Twist fields and moduli space of D-branes

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Outline

- Motivation
    - Bound states of D-branes
    - Instantons and finite size D-branes
- Twist fields in CFT
- Bosonization of twist fields
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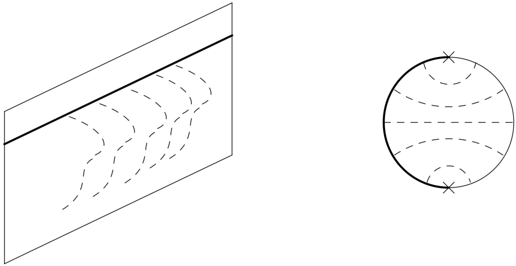
- Finite size D-branes
    - D(-1)/D15 system in bosonic string theory
    - D(-1)/D3 system in superstring theory

Bound states of D-branes

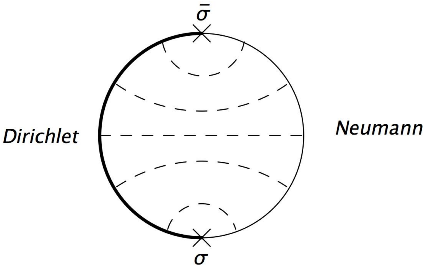
A bound state of D-branes allows for strings attached to D-branes with different dimension.



The corresponding worldsheet has mixed (Neumann and Dirichlet) boundary conditions.

Boundary twist fields and boundary conditions

A change of boundary condition from Neumann ($\partial X = \bar{\partial}\bar{X}$) to Dirichlet ($\partial X = -\bar{\partial}\bar{X}$) is described by the presence of a *twist field*.

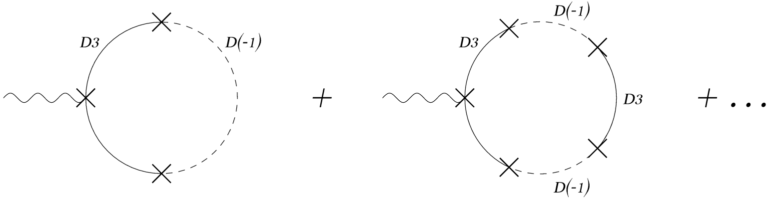


The calculation of amplitudes of strings emitted by brane systems involves twist fields.

SYM instanton in string theory

Setup: D(-1)/D3 system.
N: number of D3 branes $\longrightarrow$ Gauge group $SU(N)$.
k: number of D(-1) branes $\longrightarrow$ Pontryagin number of the instanton.

Instanton profile:



SYM instanton in string theory

Large distance (or small size) expansion $\Leftrightarrow$ Expansion in number of twist fields.

Question: Can we interpret this as a finite size D(-1)-brane?

$\mathbb{Z}_2$ twist fields

The **twist fields** defines the Ramond vacuum of a boson.

$$|\sigma\rangle = \sigma(0)|0\rangle$$

The twist field is a primary of conformal dimension $1/16$.

$$i\partial X(z)\sigma(w) \sim \frac{\sigma'(w)}{(z-w)^{1/2}} + \dots$$
$$i\partial X(z)\sigma'(w) \sim \frac{\sigma(w)}{2(z-w)^{3/2}} + \frac{2\partial\sigma(w)}{(z-w)^{1/2}} + \dots$$

σ' is called excited twist field (conformal dimension $9/16$).

Twist fields are hard to deal with

- Twist fields are not local w.r.t. the fields they twist.
- Increasing the number of twist field insertions changes the topology of the worldsheet.
- Twisting a boson has consequences for the geometry of the spacetime.
- Twist fields do not have a known representation in terms of free fields (bosonization).

Bosonization of twist fields

When the boson X is compactified (and the radius is multiple of $\sqrt{2}$) a $\mathfrak{su}(2)$ Kač-Moody algebra is present:

$$j_X^1 = i\partial X \,, \quad j_X^2 = \frac{1}{\sqrt{2}}(e^{i\sqrt{2}X} + e^{-i\sqrt{2}X}) \,, \quad j_X^3 = \frac{i}{\sqrt{2}}(e^{i\sqrt{2}X} - e^{-i\sqrt{2}X})$$

We perform a change of basis, in terms of a new boson Ω, such that $j_X^1 = j_\Omega^2$, i.e.

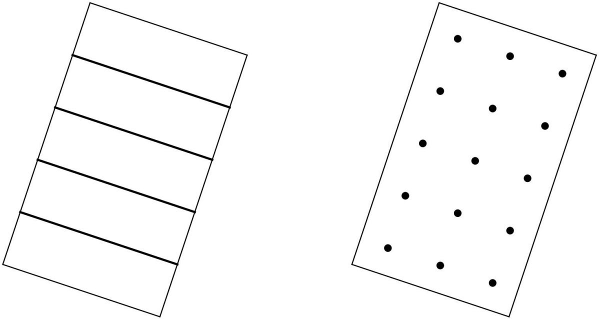
$$i\partial X = \frac{1}{\sqrt{2}}(e^{i\sqrt{2}\Omega} + e^{-i\sqrt{2}\Omega})$$

Twist field can be bosonized in terms of Ω:

$$\sigma_B = e^{i\frac{\sqrt{2}}{4}\Omega} \qquad \bar{\sigma}_B = e^{-i\frac{\sqrt{2}}{4}\Omega}$$

Bosonization of twist fields

- Bosonized twist fields represent a periodic configuration $\Rightarrow$ array or lattice of D-branes.



- Correlation functions are straightforward to compute.

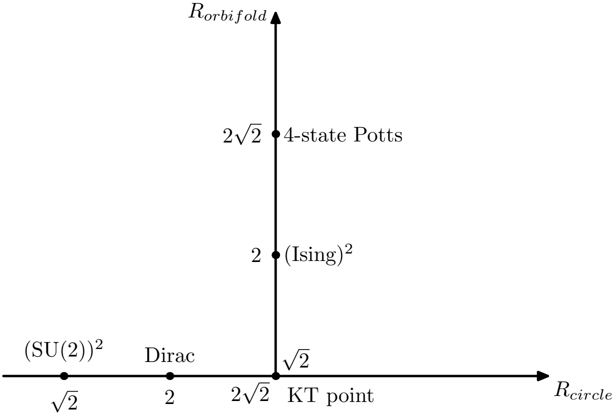
Bosonization of twist fields

We computed correlation functions and checked that

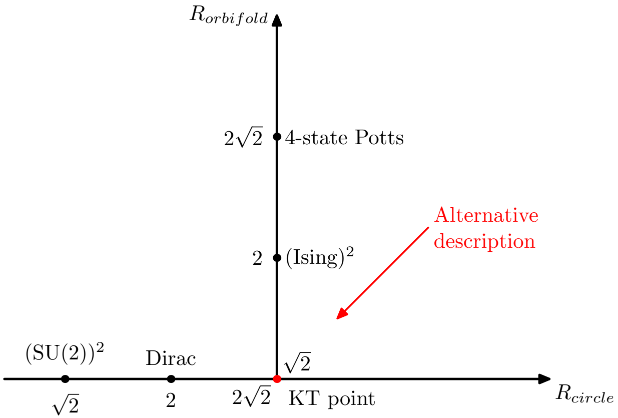
$$\sum_{x_0 = 2\sqrt{2}\pi n} \langle (\bar{\sigma}\sigma)_{x_0} \mathcal{O}_1 \mathcal{O}_2 \ldots \rangle = \langle \bar{\sigma}_B \sigma_B \mathcal{O}_1 \mathcal{O}_2 \ldots \rangle$$

$$\sum_{x_0, x_0' = 2\sqrt{2}\pi n} \langle (\bar{\sigma}\sigma)_{x_0} (\bar{\sigma}\sigma)_{x_0'} \mathcal{O}_1 \mathcal{O}_2 \ldots \rangle = \langle \bar{\sigma}_B \sigma_B \bar{\sigma}_B \sigma_B \mathcal{O}_1 \mathcal{O}_2 \ldots \rangle$$

The space of $c = 1$ CFTs

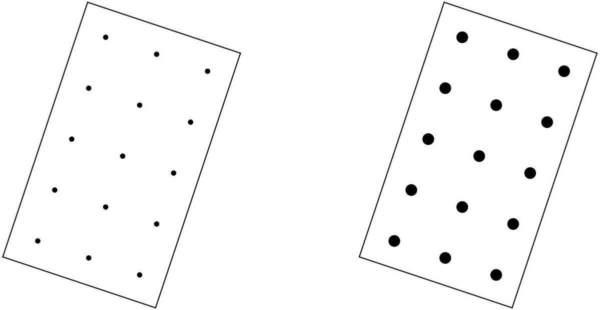


The space of $c = 1$ CFTs



Twist fields and marginal deformations

Question: Can twist fields generate a deformation of the CFT corresponding to blowing up the size of the D(-1) branes?



In CFT this is allowed if the deformation is *marginal*.

Marginal deformations

A marginal deformation in CFT is associated to the presence of a family of theories parametrized by a continuous parameter (example: boson compactified on a circle).

A set of boundary operators j^i defines a truly marginal deformation if

- j^i have conformal dimension 1;
- The operators j^i are mutually local, i.e. $[j^i, j^j] = 0$ or equivalently, there is no single pole in the OPE.

Twist fields have conformal dimension $1/16 \Rightarrow$ We need to twist 16 directions $\Rightarrow$ D(-1)/D15 system in bosonic string theory.

Obstructions to marginality

Using the OPE of bosonized twist fields we can prove that:

- Marginality is obstructed if the size of the lattice is $2\sqrt{2}\pi$ or a multiple of it.
- The D(-1) branes “feel” each other through the exchange of massless primaries.
- For a single D(-1) brane a similar obstruction is present.

Vertex operator in superstring theory

The vertex operator (at zero momentum) for the D(-1)/D3 system is actually a $(N+k) \times (N+k)$ matrix:

$$\Phi^{(-1)} = \begin{pmatrix} A_\mu \psi^\mu & w_\alpha \Delta S^\alpha \\ \bar{w}_\alpha \bar{\Delta} S^\alpha & a_\mu \psi^\mu \end{pmatrix} e^{-\phi}$$

Problem: picture ambiguity $\rightarrow$ How to exponentiate Φ?

Preliminary results: requiring marginality implies:

- ADHM constraints are satisfied.
- Twist fields can be "switched on" only together with a zero-momentum gluon.

Conclusions

Results:

- Free field representation of twist fields for lattice of D-branes.
- Study of marginality of D(-1)/D15 system in bosonic string theory.

Future developments:

- Generalization to superstring D-brane systems.
- Application to D(-1)/D3 system (finite size $\mathcal{N} = 4$ SYM instantons).

Thank you for your attention