Resummed P_T Spectrum of Drell-Yan with Massive Bottom Quark Effect

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Ongoing work with F. Tackmann & E. Bagnaschi

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

DESY.

Prologue: Heavy quark effect

- Many studies exist in inclusive cross-section to include bottom mass effect for the DIS, Higgs and DY.

  ...
  [M. Aivazis, F. Olness, W. Tung ('94)]
  [R. Thorne, R. Roberts ('97)]
  [S. Kretzer, I. Schienbein ('98)]; [J. Collins ('98)]
  [M. Cacciari, M. Greco, P. Nason ('98)]
  [S. Forte, E. Laenen, P. Nason, J. Rojo ('10)]
  [S. Forte, D. Napoletano, M. Ubiali ('15,'16,'18)]
  [M. Bonvini, A. Papanastasiou, F. Tackmann ('15)]
  ...

- Many approaches and variants to match **4F** and **5F** results.

  FONLL, FONLL-A, FONLL-B, ACOT, S-ACOT, TR …

  1. Resummation of collinear logarithms and non-singular power correction
  2. Perturbative counting
  3. Low matching scale

- Resummation of mass logarithms for qT distribution: Only certain type of diagrams (Primary) **ACOT+CSS.** [P. Nadolsky, N. Kidonakis, F. Olness, C. Yuan ('03)] [S. Berge, P. Nadolsky, F. Olness ('06)] [A. Belyaev, P. Nadolsky, C. Yuan ('06)]

- Systematic description of Secondary mass effect for threshold resum in DIS.

  [S. Gritschacher, A. Hoang, I. Jemos, P. Pietrulewicz ('13)]
  [P. Pietrulewicz, S. Gritschacher, A. Hoang, I. Jemos, V. Mateu ('14)]
  [A. Hoang, P. Pietrulewicz, D. Samitz ('16)]

- Systematic description between hierarchies with Primary and Secondary contribution to DY. [P. Pietrulewicz, D. Samitz, A. Spiering, F. Tackmann ('17)]

Plan of the Talk

- Motivation
- SCET set up
  - ✦ Factorisation
  - ✦ Resummation
- Fixed order expansion
- Resummed behaviour
- Summary

Motivation: W-mass measurement

- ~2% uncertainty in $W^{\pm}$ p_T translates into ~ 10 MeV uncertainty in $W^{\pm}$ mass.

- One way to get precise $W^{\pm}$ p_T spectrum is to precisely measure the Z p_T spectrum.

$$\frac{d\sigma_W}{dp_T} = \left[\frac{d\sigma_W/dp_T}{d\sigma_Z/dp_T}\right]_{\text{theory}} \times \left.\frac{d\sigma_Z}{dp_T}\right|_{\text{measured}}$$

Major uncertainty from QCD (See: Plenary talk by U. Blumenschein)

- Major theoretical difference in $W^{\pm}$ and Z p_T spectrum in the massive b-quark contributions.

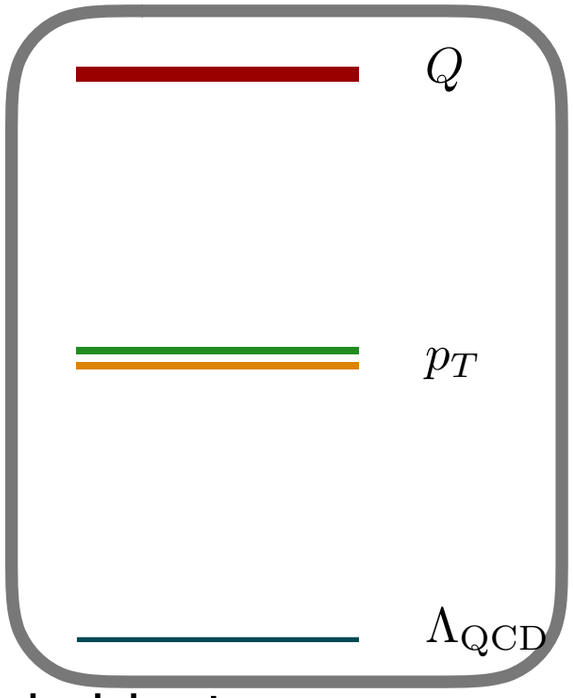
- Due to strong CKM suppression, the massive b-quark does not play any significant role in $W^{\pm}$ production.

SCET setup: Massless Factorisation

- All order SCET factorisation at Leading Power:

$$\frac{d\sigma}{dq_T^2 dQ^2 dY} = H(Q,\mu) \times B(\vec{q}_T, x_a, \mu, \nu/\omega_a) \otimes B(\vec{q}_T, x_b, \mu, \nu/\omega_b) \otimes S(\vec{q}_T, \mu, \nu)$$

$$\frac{d\sigma}{dq_T^2 dQ^2 dY} = \sum_{i,j\in\{q,\bar{q}\}} H_{ij}^{(n_f)}(Q,\mu) \int d^2p_{T_a} d^2p_{T_b} d^2p_{T_s} \delta(q_T^2 - |\vec{p}_{T_a} + \vec{p}_{T_b} + \vec{p}_{T_s}|^2)$$
$$\times B_i^{(n_f)}(\vec{p}_{T_a}, x_a, \mu, \nu/\omega_a) B_j^{(n_f)}(\vec{p}_{T_b}, x_b, \mu, \nu/\omega_b) S^{(n_f)}(\vec{q}_T, \mu, \nu)[1 + \mathcal{O}(\frac{q_T}{Q})]$$



The Hard function, Beam functions and Soft function are all renormalised objects.

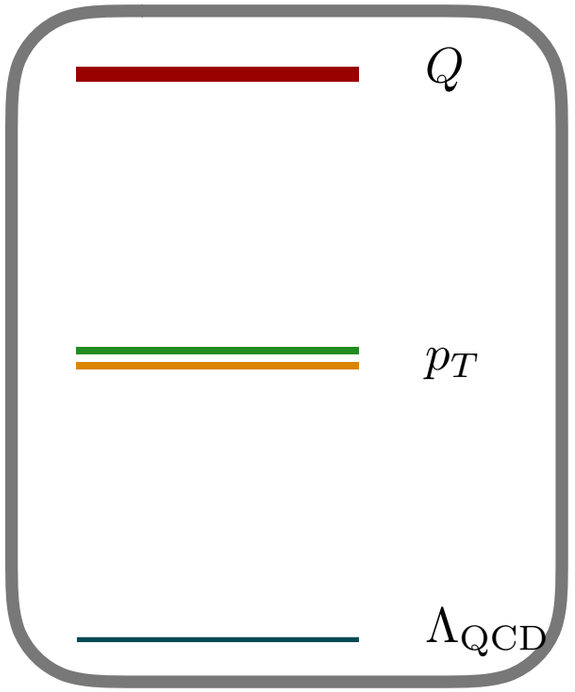
- Beam function is defined through the convolution among the matching coefficients with non-perturbative PDFs.

$$B_i^{(n_f)}(\vec{p}_T, x, \mu, \nu/\omega) = \sum_k \mathcal{I}_{ik}^{(n_f)}(\vec{p}_T, x, \mu, \nu/\omega) \otimes_x f_k^{(n_f)}(x,\mu)$$

- Beam matching coefficients are perturbative and describe the collinear initial state radiation.

- These functions depend on unphysical scale μ and ν . These dependences are cancelled among themselves.

Massless Factorisation



- Unphysical scale dependence on μ and ν are governed by corresponding RGEs.

$$\ln^2 \frac{q_T}{Q} = \ln^2 \frac{Q}{\mu} + 2 \ln \frac{q_T}{\mu} \ln \frac{\nu}{Q} + \ln \frac{q_T}{\mu} \ln \frac{\mu q_T}{\nu^2}$$

The log structure of the cross-section is encoded through the scale dependence at all orders.

- p_T distribution comes under SCET-II theory.

  Collinear and Soft modes have same virtuality $\longrightarrow$ Rapidity divergences

  Regulator introduces a rapidity renormalisation scale ν . [J.-Y. Chiu etc.all (2012)]

  Rapidity divergences cancel in each order between B and S.

  Large rapidity logarithms can be resummed to all orders.

Characteristic Scales

- Unphysical scales: virtuality scale (μ) and rapidity scale (ν)
- Hard function: $\mu_H \sim Q$
- Beam function: $\mu_B \sim q_T$ and $\nu_B \sim Q$
- Soft function: $\mu_S \sim q_T$ and $\nu_S \sim q_T$
- PDF: $\Lambda_{\rm QCD}$

$$\ln^2 \frac{q_T}{Q} = \ln^2 \frac{Q}{\mu} + 2 \ln \frac{q_T}{\mu} \ln \frac{\nu}{Q} + \ln \frac{q_T}{\mu} \ln \frac{\mu q_T}{\nu^2}$$

RGEs for qT Distribution

$$\frac{d\ln H(Q,\mu)}{d\ln\mu} = \gamma_H(Q,\mu)$$

$$\frac{d\ln B(\vec{p}_T, x, \mu, \nu/\omega)}{d\ln\mu} = \gamma_B(\mu,\nu)$$

$$\frac{d\ln S(\vec{p}_T, \mu, \nu)}{d\ln\mu} = \gamma_S(\mu,\nu)$$

$$\frac{dB(\vec{p}_T, x, \mu, \nu/\omega)}{d\ln\nu} = \int d^2\vec{k}_T \gamma_{\nu,B}(\vec{k}_T,\mu) B(\vec{p}_T - \vec{k}_T, x, \mu, \nu/\omega)$$

$$\frac{dS(\vec{p}_T, \mu, \nu)}{d\ln\nu} = \int d^2\vec{k}_T \gamma_{\nu,S}(\vec{k}_T,\mu) S(\vec{p}_T - \vec{k}_T, \mu, \nu)$$

- ν independence of the cross-section:

$$\gamma_{\nu,S}(\vec{k}_T,\mu) = -2\gamma_{\nu,B}(\vec{k}_T,\mu) \equiv \gamma_\nu(\vec{k}_T,\mu)$$

- Path independence in $\mu - \nu$ plane:

$$\frac{d}{d\ln\mu}\gamma_\nu(\vec{k}_T,\mu) = \frac{d}{d\ln\nu}\gamma_S(\mu,\nu)\delta(\vec{k}_T) \equiv -4\Gamma_{\rm cusp}(\alpha_S(\mu))\delta(\vec{k}_T)$$

Choose proper initial scale to minimise logs in boundary term

RGE of Hard Function

$$\frac{d\ln H(Q,\mu)}{d\ln\mu} = \gamma_H(Q,\mu)$$

- Hard anomalous dimension has a following structure: [Becher & Neubert (2010)]

$$\gamma_H(Q,\mu) = \Gamma_{\text{cusp}}(\alpha_S)\ln\frac{Q}{\mu} + \gamma_{\text{non-cusp}}(\alpha_S)$$

- Solution:

$$H(Q,\mu) = H(Q,\mu_H)U_H(\mu_H,\mu)$$

Boundary Term **Evolution Kernel**

- Boundary term is free from logarithms $(\mu_H \equiv Q)$
- Evolution kernel resums logarithms $\ln^n\left(\frac{Q}{\mu}\right)$ to all orders

$$U_H(\mu_H,\mu) = \text{Exp}\left[\int_{\mu_H}^{\mu}\frac{d\mu'}{\mu'}\gamma_H(Q,\mu')\right]$$

Resummation

- Goal is to set the scales in the Hard, Beam and Soft function to their natural scales in μ and ν.

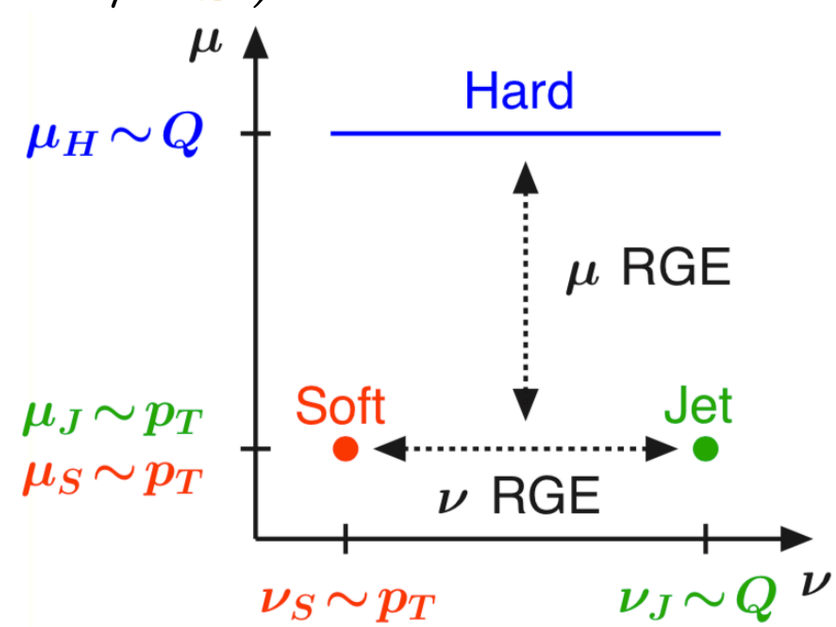
$$\mu_H = Q, \qquad \mu_B = q_T, \quad \nu_B = Q, \qquad \mu_S = \nu_S = q_T$$

- Then resum large logs $\ln^n(q_T/Q)$ through the following large logs coming from different functions:

$$\ln^n(\mu_B/\mu_H), \ \ln^n(\mu_S/\mu_H), \ \ln^n(\nu_B/\nu_S)$$

- Full resummed result:

$$\frac{d\sigma}{dq_T^2 dQ^2 dY} = H(Q,\mu_H) \times B(\vec{q}_T, x_a, \mu_B, \nu_B/\omega_a) \otimes B(\vec{q}_T, x_b, \mu_B, \nu_B/\omega_b) \otimes S(\vec{q}_T, \mu_S, \nu_S) \otimes U_{\rm tot}(\vec{q}_T; \mu_H, \mu_B, \mu_S, \nu_B, \nu_S)$$



Order Counting

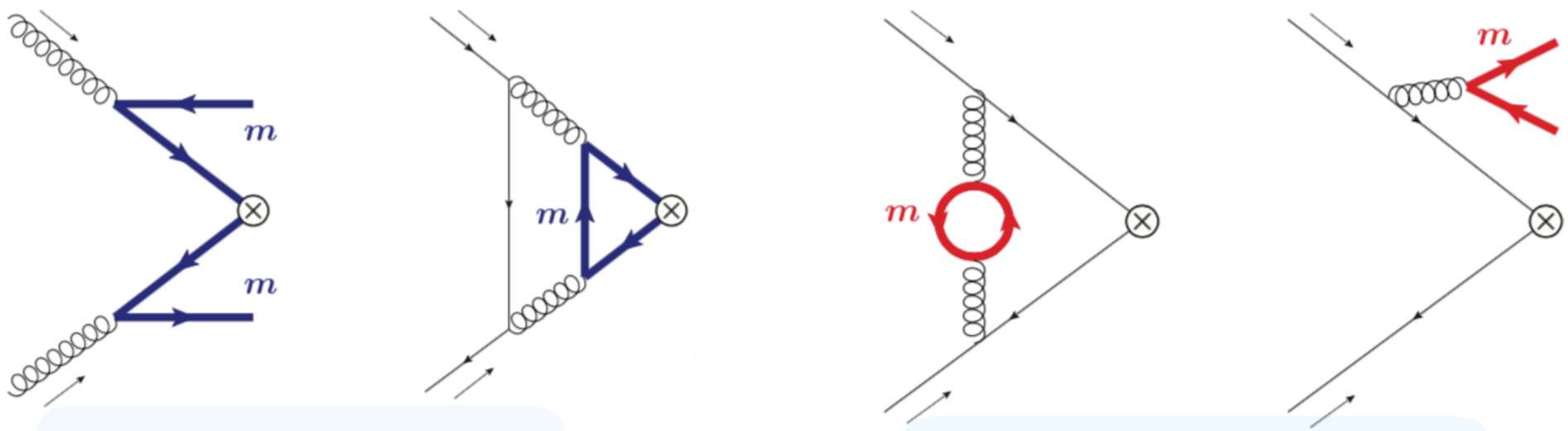
- Resummation order is determined by the order counting in the anomalous dimensions (γ) and in boundary term.

Resummed accuracy	Boundary conditions	Anomalous dimensions $\gamma_{H,B,S,\nu}$	Anomalous dimensions $\Gamma_{\rm cusp}, \beta$	FO matching
LL	1	-	1-loop	-
NLL	1	1-loop	2-loop	-
NLL` + NLO	α_s	1-loop	2-loop	α_s
NNLL+ NLO	α_s	2-loop	3-loop	α_s
NNLL`+NNLO	α_s^2	2-loop	3-loop	α_s^2

- Easy to track the resum order unlike the usual case where the log scaling $\alpha_S L \sim \mathcal{O}(1)$ in the final result determines the order.

Mass effect in DY qT spectrum

- Mass effect in Drell-Yan:



Primary Contribution **Secondary Contribution**

- **Primary Contribution**: Heavy quark initiates the hard process.
- **Secondary Contribution**: Massive corrections to light quark induced processes.
- Earlier approach to only Primary contributions using **S-ACOT** (CSS-type resummation). Estimated upto **~10 MeV** shift in W-mass.

Berge, Nadolsky, Onless ('05)

- Modified Parton Shower algorithm based numerical approach: **< 5 MeV**

E. Bagnaschi, F. Maltoni, A. Vicini, M. Zaro. ('18)

Massive b-quark effect

- Massive quark starts contributing at $\mathcal{O}(\alpha_S^2)$.
- Additional mass scale (m_b) introduces different scale hierarchies.

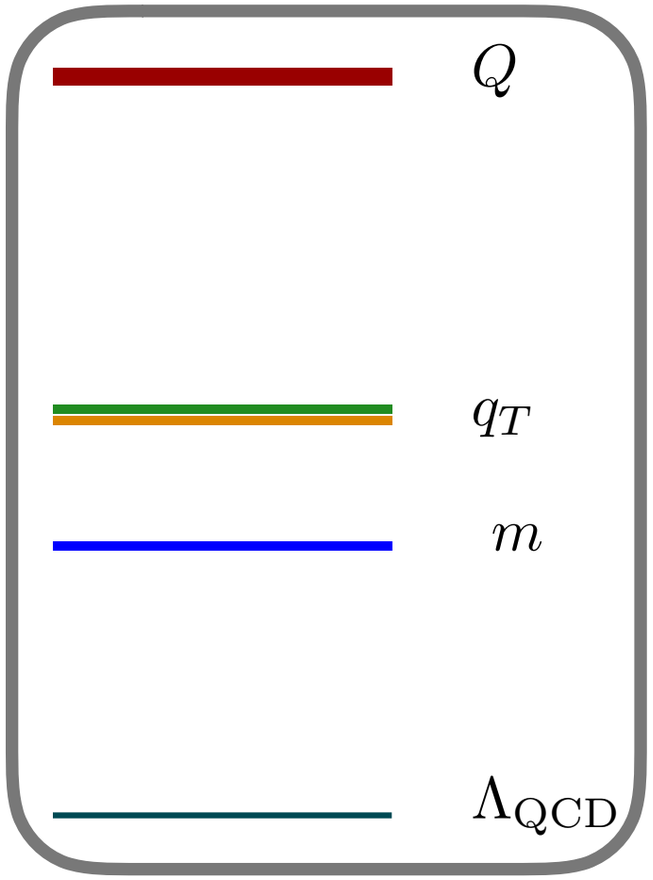
$\Lambda_{\rm QCD} \ll p_T \ll m_b \ll Q$	$\Lambda_{\rm QCD} \ll p_T \sim m_b \ll Q$	$\Lambda_{\rm QCD} \ll m_b \ll p_T \ll Q$
Heavy quark decouples (4F)	**Quark mass changes resummation structure**	**Massless limit (5F)**

- Full resummation requires complete factorisation structure for all different regions.

[P. Pietrulewicz, D. Samitz, A. Spiering, F. Tackmann ('17)]

Factorisation: qT>>m

$$\frac{d\sigma}{dq_T^2 dQ^2 dY} = \sum_{i,j\in\{q,\bar{q},b,\bar{b}\}} H_{ij}^{(n_l+1)}(Q,\mu)$$
$$\times \sum_{k\in\{q,\bar{q},b,\bar{b},g\}} I_{ik}^{(n_l+1)}(\vec{q}_T, x_a, \mu, \nu/\omega_a) \otimes_x f_k^{(n_l+1)}(x_a, m, \mu)$$
$$\otimes \sum_{k\in\{q,\bar{q},b,\bar{b},g\}} I_{jk}^{(n_l+1)}(\vec{q}_T, x_b, \mu, \nu/\omega_b) \otimes_x f_k^{(n_l+1)}(x_b, m, \mu)$$
$$\otimes S^{(n_l+1)}(\vec{q}_T, \mu, \nu) + \mathcal{O}\Big(\frac{q_T}{Q}, \frac{m^2}{q_T^2}, \frac{\Lambda_{\rm QCD}^2}{m^2}\Big)$$



- Hard, Beam, Soft functions are defined in $n_l + 1$ flavours.
- The b-quark mass effect contained in a massive b-pdf.
- b-pdf:

$$f_k^{(n_l+1)}(x, m, \mu) = \sum_{l\in\{q,\bar{q},g\}} \mathcal{M}_{kl}(x, m, \mu) \otimes_x f_l^{(n_l)}(x, \mu) + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}^2}{m^2}\Big)$$

Factorisation: qT>>m

$$\frac{d\sigma}{dq_T^2 dQ^2 dY} = \sum_{i,j\in\{q,\bar{q},b,\bar{b}\}} H_{ij}^{(n_l+1)}(Q,\mu)$$
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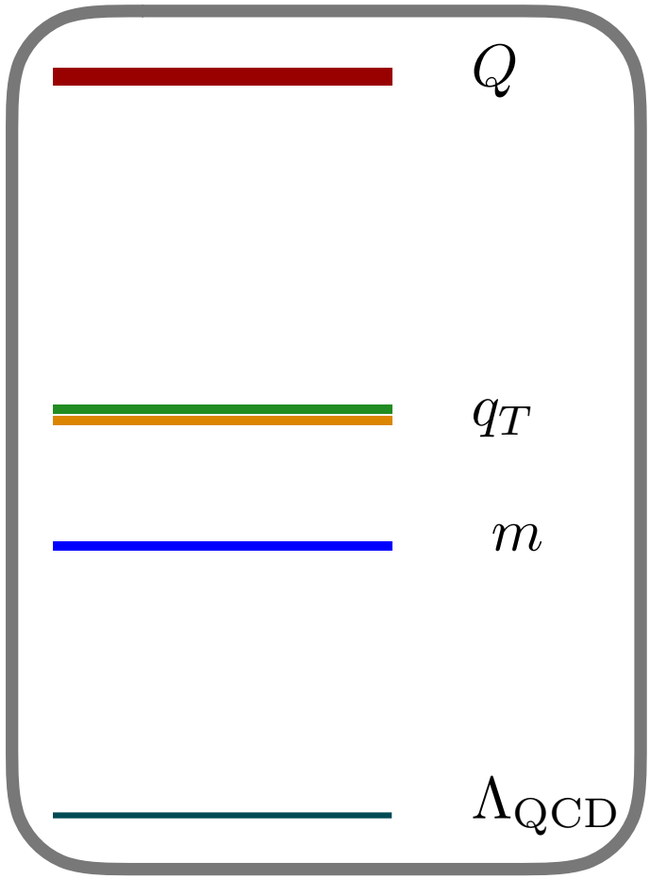
$$\otimes S^{(n_l+1)}(\vec{q}_T, \mu, \nu) + \mathcal{O}\Big(\frac{q_T}{Q}, \frac{m^2}{q_T^2}, \frac{\Lambda_{\rm QCD}^2}{m^2}\Big)$$

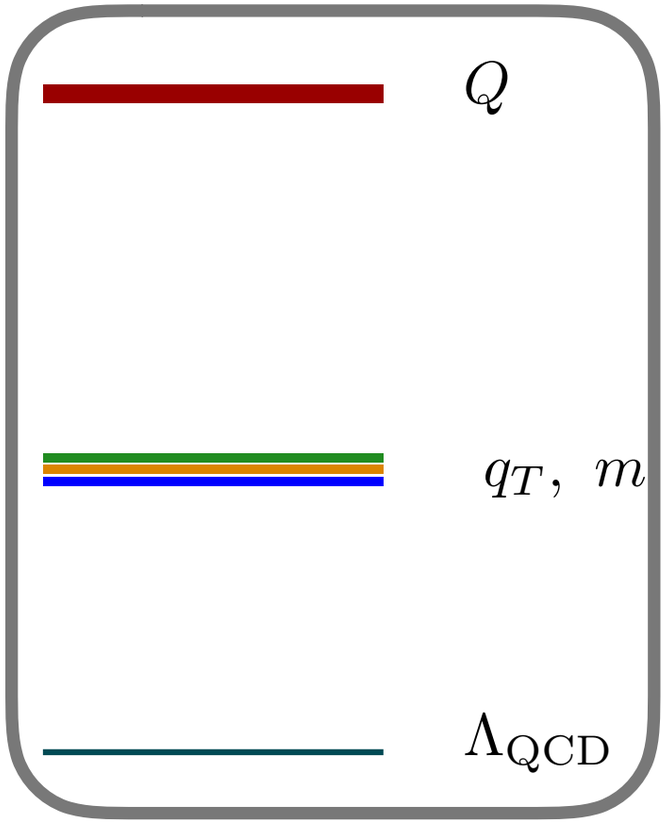


- Mass dependence is contained in PDF matching coefficients.
- $\mathcal{M}_{bg}$ to Primary contribution by $\mathcal{O}(\alpha_S \otimes \alpha_S)$. $\mathcal{M}_{qq}$ to Secondary contribution by $\mathcal{O}(\alpha_S^2)$.

  [Buza, Matiounine, Smith, van Neerven (1998)]
- No rapidity divergences in PDF matching coefficients.

Factorisation: qT~m

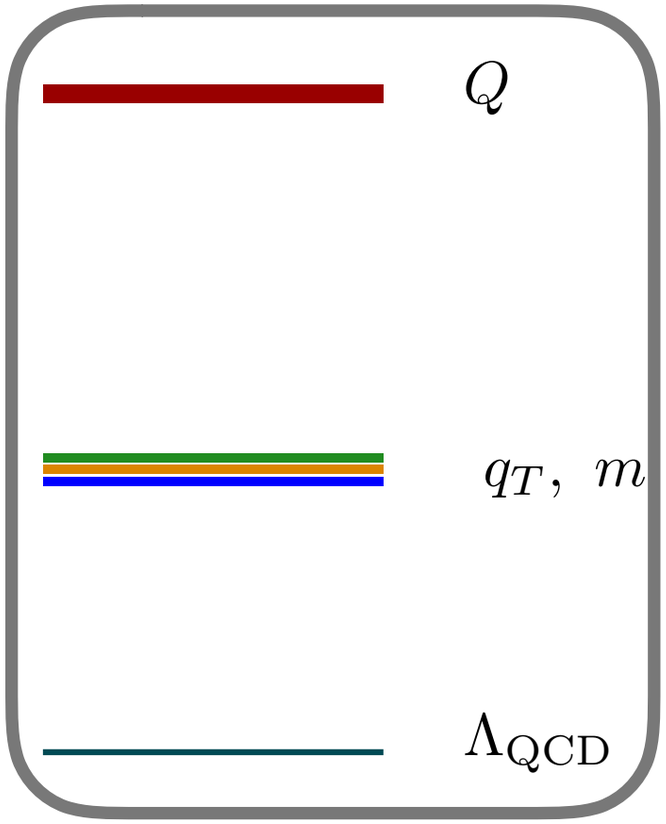
$$\frac{d\sigma}{dq_T^2 dQ^2 dY} = \sum_{i,j\in\{q,\bar{q},b,\bar{b}\}} H_{ij}^{(n_l+1)}(Q,\mu)$$
$$\times \sum_{k\in\{q,\bar{q},g\}} I_{ik}(\vec{q}_T, m, x_a, \mu, \nu/\omega_a) \otimes_x f_k^{(n_l)}(x_a,\mu)$$
$$\otimes \sum_{k\in\{q,\bar{q},g\}} I_{jk}(\vec{q}_T, m, x_b, \mu, \nu/\omega_b) \otimes_x f_k^{(n_l)}(x_b,\mu)$$
$$\otimes S(\vec{q}_T, m, \mu, \nu) + \mathcal{O}\Big(\frac{q_T}{Q}, \frac{m^2}{Q^2}, \frac{\Lambda_{\rm QCD}^2}{m^2}, \frac{\Lambda_{\rm QCD}^2}{q_T^2}\Big)$$



- Hard function in $n_l + 1$ flavour. PDF in n_l flavour.
- I and S can be expressed either in n_l or $n_l + 1$ flavour for as.
- I_{bg} to Primary contribution. Two loop I_{qq} and Soft to Secondary contribution.

Factorisation: qT~m

$$\frac{d\sigma}{dq_T^2 dQ^2 dY} = \sum_{i,j\in\{q,\bar{q},b,\bar{b}\}} H_{ij}^{(n_l+1)}(Q,\mu)$$
$$\times \sum_{k\in\{q,\bar{q},g\}} I_{ik}(\vec{q}_T, m, x_a, \mu, \nu/\omega_a) \otimes_x f_k^{(n_l)}(x_a,\mu)$$
$$\otimes \sum_{k\in\{q,\bar{q},g\}} I_{jk}(\vec{q}_T, m, x_b, \mu, \nu/\omega_b) \otimes_x f_k^{(n_l)}(x_b,\mu)$$
$$\otimes S(\vec{q}_T, m, \mu, \nu) + \mathcal{O}\Big(\frac{q_T}{Q}, \frac{m^2}{Q^2}, \frac{\Lambda_{\rm QCD}^2}{m^2}, \frac{\Lambda_{\rm QCD}^2}{q_T^2}\Big)$$

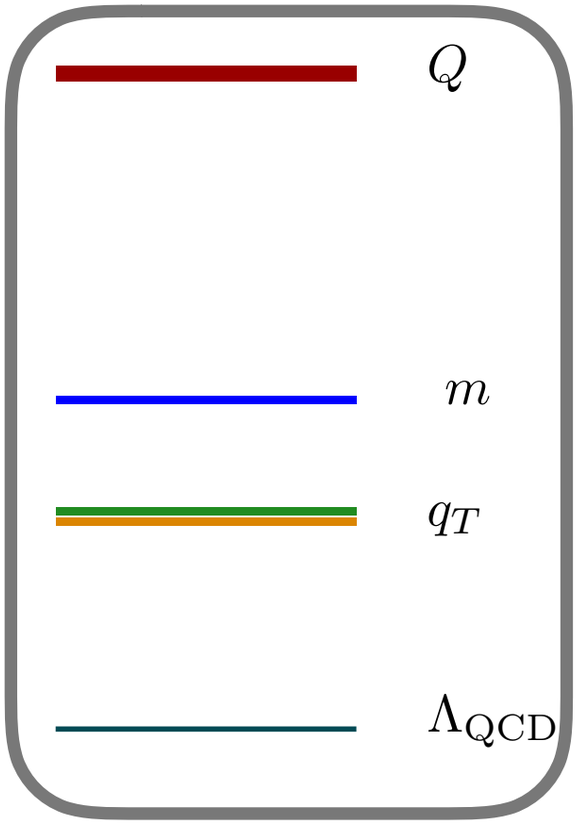


- Massive rapidity divergences from Secondary contribution.
- Additional mass-dependent rapidity logarithms.
- Needs to be resummed through rapidity RGE.

$$\frac{d}{d\ln\nu} B_i^{(n_l+1)}(\vec{q}_T, m, \mu, \nu/\omega) = \int d^2p_T \gamma_{\nu,B}^{(n_l+1)}(\vec{q}_T - \vec{p}_T, m, \mu) B_i^{(n_l+1)}(\vec{p}_T, m, \mu, \nu/\omega)$$
$$\frac{d}{d\ln\nu} S^{(n_l+1)}(\vec{q}_T, m, \mu, \nu) = \int d^2p_T \gamma_{\nu,S}^{(n_l+1)}(\vec{q}_T - \vec{p}_T, m, \mu) S^{(n_l+1)}(\vec{p}_T, m, \mu, \nu)$$

Factorisation: qT<<m

$$\frac{d\sigma}{dq_T^2 dQ^2 dY} = \sum_{i,j\in\{q,\bar{q}\}} H_{ij}^{(n_l+1)}(Q,\mu) H_c(m,\mu,\nu/\omega_a) H_{\bar{c}}(m,\mu,\nu/\omega_b) H_s(m,\mu,\nu)$$
$$\times B_i^{(n_l)}(\vec{q}_T, x_a, \mu, \nu/\omega_a) \otimes B_j^{(n_l)}(\vec{q}_T, x_b, \mu, \nu/\omega_b)$$
$$\otimes S^{(n_l)}(\vec{q}_T, \mu, \nu) + \mathcal{O}\Big(\frac{q_T}{Q}, \frac{q_T^2}{m^2}, \frac{m^2}{Q^2}\Big)$$



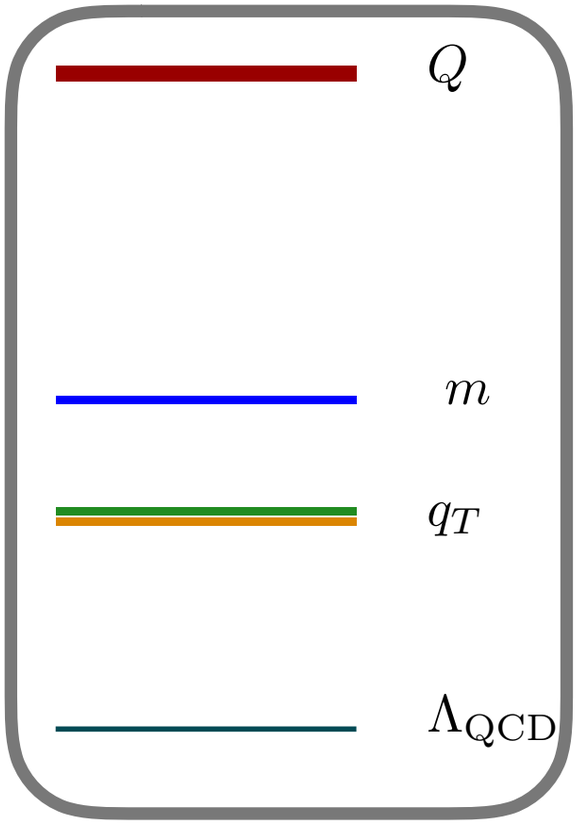
- Hard function with $n_l + 1$ flavour.
- Beam, Soft function with n_l flavour.
- Mass dependence in mass mode matching coefficients. SCET with $n_l + 1$ flavour is matched to SCET with n_l flavour at $\mu \sim m$

[Gritschacher, Hoang, Jemos, Mateu, Pietrulewicz (2014)]
[Hoang, Pathak, Pietrulewicz, Stewart (2016)]
[Hoang, Pietrulewicz, Samitz (2016)]

- Rapidity divergences in Soft and Collinear mass mode cancel among themselves.

Factorisation: qT<<m

$$\frac{d\sigma}{dq_T^2 dQ^2 dY} = \sum_{i,j\in\{q,\bar{q}\}} H_{ij}^{(n_l+1)}(Q,\mu) H_c(m,\mu,\nu/\omega_a) H_{\bar{c}}(m,\mu,\nu/\omega_b) H_s(m,\mu,\nu)$$
$$\times B_i^{(n_l)}(\vec{q}_T, x_a, \mu, \nu/\omega_a) \otimes B_j^{(n_l)}(\vec{q}_T, x_b, \mu, \nu/\omega_b)$$
$$\otimes S^{(n_l)}(\vec{q}_T, \mu, \nu) + \mathcal{O}\Big(\frac{q_T}{Q}, \frac{q_T^2}{m^2}, \frac{m^2}{Q^2}\Big)$$



- RG consistency ($m_b \sim Q$ and $m_b \ll Q$):
The μ evolution for the mass-dependent hard functions is given by the difference between n_l and $n_l + 1$ flavours in the evolution of Hard function.

$$\gamma_{H_c}(m,\mu,\nu/\omega_a) + \gamma_{H_{\bar{c}}}(m,\mu,\nu/\omega_b) + \gamma_{H_s}(m,\mu,\nu) = \gamma_H^{(n_l)}(Q,\mu) - \gamma_H^{(n_l+1)}(Q,\mu)$$

Relation among hierarchies

- Matching between $q_T \sim m_b$ and $q_T \ll m_b$:

$$I_{ij}(\vec{q}_T, x, m, \mu, \nu/\omega) = I_{ij}^{(n_l)}(\vec{q}_T, x, \mu, \nu/\omega) H_c(m, \mu, \nu/\omega) + \mathcal{O}\left(\frac{q_T^2}{m^2}\right)$$

$$S(\vec{q}_T, m, \mu, \nu) = S^{(n_l)}(\vec{q}_T, \mu, \nu) H_s(m, \mu, \nu) + \mathcal{O}\left(\frac{q_T^2}{m^2}\right)$$

$\ln\left(\frac{q_T^2}{m^2}\right)$ can be resummed to all orders in $q_T \ll m_b$ at RHS.

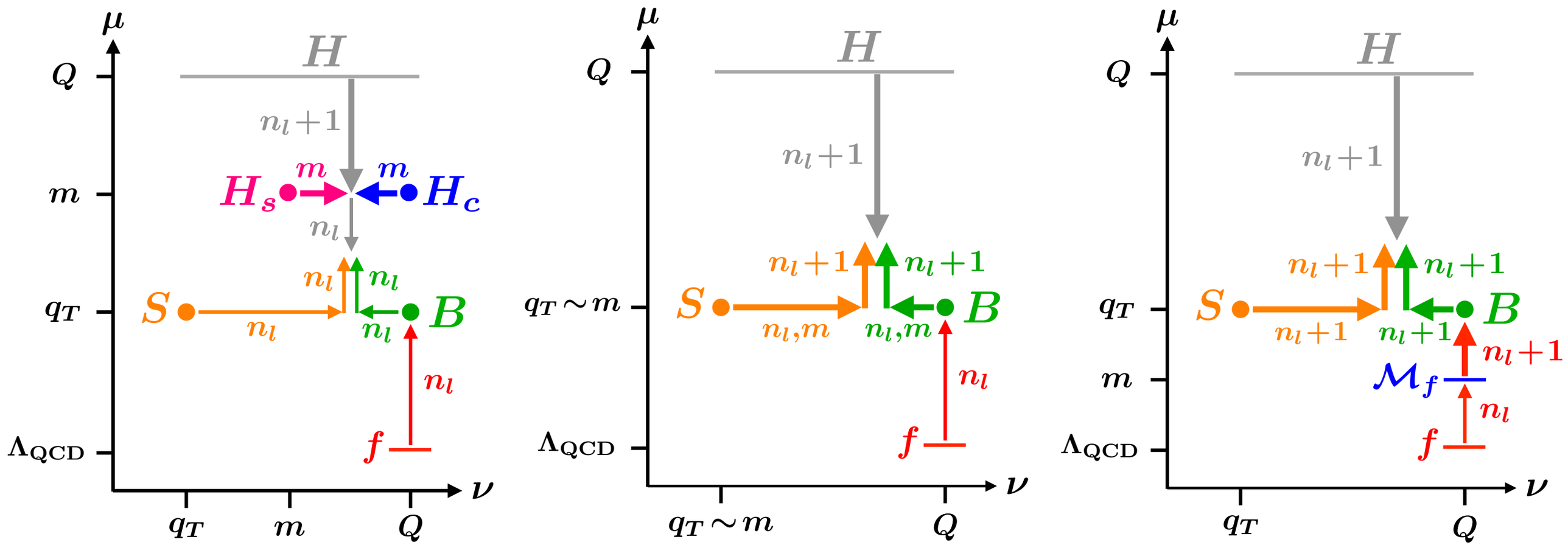
- Matching between $q_T \sim m_b$ and $q_T \gg m_b$:

$$I_{ij}(\vec{q}_T, x, m, \mu, \nu/\omega) = \sum_{k \in q, \bar{q}, g} I_{ik}^{(n_l)}(\vec{q}_T, x, \mu, \nu/\omega) \otimes_x \mathcal{M}_{kj}(m, x, \mu,) + \mathcal{O}\left(\frac{m^2}{q_T^2}\right)$$

$$S(\vec{q}_T, m, \mu, \nu) = S^{(n_l+1)}(\vec{q}_T, \mu, \nu) + \mathcal{O}\left(\frac{m^2}{q_T^2}\right)$$

$\ln\left(\frac{m^2}{q_T^2}\right)$ can be resummed to all orders in $q_T \gg m_b$ at RHS.

Resummation Structure



(a) $q_T \ll m \ll Q$ (b) $q_T \sim m \ll Q$ (c) $m \ll q_T \ll Q$

[P. Pietrulewicz, D. Samitz, A. Spiering, F. Tackmann ('17)]

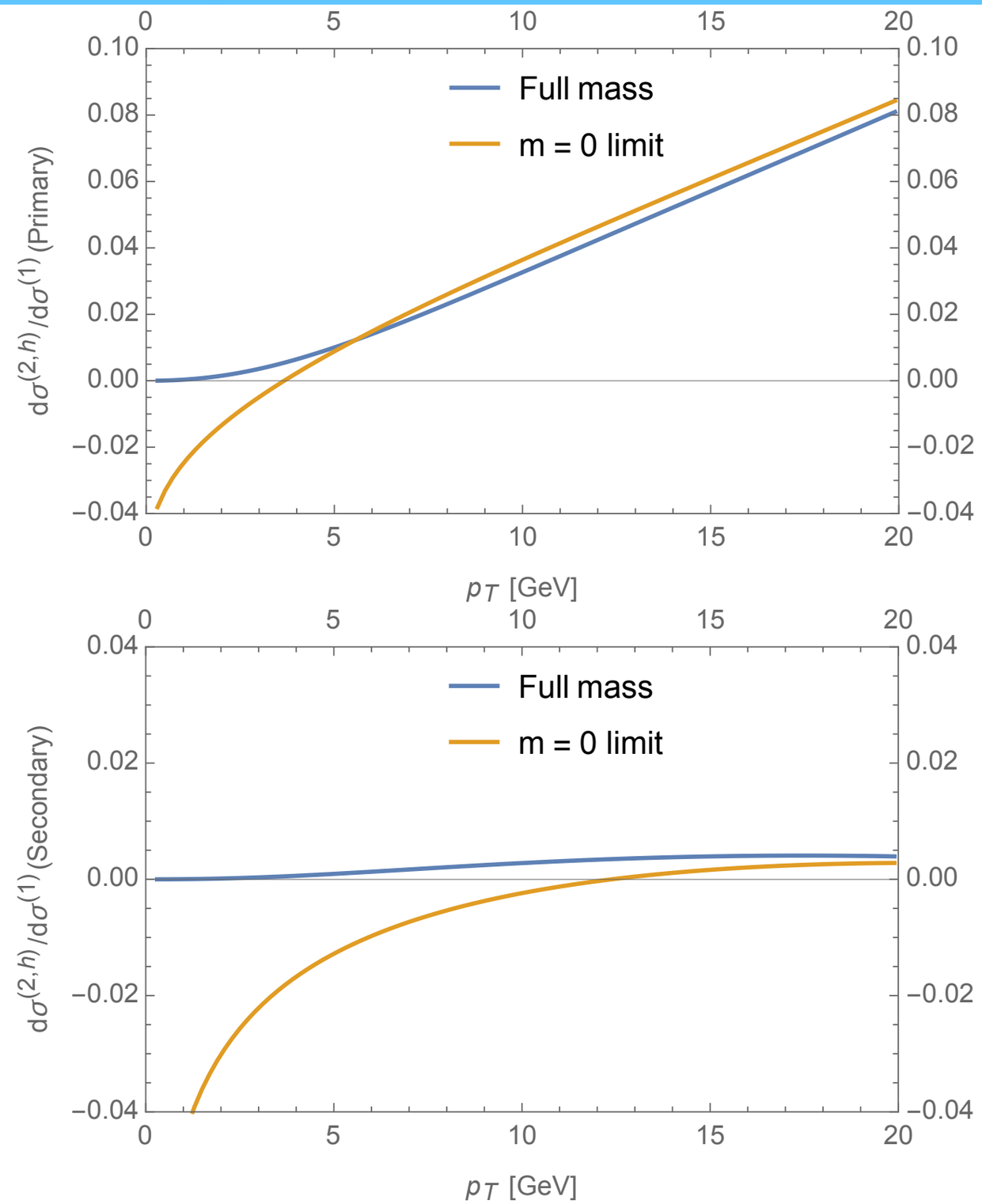
Fixed order expansion

- MMHT2014nnlo68 PDF

$$\mu = m_b = 4.8 \quad \text{GeV}$$

$$Q = m_Z = 91.19 \quad \text{GeV}$$

$$Y = 0, \qquad E_{CM} = 13 \quad \text{TeV}$$



❖ Primary contribution grows with larger q_T.

❖ Secondary contribution gives sub-percent effect.

Resummation: Massive Ano. Dim.

- The rapidity anomalous dimensions are now explicit mass dependent.

$$\tilde{\gamma}_\nu^{(2,h)}(b,m,\mu) = \frac{\alpha_S^2}{16\pi^2} C_F T_F \Bigg[-\frac{32}{3} L_b L_m - \frac{16}{3} L_m^2 - \frac{160}{9} L_m - \frac{448}{27} + \frac{8\sqrt{\pi}}{3} \left\{ 2 G_{1,3}^{3,0}\left(\begin{matrix} \frac{3}{2} \\ 0,0,0 \end{matrix} \middle| m^2 b^2 \right) + G_{1,3}^{3,0}\left(\begin{matrix} \frac{5}{2} \\ 0,0,1 \end{matrix} \middle| m^2 b^2 \right) \right\} \Bigg]$$

Meijer G functions

Large Logs:

$$L_b = \ln\left[\frac{b^2 \mu^2 e^{2\gamma_E}}{4}\right] \quad ; \quad L_m = \ln\left[\frac{m^2}{\mu^2}\right]$$

- Limiting behaviour:

$$\tilde{\gamma}_\nu^{(2,h)}(b,m,\mu) \xrightarrow{b\to 0} \frac{\alpha_S^2}{16\pi^2} C_F T_F \left[\frac{16}{3} L_b^2 + \frac{160}{9} L_b + \frac{448}{27} \right] + \mathcal{O}(m^2 b^2)$$

$$\tilde{\gamma}_\nu^{(2,h)}(b,m,\mu) \xrightarrow{b\to \infty} \frac{\alpha_S^2}{16\pi^2} C_F T_F \left[-\frac{16}{3} L_m^2 - \frac{160}{9} L_m - \frac{448}{27} \right] + \mathcal{O}\left(\frac{1}{m^2 b^2}\right)$$

Resummation: Scale Choice

- Minimise the large logarithms from the boundary term with proper scale choice.

- Scale behaviour:

$$\mu_0(b,m) \sim \mu_0(b) = \frac{2\mathrm{e}^{-\gamma_E}}{b} \qquad \text{for} \quad b \to 0$$

$$\mu_0(b,m) \sim m \qquad \text{for} \quad b \to \infty$$

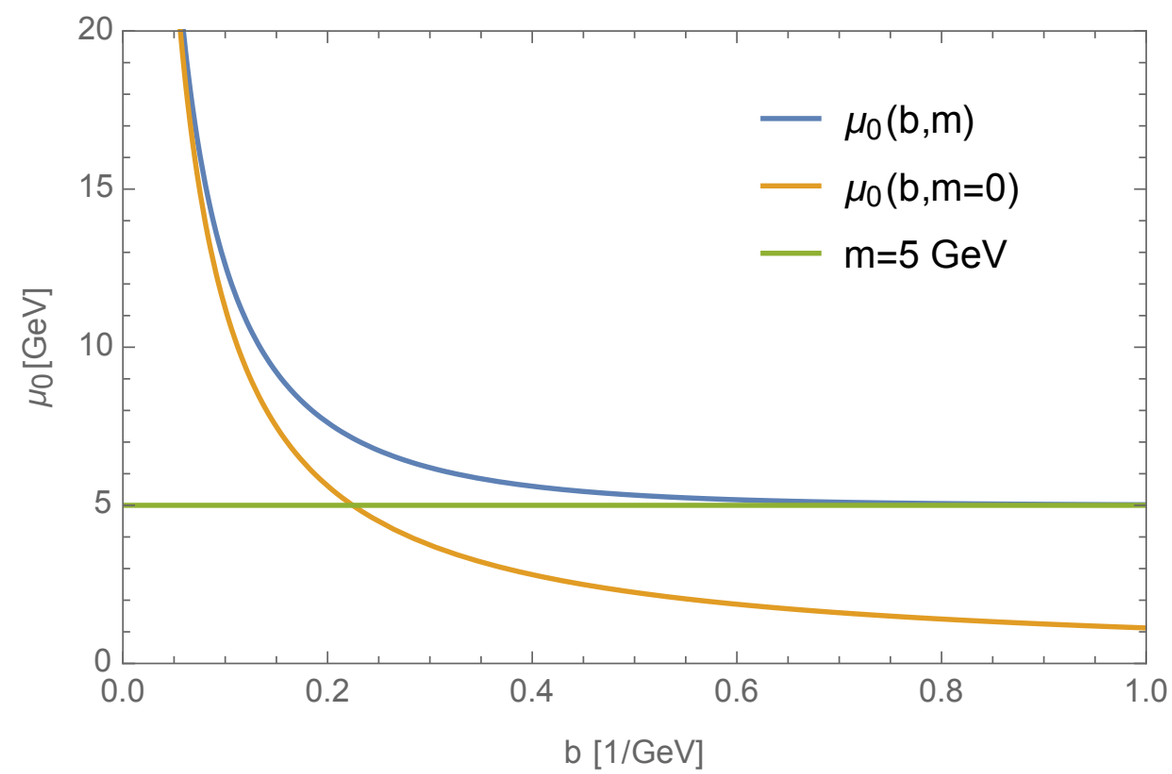
$$L_b = \ln\left[\frac{b^2\mu^2\mathrm{e}^{2\gamma_E}}{4}\right]$$

$$L_m = \ln\left[\frac{m^2}{\mu^2}\right]$$

- Natural scale choice

$$\mu_0(b,m) = m\ \mathrm{Exp}[K_0(mb)]$$

Non-perturbative sensitivity gets regulated by quark mass.



Resummation: Massive Contribution

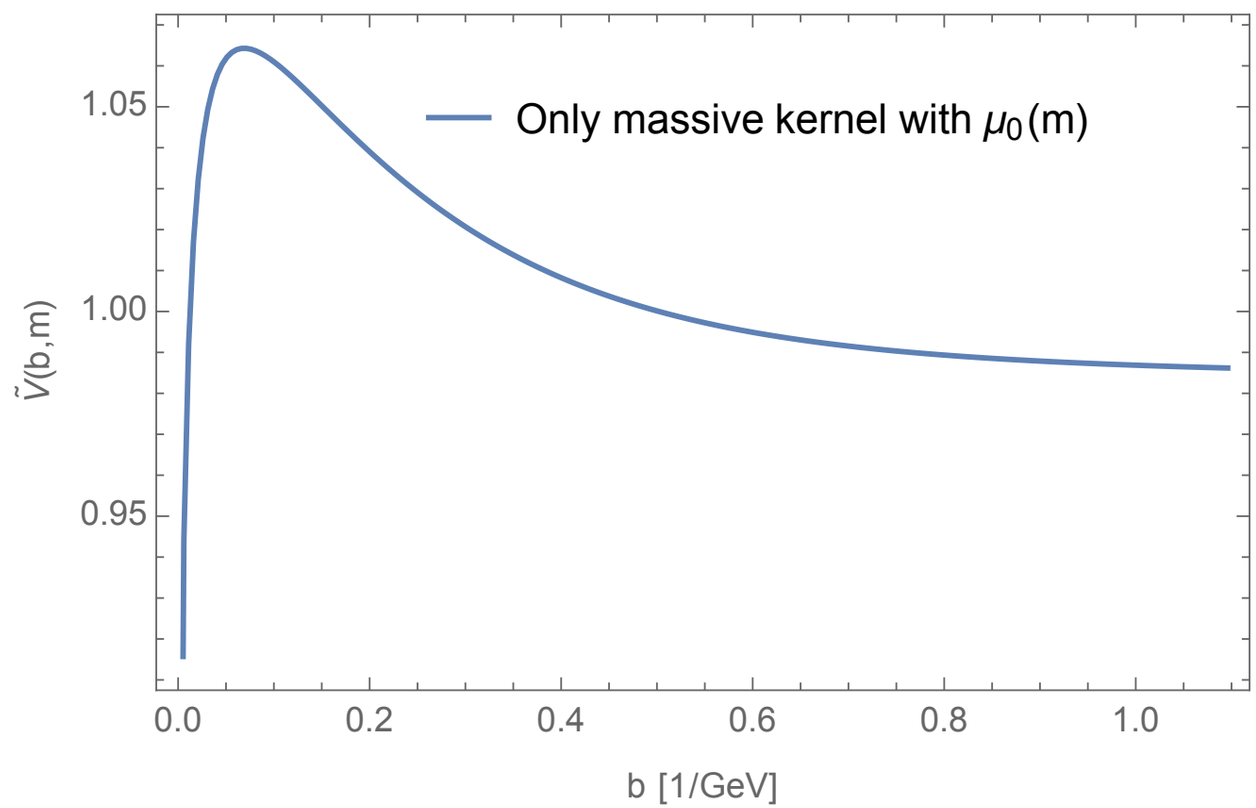
- The massive contribution to the rapidity kernel:

$$\tilde{\gamma}_\nu^{(h)}(b,m,\mu) = 4\eta_\Gamma^{(n_l)}(\mu_0(b,m),\mu) - 4\eta_\Gamma^{(n_l+1)}(\mu_0(b,m),\mu) + \tilde{\gamma}_\nu^{(2,h)}(b,m,\mu_0(b,m))$$

$$\eta_\Gamma^{(n_f)}(\mu_0,\mu) = \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{\rm cusp}^{(n_f)}[\alpha_s^{(n_f)}(\mu')]$$

Explicit mass dependence

$$m_b = 4.8 \text{ GeV}$$
$$\nu_B = Q = 91.19 \text{ GeV}$$
$$\nu_S = b_0/b \text{ GeV}$$
$$\mu = 4.8 \text{ GeV}$$



Resummation: Massive Contribution

- Complete rapidity anomalous dimension

$$\tilde{\gamma}_{\nu,S}(b,m,\mu) = -4\eta_{\Gamma}^{(n_l)}(\mu_0(b)) + \tilde{\gamma}_{\nu,S,bc}^{(n_l)} + \tilde{\gamma}_{\nu}^{(h)}(b,m,\mu)$$

Massless with massless scale

Massive with massive scale

- Behaviour of the full rapidity anomalous dimension:

$$\tilde{\gamma}_{\nu,S}(b,m,\mu) = \tilde{\gamma}_{\nu,S}^{(n_l+1)}(b,\mu) + \mathcal{O}(m^2b^2)$$

Massless limit (5F)

$$\tilde{\gamma}_{\nu,S}(b,m,\mu) = \tilde{\gamma}_{\nu,S}^{(n_l)}(b,\mu) + \gamma_{\nu,H_s}(m,\mu) + \mathcal{O}(1/m^2b^2)$$

Heavy quark decouples (4F)

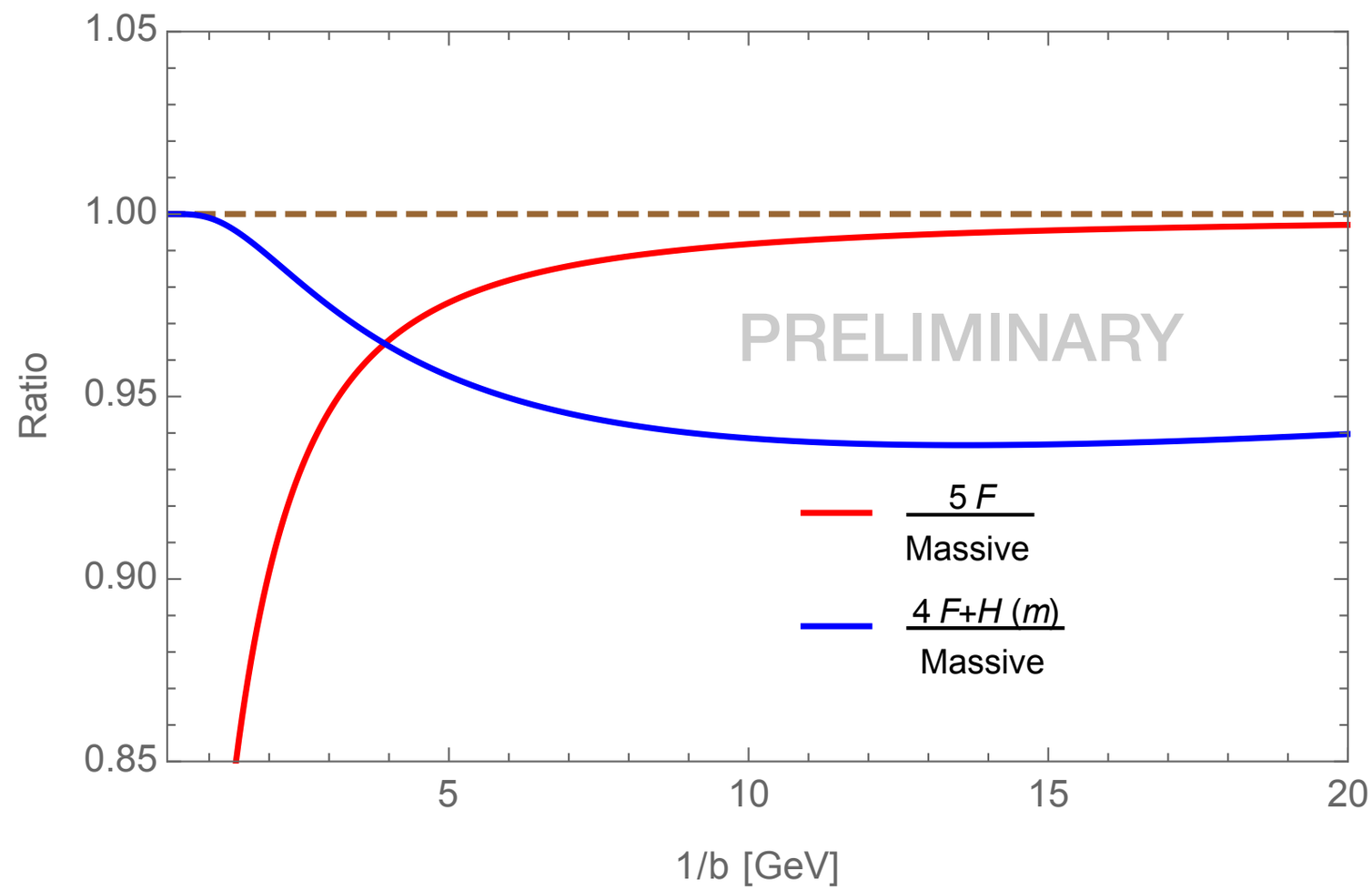
Rapidity kernel

- Full massive rapidity kernel:

$$\tilde{V}(m,b,\mu) = \mathrm{Exp}\left[\tilde{\gamma}_\nu(b,m,\mu)\ln\left(\frac{\nu_B}{\nu_S}\right)\right]$$

- Parameters:

$$m_b = 4.8 \text{ GeV}$$
$$\nu_B = Q = 91.19 \text{ GeV}$$
$$\nu_S = b_0/b \text{ GeV}$$
$$\mu = 4.8 \text{ GeV}$$



Summary

- SCET gives a better way to handling the multi-scale problem.
- We exploited the **factorisation** in **SCET** to incorporate massive b-quark effect in DY p_T distribution
- **RGE** can be used to systematically resum large logs coming in different mass hierarchies.
- The resummed prediction in SCET factorisation combines the **4F** and **5F** results in a systematic way and is expected to **improve the reliability** of the perturbative prediction.
- We look forward to a detailed study on **scale choice, scale variation, pdf variation** which will be useful at the LHC in near future.
- Full implementation will be available very soon in **SCETlib.**

[F. Tackmann, J. Michel, M. Ebert, …]