A Non-Unitary Surprise

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[ZL, Matthew Buican]

Introduction I

- We study strongly interacting 4d $\mathcal{N} = 2$ superconformal field theories (SCFTs) called Argyres-Douglas theories. These are characterized by having fractional dimension Coulomb branch operators. No Lagrangian description is known.
- They can be engineered by compactifying 6d $\mathcal{N} = (2, 0)$ theory on a Riemann surface with punctures.
- The so-called Schur sector $(r = j_2 - j_1)$ of operators in 4d $\mathcal{N} = 2$ SCFTs is related to 2d chiral algebras. This correspondence changes the sign of the c central charge, exchanging unitary and non-unitary theories.
- The superconformal index in the Schur limit corresponds to the vacuum character of the chiral algebra.

Introduction II

Schur index and vacuum character

$$q^{c_{4d}/2}\mathsf{Tr}_{\mathcal{H}_{4d}}(-1)^F q^{E-R} = q^{-c_{2d}/24}\mathsf{Tr}_{\mathcal{H}_{2d}}(-1)^{j_1-j_2}q^{L_0}$$

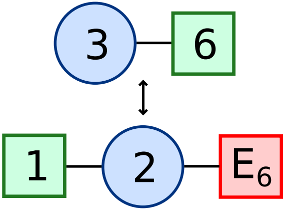
- Simplest example: Lee-Yang CFT ($(5,2)$ minimal model)

$$\chi_0^{LY}(q) = q^{\frac{11}{60}}\left(1+q^2+q^3+q^4+q^5+2q^6+\mathcal{O}\left(q^7\right)\right)$$
$$\chi_{-\frac{2}{5}}^{LY}(q) = q^{-\frac{13}{60}}\left(1+q+q^2+q^3+2q^4+2q^6+\mathcal{O}\left(q^7\right)\right)$$

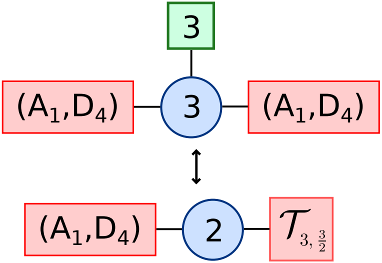
- Possibility of refinement of characters of CFTs with affine symmetry, we will turn corresponding fugacities off.
- Flavorless characters of chiral algebras satisfy Linear Modular Differential Equations (LMDEs) which can lead to classification of potential CFT characters.

Motivations

Argyres-Seiberg S-duality



S-duality for AD theories



$$\mathcal{I}_S^{\text{half-hyper}} = q^{1/6}\text{P.E.}\left(\frac{q^{1/2}}{1-q}\right)$$

$$\mathcal{I}_S^{(A_1,D_4)} = q^{1/3}\text{P.E.}\left(8\frac{q}{1-q^2}\right)$$

$$\text{P.E.}\left(f(q)\right) \equiv \text{Exp}\left(\sum_n \frac{1}{n} f(q^n)\right)$$

Character relations

- The chiral algebra corresponding to (A_1, D_4) is the $\widehat{SU(3)}_{-\frac{3}{2}}$ WZW CFT. It has two finite flavorless characters:

$$\chi_0(q) = q^{\frac{1}{3}} \left(1 + 8q + 36q^2 + 128q^3 + 394q^4 + \mathcal{O}\left(q^5\right)\right)$$

$$\chi_{-\frac{1}{2}}(q) = q^{-\frac{1}{6}} \left(1 + 28q + 134q^2 + 568q^3 + 1809q^4 + \mathcal{O}\left(q^5\right)\right)$$

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$\widehat{SO(8)}_1$ characters:

$$\chi_0(q) = q^{-\frac{1}{6}}\left(1 + 28q + 134q^2 + 568q^3 + 1809q^4 + \mathcal{O}\left(q^5\right)\right)$$

$$\chi_{\frac{1}{2}}(q) = q^{\frac{1}{3}}\left(1 + 8q + 36q^2 + 128q^3 + 394q^4 + \mathcal{O}\left(q^5\right)\right)$$

- These are the same characters, only their interpretation changed. We get a relation between a unitary a non-unitary theory!

Missing link

$$
\begin{tikzcd}
(A_1, D_4) \arrow[d, "\text{chiral}"', "\text{algebra}"] & \\
\widehat{SU(3)}_{-\frac{3}{2}} \arrow[r, "\text{swap}", "\text{characters}"'] & \widehat{SO(8)}_1
\end{tikzcd}
$$

- What is the 4d ancestor of $\widehat{SO(8)}_1$? Clue from its construction as 8 free Majorana fermions!

Missing link

$$\begin{tikzcd}
(A_1, D_4) \arrow[r, leftrightarrow] \arrow[d, "\text{chiral}"', "\text{algebra}"] & \begin{array}{c}\text{8 ghost}\\ \text{half-hypers}\end{array} \\
\widehat{SU(3)}_{-\frac{3}{2}} \arrow[r, "\text{swap}", "\text{characters}"'] & \widehat{SO(8)}_1 \arrow[u, "\text{lift}", "\text{to 4D}"']
\end{tikzcd}$$

- What is the 4d ancestor of $\widehat{SO(8)}_1$? Clue from its construction as 8 free Majorana fermions!

Ghost hypers

- The 4d theory is made up of 8 half-hypermultiplets q^I with wrong statistics i.e. scalars anticommute while fermions commute.
- We can write down a Lagrangian for these fields:

$$\mathcal{L} = -\int d^4\theta \left(q^{I\dagger}\Omega_{IJ}q^J\right) \qquad \Omega \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- The resulting OPEs between twisted-translated fields (coming from the chiral algebra correspondence) reproduce the free Majorana OPEs of $\widehat{SO(8)}_1$.
- Moment-map operators μ^A in the ghost theory constructed from bilinears of the q^I fields reproduce the $\widehat{SO(8)}_1$ current algebra OPEs:

$$\mu^A(z,\bar{z})\,\mu^B(0) \sim \frac{\delta^{AB}}{z^2} + i f^{AB}{}_C \frac{\mu^C(0,0)}{z} + \{\mathrm{Q}, \dots\}$$

Generalization

- We can easily generalize this construction to a class of AD theories called $D_2\left[SU(2N+1)\right]$ whose first element is $D_2\left[SU(3)\right] \equiv (A_1, D_4)$.
- The Schur-index of these theories is related (under the $q \to q^{\frac{1}{2}}$ map) to the index of $4N(N+1)$ half-hypers.

$$\begin{tikzcd}
D_2\left[SU(2N+1)\right] \arrow[r, leftrightarrow] \arrow[d, "\text{chiral}"', "\text{algebra}"] & \begin{array}{c} 4N(N+1)\ \text{ghost} \\ \text{half-hypers} \end{array} \\
\widehat{SU(2N+1)}_{-\frac{2N+1}{2}} \arrow[r, "\text{shuffle}", "\text{characters}"'] & \widehat{SO(4N(N+1))}_1 \arrow[u, "\text{lift}", "\text{to 4D}"']
\end{tikzcd}$$

Summary and further directions

- We started with the curious observation that part of the index of a family of AD theories is seemingly related to free fields.
- Using the 4d $\mathcal{N} = 2$ SCFT, chiral algebra correspondence we then found a relation between two sets of chiral algebras one of which corresponds to our family of AD theories while the other is constructed from free fields.
- Finally we found the 4d ancestors of these chiral algebras, these turned out to be non-unitary but free 4d theories explaining our initial observation.
- Can this be repeated for other AD theories or other sectors?
- Can it be extended to the flavor refined index?
- What can we learn about the relation between non-vacuum chiral algebra characters and the presence of defects in the 4d theory?

The End

Thank you for your attention!