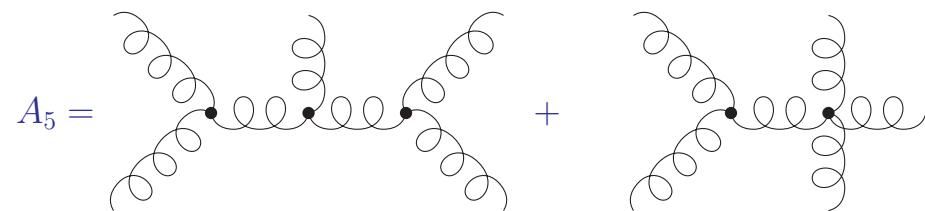
- Feynman rules for colour ordered partial amplitudes
 - vertices, propagators (omitting ghosts)

 μ	$-\frac{i}{\sqrt{2}} \gamma^\mu$	 μ	$+\frac{i}{\sqrt{2}} \gamma^\mu$
μ  p	$-i \frac{g^{\mu\nu}}{p^2}$	p	$i \frac{1}{p}$
μ  q	$\frac{i}{\sqrt{2}} ((p-q)^\rho g^{\mu\nu} + (q-r)^\mu g^{\nu\rho} + (r-p)^\nu g^{\mu\rho})$		
μ  ρ	$ig^{\mu\sigma} g^{\nu\rho} - \frac{i}{2} (g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma})$		

Colour ordered rules (IV)

- Calculate only diagrams with cyclic colour ordering

- example 5-gluon amplitude A_5
(10 diagrams instead of 25)



- In general big reduction in number of Feynman diagrams
 - example n -gluon amplitudes

n	diagrams	colour ordered diagrams
4	4	3
5	25	10
6	220	36
7	2485	133
8	34300	501
9	559405	1991
10	10525900	7335

Spinor conventions (I)

- Massless Dirac spinors $\psi(k)$ with momentum k
 - chiral representation of Dirac γ matrices

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- chiral projections $\psi_{\pm}(k) = \frac{1}{2}(1 \pm \gamma_5)\psi(k)$
- Massless Weyl spinors $u_{\pm}(k)$ with \pm -chirality
 - construction of chiral components of Dirac spinor (4-dim) $\psi(k)$ from two Weyl spinors (2-dim) $u_{\pm}(k)$
- (Weyl) spinor inner-products

$$\langle jl \rangle = \langle j^- | l^+ \rangle = \overline{u_-}(k_j) u_+(k_l)$$

$$[jl] = \langle j^+ | l^- \rangle = \overline{u_+}(k_j) u_-(k_l) = \text{sign}(k_j^0 k_l^0) \langle lj \rangle^*$$

- Scalar product of Lorentz-vectors

$$\langle ij \rangle [ji] = 2k_i \cdot k_j = s_{ij}$$

Spinor conventions (II)

- Gordon identity

$$\langle i^\pm | \gamma^\mu | i^\pm \rangle = 2 k_i^\mu$$

- Antisymmetry

$$\langle ji \rangle = -\langle ij \rangle, \quad [ji] = -[ij], \quad \langle ii \rangle = [ii] = 0$$

- Fierz rearrangement

$$\langle i^+ | \gamma^\mu | j^+ \rangle \langle k^+ | \gamma_\mu | l^+ \rangle = 2 [ik] \langle lj \rangle$$

- Charge conjugation of current

$$\langle i^+ | \gamma^\mu | j^+ \rangle = \langle j^- | \gamma^\mu | i^- \rangle$$

- Schouten identity

$$\langle ij \rangle \langle kl \rangle = \langle ik \rangle \langle jl \rangle + \langle il \rangle \langle kj \rangle$$

- Momentum conservation in n -point amplitude $\sum_{i=1}^n k_i^\mu = 0$

$$\sum_{\substack{i=1 \\ i \neq j, k}}^n [ji] \langle ik \rangle = 0$$

Spinor conventions (III)

- Polarization vector for massless gauge boson (helicity states ± 1)
 - spinor representation for boson with momentum k

$$\epsilon_\mu^\pm(k, q) = \pm \frac{\langle q^\mp | \gamma_\mu | k^\mp \rangle}{\sqrt{2} \langle q^\mp | k^\pm \rangle}$$

- massless auxiliary vector q (reference momentum) reflects on-shell gauge degrees of freedom

Properties

- ϵ^\pm transverse to k for any q , that is $\epsilon^\pm(k, q) \cdot k = 0$.
- Complex conjugation reverses helicity: $(\epsilon_\mu^+)^* = \epsilon_\mu^-$
- Normalization

$$\epsilon^+ \cdot (\epsilon^+)^* = \epsilon^+ \cdot \epsilon^- = -\frac{1}{2} \frac{\langle q^- | \gamma_\mu | k^- \rangle \langle q^+ | \gamma^\mu | k^+ \rangle}{\langle qk \rangle [qk]} = -1$$

$$\epsilon^+ \cdot (\epsilon^-)^* = \epsilon^+ \cdot \epsilon^+ = \frac{1}{2} \frac{\langle q^- | \gamma_\mu | k^- \rangle \langle q^- | \gamma^\mu | k^- \rangle}{\langle qk \rangle^2} = 0$$

Spinor conventions (IV)

- Choice of reference momentum q can simplify calculation
 - useful identities

$$\epsilon^\pm(k_i, q) \cdot q = 0$$

$$\epsilon^+(k_i, q) \cdot \epsilon^+(k_j, q) = \epsilon^-(k_i, q) \cdot \epsilon^-(k_j, q) = 0$$

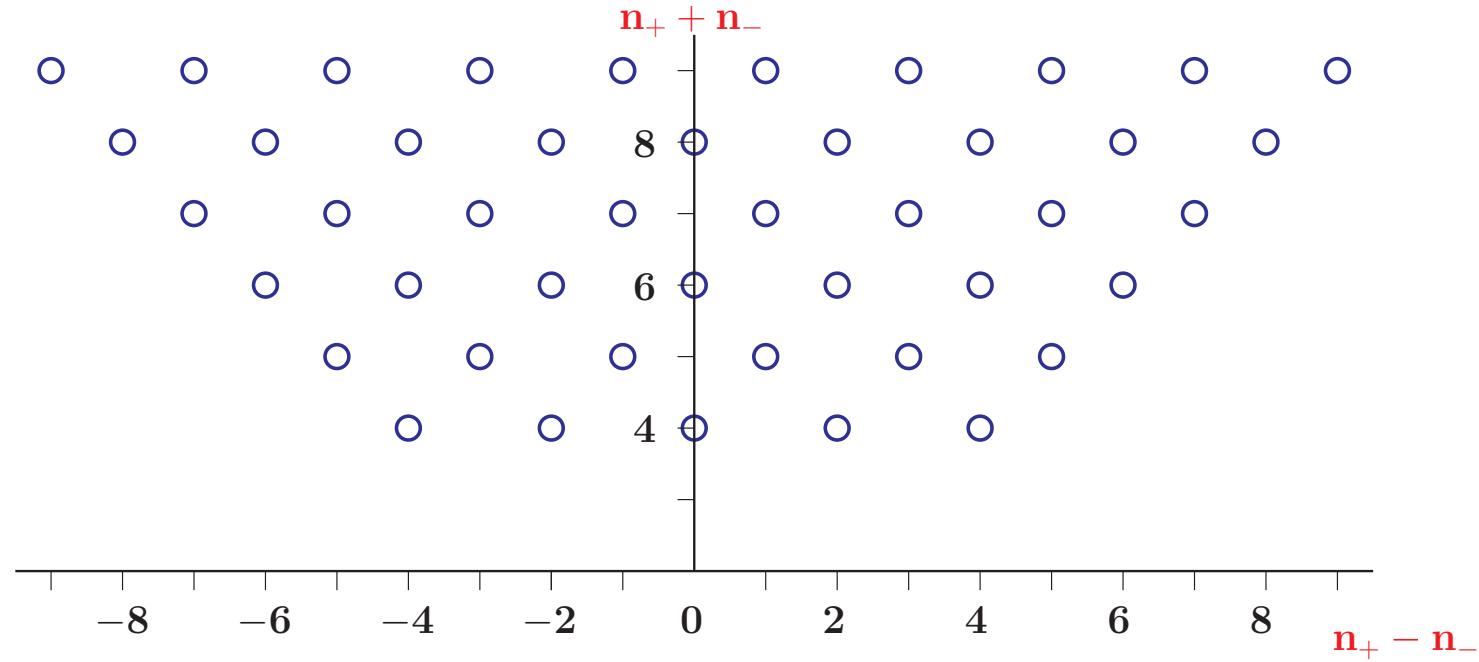
$$\epsilon^+(k_i, k_j) \cdot \epsilon^-(k_j, q) = \epsilon^+(k_i, q) \cdot \epsilon^-(k_j, k_i) = 0$$

$$\not{\epsilon}^+(k_i, k_j)|j^+\rangle = \not{\epsilon}^-(k_i, k_j)|j^-\rangle = 0$$

$$\langle j^+|\not{\epsilon}^-(k_i, k_j) = \langle j^-|\not{\epsilon}^+(k_i, k_j) = 0$$

Helicity amplitudes (I)

- n -gluon helicity amplitudes
 - difference between n_+ positive and n_- negative helicities



- each row describes scattering process with n_+ positive and n_- negative helicities
- each circle represents one allowed helicity configuration

Helicity amplitudes (II)

- Example 5-gluon amplitude A_5
 - result of computing the 25 diagrams for the five-gluon process

$$A_5^{\text{tree}}(1^\pm, 2^+, \dots, 5^+) = 0$$

$$A_5^{\text{tree}}(1^-, 2^-, 3^+, \dots, 5^+) = i \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

n -point amplitudes

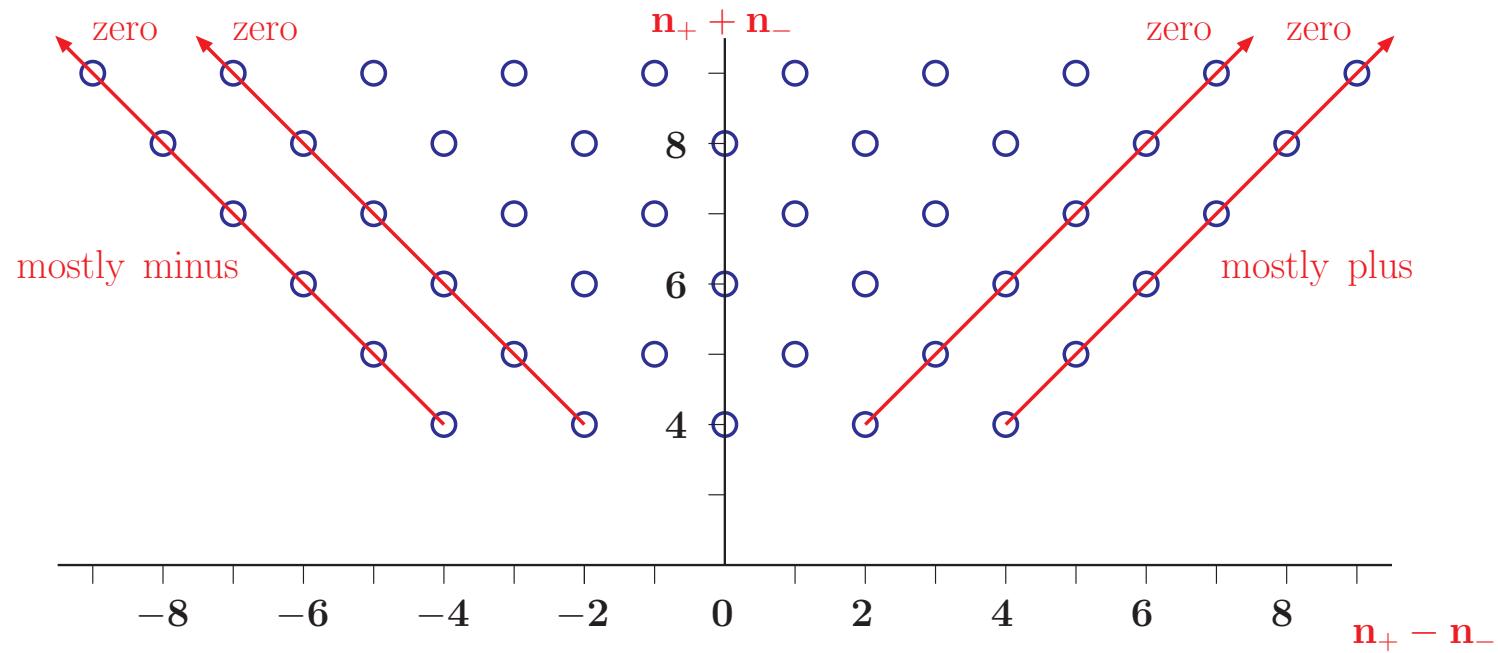
- Generally, for n -gluon amplitude A_n
 - $A_n^{\text{tree}}(1^\pm, 2^+, \dots, n^+) = 0$
 - maximal helicity violating (MHV) amplitudes

Parke, Taylor '86 Berends, Giele '87

$$A_n^{\text{tree}}(1^-, 2^-, 3^+, \dots, n^+) = i \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

Helicity amplitudes (III)

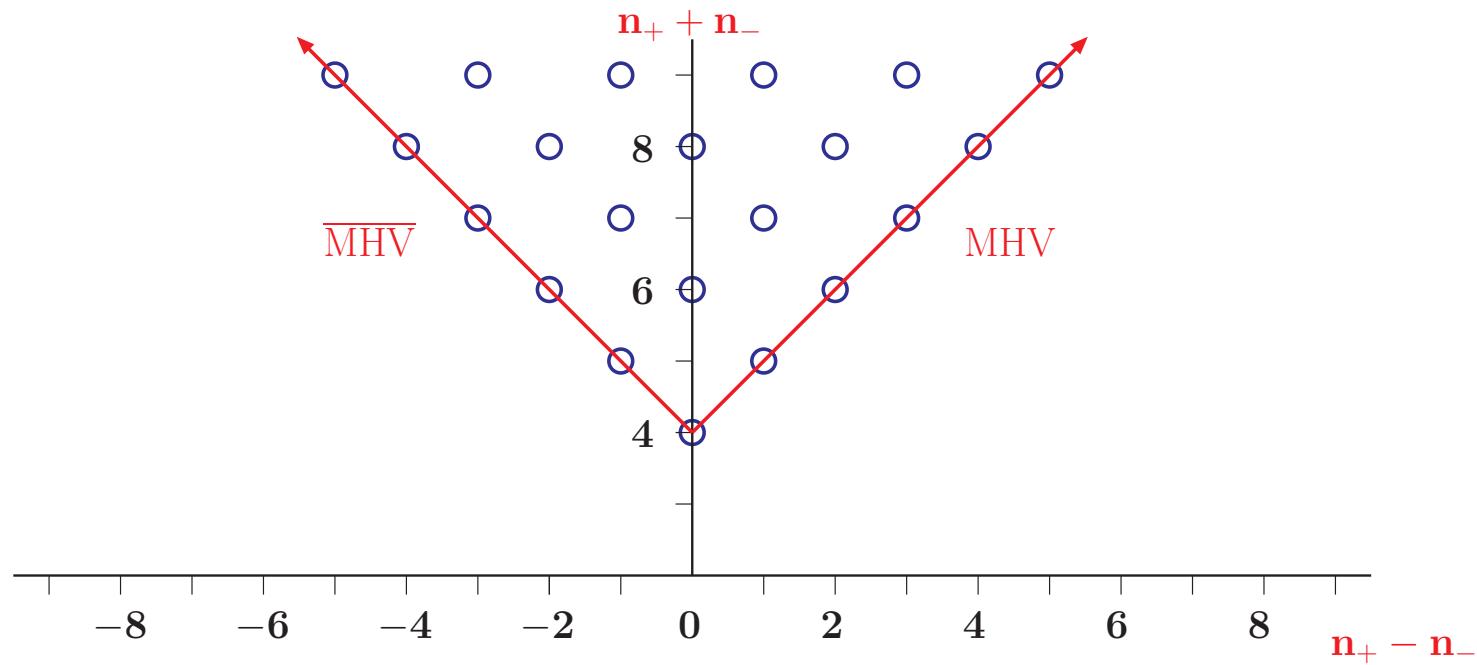
- n -gluon helicity amplitudes



- effective supersymmetry at tree level $A_n^{\text{tree}}(1^\pm, 2^+, \dots, n^+) = 0$

Helicity amplitudes (IV)

- n -gluon helicity amplitudes



- maximal helicity violating amplitudes Parke, Taylor '86

$$A_n^{\text{tree}}(1^-, 2^-, 3^+, \dots, n^+) = i \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

Helicity amplitudes (V)

Applications in practice

- Jet cross sections at LHC
 - simplest examples: hadronic di-jet production
 - e.g. $q\bar{q} \rightarrow q'\bar{q}'$
$$\frac{d\sigma}{dt du} = \alpha_s^2 \pi \frac{N_c^2 - 1}{4N_c^2} \frac{t^2 + u^2}{s^2}$$
- Exercise session today
→ S. Badger

Plan (cont'd)

Rest of this lecture

- QCD factorization
- Parton distributions
- Evolution equations

Perturbative QCD at Work

- QCD – the gauge theory of the strong interactions
- QCD covers dynamics in a large range of scales
 - asymptotically free theory of quarks and gluons at short distances
 - confining theory of hadrons at long distances
- Essential and established part of toolkit for discovering new physics
 - Tevatron and LHC
 - we no longer “test” QCD

Basic concepts of perturbative QCD

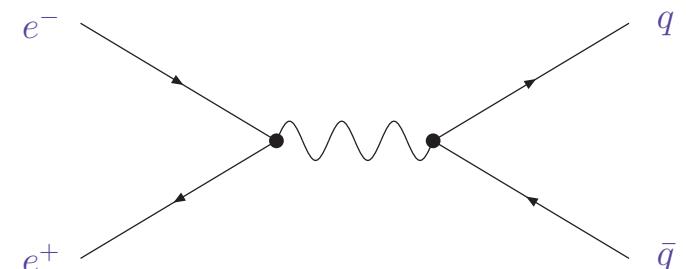
- Theoretical framework for QCD predictions at high energies relies on few basic concepts
 - infrared safety
 - factorization
 - evolution

Infrared safety

- Small class of cross sections at high energies and decay rates directly calculable in perturbation theory
- Infrared safe quantities
 - free of long range dependencies at leading power in large momentum scale Q Kinoshita '62; Lee, Nauenberg '64
- General structure of cross section
 - large momentum scale Q , renormalization scale μ
$$Q^2 \hat{\sigma} \left(Q^2, \mu^2, \alpha_s(\mu^2) \right) = \sum_n \alpha_s^n c^{(n)}(Q^2/\mu^2)$$
- Examples
 - total cross section in $e^+ e^-$ -annihilation
$$R^{\text{had}}(s) = \frac{\sigma(e^+ e^- \rightarrow \text{hadrons})}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)}$$
 - jet cross sections in $e^+ e^-$ -annihilation
 - total width of Z -boson

Soft and collinear singularities

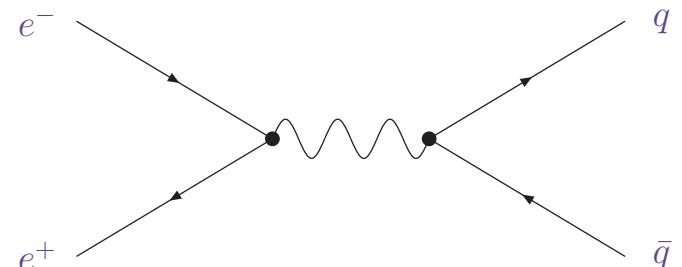
- $e^+ e^-$ -annihilation (massless quarks)
 - Born cross section $\sigma^{(0)} = \frac{4\pi\alpha^2}{3s}$



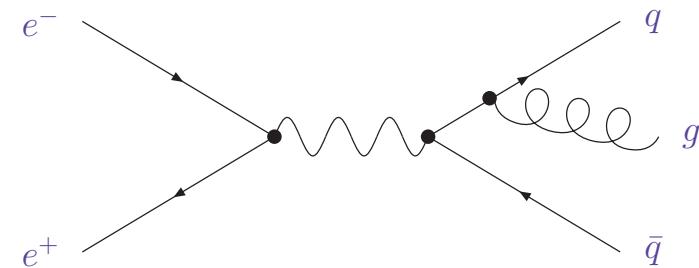
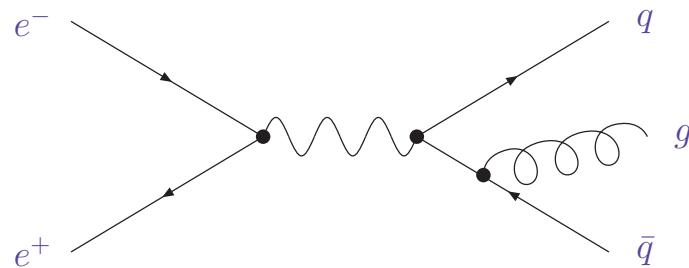
Soft and collinear singularities

- $e^+ e^-$ -annihilation (massless quarks)

- Born cross section $\sigma^{(0)} = \frac{4\pi\alpha^2}{3s}$



- Study QCD corrections (real emissions)

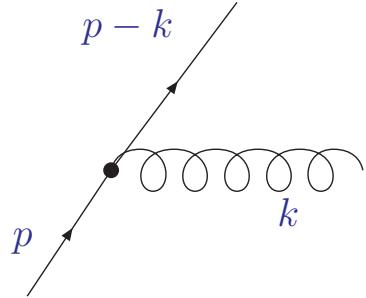


- Cross section
 - dimensional regularization $D = 4 - 2\epsilon$ (with $f(\epsilon) = 1 + \mathcal{O}(\epsilon^2)$)

$$\sigma^{q\bar{q}g} = \sigma^{(0)} 3 \sum_q e_q^2 f(\epsilon) C_F \frac{\alpha_s}{2\pi} \int dx_1 dx_2 \frac{x_1^2 + x_2^2 - \epsilon(2 - x_1 - x_2)}{(1 - x_1)^{1+\epsilon} (1 - x_2)^{1+\epsilon}}$$

- scaled energies $x_1 = 2 \frac{E_q}{\sqrt{s}}$ and $x_2 = 2 \frac{E_{\bar{q}}}{\sqrt{s}}$

- Soft and collinear divergencies ($0 \leq x_1, x_2 \leq 1$ and $x_1 + x_2 \geq 1$)



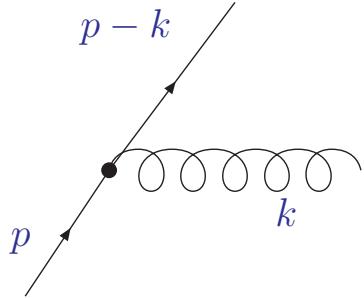
$$1 - x_1 = x_2 \frac{E_g}{\sqrt{s}} (1 - \cos \theta_{2g}) \text{ and}$$

$$1 - x_2 = x_1 \frac{E_g}{\sqrt{s}} (1 - \cos \theta_{1g})$$

- Integrate over phase space for real emission contributions

$$\sigma^{q\bar{q}g} = \sigma^{(0)} 3 \sum_q e_q^2 f(\epsilon) C_F \frac{\alpha_s}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + \mathcal{O}(\epsilon) \right)$$

- Soft and collinear divergencies ($0 \leq x_1, x_2 \leq 1$ and $x_1 + x_2 \geq 1$)



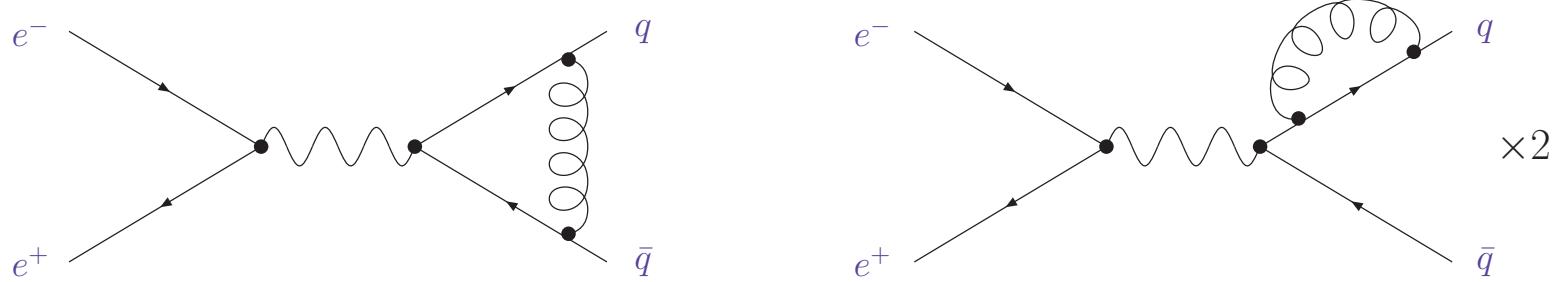
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- Integrate over phase space for real emission contributions

$$\sigma^{q\bar{q}g} = \sigma^{(0)} 3 \sum_q e_q^2 f(\epsilon) C_F \frac{\alpha_s}{2\pi} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + \mathcal{O}(\epsilon) \right)$$

- Divergencies cancel against virtual contributions



$$\sigma^{q\bar{q}(g)} = \sigma^{(0)} 3 \sum_q e_q^2 f(\epsilon) C_F \frac{\alpha_s}{2\pi} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \mathcal{O}(\epsilon) \right)$$

Infrared safety

- Total cross section ($R(s)$) is directly calculable in perturbation theory (finite)

$$R(s) = 3 \sum_q e_q^2 \left\{ 1 + \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right\}$$

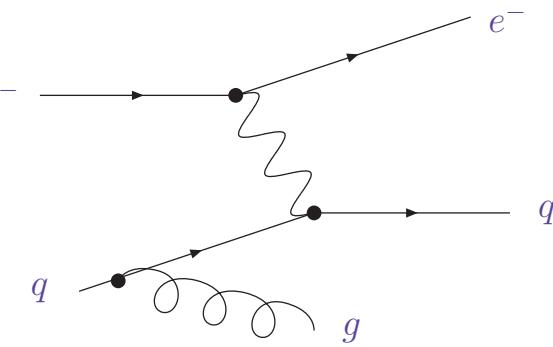
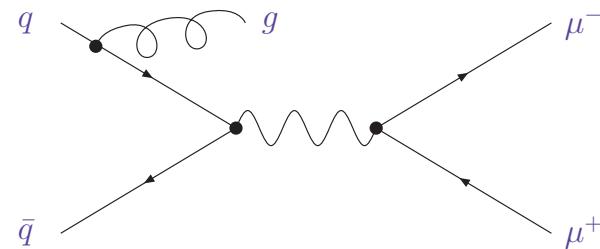
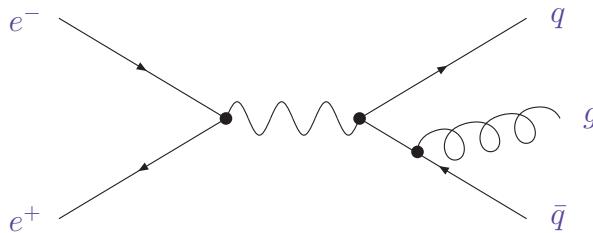
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QCD factorization

- Collinear divergencies remain for hadronic observables
→ factorization



- Left: single-hadron inclusive e^+e^- -annihilation (time-like kinematics)
- Center: Drell-Yan process in pp -scattering (space-like kinematics)
- Right: Deep-inelastic e^-p -scattering (space-like kinematics)

Factorization

- Large class of hard-scattering reactions with initial state hadrons
 - cross section not infrared safe
 - dependent on quark and gluon degrees of freedom in hadron
 - sensitive to nonperturbative processes at long distances
- Factorization of cross section
 - infrared safe hard part $\hat{\sigma}_{\text{pt}}$ calculable in perturbative QCD
 - nonperturbative function f determined from data
 - f parametrizes hadron structure
- General structure of cross section
 - large momentum scale Q , factorization scale μ , hadron scale m

$$Q^2 \sigma_{\text{phys}}(Q, m) = \hat{\sigma}_{\text{pt}}(Q/\mu, \alpha_s(\mu)) \otimes f(\mu, m)$$

- convolution \otimes in suitable kinematical variables
- Factorization
 - generalization of operator product expansion

Evolution

- Evolution formulates dependence of cross sections for observable on momentum transfer
- Classic example: QCD corrections to deep-inelastic scattering
 - scaling violations of structure functions
Gross, Wilczek '73; Politzer '73
- Physical cross section in factorization ansatz cannot depend on μ
 - factorization scale μ arbitrary

$$\mu \frac{d\sigma_{\text{phys}}}{d\mu} = 0$$

- Immediate consequence **DGLAP**: Altarelli, Parisi '77

$$\mu \frac{df(\mu, m)}{d\mu} = P(\alpha_s(\mu)) \otimes f(\mu, m) + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

$$\mu \frac{d\hat{\sigma}_{\text{pt}}(Q/\mu, \alpha_s(\mu))}{d\mu} = -P(\alpha_s(\mu)) \otimes \hat{\sigma}_{\text{pt}}(Q/\mu, \alpha_s(\mu)) + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

- PDF evolution from renormalization group equation

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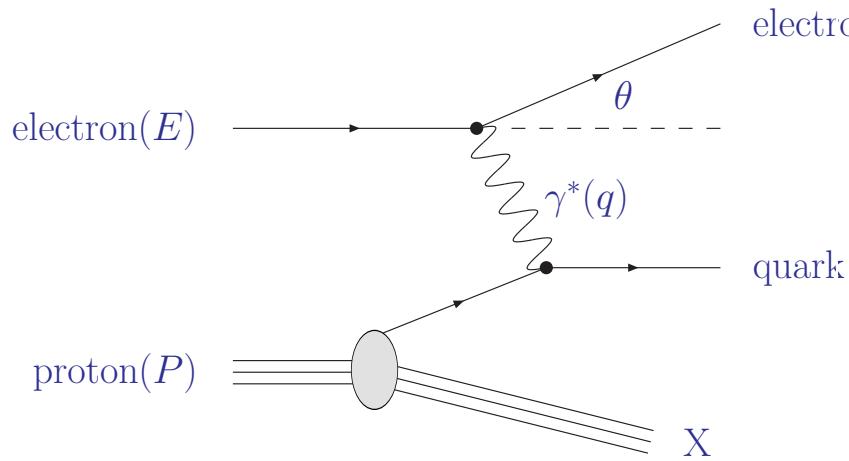
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- PDF evolution from renormalization group equation
 - splitting functions calculable in QCD

Inelastic electron-proton scattering



- Virtuality of photon: resolution

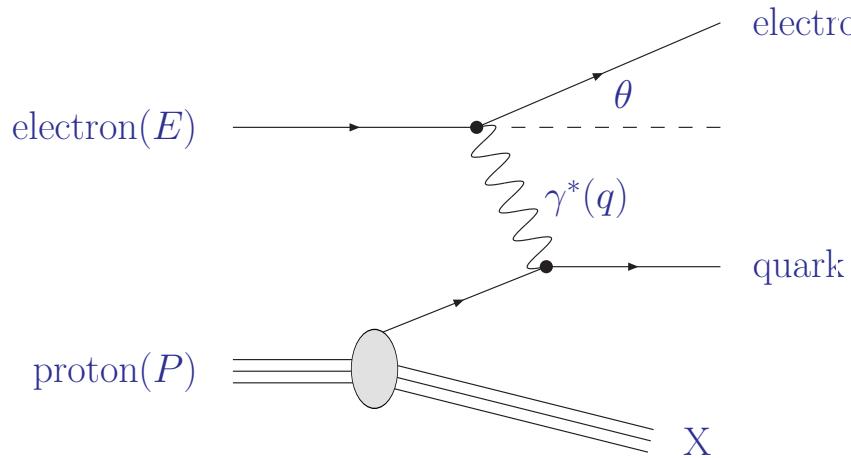
$$Q^2 \equiv -q^2 = 4EE' \sin^2(\theta/2)$$
- Bjorken variable: inelasticity

$$x = \frac{Q^2}{2P \cdot q} < 1$$

- Cross section (X inclusive): proton structure function F_i^p

$$(E - E') \frac{d\sigma}{d\Omega dE'} \stackrel{\text{lab}}{=} \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}} \underbrace{\left\{ F_2^p(x, Q^2) + \tan^2 \frac{\theta}{2} F_1^p(x, Q^2) \right\}}_{\text{Mott-scattering (point-like)}}$$

Inelastic electron-proton scattering



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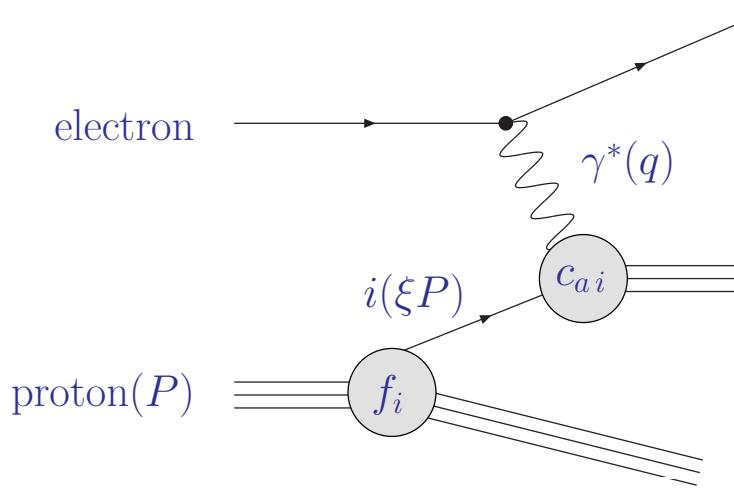
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- Deep-inelastic scattering (Bjorken limit: $Q^2 \rightarrow \infty$ and x fixed)
Parton modell (quasi-free point-like constituents, incoherence)

$$F_2(x, Q^2) \simeq F_2(x) = \sum_i e_i^2 x f_i(x)$$

- $x f_i(x)$ distribution for momentum fraction x of parton i

QCD corrections in deep-inelastic scattering



- Structure function F_2 (up to terms $\mathcal{O}(1/Q^2)$)
 - Renormalization/factorization scale $\mu = \mathcal{O}(Q)$

$$x^{-1} F_2^p(x, Q^2) = \sum_i \int_x^1 \frac{d\xi}{\xi} c_{2,i} \left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2} \right) f_i^p(\xi, \mu^2)$$

- Coefficient functions c_a

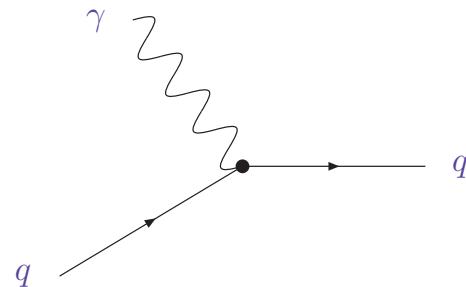
$$c_a = \underbrace{\alpha_s^{n_a} \left[c_a^{(0)} + \alpha_s c_a^{(1)} + \alpha_s^2 c_a^{(2)} + \dots \right]}_{\text{NLO: standard approximation (large uncertainties)}}$$

NLO: standard approximation (large uncertainties)

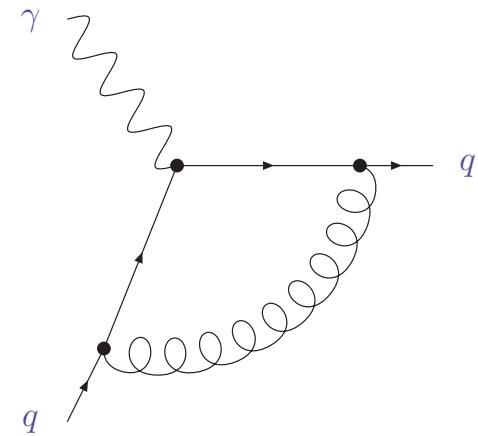
Radiative corrections in a nutshell

- Leading order
 - partonic structure function

$$\hat{F}_{2,q}^{(0)} = e_q^2 \delta(1-x)$$

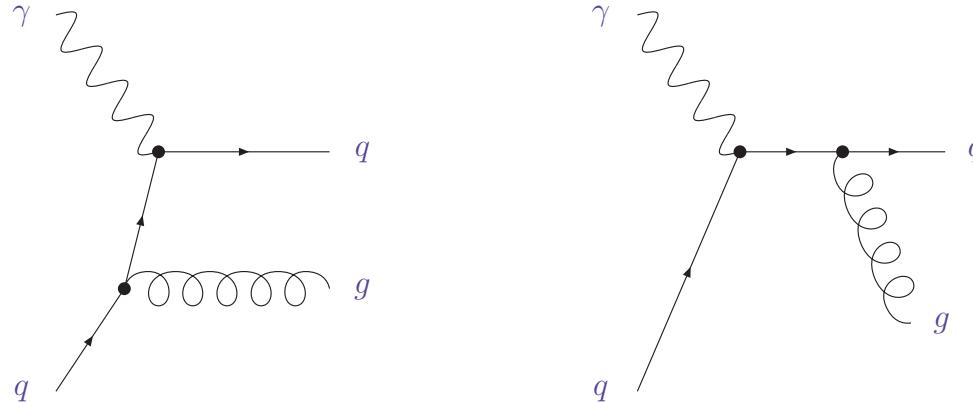


- Next-to-leading order
 - virtual correction
(infrared divergent; proportional to Born)
 - dimensional regularization $D = 4 - 2\epsilon$



$$\hat{F}_{2,q}^{(1),v} = e_q^2 C_F \frac{\alpha_s}{4\pi} \delta(1-x) \left(\frac{\mu^2}{Q^2} \right)^\epsilon \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \zeta_2 + \mathcal{O}(\epsilon) \right)$$

- Next-to-leading order



- add real and virtual corrections $\hat{F}_{2,q}^{(1)} = \hat{F}_{2,q}^{(1),r} + \hat{F}_{2,q}^{(1),v}$
- collinear divergence remains **splitting functions** $P_{qq}^{(0)}$

$$\begin{aligned}\hat{F}_{2,q}^{(1)} &= e_q^2 C_F \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^\epsilon \left\{ \frac{1}{\epsilon} \left(\frac{4}{1-x} - 2 - 2x + 3\delta(1-x) \right) \right. \\ &\quad + 4 \frac{\ln(1-x)}{1-x} - 3 \frac{1}{1-x} - (9 + 4\zeta_2)\delta(1-x) \\ &\quad - 2(1+x)(\ln(1-x) - \ln(x)) - 4 \frac{1}{1-x} \ln(x) + 6 + 4x \\ &\quad \left. + \mathcal{O}(\epsilon) \right\}\end{aligned}$$

- Structure of NLO correction

- absorb collinear divergence $P_{qq}^{(0)}$ in renormalized parton distributions

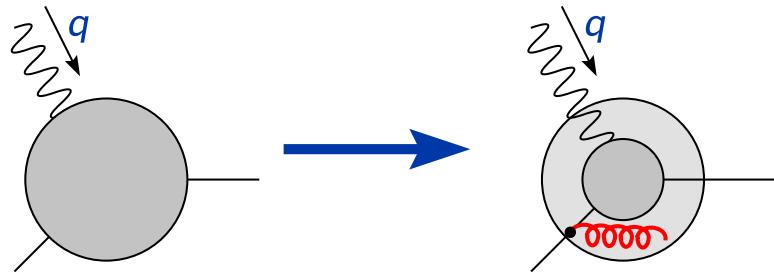
$$\hat{F}_{2,q}^{(1),bare} = e_q^2 \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{Q^2} \right)^\epsilon \left\{ \frac{1}{\epsilon} P_{qq}^{(0)}(x) + c_{2,q}^{(1)}(x) + \mathcal{O}(\epsilon) \right\}$$

$$q^{ren}(\mu_F^2) = q^{bare} - e_q^2 \frac{\alpha_s}{4\pi} \frac{1}{\epsilon} P_{qq}^{(0)}(x) \left(\frac{\mu^2}{\mu_F^2} \right)^\epsilon$$

- partonic (physical) structure function at factorization scale μ_F

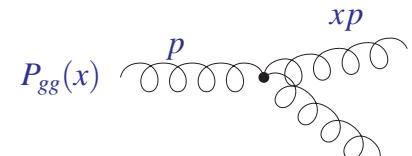
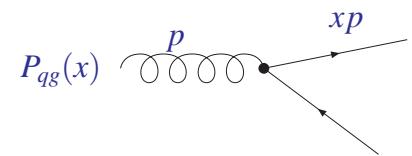
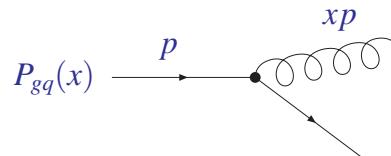
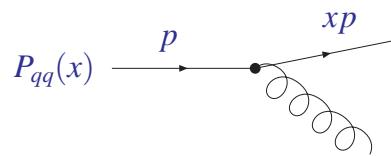
$$\hat{F}_{2,q} = e_q^2 \left(\delta(1-x) + \frac{\alpha_s}{4\pi} \left\{ c_{2,q}^{(1)}(x) - \ln \left(\frac{Q^2}{\mu_F^2} \right) P_{qq}^{(0)}(x) \right\} \right)$$

Parton luminosity

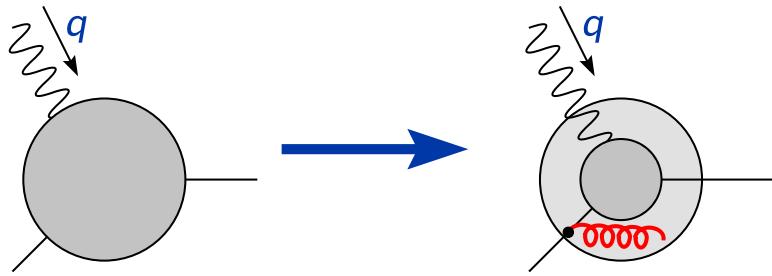


- Proton in resolution $1/Q \rightarrow$ sensitive to lower momentum partons

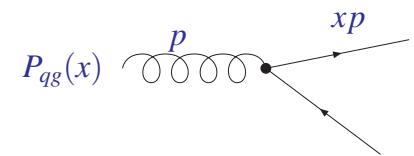
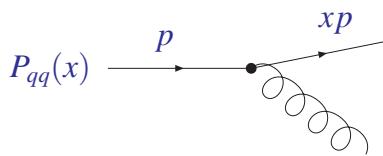
- Feynman diagrams in leading order



Parton luminosity



- Feynman diagrams in leading order



- Proton in resolution $1/Q$ → sensitive to lower momentum partons
- Evolution equations for parton distributions f_i
 - predictions from fits to reference processes (universality)

$$\frac{d}{d \ln \mu^2} f_i(x, \mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2) \right] (x)$$

- Splitting functions P

$$P = \underbrace{\alpha_s P^{(0)} + \alpha_s^2 P^{(1)}} + \underbrace{\alpha_s^3 P^{(2)}} + \dots$$

NLO: standard approximation (large uncertainties)

Complete set of splitting functions and PDFs

- Evolution equations
 - non-singlet ($2n_f - 1$ scalar) and singlet (2×2 matrix) equations

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- Non-singlet and singlet distributions q^\pm , q^v and q_s , g

$$q_{\text{ns},ik}^\pm = q_i \pm \bar{q}_i - (q_k \pm \bar{q}_k) \quad \text{flavour asymmetries}$$

$$q_{\text{ns}}^v = \sum_{r=1}^{n_f} (q_r - \bar{q}_r) \quad \text{total valence distribution}$$

$$q_s = \sum_{r=1}^{n_f} (q_r + \bar{q}_r) \quad \text{flavour singlet distribution, } f_i = \begin{pmatrix} q_s \\ g \end{pmatrix}$$

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- Splitting function combinations

$$P_{\text{ns}}^\pm, \quad P_{\text{ns}}^v = P_{\text{ns}}^- + P_{\text{ns}}^s \quad \text{non-singlet}$$

$$P_s = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix}, \quad P_{qq} = P_{\text{ns}}^+ + P_{ps} \quad \text{singlet}$$

Flavor asymmetries

- Splitting $q \rightarrow q'$ different from that of $\bar{q} \rightarrow \bar{q}'$ at higher orders
- Flavor dependence of splitting functions

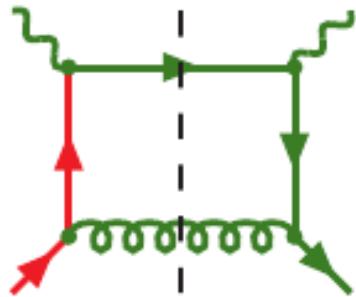
$$P_{q_i q_k} = P_{\bar{q}_i \bar{q}_k} = \delta_{ik} P_{qq}^v + P_{qq}^s$$

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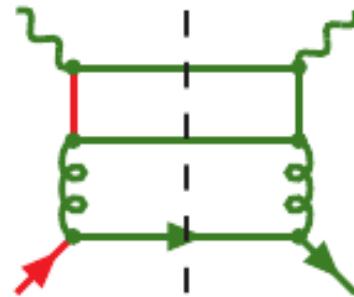
- Flavor dependence of PDFs
 - $2(n_f - 1)$ flavor asymmetries and one valence distribution

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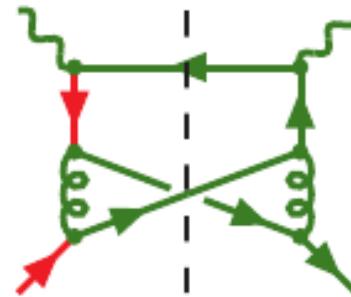
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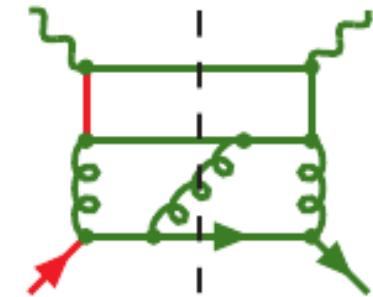
$$P_{qq}^v = \mathcal{O}(\alpha_s)$$



$$P_{qq}^s, P_{q\bar{q}}^s : \alpha_s^2$$



$$P_{q\bar{q}}^v : \alpha_s^2$$



$$P_{q\bar{q}}^s \neq P_{qq}^s : \alpha_s^3$$

Heavy quark effects

- Light quarks: $m_u, m_d \ll \Lambda_{\text{QCD}}$, $m_s < \Lambda_{\text{QCD}}$
 - neglect “light quark” masses in hard scattering process
 - mass singularities, scale-dependent u, d, s, g PDFs
- Heavy quarks: $m_c, m_b, m_t \gg \Lambda_{\text{QCD}}$
 - no mass singularities, no (evolving) PDFs
- Charm structure function F_2^c at HERA (assume no “intrinsic charm”)
 - $Q \gtrless m_c$: Fixed flavor-number scheme FFNS
 u, d, s, g partons and massive charm coeff. fcts.
 - $Q \ggg m_c$: Zero-mass variable flavor-number scheme ZM-VFNS
terms $m_c/Q \rightarrow 0$, $n_f = 4$ PDFs (matching), $m_c = 0$ coeff. fcts.
 - $Q \gg m_c$: General-mass variable flavor-number scheme GM-VFNS
terms $m_c/Q \neq 0$, but quasi-collinear logs $\ln(Q/m_c)$ large
 $n_f = 4$ PDFs, “interpolating” coeff. fcts. (matching prescriptions)

FFNS

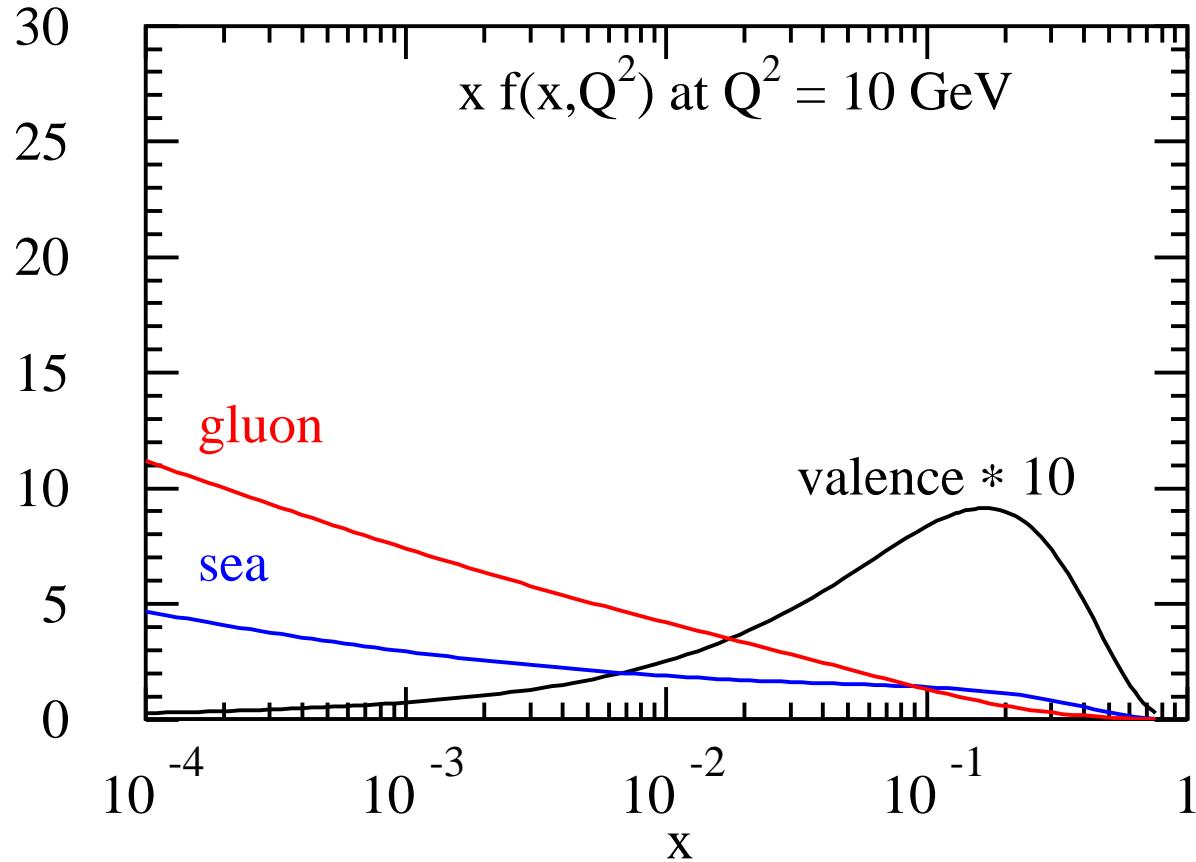
- Complete NLO QCD predictions for F_2^c and F_L^c (neutral current)
Laenen, Riemersma, Smith, van Neerven '92
 - NLO parton level Monte Carlo HVQDIS Harris, Smith '95

VFNS

- Variable flavor number schemes → matching of two distinct theories
Avazis, Collins, Olness, Tung '94; Buza, Matiounine, Smith, van Neerven '98
 - n_f light flavors + heavy quark of mass m at low scales
 - $n_f + 1$ light flavors at high scales
- Important aspects of variable flavor number schemes
 - mass factorization to be carried out before resummation
 - mass factorization involves both heavy and light component of structure function
 - matching conditions required through NNLO
Chuvakin, Smith, van Neerven '00
- Details of implementation matter in global fits
Tung, Thorne '08

Parton distributions in proton

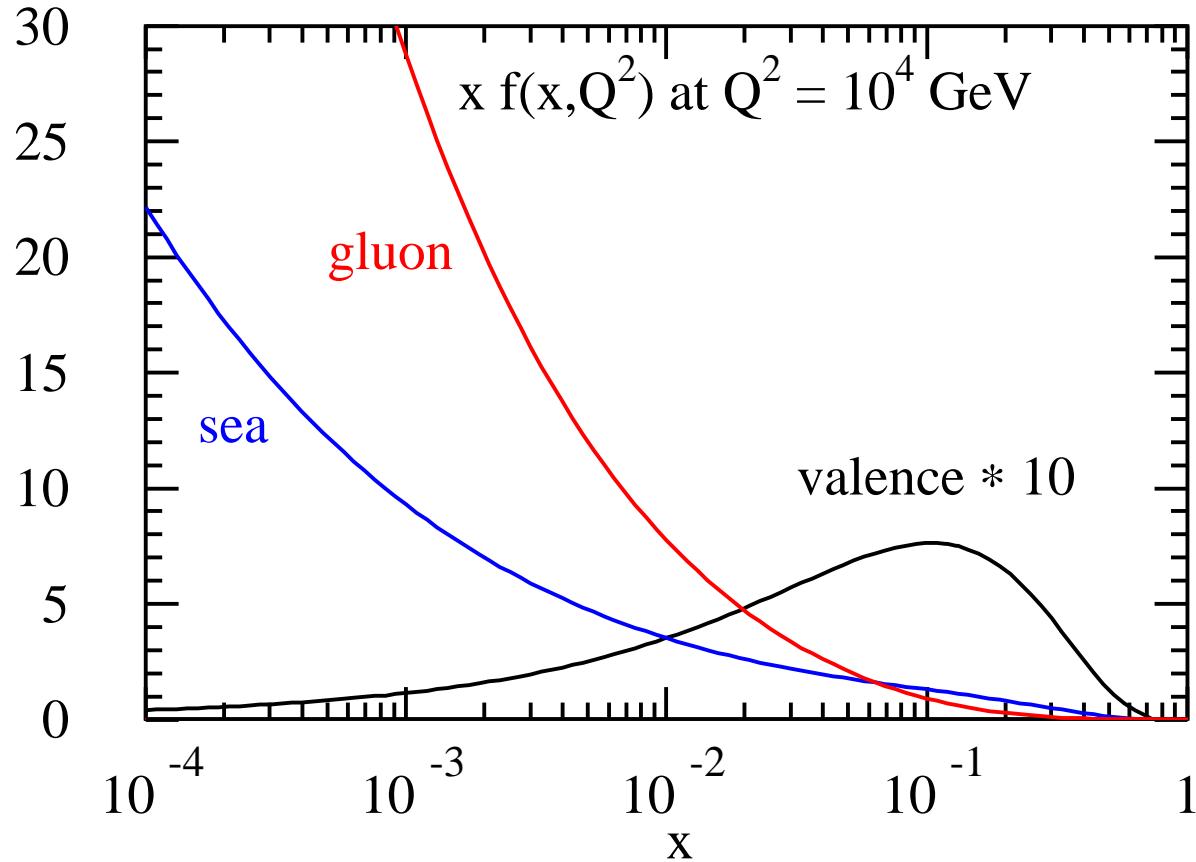
- Valence $q - \bar{q}$ (additive quantum numbers) sea (part with $q + \bar{q}$)



- Parameterization (bulk of data from deep-inelastic scattering)
 - structure function F_2 —> quark distribution
 - scale evolution (perturbative QCD) —> gluon distribution

Parton distributions in proton

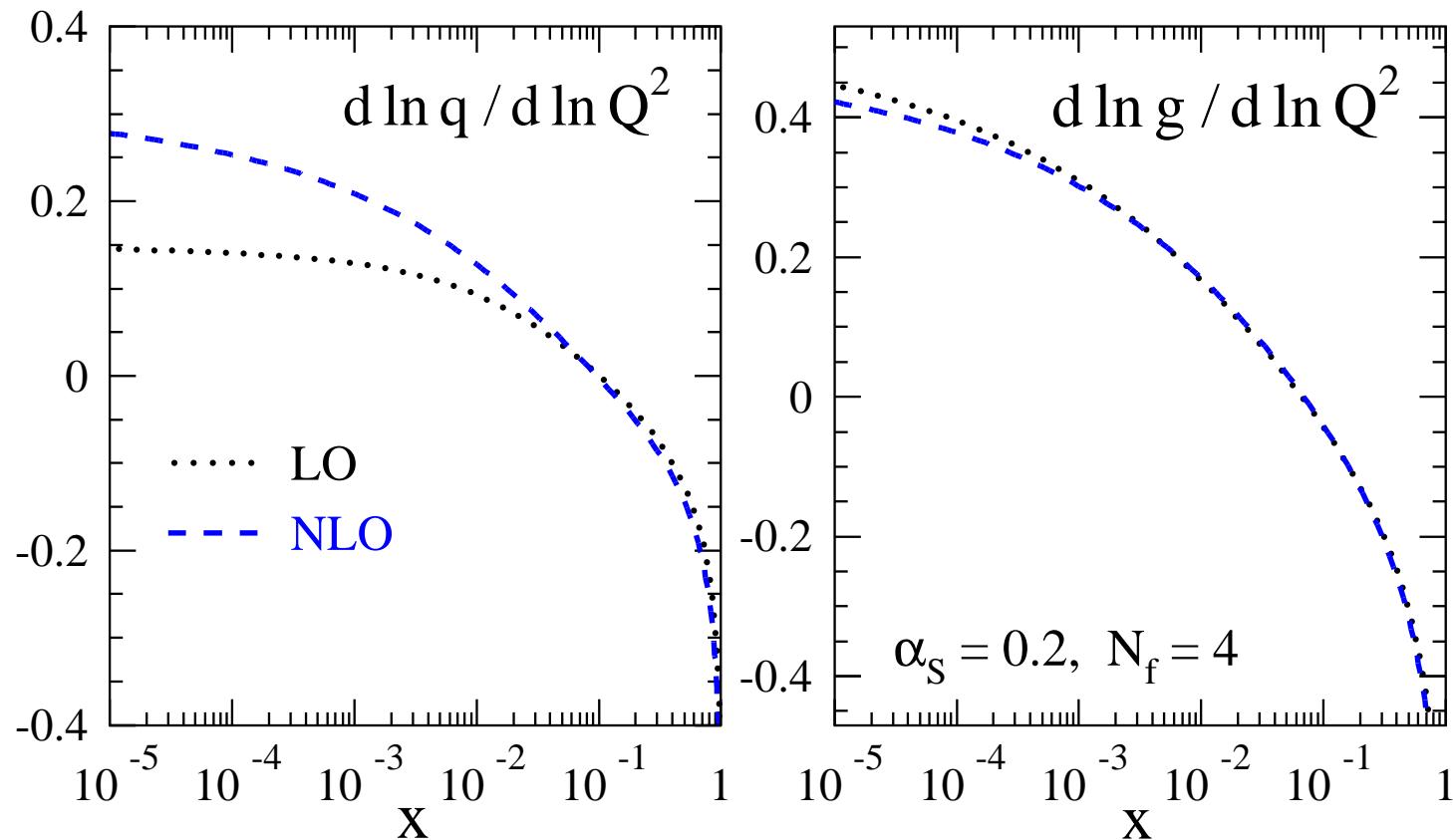
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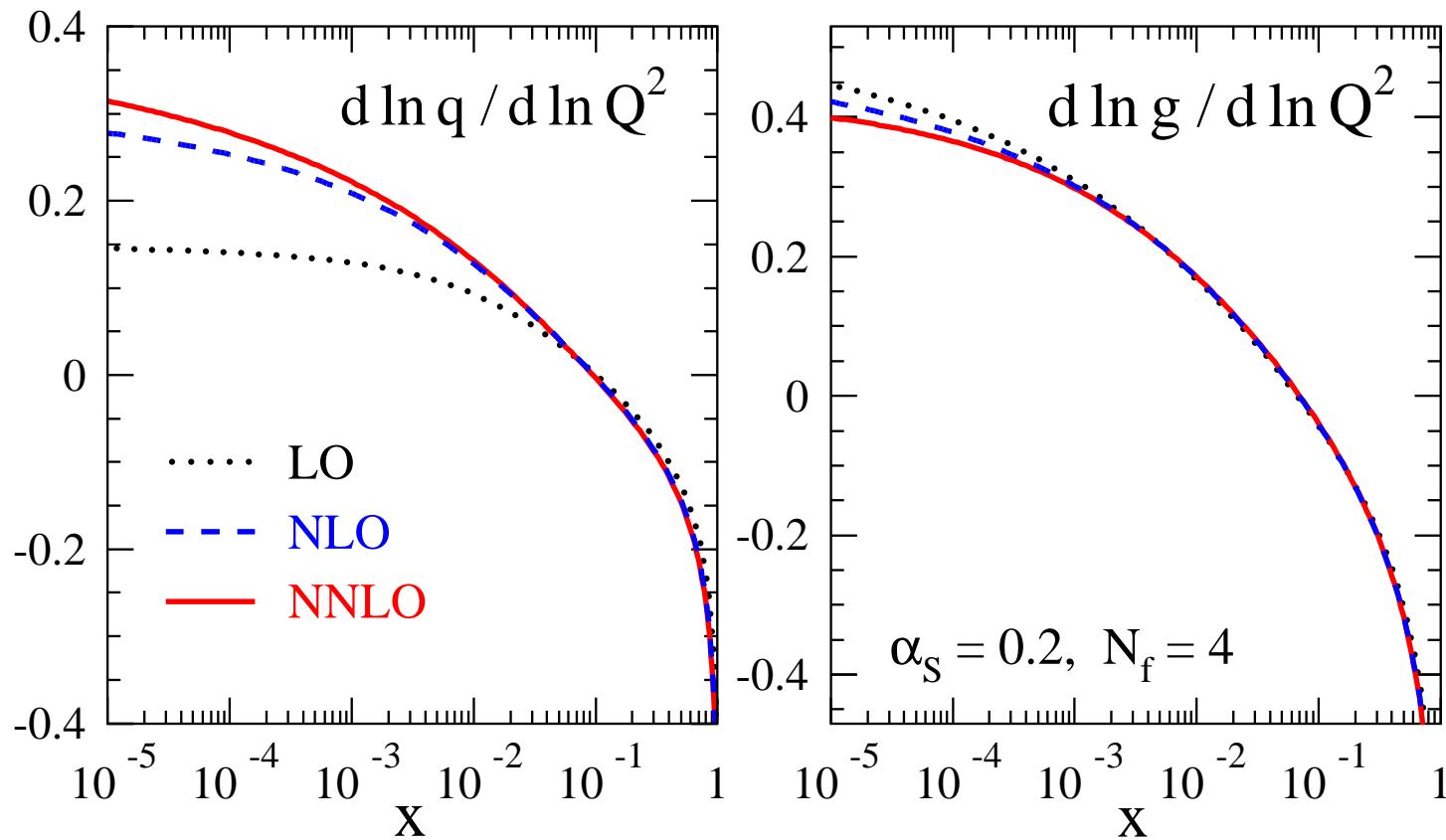
Perturbative stability of evolution

- Scale derivatives of quark and gluon distributions at $Q^2 \approx 30 \text{ GeV}^2$



Perturbative stability of evolution

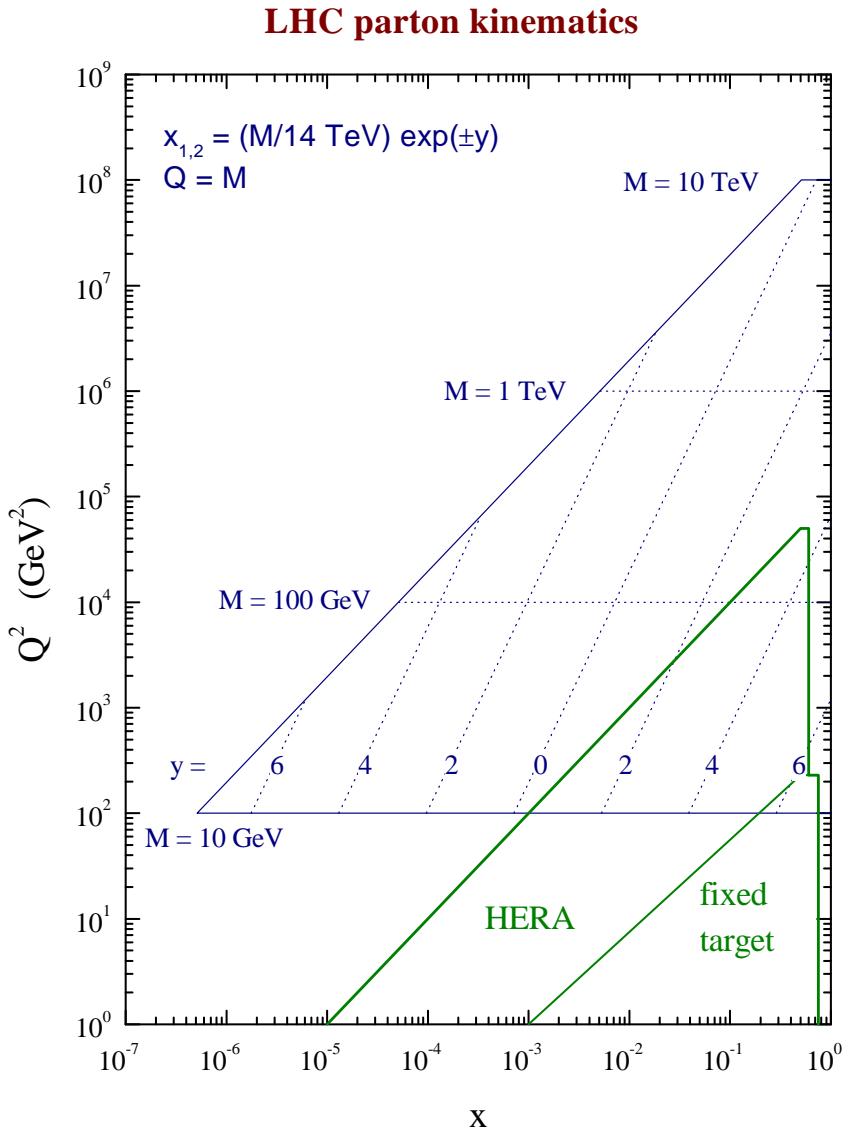
- Scale derivatives of quark and gluon distributions at $Q^2 \approx 30 \text{ GeV}^2$



- Expansion very stable except for very small momenta $x \lesssim 10^{-4}$
S.M. Vermaseren, Vogt '04

Parton luminosity at LHC

- Precision HERA data on F_2 covers most of the LHC x -range
- Scale evolution of PDFs in Q over two to three orders
- Sensitivity at LHC
 - 100 GeV physics: small- x , sea partons
 - TeV scales: large- x
 - rapidity distributions probe extreme x -values
- Stable evolution in QCD
 - splitting functions to NNLO
S.M. Vermaseren, Vogt '04



Summary

Introduction to QCD I

- Asymptotic freedom
- Review of Feynman rules
- Colour ordering
- Spinor conventions
- Helicity amplitudes
- QCD factorization
- Parton distributions
- Evolution equations

Stay tuned

- Introduction to QCD II

Wednesday, October 21, 2009

Literature

- Review

- *Expectations at LHC from hard QCD*

J. Phys. G: Nucl. Part. Phys. **35**, 073001 (2008) [arXiv:0803.0457] [hep-ph].