Inversion formula for Defects I: Defect Channel

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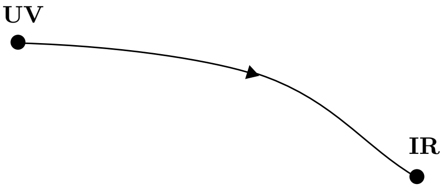
DESY Theory Workshop

1712.08185 with M. Lemos, M. Meineri, S. Sarkar

Motivation

Renormalization group flow



Renormalization group (RG) flow.

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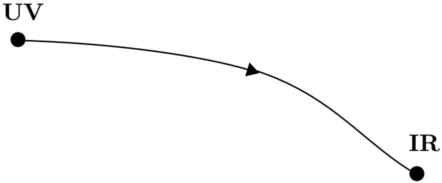

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Conformal symmetry is extremely powerful.

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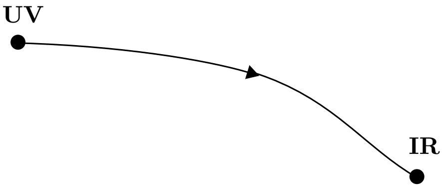

Renormalization group (RG) flow.

Conformal symmetry is extremely powerful.

It includes translations, rotations, and scale transformations (+ more)

$$x \to x + a\,, \quad x \to R \cdot x\,, \quad x \to h\,x\,.$$

CFT

Defect CFT

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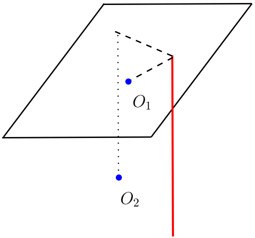

Local operatos in the presence of a defect.

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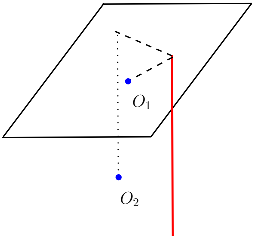

Local operatos in the presence of a defect.

We have $SO(1, d+1) \to SO(1, p+1) \times SO(q)$ where $q + p = d$.

Defect CFT correlators

The $SO(1, p+1) \times SO(q)$ symmetry preserved by the defect implies that one-point functions are non-zero:

$$\langle \mathcal{O}(x) \rangle = \frac{a_{\mathcal{O}}}{(x^i)^{\Delta}} \,.$$

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Two-point functions depend on two conformal invariants

$$\langle \phi(x_1) \phi(x_2) \rangle = \frac{1}{(z\bar{z})^{\Delta_{\phi/2}}} g(z, \bar{z}) \,,$$

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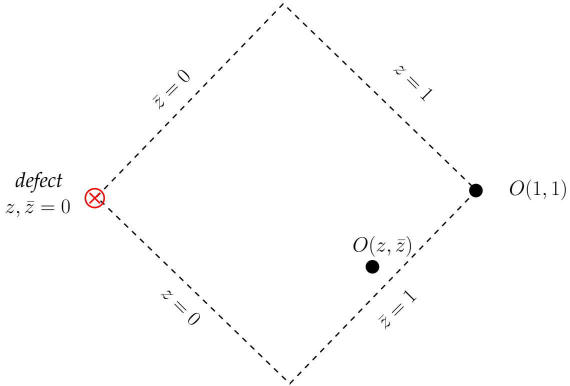
Remark. Compare with the four-point function in standard CFT.

Two-point function configuration



Configuration of the system in the plane orthogonal to the defect.

Bulk OPE

Bulk channel:
We had

$$\phi(x)\phi(0) \sim \sum_{\mathcal{O}} C_{\phi\phi O} d(x, \partial) O(0) \,.$$

recall that in the presence of a defect a scalar can have a non-zero one-point function

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The expansion for the two-point function is

$$\langle \phi(x_1)\phi(x_2) \rangle = \left(\frac{(1-z)(1-\bar{z})}{(z\bar{z})^{1/2}} \right)^{-\Delta_\phi} \sum_{\Delta,J} C_{\phi\phi O}\, a_O\, f_{\Delta,J}(z,\bar{z})$$

where the sum goes over the bulk spectrum.

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Defect channel:

We can also write a bulk operator as a sum of defect operators

$$\phi(x) = \sum_{\hat{O}} b_{\phi\widehat{O}} D(x^i, \partial_{\vec{x}}) \widehat{O}(\vec{x})$$

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Plugging this expansion into the two-point function,

$$\langle \phi(x_1)\phi(x_2) \rangle = \sum_{\hat{\Delta},s} b^2_{\phi\widehat{O}} \, \widehat{f}_{\hat{\Delta},s}(z,\bar{z}) \, .$$

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Crossing symmetry

Equality of both expansions implies

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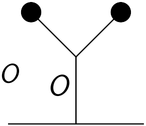
Pictorially

$$\sum_{\Delta,J} C_{\phi\phi O}\, a_O$$



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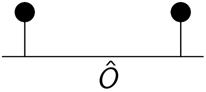

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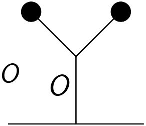

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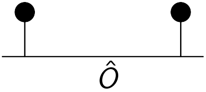

The defect blocks are known in closed-form.

[Billo, Goncalvez, Lauria, Meineri (2016)]

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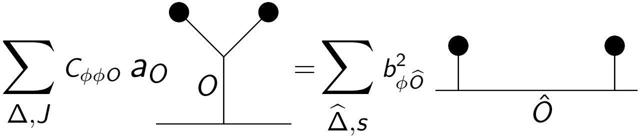

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The bulk blocks are Calogero-Sutherland wave-functions.

[Isachenkov, PL, Linke, Schomerus (2018)]

Inversion Formula

From Euclidean to Lorentzian

The idea...

$$z = rw\,, \qquad \bar{z} = \frac{r}{w}$$

$$g(r,w) = \int b(\hat{\Delta},s)h(r,w) \quad \rightarrow \quad b(\hat{\Delta},s) \sim \int g(r,w)\bar{h}(r,w)$$

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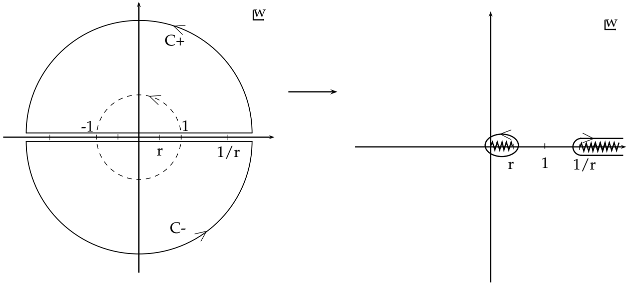

Contour deformation from Euclidean to Lorentzian configuration.

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Long story short: we need an *infinite number of defect operators*.

[Lemos, PL, Meineri, Sarkar (2018)]

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Plugging the identity in the inversion formula implies

$$\widehat{\Delta} = \Delta_\phi + s + 2m + \mathcal{O}(s^{-\alpha}) \ , \quad s \to \infty \, ,$$

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This result is **universal**!

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- We concentrated on the defect channel and prove universality at large transverse spin s.
- In the next talk Yannick will tell you about the bulk channel!

Thank you!