Polarization and Electroweak Precision Measurements at the ILC for $\sqrt{s} = 250\,\text{GeV}$

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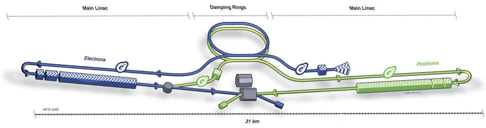
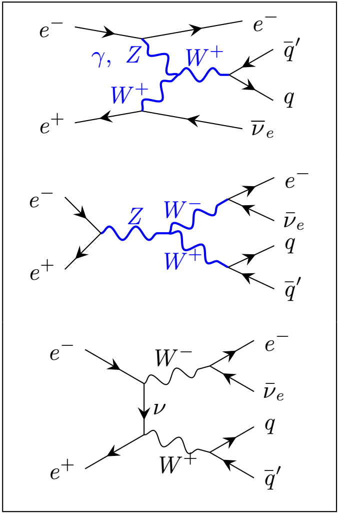
The International Linear Collider (ILC)

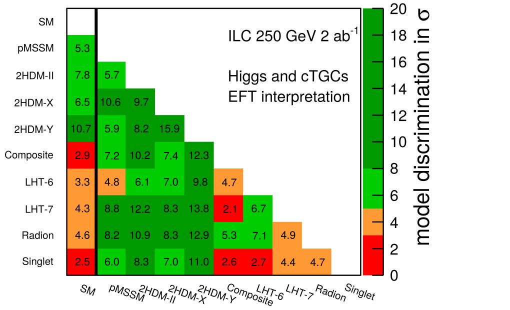


- Future linear e^+e^- Collider: $\sqrt{s} = 250\,\text{GeV}$ (As a first stage)
- Construction under political consideration in the Kitakami region, Prefecture Iwate, Japan
- **At the ILC both beams (e^+, e^-) are polarized:** $P_{e^-} = \pm 80\%, \quad P_{e^+} = \pm 30\%$
- **Switch of polarization sign (helicity reversal) $\rightarrow$ choice of spin configuration**
- Designed for precision studies for physics of the standard model and beyond

Anomalous Triple Gauge Couplings (aTGCs)

$e^+e^- \to e\nu q\bar{q}$





arXiv:1708.08912v1 [hep-ph]

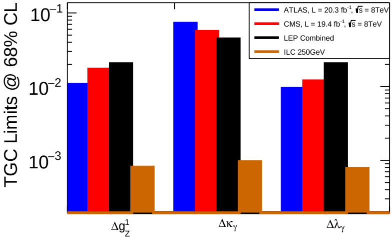
- The TGC precision is important to distinguished different Higgs-models beyond the SM
- Additional bosons (e.g. Z') will affect TGCs

⇒ TGCs have to be precisely measured

⇒ aTGCs described by an Effective Field Theory (EFT)

ILC Aims for Precision Measurements

Triple Gauge Couplings (TGC)



- ⇒ **Polarization has to be known as precisely as the luminosity!**
- ⇒ **Requirement for a permille-level precision of the luminosity-weighed average polarization**

Previously achieved polarization precision

HERA: $\Delta P/P = 2\%_{\text{stat}} \oplus 1\%_{\text{sys}}$ [1] SLAC: $\Delta P/P = 1.1\%$ [2]

- ⇒ **More than one order of magnitude better precision on both TGC and polarization measurement (for a start)**

- ⇒ **Accomplished by simultaneous measurement of both of them**

Beam Polarization Dependent Cross Section

- Theoretical polarized cross section in general:

$$\sigma_{\text{theory}}\left(P_{e^-},P_{e^+}\right)=\frac{\left(1-P_{e^-}\right)}{2}\frac{\left(1-P_{e^+}\right)}{2}\cdot\sigma_{\text{LL}}+\frac{\left(1+P_{e^-}\right)}{2}\frac{\left(1+P_{e^+}\right)}{2}\cdot\sigma_{\text{RR}}$$
$$+\frac{\left(1-P_{e^-}\right)}{2}\frac{\left(1+P_{e^+}\right)}{2}\cdot\sigma_{\text{LR}}+\frac{\left(1+P_{e^-}\right)}{2}\frac{\left(1-P_{e^+}\right)}{2}\cdot\sigma_{\text{RL}}$$

- Nominal ILC polarization values

$$\underbrace{P_{e^-}^{-}=-80\%,}_{\text{"left"-handed } e^-\text{-beam}}\quad\underbrace{P_{e^-}^{+}=80\%,}_{\text{"right"-handed } e^-\text{-beam}}\quad\underbrace{P_{e^+}^{-}=-30\%,}_{\text{"left"-handed } e^+\text{-beam}}\quad\underbrace{P_{e^+}^{+}=30\%,}_{\text{"right"-handed } e^+\text{-beam}}$$

- Cross section of the 4 polarization configurations

$$\sigma_{--}:=\sigma\left(P_{e^-}^{-},P_{e^+}^{-}\right)\qquad\sigma_{++}:=\sigma\left(P_{e^-}^{+},P_{e^+}^{+}\right)$$
$$\sigma_{-+}:=\sigma\left(P_{e^-}^{-},P_{e^+}^{+}\right)\qquad\sigma_{+-}:=\sigma\left(P_{e^-}^{+},P_{e^+}^{-}\right)$$

- σ_{LL}, σ_{RR}, σ_{LR}, σ_{RL} are theoretically calculated including ISR and beam spectrum

Polarized Cross Section Measurement

- Measured polarized cross section:

$$\sigma_{\text{data}} = \frac{D - \mathfrak{B}}{\varepsilon \cdot \mathcal{L}}$$

D:	Number of signal events	$\mathfrak{B}$:	Background expectation value
ε:	Detector selection efficiency	$\mathcal{L}$:	Integrated luminosity

Remark:
All of them can variate between the different data sets $(\sigma_{-+},\ \sigma_{+-},\ \sigma_{--},\ \sigma_{++})$

- Uncertainty of the polarized cross section is calculated via error propagation

$$\text{e.g.}\quad (\Xi_{\mathcal{L}})_{ij} = \text{corr}\left(\sigma_i^{\mathcal{L}},\ \sigma_j^{\mathcal{L}}\right) \frac{\partial \sigma_i}{\partial \mathcal{L}_i} \frac{\partial \sigma_j}{\partial \mathcal{L}_j} \Delta\mathcal{L}_i \Delta\mathcal{L}_j \qquad i,j \in \{-+,+-,--,++\}$$

$$\Xi := \underbrace{\Xi_D}_{\substack{\text{statistical} \\ \text{uncertainty}}} + \underbrace{\Xi_{\mathfrak{B}} + \Xi_{\varepsilon} + \Xi_{\mathcal{L}}}_{\text{systematic uncertainty}};$$

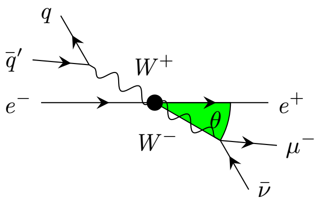
Remark:
Statistical uncertainty is always uncorrelated: corr $\left(\sigma_i^D,\ \sigma_j^D\right) \equiv \delta_{ij}$
And it is determined by Poisson fluctuations:
$\Delta D \equiv \sqrt{D}$

Usage of the Differential Polarized Cross Section

Choice of the angle:

- ✓ Individual for each channel
- ✓ High dependence of the angular distribution on the chiral structure
- ✓ Angle has to be well measurable
- ✓ Multi-angle distribution available

e.g.: $e^+e^- \to W^+W^- \to q\bar{q}'\mu^-\bar{\nu}$



Bin-wise cross section calculation:

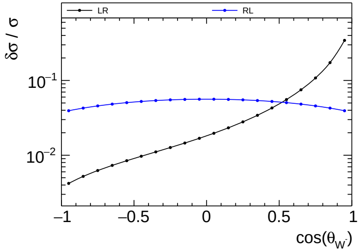
$$\overbrace{\partial\sigma/\partial\theta}^{\text{differential cross section}} \to \overbrace{\delta_i\sigma_{\text{data}}}^{\text{cross section of the } i\text{-th bin}} := \delta_i N/\mathcal{L}$$

$$\to \delta_i\sigma_{LR} := f_{LR}(\theta_i)\cdot\sigma_{LR}$$

Analog: RL, LL, RR

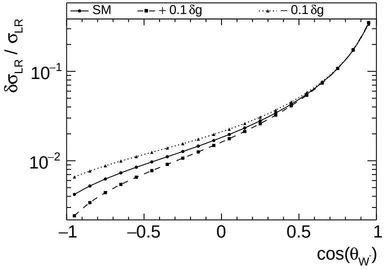
- $\delta_i N = (\delta_i D - \delta_i\mathfrak{B})/\delta_i\varepsilon$: events of i-th bin
- $f(\theta_i)$: fraction of the total cross section

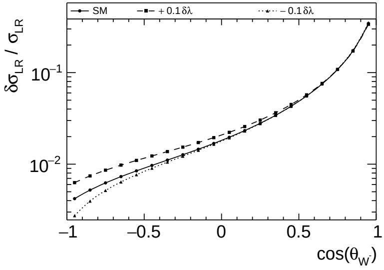
Projection on the θ_{W^-}-axis

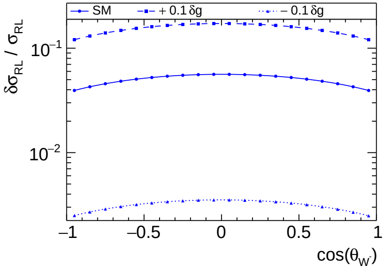


DESY

Effect of Anomalous TGC in $e^- e^+ \to \mu\nu q\bar{q}$ at 250 GeV







- The effect of a $\pm 5\sigma$ deviation of a single TGC assuming LEP precision
- TGCs are a very small effect:
  - Especial affects angular ranges of low differential cross sections
  - → But with a clear angular dependence
  - \+ Strong dependence of the chiral structure

Fit Procedure

- **Using the method of least squares:**

$$\chi^2 = \sum_{\text{channel}} \sum_i \left(\delta_i\vec{\sigma}_{\text{data}} - \delta_i\vec{\sigma}_{\text{theory}}\right)^T \left(\delta_i\Xi\right)^{-1} \left(\delta_i\vec{\sigma}_{\text{data}} - \delta_i\vec{\sigma}_{\text{theory}}\right);$$

$$\vec{\sigma} := \begin{pmatrix} \delta_i\sigma_{-+} & \delta_i\sigma_{+-} & \delta_i\sigma_{--} & \delta_i\sigma_{++} \end{pmatrix}^T$$

- **Considered Channels and their Parameters:**

channel	cross section	left-right asymmetry	TGC
$e^- e^+ \longrightarrow e^- \bar{\nu} q\bar{q}$	$\sigma_{e^-\bar{\nu}q\bar{q}}$	$A_{RR}^{e^-\bar{\nu}q\bar{q}} = \frac{\sigma_{LR}-\sigma_{RR}}{\sigma_{LR}+\sigma_{RR}}$	$g_Z^1,\ \kappa_\gamma,\ \lambda_\gamma$
$e^- e^+ \longrightarrow e^+ \nu q\bar{q}$	$\sigma_{e^+\nu q\bar{q}}$	$A_{LL}^{e^+\nu q\bar{q}} = \frac{\sigma_{LR}-\sigma_{LL}}{\sigma_{LR}+\sigma_{LL}}$	$g_Z^1,\ \kappa_\gamma,\ \lambda_\gamma$
$e^- e^+ \longrightarrow \mu\nu q\bar{q}$	$\sigma_{\mu\nu q\bar{q}}$	$A_{RL}^{\mu\nu q\bar{q}} = \frac{\sigma_{LR}-\sigma_{RL}}{\sigma_{LR}+\sigma_{RL}}$	$g_Z^1,\ \kappa_\gamma,\ \lambda_\gamma$
$e^- e^+ \longrightarrow \mu^+\mu^- q\bar{q}$	$\sigma_{\mu\mu q\bar{q}}$	$A_{RL}^{\mu\mu q\bar{q}} = \frac{\sigma_{LR}-\sigma_{RL}}{\sigma_{LR}+\sigma_{RL}}$	–
$e^- e^+ \longrightarrow q\bar{q}$	$\sigma_{q\bar{q}}$	$A_{RL}^{q\bar{q}} = \frac{\sigma_{LR}-\sigma_{RL}}{\sigma_{LR}+\sigma_{RL}}$	–
$e^- e^+ \longrightarrow ll$	σ_{ll}	$A_{RL}^{ll} = \frac{\sigma_{LR}-\sigma_{RL}}{\sigma_{LR}+\sigma_{RL}}$	–

- $P_{e^-}^-,\ P_{e^-}^+,\ P_{e^+}^-,\ P_{e^+}^+$
  determined globally for all channels

Results

Using the following parameter values:

$$\varepsilon = 0.6$$
$$\pi = \frac{D - \mathfrak{B}}{D} = 0.8$$
$$\mathcal{L} = 2\,\mathrm{ab}^{-1}$$
$$\Delta\varepsilon = \Delta\pi = \Delta\mathcal{L} = 0$$

Luminosity sharing:

$$-+ : 45\%, \quad +- : 45\%,$$
$$-- : 5\%, \quad ++ : 5\%$$

All results in $\left[10^{-4}\right]$

Cross Section

$\Delta\sigma_{e^+\nu q\bar{q}}/\sigma$	12.9
$\Delta\sigma_{e^-\bar{\nu} q\bar{q}}/\sigma$	13.3
$\Delta\sigma_{\mu\nu q\bar{q}}/\sigma$	11.4
$\Delta\sigma_{\mu\mu q\bar{q}}/\sigma$	13.8
$\Delta\sigma_{q\bar{q}}/\sigma$	3.78
$\Delta\sigma_{ll}/\sigma$	3.91

Asymmetry

$\Delta A_{RR}^{e^+\nu q\bar{q}}$	6.37
$\Delta A_{LL}^{e^-\bar{\nu} q\bar{q}}$	19.1
$\Delta A_{LR}^{\mu\nu q\bar{q}}$	3.32
$\Delta A_{LR}^{\mu\mu q\bar{q}}$	15.4
$\Delta A_{LR}^{q\bar{q}}$	6.66
ΔA_{LR}^{ll}	7.72

Polarization

$\Delta P_{e^-}^{-}$	7.68
$\Delta P_{e^-}^{+}$	3.4
$\Delta P_{e^+}^{-}$	8.11
$\Delta P_{e^+}^{+}$	10.7

TGC

Δg	8.18
$\Delta\kappa$	10.1
$\Delta\lambda$	9.33

Conclusion

- **Polarization provides a deep insight into the chiral structure of the standard model and beyond**
  - A permille-level precision of the luminosity-weighted average polarization at the IP is required
- **A full electroweak precision fit is achievable at the ILC**
  - The beam polarization, unpolarized cross section, the left-right asymmetry and anomalous Triple Gauge couplings can be determined with a relative precision of $\approx 10^{-3}$
- **Additional studies on the dependence of systematic quantities and their uncertainties will follow**

References

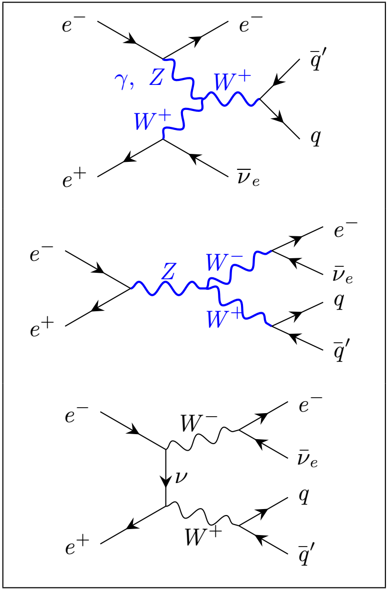
[1] S. Baudrand, M Bouchela, V Brissona, R Chichea, M Jacqueta, S Kurbasova, G Lia, C Pascauda, A Rebouxa, V Soskova, Z Zhanga, F Zomera, M Beckinghamb, T Behnkeb, N Coppolab, N Meynersb, D Pitzlb, S Schmittb, M Authierc, P Deck-Betinellic, Y Queinecc and L Pinardd, *A high precision Fabry-Perot cavity polarimeter at HERA*, Journal of Instrumentation 2010, (http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-6700.pdf)

[2] P. C. ROWSON, *PRECISION ELECTROWEAK PHYSICS WITH THE SLD/SLC: THE LEFT-RIGHT POLARIZATION ASYMMETRY*, SLAC-PUB-6700, December 1994, (http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-6700.pdf)

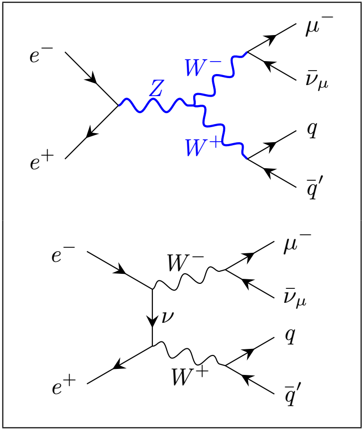
Backup Slides

TGC Contribution for the Final State

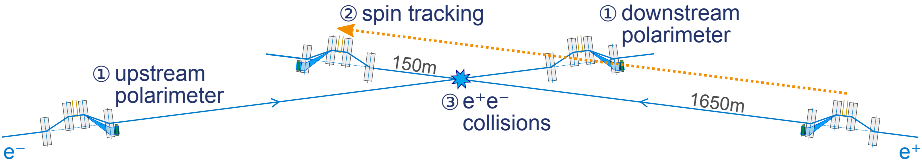
$e^+e^- \rightarrow e\nu q\bar{q}$



$e^+e^- \rightarrow \mu\nu q\bar{q}$



ILC Polarimetry Concept for Permille-Level Polarization Precision



- **The time-resolved beam polarization:**
  - Measured with 2 laser-Compton polarimeters before and after the $e^- e^+$ IP
  - Polarimeter precision $\Delta P/P = 0.25\%$ from the start
  - Extrapolated to the $e^- e^+$ IP via spin tracking

\+ **The luminosity-weighed averaged polarization:**
  - Calculated from collision data at the IP
  - Using the cross section measurement of well known standard model processes

⇒ **Combination of both measurements**

→ With the aim to reach the permille-level precision $\Delta P/P = 0.1\%$

Physics Processes for Polarization Measurement

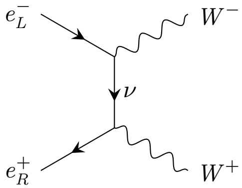
- Unpolarized cross section $\sigma_0 := \frac{1}{4}\sum \sigma$
  - Higher σ_0 increases the event rate
- Left-right-asymmetry $A_{\mathrm{RL}}^{\mathrm{LR}} := \frac{\sigma_{LR}-\sigma_{RL}}{\sigma_{LR}+\sigma_{RL}}$
  - Sensitivity to the chiral structure

$\Rightarrow$ Both should as high as possible

Examples at 500 GeV including ISR + Beamstrahlung:

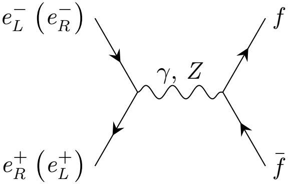
Process	$A_{\mathrm{RL}}^{\mathrm{LR}}$	σ_0[pb]
$e^+e^- \to W^+W^- \to q\bar{q}l\nu$	0.990	2.39
$e^+e^- \to Z \to q\bar{q}$	0.287	12.6
$e^+e^- \to e^-\bar{\nu}W^+ \to e^-\bar{\nu}q\bar{q}$	0.994	1.17
$e^+e^- \to e^+\nu W^- \to e^+\nu q\bar{q}$	0.994	1.17
⋮	⋮	⋮

W-pair production:



$\sigma_{\mathrm{LL}} = \sigma_{\mathrm{RR}} = \sigma_{\mathrm{RL}} = 0$

s-channel spin-1 particle:



$\sigma_{\mathrm{LL}} = \sigma_{\mathrm{RR}} = 0$

- Measured polarized cross section:

$$\sigma_{\text{data}} = \frac{D - \mathfrak{B}}{\varepsilon \cdot \mathcal{L}}$$

D: Number of the measured signal events

$\mathfrak{B}$: Background expectation value

ε: Selection efficiency of the detector

$\mathcal{L}$: Integrated luminosity provided by the accelerator

Remark:
All of them can variate between the different data sets $(\sigma_{--}, \sigma_{++}, \sigma_{-+}, \sigma_{+-})$

- Uncertainty of the polarized cross section calculated via error propagation

$$\Delta\sigma^2 = \underbrace{\left(\frac{\partial\sigma}{\partial D}\Delta D\right)^2}_{\text{statistical uncertainty}} + \underbrace{\left(\frac{\partial\sigma}{\partial\mathfrak{B}}\Delta\mathfrak{B}\right)^2 + \left(\frac{\partial\sigma}{\partial\varepsilon}\Delta\varepsilon\right)^2 + \left(\frac{\partial\sigma}{\partial\mathcal{L}}\Delta\mathcal{L}\right)^2}_{\text{systematic uncertainty}}$$

Fit Procedure

- **The 4 ILC polarization configurations**

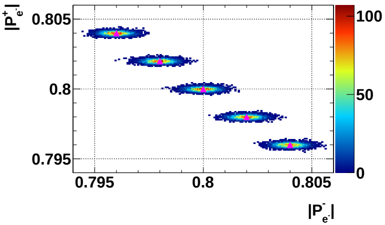
$$\sigma_{-+} := \sigma\left(P^{-}_{e^-}, P^{+}_{e^+}\right) \qquad \sigma_{+-} := \sigma\left(P^{+}_{e^-}, P^{-}_{e^+}\right)$$
$$\sigma_{--} := \sigma\left(P^{-}_{e^-}, P^{-}_{e^+}\right) \qquad \sigma_{++} := \sigma\left(P^{+}_{e^-}, P^{+}_{e^+}\right)$$

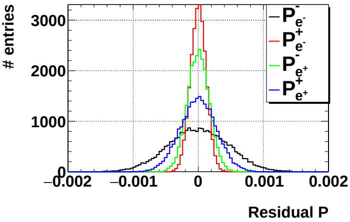
- **Defining χ^2 function:**

$$\chi^2 := \sum_{\text{processes}} \sum_{i,k} \left(\frac{\sigma^{\text{data}}_{i,k} - \sigma^{\text{theory}}_{i,k}\left(P^{i}_{e^-},\ P^{k}_{e^+}\right)}{\Delta\sigma_{i,k}} \right)^2 \qquad i,k \in \{+,-\}$$

- **Determine the polarization:**
  - Find $P^{-}_{e^-}$, $P^{+}_{e^-}$, $P^{-}_{e^+}$, $P^{+}_{e^+}$ that minimizes χ^2
  - Parameter uncertainties provides also the polarization uncertainties: $\Delta P^{-}_{e^-}$, $\Delta P^{+}_{e^-}$, $\Delta P^{-}_{e^+}$, $\Delta P^{+}_{e^+}$

Testing for a Non-Perfect Helicity Reversal





- **Variation in the absolute polarization**
  - Toy Measurement for 5 different polarization discrepancies for both beams
  - Nominal initial polarizations: $|P_{e-}| = 80\%$, $|P_{e+}| = 30\%$
  - Statistical uncertainties only
- **χ^2-Fit:**
  - Correct determination of the 4 polarization values
  - No noticeable impact on polarization precision using total cross sections

✓ **Can compensate for a non-perfect helicity reversal**

Consider Constraints from the Polarimeter Measurement

Simplified approach:
(as a first step)

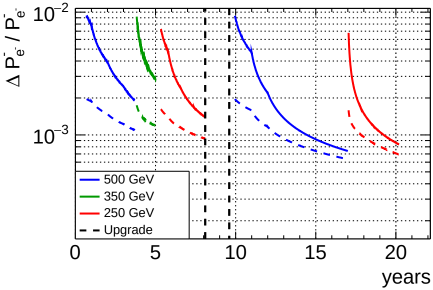
- Neglect spin transport
- Using $\Delta P/P = 0.25\%$:
- Gaussian distribution
  - Mean: $|P_{e^-}| = 80\%, |P_{e^+}| = 30\%$
  - Width: ΔP

Implementation:

$$\chi'^2 = \chi^2 + \sum_P \left[\frac{\left(P^{\pm}_{e^{\pm}} - \mathcal{P}^{\pm}_{e^{\pm}}\right)^2}{\Delta \mathcal{P}^2} \right]$$

- $P^{\pm}_{e^{\pm}}$: 4 fitted parameters
- $\mathcal{P}^{\pm}_{e^{\pm}}$: Polarimeter measurement
- $\Delta\mathcal{P}$: Polarimeter uncertainty

E[GeV]	**500**	**350**	**250**	**500**	**250**
$\mathcal{L}$[1/fb]	**500**	**200**	**500**	**3500**	**1500**
$\left[10^{-3}\right]$	**Without Constraint**				
$\Delta P^-_{e^-}/P$	1.9	2.8	1.4	0.74	0.84
$\left[10^{-3}\right]$	**With Constraint**				
$\Delta P^-_{e^-}/P$	1.1	1.2	0.93	0.63	0.69



Conclusions

- **Polarization provides a deep insight in the chiral structure of the standard model and beyond**

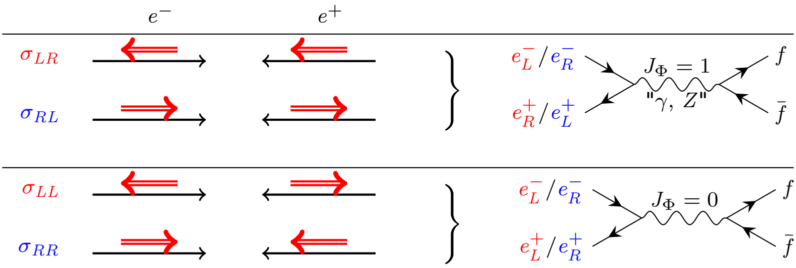
  $\Rightarrow$ A permille-level precision of the luminosity-weighted average polarization at the IP is required

- **Permille-Level polarization precision is achieved by the combination of**
  - The time-resolved beam polarization measurement with laser-Compton polarimeters

  $+$ The luminosity-weighed averaged polarization determined from collision data

$\Rightarrow$ **With the combined method, the permille-level will be reached after ≈ 3 years of data taking**

- **Consider only one electron positron pair:**
  - Helicity is the projection of the spin vector on the direction of motion
  - In case of massless particles, helicity is equal to chirality (left and right handedness)
  - If $E_{\text{kin}} \gg E_0 \quad \longrightarrow \quad m_e \approx 0$ e.g. ILC: $E_{\text{kin}}/E_0 \approx \mathcal{O}\left(10^5 - 10^6\right)$



- **For a bunch of particles the polarization P is defined as:**

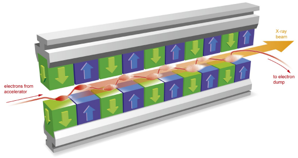
$$P := \frac{N_R - N_L}{N_R + N_L} \qquad \begin{cases} N_R: & \text{The number of right-handed particles} \\ N_L: & \text{The number of left-handed particles} \end{cases}$$

Electron beam:

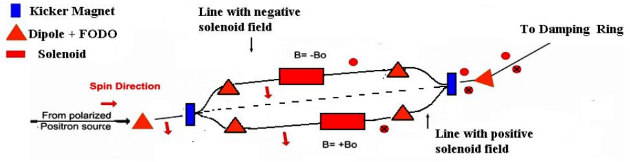
- Shooting of a circular polarized laser on a photocathode
- Switch between polarization signs (helicity reversal)
  - $\Rightarrow$ Switch between signs of the laser polarization

Positron beam:

- Production of circular polarized γ's from e^--beam propagating through a helical undulator
  - $\Rightarrow$ e^+ obtained via pair-production of the γ's
- Helicity reversal
  - $\Rightarrow$ Switch between two beam lines

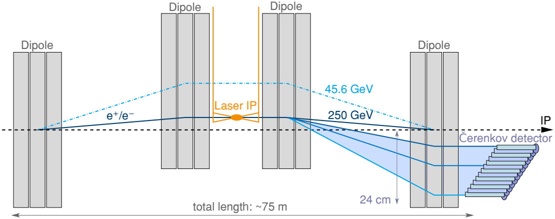


www.xfel.eu/ueberblick/funktionsweise/



Laser-Compton Polarimeters

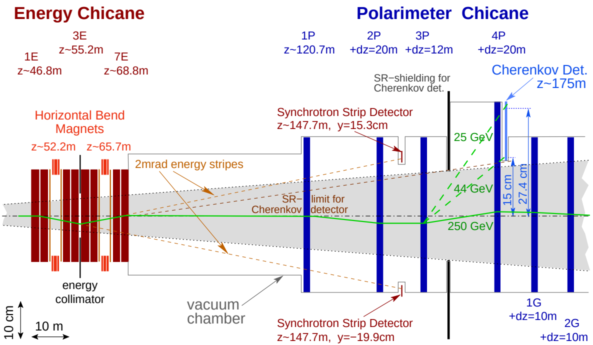
Magnetic chicane of the upstream polarimeters



- Compton scattering of the beam with a polarized Laser
- $\mathcal{O}(10^3)$ particles per bunch $(2 \cdot 10^{10})$ are scattered
- Magnetic chicane: energy spectrum $\Rightarrow$ spatial distribution

- Energy spectrum measurement:
  $\Rightarrow$ Counting the scattered particles at different positions
- Design of the magnetic Chicane:
  - Laser-bunch interaction point moves with beam energy
    $\longrightarrow$ position of the Compton edge stays the same
  - Orbit of the non-scattered particles is unaffected by the magnetic chicane

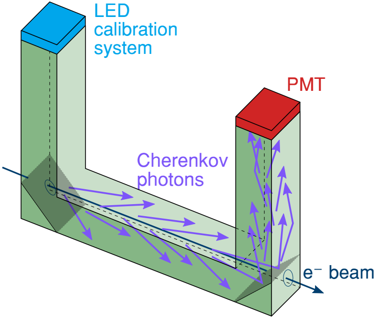
Downstream Polarimeter

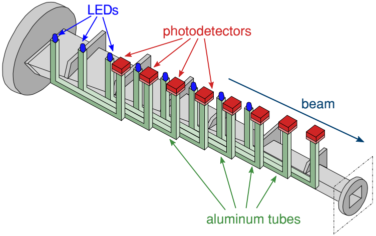


Difference to Upstream Polarimeter due to a large disturbed beam

- Stronger banding of the beam after γ-IP
- 2 additional magnets to restore the beam orbit
- Measuring one bunch per train

Cherenkov Detectors: Basic Concept

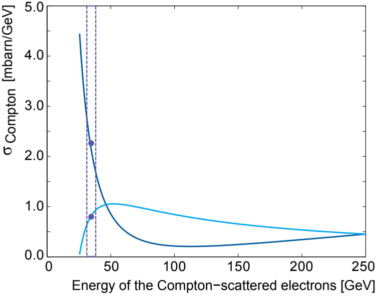




- U-shape channels filled with gas: e.g. perfluorobutane
- Concept
  - Scattered particles propagates through the bottom
  - Produced Cherenkov light is reflected to one end of the channel
  - Light measurement with photomultiplier tube (PMT)
- At the other end: LED for PMT calibration
- Sampling of the energy distribution
  → Number of Cherenkov detector
- Energy resolution
  → Thickness of a Cherenkov detector
- Quartz Cherenkov detector concept:
  Ref.: Theses Annika Vauth

http://bib-pubdb1.desy.de/record/171400

Differential Compton Cross Section



Energy dependence:

$$\frac{\mathrm{d}\sigma_C}{\mathrm{d}y_C} = \frac{2\pi r_e^2}{x_C}\left(a_C + \lambda\mathcal{P}\cdot b_C\right); \qquad y_C := 1 - \frac{E'}{E}$$

e^- Polarization: $\mathcal{P}$; Laser Polarization: λ

DarkBlue: $\lambda\mathcal{P} = +1$

Cyan: $\lambda\mathcal{P} = -1$

Calculating $\mathcal{P}_i$ of the i-th channel with asymmetry A_i, analysing power Π_i

$$A_i := \frac{N_i^- - N_i^+}{N_i^- + N_i^+}; \qquad \Pi_i = \frac{\mathcal{I}_i^- - \mathcal{I}_i^+}{\mathcal{I}_i^- + \mathcal{I}_i^+}; \qquad \mathcal{I}_i^{\pm} := \int_{E_i-\Delta/2}^{E_i+\Delta/2} \left.\frac{\mathrm{d}\sigma_C}{\mathrm{d}y_C}\right|_{\lambda\mathcal{P}=\pm 1} \mathrm{d}y_C$$

$N^{\pm} := \#e_{\text{Compton}}$ for $\lambda\mathcal{P} = \pm 1$; E_i : energy of i-th channel; Δ : energy width

$$\Rightarrow \quad \lambda\mathcal{P}_i = \frac{A_i}{\Pi_i} \quad \Rightarrow \quad \mathcal{P} = \langle \mathcal{P}_i \rangle$$

Compton Scattering Cross Section: Formulary

$$\frac{d\sigma}{dy_C} = \frac{2\pi r_e^2}{x_C}\left(a_C + \lambda\mathcal{P}\cdot b_C\right)$$

$$y_C := 1 - \frac{E'_\gamma}{E}; \quad x_C := \frac{4EE_\gamma}{m_e^2}\cos^2\left(\frac{\vartheta_0}{2}\right)$$

$$r_C := \frac{y_C}{x_C\left(1-y_C\right)}$$

$$a_C := \left(1-y_C\right)^{-1} + 1 - y_C - 4r_C\left(1-r_C\right)$$

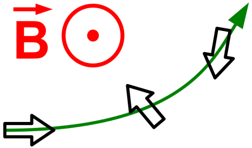
$$b_C := r_C x_C\left(1-2r_C\right)\left(2-y_C\right)$$

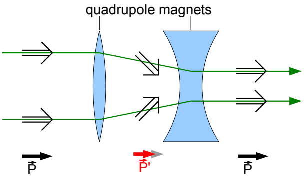
$E,\ E_\gamma$:	e^-,γ energy before Compton scattering
$E',\ E'_\gamma$:	e^-,γ energy after Compton scattering
$m_e,\ r_e$:	mass, classical radius of e^-
ϑ_0 :	crossing angle between e^-,γ
$\mathcal{P}$:	longitudinal polarization of e^-
λ :	circular polarization of γ_{Laser}

Characteristic Point:

$$E'_{\text{crossover}} = \frac{E}{1+x_C/2}, \qquad E'_{\text{ComptonEdge}} = E'_{\text{min}} = \frac{E}{1+x_C}$$

Spin Precession





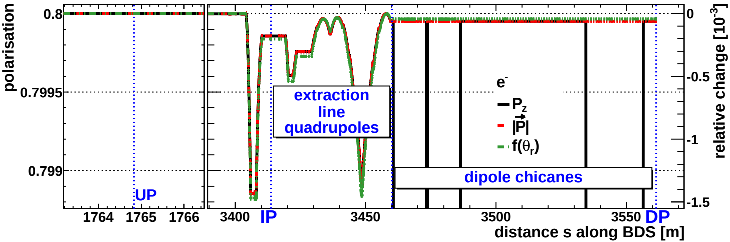
- Polarimeters are 1.65 km and 150 m away from IP
  - → Particles propagate through magnets
  - → Magnets influence the spin, as well
  - → Described by Thomas precession
- if $\vec{B}_{\parallel} = \vec{E} = 0$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{S} = -\frac{q}{m\gamma}\left((1 + a\gamma)\,\vec{B}_{\perp}\right) \times \vec{S}$$

- Effects from focusing and defocusing can cancel
- For a series of quadrupole magnets $\mathcal{P}$ described by the angular divergence θ_r

$$f(\theta_r) = |\vec{\mathcal{P}}|_{\mathrm{max}} \cdot \cos\left((1 + a\gamma) \cdot \theta_r\right)$$

Spin Tracking



Further causes of longitudinal beam polarization change:

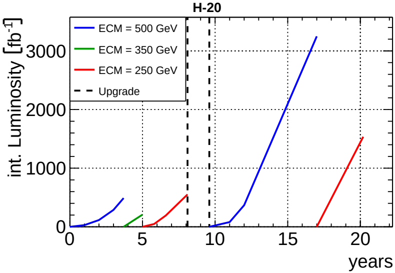
- *Bremsstrahlung:*
  Deceleration by passing through matter $\longrightarrow$ negligible for colliders
- *Beamstrahlung:*
  Deflection by the em-field of the oncoming bunch during collision
- *Synchrotron radiation:*
  Deflection by the em-field of accelerator magnets

Systematic Polarization Uncertainty

contribution	uncertainty $\left[10^{-3}\right]$
Beam and polarization alignment at polarimeters and IP ($\Delta\vartheta_{\text{bunch}} = 50\,\mu$rad, $\Delta\vartheta_{\text{pol}} = 25$ mrad)	0.72
Variation in beam parameters (10 % in the emittances)	0.03
Bunch rotation to compensate the beam crossing angle	< 0.01
Longitudinal precession in detector magnets	0.01
Emission of synchrotron radiation	0.005
Misalignments (10 μ) without collision effects	0.43
Total (quadratic sum)	0.85
Collision effects in absence of misalignments	< 2.2

[Ref.:] Thesis Moritz Beckmann (http://bib-pubdb1.desy.de/record/155874)

Reference ILC Running Scenario: H-20



- Each run have its own polarization measurement

⇒ H-20: 5 polarization measurements

- All plots will refer to ILC nominal energy of 500 GeV

$\sqrt{s}$	$(-,+)$	$(+,-)$	$(-,-)$	$(+,+)$	$\int L$ dt
GeV	[%]	[%]	[%]	[%]	[fb^{-1}]
250	67.5	22.5	5	5	2000
350	67.5	22.5	5	5	200
500	40	40	10	10	4000

Special Case: The Modified Blondel Scheme (MBS)

- Constraints for the Modified Blondel Scheme:
  - Process must fulfill: $\sigma_{\text{LL}} \equiv \sigma_{\text{RR}} \equiv 0$
  - Perfect helicity reversal: $+|P| \longleftrightarrow -|P| \quad \Rightarrow \quad |P| \equiv \text{const.}$
- Unique solution:

  4 possible cross section measurements: $\sigma_{-+},\ \sigma_{+-},\ \sigma_{--},\ \sigma_{++}$

  Maximal 4 unknown quantities: $\sigma_{\text{LR}},\ \sigma_{\text{RL}},\ |P_{e^-}|,\ |P_{e^+}|$
- Solve for $|P_{e^\mp}|$:

$$\sigma_{\pm\pm} = \frac{(1 \pm |P_{e^-}|)}{2} \frac{(1 \mp |P_{e^+}|)}{2} \cdot \sigma_{RL} + \frac{(1 \mp |P_{e^-}|)}{2} \frac{(1 \pm |P_{e^+}|)}{2} \cdot \sigma_{LR}$$

- Modified Blondel-Scheme:

$$|P_{e^\mp}| = \sqrt{\frac{(\sigma_{-+} + \sigma_{+-} - \sigma_{--} - \sigma_{++})(\pm\sigma_{-+} \mp \sigma_{+-} + \sigma_{--} - \sigma_{++})}{(\sigma_{-+} + \sigma_{+-} + \sigma_{--} + \sigma_{++})(\pm\sigma_{-+} \mp \sigma_{+-} - \sigma_{--} + \sigma_{++})}}$$

- Uncertainties are calculated via analytic error propagation

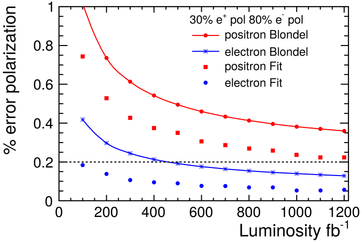
Previous W-Pair Study by Ivan Marchesini

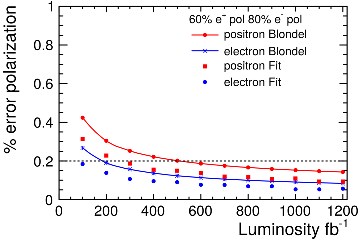
W-Pair Production:

- Using $e^- e^+ \rightarrow W^+ W^- \rightarrow q\bar{q}l\nu$
- Statistical uncertainties only
- Consider equal absolute polarizations
- Including full background study

Analyses techniques: (overview)

- Modified Blondel Scheme
- Angular Fit:
  - Using a χ^2-minimization
  - Considering the production at different angles
  - Studied effects on deviations of the absolute polarization value
  - Measurement of triple gauge couplings



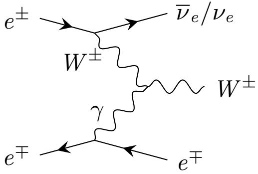


Ref.: Theses Ivan Marchesini (`http://pubdb.xfel.eu/record/94888`)

Previous Single $W^\pm$, Z, γ Study by Graham W. Wilson

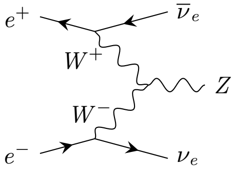
Single $W^\pm$

$e^- e^+ \to e^\pm \nu \mu^\mp \nu$



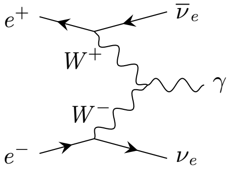
Single Z

$e^- e^+ \to \nu\bar{\nu}Z$



Single γ

$e^- e^+ \to \nu\bar{\nu}\gamma;$



#	P	$\Delta P/P$
2	P_{e^-}	0.07%
	P_{e^+}	0.22%
4	P_{e^-}	0.085%
	δ_{e^-}	0.12%
	P_{e^+}	0.22%
	δ_{e^+}	0.32%

- Using a χ^2-minimization
  - Total cross sections only
  - Simultaneous cross section measurement
  - Using 4 processes simultaneously
- Only statistical error with fiducial cuts on cross sections
- Measuring absolute polarization deviation

Ref.: Talk Graham W. Wilson (https://agenda.linearcollider.org/event/5468/contributions/24027/)

Current Work on the Determination of the Polarization from Collision Data

Goal:

General strategy for the polarization determination which yields the best precision per measurement time

- General and flexible method combining all relevant processes
- Including all uncertainties and their correlations
- Compensating for a non-perfect helicity reversal
- Considering the additional information from the angular distributions
- Using constraints from the polarimeter measurement for further improvement

Expected Polarized Cross Section

- Theoretical polarized cross section in general:

$$\sigma_{\text{theory}}\left(P_{e^-}, P_{e^+}\right) = \frac{\left(1-P_{e^-}\right)}{2}\frac{\left(1-P_{e^+}\right)}{2}\cdot\sigma_{\text{LL}} + \frac{\left(1+P_{e^-}\right)}{2}\frac{\left(1+P_{e^+}\right)}{2}\cdot\sigma_{\text{RR}} + \frac{\left(1-P_{e^-}\right)}{2}\frac{\left(1+P_{e^+}\right)}{2}\cdot\sigma_{\text{LR}} + \frac{\left(1+P_{e^-}\right)}{2}\frac{\left(1-P_{e^+}\right)}{2}\cdot\sigma_{\text{RL}}$$

- Nominal ILC Polarization values

$$\underbrace{P_{e^-}^{-} = -80\%,}_{\text{"left"-handed } e^- \text{-beam}} \quad \underbrace{P_{e^-}^{+} = 80\%,}_{\text{"right"-handed } e^- \text{-beam}} \quad \underbrace{P_{e^+}^{-} = -30\%,}_{\text{"left"-handed } e^+ \text{-beam}} \quad \underbrace{P_{e^+}^{+} = 30\%,}_{\text{"right"-handed } e^+ \text{-beam}}$$

- Cross section of the 4 polarization configurations

$$\sigma_{--} := \sigma\left(P_{e^-}^{-}, P_{e^+}^{-}\right) \qquad \sigma_{++} := \sigma\left(P_{e^-}^{+}, P_{e^+}^{+}\right)$$

$$\sigma_{-+} := \sigma\left(P_{e^-}^{-}, P_{e^+}^{+}\right) \qquad \sigma_{+-} := \sigma\left(P_{e^-}^{+}, P_{e^+}^{-}\right)$$

Comparison to Previous Analyses (Statistical Uncertainty Only)

E	**500**	**250**
$\mathcal{L}$	**3500**	**1500**
$P^{-}_{e^-}$	0.08	0.09
$P^{+}_{e^-}$	0.02	0.02
$P^{-}_{e^+}$	0.04	0.04
$P^{+}_{e^+}$	0.08	0.08

E	**500**	**350**	**250**
$\mathcal{L}$	**500**	**200**	**500**
$P^{-}_{e^-}$	0.2	0.3	0.1
$P^{+}_{e^-}$	0.05	0.06	0.03
$P^{-}_{e^+}$	0.1	0.1	0.06
$P^{+}_{e^+}$	0.2	0.3	0.1

Single boson:

- $\mathcal{L} = 2000\,\text{fb}^{-1},\ E = 500\ GeV$
- No background estimation
- Fiducial cross section cuts
- Limitation on δ: $\Delta\delta < 10^{-3}$

P_{e^-} : 0.085% δ_{e^-} : 0.12%
P_{e^+} : 0.22% δ_{e^+} : 0.32%

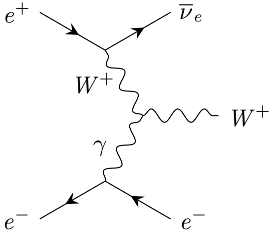
W-pairs:

- $\mathcal{L} = 500\,\text{fb}^{-1},\ E = 500\ GeV$
- Full background estimation
- Differential cross section (angular fit)
- Only using 2 free parameters

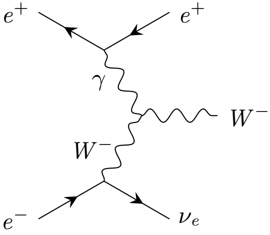
P_{e^-} : 0.08% P_{e^+} : 0.34%

Previous Single $W^{\pm}$, Z, γ Study: Leading Diagrams

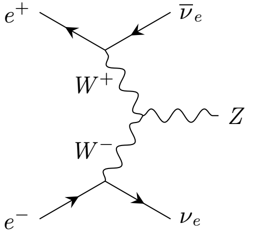
Single W^+



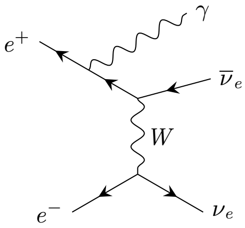
Single W^-

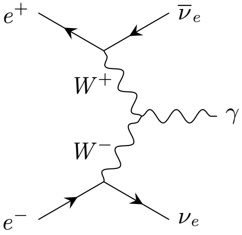


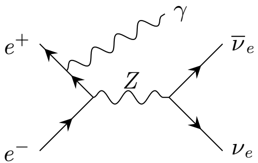
Single Z



Single γ



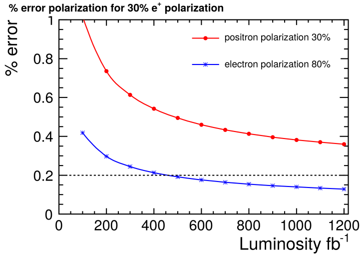




Comparison to the Previous W-Pair Study

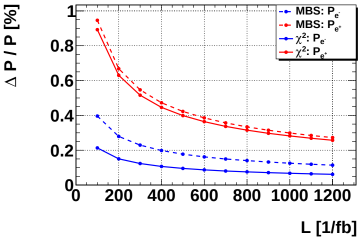
Study by Ivan Marchesini:

- Using $e^- e^+ \to W^+ W^- \to q\bar{q}l\nu$
- Statistical uncertainties only
- Consider equal absolute polarizations (MBS)
- Including full background study



Adjustment of the current study:

- Limited to $e^- e^+ \to W^+ W^- \to q\bar{q}l\nu$
- Forced equal absolute polarizations $\left(\left|P^L\right| \equiv \left|P^R\right|\right)$
- Including same background estimation and selection efficiency



Comparison:

⇒ χ^2-method yields better precision under same conditions than the MBS

Comparison to Previous Single $W^{\pm}$, γ, Z Study

Study by Graham W. Wilson

- Using 4 Processes simultaneously:

$$e^- e^+ \to \nu\bar{\nu}\gamma; \qquad e^- e^+ \to \nu\bar{\nu}Z$$
$$e^- e^+ \to e^+ \nu W^- \to e^+ \nu \mu^- \bar{\nu}$$
$$e^- e^+ \to e^- \bar{\nu} W^+ \to e^- \bar{\nu} \mu^+ \nu$$

- Consider equal absolute polarizations
  2 Parameters: P_{e^-}, P_{e^+}

- Consider deviations: 4 Parameters

$$P^L_{e^\pm} = -|P_{e^\pm}| + \tfrac{1}{2}\delta_\pm$$
$$P^R_{e^\pm} = \quad |P_{e^\pm}| + \tfrac{1}{2}\delta_\pm$$

parameters		$\Delta P/P,\ \mathcal{L} = 2\text{ab}^{-1}$	
#	P	**Previous**	**Current**
2	P_{e^-}	0.07%	0.051%
	P_{e^+}	0.22%	0.21%
4	P_{e^-}	0.085%	0.088%
	δ_{e^-}	0.12%	0.19%
	P_{e^+}	0.22%	0.23%
	δ_{e^+}	0.32%	0.56%

$\mathcal{L}$ equally distributed between $\sigma_{\pm\pm}$

Statistical precision only

Comparison to Current analysis

- Differences:

Previous: Constraint on δ: $\Delta\delta < 10^{-3}$

Current: direct fit of $P^{L,R}_{e^\pm}$

- Very similar precision even without additional constraint on δ

DESY

Combining W-Pair + Single W, Z, γ

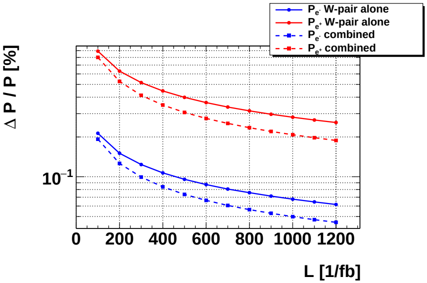
Combined vs. W-Pairs alone

- W-Pair yields only enough information for 2 parameter fit P_{e^-}, P_{e^+}
- Large improvement
  $\rightarrow$ due to additional processes
- Combined: fit of 4 parameters is possible $P^L_{e^-}, P^R_{e^-}, P^L_{e^+}, P^R_{e^+}$

$\Rightarrow$ Compensation for a non-perfect helicity reversal

Combined vs. Single Boson

- The 4 processes Single $W^\pm$, Single Z, Single γ yields a large analysis power
- Combined precision dominated by single boson processes



$\Delta P/P,\ \mathcal{L} = 2\text{ab}^{-1}$

	single W, Z, γ	Combined
P_{e-}	0.088%	0.079%
δ_{e-}	0.19%	0.18%
P_{e+}	0.23%	0.16%
δ_{e+}	0.56%	0.51%

Polarized Cross Section Calculation: Basic Concept

Theory:

$$\sigma^{\text{theory}}(P_{e^-}, P_{e^+}) = \frac{(1-P_{e^-})}{2}\frac{(1+P_{e^+})}{2}\cdot\sigma_{\text{LR}} + \frac{(1+P_{e^-})}{2}\frac{(1-P_{e^+})}{2}\cdot\sigma_{\text{RL}} + \frac{(1-P_{e^-})}{2}\frac{(1-P_{e^+})}{2}\cdot\sigma_{\text{LL}} + \frac{(1+P_{e^-})}{2}\frac{(1+P_{e^+})}{2}\cdot\sigma_{\text{RR}}$$

Experiment:

$$\sigma^{\text{data}}(P_{e^-}, P_{e^+}) = \frac{N(P_{e^-}, P_{e^+})}{\mathcal{L}(P_{e^-}, P_{e^+})}$$

N: number of events
$\mathcal{L}$: integrated luminosity
Uncertainty via propagation of errors

$$\Delta\sigma^2 = \underbrace{\left(\frac{\partial\sigma}{\partial N}\Delta N\right)^2}_{\text{statistical uncertainty}} + \underbrace{\left(\frac{\partial\sigma}{\partial \mathcal{L}}\Delta \mathcal{L}\right)^2}_{\text{systematical uncertainty}}$$

Nominal ILC Polarization values

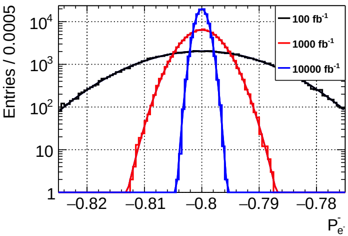
$$\underbrace{P^-_{e^-} = -80\%,}_{\text{"left"-handed } e^-\text{-beam}} \quad \underbrace{P^+_{e^-} = 80\%,}_{\text{"right"-handed } e^-\text{-beam}} \quad \underbrace{P^-_{e^+} = -30\%,}_{\text{"left"-handed } e^+\text{-beam}} \quad \underbrace{P^+_{e^+} = 30\%,}_{\text{"right"-handed } e^+\text{-beam}}$$

χ^2-Minimization

- **Defining χ^2 function:**

$$\chi^2 := \sum_{\text{process}} \sum_{\pm\pm} \frac{\left(\sigma_{\text{data}} - \sigma_{\text{theory}}\left(P_{e-}^{-},\, P_{e-}^{+},\, P_{e+}^{-},\, P_{e+}^{+}\right)\right)^2}{\Delta\sigma^2}$$

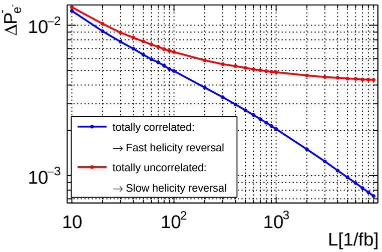
- **Variating** $\left(P_{e-}^{-},\, P_{e-}^{+},\, P_{e+}^{-},\, P_{e+}^{+}\right) \longrightarrow$ **Minimizes** χ^2
- **Toy measurement:**
  - Signal expectation value: $\langle D \rangle = \sigma_{\text{theory}} \cdot \varepsilon \cdot \mathcal{L} + \mathfrak{B}$
  - One toy experiment: Random Poisson number around each $\langle D \rangle$
  - Determine $P_{e\pm}^{\pm}$ by minimizing χ^2
  - Simplified case for illustration:
    - $\mathfrak{B} = 0$ and $\varepsilon = 1$
    - Statistical uncertainties only
    - Using 10^5 toy measruements



Systematic Uncertainties and their Correlations

- **Systematic Uncertainties are influenced by**
  - Detector calibration and alignment
  - Machine performance
  - ...

⇒ **Time dependent uncertainties**



- **Data set are taken one at a time:**
  - Slow frequency of helicity reversals: $\mathcal{O}$ (weeks to months)
  - Data sets are independent

  → Completely uncorrelated

  ✘ Lead to saturation at systematic precision

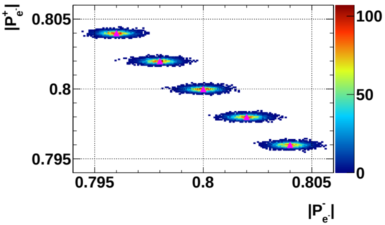
- **Data sets taken concurrently:**
  - Fast frequency of helicity reversals: $\mathcal{O}$ (train-by-train)

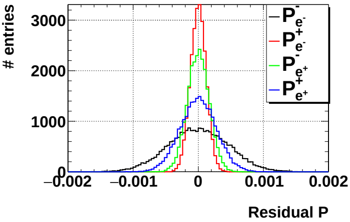
  → Faster than changes in calibration/alignment

  → Generate correlations

  ✓ Lead to cancellation of systematic uncertainties

Testing for a Non-Perfect Helicity Reversal





- **Variation in the absolute polarization**
  - Toy Measurement for 5 different polarization discrepancies for both beams
  - Nominal initial polarizations: $|P_{e-}| = 80\%$, $|P_{e+}| = 30\%$
  - Statistical uncertainties only
- **χ^2-Fit:**
  - Correct determination of the 4 polarization values
  - No noticeable impact on polarization precision using total cross sections

✓ **Can compensate for a non-perfect helicity reversal**

Theoretical Limit of the Statistical Precision

Consider most relevant processes:

Process	Channel
single $W^{\pm}$	$e\nu l\nu$, $e\nu q\bar{q}$
WW	$q\bar{q}q\bar{q}$, $q\bar{q}l\nu$, $l\nu l\nu$
ZZ	$q\bar{q}q\bar{q}$, $q\bar{q}ll$, $llll$
$ZZWW$Mix	$q\bar{q}q\bar{q}$, $l\nu l\nu$
Z	$q\bar{q}$, ll

- Same processes as for physics analysis (DBD)
- Tree-level cross sections + ISR
- Any combination of processes can be used
- Further process can easily added

Consider best case scenario using σ_{tot}:

- Assumption of a perfect 4π detector
- No background
- No systematic uncertainties
- Using all considered processes

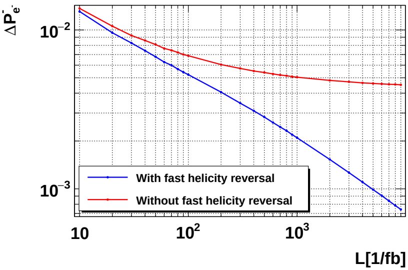
Statistical precision H-20: $\Delta P/P\,[\%]$

E	500	350	250	500	250
$\mathcal{L}$	500	200	500	3500	1500
$P^-_{e^-}$	0.2	0.3	0.1	0.08	0.09
$P^+_{e^-}$	0.05	0.06	0.03	0.02	0.02
$P^-_{e^+}$	0.1	0.1	0.06	0.04	0.04
$P^+_{e^+}$	0.2	0.3	0.1	0.08	0.08

DESY

Systematic Uncertainties and their Correlations

Systematic quantity		related to:
Integrated luminosity	$\mathcal{L}$	accelerator
Selection efficiency	ε	detector
Background estimate	$\mathfrak{B}$	theory



Remark:
A non-perfect helicity reversal has close to no influence on the precision due to compensation

- **Uncertainties influenced by**
  - Detector calibration and alignment
  - Machine performance
  - $\mathfrak{B}$ assumed constant and small
  - ⇒ $\Delta\mathcal{L}$, $\Delta\varepsilon$ are time dependent
- **One data set at a time:**
  - Data sets are independent
  - Completely uncorrelated
  - ⇒ Lead to saturation at systematic precision
- **Fast helicity reversal:**
  - Data sets taken concurrently
  - Generate correlations
  - ⇒ Lead to cancellation of systematic uncertainties

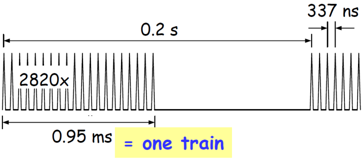
Generation of Correlated Uncertainties: Fast Helicity Reversal

Generation of Correlated Uncertainties

⇒ Change between data sets $(\sigma_{-+},\ \sigma_{+-},\ \sigma_{--},\ \sigma_{++})$
faster than change in detector and accelerator calibration

⇒ Change between data sets during normal run without additional breaks

ILC Bunch Structure



Two possible frequency:

- bunch-by-bunch:
  switch between tow bunches
- train-by-train:
  switch between two trains

- Technical feasibility much easier for train-by-train
- Switch train-by-train should be sufficient for polarization precision

⇒ Precise correlation coefficient still to do

Implementing Correlated Uncertainties

$$\chi^2 = \sum_{\text{process}} \left(\vec{\sigma}_{\text{data}} - \vec{\sigma}_{\text{theory}}\right)^T \Xi^{-1} \left(\vec{\sigma}_{\text{data}} - \vec{\sigma}_{\text{theory}}\right); \qquad \vec{\sigma} := \begin{pmatrix} \sigma_{-+} & \sigma_{+-} & \sigma_{--} & \sigma_{++} \end{pmatrix}^T$$

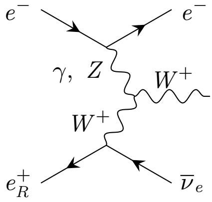
$$\Xi := \Xi_D + \Xi_{\mathfrak{B}} + \Xi_\varepsilon + \Xi_{\mathcal{L}}; \qquad \text{e.g.} \quad \left(\Xi_\varepsilon\right)_{ij} = \text{corr}\left(\vec{\sigma}_i^\varepsilon,\ \vec{\sigma}_j^\varepsilon\right) \frac{\partial \vec{\sigma}_i}{\partial \varepsilon_i} \frac{\partial \vec{\sigma}_j}{\partial \varepsilon_j} \Delta\varepsilon_i \Delta\varepsilon_j$$

- Using the inverse covariance matrix Ξ^{-1}
- Correlation coefficients:
  - Identical for each process
  - Can be different for each quantity $(\mathfrak{B},\ \varepsilon,\ \mathcal{L})$
  - Statistical uncertainties are always uncorrelated $\text{corr}\left(\vec{\sigma}_i^D,\ \vec{\sigma}_j^D\right) = 0 \quad \forall i \neq j$
  - Correlation coefficients are no free parameter
    $\Rightarrow$ They are an external input parameter

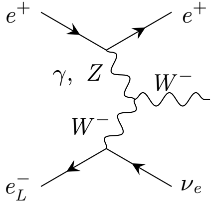
Consideration of the Addition Information from the Angular Distribution

- Total cross section
  - Rely on theoretical calculation
  - ⇒ Susceptible to BSM effects
- Differential cross section
  - Additional usage of the angular information
  - ⇒ Increase of the robustness against BSM effects
- Starting with Single W Process
  - Angular distribution has a large dependence on the chirality
  - Separated in W^+ and W^- production
  - ⇒ Sensitive to individual beam polarization
    - W^+: only sensitive to P_{e+}
    - W^-: only sensitive to P_{e-}
- Further processes can easily be included

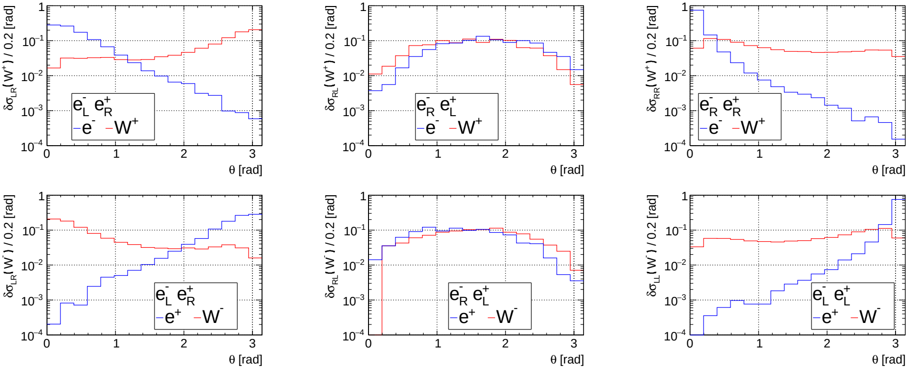
$e^- e^+ \longrightarrow W^+ e^- \bar{\nu}_e$



$e^- e^+ \longrightarrow W^- e^+ \nu_e$



Single $W^{\pm}$: Polar Production Angle Distribution

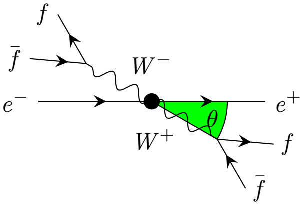


- Single differential cross section: $\partial\sigma/\partial\theta$
  - Two independent angles: $\theta_e,\ \theta_W$
  - For now start with $\theta_e \rightarrow e^{\pm}$ also needed for separation between $W^{\pm}$
- $\partial\sigma/\partial\theta$ will be calculated via $\Delta\sigma_i/\Delta\theta_i$ ("cross section for the i-th bin in θ")

Usage of the Differential Polarized Cross Section

- Total cross section
  - Rely on theoretical calculation
  - ⇒ Susceptible to BSM effects
- Differential cross section
  - Additional usage of the angular information
  - ⇒ Increase of the robustness against BSM effects

e.g.: $e^+e^- \to W^+W^- \to q\bar{q}l\nu$



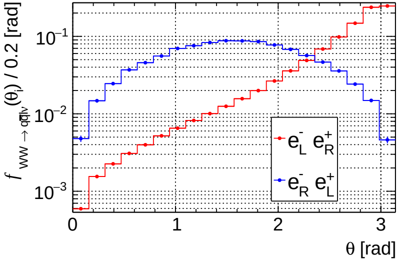
Bin-wise cross section calculation:

$$\underbrace{\partial\sigma/\partial\theta}_{\text{differential cross section}} \longrightarrow \underbrace{\delta_i\sigma_{\text{data}}}_{\text{cross section of the } i\text{-th bin}} := \delta_i N/\mathcal{L}$$

$$\longrightarrow \delta_i\sigma_{LR} := f_{LR}(\theta_i)\cdot\sigma_{LR}$$

Analog: $RL,\ LL,\ RR$

- $\delta_i N$: events of i-th bin
- $f(\theta_i)$: fraction of the total cross section



Defining Differential Cross Sections

Measured cross section:

$$\overbrace{\frac{\partial\sigma}{\partial\theta}}^{\text{differential cross section}} \longrightarrow \overbrace{\delta_i\sigma_{\text{data}}}^{\text{cross section per } i\text{th bin}} := \frac{\delta_i D - \delta_i \mathfrak{B}}{\delta_i\varepsilon \cdot \mathcal{L}}$$

$\delta_i D$	Number of signal events	of the ith bin
$\delta_i \mathfrak{B}$	Number of expected background events	of the ith bin
$\delta_i \varepsilon$	Selection efficiency	of the ith bin
$\mathcal{L}$	Integrated luminosity	

Theoretical cross section:

$$\begin{aligned}\delta_i\sigma_{\pm\pm} = &\frac{\left(1 \pm \left|P^{\pm}_{e^-}\right|\right)}{2}\frac{\left(1 \pm \left|P^{\pm}_{e^+}\right|\right)}{2}\delta_i\sigma_{RR} + \frac{\left(1 \mp \left|P^{\pm}_{e^-}\right|\right)}{2}\frac{\left(1 \mp \left|P^{\pm}_{e^+}\right|\right)}{2}\delta_i\sigma_{LL} \\ + &\frac{\left(1 \pm \left|P^{\pm}_{e^-}\right|\right)}{2}\frac{\left(1 \mp \left|P^{\pm}_{e^+}\right|\right)}{2}\delta_i\sigma_{RL} + \frac{\left(1 \mp \left|P^{\pm}_{e^-}\right|\right)}{2}\frac{\left(1 \pm \left|P^{\pm}_{e^+}\right|\right)}{2}\delta_i\sigma_{LR}\end{aligned}$$

$$\delta_i\sigma_{\text{theory}} := f(\theta_i) \cdot \sigma_{\text{theory}}$$

$f(\theta_i)$ is directly obtained from the angular distributions

Implementing Differential Cross Sections in the χ^2 Minimization

Replacing: $\sigma \longrightarrow \delta_k\sigma$ + Sum over all bins

$$\chi^2 = \sum_{\text{process}} \sum_{\theta_k} \left(\delta_k\vec{\sigma}_{\text{data}} - \delta_k\vec{\sigma}_{\text{theory}}\right)^T \left(\delta_k\Xi\right)^{-1} \left(\delta_k\vec{\sigma}_{\text{data}} - \delta_k\vec{\sigma}_{\text{theory}}\right)$$

$$\delta_k\vec{\sigma} := \begin{pmatrix}\delta_k\sigma_{-+} & \delta_k\sigma_{+-} & \delta_k\sigma_{--} & \delta_k\sigma_{++}\end{pmatrix}^T$$

$$\delta_k\Xi := \delta_k\Xi_N + \delta_k\Xi_{\mathfrak{B}} + \delta_k\Xi_\varepsilon + \delta_k\Xi_{\mathcal{L}};$$

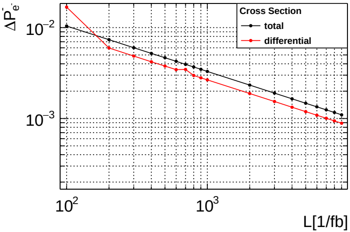
$$\left(\delta_k\Xi_\varepsilon\right)_{ij} = \text{corr}\left(\vec{\sigma}_i^\varepsilon,\ \vec{\sigma}_j^\varepsilon\right) \frac{\partial\left(\delta_k\vec{\sigma}_i\right)}{\partial\left(\delta_k\varepsilon_i\right)} \frac{\partial\left(\delta_k\vec{\sigma}_j\right)}{\partial\left(\delta_k\varepsilon_j\right)} \Delta\left(\delta_k\varepsilon_i\right) \Delta\left(\delta_k\varepsilon_j\right)$$

Remarks:

- Due to correlations, the binning in θ has to be equal for all cross sections
- It can differ between processes and decay-channels
- Range and number of bins of θ can be changed externally for each process

First Toy Measurements: Preliminary Results

Single $W^{\pm}$ only



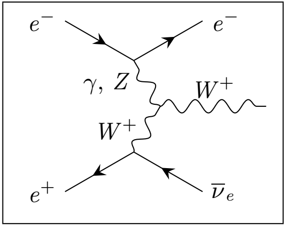
Using the following configuration:

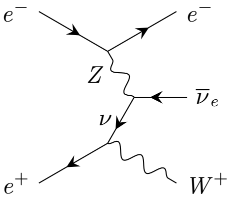
- Using 16 equal bins in a θ range of $[0, \pi]$
- Signal determination bin-by-bin: $\langle \delta_k D \rangle = \delta_k \sigma_{\text{theory}} \cdot \delta_k \varepsilon \cdot \mathcal{L} + \delta_k \mathfrak{B}$
- For the start: Statistical error only + no background
- Using H-20 integrated luminosity sharing due to energy

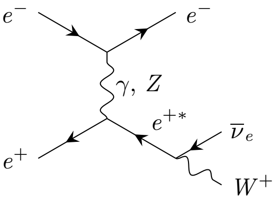
- Differential cross section have a lower statistic uncertainty:
  - Expectation of $\delta_k D$ can be for some bins $\mathcal{O}(1)$
  - Some zero diagonal entries of the covariance matrix → not invertible

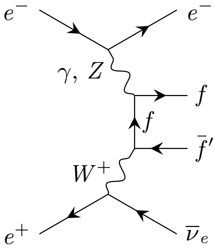
  ⇒ Dropping χ^2−terms with $\delta_k D = 0$
- Further steps:
  - Optimizing the θ range and binning
  - Including further processes

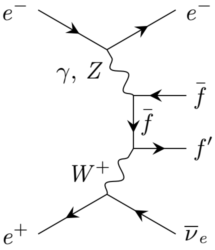
Contributions to "Actual" Single W Production









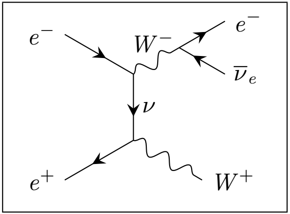


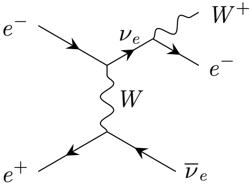
Polarization dependence:

- Positrons:
  - $\Rightarrow$ only right-handed: e^+_R
- Electrons:
  - $e^-_L + e^-_R$
  - difference for $\gamma,\ Z$

Further Contributions to Single W Production

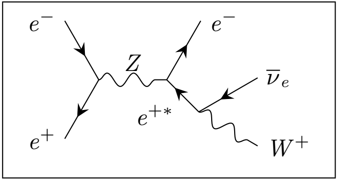
Contribution from t-channel process:

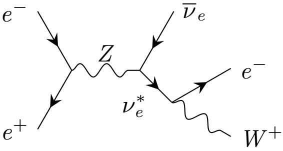


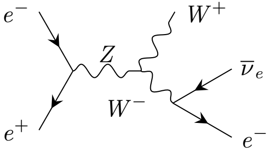


- t-channel contributions:
  - W-pair production
  - W exchange
- Polarization dependence:
  - → only e_R^+
  - + only e_L^-

Contribution from s-channel Z process (only for opposite chirality):







E	500	350	250	500	250
$\mathcal{L}$	500	200	500	3500	1500
Without Constraint					
$P_{e^-}^-$	0.2	0.3	0.1	0.08	0.09
$P_{e^-}^+$	0.05	0.06	0.03	0.02	0.02
$P_{e^+}^-$	0.1	0.1	0.06	0.04	0.04
$P_{e^+}^+$	0.2	0.3	0.1	0.08	0.08
With Constraint					
$P_{e^-}^-$	0.1	0.1	0.1	0.07	0.07
$P_{e^-}^+$	0.04	0.05	0.03	0.02	0.02
$P_{e^+}^-$	0.09	0.09	0.05	0.04	0.03
$P_{e^+}^+$	0.1	0.1	0.09	0.07	0.07

Systematic Uncertainty Calculation

Selection Efficiency ε

- Realistic values for ε and $\Delta\varepsilon$ still missing
- ε values for:
  - W-pair production: Thesis Ivan
  - Single W: Accessible due to the work of our former intern Sebastian Garcia

Expected Background Prediction $\mathfrak{B}$

- Background estimate $\mathfrak{B} \pm \Delta\mathfrak{B}$ is also missing for each process
- Full background analysis for every process is not feasible
- $WW \rightarrow qql\nu$: $\mathfrak{B}$ can be taken from Ivan's thesis

  $\Rightarrow$ Assuming similar $\mathfrak{B}$ for other boson pair

Simultaneous Fitting of chiral cross sections

Similar to G. Wilson's single boson study:

$$\begin{pmatrix} \sigma^{\gamma}_{LR} & \sigma^{Z}_{LR} & \sigma^{\mu}_{LR} & \sigma^{\mu}_{LR} \\ \sigma^{\gamma}_{RL} & 0.465 & 0.066 & 0.066 \\ 0.0 & 0.0 & \sigma^{\mu}_{SS} & 0.0 \\ 0.0 & 0.0 & 0.0 & \sigma^{\mu}_{SS} \end{pmatrix}$$

Ongoing Work for the current study:

- Implementing of free linear cross section scaling parameters
- Each chiral cross section can be scaled individually or fixed
- Chiral cross section can have the same scaling parameter
- Which cross section is scaled will be changeable

7-parameter single boson fit

$2\,\text{ab}^{-1}$ equally distributed (statistical errors only)

$$|P_{e^-}| = 80.000 \pm 0.056\%$$
$$|P_{e^-}| = 30.000 \pm 0.065\%$$
$$\sigma^{\gamma}_{LR} = 3098.0 \pm 2.7\,\text{fb}$$
$$\sigma^{\gamma}_{RL} = 25.3 \pm 1.1\,\text{fb}$$
$$\sigma^{Z}_{LR} = 159.40 \pm 0.57\,\text{fb}$$
$$\sigma^{\mu}_{LR} = 580.9 \pm 1.1\,\text{fb}$$
$$\sigma^{\mu}_{SS} = 657.4 \pm 1.1\,\text{fb}$$

Beam polarizations correlation:

$$\rho\left(|P_{e^-}|, |P_{e^-}|\right) = 14\%$$