Models of flavour Part I: Froggatt - Nielsen

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1. There is a (small) hierarchy problem in SM fermion masses
2. There is a hierarchy in the quark mixing matrix

$$|V_{CKM}| = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\lambda = \sin\Theta_{12} = 0.22$$

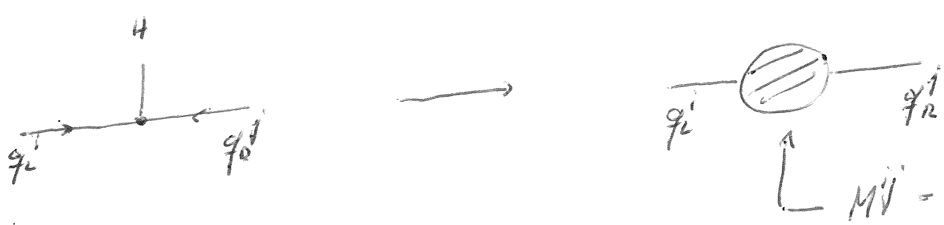
	m_u	m_d	m_s	m_c	m_b	m_t
m_u	1					
m_d	$\lambda^{0.5}$	1				
m_s	$\lambda^{2.4}$	$\lambda^{2.0}$	1			
m_c	$\lambda^{4.1}$	$\lambda^{3.7}$	$\lambda^{1.7}$	1		
m_b	$\lambda^{4.9}$	$\lambda^{4.5}$	$\lambda^{2.5}$	$\lambda^{0.8}$	1	
m_t	$\lambda^{7.3}$	$\lambda^{6.8}$	$\lambda^{5.0}$	$\lambda^{3.2}$	$\lambda^{2.5}$	1

Historically, consider quark mixing first, look at the lepton sector later

Free parameters:
- 6 masses m_i
- 3 mixing angles $\Theta_{12}, \Theta_{13}, \Theta_{23}$
- 1 CP violating phase δ

$\longrightarrow$ This is the flavour puzzle

Classical solution approach: Replace yukawa coupling yH by effective operator M depending on new degrees of freedom Θ



$$M^{ij} = M^{ij}(H_k, \Theta_k \ldots \Sigma_k, Y_{ij})$$

flavons

higgses

Order 1 coefficients

The form of M is dictated by a new flavour symmetry G_F which in principle can be abelian, non-abelian, discrete, continuous, global, gauged

Alternatively: No flavour symmetry, but Extra-Dimensions
$\longrightarrow$ See Part II by Benedict

Froggatt - Nielsen

First a little Taxonomy of Froggatt - Nielsen models on the market:

$G_F = U(1)_{gauged}$: Original proposal, possibly dangerous because of anomalies. Constrained at low energy because of absence of new heavy gauge bosons

$= U(1)_{global}$: Massless Goldstone mode can be explained away by explicit symmetry breaking

$= Z_n$: Discrete Froggatt - Nielsen

$\boxed{\text{F.N. Nucl. Phys '79}}$

Set - Up for $U(1)_{FN} = U(1)_{global}$

Simplest variant : Introduce 1 flavon θ, conventionally charged $a_\theta = FN(\theta) = -1$ under $U(1)_{FN}$

Assign charges to SM fermions as well such that the effective mass Lagrangian reads

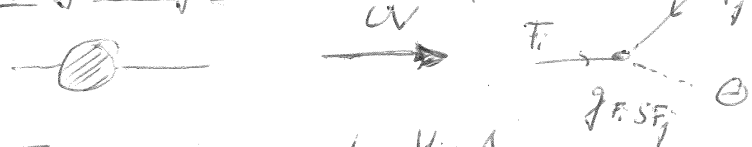
$$-\mathcal{L} = y^d_{ij}\, q^i_L\, H \left(\frac{\theta}{\Lambda}\right)^{n^d_{ij}} d^j_R + y^u_{ij}\, q^i_L\, \tilde{H} \left(\frac{\theta}{\Lambda}\right)^{n^u_{ij}} u^j_R$$

$$+ y^e_{ij}\, l^i_L\, H \left(\frac{\theta}{\Lambda}\right)^{n_{ij}} e^j_R + \text{h.c.}$$

SM yukawa couplings are suppressed $y^{ij} = \underset{\substack{\uparrow \\ \mathcal{O}(1)}}{Y_{ij}}\, \epsilon^{n_{ij}}$ where in the Standard FN, we identify $\lambda = \epsilon = \frac{\theta}{\Lambda}$

$\longrightarrow$ hierarchical structure

Relating charges to flavour patterns



F_i : new heavy quarks $M_i \sim \Lambda_F$
(play role of messenger fields)

Charges:

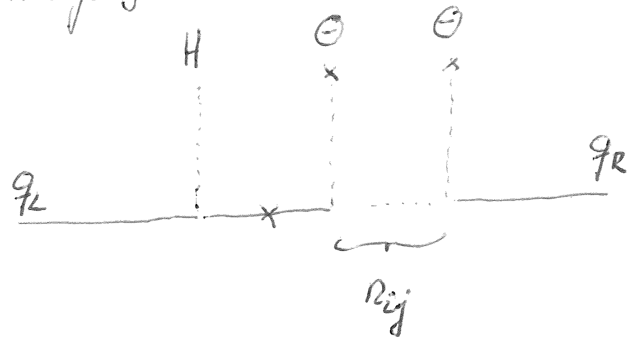
$a_\Theta = FN(\Theta) = -1$

$a_H = FN(H) = 0$

$c + a_q = FN(q_L)$

$c - a_d = FN(d_R)$

$c - a_u = FN(u_R)$



$\longrightarrow$ Get n_{ij} from FN charges :
$$n^d_{ij} = a_q + a_d - a_H$$
$$n^u_{ij} = a_q + a_u + a_H$$

$$y^{u,d}_{ij} = Y^{u,d}_{ij} \, \epsilon^{a_q + a_{d,u} \mp a_H}$$

Yukawa matrices are diagonalized by

$$y^{u,d}_{ij} = U^{u,d}_{ik} \underbrace{\lambda_{kl}}_{\text{diagonal}\left(\frac{\sqrt{2}}{v} m_{u,d}\right)} (V^{u,d}_{lj})^\dagger \quad \longrightarrow \text{CKM matrix} \quad V_{CKM} = U_u^\dagger U_d$$

For $a_H = 0$, write Yukawa matrix as

$$y^{u,d}_{ij} = \underbrace{\begin{pmatrix} \epsilon^{a_{q_1}} & & \\ & \epsilon^{a_{q_2}} & \\ & & \epsilon^{a_{q_3}} \end{pmatrix}_{ik}}_{A_{q_1}} Y^{u,d}_{kl} \underbrace{\begin{pmatrix} \epsilon^{a_{u,d_1}} & & \\ & \epsilon^{a_{u,d_2}} & \\ & & \epsilon^{a_{u,d_3}} \end{pmatrix}_{lj}}_{A_{u,d}}$$

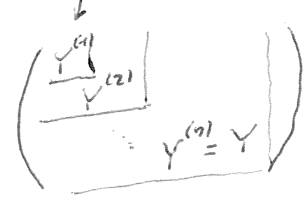
Then the product of eigenvalues is

$$\prod_{i}^{3} \lambda_i = \det Y^{u,d}_{kl} \, \epsilon^{\sum_{i=1}^{3} a_{q_i} + a_{u_i}}$$

Now find eigenvalues perturbatively. Assume hierarchy

$a_{q_i} > a_{q_j}$ $\quad i > j$, $\quad a_{u_i} > a_{u_j}$ $\quad a_{d_i} > a_{d_j}$

first submatrix of Y



$$\lambda_1 = \det Y^{(1)} \, \epsilon^{a_1 + a_{u_1}}$$
$$\lambda_1 \lambda_2 = \det Y^{(2)} \, \epsilon^{a_1 + a_{u_1} + a_2 + a_{u_2}}$$

$$\boxed{\longrightarrow \lambda_i = \frac{\det Y^{(i)}}{\det Y^{(i-1)}} \, \epsilon^{a_i + a_{u_i}}}$$

$$\longrightarrow \frac{m_i}{m_j} = \epsilon^{a_i - a_j + a_{R_i} - a_{R_j}}$$

In order to find CKM-matrix, compute $U^{u,d}$.

U^u diagonalizes $\quad y_u y_u^\dagger = U_u \lambda_u^2 U_u^\dagger$

$$(y_u y_u^\dagger)_{ij} = \underbrace{\left(Y_{ik} Y_{jk}^* \epsilon^{2a_{uk}}\right)}_{c_{ij}} \epsilon^{a_{q_i} + a_{q_j}}$$

Find eigenvectors perturbatively by assuming $(U_u)_{ij} = \begin{cases} 1 & i=j \\ u_{ij}\epsilon^{k_{ij}} & i>j \\ -u_{ij}^* \epsilon^{k_{ji}} & i<j \end{cases}$

By comparing orders of ϵ (neglecting prefactors c_{ij} / u_{ij}), one finds $k_{ij} = |a_{q_i} - a_{q_j}| = k_{ji}$

$$\boxed{\begin{matrix} U_{ij}^u \sim \epsilon^{|a_{q_i} - a_{q_j}|} & U_{ij}^d \sim \epsilon^{|a_{q_i} - a_{q_j}|} \\ V_{ij}^u \sim \epsilon^{|a_{u_i} - a_{u_j}|} & V_{ij}^d \sim \epsilon^{|a_{d_i} - a_{d_j}|} \end{matrix}} \qquad V_{CKM} = U_u^\dagger U_d$$

$t = 1 \to 3$
$u = 3 \to 1$

$$\longrightarrow \quad (V_{CKM})_{ii} = \mathcal{O}(1) \qquad (V_{CKM})_{ub} \approx (V_{CKM})_{td}$$
$$\approx V_{us} \times V_{cb}$$

Hierarchical CKM matrix just from $FN(q_3) > FN(q_2) > FN(q_1)$. Fitting also the mass hierarchies, we get something like

$$\begin{pmatrix} 3 & 2 & 0 \\ 4 & 2 & 0 \\ 1+p & p & p \end{pmatrix} = \begin{pmatrix} a_{q_1} & a_{q_2} & a_{q_3} \\ a_{u_1} & a_{u_2} & a_{u_3} \\ a_{d_1} & a_{d_2} & a_{d_3} \end{pmatrix} \qquad p = 2 \text{ for SM case}$$

Generalization to leptons and more

From Babu, TASI lectures

- SUSY SU(5) GUT model ($\Lambda_F > \Lambda_{GUT}$)
- Includes ν_R - Majorana and a type I Seesaw.

Charge Assignment:

Q_i	FN(Q_i)
q	4, 2, 0
u^c	4, 2, 0
d^c	1+p, p, p

L_i	FN(L_i)
ℓ	1+s ; s, s
e^c	4+p-s, 2+p-s, p-s
ν^c	1 ; 0, 0

$$FN(H_u, H_d, \Theta) = 0, 0, 1$$

Resulting mass matrices:

$$M_u \sim \langle H_u \rangle \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \qquad M_d \sim \langle H_d \rangle \epsilon^p \begin{pmatrix} \epsilon^5 & \epsilon^4 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix},$$

$$M_e \sim \langle H_d \rangle \epsilon^p \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon \\ \epsilon^4 & \epsilon^2 & 1 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \qquad M_{\nu_D} \sim \langle H_u \rangle \epsilon^s \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix},$$

$$M_{\nu^c} \sim M_R \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \quad \Rightarrow \quad M_\nu^{light} \sim \frac{\langle H_u \rangle^2}{M_R} \epsilon^{2s} \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}$$

$p = 0, 1, 2$ depending on the value of $\tan\beta$

$s \overset{!}{=} p$ for SU(5) unification

$s = p$ also allows for anomaly cancellation via Green-Schwarz mechanism (see [2] for more)

This gives good fits for $\frac{m_i}{m_j}$ and CKM matrix elements.

What are Froggatt-Nielsen models good for?

- Do they explain the observed number of families?
  → No, hierarchy generation works for any number of families

- Do they quantify/explain (at least some of) the observed patterns in the SM flavour sector?
  → Yes, we discussed quark mass hierarchies, CKM
  Also easily generalizable to leptons (you need to relax $FN(q_i) > FN(q_j)$ to $FN(\ell_i) \geq FN(\ell_j)$)

- Do they point to new physics scale?
  No. CKM hierarchy is fixed by $\frac{f}{\sqrt{2}\Lambda} \sim 0.2$
  Λ is a free parameter.
  → Build Froggatt-Nielsen into another BSM framework at see what it gets you. For instance:
  - Relate flavour symmetry breaking to EW symmetry $\Theta \sim H$ breaking $\left(\frac{\Theta}{\Lambda}\right)^{n_{ij}} \rightarrow \left(\frac{H^\dagger H}{\Lambda}\right)^{n_{ij}}$
    Does not work in the SM: $FN(H^\dagger H) = 0$
    Therefore 2HDM/SUSY $\Lambda = \mathcal{O}(\text{TeV})$
  - Requirements for EW baryogenesis:
    B number violation (Yes, in the SM t'Hooft '76)
    CP violation (CKM phase, however CPV too small in the SM)
    Departure from thermodynamical equilibrium (first order EW phase transition, requires new TeV scale physics)

- Predict new observables ? $\longrightarrow$ flavor hunt

Flavor hunt [Bauer, Schell, Plehn PR D94 (2016)]

$$-\mathcal{L}_Y = y_{ij}^d \left(\frac{S}{\Lambda}\right)^{n_{ij}^d} \bar{Q}_i H d_{R_j} + y_{ij}^u \left(\frac{S}{\Lambda}\right)^{n_{ij}^u} \bar{Q}_i \tilde{H} u_{R_j}$$

$$+ y_{ij}^l \left(\frac{S}{\Lambda}\right)^{n_{ij}^l} \bar{L}_i H l_{R_j} + y_{ij}^\nu \left(\frac{S}{\Lambda}\right)^{n_{ij}^\nu} \bar{L}_i \tilde{H} \nu_{R_j} + h.c.$$

$$-\mathcal{L}_{pot} = -\mu_S^2 S^* S + \lambda_S (S^* S)^2 + b(S^2 + S^{*2}) + \lambda_{HS} (S^* S)(H^\dagger H) + V(H)$$

In the completly broken phase: $\lambda_{HS} = 0$ If not look, for deviations in SM Higgs couplings

$$S(x) = \frac{f + s(x) + i a(x)}{\sqrt{2}} \qquad H^T(x) = \left(0, \frac{v + h(x)}{\sqrt{2}}\right)$$

$$m_s = \sqrt{\frac{\lambda_S}{2}} f \qquad m_a = \sqrt{2b}$$

Expected hierarchy: $m_a < m_s \approx f < \Lambda$

The expansion parameter is $\epsilon = \frac{f}{\sqrt{2}\Lambda} \simeq 0.22$

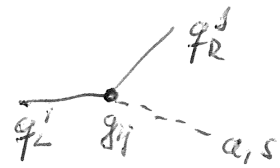
Benchmark point:

a_{Q_i}	3	2	0
a_{u_i}	5	2	0
a_{d_i}	4	3	3
a_{L_i}	1	0	0
a_{ν_i}	24	21	20
a_{e_i}	8	5	3

Expanding $\mathcal{L}_Y$ in the broken phase yields:

$$\frac{v}{\sqrt{2}} y_{ij} \epsilon^{n_{ij}} \bar{d}_L^i \left(1 + \frac{h}{v} + \frac{n_{ij} s + i n_{ij} a}{f}\right) d_R + \dots$$

$\longrightarrow$ FCNC's at tree level



fermion flavon couplings:

$$m_t \approx \frac{v}{\sqrt{2}} \qquad \frac{m_b}{m_t} \approx \epsilon^3 \qquad \frac{m_c}{m_t} \approx \epsilon^4 \qquad \frac{m_s}{m_t} \approx \epsilon^5 \qquad \frac{m_d}{m_t} \approx \epsilon^7 \qquad \frac{m_u}{m_t} \approx \epsilon^8$$

In mass eigenbasis, we have approximately

$$g^u_a f_{iL} f_{jR} = g^u_{aij} = \frac{1}{f}\begin{pmatrix} 8\, m_u & \epsilon\, m_c & \epsilon^3 m_t \\ \epsilon^3 m_c & 4\, m_c & \epsilon^2 m_t \\ \epsilon^5 m_t & \epsilon^2 m_t & \sim 0 \end{pmatrix}$$

$$g^d_{aij} = \frac{1}{f}\begin{pmatrix} 7\, m_d & \epsilon\, m_s & \epsilon^3 m_b \\ \epsilon\, m_s & 5\, m_s & \epsilon^2 m_b \\ \epsilon\, m_b & \epsilon^2 m_b & 3\, m_b \end{pmatrix}$$

for the quark masses.

$$\boxed{g_{ij} = g_{sij} = i g_{aij}}$$

General strategies:

- flavon decay / decay to flavon, e.g. $t \to ac$
- quark flavour physics (e.g $\bar{K}$-K mixing)
- lepton flavour physics (e.g. μe conversion)

Decay width for flavon decay:

$$\Gamma(a \to f_i \bar{f}_j) = \frac{N_c}{16\pi}\left[\frac{(m_a^2-(m_i+m_j)^2)(m_a^2-(m_i-m_j)^2)}{m_a^4}\right]^{\frac{1}{2}} \times$$

$$\times \left[(|g_{ij}|^2+|g_{ji}|^2)\left(1-\frac{m_i^2+m_j^2}{m_a^2}\right) - 2(g_{ij}g_{ji} + g^*_{ij}g^*_{ji})\frac{m_i m_j}{m_a^2}\right]$$

Above top threshold: $a \to t_j + \bar{t}_j$

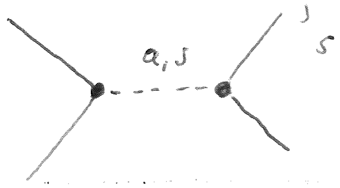
$$\frac{\Gamma(a \to t\bar{u})}{\Gamma(a \to t\bar{c})} \simeq \epsilon^2 \simeq \frac{1}{20}$$

$a \to t\bar{t}$ forbidden by charge assignment

K-$\bar{K}$ mixing:

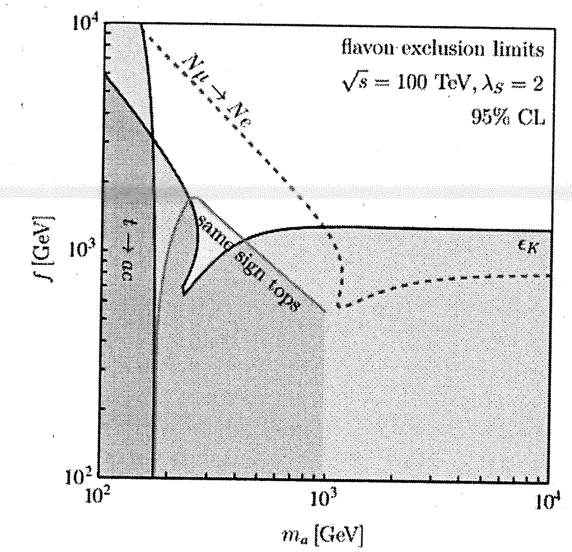
Parametrize contributions to meson mixing via

$$\mathcal{H}_{NP}^{\Delta F=2} = C_1^{ij}(\bar{q}_L^i \gamma_\mu q_L^j)^2 + \tilde{C}_1^{ij}(\bar{q}_R^i \gamma_\mu q_R^j)^2 + C_2^{ij}(\bar{q}_R^i q_L^j)^2$$
$$+ C_4^{ij}(\bar{q}_R^i q_L^j)(\bar{q}_L^i q_R^j) + C_5^{ij}(\bar{q}_L^i \gamma_\mu q_R^j)(\bar{q}_R^i \gamma^\mu q_R^j) + h.c.$$



lead to

$$C_2^{ij} = -(g_{ji}^*)^2\left(\frac{1}{m_s^2} - \frac{1}{m_a^2}\right)$$
$$\tilde{C}_2^{ij} = -g_{ij}^2\left(\frac{1}{m_s^2} - \frac{1}{m_a^2}\right)$$
$$C_4^{ij} = -\frac{g_{ij}g_{ji}^*}{2}\left(\frac{1}{m_s^2} - \frac{1}{m_a^2}\right)$$



To the left: Best constraints from quark mixing, future μ-beam experiments, and collider searches (last two projected) [4]

$$C_{\epsilon_K} = \frac{\mathrm{Im}\langle K^0|\mathcal{H}^{\Delta F=2}|\bar{K}^0\rangle}{\mathrm{Im}\langle K^0|\mathcal{H}_{SM}^{\Delta F=2}|\bar{K}^0\rangle} = 1.05^{+0.36}_{-0.28}$$

$$C_{\Delta m_K} = \frac{\mathrm{Re}\langle K^0|\mathcal{H}^{\Delta F=2}|\bar{K}^0\rangle}{\mathrm{Re}\langle K^0|\mathcal{H}_{SM}^{\Delta F=2}|\bar{K}^0\rangle} = 0.93^{+1.14}_{-0.42}$$

The Higgs as the flavon

already excluded

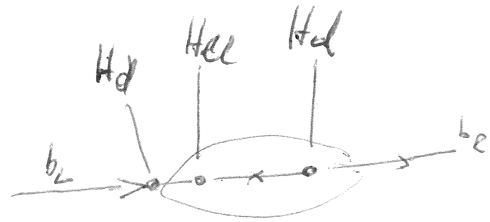
Original paper by Babu, Nandi '99
The same case discussed by Giudice, Lebedev '08

Current analysis : Bauer, Carena '15

$$\mathcal{Y}_{ij} = Y_{ij} \left(\frac{H_u H_d}{\Lambda^2} \right)^{n_{ij}}$$

$\underbrace{FN(H_u)=1}$ $FN(H_d)=0$

$$\frac{f}{\sqrt{2}\Lambda} \longrightarrow \frac{\langle H_u H_d \rangle}{\Lambda^2} = \frac{\tan\beta}{1+\tan^2\beta}\frac{v^2}{2\Lambda^2}$$



Expand in $\frac{m_b}{m_t} \simeq \frac{1}{60}$

For $\tan\beta = 1$ $\Lambda \to 4v = 1\,TeV$

$\frac{v_u}{v_d} = \tan\beta$

$v^2 = v_u^2 + v_d^2$

- We do not expand in Cabibbo angle anymore so the charge assignments significantly change.
- All effects we observed for the flavon, now are true for the Higgs : • FCNC's at tree level • enhanced couplings on the diagonal

$\rightarrow$ $m_t = \frac{v_u}{\sqrt{2}}$ $\frac{m_b}{m_t} \simeq \frac{m_c}{m_t} \approx \epsilon^1$ $\frac{m_s}{m_t} \simeq \epsilon^2$ $\frac{m_{u,d}}{m_t} \approx \epsilon^3$

CKM matrix $V_{12} \approx \epsilon^0$ $V_{13} \approx V_{23} \approx \epsilon^1$

Fixes two parameters. A better choice is

$V_{12} = V_{13} = V_{23} \simeq \epsilon^0$ evades constraints from flavor mixing

References

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