Next to Leading Power Corrections: Resummation and Improving Fixed Order Subtractions

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DESY Theory Workshop 2018
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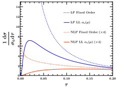
[Moult, Stewart, GV, Zhu] 1804.04665 and

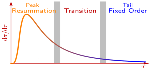
[Ebert, Moult, Stewart, Tackmann, GV, Zhu] 1807.10764

Outline

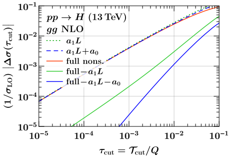
- Motivations for studying Perturbative Power Corrections



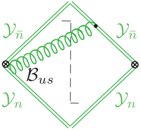
Constraint	$L=2$	$L=3$	$L=4$
1. Integrability	75	643	5897
2. Total S_3 symmetry	20	151	1224
3. Parity invariance	18	120	874
4. Collinear vanishing (T^0)	4	59	622
5. OPE leading discontinuity	0	26	482
6. Final entry	0	2	113
7. Multi-Regge limit	0	2	80
8. Near-collinear OPE (T^1)	0	0	4
9. Near-collinear OPE (T^2)	0	0	0



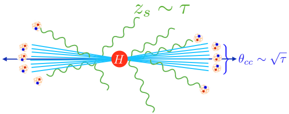
- Power Corrections for N-Jettiness Subtractions at Fixed Order for DY and Higgs production



- Resummation at Subleading Power

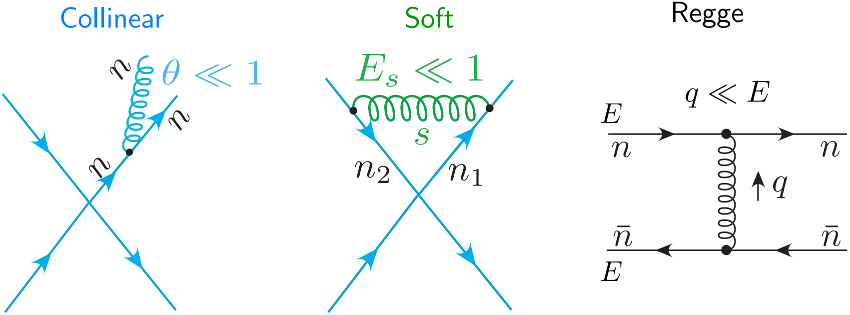


- Leading Log Resummation at Next-to-Leading Power for Thrust



Limits of QCD

- Significant progress in understanding QCD made by considering limits where we have a power expansion in some small kinematic quantity.



- All orders behavior described by factorization theorems (eg. thrust):

$$\frac{d\sigma^{(0)}}{d\tau} = H^{(0)} J_\tau^{(0)} \otimes J_\tau^{(0)} \otimes S_\tau^{(0)} + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q\tau}, \tau\right)$$

Power Corrections for Event Shapes

- Standard factorization theorems describe only leading power term.
- More generally, can consider expanding an observable in τ

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \sum_{n=0}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(0)} \left(\frac{\log^m \tau}{\tau}\right)_+$$

Leading Power (LP)

$$+ \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(2)} \log^m \tau$$

Next to Leading Power (NLP)

$$+ \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(4)} \tau \log^m \tau$$

$$+ \cdots$$

$$= \frac{d\sigma^{(0)}}{d\tau} + \frac{d\sigma^{(2)}}{d\tau} + \frac{d\sigma^{(4)}}{d\tau} + \cdots$$

Power Corrections for Event Shapes

- Standard factorization theorems describe only leading power term.
- More generally, can consider expanding an observable in τ

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} &= \sum_{n=0}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(0)} \left(\frac{\log^m \tau}{\tau}\right)_+ && \text{Leading Power (LP)} \\
&+ \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(2)} \log^m \tau && \text{Next to Leading Power (NLP)} \\
&+ \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(4)} \tau \log^m \tau \\
&+ \cdots \\
&= \frac{d\sigma^{(0)}}{d\tau} + \frac{d\sigma^{(2)}}{d\tau} + \frac{d\sigma^{(4)}}{d\tau} + \cdots
\end{aligned}
$$

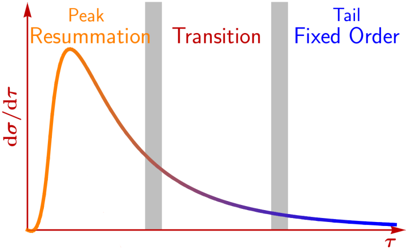
- Why do we want to understand power corrections?

Some applications of Next to Leading Power calculations

Matching resummation with FO

If an observable τ needs resummation:

- Use Leading Power EFT for resummed XS at small τ
- For large τ use Fixed Order calculation to get full $\mathcal{O}(\alpha_s^n)$ contribution
- Need **matching procedure** in transition region between the two.
- Computing **Power Corrections** analytically **extends** domain of **validity** of the EFT to larger values of $\tau \implies$ **smaller transition regions** $\implies$ smaller uncertainties from matching procedure



Bootstrap

Power corrections provide constraints to completely reconstruct amplitudes or cross sections from limits.

Remaining Parameters in 6-Point MHV Remainder Function

LL All Powers →

NLP, NNLP →

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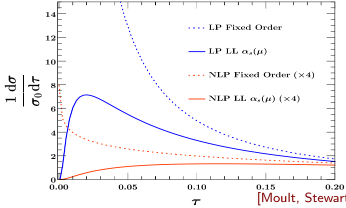
[Basso, Sever, Vieira] [Dixon et al.]

Taming log divergence of NLP

Issue in adding log divergent fixed order power correction to resummed LP cross section demands resummation also at NLP



[Moult, Stewart, GV, Zhu]

More Applications: Fixed Order Subtractions

- IR divergences in fixed order calculations can be regulated using event shape observables. [Boughezal, Focke, Petriello, Liu], [Gaunt, Stahlhofen, Tackmann, Walsh]

$$\boxed{\sigma(X) = \int_0 d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} = \int_0^{\mathcal{T}_N^{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} + \int_{\mathcal{T}_N^{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}}$$

- Want $\mathcal{T}_N$ to isolate collinear and soft singularities around an N-jet configuration.

$$\int_0^{\mathcal{T}_N^{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} \qquad\qquad \int_{\mathcal{T}_N^{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$

Compute using factorization in soft/collinear limits:

$$\frac{d\sigma}{d\tau_N} = HB_a \otimes B_b \otimes S \otimes J_1 \otimes \cdots \otimes J_{N-1}\,[1 + \mathcal{O}(\tau_N)]$$

Additional jet resolved.

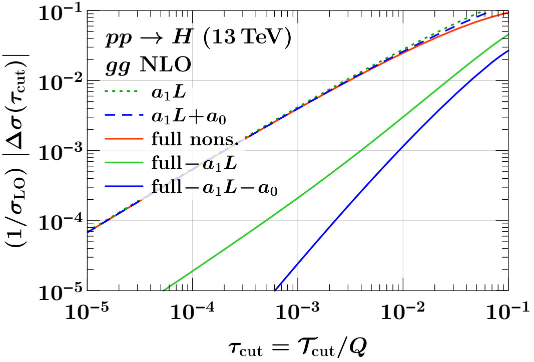
Power Corrections for NLO Subtractions

$$\int_0^{\mathcal{T}_N^{\rm cut}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}\,, \qquad \frac{d\sigma}{d\tau_N} = H B_a \otimes B_b \otimes S \otimes J_1 \otimes \cdots \otimes J_{N-1} + \mathcal{O}(\tau_{\mathbf{N}})$$

Power Correction

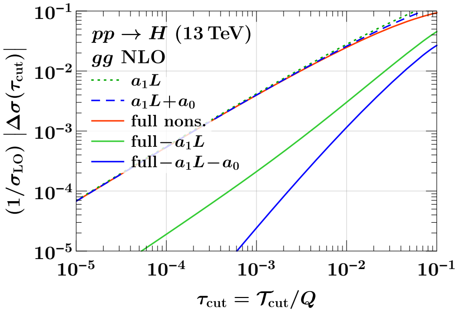
- Error, $\Delta\sigma(\tau_{\rm cut})$, (or computing time) can be exponentially improved by analytically computing power corrections.

- Understanding of power corrections crucial for applications to more complicated processes.



[Ebert, Moult, Stewart, Tackmann, GV, Zhu] [1807.10764]

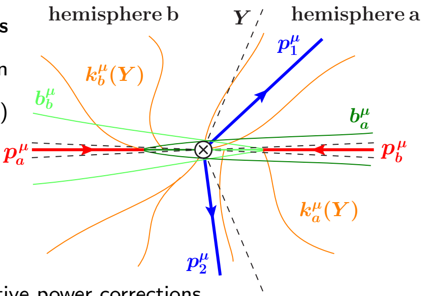
Power corrections at Fixed Order



(Ebert, Moult, Stewart, Tackmann, GV, Zhu) [1807.10764]

Power corrections at FO: General Setup

- We want to compute **fully differential cross section** $\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\mathcal{T}}$ for color singlet production (0-jettiness) including $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\mathcal{T}/Q)$ corrections to LO.



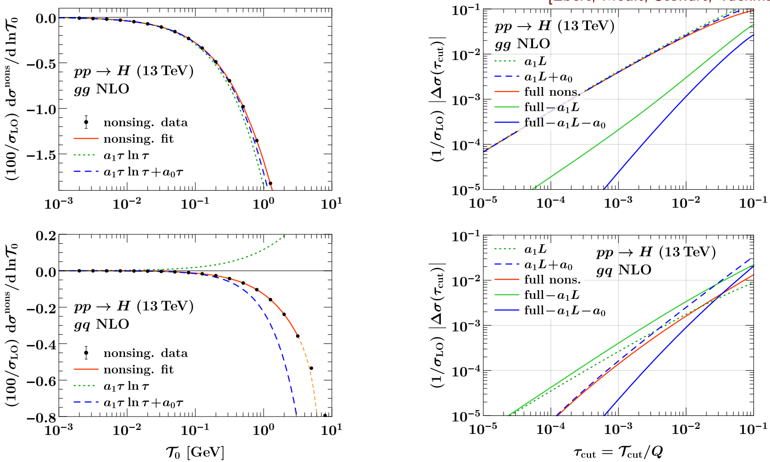
- Power corrections in $\mathcal{O}(\mathcal{T}/Q)$:
  - **Perturbative**
  - **NOT** higher twist PDFs/non-perturbative power corrections.

- $\mathcal{O}(\mathcal{T}/Q)$ corrections contained in:
  - **Phase space:** $\Phi = \Phi^{(0)} + \frac{\mathcal{T}}{Q}\Phi^{(2)} + \mathcal{O}(\frac{\mathcal{T}^2}{Q^2})$
  - **Matrix element squared:** $|\mathcal{M}|^2 = A^{(0)} + \frac{\mathcal{T}}{Q}A^{(2)} + \mathcal{O}(\frac{\mathcal{T}^2}{Q^2})$

Schematically: $$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\mathcal{T}} \sim \int \frac{\mathrm{d}z}{z}\left[A^{(0)}\Phi^{(0)} + \frac{\mathcal{T}}{Q}A^{(0)}\Phi^{(2)} + \frac{\mathcal{T}}{Q}A^{(2)}\Phi^{(0)}\right] + \mathcal{O}\left(\frac{\mathcal{T}^2}{Q^2}, \alpha_s^2\right)$$

Power corrections at FO: full NLO results for $pp \to H$

[Ebert, Moult, Stewart, Tackmann, GV, Zhu] [1807.10764]



$$F_{\rm NLO}(\tau) = \frac{d}{d\ln\tau}\Big\{\tau\big[a_1 \ln\tau + a_0 + \mathcal{O}(\tau)\big]\Big\}$$

NLO $\mathcal{T}_0^{\rm lep}$ $gg \to Hg$	a_1	a_0
earlier fit	$+0.6090 \pm 0.0060$	$+0.1824 \pm 0.0043$
analytic	$+0.6040$	$+0.1863$
NLO $\mathcal{T}_0^{\rm lep}$ $gq \to Hq$	a_1	a_0
earlier fit	-0.0373 ± 0.0007	-0.42552 ± 0.00032
analytic	-0.0381	-0.42576

Numerical fit matches analytic calculation within 1 σ at percent level.

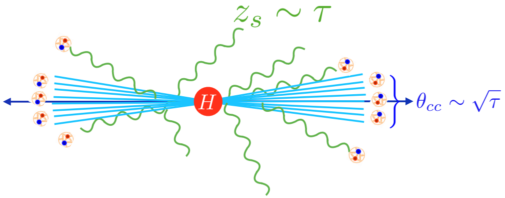
Power Corrections for Event Shapes: what next?

- So far, we have seen FO calculation of NLO Next to Leading Power (NLP) term

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = & \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(0)} \left(\frac{\log^m \tau}{\tau}\right)_+ && \text{Leading Power (LP)} \\
& + \left(\frac{\alpha_s}{4\pi}\right) (a_1 \log \tau + a_0) && \text{NLO NLP} \\
& + \left(\frac{\alpha_s}{4\pi}\right)^2 (a_3 \log^3 \tau + a_2 \log^2 \tau + \dots) && \text{NNLO NLP} \\
& + \left(\frac{\alpha_s}{4\pi}\right)^3 (a_5 \log^5 \tau + a_4 \log^4 \tau + \dots) && \text{N}^3\text{LO NLP} \\
& + \quad \dots && \vdots
\end{aligned}
$$

- Can we predict these logs using resummation techniques at subleading powers?
- Let's start with the LL series

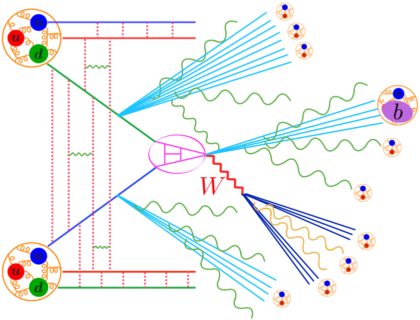
Leading Log Resummation at Next-to-Leading Power for Thrust in $H \to gg$



(Moult, Stewart, GV, Zhu) [1804.04665]

SCET [Bauer, Fleming, Pirjol, Stewart]

- SCET describes soft and collinear radiation in the presence of a hard scattering.



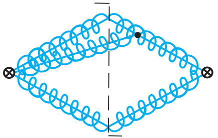
- Allows for a factorized description: Hard, Jet, Beam, Soft functions

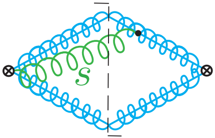
$$\frac{d\sigma}{d\mathcal{M}_1\cdots} = \sum_{\{\kappa\}} \mathrm{tr} H_\kappa \mathcal{I}\mathcal{I} J_{\kappa_i} \otimes \cdots \otimes J_{\kappa_j} S_{\kappa_s} \otimes f_{p/i} f_{p/j} \otimes f_{k\to H} \otimes \cdots \otimes f_{l\to H} \otimes F$$

Fixed Order Calculation

[Freedman], [Moult, Rothen, Stewart, Tackmann, Zhu]

- Compute power corrections for Higgs thrust ($H \to gg$) at lowest order



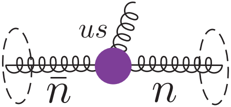


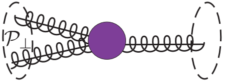
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}}{\mathrm{d}\tau} = 8C_A\left(\frac{\alpha_s}{4\pi}\right)\left[\left(\frac{1}{\epsilon}+\log\frac{\mu^2}{Q^2\tau}\right)-\left(\frac{1}{\epsilon}+\log\frac{\mu^2}{Q^2\tau^2}\right)\right]\theta(\tau)+\mathcal{O}(\alpha_s^2)$$
$$= 8C_A\left(\frac{\alpha_s}{4\pi}\right)\log\tau\ \theta(\tau)+\mathcal{O}(\alpha_s^2)$$

- No virtual corrections at lowest order ($\delta(\tau) \sim 1/\tau$).
- Divergences cancel between soft and collinear.
- Log appears at first non-vanishing order:
  - At LP, $\log(\tau)/\tau$ arises from RG evolution of $\delta(\tau)$
  - At NLP $\log(\tau)$ arises from RG evolution of "nothing"?

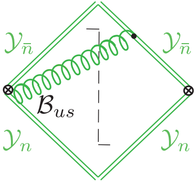
- Analogously to what we have seen at FO, power corrections arise from two distinct sources:
    - Power corrections to scattering amplitudes.
    - Power corrections to kinematics.

- Power corrections to scattering amplitudes can be computed from subleading SCET operators [Moult, Stewart, GV]



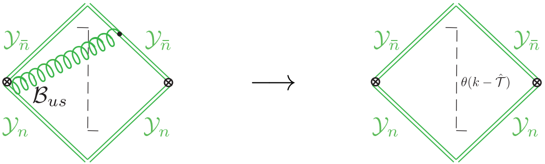


- They give rise to new jet and soft functions, whose renormalization was not previously known



Renormalization of subleading soft functions

[Moult, Stewart, Vita, Zhu]



- The subleading jet and soft functions satisfy a 2×2 mixing RG

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{pmatrix} \tilde{S}^{(2)}_{g,\mathcal{B}_{us}}(y,\mu) \\ \tilde{S}^{(2)}_{g,\theta}(y,\mu) \end{pmatrix} = \begin{pmatrix} \gamma_{11}(y,\mu) & \gamma_{12} \\ 0 & \gamma_{22}(y,\mu) \end{pmatrix} \begin{pmatrix} \tilde{S}^{(2)}_{g,\mathcal{B}_{us}}(y,\mu) \\ \tilde{S}^{(2)}_{g,\theta}(y,\mu) \end{pmatrix}$$

- Solving this equation to renormalize the operators, and resum subleading power logarithms.

$$S^{(2)}_{g,\theta}(\tau,\mu) = \frac{1}{(N_c^2-1)} \mathrm{tr}\langle 0|\mathcal{Y}^T_{\bar{n}}(0)\mathcal{Y}_n(0)\theta(\tau-\hat{\tau})\mathcal{Y}^T_n(0)\mathcal{Y}_{\bar{n}}(0)|0\rangle$$

  - They are power suppressed due to $\theta(\tau) \sim 1$ instead of $\delta(\tau) \sim 1/\tau$.
  - We find this type of mixing is a generic behavior at subleading power. (see also S.Jaskiewicz's talk)

Resumed Soft Function

- We find the final result for the renormalized subleading power soft function:

$$S^{(2)}_{g,\mathcal{B}_{us}}(Q\tau,\mu) = \theta(\tau)\gamma_{12}\log\left(\frac{\mu}{Q\tau}\right)e^{\frac{1}{2}\gamma_{11}\log^2\left(\frac{\mu}{Q\tau}\right)}$$

- Expanded perturbatively, we see a simple series:

$$S^{(2)}_{g,\mathcal{B}_{us}}(Q\tau,\mu) = \theta(\tau)\left[\gamma_{12}\log\left(\frac{\mu}{Q\tau}\right) + \frac{1}{2}\gamma_{12}\gamma_{11}\log^3\left(\frac{\mu}{Q\tau}\right) + \cdots\right]$$

- In particular, we find
  - First log generated by **mixing** with the θ function operators.
  - The single log is then dressed by Sudakov double logs from the diagonal anomalous dimensions.

- Example also useful for understanding power suppressed RG consistency.

- Complete result given by sum of two contributions.

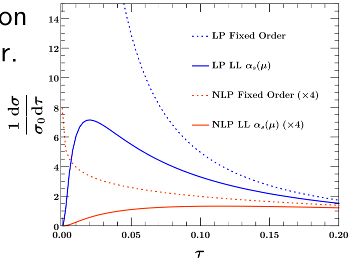
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{LL}}}{\mathrm{d}\tau} = \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{kin,LL}}}{\mathrm{d}\tau} + \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{hard,LL}}}{\mathrm{d}\tau}$$

- Both have same Sudakov $\implies$ can be directly added.
- Obtain the LL resummed result for pure glue $H \to gg$ thrust

$$\boxed{\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{LL}}}{\mathrm{d}\tau} = \left(\frac{\alpha_s}{4\pi}\right) 8 C_A \log(\tau) e^{-\frac{\alpha_s}{4\pi}\Gamma^g_{\mathrm{cusp}}\log^2(\tau)}}$$

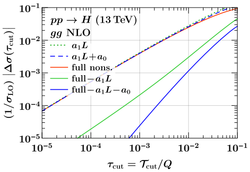
checked with FO calculation up to $\mathcal{O}(\alpha_s^3)$

- Provides the first all orders resummation for an event shape at subleading power.
- Very simple result. Subleading power LL driven by cusp anomalous dimension!



Conclusions

- Computed $\mathcal{O}(\alpha_s)$ **power correction** of differential cross section for color singlet production including LL and NLL



- Cross section level **renormalization at subleading power** involves a new RG structure involving **mixing** in crucial way.

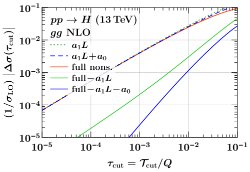
$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{pmatrix} \tilde{S}^{(2)}_{g,\mathcal{B}_{us}}(y,\mu) \\ \tilde{S}^{(2)}_{g,\theta}(y,\mu) \end{pmatrix} = \begin{pmatrix} \gamma_{11}(y,\mu) & \gamma_{12} \\ 0 & \gamma_{22}(y,\mu) \end{pmatrix} \begin{pmatrix} \tilde{S}^{(2)}_{g,\mathcal{B}_{us}}(y,\mu) \\ \tilde{S}^{(2)}_{g,\theta}(y,\mu) \end{pmatrix}$$

- Achieved **first** all orders resummation at subleading power for an event shape observable.

$$\boxed{\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\mathrm{LL}}}{\mathrm{d}\tau} = \left(\frac{\alpha_s}{4\pi}\right) 8 C_A \log(\tau) e^{-\frac{\alpha_s}{4\pi}\Gamma^g_{\mathrm{cusp}}\log^2(\tau)}}$$

Conclusions

- Computed $\mathcal{O}(\alpha_s)$ **power correction** of differential cross section for color singlet production including LL and NLL



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- Achieved **first** all orders resummation at subleading power for an event shape observable.

$$\boxed{\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(2)}_{\rm LL}}{\mathrm{d}\tau} = \left(\frac{\alpha_s}{4\pi}\right) 8 C_A \log(\tau) e^{-\frac{\alpha_s}{4\pi}\Gamma^g_{\rm cusp}\log^2(\tau)}}$$

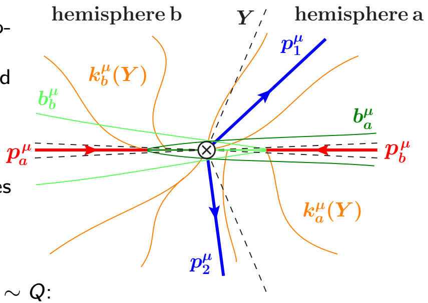
THANK YOU!

Backup slides

Power corrections at FO: PDF expansion

- Need to keep track of $\mathcal{O}(\mathcal{T})$ component of momenta: both for phase space expansion and mandelstams entering $|\mathcal{M}|^2$.

- Solving Q and Y measurements uniquely fixes how factors of $\mathcal{T}$ enters the PDFs.



Example n-collinear emission, $k^+ \sim \mathcal{T}$, $k^- \sim Q$:

$$p_a^\mu = Qe^Y\left[\left(1+\frac{k^- e^{-Y}}{Q}\right)+\frac{\mathcal{T}}{Q}\frac{k^-}{2Q}+\mathcal{O}\left(\frac{\mathcal{T}^2}{Q^2}\right)\right]\frac{n^\mu}{2}$$

$$p_b^\mu = Qe^{-Y}\left[1+\frac{\mathcal{T}}{Q}\left(e^Y+\frac{k^-}{2Q}\right)+\mathcal{O}\left(\frac{\mathcal{T}^2}{Q^2}\right)\right]\frac{\bar{n}^\mu}{2}$$

$$n^\mu = (1,0,0,1)$$
$$\bar{n}^\mu = (1,0,0,-1)$$

- At subleading power **both** PDF momenta contain power corrections **regardless** of the direction of the emission $\Longrightarrow$ derivative of both PDFs

$\mathcal{T}$ power corrections from residual momenta in PDFs for an n-collinear emission:

$$f_a\left(\frac{p_a}{E_{cm}}\right) \sim f_a\left(\frac{x_a}{z_a}+\frac{\mathcal{T}}{Q}\Delta_a\right) = f_a\left(\frac{x_a}{z_a}\right)+\frac{\mathcal{T}}{Q}\Delta_a f_a'\left(\frac{x_a}{z_a}\right)$$

$$f_b\left(\frac{p_b}{E_{cm}}\right) \sim f_b\left(x_b+\frac{\mathcal{T}}{Q}\Delta_b\right) = f_b\left(x_b\right)+\frac{\mathcal{T}}{Q}\Delta_b f_b'\left(x_b\right)$$

Power corrections at FO: Master formulae

- Expansion of **phase space** and **matrix element squared** in **soft** and **collinear** limits has a general (universal) structure

n-Collinear Master Formula for 0-Jettiness power corrections

$$\frac{\mathrm{d}\sigma_n^{(2)}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\mathcal{T}} \sim \int_{x_a}^1 \frac{\mathrm{d}z_a}{z_a}\frac{z_a^\epsilon}{(1-z_a)^\epsilon}\left(\frac{Q\mathcal{T}e^Y}{\rho}\right)^{-\epsilon}\Bigg\{f_a f_b\, A^{(2)}(Q,Y,z_a) + \frac{e^Y}{\rho}A^{(0)}\frac{\mathcal{T}}{Q}\left[f_a f_b\frac{(1-z_a)^2-2}{2z_a} + x_a\frac{1-z_a}{2z_a}f_a' f_b + x_b\frac{1+z_a}{2z_a}f_a f_b'\right]\Bigg\}$$

Soft Master Formula for 0-Jettiness power corrections

$$\frac{\mathrm{d}\sigma_s^{(2)}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\mathcal{T}} \sim \frac{1}{\epsilon}\frac{\mathcal{T}^{-2\epsilon}}{Q}\Bigg\{\bar{A}^{(0)}(Q,Y)\left[f_a f_b\left(-\frac{\rho}{e^Y}-\frac{e^Y}{\rho}\right) + x_a\frac{\rho}{e^Y}f_a' f_b + x_b\frac{e^Y}{\rho}f_a f_b'\right] + f_a f_b\left[\rho Q\,\bar{A}_+^{(2)}(Q,Y) + \frac{Q}{\rho}\,\bar{A}_-^{(2)}(Q,Y)\right]\Bigg\}$$

Power corrections at FO: Cross section results

- Combining **soft** and **collinear** kernels, $\frac{1}{\epsilon}$ poles cancel (consistency check) and the differential cross section takes the form:

$$\frac{\mathrm{d}\sigma^{(2,n)}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\mathcal{T}} = \hat{\sigma}^{\mathrm{LO}}\left(\frac{\alpha_s}{4\pi}\right)^n \int_{x_a}^1 \int_{x_b}^1 \frac{\mathrm{d}z_a}{z_a}\frac{\mathrm{d}z_b}{z_b}\left[f_i f_j C^{(2,n)}_{f_i f_j}(z_a, z_b, \mathcal{T}) + \frac{x_a}{z_a} f_i' f_j C^{(2,n)}_{f_i' f_j}(z_a, z_b, \mathcal{T}) + \frac{x_b}{z_b} f_i f_j' C^{(2,n)}_{f_i f_j'}(z_a, z_b, \mathcal{T})\right]$$

- Example for gg channel in H production:

$$C^{(2,1)}_{f_g' f_g}(z_a, z_b, \mathcal{T}) = 4C_A \frac{\rho}{Qe^Y}\,\delta(1-z_a)\left[\left(-\ln\frac{\mathcal{T}e^Y}{Q\rho} - 1\right)\delta(1-z_b) + \frac{(1+z_b)(1-z_b+z_b^2)^2}{2z_b^2}\,\mathcal{L}_0(1-z_b)\right]$$
$$+\, 4C_A \frac{e^Y}{Q\rho}\,\frac{(1-z_a+z_a^2)^2}{2z_a}\,\delta(1-z_b)$$

- Extension to NNLO has been computed for the LL term
[Moult, Rothen, Stewart, Tackmann, Zhu], [Boughezal, Liu, Petriello]

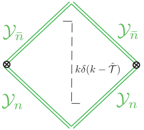
[Moult, Stewart, Vita, Zhu]

- Consider the power suppressed soft function:

$$S^{(2)}_{g,\tau\delta}(\tau,\mu) = \frac{1}{(N_c^2-1)} \mathrm{tr}\langle 0|\mathcal{Y}^T_{\bar{n}}(0)\mathcal{Y}_n(0)\,\tau\;\delta(\tau-\hat{\tau})\mathcal{Y}^T_n(0)\mathcal{Y}_{\bar{n}}(0)|0\rangle$$

- This soft function vanishes at lowest order

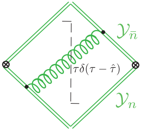
$$S^{(2)}_{g,\tau\delta}(\tau,\mu)\Big|_{\mathcal{O}(\alpha_s^0)} =$$



$$= \tau\delta(\tau) = 0$$

- It has a UV divergence at the first order

$$S^{(2)}_{g,\tau\delta}(\tau,\mu)\Big|_{\mathcal{O}(\alpha_s)} = 2$$



$$= g^2\theta(\tau)\left(\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{(Q\tau)^2}\right) + \mathcal{O}(\epsilon)\right)$$

- What renormalizes this function?

An Important Illustrative Example

[Moult, Stewart, Vita, Zhu]

- Consider the power suppressed soft function:

$$S^{(2)}_{g,\tau\delta}(\tau,\mu) = \frac{1}{(N_c^2-1)}\mathrm{tr}\langle 0|\mathcal{Y}^T_{\bar n}(0)\mathcal{Y}_n(0)\,\tau\;\delta(\tau-\hat\tau)\mathcal{Y}^T_n(0)\mathcal{Y}_{\bar n}(0)|0\rangle$$

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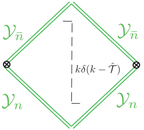

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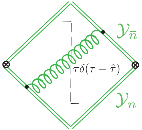


$$= g^2\theta(\tau)\left(\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{(Q\tau)^2}\right) + \mathcal{O}(\epsilon)\right)$$

- What renormalizes this function?

$\Longrightarrow$ Mixing with another operator!

An Important Illustrative Example

[Moult, Stewart, Vita, Zhu]

- We can use a simple trick to find the missing operator.
- The RG for the leading power soft function is known:

$$\mu \frac{dS^{(0)}_{g,\delta}(\tau,\mu)}{d\mu} = \int d\tau' \, 2\Gamma^g_{\rm cusp} \left(2\left[\frac{\theta(\tau-\tau')}{\tau-\tau'}\right]_+ - \log\left(\frac{\mu^2}{Q^2}\right)\delta(\tau-\tau')\right) S^{(0)}_{g,\delta}(\tau',\mu)$$

- Multiplying by τ, we find

$$\mu \frac{d}{d\mu}\tau S^{(0)}_{g,\delta}(\tau,\mu) = \int d\tau'((\tau-\tau')+\tau') \, 2\Gamma^g_{\rm cusp} \left(2\left[\frac{\theta(\tau-\tau')}{\tau-\tau'}\right]_+ - \log\left(\frac{\mu^2}{Q^2}\right)\delta(\tau-\tau')\right) S^{(0)}_{g,\delta}(\tau',\mu)$$

- Simplifying, we have

$$\mu \frac{d}{d\mu}\tau S^{(0)}_{g,\delta}(\tau,\mu) = \int d\tau' \, 4\Gamma^g_{\rm cusp}\theta(\tau-\tau') S^{(0)}_{g,\delta}(\tau',\mu) + \int d\tau' \gamma^S_g(\tau-\tau')\tau' S^{(0)}_{g,\delta}(\tau',\mu)$$

- Performing the integral, we have

$$\mu \frac{d}{d\mu}\tau S^{(0)}_{g,\delta}(\tau,\mu) = 4\Gamma^g_{\rm cusp} \, S^{(2)}_{g,\theta}(\tau,\mu) + \int d\tau' \gamma^S_g(\tau-\tau',\mu)\tau' S^{(0)}_{g,\delta}(\tau',\mu)$$

- Here we have defined a new power suppressed soft function

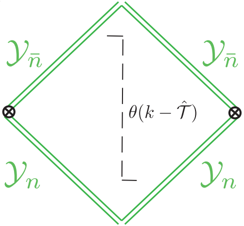
$$S^{(2)}_{g,\theta}(\tau,\mu) = \frac{1}{(N_c^2-1)} {\rm tr}\langle 0|\mathcal{Y}^T_{\bar n}(0)\mathcal{Y}_n(0)\theta(\tau-\hat\tau)\mathcal{Y}^T_n(0)\mathcal{Y}_{\bar n}(0)|0\rangle$$

θ-Function Operators

- At subleading power we require θ-jet and θ-soft functions

$$J^{(2)}_{\mathcal{B}_n,\theta}(\tau,\mu) = \frac{(2\pi)^3}{(N_c^2-1)} \mathrm{tr}\Big\langle 0 \Big| \mathcal{B}^{\mu a}_{n\perp}(0)\,\delta(Q+\bar{\mathcal{P}})\delta^2(\mathcal{P}_\perp)\,\theta(\tau-\hat{\tau})\,\mathcal{B}^{\mu a}_{n\perp,\omega}(0)\Big|0\Big\rangle$$

$$S^{(2)}_{g,\theta}(\tau,\mu) = \frac{1}{(N_c^2-1)} \mathrm{tr}\langle 0|\mathcal{Y}^T_{\bar{n}}(0)\mathcal{Y}_n(0)\theta(\tau-\hat{\tau})\mathcal{Y}^T_n(0)\mathcal{Y}_{\bar{n}}(0)|0\rangle$$



- They are power suppressed due to $\theta(\tau)\sim 1$ instead of $\delta(\tau)\sim 1/\tau$.

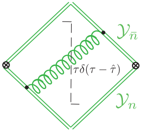
- Arise only through mixing at cross section level.

- We find this type of mixing is a generic behavior at subleading power.

Perturbative View

- Returning to our perturbative calculation of the subleading power soft function

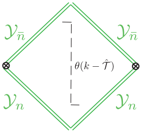
$$S^{(2)}_{g,\tau\delta}(\tau,\mu)\Big|_{\mathcal{O}(\alpha_s)} = 2$$



$$= g^2\theta(\tau)\left(\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{(Q\tau)^2}\right) + \mathcal{O}(\epsilon)\right)$$

- UV divergence now easily understood as mixing with θ function operator, which is non-vanishing at lowest order

$$S^{(2)}_{g,\theta}(\tau,\mu)\Big|_{\mathcal{O}(\alpha_s^0)} =$$

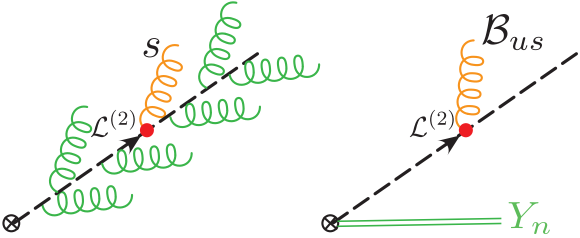


$$= \theta(\tau)$$

- Similar θ function counterterm observed by Paz in subleading power jet function at one-loop. Our example enables us to prove their all orders structure.

Gauge Invariant Ultrasoft Fields

- At subleading power, explicit ultrasoft fields appear.
- Wilson lines from field redefinition can be arranged into gauge invariant "gluon" operators plus Wilson lines (analogous to $\mathcal{B}_{\perp n}$ at leading power).

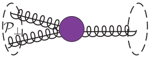


$$Y_{n_i}^{(r)\,\dagger} iD_{us}^{(r)\,\mu} Y_{n_i}^{(r)} = i\partial_{us}^{\mu} + [Y_{n_i}^{(r)\,\dagger} iD_{us}^{(r)\,\mu} Y_{n_i}^{(r)}] = i\partial_{us}^{\mu} + T_{(r)}^{a} g \mathcal{B}_{us(i)}^{a\mu}$$

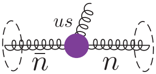
- Provides gauge invariant description of soft sector at subleading power.

[Moult, Stewart, Vita]

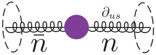
- Matrix element corrections arise from operators involving an additional $\mathcal{B}_{n\perp}$, $\mathcal{B}_{us}$ or ∂_{us}.
- We have performed an explicit matching to the required operators



$$\mathcal{O}^{(2)}_{\mathcal{P}\mathcal{B}1} = C^{(2)}_{\mathcal{P}\mathcal{B}1} if^{abc} \mathcal{B}^{a}_{n\perp,\omega_1} \cdot \left[\mathcal{P}_\perp \mathcal{B}^{b}_{\bar{n}\perp,\omega_2} \cdot\right] \mathcal{B}^{c}_{\bar{n}\perp,\omega_3} H\,,$$
$$\mathcal{O}^{(2)}_{\mathcal{P}\mathcal{B}2} = C^{(2)}_{\mathcal{P}\mathcal{B}2} if^{abc} \left[\mathcal{P}_\perp \cdot \mathcal{B}^{a}_{\bar{n}\perp,\omega_3}\right] \mathcal{B}^{b}_{n\perp,\omega_1} \cdot \mathcal{B}^{c}_{\perp\bar{n},\omega_2} H$$



$$\mathcal{O}^{(2)}_{\mathcal{B}(us(n))} = C^{(2)}_{\mathcal{B}(us(n))} \left(if^{abd} \left(\mathcal{Y}_n^T \mathcal{Y}_{\bar{n}}\right)^{dc}\right) \left(\mathcal{B}^{a}_{n\perp,\omega_1} \cdot \mathcal{B}^{b}_{\bar{n}\perp,\omega_2} \bar{n} \cdot g\mathcal{B}^{c}_{us(n)}\right)\,,$$
$$\mathcal{O}^{(2)}_{\mathcal{B}(us(\bar{n}))} = C^{(2)}_{\mathcal{B}(us(\bar{n}))} \left(if^{abd} \left(\mathcal{Y}_{\bar{n}}^T \mathcal{Y}_{n}\right)^{dc}\right) \left(\mathcal{B}^{a}_{n\perp,\omega_1} \cdot \mathcal{B}^{b}_{\bar{n}\perp,\omega_2} n \cdot g\mathcal{B}^{c}_{us(\bar{n})}\right)$$



$$\mathcal{O}^{(2)}_{\partial\mathcal{B}(us)(0)} = C^{(2)}_{n\cdot\partial} \mathcal{B}^{\mu a}_{\perp n,\omega_1}\, in\cdot\partial \mathcal{B}^{\mu b}_{\perp\bar{n},\omega_2} \left(\mathcal{Y}_{\bar{n}}^T \mathcal{Y}_n\right)^{ab} H\,,$$
$$\mathcal{O}^{(2)}_{\partial\mathcal{B}(us)(\bar{0})} = C^{(2)}_{\bar{n}\cdot\partial} \mathcal{B}^{\mu a}_{\perp\bar{n},\omega_2}\, i\bar{n}\cdot\partial \mathcal{B}^{\mu b}_{\perp n,\omega_1} \left(\mathcal{Y}_{\bar{n}}^T \mathcal{Y}_n\right)^{ab} H$$

- Wilson coefficients of soft operators are fixed to all orders using RPI:

$$C^{(2)}_{\mathcal{B}(us(n))} = -\frac{\partial C^{(0)}}{\partial\omega_1}$$

Factorization for Matrix Element Corrections

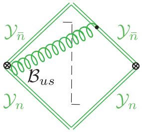
- By RG consistency, it is sufficient to consider the power suppressed soft function, involving a ∂_{us} or $\mathcal{B}_{us}$

$$\frac{1}{N_c}\mathrm{tr}\langle 0|\mathcal{Y}_{\bar{n}}^T(x)\mathcal{Y}_n(x)\bar{n}\cdot\mathcal{B}_{us(n)}(x)\delta(\tau_{us}-\hat{\tau}_{us})\mathcal{Y}_n^T(0)\mathcal{Y}_{\bar{n}}(0)|0\rangle = \int\frac{d^4r}{(2\pi)^4}e^{-ir\cdot x}S^{(2)}_{n\mathcal{B}_{us}}(\tau_{us},r)$$

which appears in the factorization as

$$\frac{d\sigma^{(2)}_{\mathcal{B}_{us},n}}{d\tau} = H_{\bar{n}\cdot\mathcal{B}}(Q^2)\int d\tau_n d\tau_{\bar{n}} d\tau_{us}\delta(\tau-\tau_n-\tau_{\bar{n}}-\tau_{us})$$
$$\cdot\left[\int\frac{d^4r}{(2\pi)^4}S^{(2)}_{n\mathcal{B}_{us}}(\tau_{us},r)\right]\cdot\left[\int\frac{dk^-}{2\pi}\mathcal{J}_{\bar{n}}(\tau_{\bar{n}},k^-)\right]\cdot\left[\int\frac{dl^+}{2\pi}\mathcal{J}_n(\tau_n,l^+)\right]$$

- These operators mix with a θ function soft function just as with the 'illustrative' example considered above. Resummation is identical.



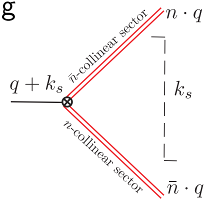
$$= \frac{\gamma_{n\mathcal{B}_{us}\to\theta}}{\epsilon}\theta(\tau)$$

$$\mu\frac{d}{d\mu}\begin{pmatrix} S_{n\mathcal{B}_{us}}(\tau,\mu) \\ S_{g,\theta}(\tau,\mu)\end{pmatrix} = \int d\tau'\begin{pmatrix}\gamma^S_{g,\delta}(\tau-\tau',\mu) & \gamma_{n\mathcal{B}_{us}\to\theta}\delta(\tau-\tau') \\ 0 & \gamma^S_{g,\delta}(\tau-\tau',\mu)\end{pmatrix}\begin{pmatrix} S_{n\mathcal{B}_{us}}(\tau',\mu) \\ S_{g,\theta}(\tau',\mu)\end{pmatrix}$$

Kinematic Corrections

- Kinematic corrections arise from
  - Phase space
  - Thrust observable definition (does not contribute at LL)

- Phase space corrections can be treated through choice of routing



$$\frac{1}{(Q+k_s)^2} = \frac{1}{Q^2} - \frac{n \cdot k_s}{Q^3} - \frac{\bar{n} \cdot k_s}{Q^3} + \mathcal{O}(\tau^2)$$

$$S^{(2)}_{g,\tau\delta}(\tau,\mu) = \frac{1}{(N_c^2-1)} \mathrm{tr}\langle 0|\mathcal{Y}^T_{\bar{n}}(0)\mathcal{Y}_n(0)\tau\ \delta(\tau-\hat{\tau})\mathcal{Y}^T_n(0)\mathcal{Y}_{\bar{n}}(0)|0\rangle$$

- Are described by the ‘illustrative’ example considered above

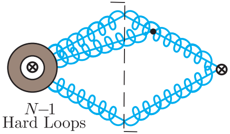
$$\mu\frac{d}{d\mu}\begin{pmatrix} S^{(2)}_{g,\tau\delta}(\tau,\mu) \\ S^{(2)}_{g,\theta}(\tau,\mu) \end{pmatrix} = \int d\tau' \begin{pmatrix} \gamma^S_{g,\tau\delta\to\tau\delta}(\tau-\tau',\mu) & \gamma^S_{g,\tau\delta\to\theta}\delta(\tau-\tau') \\ 0 & \gamma^S_{g,\theta\to\theta}(\tau-\tau',\mu) \end{pmatrix} \begin{pmatrix} S^{(2)}_{g,\tau\delta}(\tau',\mu) \\ S^{(2)}_{g,\theta}(\tau',\mu) \end{pmatrix}$$

Fixed Order Check

- We can explicitly check this result by **fixed order** calculation of the power corrections.

- **RG consistency** for $1/\epsilon$ poles implies that the LL power correction can be computed only from **hard-collinear** contributions:

[Moult, Rothen, Stewart, Tackmann, Zhu]



- Expanding known results for $H \to 3$ partons at NNLO [Gehrmann et al.], we can analytically compute the power corrections to $\mathcal{O}(\alpha_s^3)$:

$$\frac{1}{\sigma_0^H}\frac{\mathrm{d}\sigma^H}{\mathrm{d}\tau} = \frac{\alpha_s}{4\pi} 8 C_A \log\tau - \left(\frac{\alpha_s}{4\pi}\right)^2 32 C_A^2 \log^3\tau + \left(\frac{\alpha_s}{4\pi}\right)^3 64 C_A^3 \log^5\tau + \mathcal{O}(\alpha_s^4)$$

- Provides a highly **non-trivial check** on the correctness of our all orders resummation.