

5d $N=1$ gauge theories via 5-brane web

Futoshi Yagi (Technion)

@ DESY

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It is ill-defined. What are you doing?!**

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(Unique/Minimal/Natural UV completion)**

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- String theory gives UV completion.
(**Brane setup**/Calabi-Yau compactification)
- We compute the quantity insensitive to the UV completion. (BPS/Suppressed by cut-off scale)
- "The answer is space-time dependent".

UV

**5d SCFT
(6d SCFT)**

RG flow



IR

**5d SUSY
gauge theory**

Relevant deformation
(or S^1 compactification)

UV

**5d SCFT
(6d SCFT)**

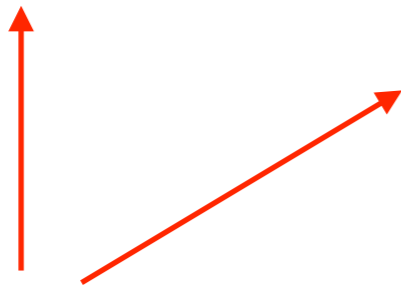
RG flow



IR

**5d SUSY
gauge theory**

Relevant deformation
(or S^1 compactification)



“Define” by brane setup

(by 5-brane web in this talk)

Motivation for studying 5-brane web

Existence of UV fixed point

“UV duality”

Two or more different gauge theories have identical UV fixed point

BPS spectrum

Seiberg-Witten curve, prepotential

Nekrasov partition function, Superconformal Index

...

Existence of the UV fixed point of 5d $N=1$ gauge theories via 5-brane web

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Preliminary Example

4D N=2 SU(2) SYM

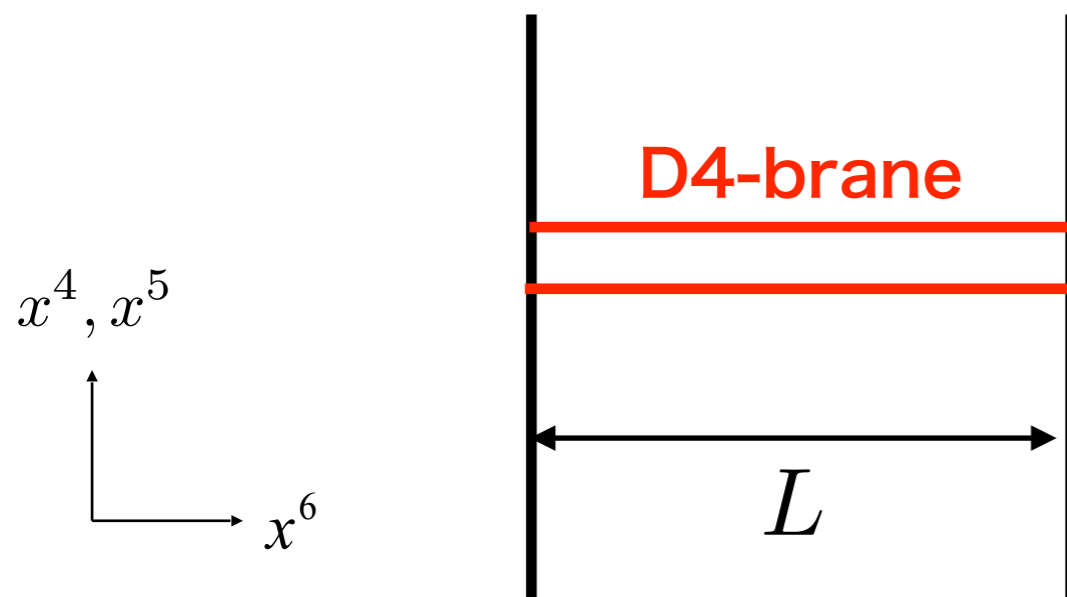
Hanany-Witten brane setup

[Hanany, Witten '96]

[Witten '97]

NS5-brane

NS5-brane



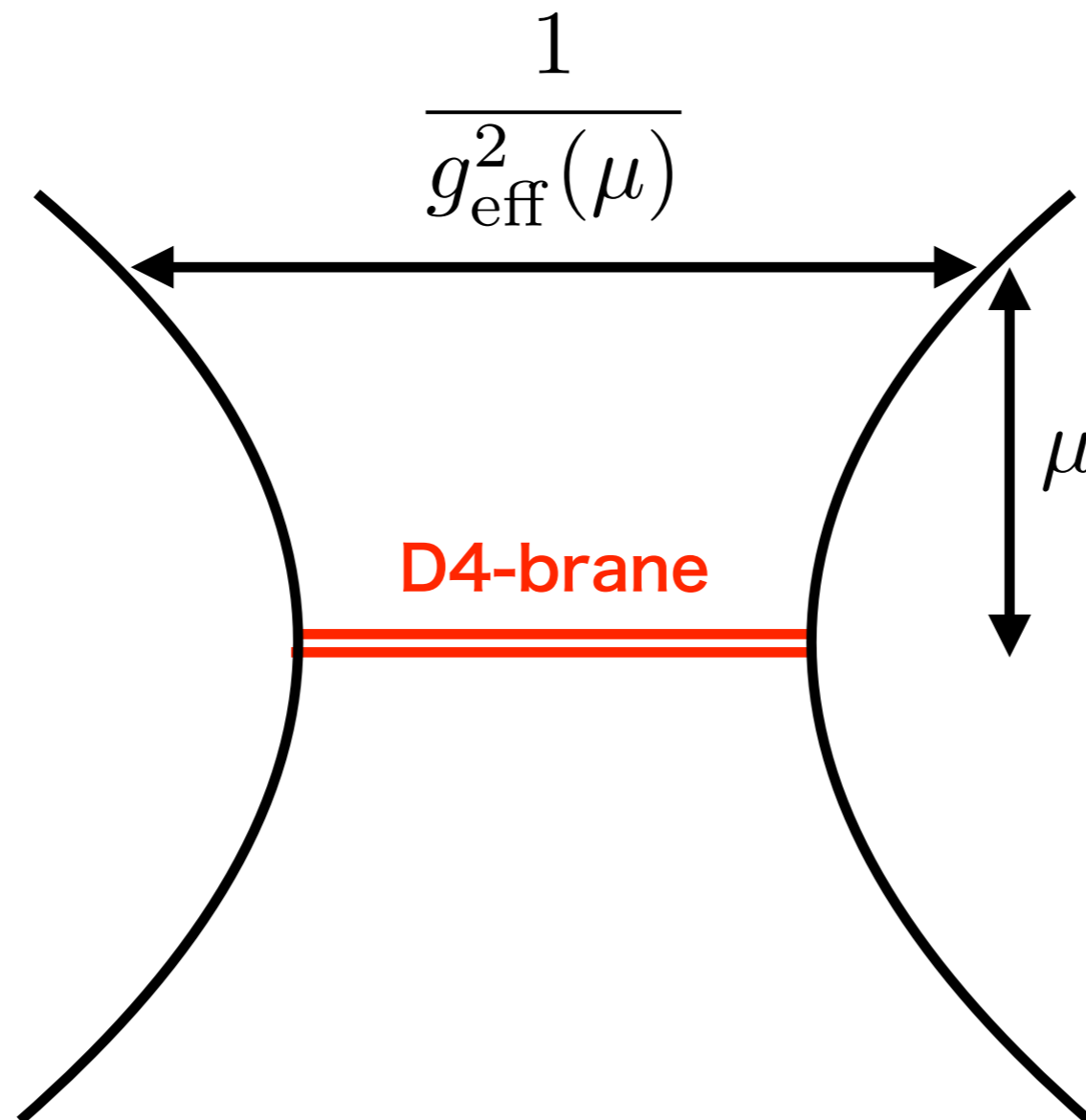
	0	1	2	3	4	5	6	7	8	9
NS5-brane	-	-	-	-	-	-
D4-brane	-	-	-	-	.	.	-	.	.	.

$$\int d^5 x F^{\mu\nu} F_{\mu\nu} \sim \underbrace{L}_{\uparrow 1/g_{\text{YM}}^2} \int d^4 x F^{\mu\nu} F_{\mu\nu}$$

Preliminary Example

Tension \rightarrow 1-loop renormalization

[Witten '97]



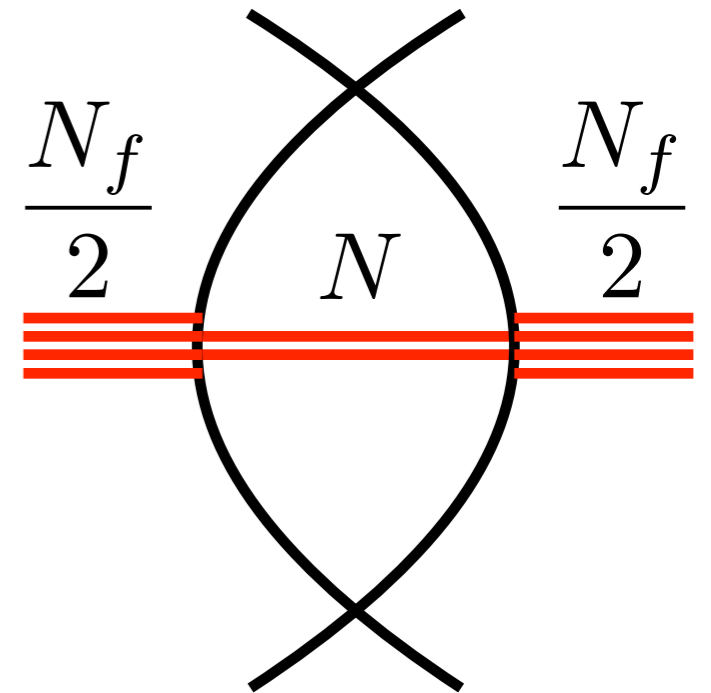
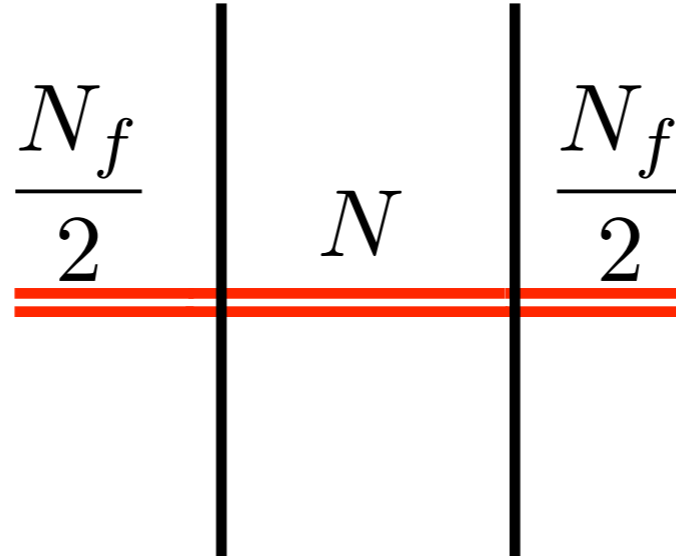
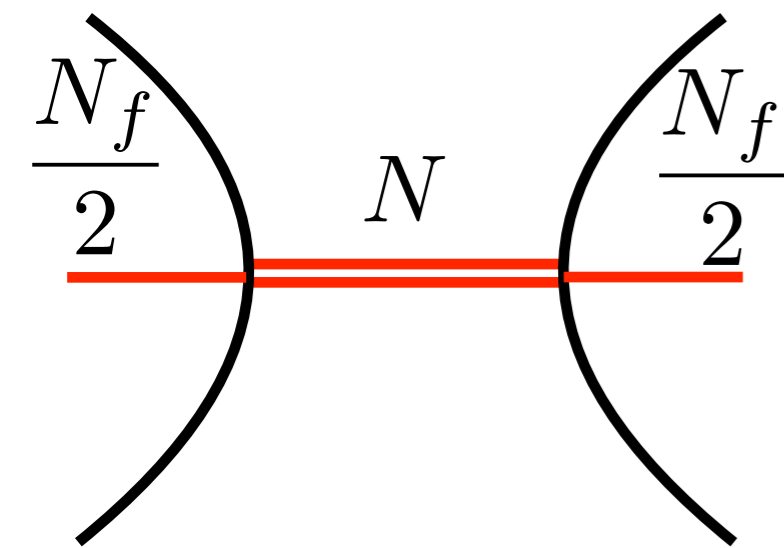
Preliminary Example

4d $\mathcal{N} = 2$ $SU(N)$ N_f flavor

$$\beta = 2N - N_f > 0$$

$$\beta = 2N - N_f = 0$$

$$\beta = 2N - N_f < 0$$



Asymptotically
free

Conformal

Landau pole
appears

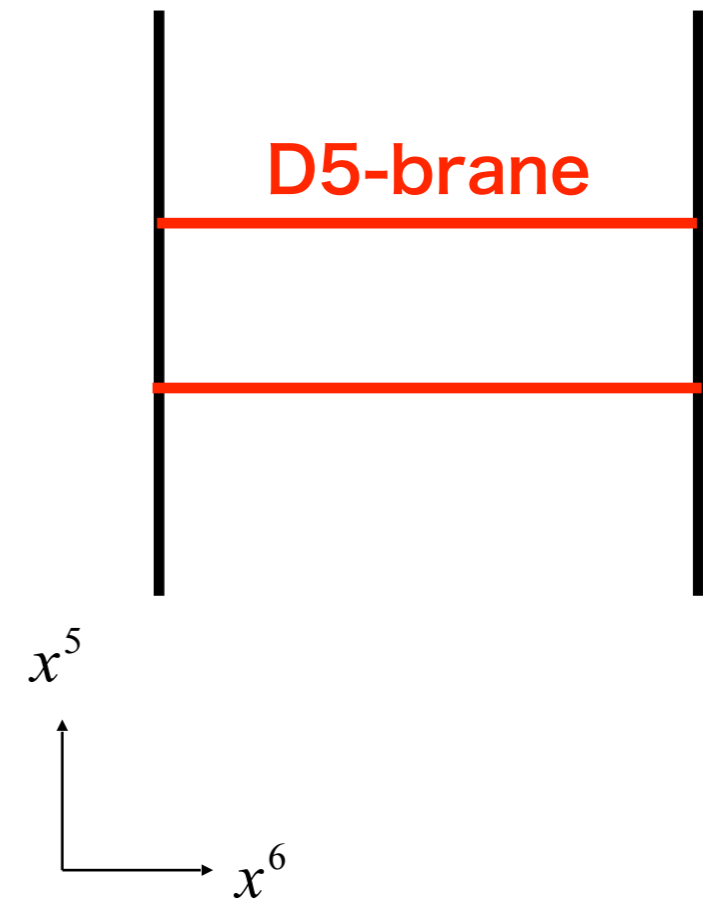
UV fixed point exists

No UV fixed point

5D N=1 SU(2) SYM

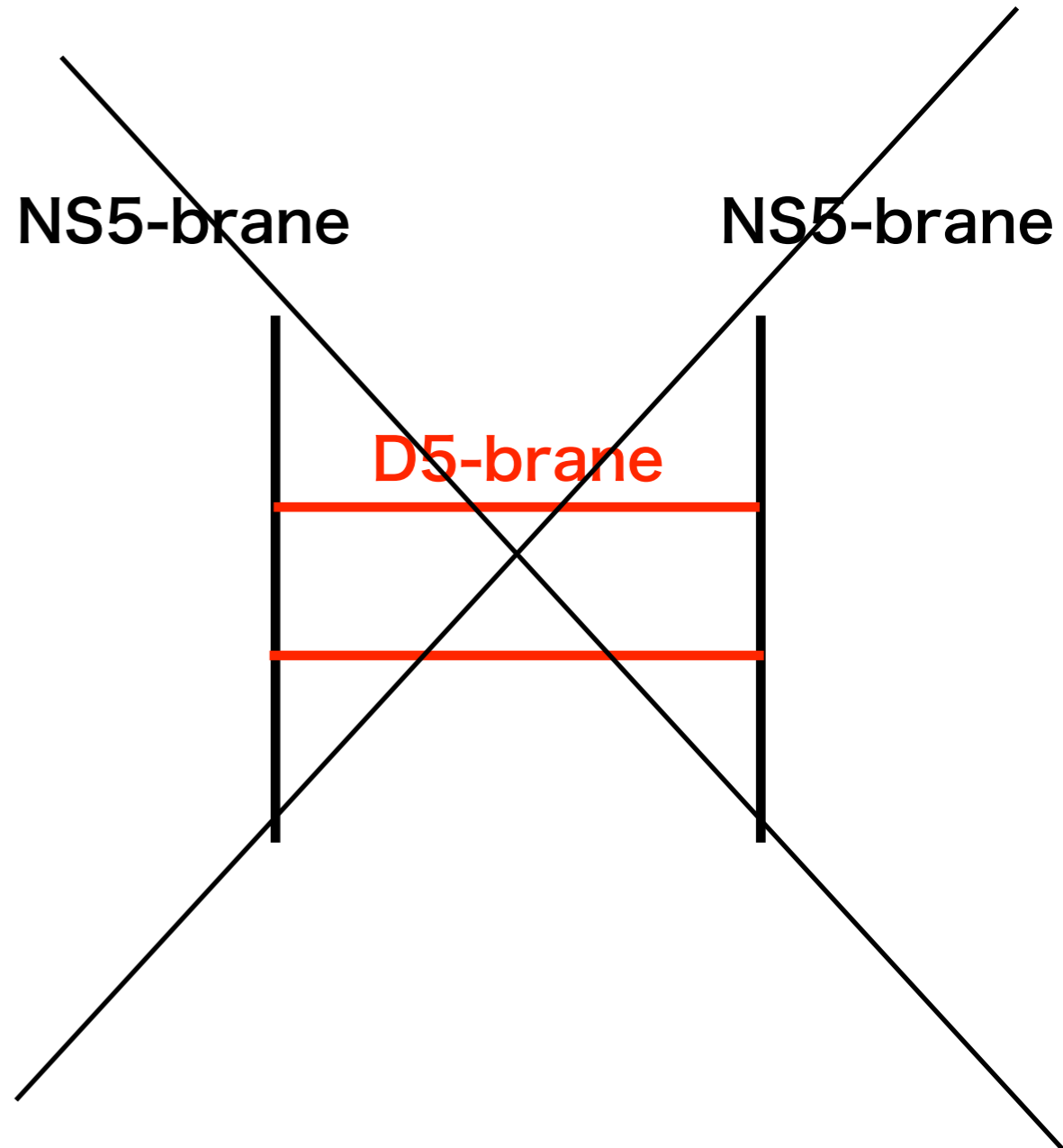
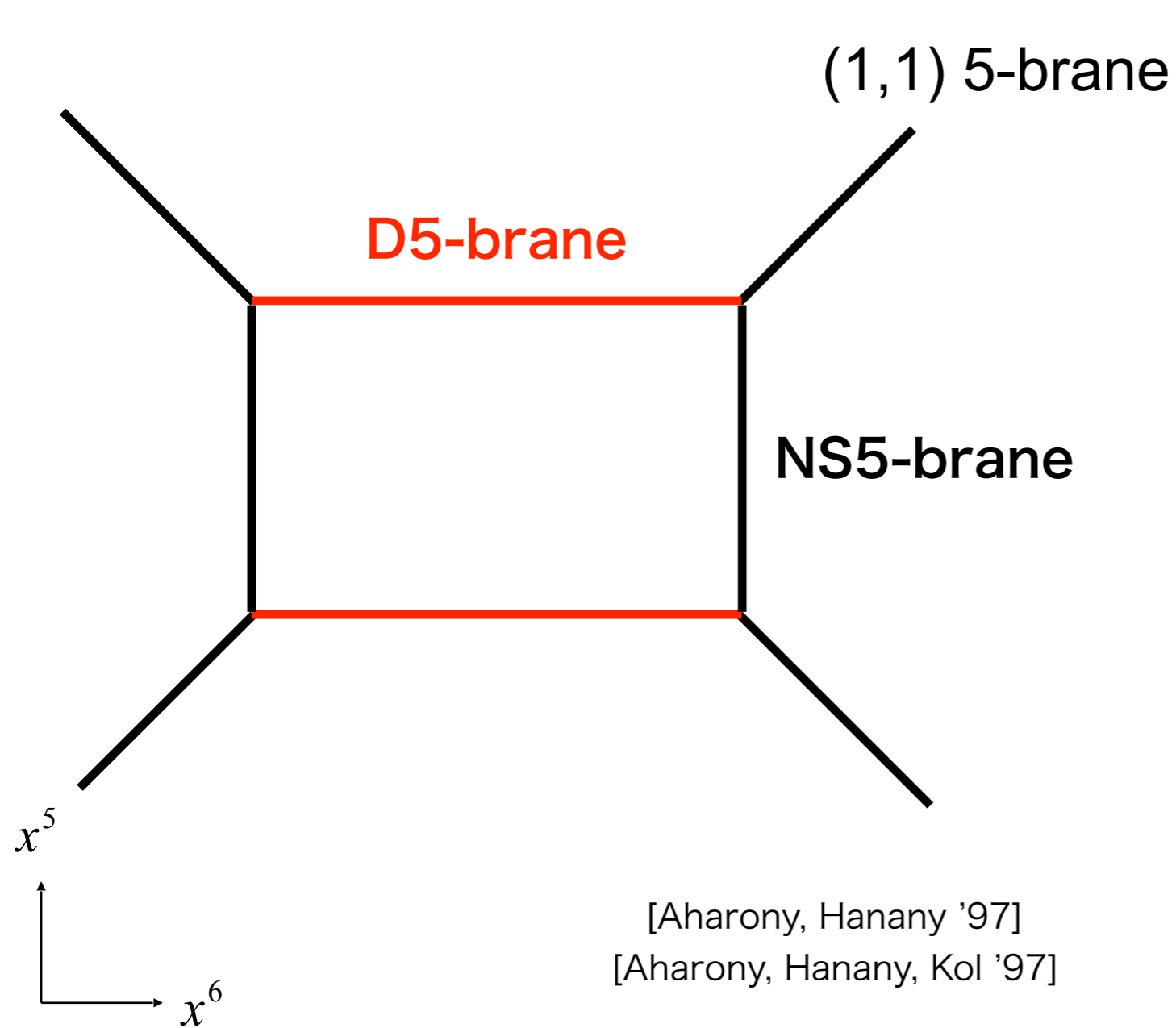
NS5-brane

NS5-brane



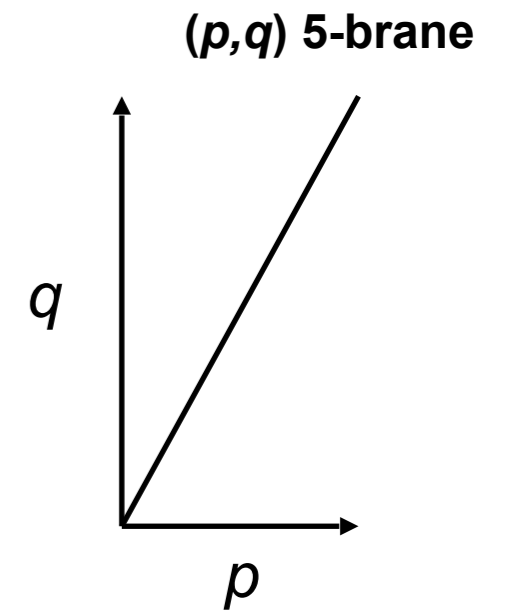
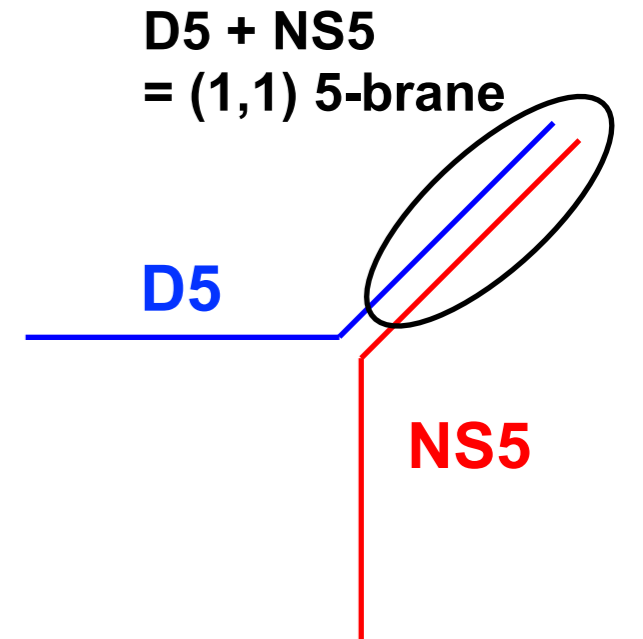
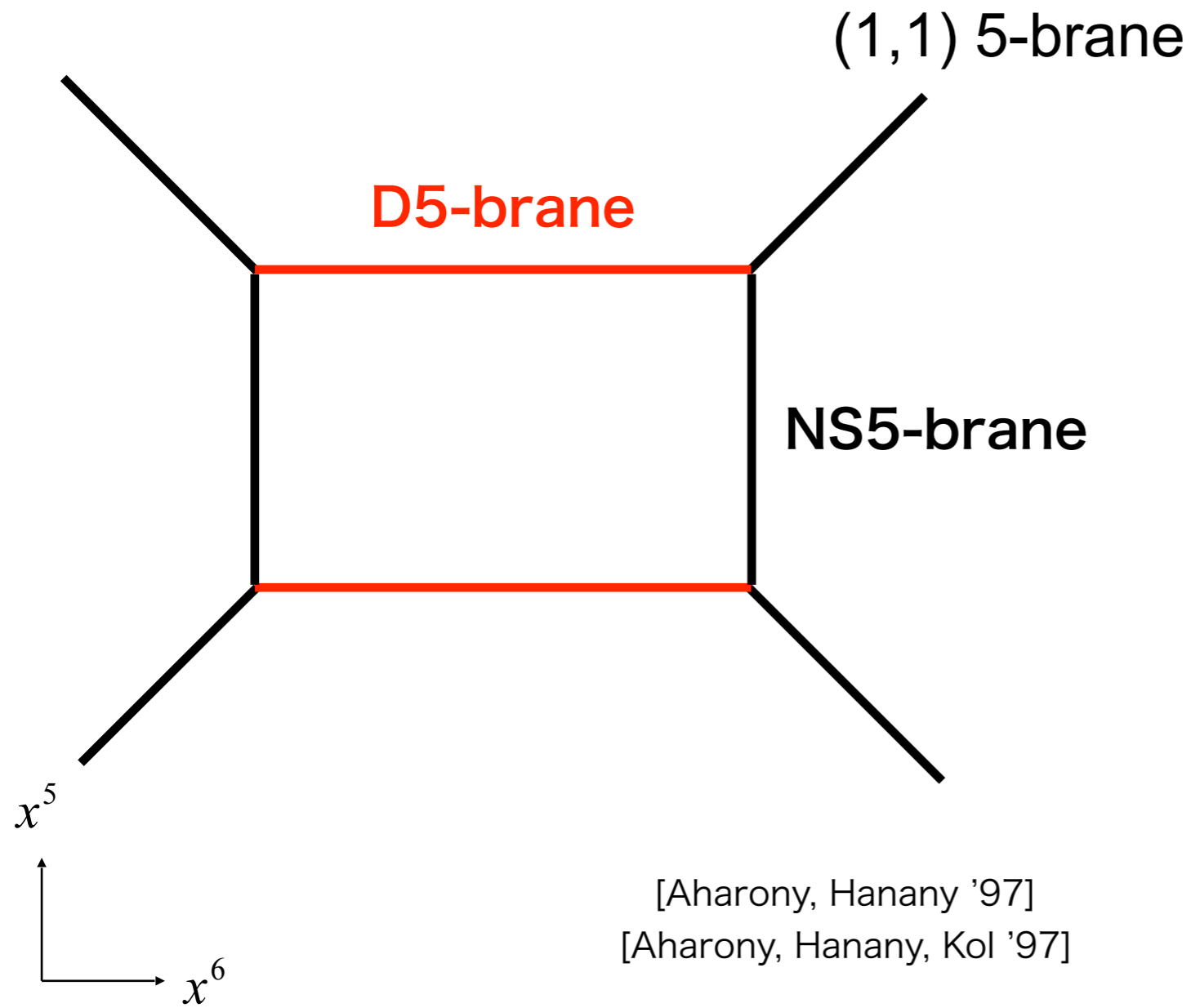
	0	1	2	3	4	5	6	7	8	9
5-brane	-	-	-	-	-	web		.	.	.

5D N=1 SU(2) SYM



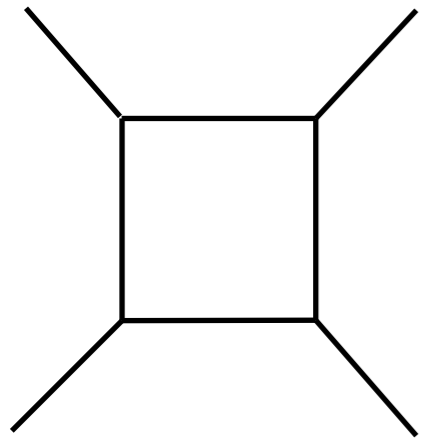
	0	1	2	3	4	5	6	7	8	9
5-brane	-	-	-	-	-	web		.	.	.

5D N=1 SU(2) SYM

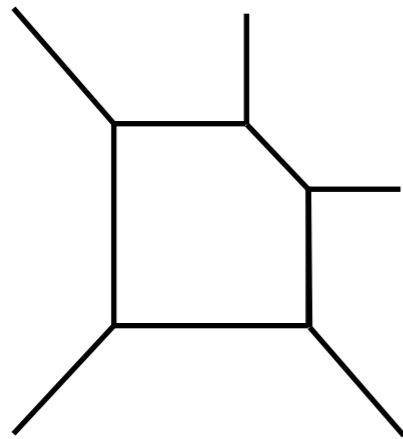


	0	1	2	3	4	5	6	7	8	9
5-brane	-	-	-	-	-	web		.	.	.

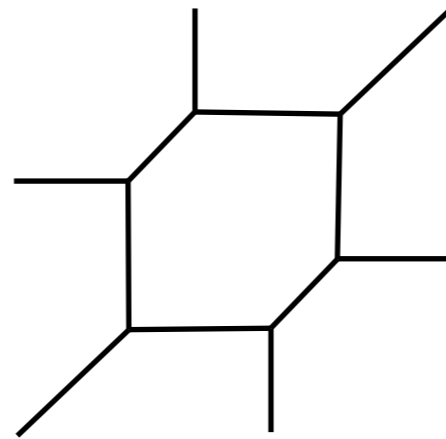
5D N=1 SU(2) SYM with N_f flavor



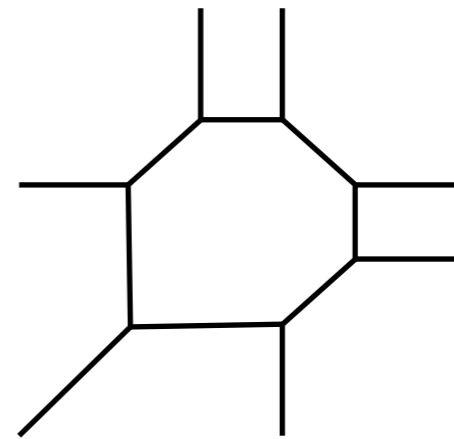
$$N_f = 0$$



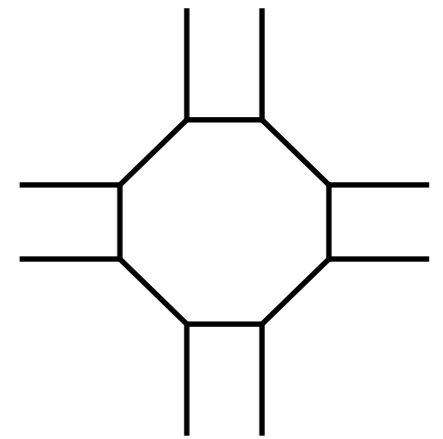
$$N_f = 1$$



$$N_f = 2$$



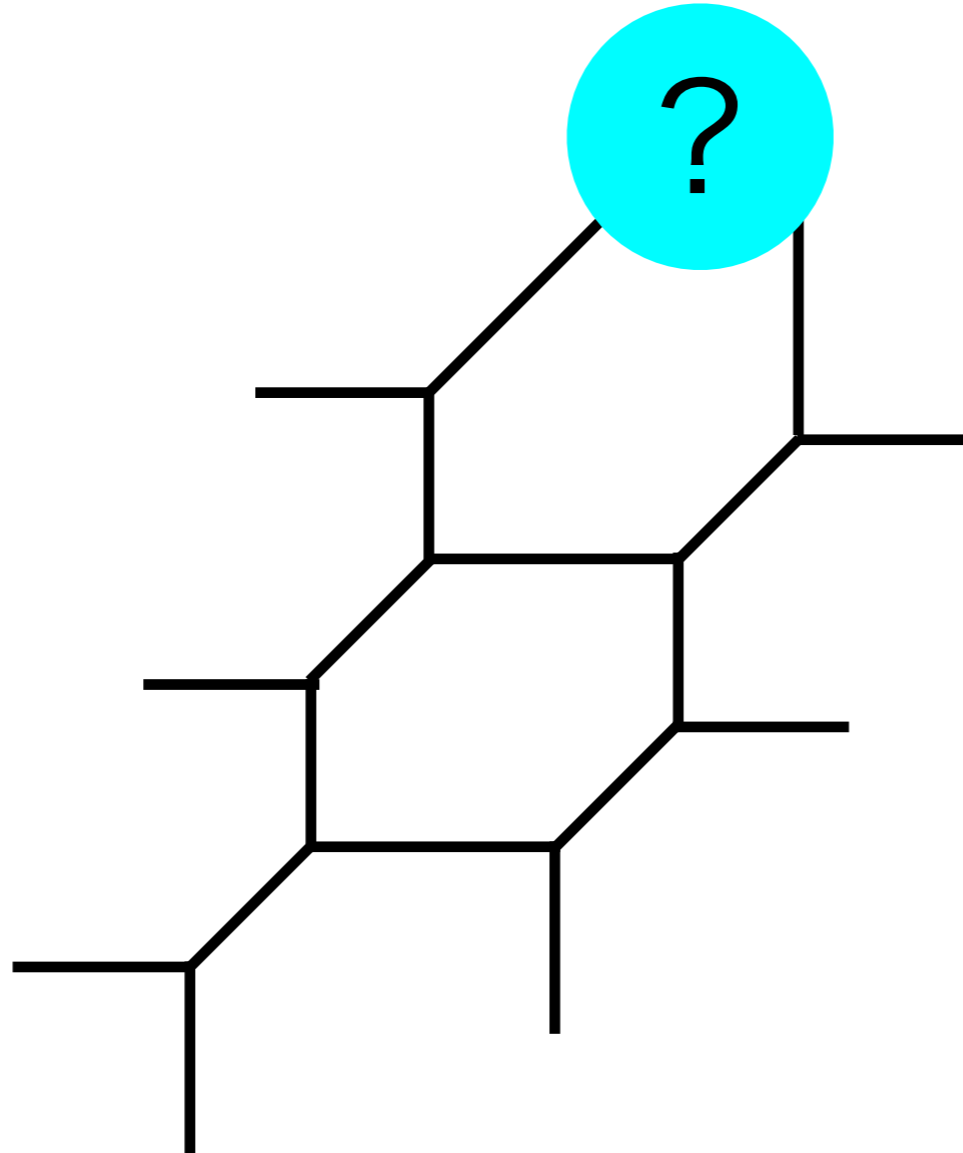
$$N_f = 3$$



$$N_f = 4$$

[Aharony, Hanany '97]

How about $N_f=5$?



No UV fixed point for $N_f=5$??

5d N=1 SU(2) gauge theory with N_f flavor

[Seiberg '97]

Renormalization of the gauge coupling constant is 1-loop exact!
(Prepotential is at most cubic)

$$\frac{1}{g_{\text{eff}}(a)^2} \propto \frac{\partial^2 F}{\partial a^2} > 0 \quad \forall a$$

$0 \leq N_f \leq 7$ **5D UV fixed point**

$N_f = 8$ **6D UV fixed point**
(E-string) [Ganor, Hanany '96]

$N_f \geq 9$ **No UV fixed point**

Understanding on UV fixed point about 20 years ago

UV fixed point exist for...

	Brane	Field theory
4d N=2 SU(N) N _f flavor	$N_f \leq 2N$ [Witten '97]	$N_f \leq 2N$
5d N=1 SU(2) N _f flavor	$N_f \leq 4?$ [Aharony, Hanany '97]	$N_f \leq 7$ [Seiberg '97] $N_f = 8$ (6d)
5d N=1 SU(N) N _f flavor (N>2)	$N_f \leq 2N$ [Aharony, Hanany '97]	$N_f \leq 2N$ [Intriligator, Morrison, Seiberg '97]

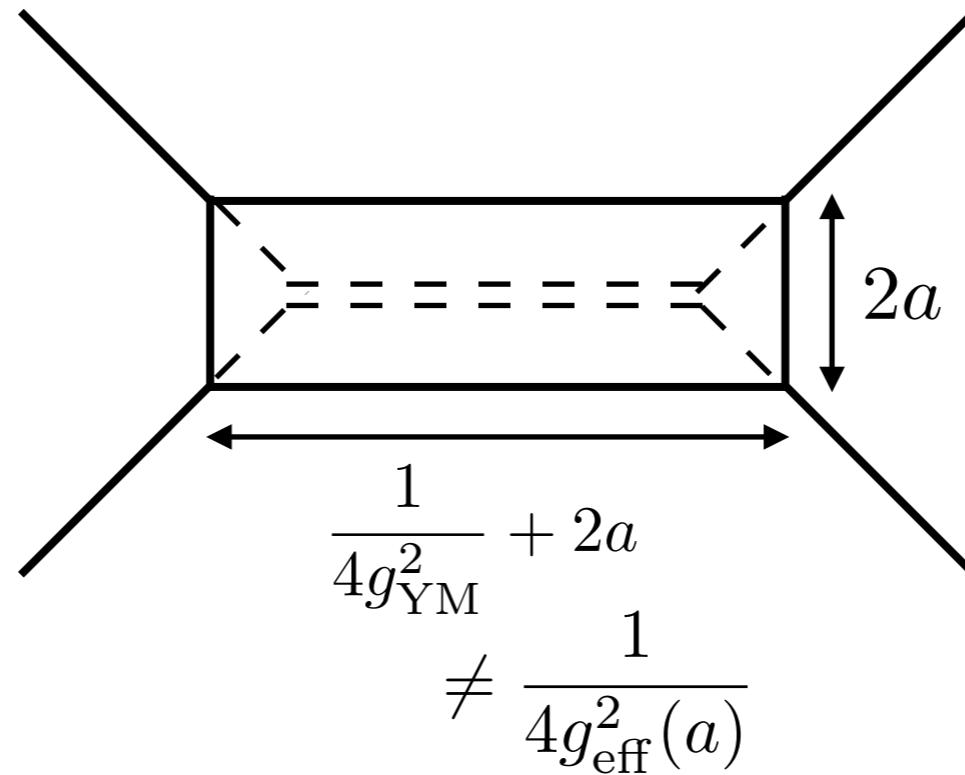


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5d N=1 SU(N) N _f flavor (N>2)	$N_f \leq 2N$ [Aharony, Hanany '97]	$N_f \leq 2N$ [Intriligator, Morrison, Seiberg '97]

How to read off gauge theory parameters?

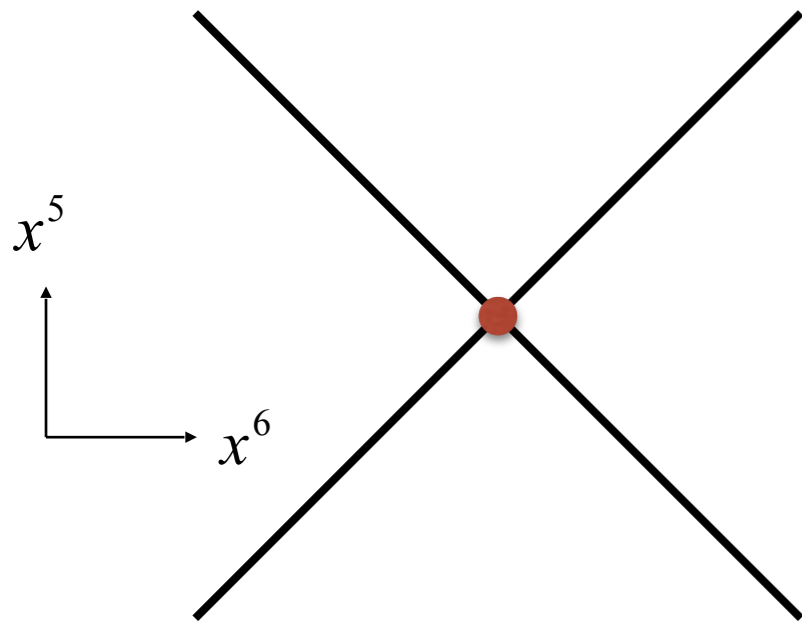


Monopole tension: $\frac{\partial F}{\partial a} = (\text{Area}) = 2a \left(\frac{1}{4g_{\text{YM}}^2} + 2a \right)$

Effective coupling: $\frac{1}{4g_{\text{eff}}^2} = \frac{1}{2} \frac{\partial^2 F}{\partial a^2} = \frac{1}{4g_{\text{YM}}^2} + 4a$

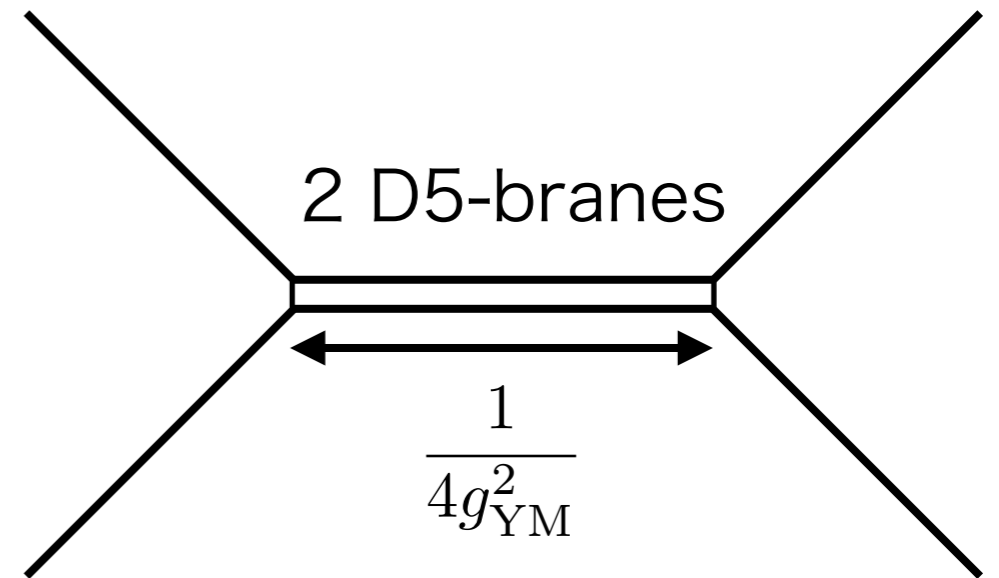
Collision of 5-brane does not indicate $\frac{1}{g_{\text{eff}}^2} < 0$

5d N=1 SCFT

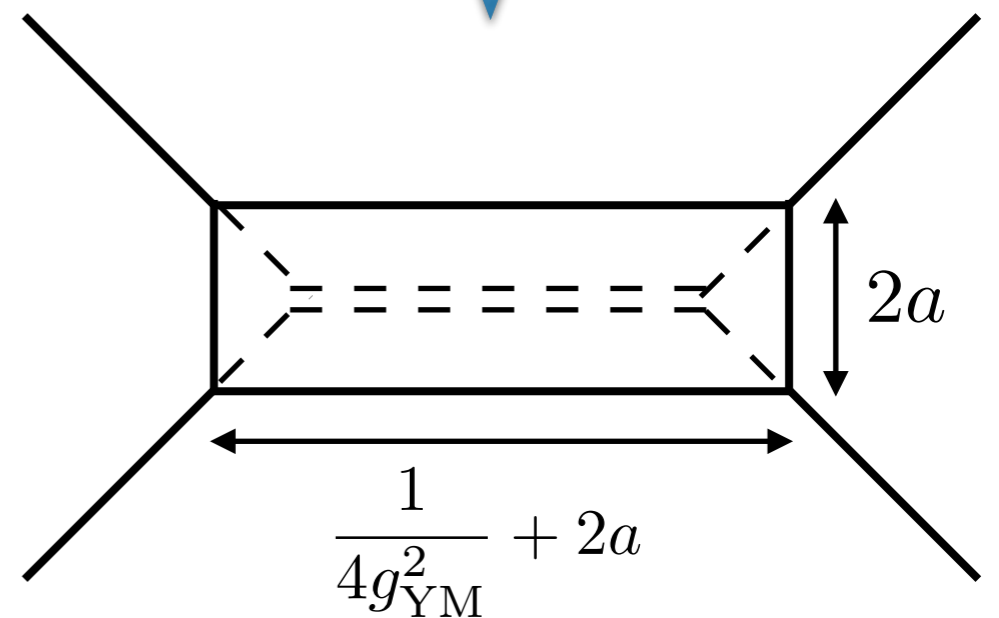


Relevant deformation
(global deformation)

5d N=1 SU(2) gauge theory



Coulomb branch
(Local deformation)

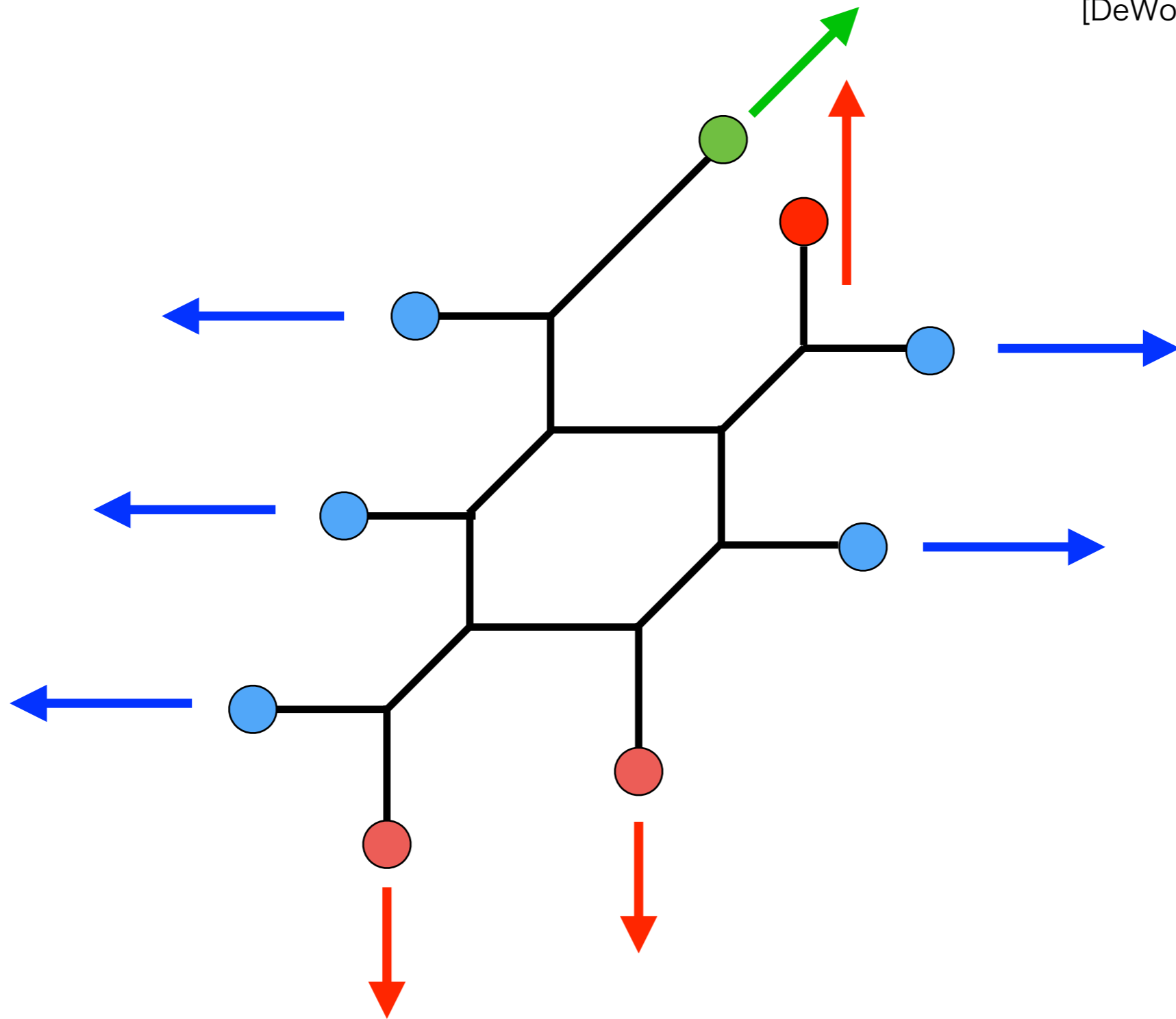


	0	1	2	3	4	5	6	7	8	9
5-brane	-	-	-	-	-	web		.	.	.

Existence of 5-brane web \rightarrow UV fixed point

Terminate (p,q) 5-brane by (p,q) 7-brane

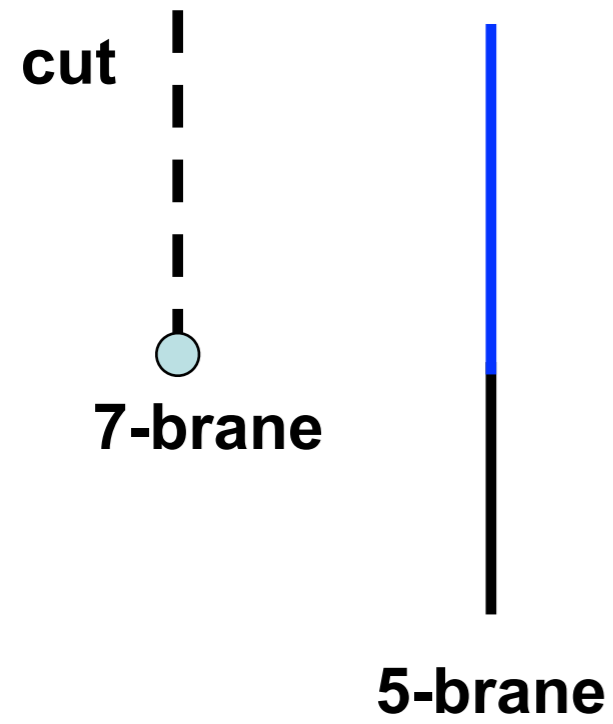
[DeWolfe, Hanany, Iqbal, Katz '99]



and move (p,q) 7-brane to (p,q) -direction

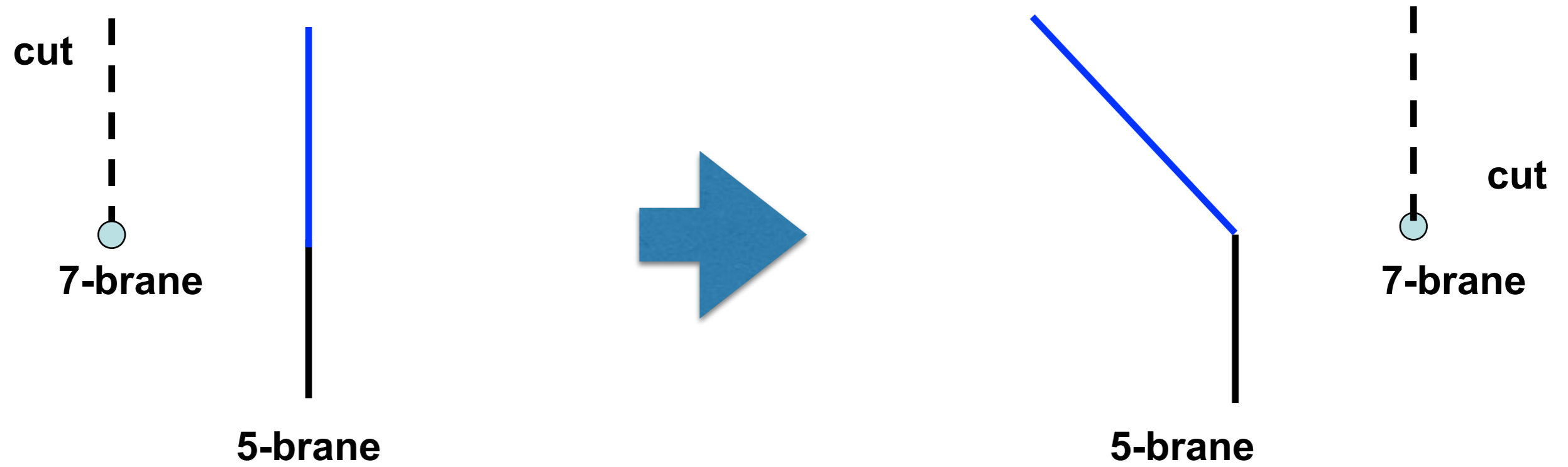
Hanany-Witten transition

[Hanany, Witten '96]



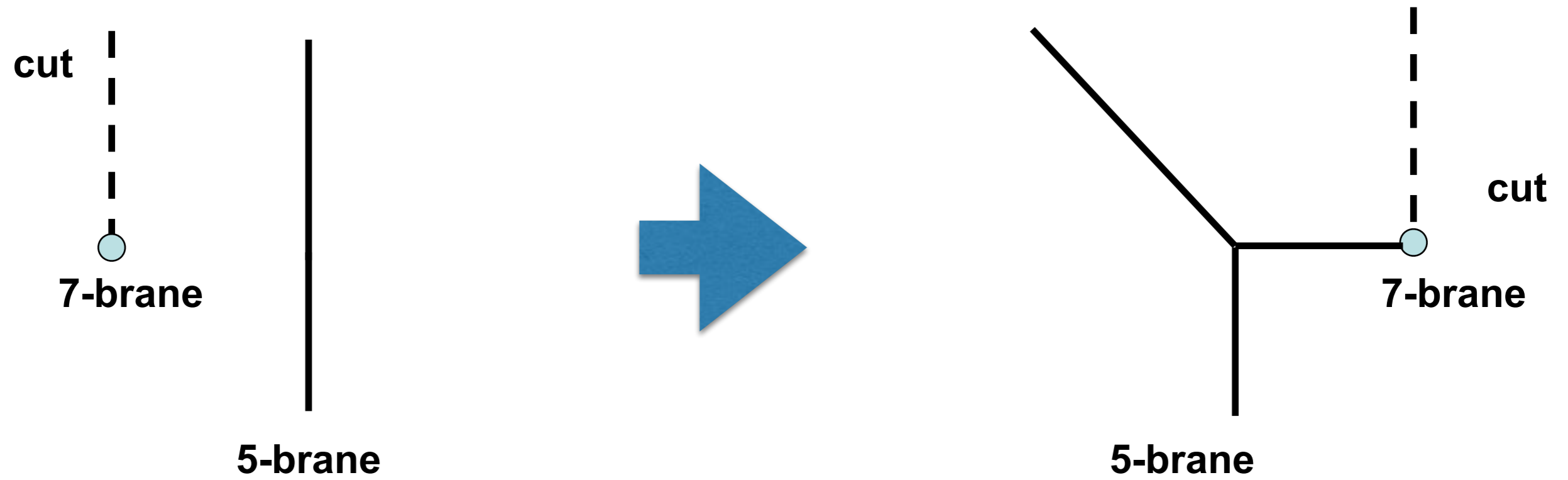
Hanany-Witten transition

[Hanany, Witten '96]

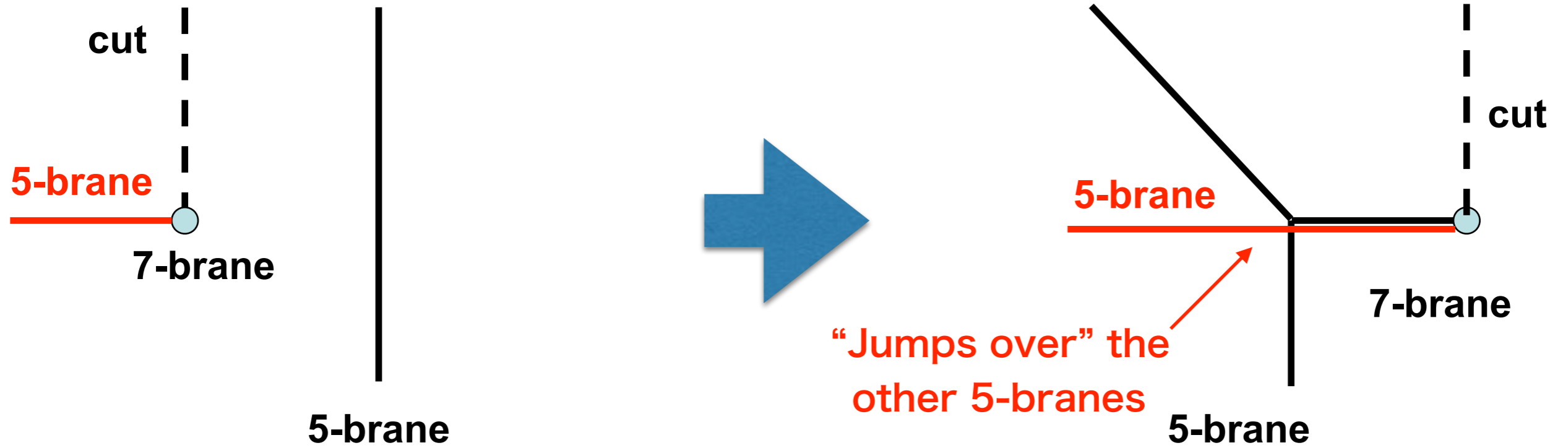


Hanany-Witten transition

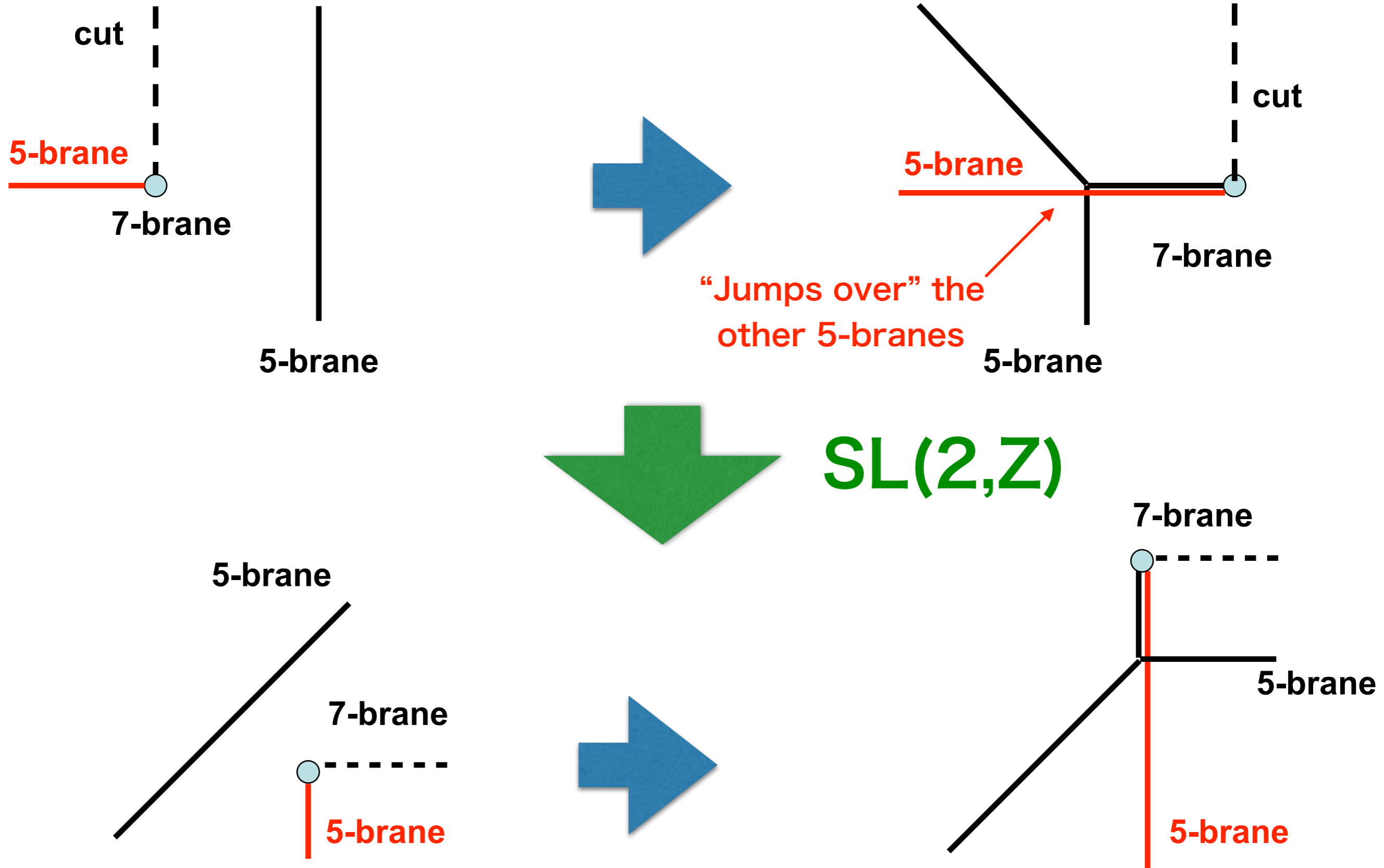
[Hanany, Witten '96]



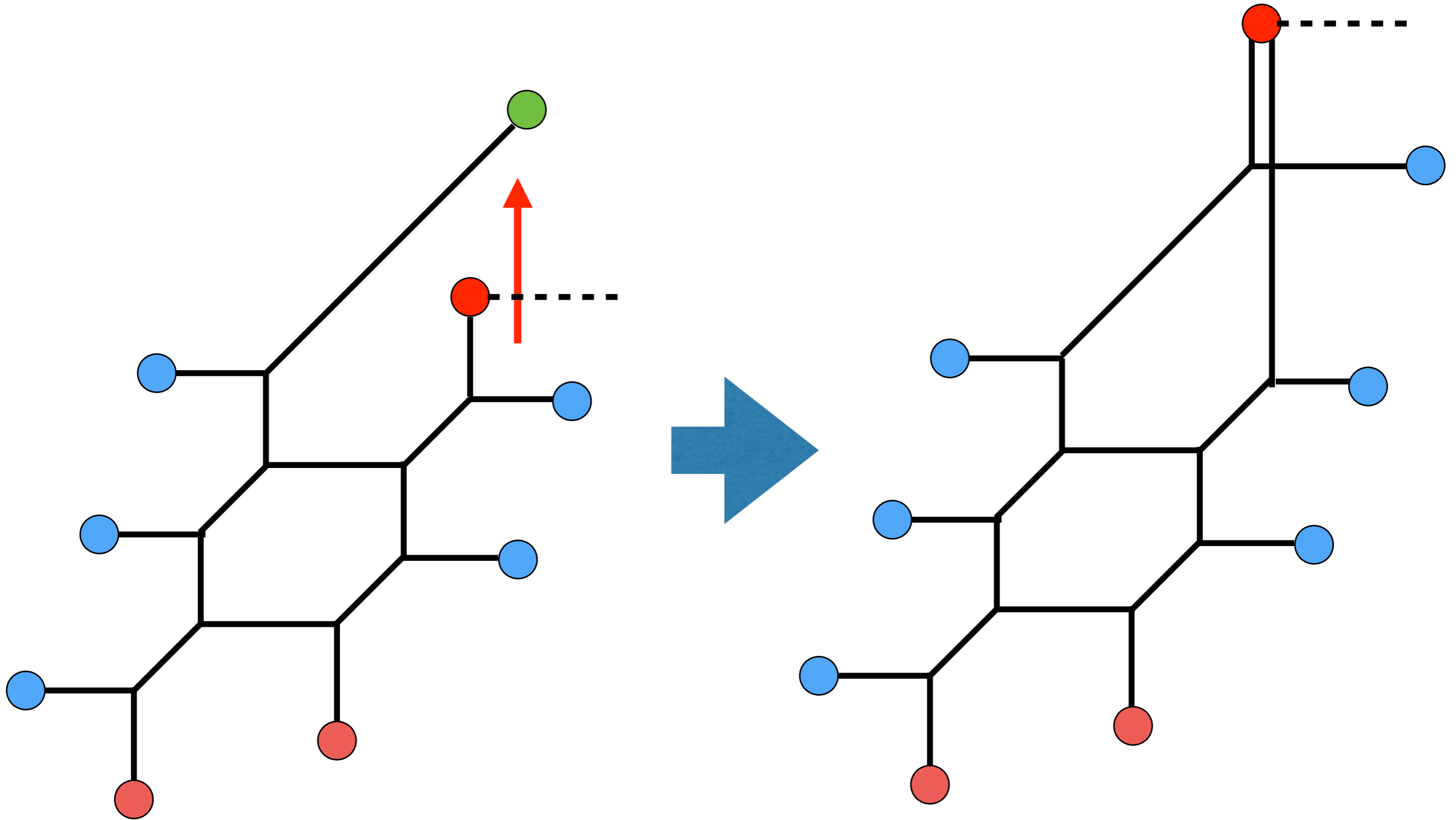
Hanany-Witten transition



Hanany-Witten transition



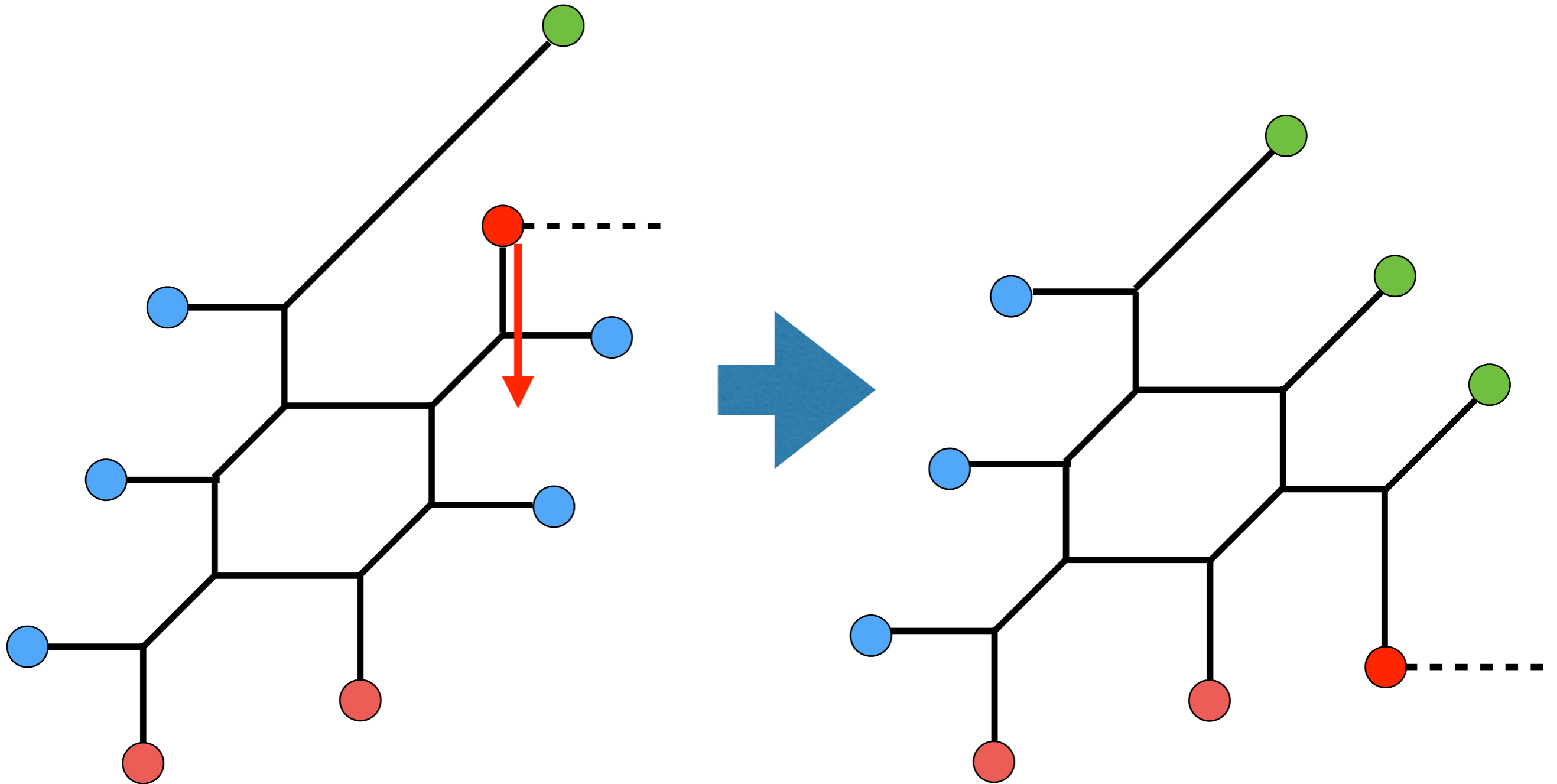
5-brane web for $N_f=5$



[Benini, Benvenuti, Tachikawa 09']

[Bao, Mitev, Pomoni, Taki, FY 13']

5-brane web for $N_f=5$

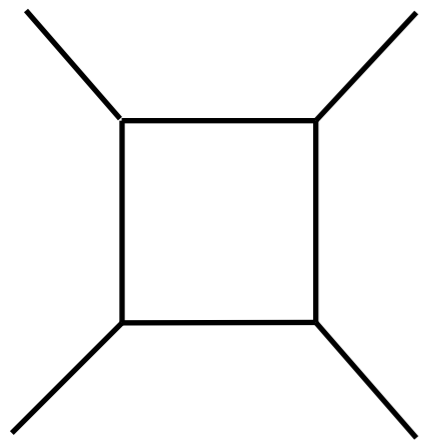


“5D T_3 theory”

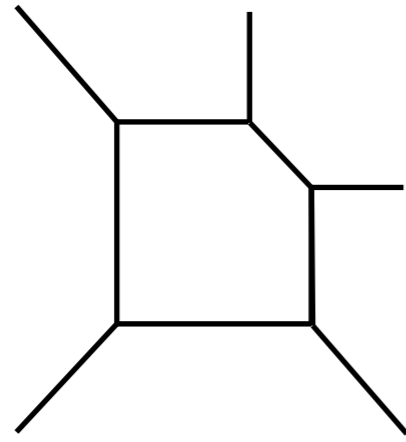
[Benini, Benvenuti, Tachikawa 09’]

[Bao, Mitev, Pomoni, Taki, FY 13’]

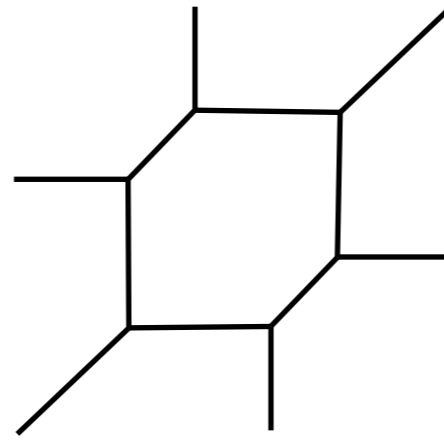
5D N=1 SU(2) SYM with N_f flavor



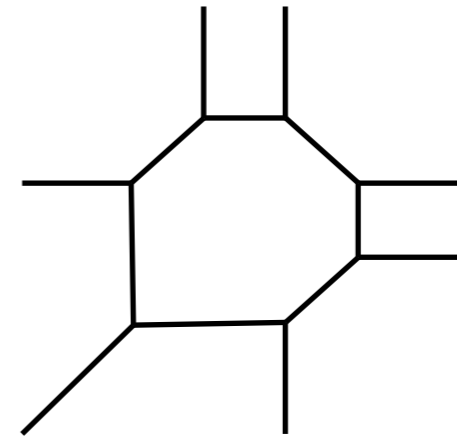
$$N_f = 0$$



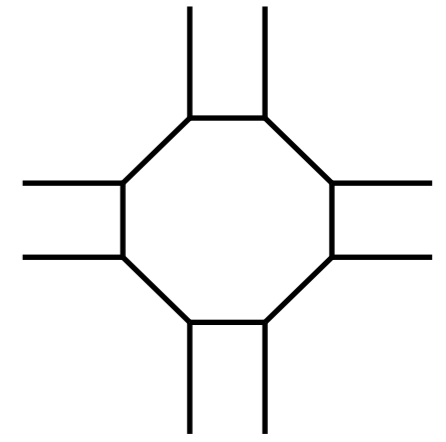
$$N_f = 1$$



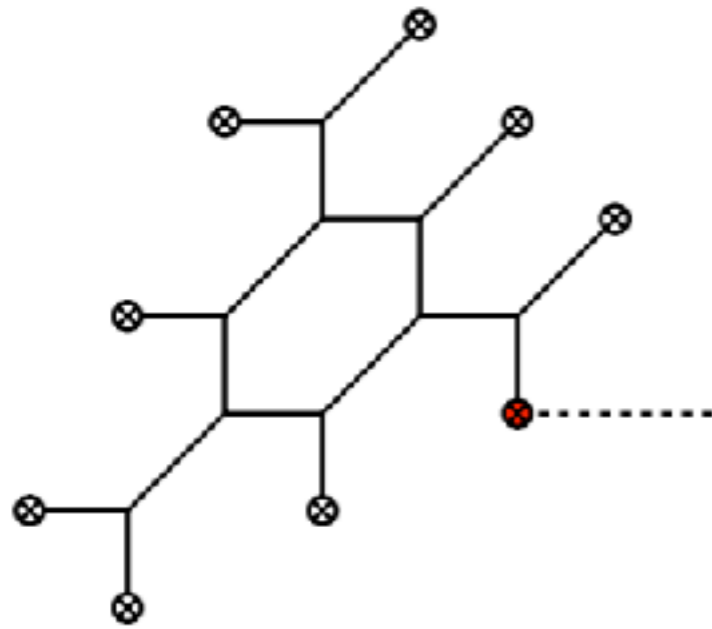
$$N_f = 2$$



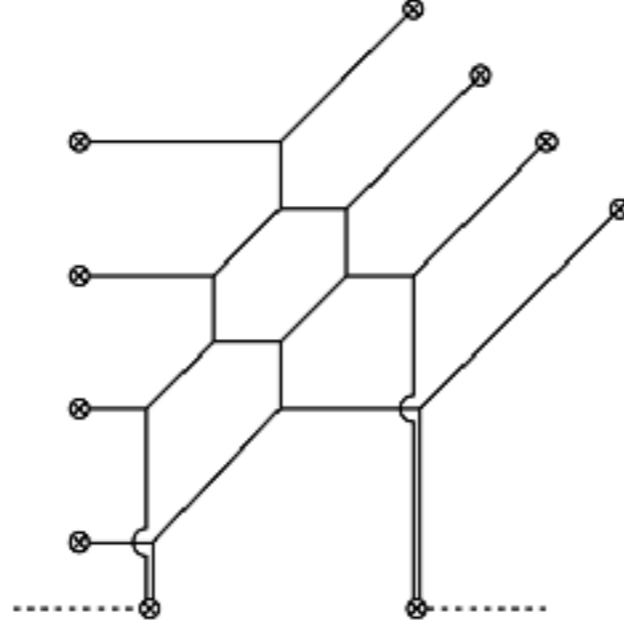
$$N_f = 3$$



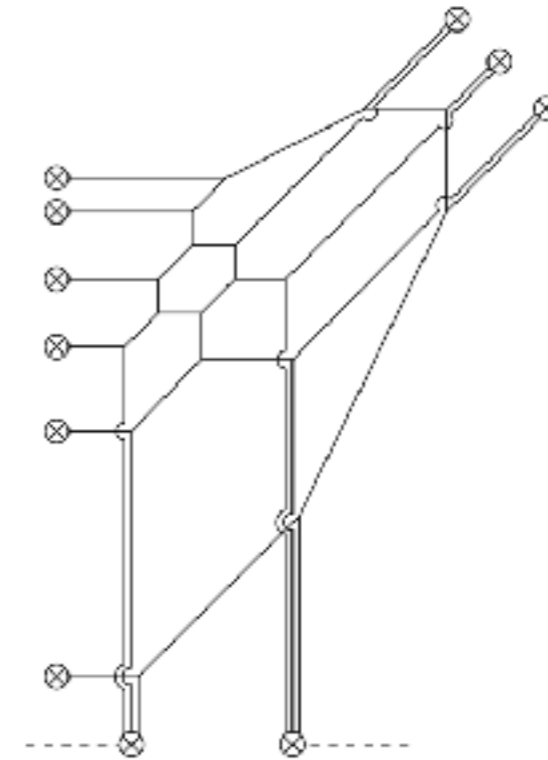
$$N_f = 4$$



$$N_f = 5$$



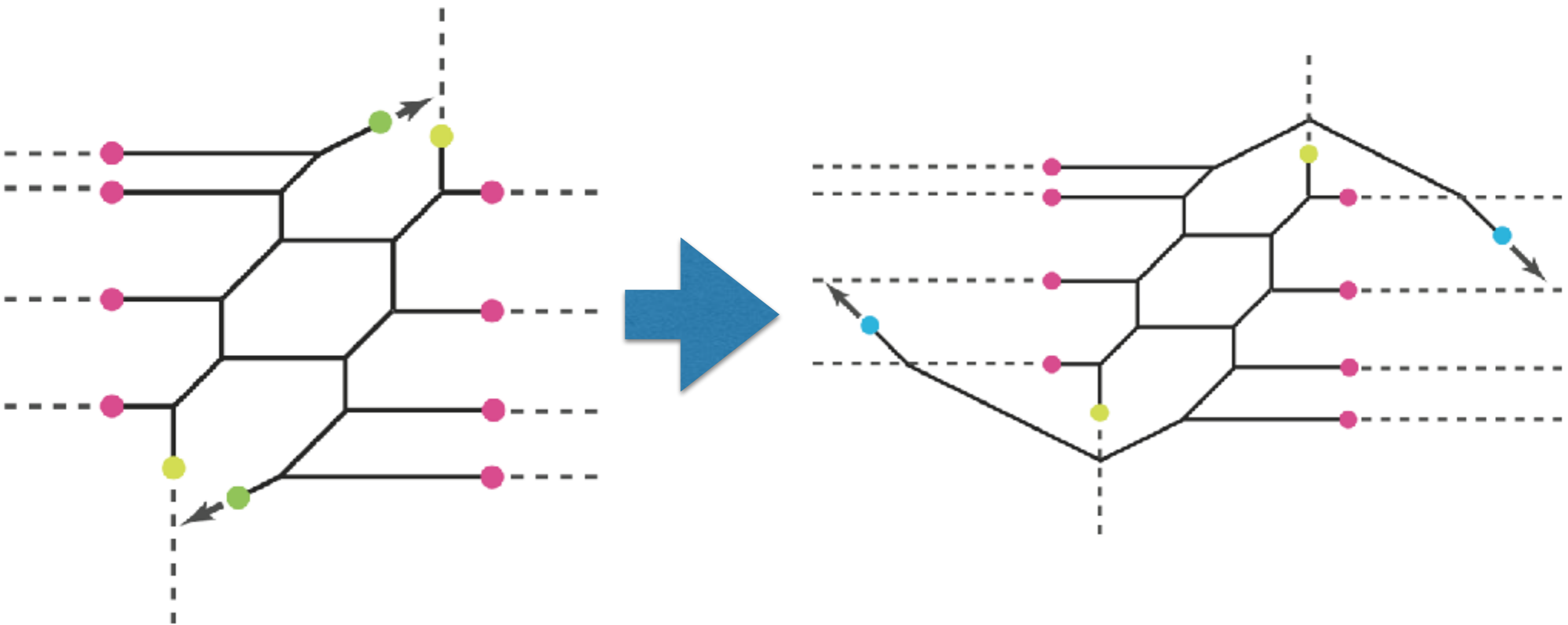
$$N_f = 6$$



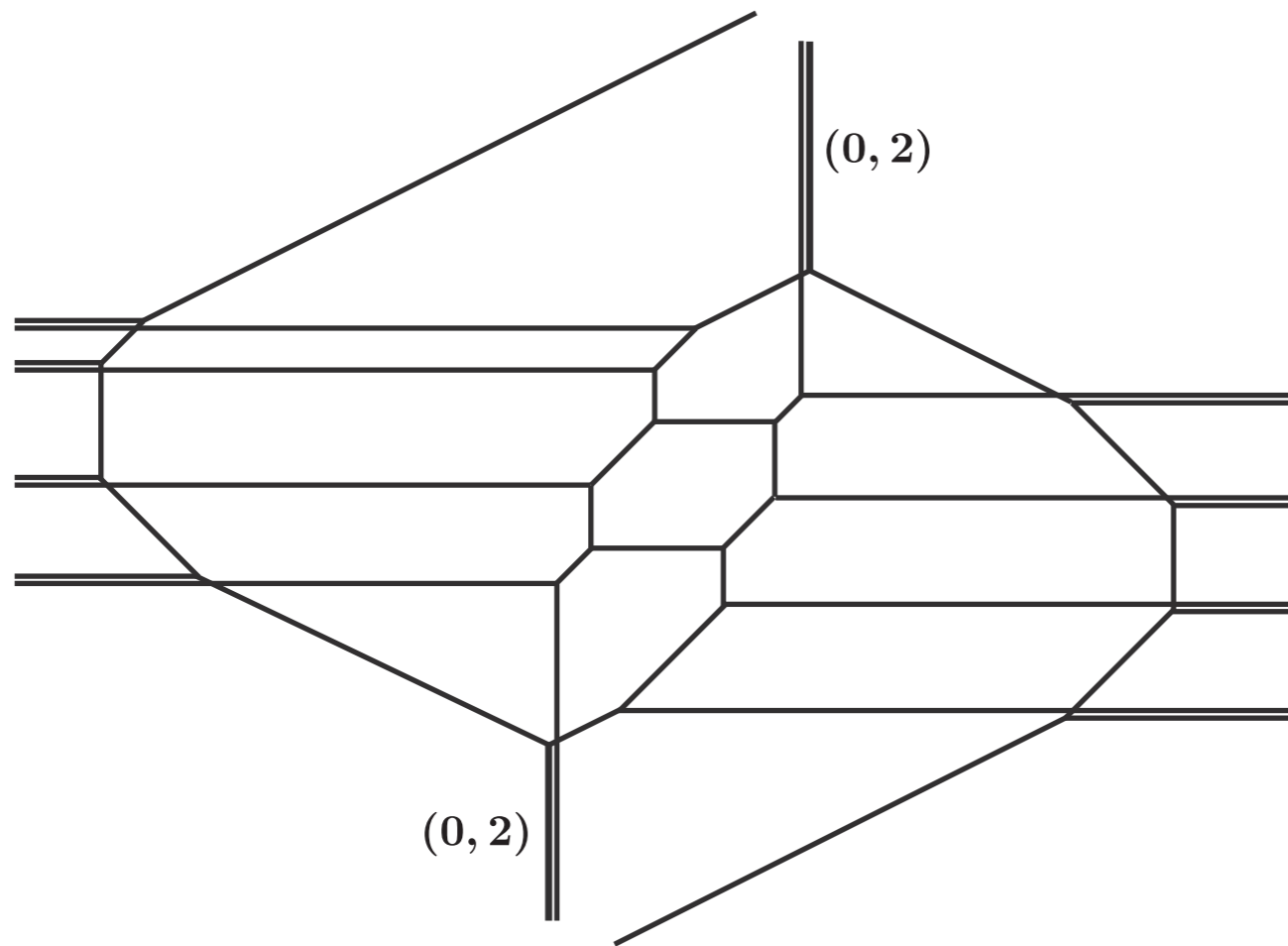
$$N_f = 7$$

[Benini Benvenuti
Tachikawa 09']

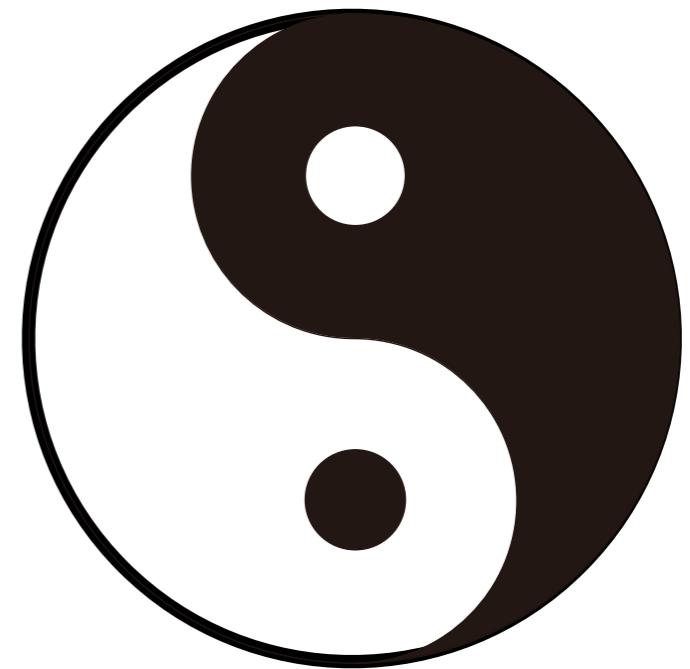
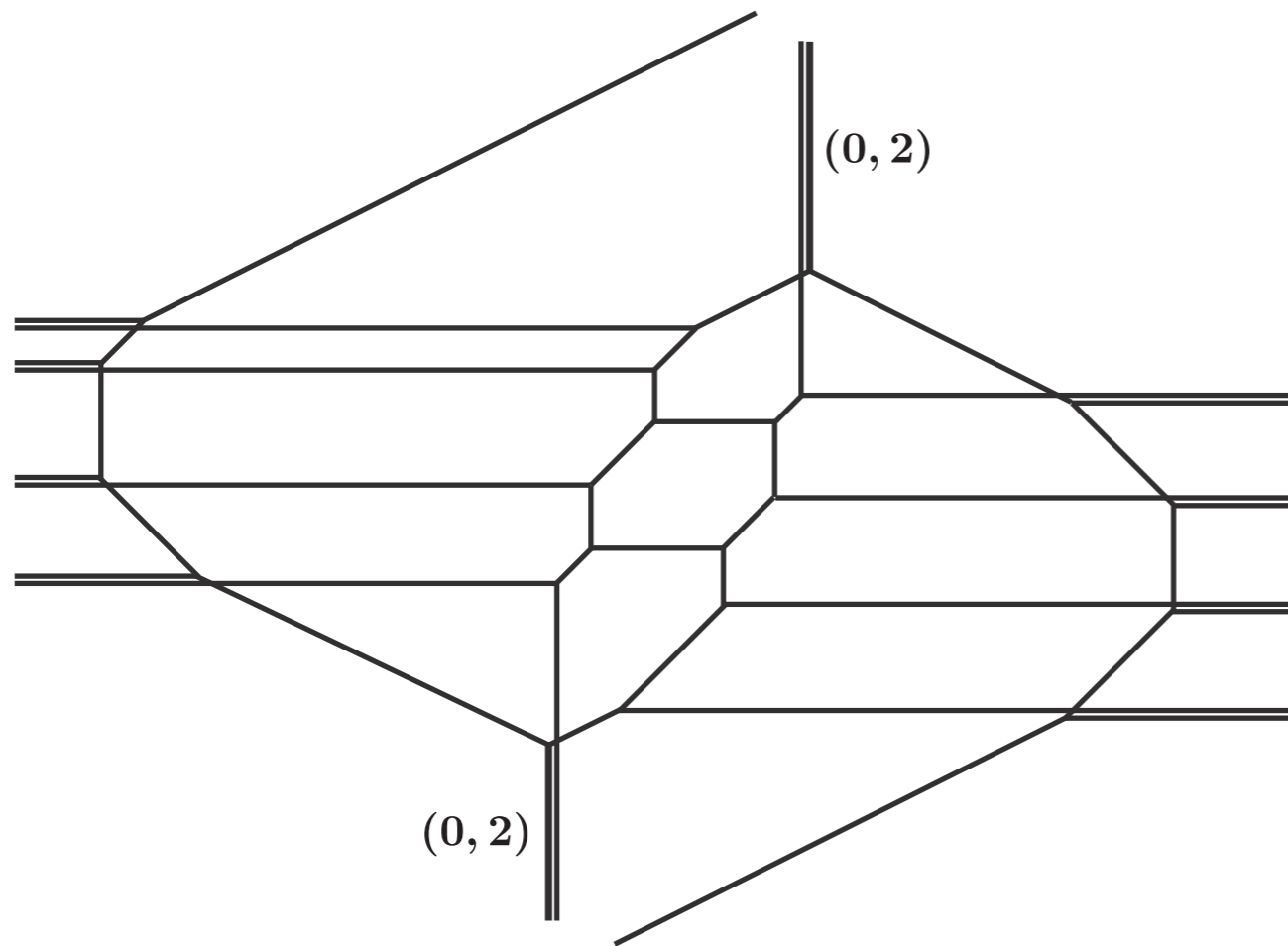
5D N=1 SU(2) SYM with $N_f = 8$ flavor



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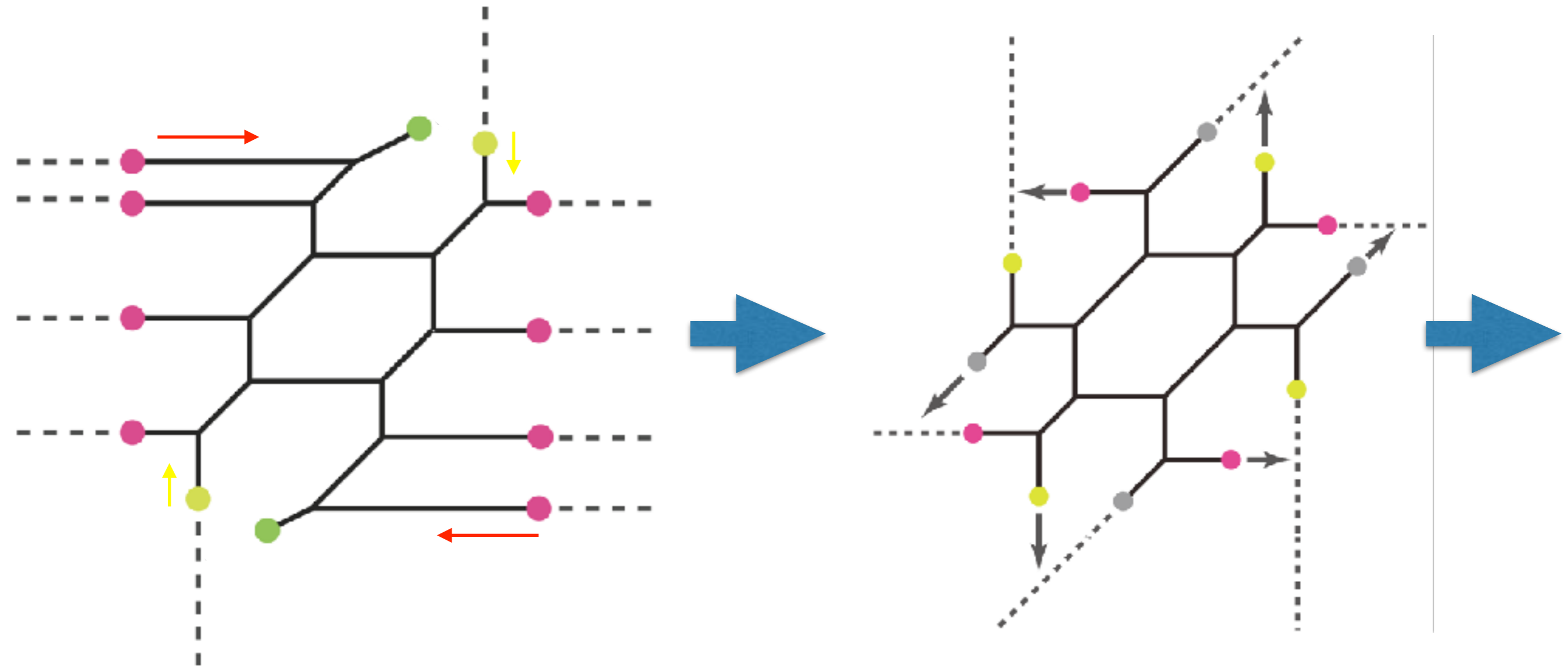
5D N=1 SU(2) SYM with $N_f = 8$ flavor



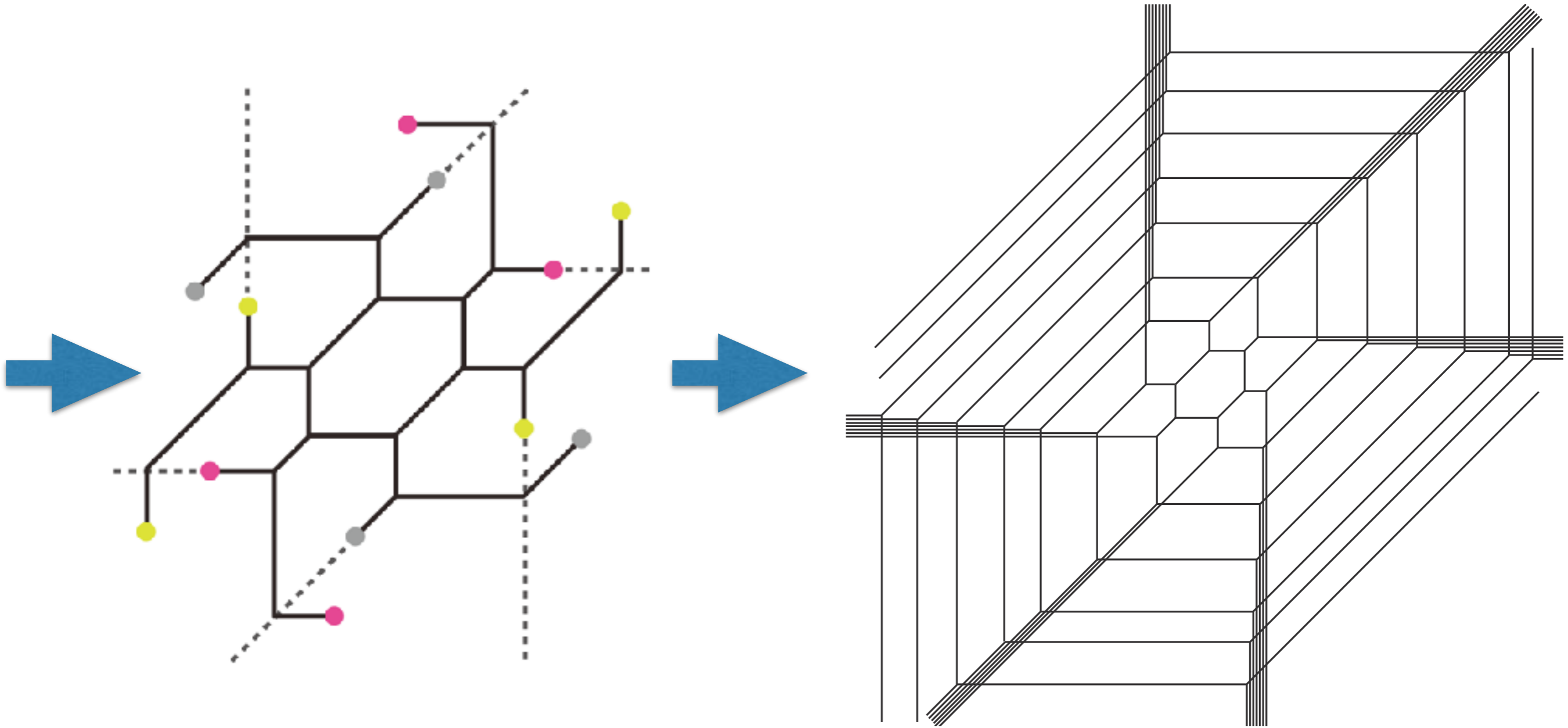
Tao Diagram

[SS.Kim, M.Taki, FY '15]

Other Tao diagram



Other Tao diagram



Periodic Structure of Tao diagram

BPS spectrum

$$m_{(n)} = m_{(0)} + n d$$

→ KK mode

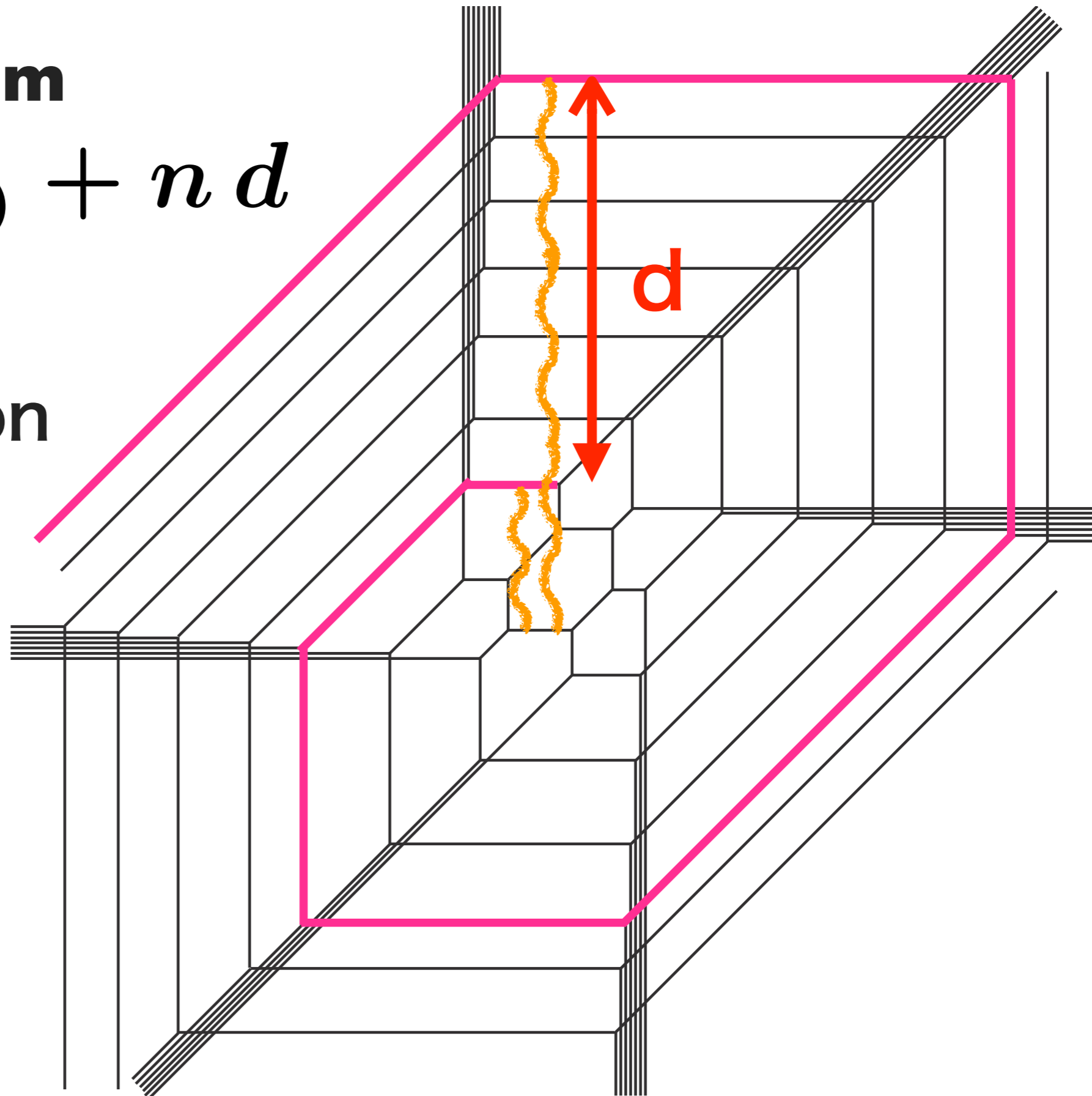
→ 6-th direction

$$d = g_{YM}^{-2}$$

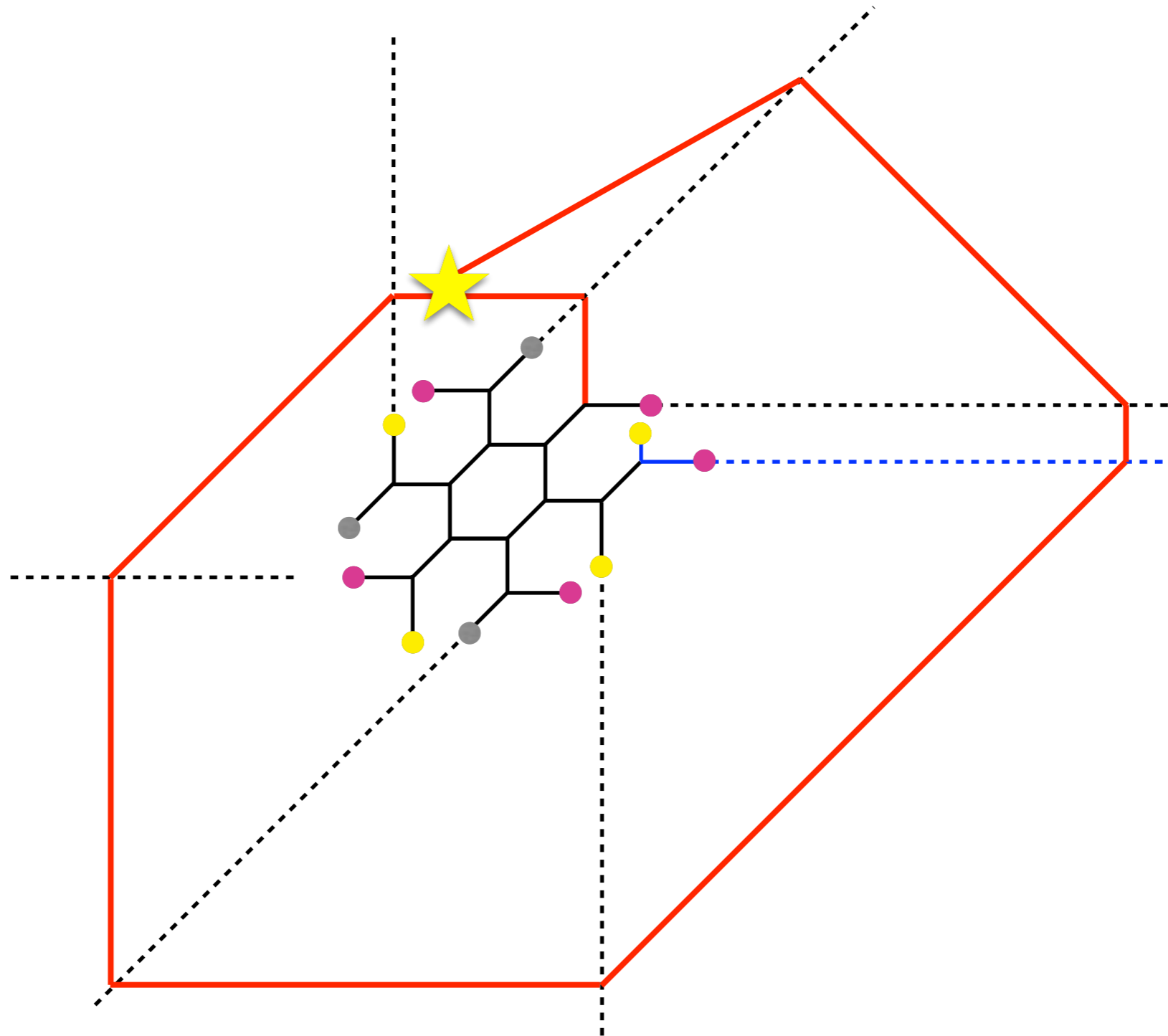
6d KK mode

||

5d Instanton



5D N=1 SU(2) SYM with $N_f=9$ flavor



We cannot move all the 7-branes to infinity



No consistent 5-brane web diagram

Observation

For 5d $\mathcal{N} = 1$ $SU(2)$, N_f flavor

[Seiberg '96]

$0 \leq N_f \leq 7$ **5D UV fixed point**  **“Finite” diagram**

$N_f = 8$ **6D UV fixed point**  **“Tao diagram”**

$N_f \geq 9$ **No UV fixed point**  **No diagram**

Understanding on UV fixed point about 10 years ago

UV fixed point exist for...

	Brane	Field theory
4d N=2 SU(N) N _f flavor	<p>Correct</p> $N_f \leq 2N$ <p>[Witten '97]</p>	<p>Correct</p> $N_f \leq 2N$
5d N=1 SU(2) N _f flavor	<p>Correct [DeWolfe, Hanany, Iqbal, Katz '99]</p> $N_f \leq 7$ <p>[Benini, Benvenuti, Tachikawa 09']</p> <p>(N_f = 8 [Kim, Taki, FY 15'])</p>	<p>Correct</p> $N_f \leq 7$ [Seiberg '97] $N_f = 8 \text{ (6d)}$
5d N=1 SU(N) N _f flavor (N>2)	$N_f \leq 2N$ <p>[Aharony, Hanany '97]</p>	$N_f \leq 2N$ <p>[Intriligator, Morrison, Seiberg '97]</p>

Understanding on UV fixed point about 10 years ago

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5d N=1 SU(2) N _f flavor	<p>Correct [DeWolfe, Hanany, Iqbal, Katz '99]</p> $N_f \leq 7$ <p>[Benini, Benvenuti, Tachikawa 09']</p> <p>(N_f = 8 [Kim, Taki, FY 15'])</p>	<p>Correct</p> $N_f \leq 7$ [Seiberg '97] $N_f = 8 \text{ (6d)}$
5d N=1 SU(N) N _f flavor (N>2)	<p>???</p> $N_f \leq 2N$ <p>[Aharony, Hanany '97]</p>	$N_f \leq 2N$ <p>[Intriligator, Morrison, Seiberg '97]</p>

Conjecture

“Finite” diagram:



5D UV fixed point

“**Tao diagram**”:



6D UV fixed point

No diagram:



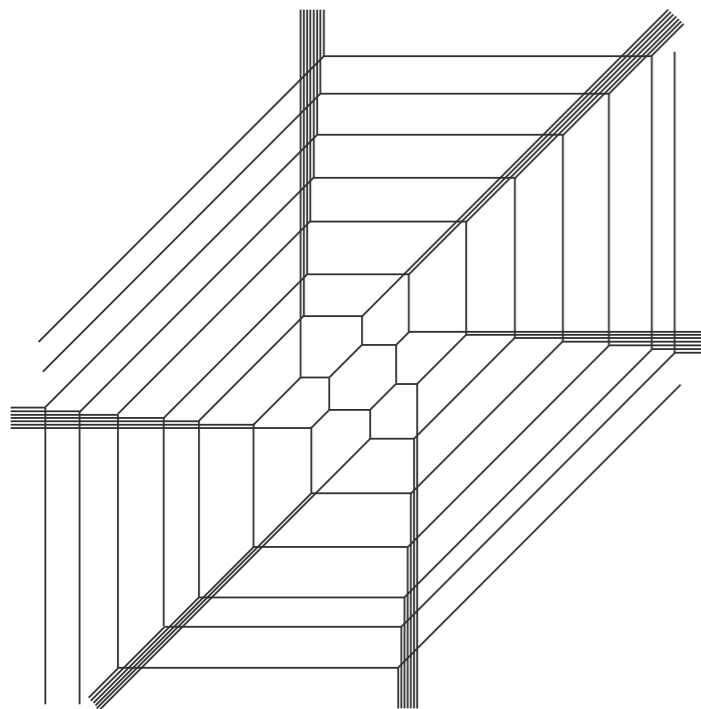
No UV fixed point

[Bergman, Zafrir '14'15]

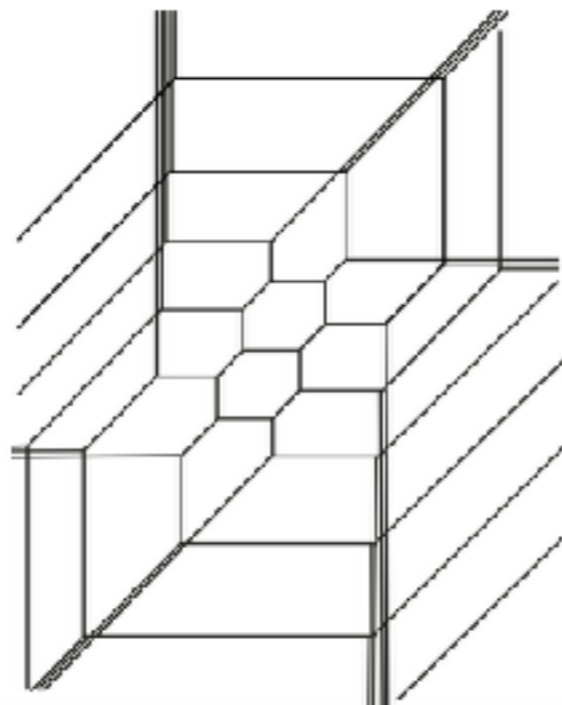
[SS.Kim, M.Taki, FY '15]

[H.Hayashi, SS.Kim, K.Lee, M.Taki, FY '15]

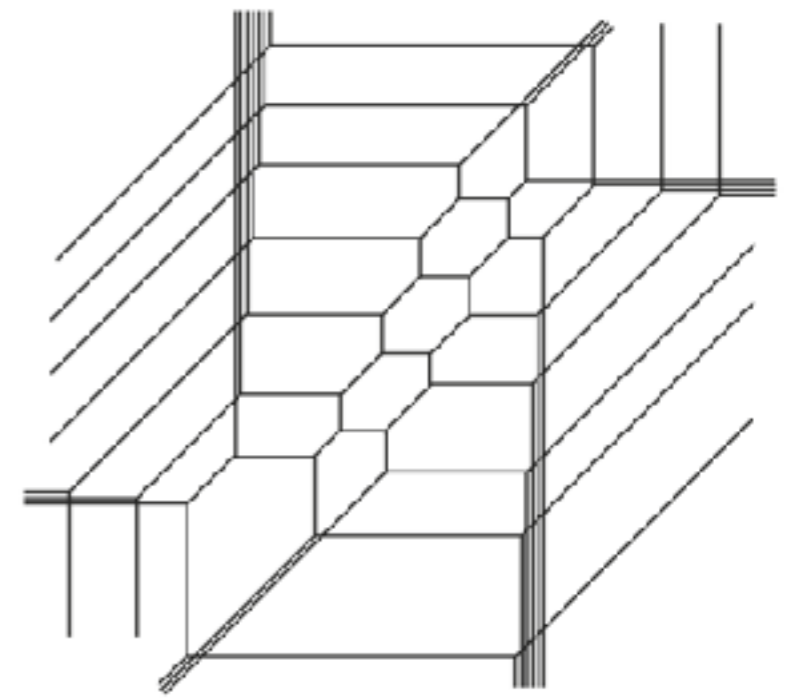
Examples of “**Tao diagrams**”



$SU(2), N_f = 8$



$SU(3), N_f = 10$



$SU(4), N_f = 12$

Understanding on UV fixed point about 3 years ago

UV fixed point exist for...

	Brane	Field theory
4d N=2 SU(N) N _f flavor	<p>Correct</p> $N_f \leq 2N$ <p>[Witten '97]</p>	<p>Correct</p> $N_f \leq 2N$
5d N=1 SU(2) N _f flavor	<p>Correct</p> $N_f \leq 7$ <p>(N_f = 8</p> <p>[DeWolfe, Hanany, Iqbal, Katz '99] [Benini, Benvenuti, Tachikawa 09'] [Kim, Taki, FY 15']</p>	<p>Correct</p> $N_f \leq 7$ [Seiberg '97] $N_f = 8$ (6d)
5d N=1 SU(N) N _f flavor (N>2)	<p>Plausible</p> $N_f \leq 2N + 3$ [Bergman, Zafrir '14'15] $N_f = 2N + 4$ [H.Hayashi, SS.Kim, K.Lee, M.Taki, FY '15]	<p>????</p> $N_f \leq 2N$ [Intriligator, Morrison, Seiberg '97]

Brane analysis or Instanton operator analysis

[Bergman, Zafrir '14'15]

[Yonekura '15]

[H.Hayashi, S.S.Kim, K.Lee, M.Taki, FY '15]

UV fixed point exists for $N_f \leq 2N + 4$

(6d for $N_f = 2N + 4$)



Discrepancy

“Exact” 1-loop beta function Computation

Intriligator, Morisson, Seiberg '97]

(Positive definiteness of effective coupling at Coulomb moduli)

$$\frac{\partial^2 F}{\partial a_i \partial a_j} > 0 \quad \forall a_i, a_j \quad \text{(Eigenvalue)}$$

UV fixed point exists for $N_f \leq 2N$ (for $N > 2$)

Improved version of “Exact” 1-loop beta function Computation

[Jefferson, Kim, Vafa, Zafrir '17]

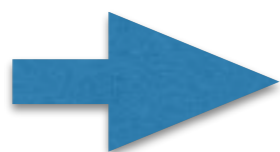
$$\frac{\partial^2 F}{\partial a_i \partial a_j} > 0 \quad \cancel{\forall a_i, a_j}$$

inside the region of Coulomb branch parameter which satisfies

$$\frac{\partial F}{\partial a_i} > 0$$

“Physical Coulomb moduli” is narrower
than the naive expectation!

Require non-empty Coulomb moduli



$$N_f \leq 2N + 4$$

My Conclusion

UV fixed point exist for...

	Brane	Field theory
4d N=2 SU(N) N _f flavor	<p>Correct</p> $N_f \leq 2N$ <p>[Witten '97]</p>	<p>Correct</p> $N_f \leq 2N$
5d N=1 SU(2) N _f flavor	<p>Correct [DeWolfe, Hanany, Iqbal, Katz '99]</p> $N_f \leq 7$ <p>[Benini, Benvenuti, Tachikawa 09']</p> <p>(N_f = 8 [Kim, Taki, FY 15'])</p>	<p>Correct</p> $N_f \leq 7$ [Seiberg '97] $N_f = 8 \text{ (6d)}$
5d N=1 SU(N) N _f flavor (N>2)	<p>Correct!!</p> $N_f \leq 2N + 3$ <p>[Bergman, Zafrir '14'15]</p> $N_f = 2N + 4$ <p>[H.Hayashi, SS.Kim, K.Lee, M.Taki, FY '15]</p>	<p>Correct!!</p> $N_f \leq 2N + 3$ $N_f = 2N + 4 \text{ (6d)}$ <p>[Jefferson, Kim, Vafa, Zafrir '17]</p>