Integrability in $\mathcal{N} = 1$ superconformal gauge theories

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Motivation:

- Dilatation operator in $\mathcal{N} = 4$ SYM integrable in the planar limit [Beisert, Dippel, Staudacher, 2004]
- All loop argument for $\mathcal{N} = 2$ SCFTs in the gluonic subsector $SU(2,1|2)$ [Pomoni, 2014]

Idea:

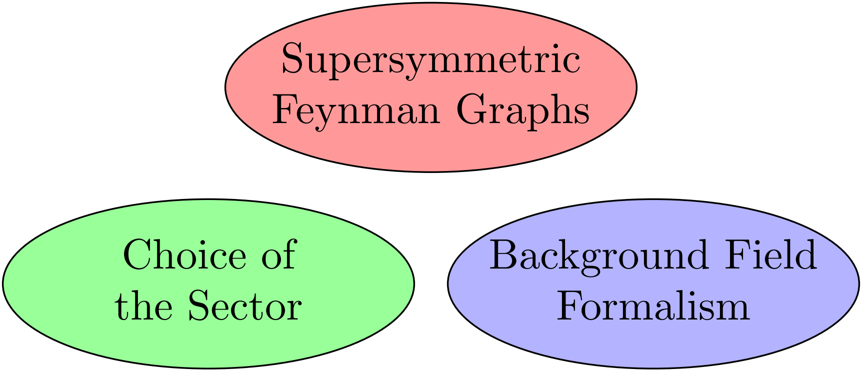
- Assume integrability in $\mathcal{N} = 4$ SYM
- Compare general $\mathcal{N} = 1$ with $\mathcal{N} = 4$ SYM in supersymmetric perturbation theory

The main statement

1. The dilatation operator in the gluonic subsector $SU(2,1|1)$ of any $\mathcal{N}=1$ superconformal gauge theory is integrable in the planar limit up to at least three loops.
2. There is a function f such that ($\lambda = g^2 N_c$)

$$\mathcal{D}_{\mathcal{N}=1}(\lambda) = \mathcal{D}_{\mathcal{N}=4}(f(\lambda)) \; .$$

Main pillars of the argument

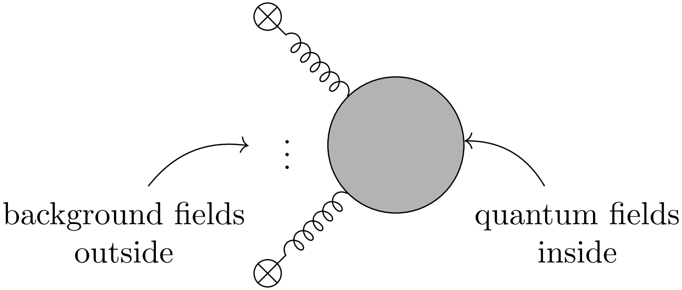


The $SU(2,1|1)$ sector

- Intuition: All fields in $\mathcal{N}=4$ connected to the gauge field by supersymmetry
- Proposal for $\mathcal{N}=2$: Gauge invariant local single trace operators built from $(\phi, \lambda_+, F_{++}, D_{+\dot{\alpha}})$ [Pomoni 2014]
- Analogous sector in $\mathcal{N}=1$: $(\lambda_+, F_{++}, D_{+\dot{\alpha}})$
- Closed by R-charge and spin conservation:

$$\Delta = \frac{r}{2} + j \text{ for operators in this sector}$$
$$\Delta \geq \frac{r}{2} + j + \frac{1}{2} \text{ for all other operators}$$

Background field formalism



- Background gauge invariance $\rightarrow$ improved convergence
- Claim: "new" effective vertices only in UV finite diagrams
- Reminder: Dilatation operator in $d = 4 - 2\epsilon$:

$$\mathcal{D} = \lim_{\epsilon \to 0} \left[2\epsilon\lambda \frac{\mathrm{d}}{\mathrm{d}\lambda} \ln \mathcal{Z}_{\mathrm{op}}(\lambda, \epsilon) \right]$$

- "New" effective vertices cannot contribute
- Shown in perturbation theory up to three loops

Background field formalism

Structure of "old" vertices very restricted:

- Counterterm relations due to gauge invariance:

$$\sqrt{\mathcal{Z}_\lambda}\mathcal{Z}_V = 1$$

- **Unique** counterterm $\mathcal{Z}_\lambda$ for vertex- and self-energy corrections

Differences between $\mathcal{N} = 4$ and $\mathcal{N} = 1$ in this sector encoded in $\mathcal{Z}_\lambda$

- Relative finite renormalization of λ:

$$f(\lambda) = \lambda + \lambda(\mathcal{Z}_{\lambda,\mathcal{N}=1} - \mathcal{Z}_{\lambda,\mathcal{N}=4})$$

Conclusions

Summary:

- Integrable subsector $SU(2,1|1)$ in any $\mathcal{N}=1$ superconformal gauge theory
- In particular

$$\mathcal{D}_{\mathcal{N}=1}(\lambda) = \mathcal{D}_{\mathcal{N}=4}(f(\lambda))$$

Future directions:

- Calculate $f(\lambda)$ perturbatively
- Devise an all loop argument