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Voluntary computer exercise: Synchrotron radiation and FELs

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Calculate the radiation spectrum of a 1.5 GeV electron traveling through an undulator using

$$\frac{d^2W}{d\Omega d\omega}(\omega) \sim \omega^2 \left| \int_{-\infty}^{\infty} \left[\vec{n} \times \left(\vec{n} \times \vec{\beta}\right)\right] \cdot e^{i\omega(t'-r/c)} dt' \right|^2 \qquad \text{(in arbitrary units)}$$

with all symbols as defined in the lecture: ω is the photon frequency, $\vec{\beta}$ is the electron velocity, $\vec{n}$ is the unit vector from the electron to the observer, and r is the distance between them. Assume a planar undulator with the electron trajectory in the horizontal mid-plane, consisting of 10 periods, each 50 mm long, and let the amplitude of the vertical magnetic field be 0.3 T. In the following, the coordinate along the undulator axis is *s*, the horizontally and vertically transverse coordinates are *x* and *z*, respectively.

a) How large is the undulator parameter *K*, what is the wavelength [nm], frequency [1/s] and energy [eV] of the fundamental undulator line? What is a meaningful photon energy range [eV] to calculate and display the line shape?

b) Fill an array **t(i)** with equidistant time steps, using 1/200 of the total time in the undulator as step size. Fill **s(i)** and **x(i)** with points along the electron trajectory, as well as **bs(i)** and **bx(i)** with the respective components of $\vec{\beta}$ (you may use a small-angle approximation). How large is the horizontal displacement [mm] of the electrons, what is the maximum angle [mrad], what is the maximum longitudinal displacement in a frame co-moving with constant velocity? Plot the computed quantities as functions of time and check whether they deviate from your expectation.

c) Consider an observer on the undulator axis 10 m away from the undulator center and calculate the distance **d(i)** between electron and observer as function of time. Fill the arrays **ns(i), nx(i), nz(i)** with the respective components of the unit vector $\vec{n}$ (small-angle approximation). Fill the arrays **vs(i), vx(i), vz(i)** with the components of the factor in square brackets of the equation given above. Note that the vertical component of $\vec{\beta}$ is zero, which simplifies the vector products. Plot all quantities and check for errors.

d) Compute the real and imaginary part of the exponential function in the equation given above and plot them as functions of time. For now, use the central frequency of the undulator line. Inspect the argument of the exponential function.

e) Compute and plot the integrand of the equation given above.

f) Finally, remove the plot statements (if any) from your program and enclose the ω-dependent steps by a loop over, say, 50 photon energy values within the range determined in exercise a). For each photon energy, add up the integrand expressions of each time step, calculate the absolute value squared and multiply by the respective ω^2. Since the result is in arbitrary units, you may also multiply by $(\hbar\omega)^2$ and you don't have to apply a factor dt'. Use a constant factor instead to get simple numbers, e.g. unity at the central frequency. Plot the result as function of photon energy and inspect the shape of the undulator line. Does it match your expectation?

g) You have created a powerful program which allows to calculate spectra and angular distributions of radiation from an electron moving on a sinusoidal path in an undulator (but also other trajectories). There are many things to explore:

- change undulator settings (magnetic field, period length, number of periods).
- calculate the shape of higher harmonics, i.e. integer multiples of the fundamental frequency, under different conditions.
- change the observer's transverse coordinate for a fixed frequency and distance to study the angular distribution horizontally, vertically or in both dimensions. Even harmonics should be strongly suppressed at the undulator axis but appear at non-zero angles.
- study the spectral shape of the fundamental and higher harmonics as function of angle by displacing the observer. The lines should be red-shifted. For small angles, the wavelength increasing quadratically with angle. Plot the intensity in a 2-dimensional angle-energy plane.
- Try different trajectories, e.g. the circular path in a dipole magnet. Do not only look at the final result, but also inspect e.g. the behavior of the exponential function, and watch out for numerical instabilities (for undulators, the simulation is usually behaves well, which is not necessarily true for dipoles and strong wigglers).