

Threshold resummation of the Drell-Yan process at next-to-leading power

Sebastian Jaskiewicz



based on 1810.xxxxx with Martin Beneke, Alessandro Broggio, Mathias Garny,
Robert Szafron, Leonardo Vernazza and Jian Wang

DESY Particle Physics Challenges September 26, 2018

Overview

Motivations

Drell-Yan process at leading power

Next-to-leading power

Introduction

- ▶ Fixed order
 - ▶ Higgs Boson Gluon-Fusion Production in QCD at Three Loops
[C. Anastasiou *et al.* 2015]
- ▶ Resummation to all orders
 - ▶ Resummation of logarithmic enhancements at threshold
[G. Sterman, 1987, S. Catani *et al.*, 9604351]
 - ▶ Leading power (LP) resummation of Drell-Yan (DY) at NNLL
[T. Becher *et al.*, 0710.0680, S. Moch *et al.*, 0508265]

The Drell-Yan process - LP review

$$A(p_A)B(p_B) \rightarrow \text{DY}(Q) + X$$

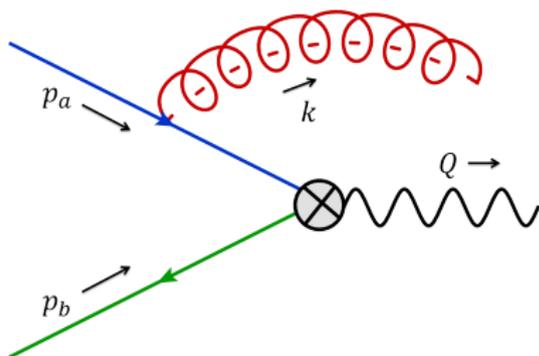
$$z = \frac{Q^2}{\hat{s}} \quad \lambda = \sqrt{(1-z)}$$

$$p_c = (n_+ p_c, n_- p_c, p_{c\perp}) \sim Q(1, \lambda^2, \lambda)$$

$$p_{c\text{-PDF}} \sim (Q, \Lambda/Q, \Lambda)$$

$$\begin{aligned} & \bar{\psi} \gamma_\mu \psi \\ &= \int dt d\bar{t} \tilde{C}^{A0}(t, \bar{t}) J_\mu^{A0}(t, \bar{t}) \end{aligned}$$

$$J_\mu^{A0}(t, \bar{t}) = \bar{\chi}_{\bar{c}}(\bar{t}n_-) \gamma_{\perp\mu} \chi_c(tn_+)$$



The Drell-Yan process - Decoupling transformation

$$\chi_c^{(0)}(tn_+) = Y_+^\dagger(0)\chi_c(tn_+)$$

where

$$Y_\pm(x) = \mathbf{P} \exp \left[ig_s \int_{-\infty}^0 ds n_\mp A_s(x + sn_\mp) \right]$$

From now on use decoupled fields. Leading power current becomes

$$J_\mu^{A0}(t, \bar{t}) = \bar{\chi}_{\bar{c}}(\bar{t}n_-) Y_-^\dagger(0) \gamma_{\perp\mu} Y_+(0) \chi_c(tn_+)$$

The Drell-Yan process - LP amplitude

$$\begin{aligned}
 \langle X | \bar{\psi} \gamma^\mu \psi(0) | A(p_A) B(p_B) \rangle &= \int \frac{d(n_+ p)}{2\pi} \frac{d(n_- \bar{p})}{2\pi} \int d(n_+ p_a) \\
 \times \int d(n_- p_b) \delta(n_+ p - (n_+ p_a)) \delta(n_- \bar{p} + (n_- p_b)) &C^{A0}(n_+ p, n_- \bar{p}) \\
 \times \langle X_{\bar{c}}^{\text{PDF}} | \hat{\chi}_{\bar{c}, \alpha a}^{\text{PDF}}(n_- p_b) | B(p_B) \rangle \gamma_{\perp, \alpha \beta}^\mu &\langle X_c^{\text{PDF}} | \hat{\chi}_{c, \beta b}^{\text{PDF}}(n_+ p_a) | A(p_A) \rangle \\
 \times \langle X_s | \mathbf{T} [Y_-^\dagger(0) Y_+(0)]_{ab} | 0 \rangle &
 \end{aligned}$$

LP result

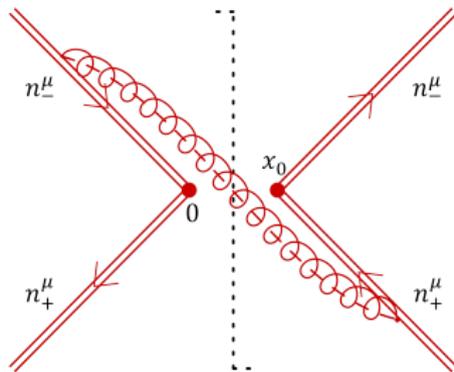
$$\frac{d\sigma_{\text{DY}}}{dQ^2} = \frac{4\pi\alpha_{\text{em}}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) \hat{\sigma}_{ab}(z)$$

where

[G. P. Korchemsky et al., 1993]

$$\hat{\sigma}(z) = |C(Q^2)|^2 Q S_{\text{DY}}(Q(1-z))$$

$$S_{\text{DY}}(\Omega) = \int \frac{dx^0}{4\pi} e^{ix^0\Omega/2} \frac{1}{N_c} \text{Tr} \langle 0 | \bar{\mathbf{T}}(Y_+^\dagger(x^0)Y_-(x^0)) \mathbf{T}(Y_-^\dagger(0)Y_+(0)) | 0 \rangle$$



NLP factorization formula

$$\frac{d\sigma_{\text{DY}}}{dQ^2} = \frac{4\pi\alpha_{\text{em}}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) \hat{\sigma}_{ab}(z)$$

The $\hat{\sigma}_{ab}(z)$ is now

$$\hat{\sigma} = [C \otimes J \otimes \bar{J}]^2 \otimes S$$

- ▶ C is the hard Wilson matching coefficient
- ▶ S is the soft function
- ▶ J is the collinear function

Matching at NLP

N -jet operators are built out of following relevant building blocks

[M. Beneke, M. Garry, R. Szafron, J. Wang, 1712.04416, 1808.04742.]

(A1-type)

$$\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_{\pm}^{\mu} i\cancel{\partial}_{\perp}] \chi_c(tn_+), \quad \bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_{\pm}^{\mu} (-i) \overleftarrow{\cancel{\partial}}_{\perp}] \chi_c(tn_+)$$

(B1-type)

$$\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_{\pm}^{\mu} \mathcal{A}_{c\perp}(t_2n_+)] \chi_c(t_1n_+), \quad \bar{\chi}_{\bar{c}}(\bar{t}_1n_-)[n_{\pm}^{\mu} \mathcal{A}_{\bar{c}\perp}(\bar{t}_2n_-)] \chi_c(tn_+)$$

With the the scaling

$$\chi_c(tn_+) \sim \lambda$$

$$\bar{\chi}_{\bar{c}}(\bar{t}n_-) \sim \lambda$$

$$[n_{\pm}^{\mu} i\cancel{\partial}_{\perp}] \sim \lambda$$

$$[n_{\pm}^{\mu} \mathcal{A}_{c\perp}(t_2n_+)] \sim \lambda$$

Time-ordered products

$$(J_{A0,V}^{T2}(s,t))^{\mu} = i \int d^4x \mathbf{T} \left[J_{A0}^{\mu}(s,t) \mathcal{L}_V^{(2)}(x) \right]$$

[M. Beneke, M. Garry, R. Szafron, J. Wang, 1712.04416, 1808.04742.]

The NLP soft-collinear SCET quark-gluon interaction Lagrangian written in terms of building blocks $\mathcal{B}_{\pm}^{\mu} = Y_{\pm}^{\dagger} [iD_s^{\mu} Y_{\pm}]$ and $q^{\pm}(x) = Y_{\pm}^{\dagger}(x) q_s(x)$ is [M. Beneke *et al.*, 0211358]

$$\begin{aligned} \mathcal{L}_{\xi}^{(1)} &= \bar{\chi}_c i x_{\perp}^{\mu} [in_{-} \partial \mathcal{B}_{\mu}^{+}] \frac{\not{n}_{+}}{2} \chi_c \\ \mathcal{L}_{1\xi}^{(2)} &= \frac{1}{2} \bar{\chi}_c i n_{-} x n_{+}^{\mu} [in_{-} \partial \mathcal{B}_{\mu}^{+}] \frac{\not{n}_{+}}{2} \chi_c \\ \mathcal{L}_{2\xi}^{(2)} &= \frac{1}{2} \bar{\chi}_c x_{\perp}^{\mu} x_{\perp}^{\rho} [i \partial_{\rho} in_{-} \partial \mathcal{B}_{\mu}^{+}] \frac{\not{n}_{+}}{2} \chi_c \\ \mathcal{L}_{3\xi}^{(2)} &= \frac{1}{2} \bar{\chi}_c x_{\perp}^{\mu} x_{\perp}^{\rho} [\mathcal{B}_{\rho}^{+}, in_{-} \partial \mathcal{B}_{\mu}^{+}] \frac{\not{n}_{+}}{2} \chi_c \\ &\dots \end{aligned}$$

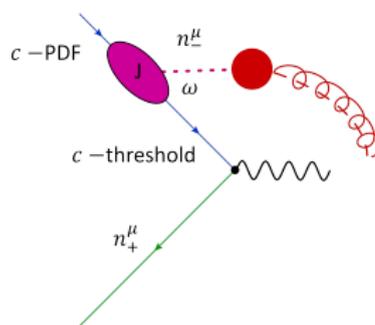
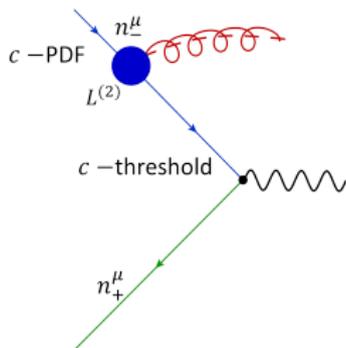
Collinear functions

We match generic collinear fields to collinear-PDF fields

$$\int dt e^{i(n+p)t} i \int d^4 z e^{i\omega(n+z)/2} \mathbf{T} \left[\chi_c(tn_+) \bar{\chi}_c(z) \frac{\not{n}_+}{2} \chi_c(z) \right]$$

$$= \int d(n_+p') \int dt e^{i(n+p')t} J(n_+p, n_+p'; \omega) \chi_c^{\text{PDF}}(tn_+)$$

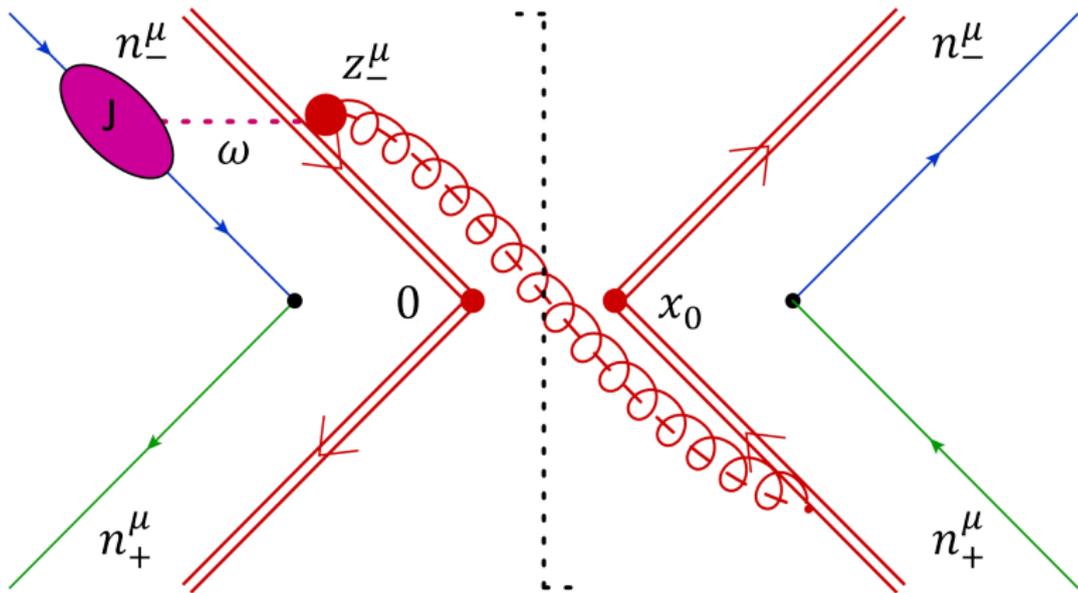
This is done at amplitude level. [D. Bonocore *et al.*, 1503.05156, 1610.06842]



Power suppressed amplitude

$$\begin{aligned}
\langle X | \bar{\psi} \gamma^\mu \psi(0) | A(p_A) B(p_B) \rangle &= \int \frac{dn_+ p_a}{2\pi} \frac{dn_- p_b}{2\pi} C^{A0}(n_+ p_a, -n_- p_b) \\
&\times \langle X_{\bar{c}, \text{PDF}} | \hat{\chi}_{\bar{c}, \alpha a}^{\text{PDF}}(n_- p_b) | B(p_B) \rangle \gamma_{\perp \alpha \beta}^\mu \langle X_{c, \text{PDF}} | \hat{\chi}_{c, \beta b}^{\text{PDF}}(n_+ p_a) | A(p_A) \rangle \\
&\times \left\{ \langle X_s | \mathbf{T} \left[Y_-^\dagger(0) Y_+(0) \right]_{ab} | 0 \rangle \right. \\
&\quad + \frac{1}{2} \int \frac{d\omega}{4\pi} J_{2\xi}^{(O)}(n_+ p_a; \omega) \int d(n_+ z) e^{-i\omega(n_+ z)/2} \\
&\quad \times \left. \langle X_s | \mathbf{T} \left(\left[Y_-^\dagger(0) Y_+(0) \right]_{af} \frac{i\partial_\perp^\nu}{in_- \partial} \mathcal{B}_{\perp \nu; fb}^+(z_-) \right) | 0 \rangle \right\} + \bar{c}\text{-term}
\end{aligned}$$

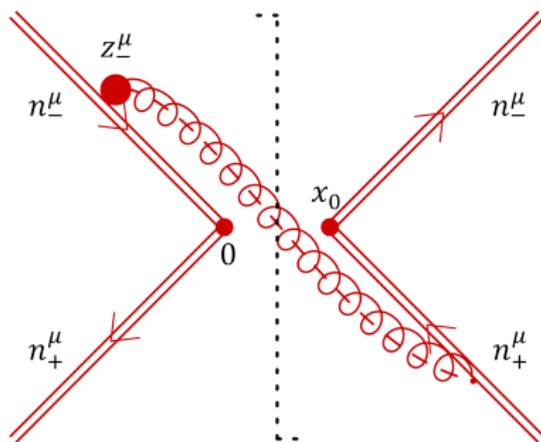
Cross section: $\mathcal{L}_{2\xi}^{(2)}$ example



Soft functions

The generalised soft function at cross section level here is

$$S_{2\xi}(\Omega, \omega) = \int \frac{dx^0}{4\pi} \int \frac{d(n_+ z)}{4\pi} e^{ix^0 \Omega/2 - i\omega(n_+ z)/2} \\
 \times \frac{1}{N_c} \text{Tr} \langle 0 | \bar{\mathbf{T}} \left[Y_+^\dagger(x^0) Y_-(x^0) \right] \mathbf{T} \left[Y_-^\dagger(0) Y_+(0) \frac{i\partial_\perp^\nu}{in_- \partial} \mathcal{B}_{\perp\nu}^+(z_-) \right] | 0 \rangle$$



The x_0 soft function

$$S_{2\xi}(\Omega, \omega) = \frac{\alpha_s C_F}{2\pi} \left\{ \theta(\Omega) \delta(\omega) \left(-\frac{1}{\epsilon} + \ln \frac{\Omega^2}{\mu^2} \right) + \dots \right\}$$

The soft function starts at α_s order and is divergent. We need to introduce the following for renormalization

$$S_{x_0}(\Omega) = \int \frac{dx^0}{4\pi} e^{ix^0\Omega/2} \frac{-2i}{x^0 - i\epsilon} \frac{1}{N_c} \text{Tr} \langle 0 | \bar{\mathbf{T}} \left[Y_+^\dagger(x^0) Y_-(x^0) \right] \mathbf{T} \left[Y_-^\dagger(0) Y_+(0) \right] | 0 \rangle$$

Leading logarithmic RG equation

$$\frac{d}{d \ln \mu} \begin{pmatrix} S_{2\xi}(\Omega, \omega) \\ S_{x_0}(\Omega) \end{pmatrix} = \frac{\alpha_s}{\pi} \begin{pmatrix} 4C_F \ln \frac{\mu}{\mu_s} & -C_F \delta(\omega) \\ 0 & 4C_F \ln \frac{\mu}{\mu_s} \end{pmatrix} \begin{pmatrix} S_{2\xi}(\Omega, \omega) \\ S_{x_0}(\Omega) \end{pmatrix}$$

where μ_s denotes an arbitrary soft scale of order $Q(1-z)$. The LL solution is

$$S_{2\xi}^{\text{LL}}(\Omega, \omega, \mu) = \frac{2C_F}{\beta_0} \ln \frac{\alpha_s(\mu)}{\alpha_s(\mu_s)} \exp[-4S^{\text{LL}}(\mu_s, \mu)] \theta(\Omega) \delta(\omega)$$

Final results

The $\Delta(z)$ is defined as $\Delta(z) = \hat{\sigma}(z)/z$. We find

$$\begin{aligned} \Delta_{\text{NLP}}^{\text{LL}}(z) &= -\exp \left[4S^{\text{LL}}(\mu_h, \mu) - 4S^{\text{LL}}(\mu_s, \mu) \right] \\ &\quad \times \frac{8C_F}{\beta_0} \ln \frac{\alpha_s(\mu)}{\alpha_s(\mu_s)} \theta(1-z), \end{aligned}$$

Final results

$$\begin{aligned}
 \Delta_{\text{NLP}}^{\text{LL}}(z, \mu) = & -\theta(1-z) \left\{ 4C_F \frac{\alpha_s}{\pi} \left[\ln(1-z) - L_\mu \right] \right. \\
 & + 8C_F^2 \left(\frac{\alpha_s}{\pi} \right)^2 \left[\ln^3(1-z) - 3L_\mu \ln^2(1-z) + 2L_\mu^2 \ln(1-z) \right] \\
 & + 8C_F^3 \left(\frac{\alpha_s}{\pi} \right)^3 \left[\ln^5(1-z) - 5L_\mu \ln^4(1-z) + 8L_\mu^2 \ln^3(1-z) \right. \\
 & \quad \left. - 4L_\mu^3 \ln^2(1-z) \right] \\
 & + \frac{16}{3} C_F^4 \left(\frac{\alpha_s}{\pi} \right)^4 \left[\ln^7(1-z) - 7L_\mu \ln^6(1-z) + 18L_\mu^2 \ln^5(1-z) \right. \\
 & \quad \left. - 20L_\mu^3 \ln^4(1-z) + 8L_\mu^4 \ln^3(1-z) \right] \\
 & + \frac{8}{3} C_F^5 \left(\frac{\alpha_s}{\pi} \right)^5 \left[\ln^9(1-z) - 9L_\mu \ln^8(1-z) + 32L_\mu^2 \ln^7(1-z) \right. \\
 & \quad \left. - 56L_\mu^3 \ln^6(1-z) + 48L_\mu^4 \ln^5(1-z) - 16L_\mu^5 \ln^4(1-z) \right] \left. \right\} + \mathcal{O}(\alpha_s^6 \times (\log)^{11})
 \end{aligned}$$

where we define $L_\mu = \ln(\mu/Q)$. Comparison to [R. Hamberg *et al.*, 1991] and [D. de Florian J. Mazzitelli, S. Moch, A. Vogt, 1408.6277]

Conclusion

- ▶ General factorization formula at next-to-leading power
- ▶ Appearance of collinear functions at next-to-leading power in DY
- ▶ Generalised soft functions
- ▶ Leading Logarithmic resummation

Thank you

Back up slides

Expansion

Near the partonic threshold $z = 1$, $\hat{\sigma}_{ab}$ has the singular expansion

$$\hat{\sigma}_{ab}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \dots \right]$$

NLP factorization formula

$$\frac{d\sigma_{\text{DY}}}{dQ^2} = \frac{4\pi\alpha_{\text{em}}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) \hat{\sigma}_{ab}(z)$$

The $\hat{\sigma}_{ab}(z)$ is now

$$\begin{aligned} \hat{\sigma}(z) &= \sum_{\text{terms}} \int d\omega_i d\bar{\omega}_i d\omega'_i d\bar{\omega}'_i D(-\hat{s}; \omega_i, \bar{\omega}_i) D^*(-\hat{s}; \omega'_i, \bar{\omega}'_i) \\ &\times Q^2 \int \frac{d^3\vec{q}}{(2\pi)^3 2\sqrt{Q^2 + \vec{q}^2}} \frac{1}{2\pi} \int d^4x e^{i(x_a p_A + x_b p_B - q) \cdot x} \\ &\times \tilde{S}(x; \omega_i, \bar{\omega}_i, \omega'_i, \bar{\omega}'_i) \end{aligned}$$

and

$$\begin{aligned} D(-\hat{s}; \omega_i, \bar{\omega}_i) &= \int d(n_+ p_i) d(n_- \bar{p}_i) C(n_+ p_i, n_- \bar{p}_i) \\ &\times J(n_+ p_i, x_a n_+ p_A; \omega_i) \bar{J}(n_- \bar{p}_i, -x_b n_- p_B; \bar{\omega}_i) \end{aligned}$$

Matching at NLP

N -jet operators have the general structure at NLP

[M. Beneke *et al.*, 1712.04416, 1808.04742.]

$$J = \int \prod_{i_k} dt_{i_k} C(\{t_{i_k}\}) \prod_{i=1}^N J_i(t_{i_1}, t_{i_2}, \dots)$$

where

$$J_i(t_{i_1}, t_{i_2}, \dots) = \prod_{k=1}^{n_i} \psi_{i_k}(t_{i_k}, n_{i+}) \quad \text{where} \quad \psi_i(t_i, n_{i+}) \in \chi_i = W_i^\dagger \xi_i, \dots$$

The relevant building blocks are

(A1-type)

$$\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_{\pm}^\mu i \not{\partial}_\perp] \chi_c(tn_+), \quad \bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_{\pm}^\mu (-i) \overleftarrow{\not{\partial}}_\perp] \chi_c(tn_+)$$

(B1-type)

$$\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_{\pm}^\mu \mathcal{A}_{c\perp}(t_2 n_+)] \chi_c(t_1 n_+), \quad \bar{\chi}_{\bar{c}}(\bar{t}_1 n_-)[n_{\pm}^\mu \mathcal{A}_{\bar{c}\perp}(\bar{t}_2 n_-)] \chi_c(tn_+)$$

Factorization example: $\mathcal{L}_{2\xi}^{(2)}$ time-ordered product

$$\hat{\sigma}(z) = H(\hat{s}) \times Q^2 \int \frac{d^3\vec{q}}{(2\pi)^3 2\sqrt{Q^2 + \vec{q}^2}} \frac{1}{2\pi} \int d^4x e^{i(x_a p_A + x_b p_B - q) \cdot x}$$

$$\times \left\{ \tilde{S}_0(x) + 2 \cdot \frac{1}{2} \int d\omega J_{2\xi}^{(O)}(x_a n_{+p_A}; \omega) \tilde{S}_{2\xi}(x, \omega) + \bar{c}\text{-term} \right\}$$