Energetic γ-rays from TeV scale dark matter resummed

Caspar Hasner

DESY Theory Workshop

September 27, 2018

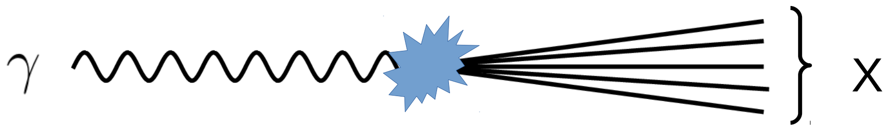
Based on:
Beneke, Broggio, Hasner, Vollmann
arXiv:1805.07367

Outline

- Introduction
- Factorization formula
- Ingredients
- Results
- Conclusion

Introduction

Consider dark matter (DM) annihilation process $\chi(p_1) + \chi(p_2) \longrightarrow \gamma(p_\gamma) + X(p_X)$



DM mass lies in the range 100 GeV < m_χ < 10 TeV.

In these processes, two effects need to be taken into account:

- DM can exchange EW gauge bosons prior to annihilation which modify the cross section. Diagrammatically, this corresponds to ladder diagrams.
  $\longrightarrow$ **Sommerfeld enhancement**

- Large logarithmically enhanced quantum corrections arise due to hierarchy of scales.
  $\longrightarrow$ **Sudakov logarithms**

Naive computation leads to a breakdown of perturbation theory
$\longrightarrow$ **Resummation** is needed

[Hryczuk,Iengo `12],[Bauer,Cohen,Hill,Solon `14],[Ovanesyan,Slatyer,Stewart `14],[Baumgart,Rothstein,Vaidya `15], [Baumgart,Vaidya `15],[Ovanesyan,Rodd,Slatyer,Stewart `15],[Baumgart,Cohen,Moult,Rodd,Slatyer,Solon,Stewart,Vaidya `17]

Non-relativistic initial state particles and energetic small invariant mass objects are described by a combination of non-relativistic and soft collinear effective theory.

Introduction

Add to SM Lagrangian a Majorana multiplet of the EW SU(2) gauge group and zero hypercharge (Y=0)

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\overline{\chi}\left(i\not{D} - m_\chi\right)\chi\,, \quad D_\mu = \partial - ig_2 A^C_\mu T^C$$

The DM particle is the electrically neutral element of the multiplet.

We assume that the energy resolution is parametrically of order $E^\gamma_{\rm res} \sim m_W^2/m_\chi$
and due to $m_X^2 = (p_1 + p_2 - p_\gamma)^2 = 4m_\chi(m_\chi - E_\gamma) = 4m_\chi E^\gamma_{\rm res} \ll m_\chi^2$
this implies the scale hierarchy $E^\gamma_{\rm res} \ll m_W,\, m_X \ll m_\chi$

Resummation requires separation of multi-scale Feynman diagrams into single-scale contributions. This is done by introducing the momentum modes

$$\begin{aligned}
\text{hard}\,(h):&\quad k^\mu \sim m_\chi(1,1,1)\\
\text{collinear}\,(c):&\quad k^\mu \sim m_\chi(1,\lambda^2,\lambda)\\
\text{anti-collinear}\,(\bar{c}):&\quad k^\mu \sim m_\chi(\lambda^2,1,\lambda)\\
\text{soft}\,(s):&\quad k^\mu \sim m_\chi(\lambda,\lambda,\lambda)\\
\text{potential}\,(p):&\quad k^0 \sim m_W^2/m_\chi,\ \mathbf{k} \sim m_W\\
\text{ultrasoft}\,(s):&\quad k^\mu \sim m_\chi(\lambda^2,\lambda^2,\lambda^2)
\end{aligned}$$

SCET-II scenario
→ rapidity regulator needed

$$\lambda = \sqrt{E^\gamma_{\rm res}/m_\chi}$$
$$k^\mu \sim (n_+\cdot k, n_-\cdot k, k_\perp)$$
$$p^\mu_\gamma = E_\gamma n^\mu_+$$
$$n_+\cdot n_- = 2$$

Factorization formula

Process is described by operators for S-wave annihilation of DM particles into EW gauge bosons. Integrate out hard modes into the coefficient functions C → collinear, anti-collinear and potential fields can no longer interact directly.

The collinear modes build up the unobserved final state X, anti-collinear modes result in a single observed photon. The non-relativistic DM particles are described by potential fields exchanging potential EW gauge bosons (Sommerfeld effect).

Soft attachments factorize from the ladder diagrams since a soft momentum throws the potential heavy particle off-shell, removing the enhancement of the ladder rungs between the soft attachment and the hard vertex. The Sommerfeld effect factorizes from the Sudakov annihilation rate

$$\frac{d(\sigma v_{\text{rel}})}{dE_\gamma} = \sum_{I,J} S_{IJ}\, \Gamma_{IJ}(E_\gamma)$$

I,J electrically neutral two-particle states

Sommerfeld factor

Sudakov resummed annihilation rate

$$S_{IJ} \sim \langle \chi^0 \chi^0 | \, [\chi\chi]_J \, |0\rangle\langle 0| \, [\chi\chi]_I \, |\chi^0 \chi^0\rangle$$

Factorization formula

The coupling of soft gauge fields to (anti-)collinear fields is removed via decoupling transformations. Since the small energy resolution forbids soft radiation, the soft function is a vacuum matrix element of Wilson lines, which can be regarded as a soft Wilson coefficient D.

C and D are hard and soft matching coefficients

V, W, X,Y are SU(2) adjoint indices

$$\Gamma_{IJ}(E_\gamma) = \frac{1}{4}\frac{2}{\pi m_\chi}\sum_{i,j=1,2}\sum_{V,W,X,Y} C_j^*(\mu_W)C_i(\mu_W)D^{j\,*}_{J,XY}(\mu_W,\nu_s)D^i_{I,VW}(\mu_W,\nu_s)$$

$$\times\, V(\mu_W,\nu_s,\nu_j)\, Z_\gamma^{YW}\int d\omega\, J^{XV}(4m_\chi(m_\chi - E_\gamma-\omega),\mu_W,\nu_j)\, S_\gamma(\omega)$$

Rapidity evolution factor, (anti-)collinear and soft modes have same virtuality

Anti-collinear factor for observed photon

Jet function of unobserved final state X, convoluted with us function

Ingredients: Hard + Soft coefficients

Hard coefficients

Effective annihilation Lagrangian and operator basis. $\frac{1}{2m_\chi}\sum_{i=1,2} C_i(\mu)\mathcal{O}_i$

$$\mathcal{O}_1 = \chi_v^{c\dagger}\Gamma^{\mu\nu}\chi_v\,\mathcal{A}^B_{\perp c,\mu}(sn_+)\mathcal{A}^B_{\perp\bar{c},\nu}(tn_-)$$

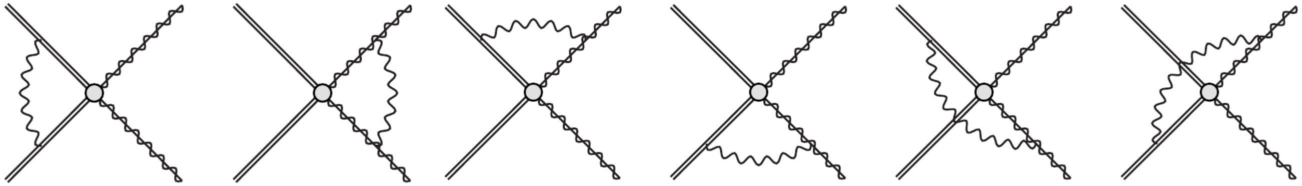
$$\mathcal{O}_2 = \frac{1}{2}\,\chi_v^{c\dagger}\Gamma^{\mu\nu}\{T^B,T^C\}\chi_v\,\mathcal{A}^B_{\perp c,\mu}(sn_+)\mathcal{A}^C_{\perp\bar{c},\nu}(tn_-)$$

C are computed in EW symmetric limit, i.e. gauge bosons are massless.

Evolution of hard matching coefficients is computed by solving the renormalization group equation.

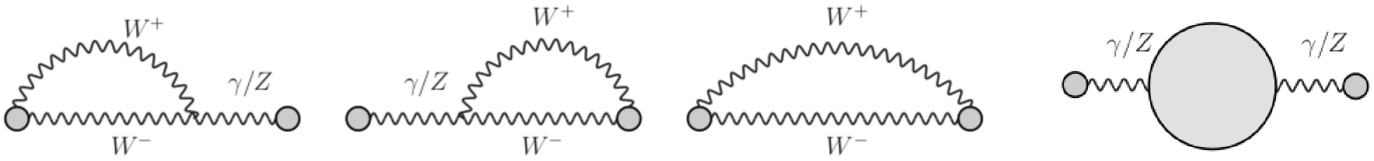
$$\frac{d}{d\log\mu}C(\mu) = \Gamma(\mu)C(\mu) \longrightarrow C(\mu_W) = \underbrace{\exp\left[\int_{\mu_h}^{\mu_W}\frac{\Gamma(\mu')}{\mu'}d\mu'\right]}_{U(\mu_h,\mu_W)}C(\mu_h)$$

Soft coefficients



- Soft coefficients $D^i_{I,VW} = K_{I,ab}\,\langle 0|[Y_v^\dagger T_i^{AB} Y_v]_{ab}\,Y_{n_-}^{AV}Y_{n_+}^{BW}|0\rangle$ (Y – Wilson lines from decoupl. transf.)
- Soft and jet functions have same virtuality and require an additional regulator [Chiu,Jain,Neill,Rothstein `12]
- Soft function lives at EW scale and must be computed with the Feynman rules of the SM after symmetry breaking (including gaube boson masses). Requiring an observed photon in the anti-collinear final state and electric charge conservation implies V = W = X = Y = 3.

Ingredients: Jet functions



Jet function of unobserved final state
is defined as the total discontinuity of the gauge boson two-point function

$$J^{BC}(p^2) = \frac{1}{\pi}\,\mathrm{Im}\big[\, i\mathcal{J}^{BC}(p^2)\big]$$

$$-\, g_{\mu\nu}\, \mathcal{J}^{BC}(p^2) \equiv \int d^4x\, e^{ip\cdot x} \langle 0|\mathbf{T}\big\{\mathcal{A}^B_{\perp\mu}(x)\, \mathcal{A}^C_{\perp\nu}(0)\big\}|0\rangle$$

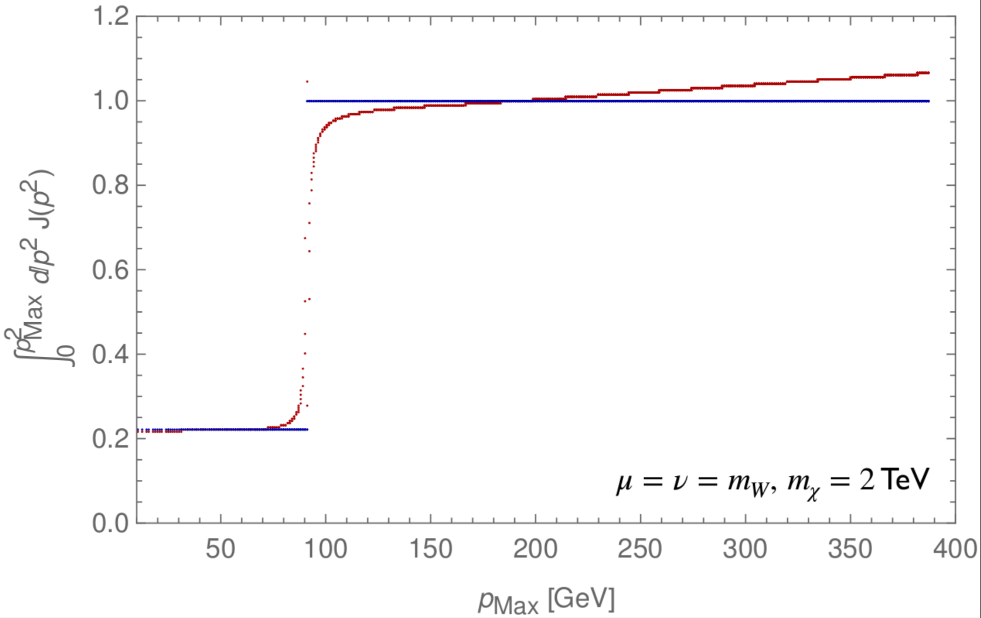
Photon jet function
is defined as the squared matrix element

$$-\, g_{\perp,\mu\nu}\, Z^{BC}_\gamma = \sum_\lambda \langle 0|\mathcal{A}^B_{\perp\mu}(0)|\gamma(p_\gamma,\lambda)\rangle\langle\gamma(p_\gamma,\lambda)|\mathcal{A}^C_{\perp\nu}(0)|0\rangle$$

$$g_2\mathcal{A}^B_{\perp\mu}T^B = W^\dagger_{\bar c}[iD_{\perp\mu}W_{\bar c}]$$

Integrated jet function on the invariant mass of the unobserved final state.

The integrated jet function jumps from a value around $s_W^2(m_W)$ to a value around 1 as the invariant mass passes through m_Z and then slowly increases. The range shown contains the WW, ZH, tt thresholds (barely visible).



Ingredients:
Sommerfeld + rapidity + us function

Rapidity evolution factor

- Rapidity logarithms can be resummed by solving the rapidity renormalization group equations at NLL’

$$\frac{d}{d\log\nu}D(\mu_W,\nu) = \gamma_\nu D(\mu_W,\nu)$$

$$D(\mu_W,\nu_j) = V(\mu_W,\nu_s,\nu_j)D(\mu_W,\nu_s)$$

Ultrasoft function

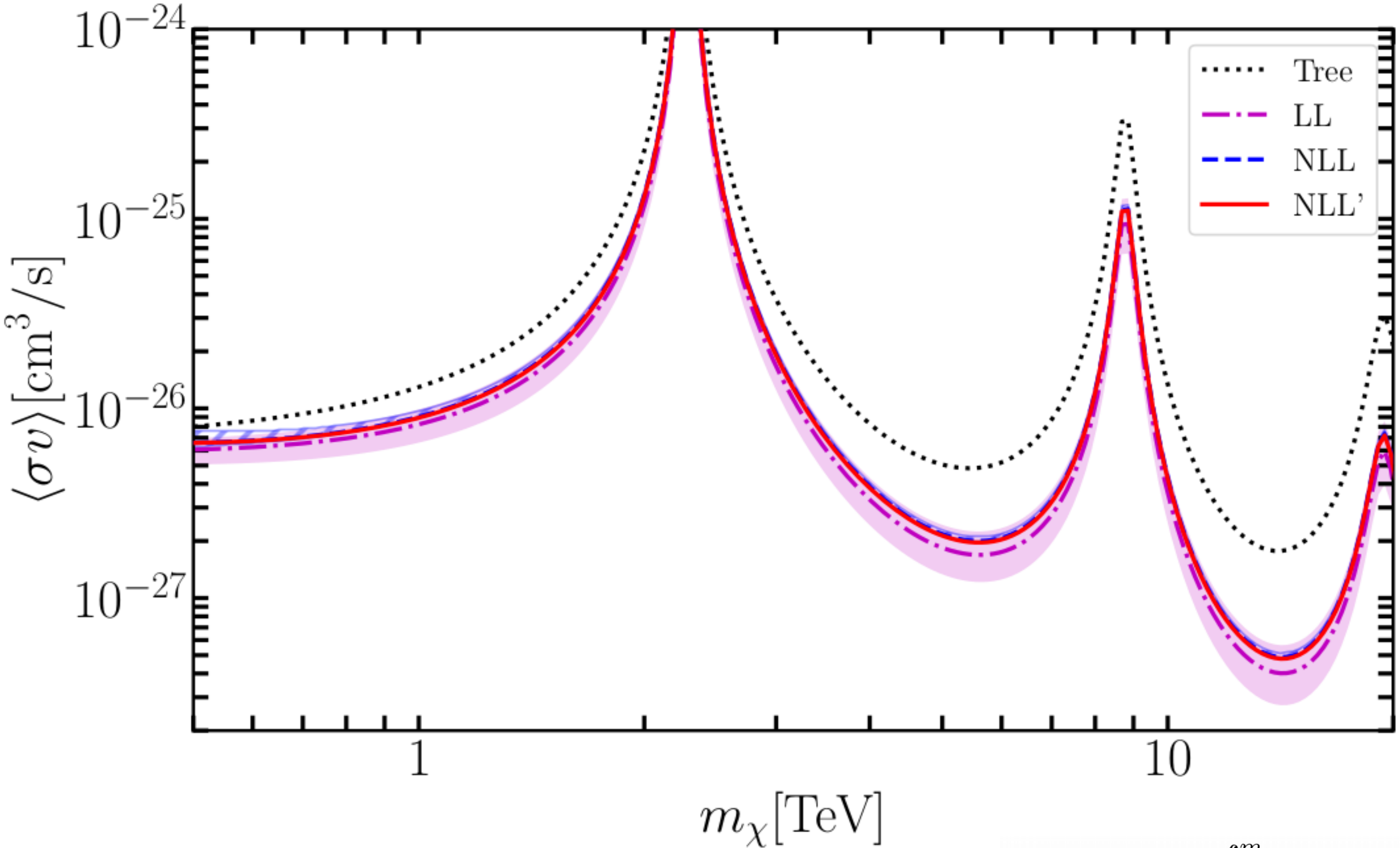
- Kinematics of process prohibit EW gauge boson radiation (photon/light fermion radiation still possible)
- Due to nature of annihilation process, us function turns out to be trivial S(ω) = δ(ω)

Sommerfeld factor

- Resum ladder diagrams by using an NREFT to describe the EW gauge boson exchange prior to the annihilation. For fermionic DM triplet, this was first computed by [Hisano,Matsumoto,Nojiri,Saito `14]
- This requires the computation of the wave function at the origin
- Can be achieved by solving the Schrödinger equation with the potential V(r)

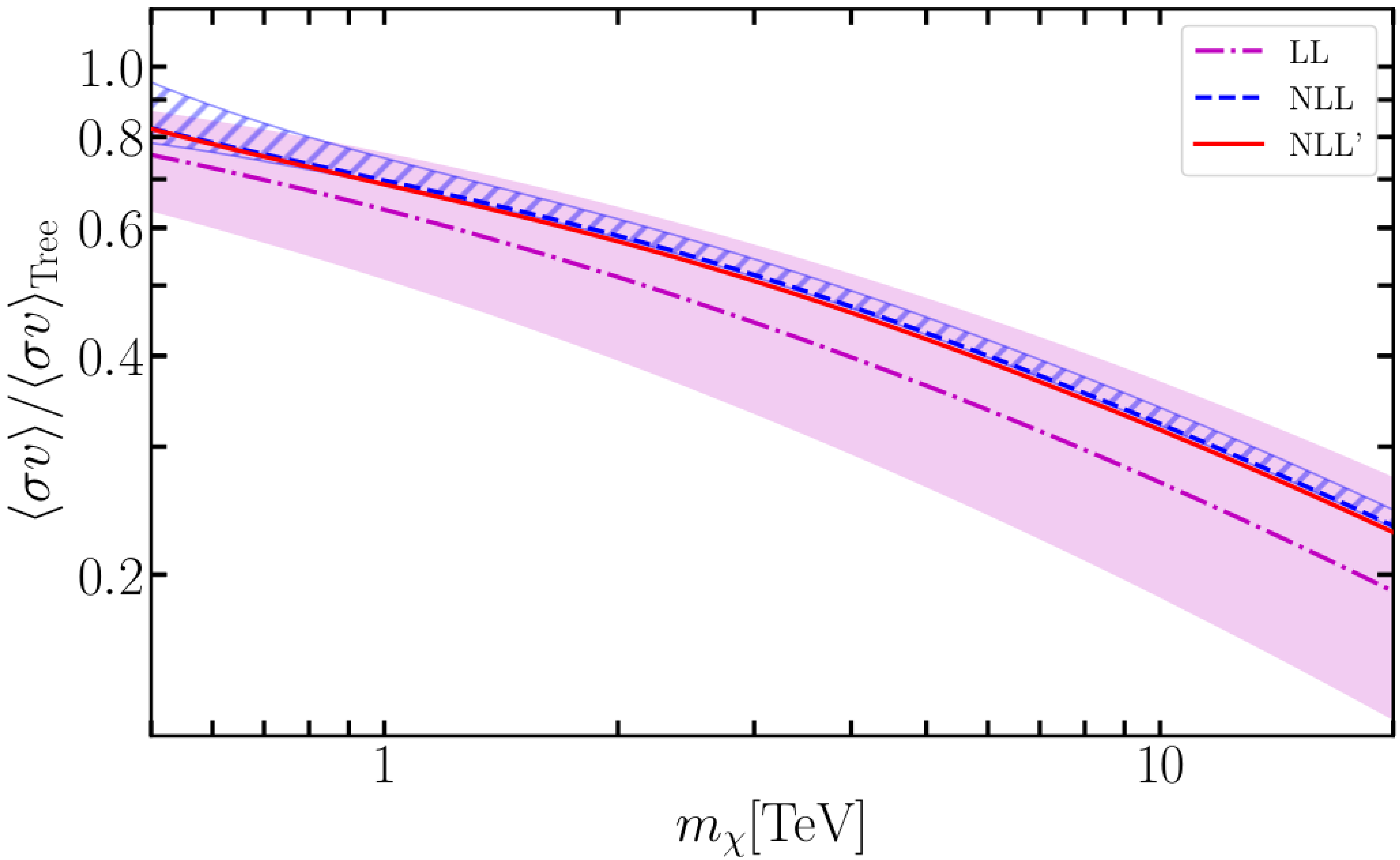
$$V(r) = \begin{pmatrix} 0 & -\sqrt{2}\alpha_2\frac{e^{-m_W r}}{r} \\ -\sqrt{2}\alpha_2\frac{e^{-m_W r}}{r} & -\frac{\alpha}{r} - \alpha_2 c_W^2\frac{e^{-m_Z r}}{r} \end{pmatrix}$$

Results



- We perform the one-dimensional integral in a photon energy bin size $\langle\sigma v\rangle(E^\gamma_{\rm res}) = \int_{m_\chi - E^\gamma_{\rm res}}^{m_\chi} dE_\gamma \frac{d(\sigma v)}{dE_\gamma}$
- Assume $4m_\chi E^\gamma_{\rm res} = (300\,\mathrm{GeV})^2$ → unobserved final state includes γ, Z, WW, ZH and light fermions in the collinear jet function
- The resummed predictions are shown with theoretical uncertainty bands computed from the separate variation of scales μ_h, ν_j in the interval $[m_\chi, 4m_\chi]$ and the scales μ_W, ν_s in $[m_W/2, 2m_W]$ added in quadrature
- We find that at the NLL' order the residual theoretical uncertainty from scale dependence is negligible

Results



- Results normalized to the Sommerfeld-only result. Resummation reduces annihilation rate. In the interesting mass range (around 3 TeV), the rate is suppressed by a factor of two.
- Scale uncertainty reduces from 23% (LL) to 3% (NLL) to 0.3% (NLL') for m_χ = 2 TeV
- At 2 TeV (10 TeV), the ratio of the resummed annihilation rate at NLL' to the Sommerfeld-only rate is $0.575^{+0.003}_{-0.000}$ $(0.316^{+0.002}_{-0.000})$

Conclusion

- Presented results for the semi-inclusive photon energy spectrum for $\chi(p_1) + \chi(p_2) \longrightarrow \gamma(p_\gamma) + X(p_X)$ at NLL’ accuracy
- Focused on gamma-ray telescopes with photon energy resolution of parametric order $E^\gamma_{\text{res}} \sim m_W^2/m_\chi$
- Due to scale hierarchies present in indirect TeV scale DM detection, large radiative corrections (besides the Sommerfeld factor) need to be resummed
- Used SCET approach to resum large Sudakov logarithms
- Obtained factorization formula, which is valid to all orders in perturbation theory
- Theoretical uncertainty was found to be negligible at NLL’