The spin chain for the AdS_3 WZW model

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Based on

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Strings on AdS_3

- Low-dimensional holographic dual $\mathrm{AdS}_3/\mathrm{CFT}_2$
- Maximally supersymmetric AdS_3 background $\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$
- Supported by a mixture of RR and NSNS three-form fluxes
- The theory is characterized by a large moduli space
- Very rich dynamics: different techniques apply at different points of the moduli space

Different perspectives: Integrable non linear sigma model

- Integrable 2D QFT in lightcone gauge
- S-matrix for infinite worldsheet size limit
- Spectrum described by Bethe equations and dispersion relation

$$e^{ip_i R} \prod_{j \neq i} S(p_i, p_j) = 1 \qquad H = \sum_i \left| \frac{k}{2\pi} p_i + \mu_i \right|$$

[Borsato, O.Sax, Sfondrini 12][Hoare, Tseytlin 13] [Borsato, O.Sax, Sfondrini, Stefanski, Torrielli 13-15]

Different perspectives: Worldsheet CFT

- RNS formulation of superstrings in AdS_3
- Exactly solvable model for pure NSNS flux
- Worldsheet energy of a state

$$\boxed{H = \sqrt{J^2 + 4k\mathcal{N}} - J + \hat{\mu}}$$

[Maldacena, Ooguri 00, ...]

Comparison of the two descriptions

Worldsheet CFT

- Exactly solvable model!
- **BUT** it's difficult to describe RR flux...

Integrability

- Applies for any mixture of RR and NSNS flux!
- **BUT** finite size corrections are difficult to compute...
- **Surprisingly** the usual integrability approach fails for pure NSNS flux ...precisely where CFT gives exact treatment...

Relating the two descriptions can teach us a lot!

Integrability: classical worldsheet Hamiltonian for T^4

Uniform light-cone gauge: free gauge parameter a

$$x^{+} = (1-a)t + a\phi = \tau, \qquad x^{-} = \phi - t, \qquad p_{-} = 1,$$

$$R(a) = J + aH, \qquad H = \int_0^{R(a)} d\sigma \mathcal{H}(a), \qquad \frac{d}{da} H = 0.$$

At classical level we obtain two equivalent descriptions

- $a = \frac{1}{2} \implies$ **Free theory** in volume $R = J + \frac{H}{2}$
- $a = 0 \implies$ **Non-linear theory** in volume $R = J$

[Arutyunov, Frolov 05] [Cavagliá et al. 16] [Smirnov, Zamolodchikov 16] [Baggio, Sfondrini 18]

From classical to quantum

Conjecture: free theory also at quantum level

$$S(p_i, p_j) = 1 \quad \Longrightarrow \quad e^{ip_i(J+H/2)} = 1 \quad \Longrightarrow \quad p_i = \frac{2\pi n_i}{R + H/2}$$

From the dispersion relation

$$H = \sum_i \frac{k}{2\pi}|p_i| = \frac{2k\mathcal{N}}{J + H/2}, \qquad \mathcal{N} = \sum_i n_i .$$

We have a quadratic equation for H! The solution

$$\boxed{H = \sqrt{J^2 + 4k\mathcal{N}} - J}$$

matches the WZW worldsheet energy for T^4 modes!

How to extend to AdS_3 and S^3 excitations?

Conjecture for the S-matrix

What about the non-linear theory in volume $R = J$?

$$e^{ip_i(J+H/2)} = 1 \iff e^{ip_iJ}\prod_j S(p_i,p_j) = 1,$$

$$S(p_i,p_j) = e^{\frac{i}{2}\Phi}, \qquad \Phi = p_iH_j - p_jH_i$$

Extending to arbitrary excitations, conjecture [Baggio, Sfondrini 18]

$$\Phi = p_i(H_j - \hat{\mu}_j) - p_j(H_i - \hat{\mu}_i), \qquad \hat{\mu} = \mu \ \mathrm{sgn}\left[\tfrac{k}{2\pi}p + \mu\right]$$

Proceeding as before, one can solve for H

$$H = \sqrt{J^2 + 4k\mathcal{N}} - J + \hat{\mu}$$

and again find **agreement with the WZW worldsheet energy!**

[Baggio, Sfondrini 18] [AD, Sfondrini 18]

Spin-chain interpretation

All this strongly suggests that

The WZW model can be described as a simple, integrable spin chain

This is a rather strong claim since

- A spin chain is a QM system while worldsheet theory is a QFT
- Even for integrable systems, the spin chain is only approximate: finite-volume corrections spoil it
- The WZW model gives a closed formula for the spectrum, while Bethe equations are usually very difficult to solve

Spin-chain interpretation

All this strongly suggests that

The WZW model can be described as a simple, integrable spin chain

However,

- We have shown that finite-volume corrections cancel exactly
- Bethe equations can be solved and give a closed formula for the worldsheet energy
- We obtain intriguing matching with the WZW worldsheet energy

[AD, Sfondrini 18]

Future directions

But much more remains to be done

- Extend the correspondence to fully describe the spectrum, including degeneracies
- Explore other backgrounds, like $AdS_3 \times S^3 \times S^3 \times S^1$
- Study integrable deformations of these backgrounds
- Go beyond the spectrum: three- and higher- point functions
- Build on the spin-chain to better understand the integrable nature of the dual CFT_2