Jets and threshold summation in Deductor

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work with Zoltan Nagy, DESY

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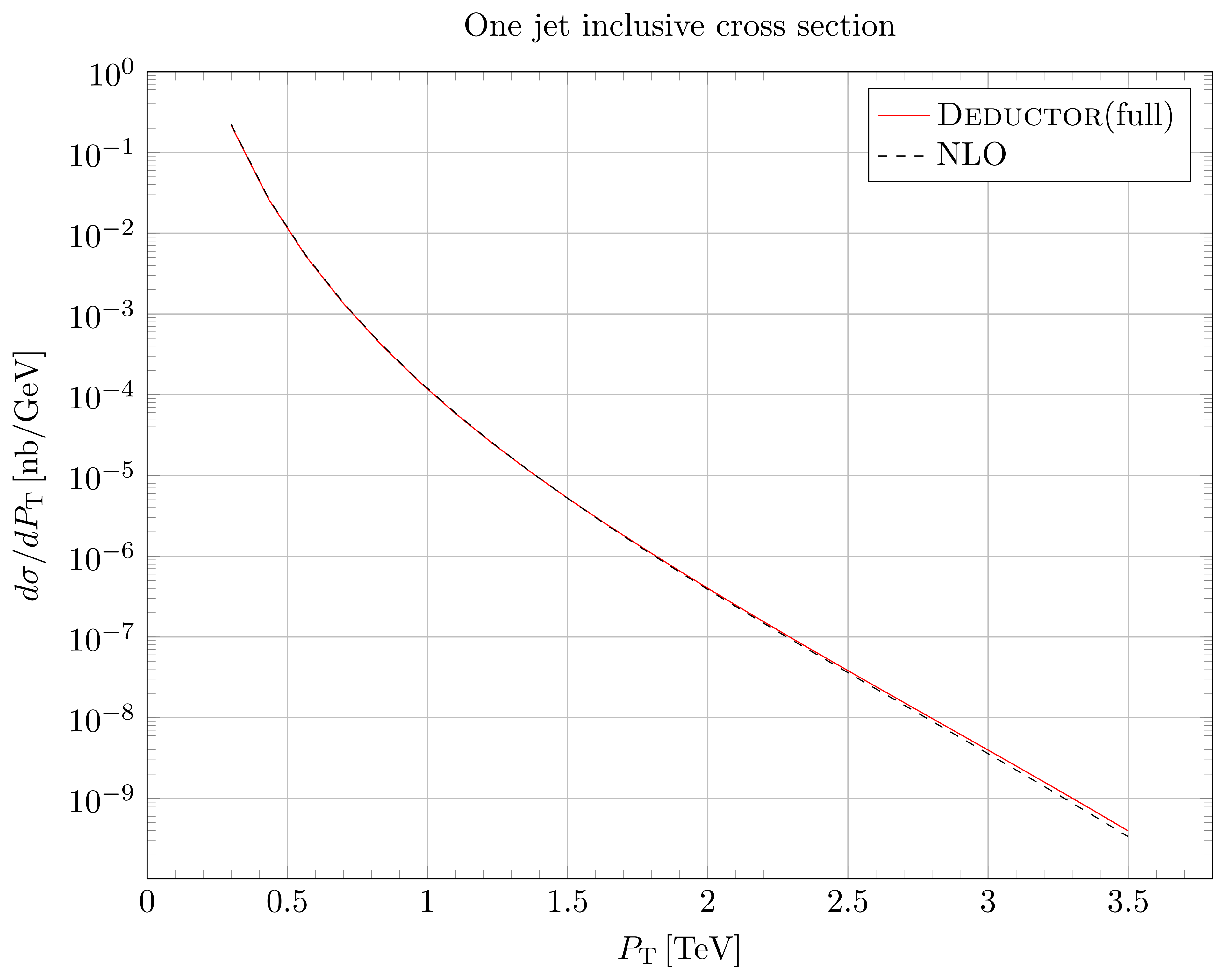
Prequel

- Zoltan Nagy (DESY) and I have a parton shower event generator, Deductor.
- Deductor now includes:
  - Summation of threshold logarithms.
  - Λ ordering (virtuality based) and k_T ordering.
  - User code for checking effect of non-perturbative physics on an infrared safe observbable (with the help of Pythia).


One jet inclusive cross section
Deductor(full)
NLO
$d\sigma/dP_T$ [nb/GeV]
P_T [TeV]
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
0
0.5
1
1.5
2
2.5
3
3.5

General theory for Deductor

- Perturbative calculations at order α_s^k can be systematically improved by working to higher order.

- But what about parton showers?

- Are they just “QCD inspired” or do they fit into a scheme that can by systematically improved?

The statistical space

- Describe momenta and flavors for m final state partons with

$$\{p,f\}_m = \{\eta_{\rm a}, a, \eta_{\rm b}, b, p_1, f_1, \cdots, p_m, f_m\}$$

- Use amplitudes $\big|M(\{p,f\}_m)\big\rangle$.
- This is a vector in spin and color space.
- Expand in color$\otimes$spin basis vectors $\big|\{s,c\}_m\big\rangle$.
- Also need conjugate amplitudes $\big\langle M(\{p,f\}_m)\big|$.
- Expand these in basis vectors $\big\langle\{s',c'\}_m\big|$.

- Describe the statistical state of an ensemble of simulations of the scattering.

- The spins and colors are quantum.

- So we need quantum statistical mechanics.

- Use the density operator

$$\rho(\{p,f\}_m) = \sum_{\{s,s',c,c'\}_m} \rho(\{p,f,s,s',c,c'\}_m) \big|\{s,c\}_m\big\rangle\big\langle\{s',c'\}_m\big|$$

$$\rho(\{p,f\}_m) = \sum_{\{s,s',c,c'\}_m} \rho(\{p,f,s,s',c,c'\}_m)\big|\{s,c\}_m\big\rangle\big\langle\{s',c'\}_m\big|$$

- Think of this as a function named ρ.

- The space of such functions is a linear vector space.

- Call this the statistical space.

- Call a vector $\big|\rho\big)$.

- Everything happens in the statistical space.

$$\rho(\{p,f\}_m) = \sum_{\{s,s',c,c'\}_m} \rho(\{p,f,s,s',c,c'\}_m)\big|\{s,c\}_m\big\rangle\big\langle\{s',c'\}_m\big|$$

- Basis vectors $\big|\{p,f,s,s',c,c'\}_m\big)$:

$$\big(\{p,f,s,s',c,c'\}_m\big|\rho\big) = \rho(\{p,f,s,s',c,c'\}_m)$$

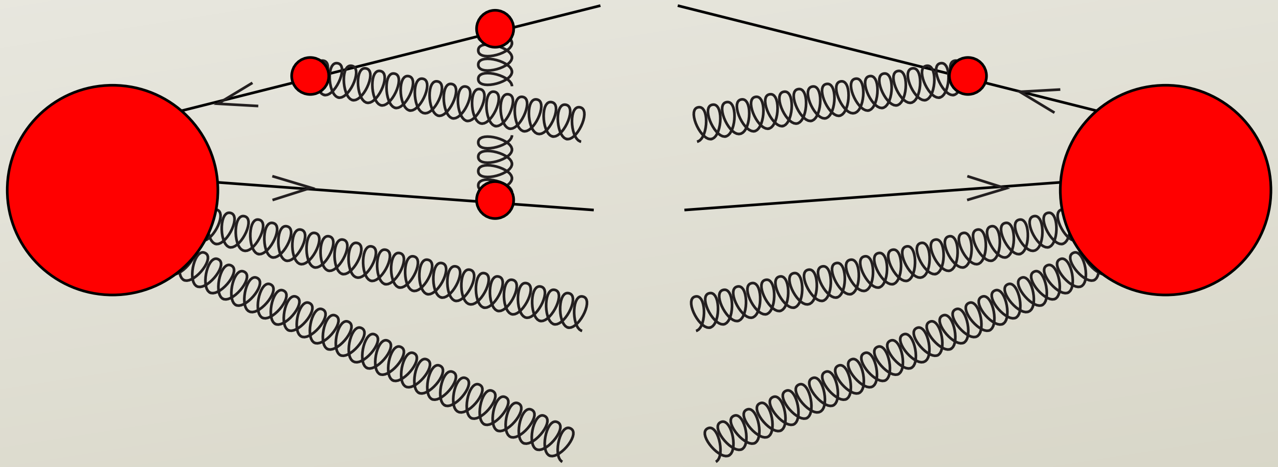
- Inclusive measurement

$$\big(1\big|\rho\big) = \sum_m \frac{1}{m!}\int[d\{p\}_m] \sum_{\{f\}_m}\sum_{\{s,s',c,c'\}_m} \big\langle\{s',c'\}_m\big|\{s,c\}_m\big\rangle \times \big(\{p,f,s,s',c,c'\}_m\big|\rho\big)$$

The infrared sensitive operator

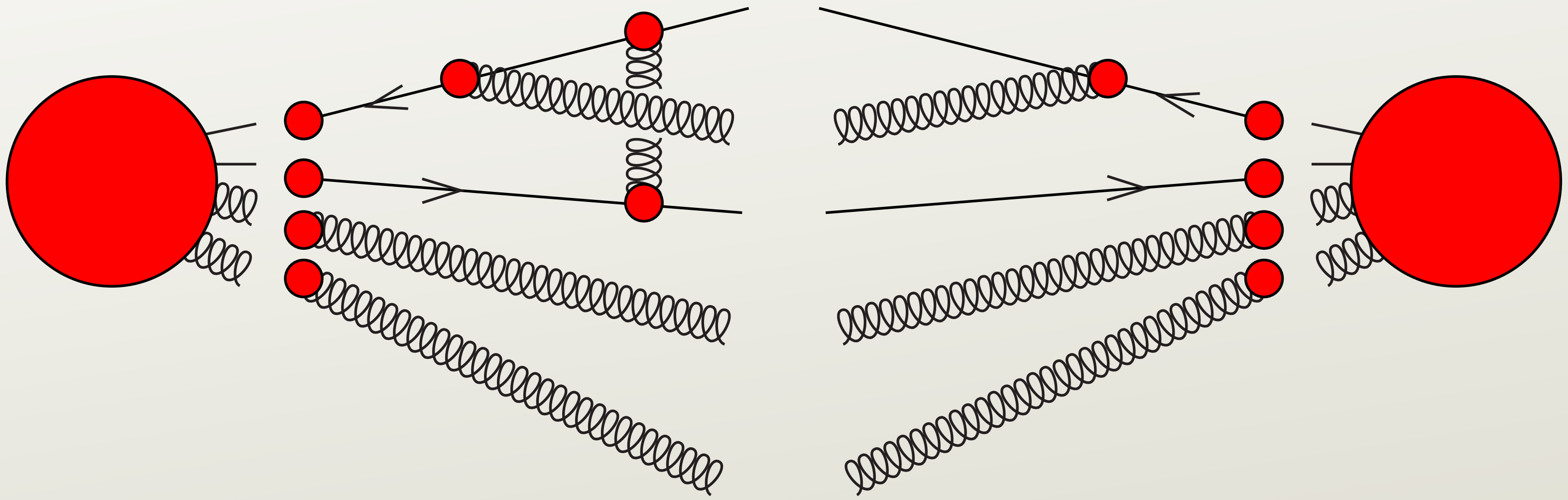
$$\mathcal{D}(\mu^2)$$

- Amplitudes have singularities when partons are soft or collinear.
- They have divergences $1/\epsilon$ from loops.



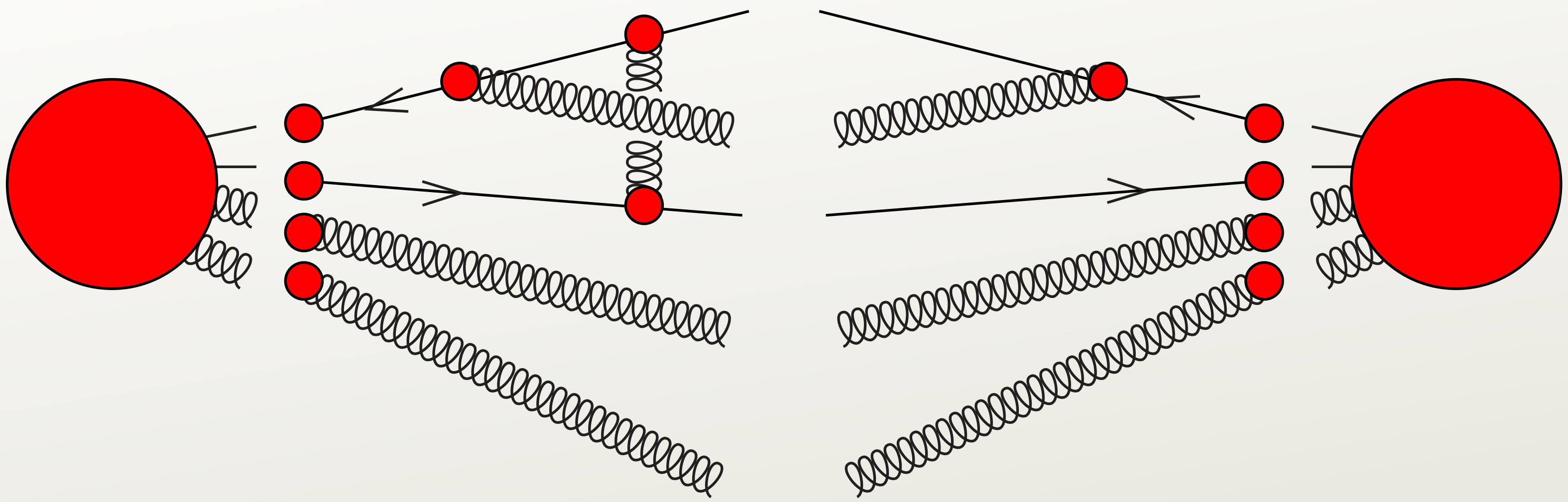
- We want to describe the singularity structure.
- Consider that everything inside the red subamplitudes is hard.

- Approximate the momenta coming from the hard part as fixed and on shell.



- This gives us an operator $\mathcal{D}(\mu^2)$.

$$
\begin{aligned}
&\big(\{\hat{p},\hat{f},\hat{s},\hat{s}',\hat{c},\hat{c}'\}_{m+n}\big|\rho(\mu^2)\big) \\
&\qquad \sim \frac{1}{m!}\int[d\{p\}_m]\sum_{\{f\}_m}\sum_{\{s,s',c,c'\}_m} \\
&\qquad\qquad \times\big(\{\hat{p},\hat{f},\hat{s},\hat{s}',\hat{c},\hat{c}'\}_{m+n}\big|\mathcal{D}(\mu^2)\big|\{p,f,s,s',c,c'\}_m\big) \\
&\qquad\qquad \times\big(\{p,f,s,s',c,c'\}_m\big|\rho_{\rm hard}(\mu^2)\big)
\end{aligned}
$$



- In $\mathcal{D}(\mu^2)$ we need an ultraviolet cutoff $\mu_{\rm s}^2$.

- *e.g.* $\Lambda^2 < \mu_{\rm s}^2$, where $\Lambda^2 \propto$ virtuality.

- We set $\mu_{\rm s}^2 = \mu^2$.

- We also need a momentum mapping.

- We assume that $\mathcal{D}(\mu^2)$ is available and investigate what to do with it.

Standard perturbation theory

cancel initial state poles

"Add back" the subtractions

Subtractions

$$\sigma[J] = \big(1\big|\big[\mathcal{F}_{\overline{\mathrm{MS}}}(\mu_{\mathrm{H}}^2) \circ \mathcal{Z}_F(\mu_{\mathrm{H}}^2)\big]\,\mathcal{D}(\mu_{\mathrm{H}}^2)\,\mathcal{D}^{-1}(\mu_{\mathrm{H}}^2)\,\mathcal{O}_J\big|\rho(\mu_{\mathrm{H}}^2)\big) + \mathcal{O}(\alpha_{\mathrm{s}}^{B+k+1}) + \mathcal{O}(\mu_{\mathrm{f}}^2/Q[J]^2)$$

Operator to measure desired cross section

Feynman diagrams to $\alpha_{\mathrm{s}}^{B+k}$

- Normally $\mathcal{D}^{-1}(\mu_{\mathrm{H}}^2)$ is constructed by hand and $\mathcal{D}(\mu_{\mathrm{H}}^2)$ is its inverse.

- It is useful to rewrite our cross section a bit:

$$\sigma[J] = \big(1\big|\big[\mathcal{F}_{\overline{\rm MS}}(\mu_{\rm H}^2) \circ \mathcal{Z}_F(\mu_{\rm H}^2)\big]\,\mathcal{D}(\mu_{\rm H}^2)\,\mathcal{D}^{-1}(\mu_{\rm H}^2)\,\mathcal{O}_J\big|\rho(\mu_{\rm H}^2)\big)$$

becomes

$$\sigma[J] = \big(1\big|\big[\mathcal{F}_{\overline{\rm MS}}(\mu_{\rm H}^2) \circ \mathcal{Z}_F(\mu_{\rm H}^2)\big]\,\mathcal{O}_J\,\mathcal{D}(\mu_{\rm H}^2)\big|\rho_{\rm H}\big)$$

where

$$\big|\rho_{\rm H}\big) = \mathcal{D}^{-1}(\mu_{\rm H}^2)\big|\rho(\mu_{\rm H}^2)\big)$$

Shower oriented parton distribution functions

- Standard $\overline{\mathrm{MS}}$ parton distribution functions are not quite right for use in a shower.
- If you use k_{T} for the shower hardness parameter, $\overline{\mathrm{MS}}$ is almost OK, although imposing a UV cut is not the same as subtracting a UV pole.
- If you use Λ for the shower hardness parameter, $\overline{\mathrm{MS}}$ parton distributions need substantial changes.
- Define

$$\mathcal{F}_{\overline{\mathrm{MS}}}(\mu^2) = [\mathcal{F}(\mu^2) \circ \mathcal{K}(\mu^2)]$$

The inclusive infrared finite operator $\mathcal{V}(\mu^2)$

- Define

$$\mathcal{X}(\mu^2) = \left[\mathcal{F}(\mu^2) \circ \mathcal{K}(\mu^2) \circ \mathcal{Z}_F(\mu^2)\right] \mathcal{D}(\mu^2)\, \mathcal{F}^{-1}(\mu^2)$$

- Then $\big(1\big|\mathcal{X}(\mu^2)\big|\{p, f, s, s', c, c'\}_m\big)$ is IR finite.

- Define an operator $\mathcal{V}(\mu^2)$ that leaves $\{p, f\}_m$ unchanged, with

$$\big(1\big|\mathcal{V}(\mu^2) = \big(1\big|\mathcal{X}(\mu^2)$$

- Then $\mathcal{V}(\mu^2)$ is IR finite.

The result

. . . without the derivation.

Operator to measure desired cross section

parton distribution functions and luminosity factor

$$\sigma[J] = \big(1\big|\mathcal{O}_J\,\mathcal{U}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)\,\mathcal{U}_\mathcal{V}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)\,\mathcal{F}(\mu_\mathrm{H}^2)\big|\rho_\mathrm{H}\big) + \mathcal{O}(\alpha_\mathrm{s}^{B+k+1}) + \mathcal{O}(\mu_\mathrm{f}^2/Q[J]^2)$$

Feynman diagrams to α_s^{B+k} with subtractions

- The most important parts are $\mathcal{U}_\mathcal{V}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)$ and $\mathcal{U}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)$.

$$\sigma[J] = \big(1\big|\mathcal{O}_J\,\mathcal{U}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)\,\mathcal{U}_\mathcal{V}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)\,\mathcal{F}(\mu_\mathrm{H}^2)\big|\rho_\mathrm{H}\big) + \mathcal{O}(\alpha_\mathrm{s}^{B+k+1}) + \mathcal{O}(\mu_\mathrm{f}^2/Q[J]^2)$$

$$\mathcal{U}_\mathcal{V}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2) = \mathcal{V}^{-1}(\mu_\mathrm{f}^2)\,\mathcal{V}(\mu_\mathrm{H}^2) = \mathbb{T}\exp\left(\int_{\mu_\mathrm{f}^2}^{\mu_\mathrm{H}^2}\frac{d\mu^2}{\mu^2}\,\mathcal{S}_\mathcal{V}(\mu^2)\right)$$

$$\mathcal{S}_\mathcal{V}(\mu^2) = \mathcal{V}^{-1}(\mu^2)\,\mu^2\frac{d}{d\mu^2}\,\mathcal{V}(\mu^2)$$

- Does not create new partons.
- Provides perturbative corrections to the hard scattering state $\big|\rho_\mathrm{H}\big)$.
- Sums threshold logarithms associated with $\big|\rho_\mathrm{H}\big)$.

$$\sigma[J] = \big(1\big|\mathcal{O}_J\,\mathcal{U}(\mu_{\rm f}^2,\mu_{\rm H}^2)\,\mathcal{U}_{\mathcal{V}}(\mu_{\rm f}^2,\mu_{\rm H}^2)\,\mathcal{F}(\mu_{\rm H}^2)\big|\rho_{\rm H}\big) + \mathcal{O}(\alpha_{\rm s}^{B+k+1}) + \mathcal{O}(\mu_{\rm f}^2/Q[J]^2)$$

$$\begin{aligned}\mathcal{U}(\mu_{\rm f}^2,\mu_{\rm H}^2) &= \mathcal{V}(\mu_{\rm f}^2)\,\mathcal{X}^{-1}(\mu_{\rm f}^2)\,\mathcal{X}(\mu_{\rm H}^2)\,\mathcal{V}^{-1}(\mu_{\rm H}^2)\\ &= \mathbb{T}\exp\left(\int_{\mu_{\rm f}^2}^{\mu_{\rm H}^2}\frac{d\mu^2}{\mu^2}\,\mathcal{S}(\mu^2)\right)\end{aligned}$$

$$\begin{aligned}\mathcal{S}(\mu^2) = {} & \mathcal{V}(\mu^2)\,\mathcal{F}(\mu^2)\,\mathcal{D}^{-1}(\mu^2)\left[\mu^2\frac{d}{d\mu^2}\mathcal{D}(\mu^2)\right]\mathcal{F}^{-1}(\mu^2)\,\mathcal{V}^{-1}(\mu^2)\\ & - \left[\mu^2\frac{d}{d\mu^2}\mathcal{F}(\mu^2)\right]\mathcal{F}^{-1}(\mu^2) - \left[\mu^2\frac{d}{d\mu^2}\mathcal{V}(\mu^2)\right]\mathcal{V}^{-1}(\mu^2)\;.\end{aligned}$$

- Creates new partons.
- Preserves probabilities: $\big(1\big|\mathcal{U}(\mu_2^2,\mu_1^2) = \big(1\big|$.

- What about the error estimate?

$$\sigma[J] = \big(1\big|\mathcal{O}_J\,\mathcal{U}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)\,\mathcal{U}_\mathcal{V}(\mu_\mathrm{f}^2,\mu_\mathrm{H}^2)\,\mathcal{F}(\mu_\mathrm{H}^2)\big|\rho_\mathrm{H}\big) + \mathcal{O}(\alpha_\mathrm{s}^{B+k+1}) + \mathcal{O}(\mu_\mathrm{f}^2/Q[J]^2)$$

- There is a power suppressed error $\mathcal{O}(\mu_\mathrm{f}^2/Q[J]^2)$.

- Such an error is part of factorization.

- There is a perturbative error $\mathcal{O}(\alpha_\mathrm{s}^{B+k+1})$.

- This is because we calculate only to order α_s^{B+k}.

- If $\mathcal{O}_J$ involves different scales, say μ_H^2 and μ_L^2 we could have $\alpha_\mathrm{s}^B[\alpha_\mathrm{s}\log^2(\mu_\mathrm{H}^2/\mu_\mathrm{L}^2)]^k$.

- We can hope that the shower sums the most important large logarithms and leaves us with a smaller error.

- Implementations will involve further approximations.

Summary

- Perturbative calculations

$$\sigma[J] = \big(1\big|\big[\mathcal{F}_{\overline{\rm MS}}(\mu^2)\circ \mathcal{Z}_F(\mu^2)\big]\,\mathcal{D}(\mu^2)\,\mathcal{D}^{-1}(\mu^2)\,\mathcal{O}_J\big|\rho(\mu^2)\big) + \mathcal{O}(\alpha_{\rm s}^{B+k+1}) + \mathcal{O}(\mu_{\rm f}^2/Q[J]^2)$$

can be systematically improved by working to higher order.

- Parton shower calculations

$$\sigma[J] = \big(1\big|\mathcal{O}_J\,\mathcal{U}(\mu_{\rm f}^2,\mu_{\rm H}^2)\,\mathcal{U}_{\mathcal{V}}(\mu_{\rm f}^2,\mu_{\rm H}^2)\,\mathcal{F}(\mu_{\rm H}^2)\big|\rho_{\rm H}\big) + \mathcal{O}(\alpha_{\rm s}^{B+k+1}) + \mathcal{O}(\mu_{\rm f}^2/Q[J]^2)$$

can be systematically improved by working to higher order.

Deductor shower

$$\sigma[J] = \big(1\big|\mathcal{O}_J\,\mathcal{U}(\mu_{\rm f}^2,\mu_{\rm H}^2)\,\mathcal{U}_{\mathcal{V}}(\mu_{\rm f}^2,\mu_{\rm H}^2)\,\mathcal{F}(\mu_{\rm H}^2)\big|\rho_{\rm H}\big) + \mathcal{O}(\alpha_{\rm s}^{B+2}) + \mathcal{O}(\mu_{\rm f}^2/Q[J]^2)$$

- The hard scattering statistical state is

$$\big|\rho_{\rm H}\big) = \mathcal{D}^{-1}(\mu_{\rm H}^2)\big|\rho(\mu_{\rm H}^2)\big)$$

  expanded to NLO, $\alpha_{\rm s}^{B+1}$.

- We don't have the code for this, so we use just LO, $\alpha_{\rm s}^{B}$.
- We simply average over spins.
- We approximate color (LC+).

$$\sigma[J] \approx \big(1\big|\mathcal{O}_J\,\mathcal{U}(\mu_{\rm f}^2,\mu_{\rm H}^2)\,\mathcal{U}_{\mathcal{V}}(\mu_{\rm f}^2,\mu_{\rm H}^2)\,[\mathcal{F}(\mu_{\rm H}^2)/\mathcal{F}_{\overline{\rm MS}}(\mu_{\rm H}^2)]\;\mathcal{F}_{\overline{\rm MS}}(\mu_{\rm H}^2)\big|\rho_{\rm H}\big)$$

- For the shower, we have a first order splitting kernel:

$$\mathcal{U}(\mu_{\rm f}^2,\mu_{\rm H}^2) = \mathbb{T}\exp\left(\int_{\mu_{\rm f}^2}^{\mu_{\rm H}^2}\frac{d\mu^2}{\mu^2}\,\frac{\alpha_{\rm s}(\mu^2)}{2\pi}\mathcal{S}^{(1)}(\mu^2)\right)$$

- The threshold factor is

$$\mathcal{U}_{\mathcal{V}}(\mu_{\rm f}^2,\mu_{\rm H}^2)\,[\mathcal{F}(\mu_{\rm H}^2)/\mathcal{F}_{\overline{\rm MS}}(\mu_{\rm H}^2)]$$

- In $\mathcal{U}_{\mathcal{V}}(\mu_{\rm f}^2,\mu_{\rm H}^2)$, we have a first order exponent,

$$\mathcal{U}_{\mathcal{V}}(\mu_{\rm f}^2,\mu_{\rm H}^2) = \mathbb{T}\exp\left(\int_{\mu_{\rm f}^2}^{\mu_{\rm H}^2}\frac{d\mu^2}{\mu^2}\,\frac{\alpha_{\rm s}(\mu^2)}{2\pi}\mathcal{S}_{\mathcal{V}}^{(1)}(\mu^2)\right)$$

- Most important term in the threshold exponent:

$$
\begin{aligned}
&\mathcal{S}_{\mathcal{V}}(\mu^2)\big|\{p,f,c,c'\}_m\big) = \\
&\qquad \frac{\alpha_{\rm s}}{2\pi}\int_{1/(1+\mu^2/\mu_{\rm H}^2)}^{1} dz\left[\left(1-\frac{f_{a/A}(\eta_{\rm a}/z,\mu^2)}{f_{a/A}(\eta_{\rm a},\mu^2)}\right)\frac{2C_a}{1-z}\right. \\
&\qquad\qquad \left. + \left(1-\frac{f_{b/B}(\eta_{\rm b}/z,\mu^2)}{f_{b/B}(\eta_{\rm b},\mu^2)}\right)\frac{2C_b}{1-z}\right][1\otimes 1]\,\big|\{p,f,c,c'\}_m\big) \\
&\qquad + \cdots\ ,
\end{aligned}
$$

- This is not important for $\mu^2 \ll \mu_{\rm H}^2$, unless the parton distribution functions are falling steeply with η/z.

Some results

Gaps between jets

The measurement

- $\sqrt{s} = 7$ TeV.
- Measure jets with the anti-k_T algorithm with $R = 0.6$.
- Jets must have $P_\mathrm{T} > p_\mathrm{T}^\mathrm{cut} = 20$ GeV, $|y| < 4.4$.
- Pick the two highest P_T jets, labeled so that $y_1 > y_2$.
- Define $\bar{p}_\mathrm{T} = (P_{\mathrm{T},1} + P_{\mathrm{T},2})/2$, $\Delta y = y_1 - y_2$.
- "Gap": no jets with $p_\mathrm{T} > p_\mathrm{T}^\mathrm{cut}$ and $y_2 < y < y_1$.
- $f(\bar{p}_\mathrm{T}, \Delta y)$ is the fraction of events with a gap.

Motivation

- This is a problem of considerable theoretical interest.
- There are large logarithms: $\log(\bar{p}_\mathrm{T}/p_\mathrm{T}^\mathrm{cut})$ and Δy.
- There is data from Atlas at 7 TeV.
- For the most accurate prediction, one should combine NLO with a parton shower. (Höche and Schönherr.)
- Here we check what perturbation theory can do by itself
- and we see what DEDUCTOR can do by itself
- and we see if the ordering variable matters.

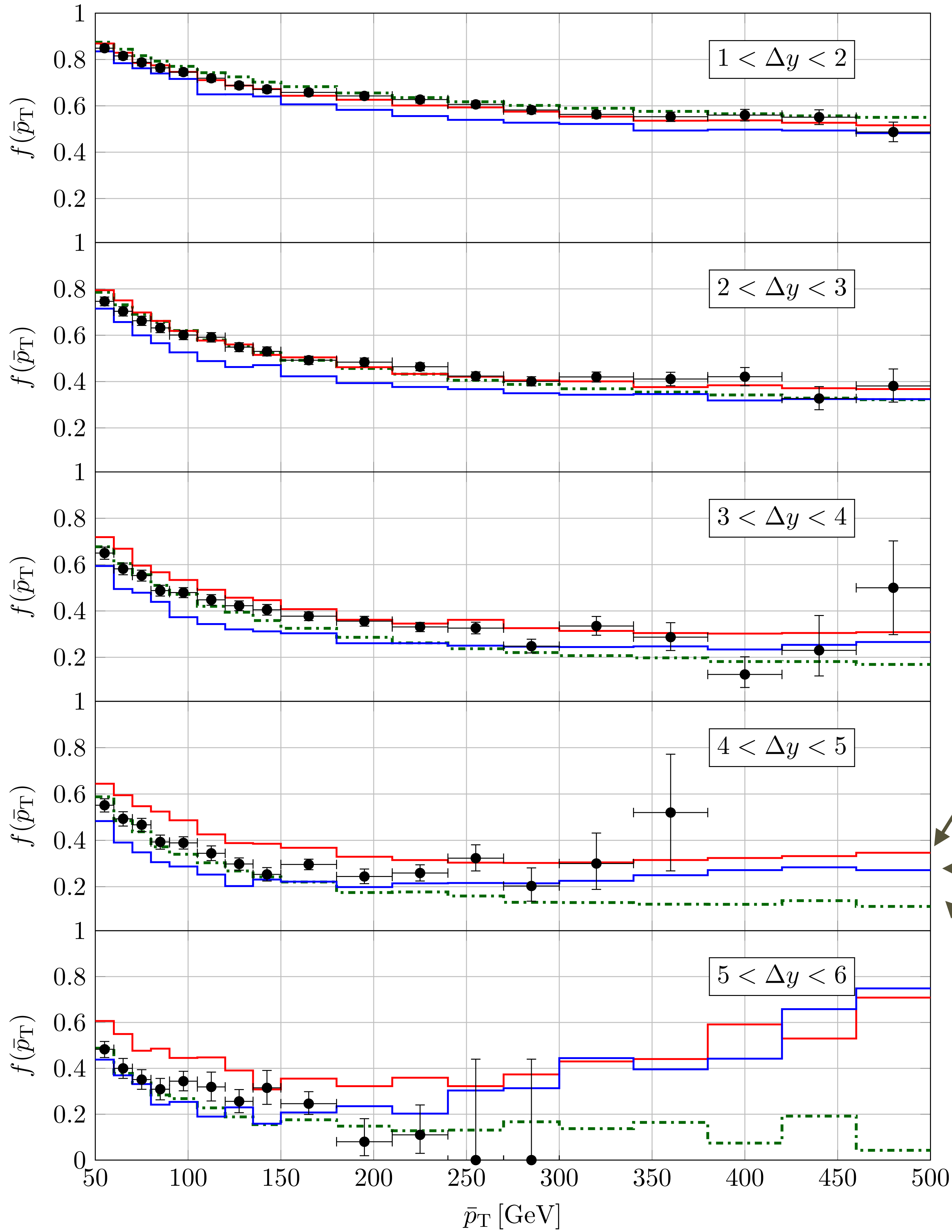
Calculations

- Use DEDUCTOR with either Λ or $k_{\rm T}$ ordering.
- Perturbative calculation (with NLOjet++).

$$f(\bar p_{\rm T}, \Delta y) = 1 - \frac{d\sigma_3/[d\bar p_{\rm T}\, d\Delta y]}{d\sigma_2/[d\bar p_{\rm T}\, d\Delta y]}$$

- $d\sigma_2/[d\bar p_{\rm T}\, d\Delta y]$ is the inclusive 2-jet cross section at NLO.
- $d\sigma_3/[d\bar p_{\rm T}\, d\Delta y]$ is the inclusive 3-jet cross section for the third jet with $p_{\rm T} > p_{\rm T}^{\rm cut}$, at NLO.

Results



Data from Atlas

DEDUCTOR-Λ

DEDUCTOR-k_T

perturbative

One jet inclusive cross section

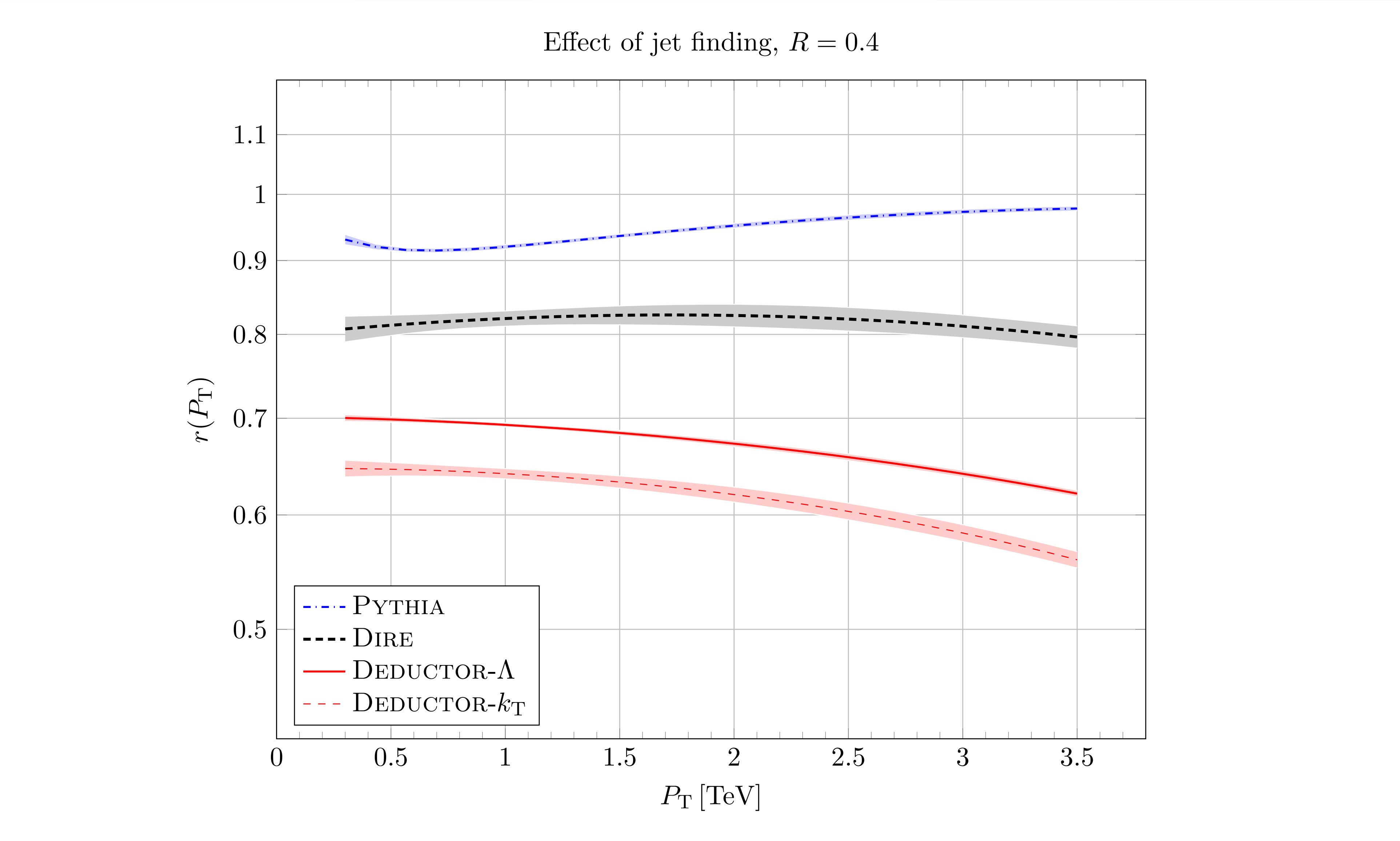
Effect of jet finding

- A parton is not the same as a jet.
- Use the anti-k_{T} jet algorithm with $R = 0.4$.
- Examine

$$r(P_{\mathrm{T}}) = \frac{d\sigma(\mathrm{std.})/dP_{\mathrm{T}}}{d\sigma(\mathrm{LO})/dP_{\mathrm{T}}}$$

- $d\sigma(\mathrm{std.})/dP_{\mathrm{T}}$ from DEDUCTOR
  with no summation of threshold logs.

- Compare to PYTHIA and DIRE.



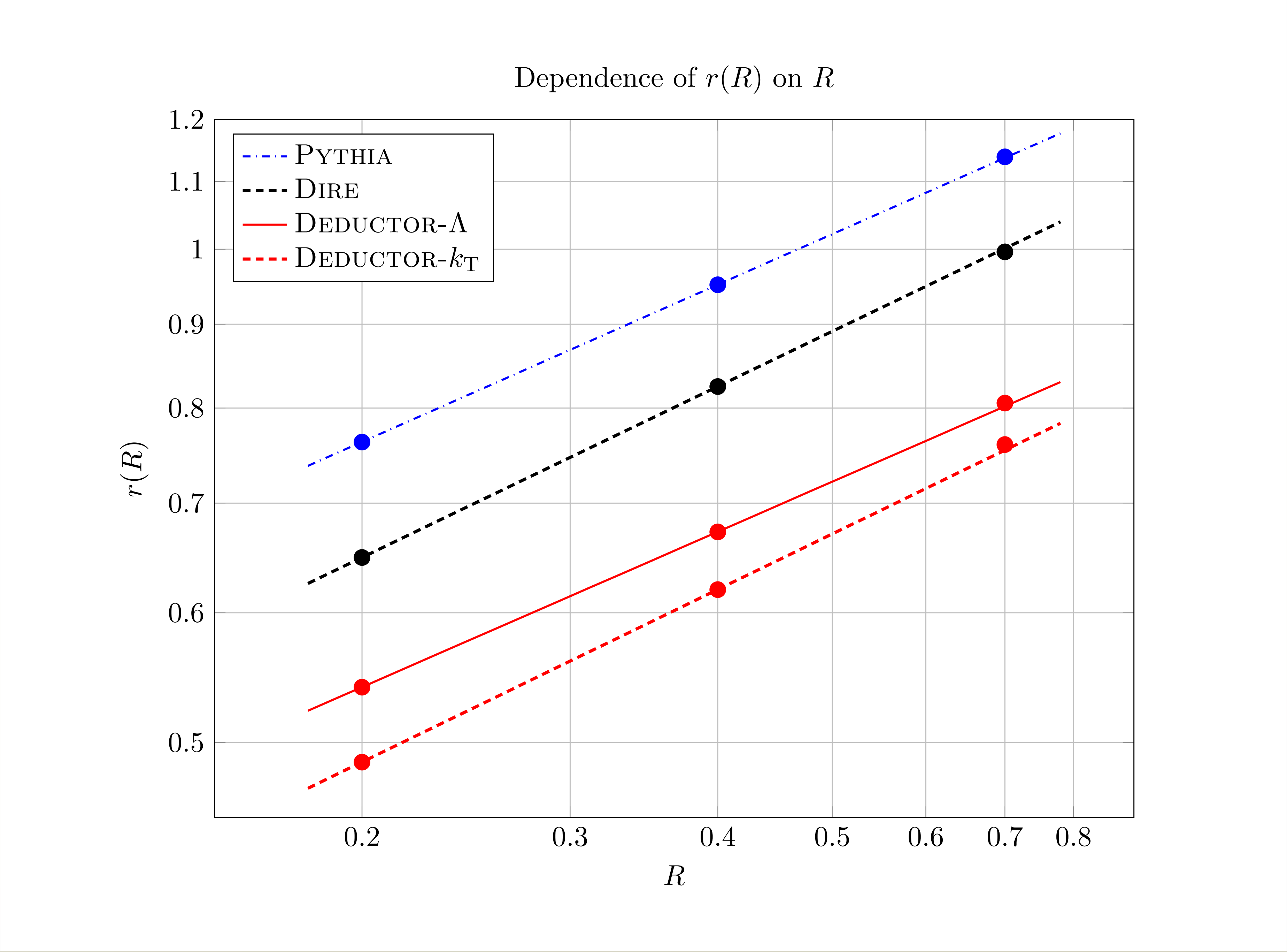
- Differences among parton shower algorithms can yield different results.

  - splitting functions away from soft and collinear limits.

  - ordering variable.

  - global vs. local momentum mapping.

- Look at dependence on R for $P_{\mathrm{T}} = 2$ TeV.



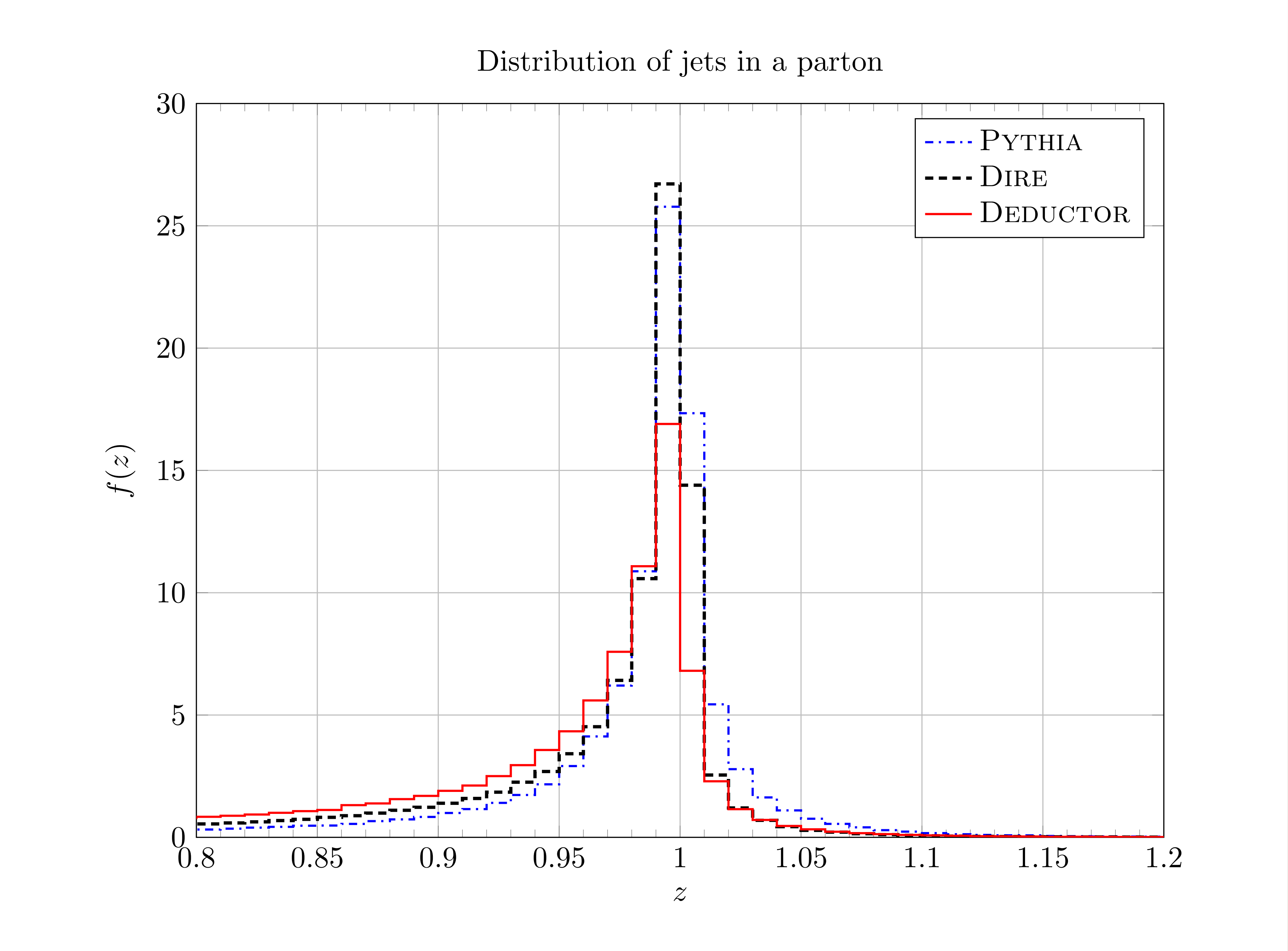
- The slopes agree pretty well.
- The values of $r(0.4)$ do not agree.

- Examine the distribution $f(z)$ of jets in a parton.

- Choose events with $P_{\rm T}^{\rm Born} \approx 3$ TeV, $|y_i| < 2$.

- Let $f(z)$ be the distribution of jets as a function of z,

$$z = \frac{P_{\rm T}^{\rm jet}}{P_{\rm T}^{\rm Born}}$$



- The distributions are pretty similar.
- A DEDUCTOR shower is more likely to radiate some p_{T} outside of the jet.

Effect of threshold logarithms

- DEDUCTOR contains a summation of threshold logarithms.

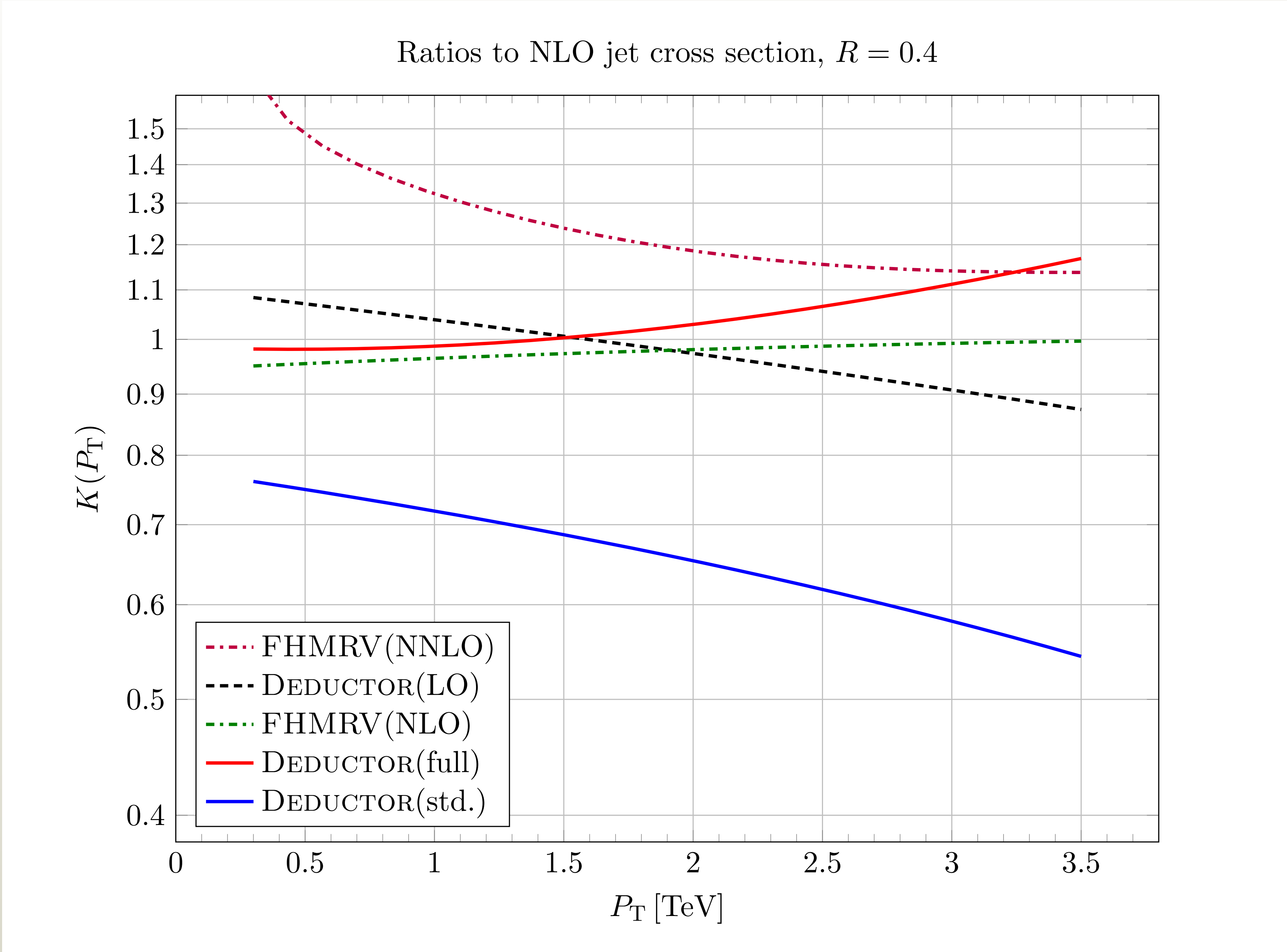
- These are important for high $P_{\rm T}$.

- Define ratios to NLO

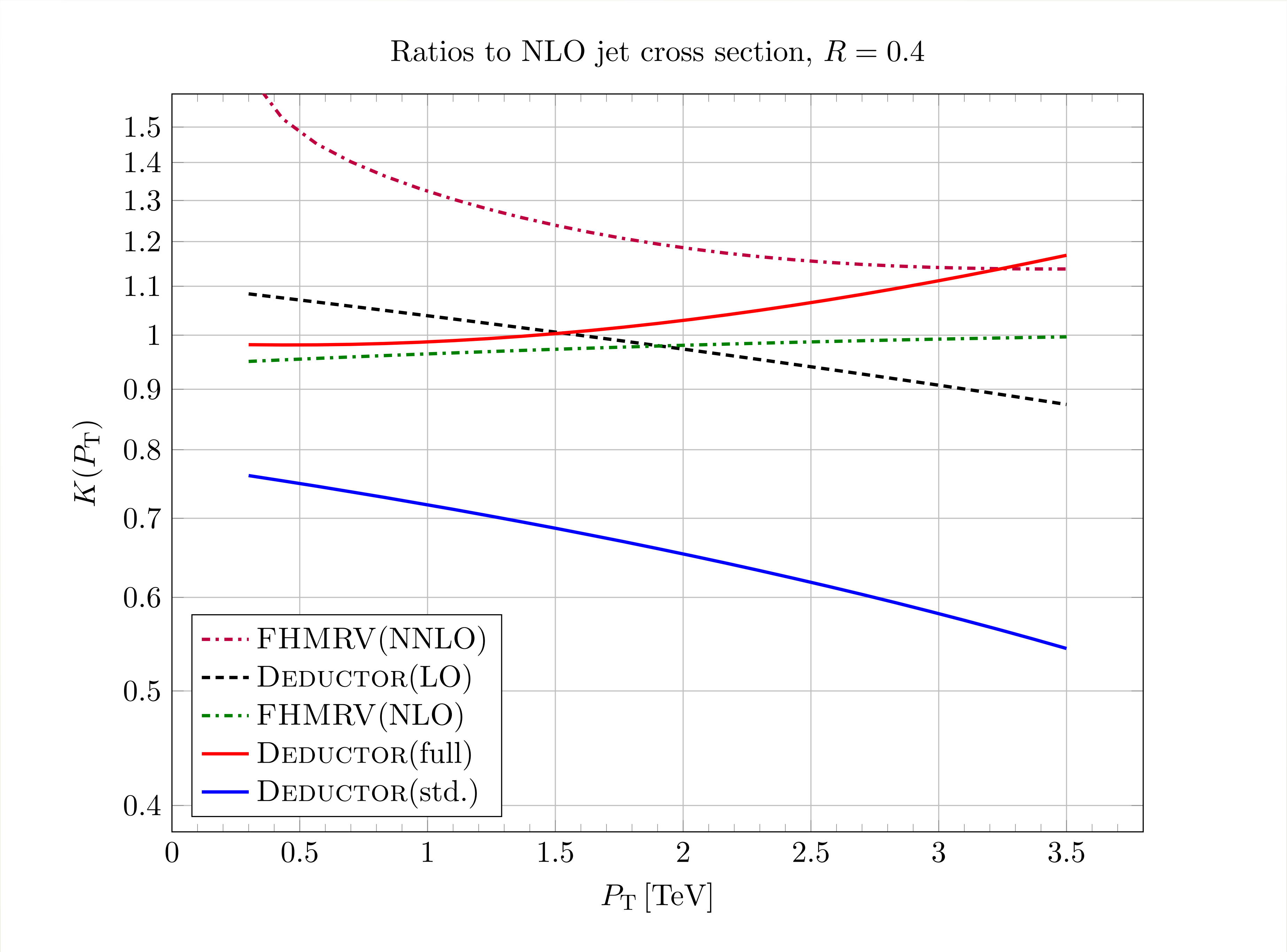
$$K(\text{“A”}) = \frac{d\sigma(\text{“A”})/dP_{\rm T}}{d\sigma(\text{NLO})/dP_{\rm T}}$$

- This includes analytical summations by FHMRV:

  de Florian, Hinderer, Mukherjee, Ringer and Vogelsang.



- LO is close to NLO by choice of scales.
- DEDUCTOR(std.) is smaller, as we have seen.



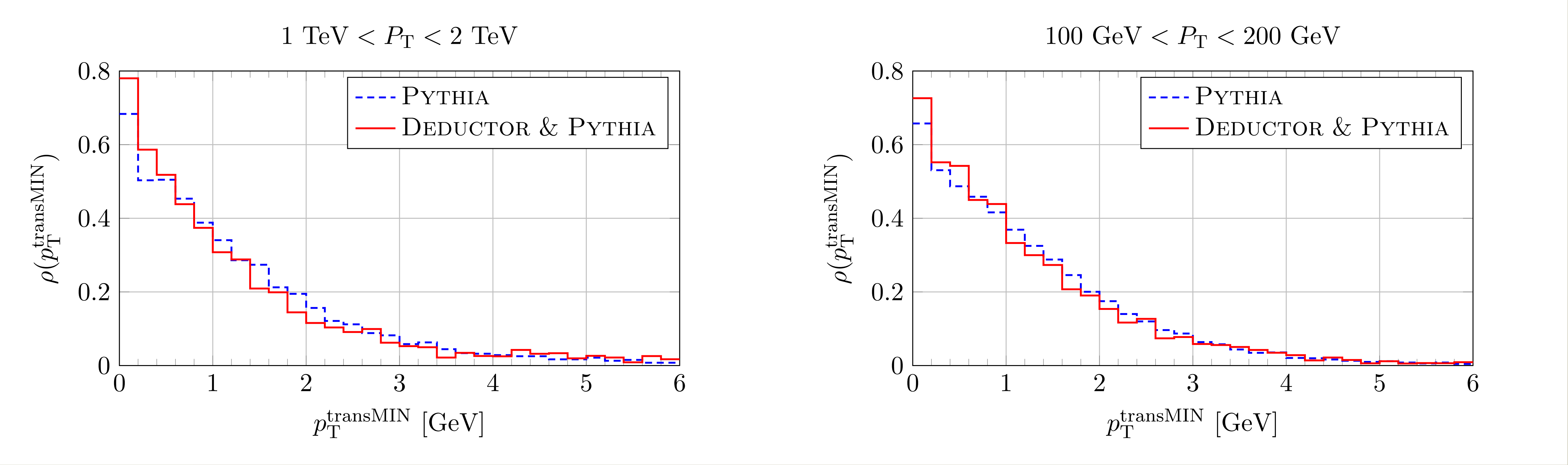
- DEDUCTOR(full)/ DEDUCTOR(std.) is quite large.
- FHMRV(NLO) is close to DEDUCTOR(full).
- New Liu, Moch, Ringer summation is similar to FHMRV(NLO).

Conclusion on the jet cross section

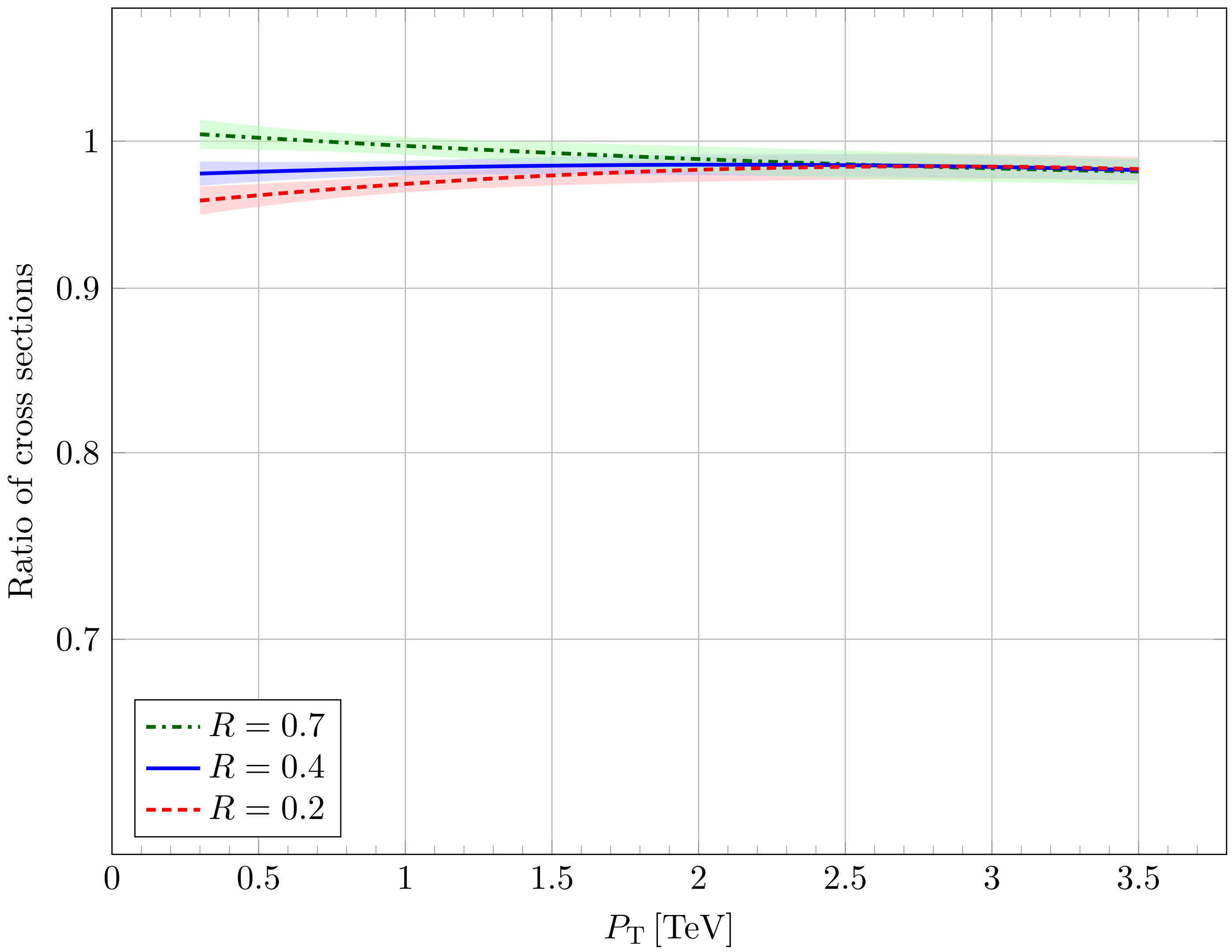
- Differences among parton shower algorithms can yield different results.
- The threshold summation factor is important at large P_{T}.
- The one jet inclusive cross section contains factors that are substantially different from 1 and largely cancel.

Additional plots

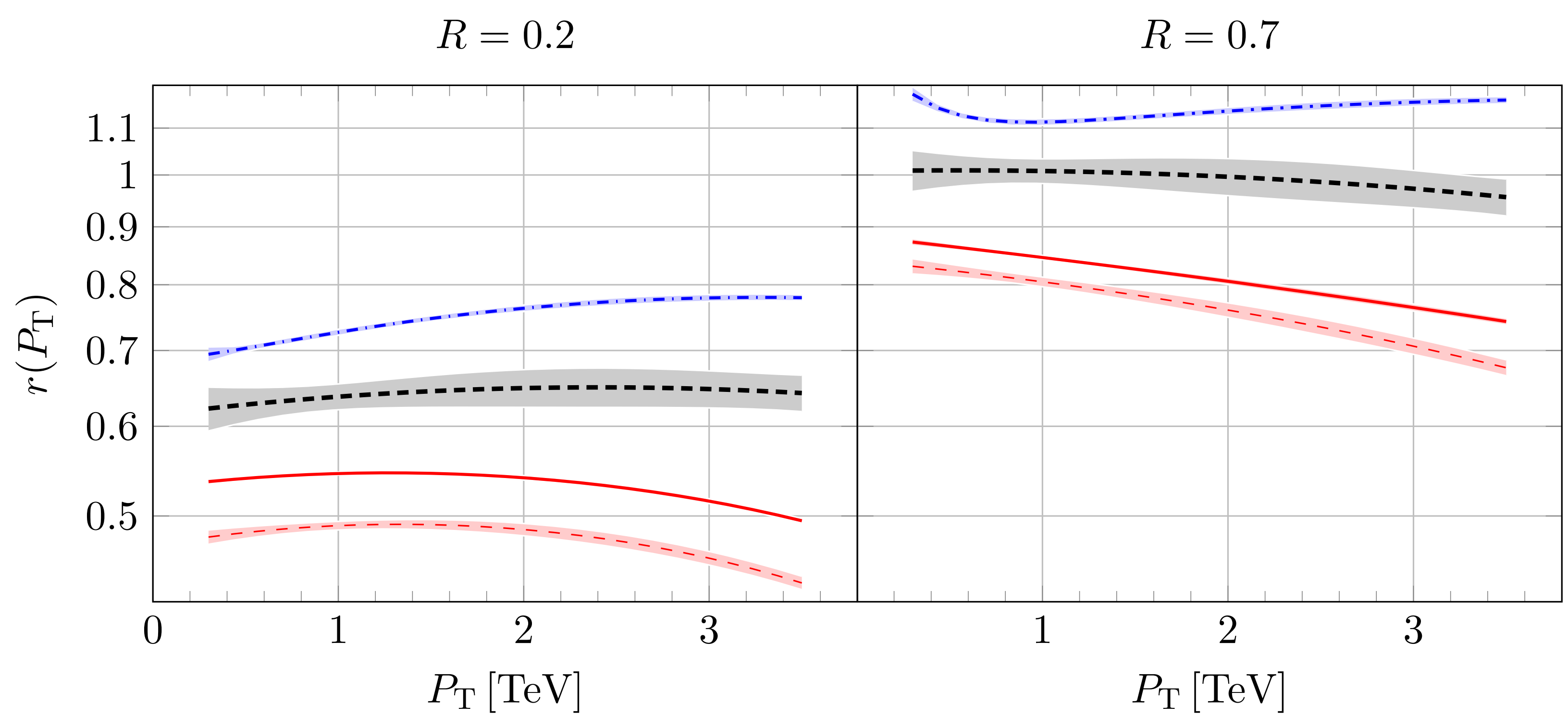
- Distribution of $p_{\mathrm{T}}^{\mathrm{transMIN}}$ after underlying event and PYTHIA hadronization.



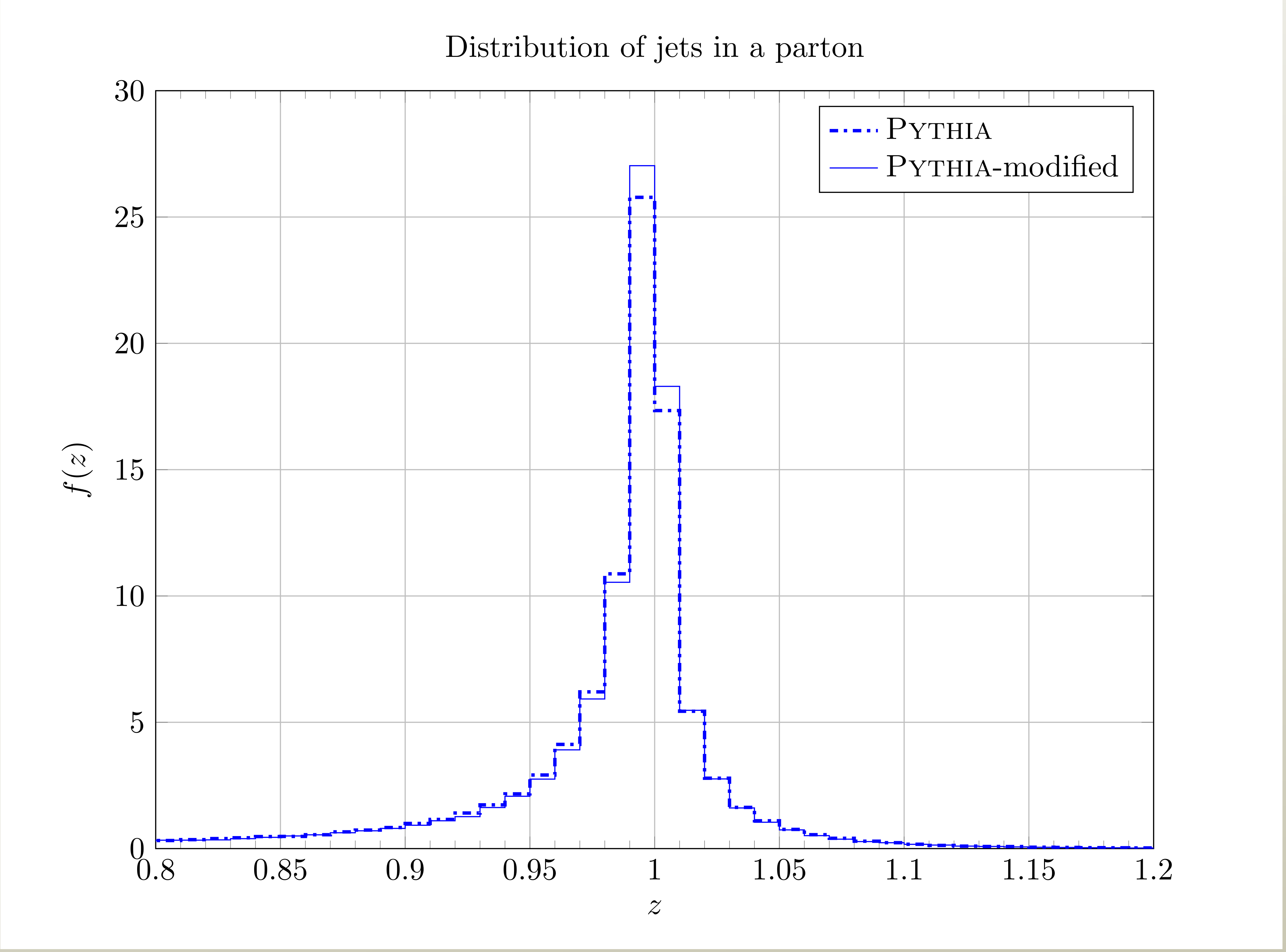
- Nonperturbative effects on the one jet inclusive cross section.



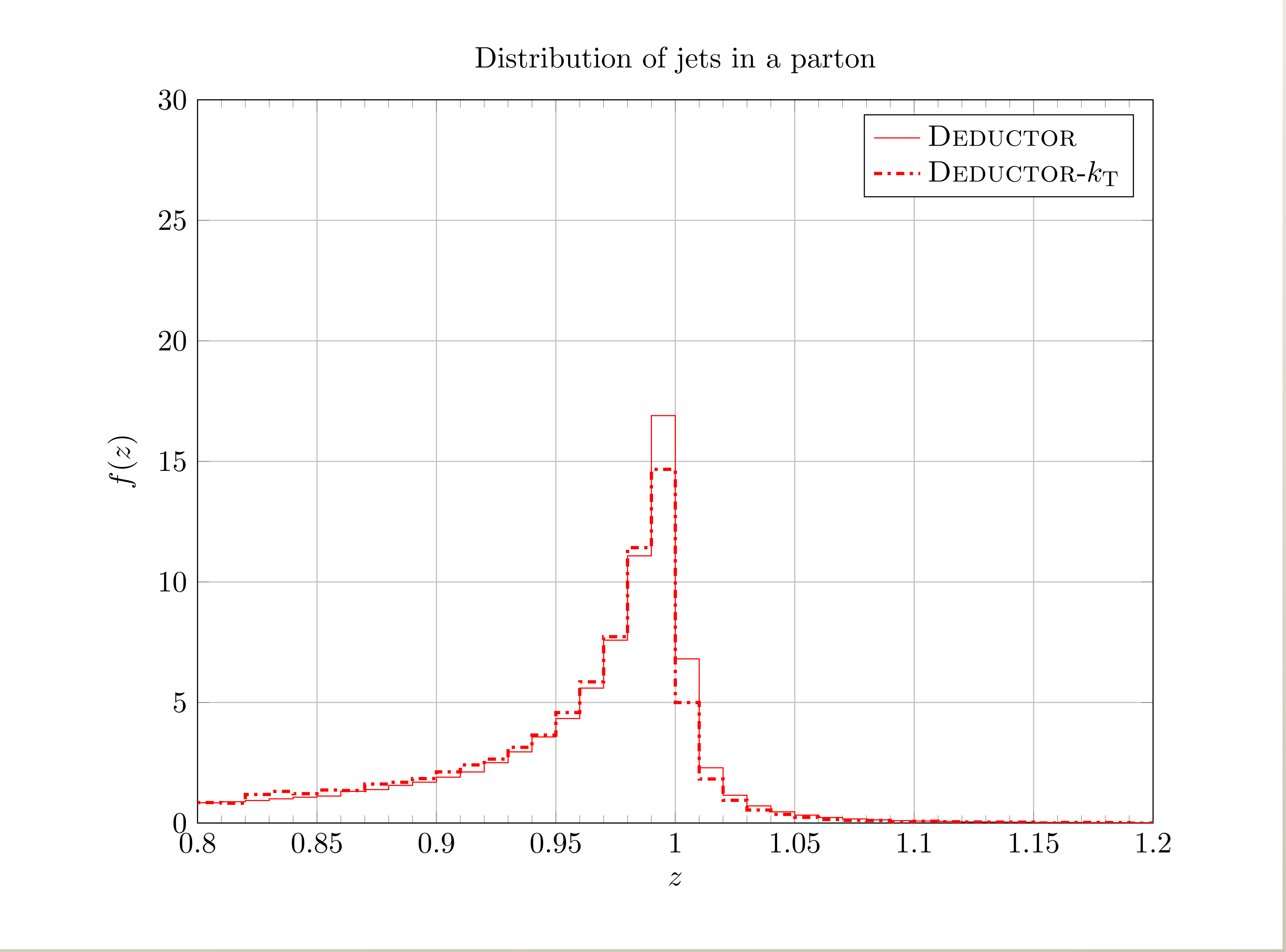
- Jet finding ratio $r(P_\mathrm{T})$ for $R = 0.2$ and $R = 0.7$.



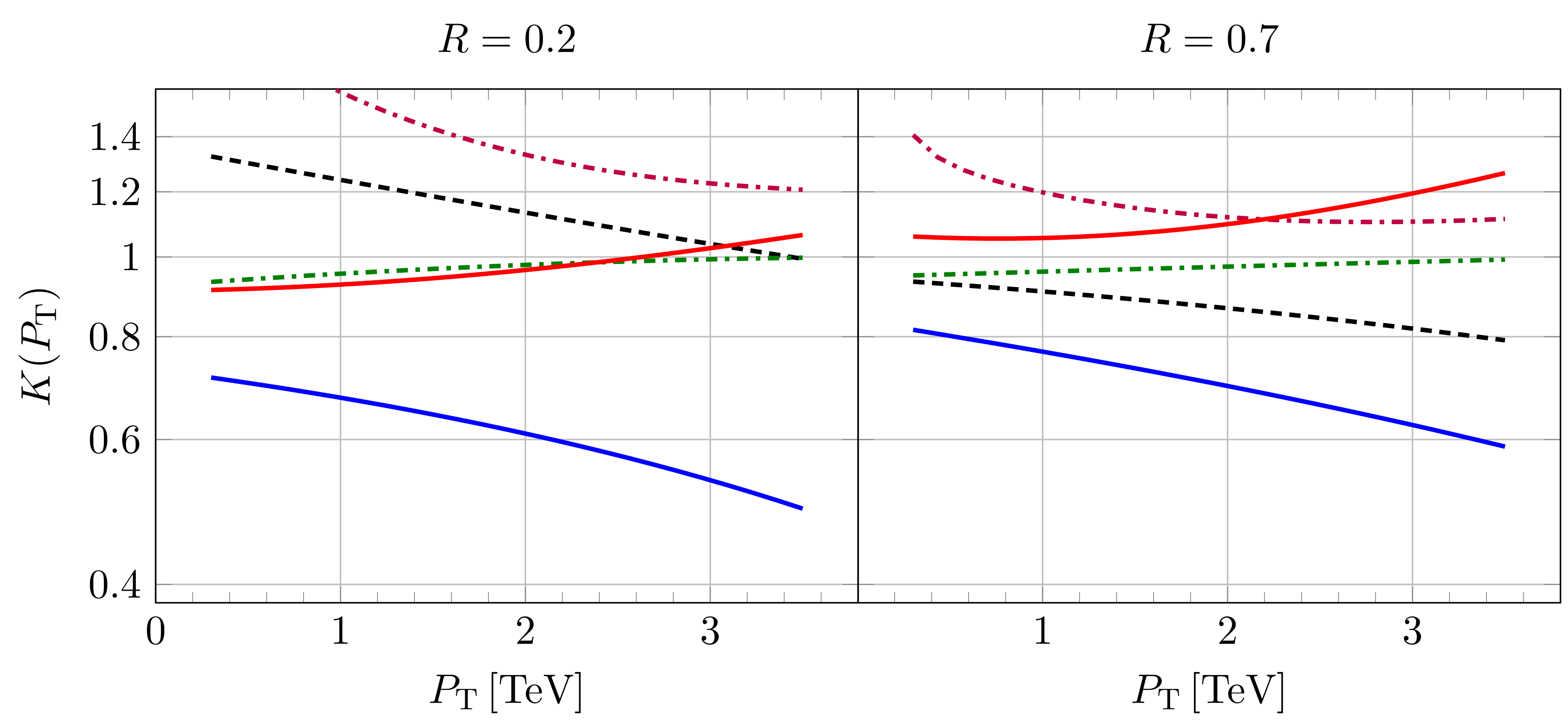
- Effect of PYTHIA conventions for α_s on distribution of jets in a parton $f(z)$.



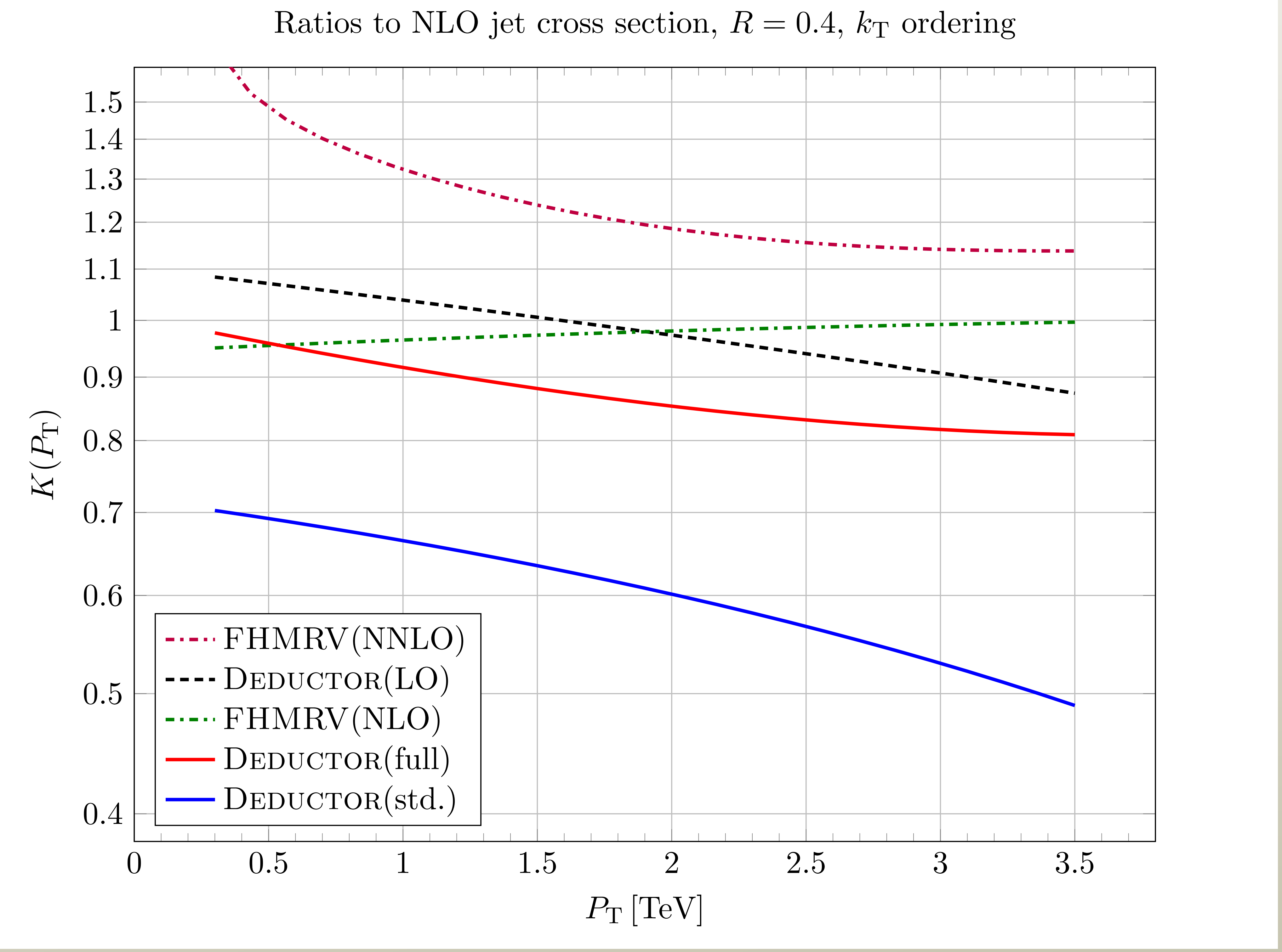
- Effect of DEDUCTOR ordering parameter choice on distribution of jets in a parton $f(z)$.



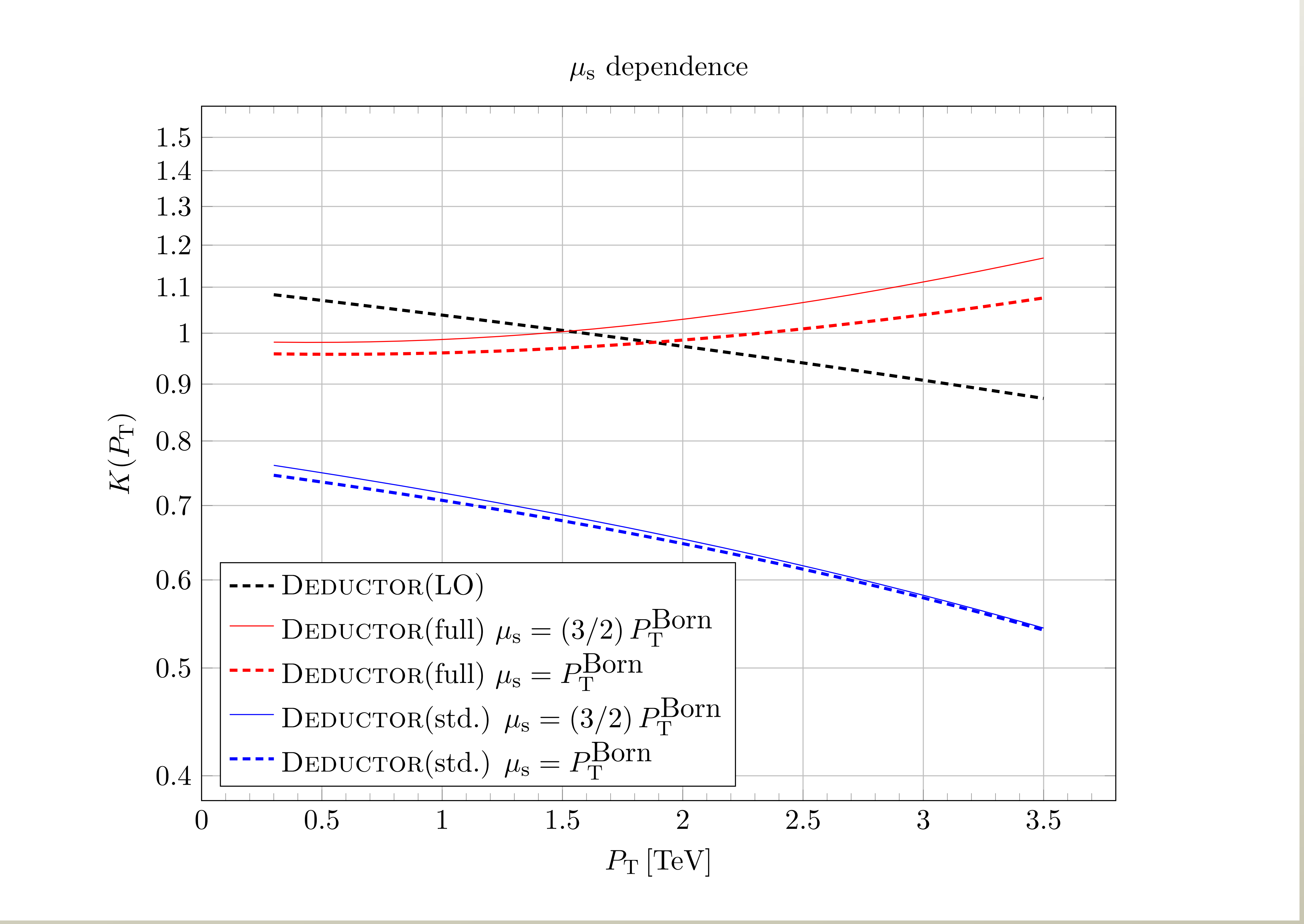
- Ratios $K(P_{\mathrm{T}})$ of the one jet inclusive cross section $d\sigma/dP_{\mathrm{T}}$ to the NLO cross section with $R = 0.2$ and $R = 0.7$.



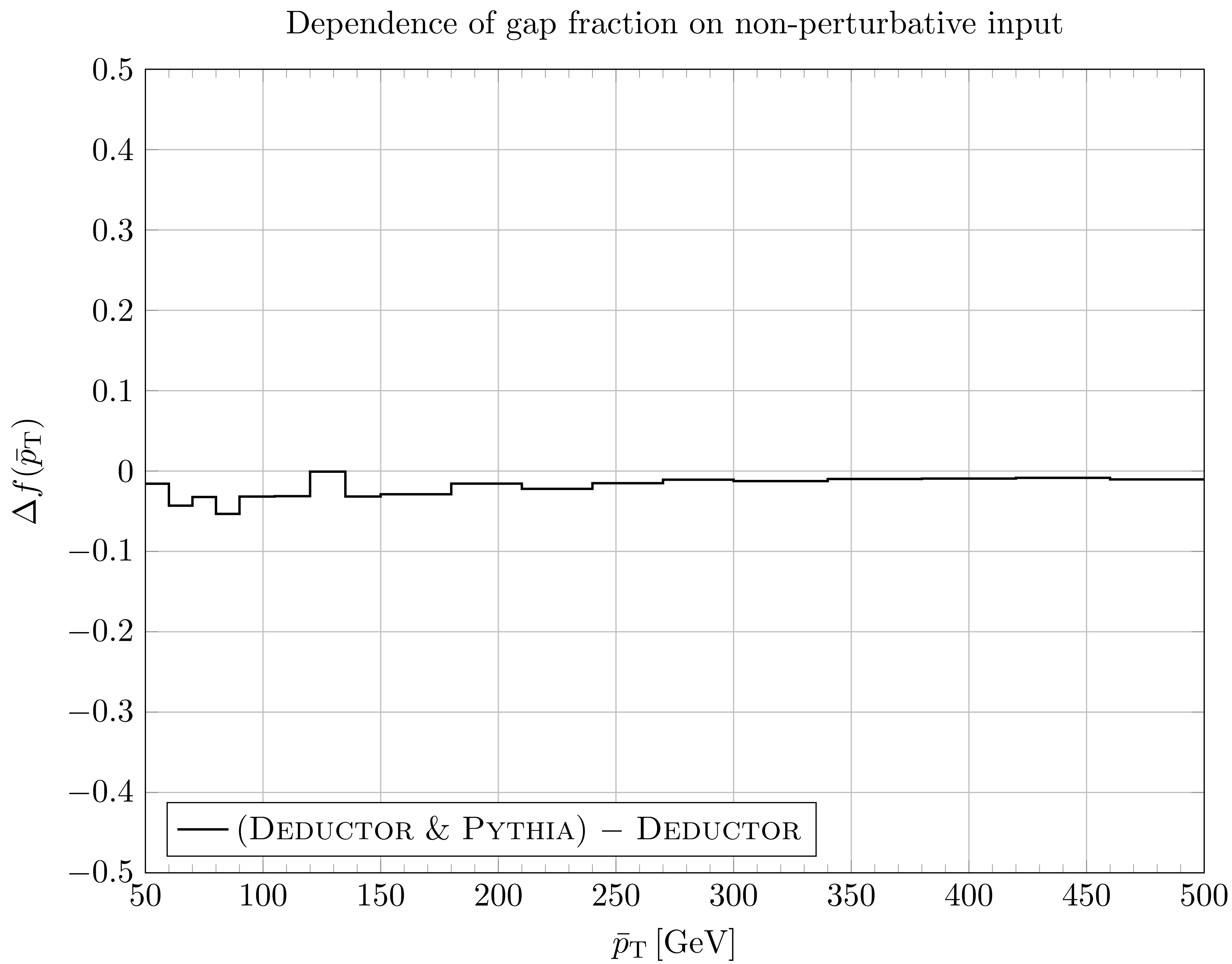
- Ratios $K(P_\mathrm{T})$ of the one jet inclusive cross section $d\sigma/dP_\mathrm{T}$ to the NLO cross section calculated using DEDUCTOR in different approximations with k_T ordering.



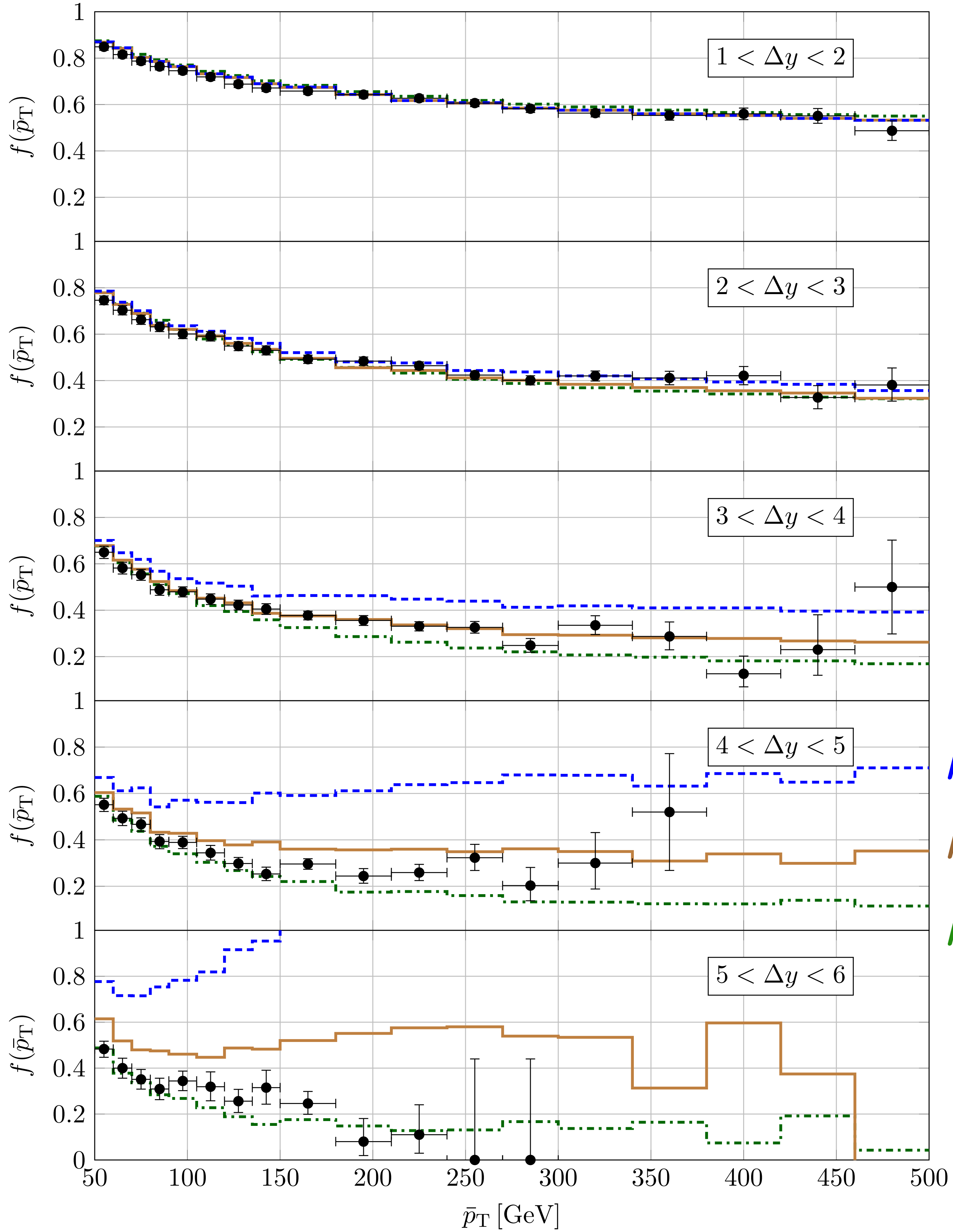
- Ratios $K(P_\mathrm{T})$ of the one jet inclusive cross section $d\sigma/dP_\mathrm{T}$ to the NLO cross section with the shower start scale set to $\mu_\mathrm{s} = P_\mathrm{T}^\mathrm{Born}$. These are compared to the results with our standard choice $\mu_\mathrm{s} = (3/2)\, P_\mathrm{T}^\mathrm{Born}$.



- Change Δf in the gap fraction for $2 < \Delta y < 3$ when nonperturbative effects are added.



- Dependence of perturbative gap fraction result on μ_{R} and μ_{F}.

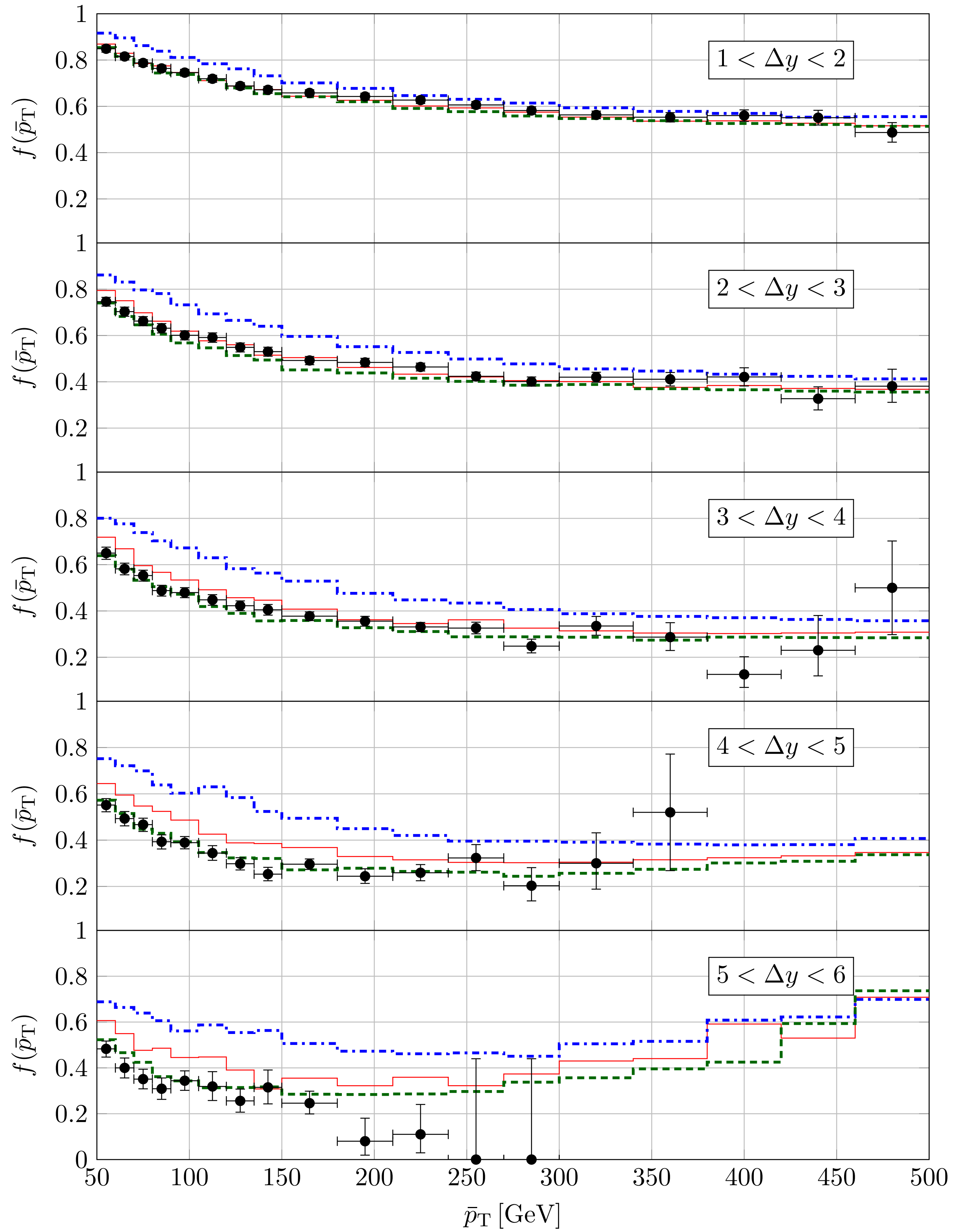


$\mu_{\mathrm{R}} = \mu_{\mathrm{F}} = \bar{p}_{\mathrm{T}}/2$

$\mu_{\mathrm{R}} = \mu_{\mathrm{F}} = \bar{p}_{\mathrm{T}}$

$\mu_{\mathrm{R}} = \mu_{\mathrm{F}} = 2\bar{p}_{\mathrm{T}}$

- Dependence of DEDUCTOR gap fraction result on shower start scale μ_{s}.



$\mu_{\mathrm{s}} = P_{\mathrm{T}}^{\mathrm{Born}}$

$\mu_{\mathrm{s}} = (3/2)\, P_{\mathrm{T}}^{\mathrm{Born}}$

$\mu_{\mathrm{s}} = 2\, P_{\mathrm{T}}^{\mathrm{Born}}$