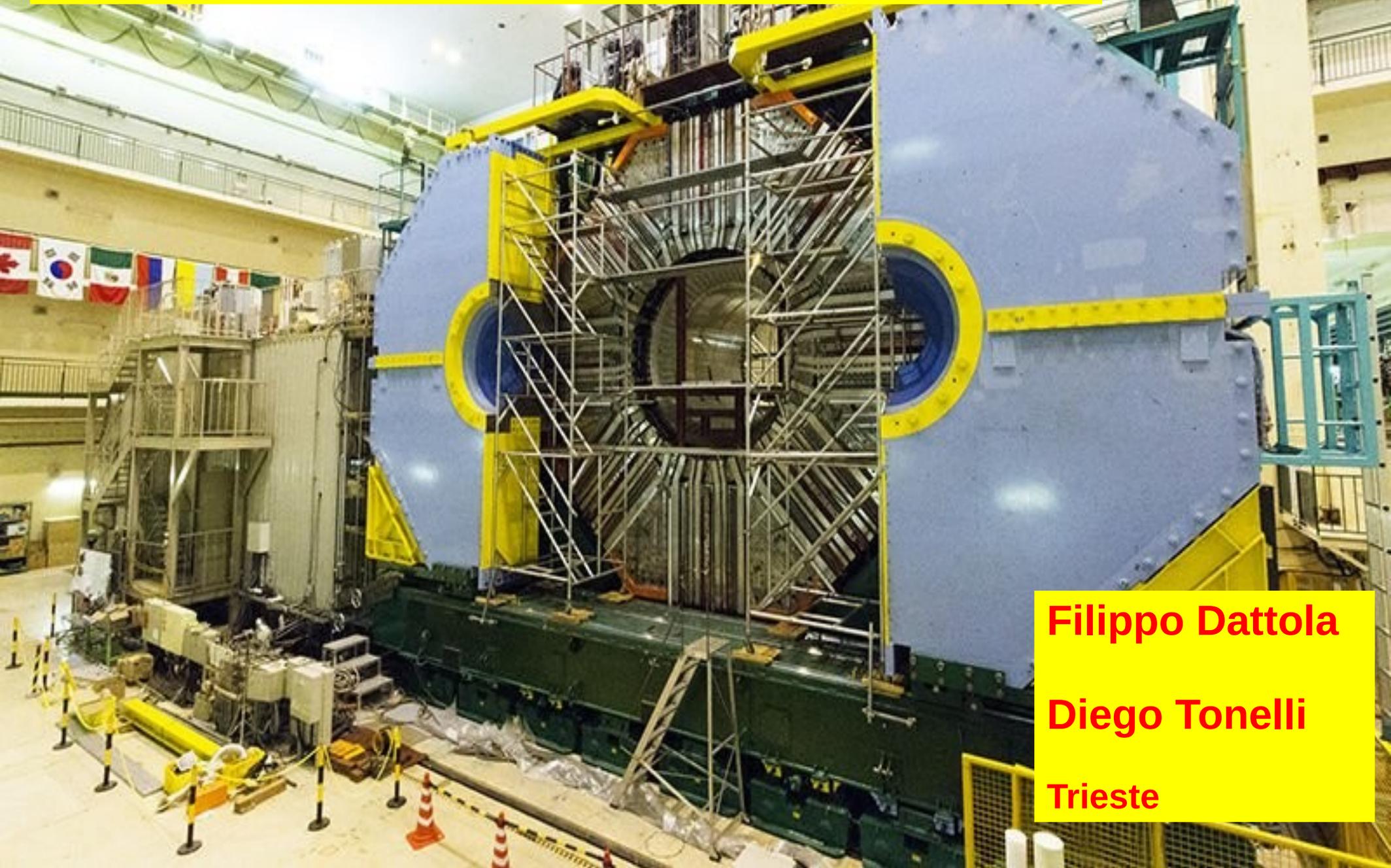


Alignment studies for the Belle II tracker



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Diego Tonelli

Trieste

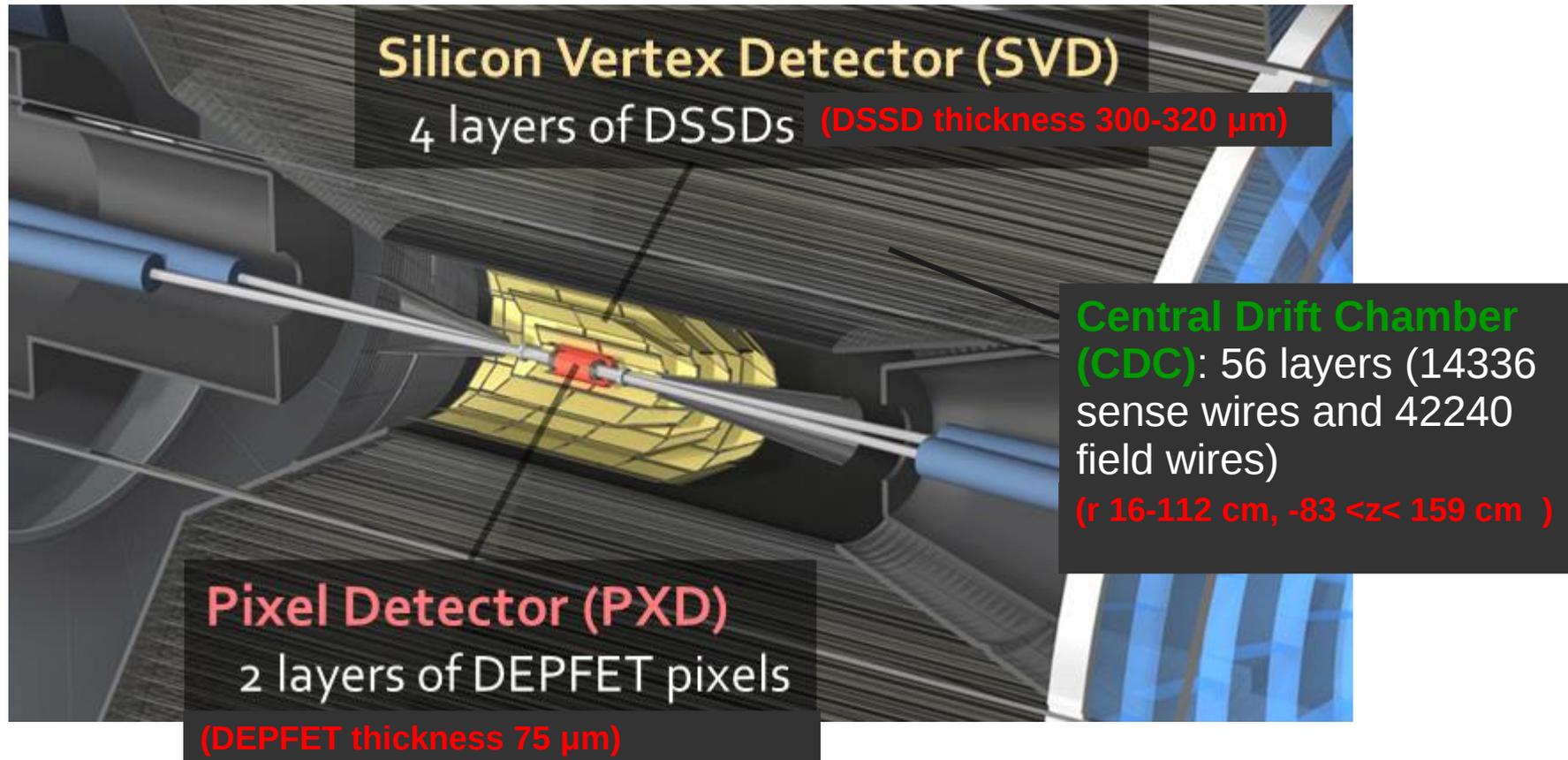
Overview

- My master thesis: a data-analysis project to study **charged-particle tracking performances**.
- Simulate **a set of VXD misalignments** using a dedicated code, developed by Prague. Then study their effects on:
 - reconstructed track parameters from $e^+e^- \rightarrow \mu^+\mu^-$ events;
 - hits in overlapping sensors of a same VXD layer.
- **Develop procedures to identify and fix misalignments**, hoping to contribute to a general improvement of the Belle II tracking reconstruction performance.
- Many thanks to **Eugenio Paoloni, Giulia Casarosa, Tadeas Bilka and Jakub Kandra** for their invaluable help with technical matters.

Tracking at Belle II

B , D and τ decays are dominated by the presence of final-state charged particles.

Precise tracking with accurate and precise momentum and impact parameter determination is essential



- Pixel Detector (PXD): 2 layers of pixels, unambiguous 2D position.
- Silicon Vertex Detector (SVD): 4 layers of double sided silicon strip detector, excellent position resolution.
- Central Drift Chamber (CDC): 8 inner layers, good momentum resolution.

Track parametrization

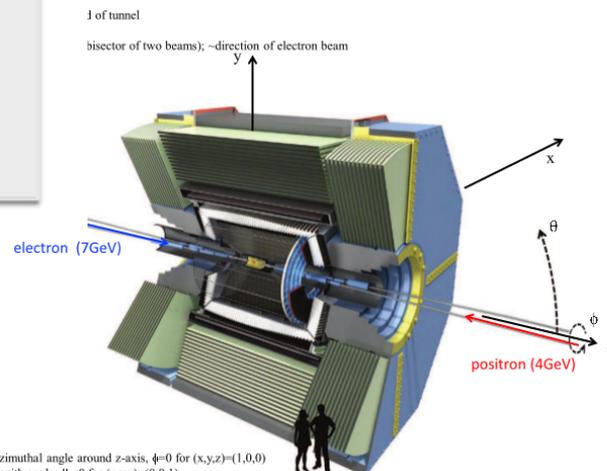
The trajectory of a charged particle (track) moving into an axial magnetic field parallel to the beam approximates an helix with 5 independent parameters

TRANSVERSE PLANE

- POCA = Point Of Closest Approach
- d_0 is the 2d signed distance of the POCA from the z axis, the sign depends on the angular momentum of the track (>0 in the fig.) : **transverse displacement.**
- φ_0 is the angle between p_t and the x axis at the POCA, $\varphi_0 \in [-\pi, \pi]$: **azimuthal information.**
- the sign of ω , the curvature, is the same as the charge of the track (>0 in the fig.) : **momentum and charge.**

LONGITUDINAL VIEW

- $\tan\lambda$ is the ratio of p_z and p_t , $\lambda \in [-\pi, \pi]$: **polar information.**
- z_0 is the signed distance of the POCA from the transverse plane : **longitudinal displacement.**



ϕ : azimuthal angle around z-axis, $\phi=0$ for $(x,y,z)=(1,0,0)$
 θ : zenith angle; $\theta=0$ for $(x,y,z)=(0,0,1)$

Overview

- The vertex detector system (VXD) samples tracks very precisely.
- The reconstruction software needs the actual position in space of the silicon sensors.
- These are assembled and installed with tight tolerances around the nominal positions, but small misalignments occur and, if corrected, yield better tracking performances.
- Such residual misalignments are typically identified and measured using tracks.
- Our task: **target a specific set of VXD global misalignments (weak modes)** by recognizing distinctive patterns in the distribution of reconstructed track variables and in hit coordinates residuals for overlapping sensors.
- Goal: identify the patterns that allow a more direct, possibly one-to-one, connection with specific misalignment weak modes.
- Once applied to data, this procedure will hopefully allow to quickly detect possible non-alignment of VXD in the configuration used to perform track reconstruction.
- This work is in progress and benefits from the collaboration with Tadeas Bilka and Jakub Kandra from the Belle II group in Prague.

Standard VXD alignment

Recursive algorithm (Millipede II) that crunches many raw data and minimizes a chi2-like expression:

$$\chi^2(\bar{\tau}, \bar{a}) = \sum_j^{tracks} \sum_i^{hits} z_{ij}^2(\bar{\tau}_j, \bar{a})$$

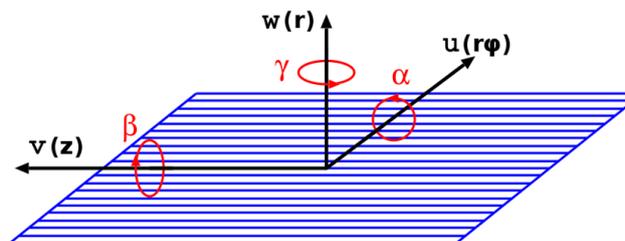
which uses the normalized tracking residuals

$$z_{ij} = \frac{u_{ij}^m - u_{ij}^p(\bar{\tau}_j, \bar{a})}{\sigma_{ij}}$$

- Where:
- u_{ij}^m is a recorded measurement of hit i on the track j ;
 - u_{ij}^p is a predicted measurement from a track model;
 - $\bar{\tau}_j$ are the track parameters;
 - \bar{a} are the alignment parameters characterizing the model.

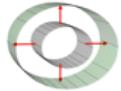
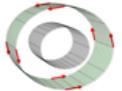
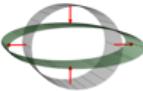
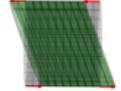
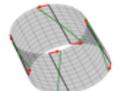
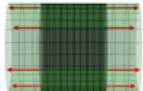
And for each of the planar sensors

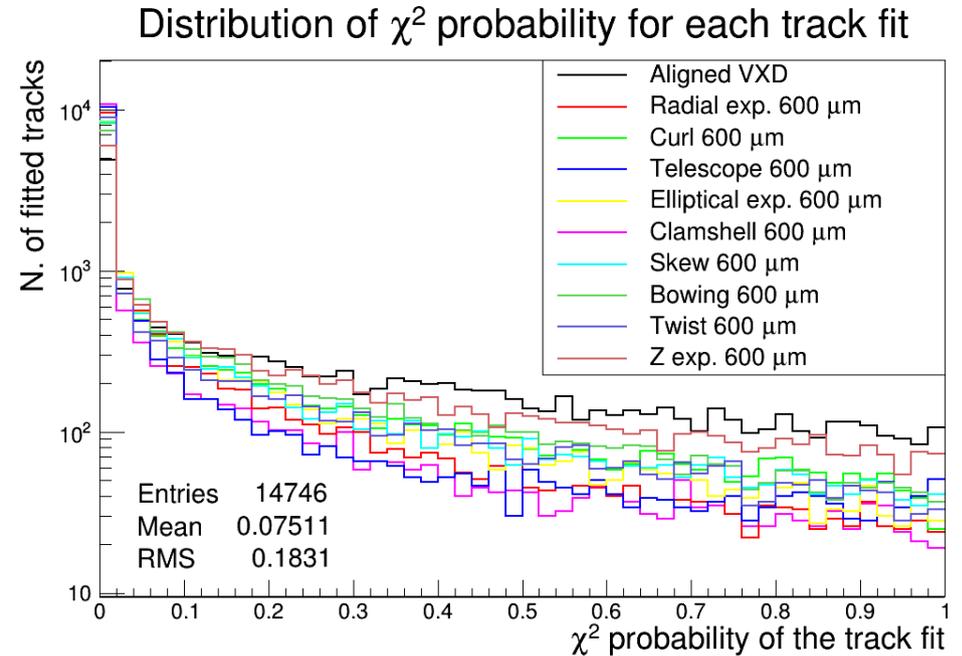
$$\bar{a} = (\Delta u, \Delta v, \Delta w, \Delta \alpha, \Delta \beta, \Delta \gamma)$$



Weak modes

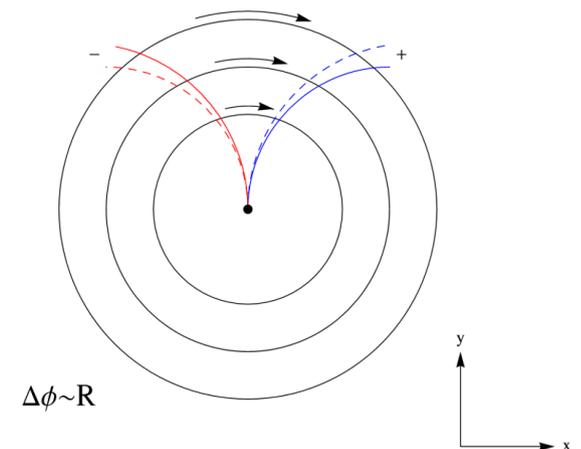
Weak modes are residual misalignments arising from global, coherent movements of sets of sensors that cannot be detected conclusively by χ^2 minimization procedures because they leave the overall χ^2 unaffected.

	Δr	$r\Delta\phi$	Δz
r	Radial expansion $\Delta r = c_{scale} \cdot r$ 	Curl $r\Delta\phi = c_{scale} \cdot r + c_0$ 	Telescope $\Delta z = c_{scale} \cdot r$ 
ϕ	Elliptical expansion $\Delta r = c_{scale} \cdot \cos(2\phi) \cdot r$ 	Clamshell $\Delta\phi = c_{scale} \cdot \cos(\phi)$ 	Skew $\Delta z = c_{scale} \cdot \cos(\phi)$ 
z	Bowing $\Delta r = c_{scale} \cdot z $ 	Twist $r\Delta\phi = c_{scale} \cdot z$ 	Z expansion $\Delta z = c_{scale} \cdot z$ 



Example: curl.

- Arrows indicate layer misalignment (proportional to its radius)
- Dashed lines indicate true trajectories of charged particles (**positively charged** and **negatively**)
- Solid lines show how trajectories are reconstructed with the deformation



My analysis

Generate 5000 $e^+e^- \rightarrow \mu^+\mu^-$ signal-only events using the KKMC generator.

Simulate their interactions with the Belle II detector in the Phase III geometry, thus generating a set of hits and trajectories.

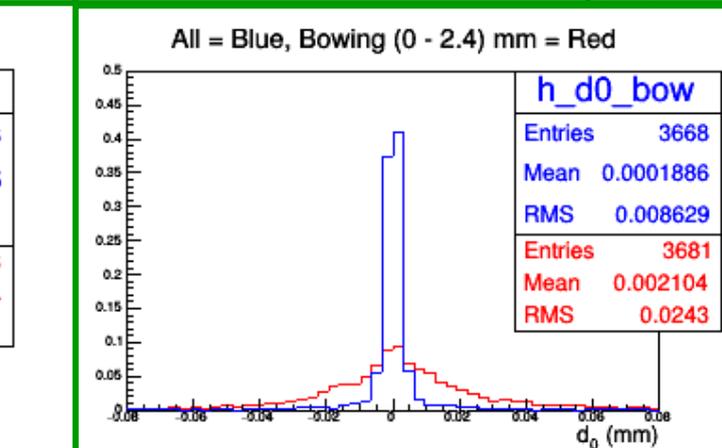
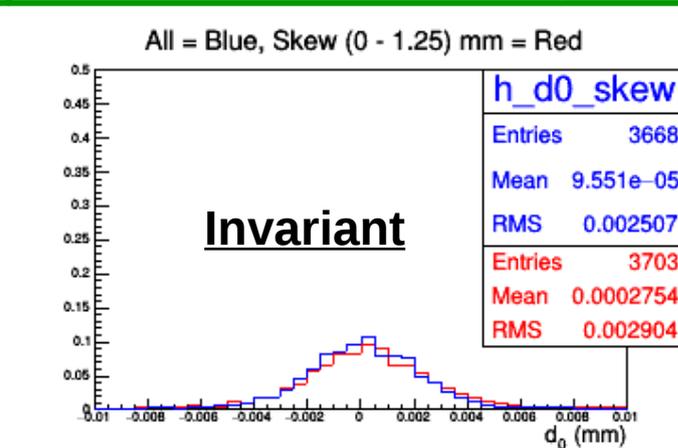
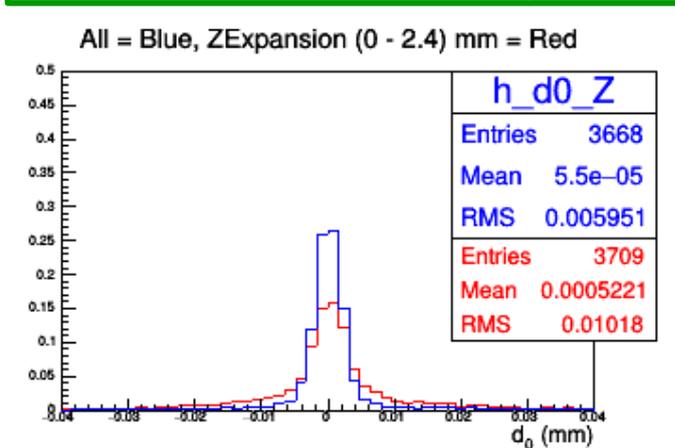
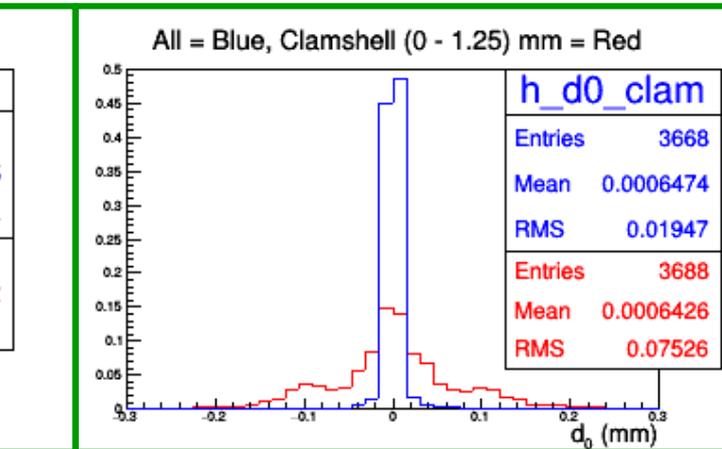
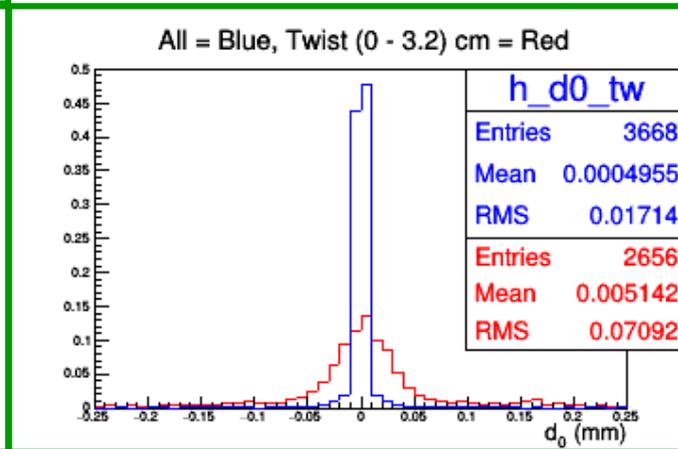
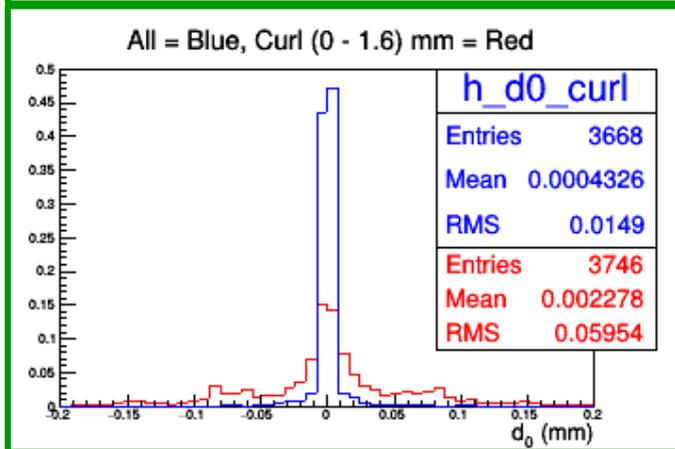
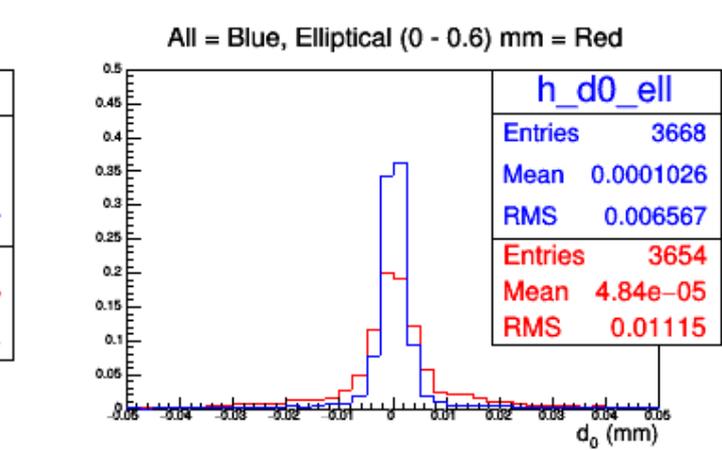
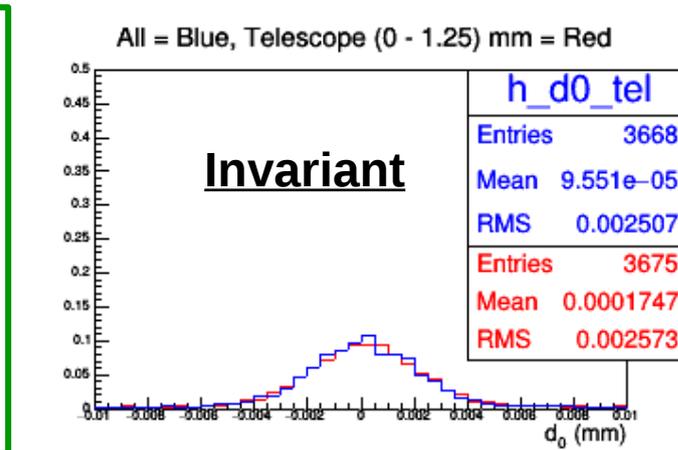
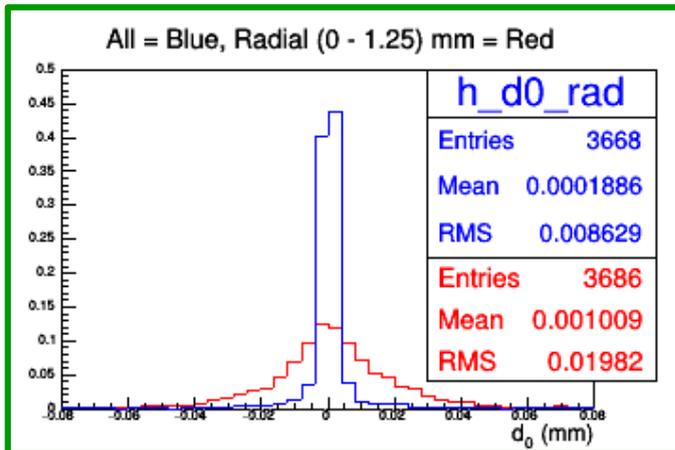
Prior to reconstruction, simulate various systematic misalignments using a python script that takes a scale factor as input and, given the geometry of the vertex detector, changes each sensors' global coordinates accordingly.

Test the effects of various VXD weak modes, by looking at the **normalized distributions** of appropriately chosen tracking variables and compare them with the distribution expected for perfectly aligned VXD.

The variables I use are combinations of track parameters of the two muons, and their values are those extrapolated from track fit at POCA.

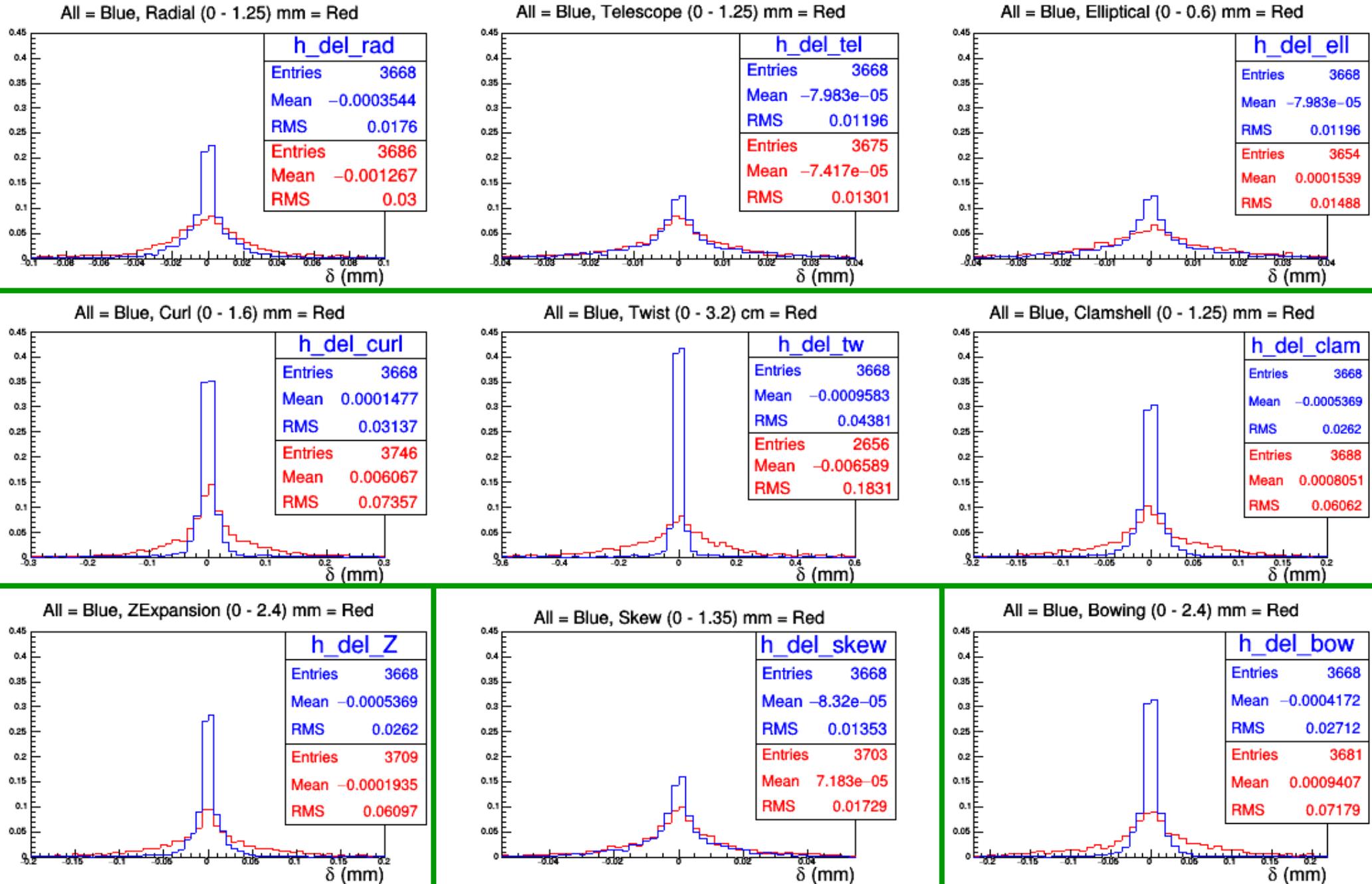
Examples

δd_0 comparison for VXD ideally **aligned** or realistically **misaligned** according to each of the weak modes.



$$\delta = \frac{\bar{p}_0^{(-)} \times \bar{p}_0^{(+)}}{|\bar{p}_0^{(-)} \times \bar{p}_0^{(+)}|} \left[d_0^{(+)} \hat{t}^{(+)} - d_0^{(-)} \hat{t}^{(-)} + (z_0^{(+)} - z_0^{(-)}) \hat{z} \right] \text{ comparison for VXD ideally aligned}$$

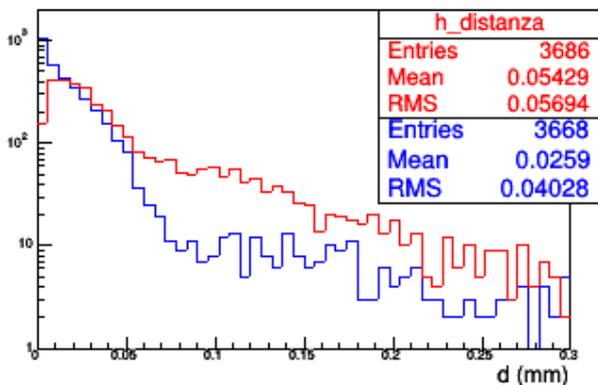
or realistically **misaligned** according to each of the weak modes.



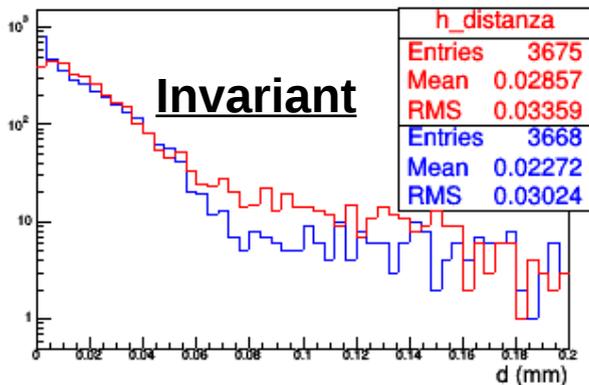
Examples

Distance between POCAs d comparison for VXD ideally **aligned** or realistically **misaligned** according to each of the weak modes.

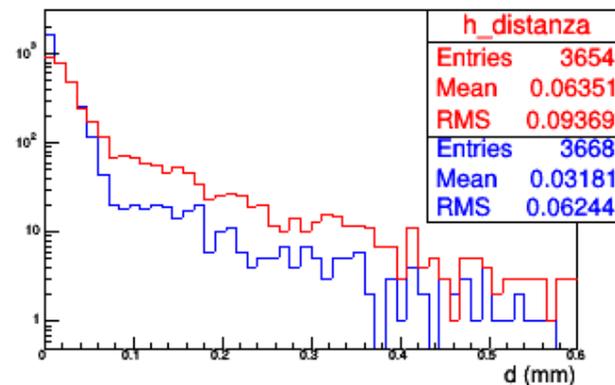
All = Blue, Radial (0 - 1.25) mm = Red



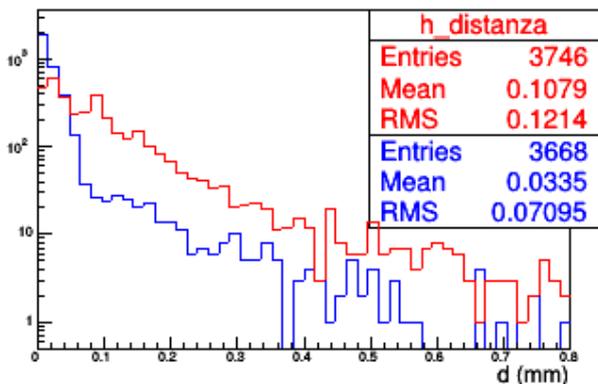
All = Blue, Telescope (0 - 1.25) mm = Red



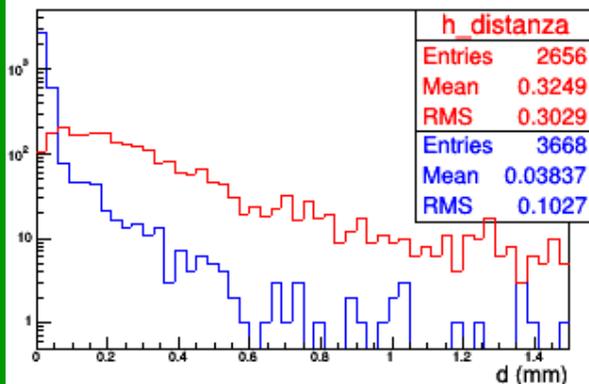
All = Blue, Elliptical (0 - 0.6) mm = Red



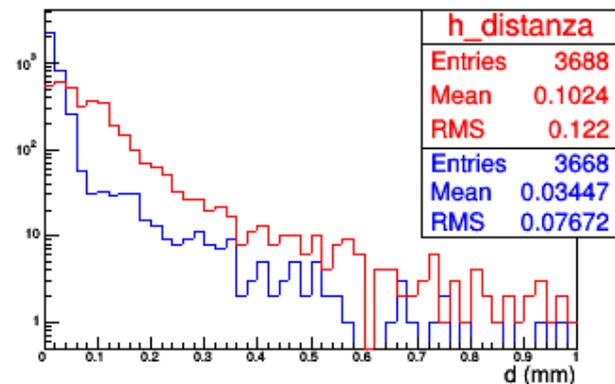
All = Blue, Curl (0 - 1.6) mm = Red



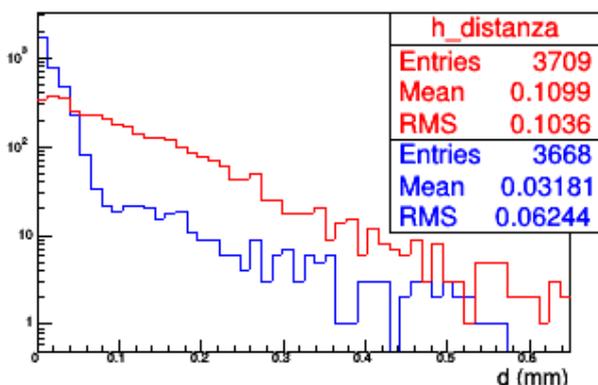
All = Blue, Twist (0 - 3.2) cm = Red



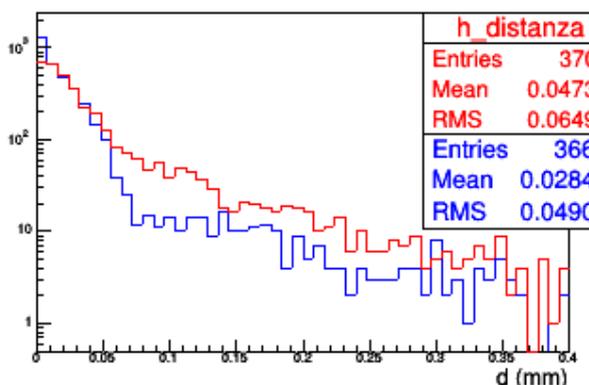
All = Blue, Clamshell (0 - 1.25) mm = Red



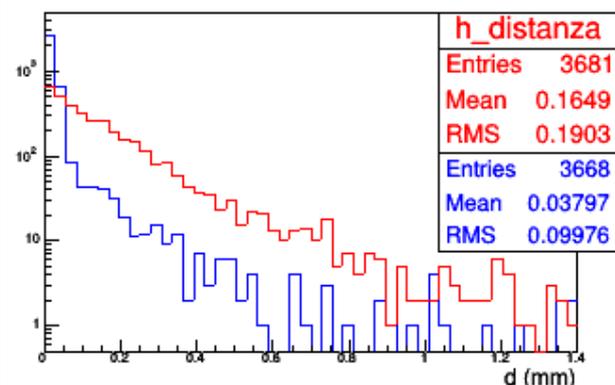
All = Blue, ZExpansion (0 - 2.4) mm = Red



All = Blue, Skew (0 - 1.25) mm = Red

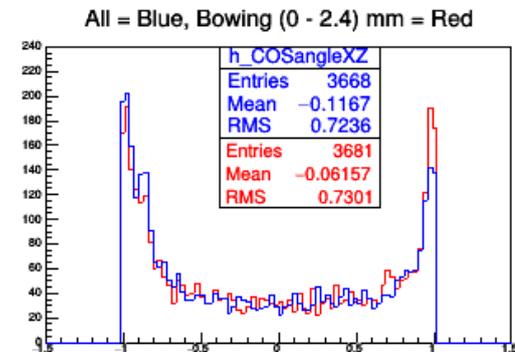
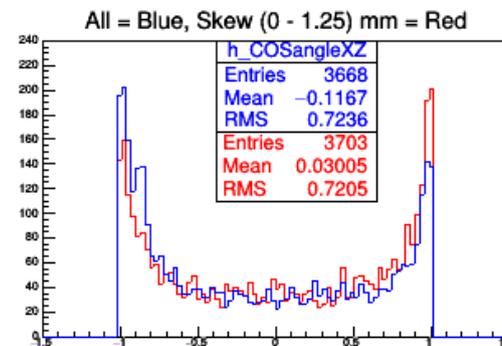
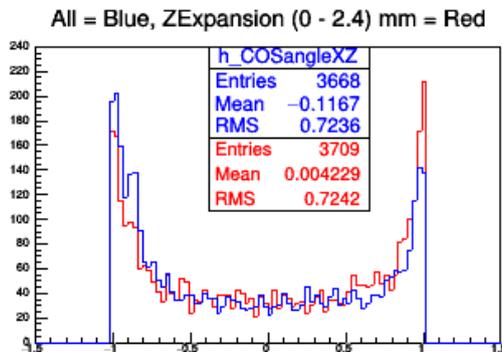
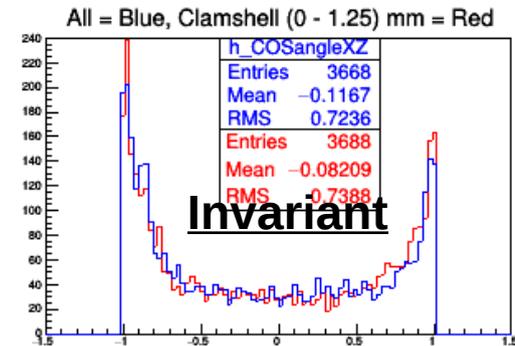
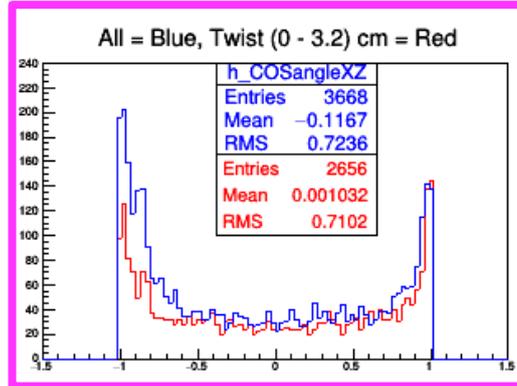
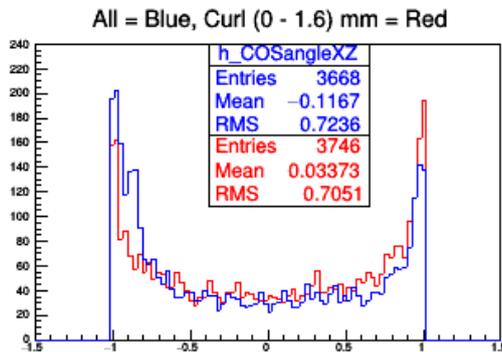
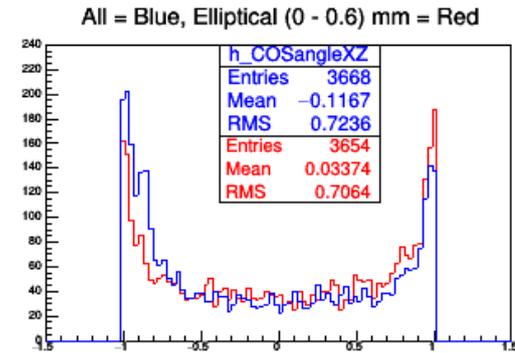
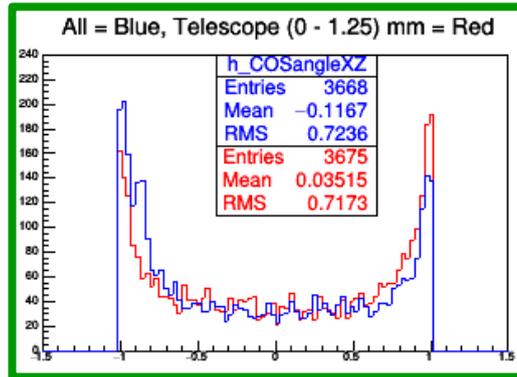
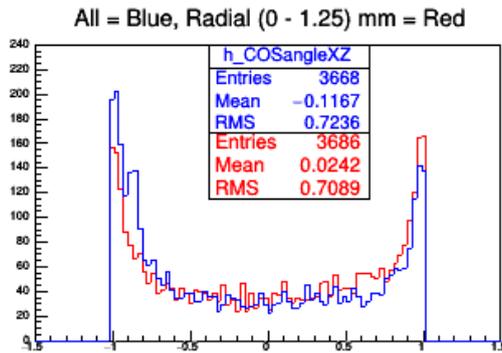


All = Blue, Bowing (0 - 2.4) mm = Red



Examples

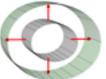
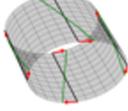
Cosine of muons momenta projections in the xz plane at POCA $\cos\theta_{xz}$ for VXD ideally **aligned** or realistically **misaligned** according to each of the weak modes.



Unambiguously Twist: Twist $\cos\theta_{xz}$ distribution is very symmetrical and the mean is ~ 0.001 .

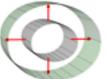
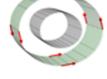
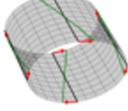
Global picture

-  : highly sensitive to the weak-mode.
-  : poorly sensitive to the weak-mode.
-  : invariant to the weak-mode.

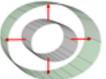
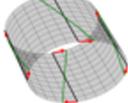
	Radial expansion $\Delta r = c_{scale} \cdot r$ 	Curl $r\Delta\phi = c_{scale} \cdot r + c_0$ 	Telescope $\Delta z = c_{scale} \cdot r$ 	Elliptical expansion $\Delta r = c_{scale} \cdot \cos(2\phi) \cdot r$ 	Clamshell $\Delta\phi = c_{scale} \cdot \cos(\phi)$ 	Skew $\Delta z = c_{scale} \cdot \cos(\phi)$ 	Bowing $\Delta r = c_{scale} \cdot z $ 	Twist $r\Delta\phi = c_{scale} \cdot z$ 	Z expansion $\Delta z = c_{scale} \cdot z$ 
δd_0									
δz_0									
δ									
δr_0									
δr_x									
δr_y									
δr_z									
$d_0^+ \cdot d_0^-$									
$z_0^+ \cdot z_0^-$									

Global picture

 Allows for unambiguous identification of a weak mode

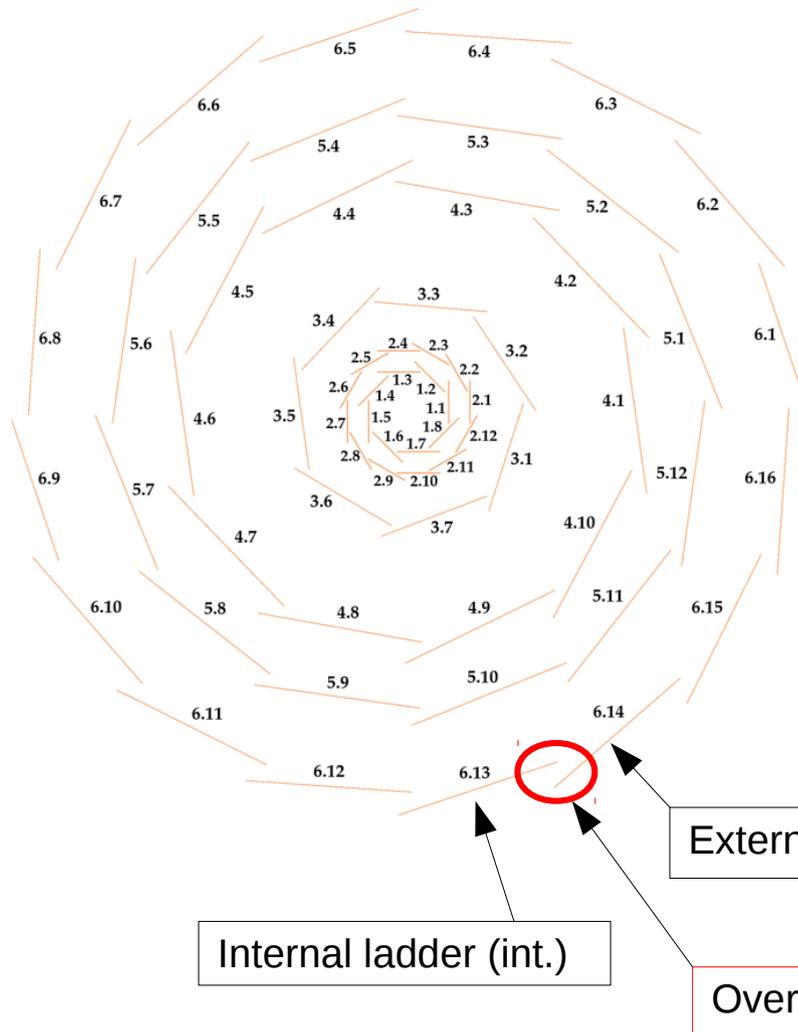
	Radial expansion $\Delta r = c_{scale} \cdot r$ 	Curl $r\Delta\phi = c_{scale} \cdot r + c_0$ 	Telescope $\Delta z = c_{scale} \cdot r$ 	Elliptical expansion $\Delta r = c_{scale} \cdot \cos(2\phi) \cdot r$ 	Clamshell $\Delta\phi = c_{scale} \cdot \cos(\phi)$ 	Skew $\Delta z = c_{scale} \cdot \cos(\phi)$ 	Bowing $\Delta r = c_{scale} \cdot z $ 	Twist $r\Delta\phi = c_{scale} \cdot z$ 	Z expansion $\Delta z = c_{scale} \cdot z$ 
d									
d_{xy}									
d_{xz}									
d_{yz}									
$\cos\theta$									
$\cos\theta_{xy}$									
$\cos\theta_{xz}$									
$\cos\theta_{yz}$									
$d_0^+ + d_0^-$									

Global picture

	Radial expansion $\Delta r = c_{scale} \cdot r$ 	Curl $r\Delta\phi = c_{scale} \cdot r + c_0$ 	Telescope $\Delta z = c_{scale} \cdot r$ 	Elliptical expansion $\Delta r = c_{scale} \cdot \cos(2\phi) \cdot r$ 	Clamshell $\Delta\phi = c_{scale} \cdot \cos(\phi)$ 	Skew $\Delta z = c_{scale} \cdot \cos(\phi)$ 	Bowing $\Delta r = c_{scale} \cdot z $ 	Twist $r\Delta\phi = c_{scale} \cdot z$ 	Z expansion $\Delta z = c_{scale} \cdot z$ 
$z_0^+ + z_0^-$									
$\tan\lambda^+$ + $\tan\lambda^-$									

Residuals in overlapping module regions

A further development to complement and validate the results is to use hits from tracks traversing regions **where modules overlap within a layer of the VXD.**



The difference in residual values for the two measurements in the overlapping modules is:

$$res = \Delta u_{ext.} - \Delta u_{int.}$$

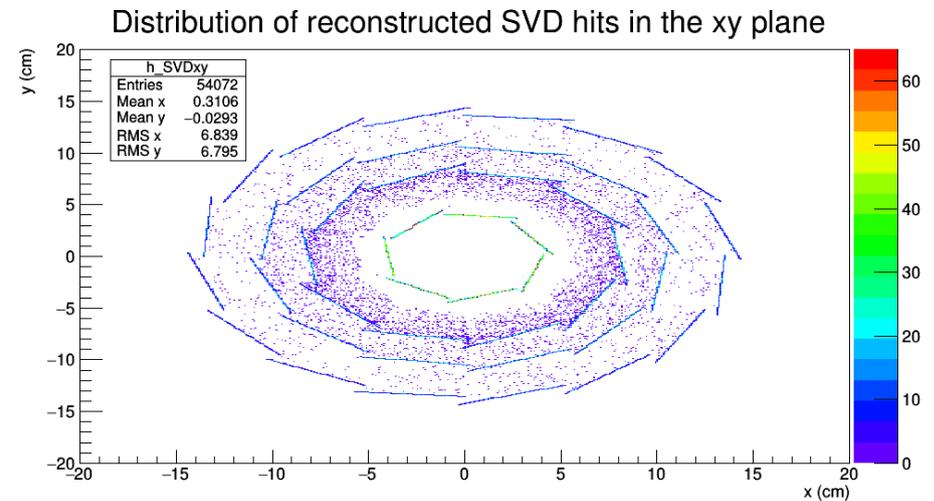
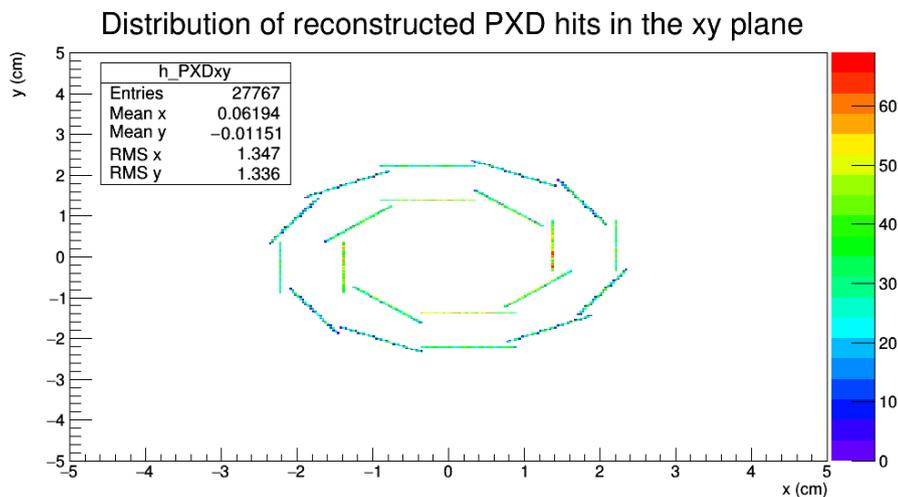
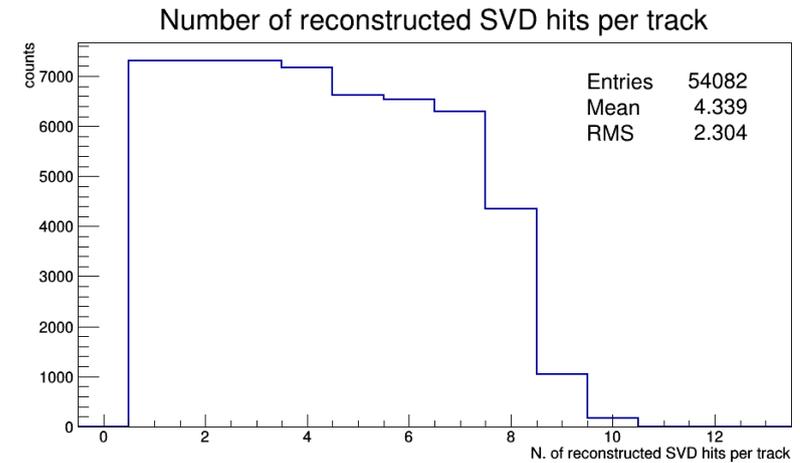
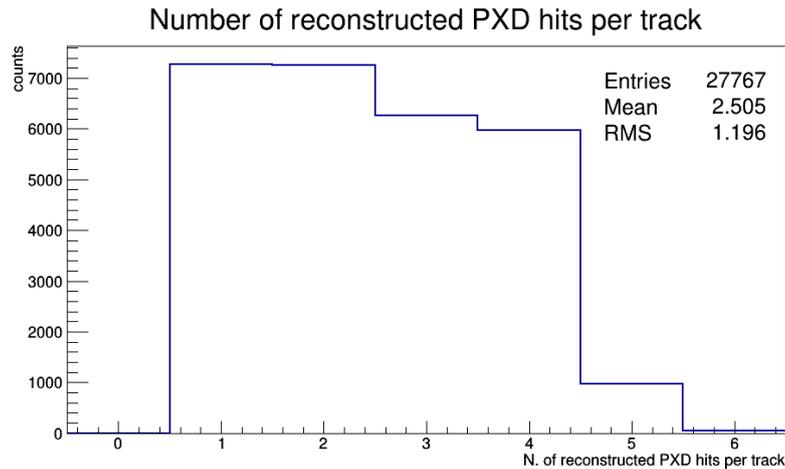
Where $\Delta u = u^{meas.} - u^{pred.}$.

u is the local coordinate parallel to the short side of the sensor.

Simulation and event reconstruction

Events with tracks traversing two overlapping sensors of a same layer are a small subset of the total. Generate $10^4 e^+e^- \rightarrow \mu^+\mu^-$, for sufficient statistics.

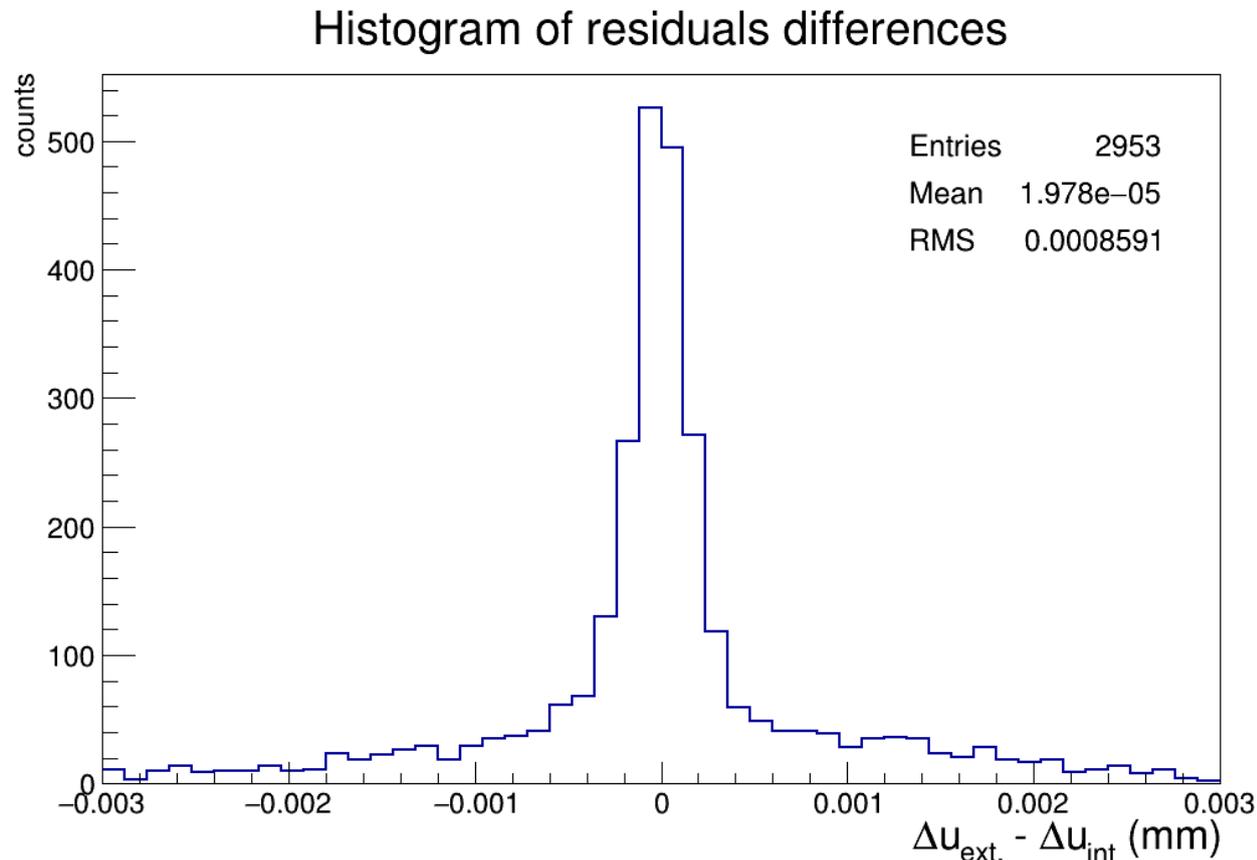
After simulation, a VXD weak-mode misalignment is simulated and tracks are reconstructed in this geometry. A few sanity checks ensure we are not missing major things (aligned reconstruction is shown).



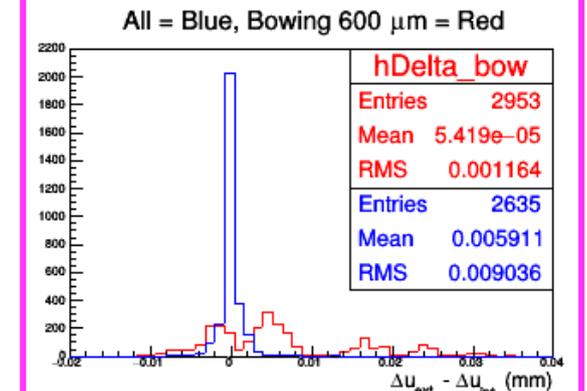
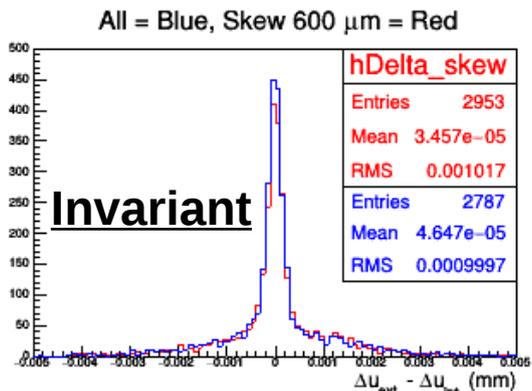
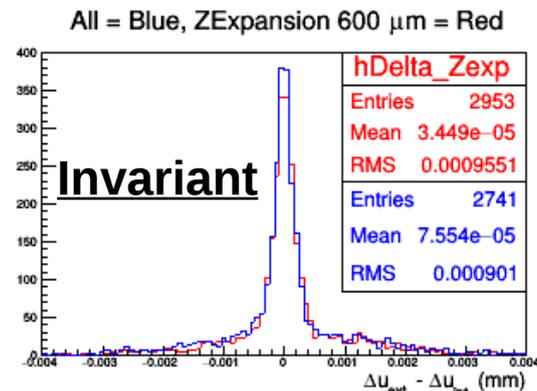
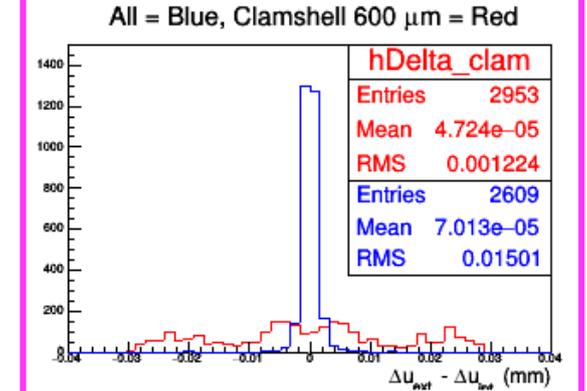
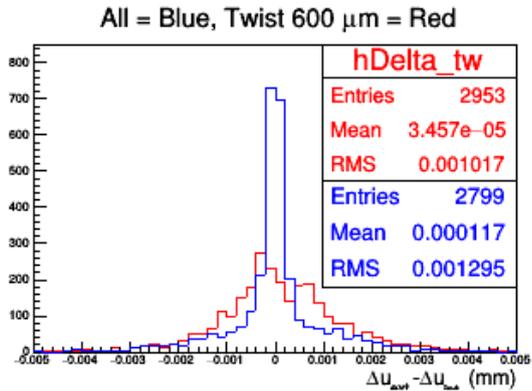
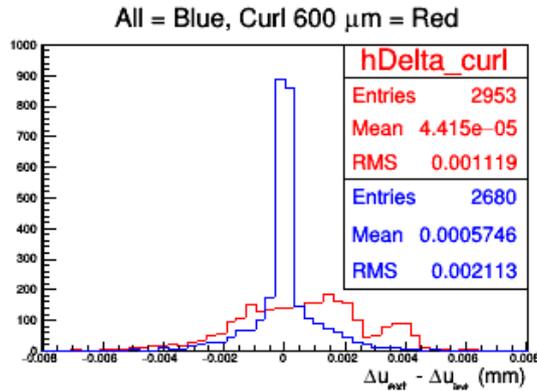
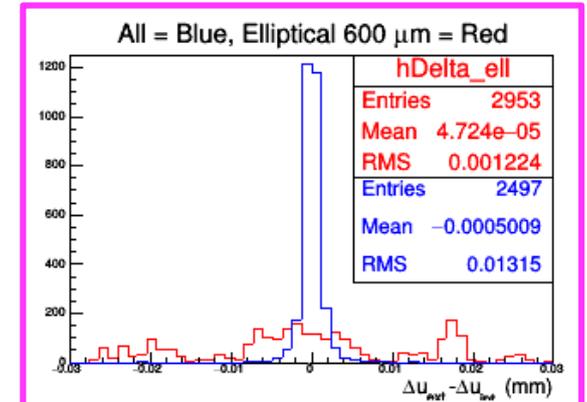
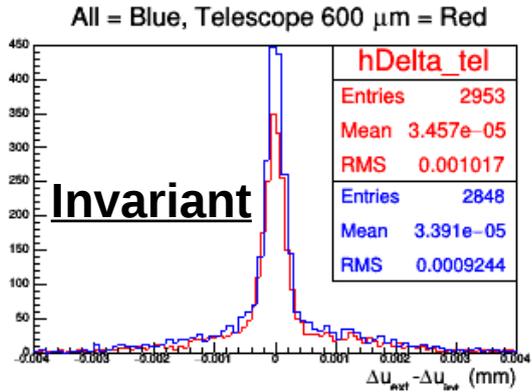
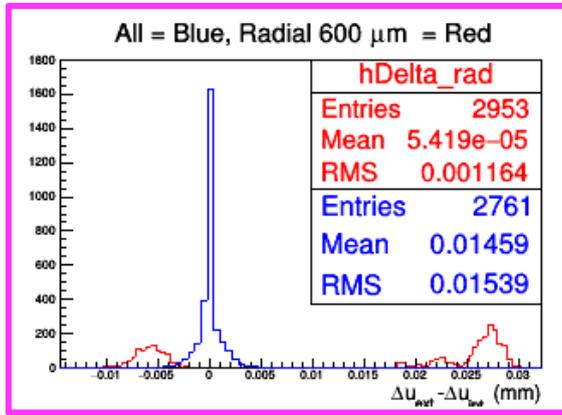
Residuals in overlapping module regions

$res = \Delta u_{ext.} - \Delta u_{int.}$ is marginally sensitive to **the effect of multiple scattering** because of the little radial separation between overlapping sensors belonging to the same layer.

When reconstruction is performed with VXD perfectly aligned we observe an unimodal, symmetric distribution as expected:



$\Delta u_{ext.} - \Delta u_{int.}$ comparison for VXD ideally **aligned** or realistically **misaligned** according to each of the weak modes.



Radial expansion, elliptical expansion, clamshell and bowing show striking distinctive features in the residual differences, with asymmetric and/or multi-modal distributions.

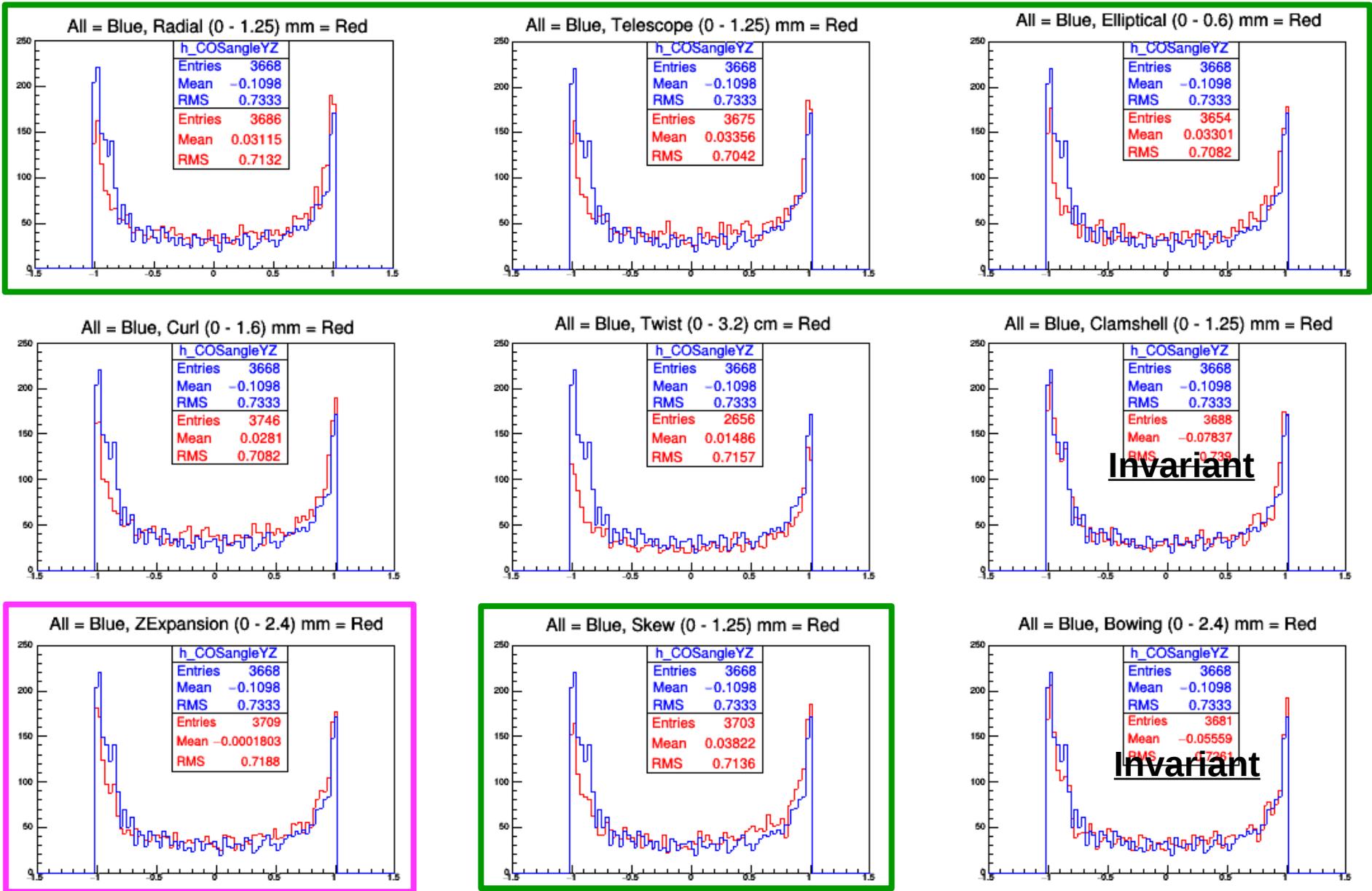
Summary

- Charged particle tracking is a key capability for the success of the Belle II program.
- I am exploring the possibility to quickly spot misalignments of the VXD, using samples of $e^+e^- \rightarrow \mu^+\mu^-$ events.
- Preliminary findings show that some weak-modes might unambiguously be connected to observed patterns in the distribution of appropriately chosen reconstructed variables.
- work still in progress: refine studies with backgrounds and then look at real data.

Backup slides

Examples

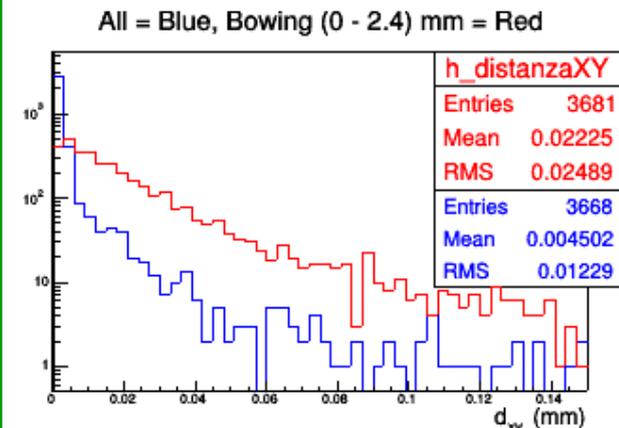
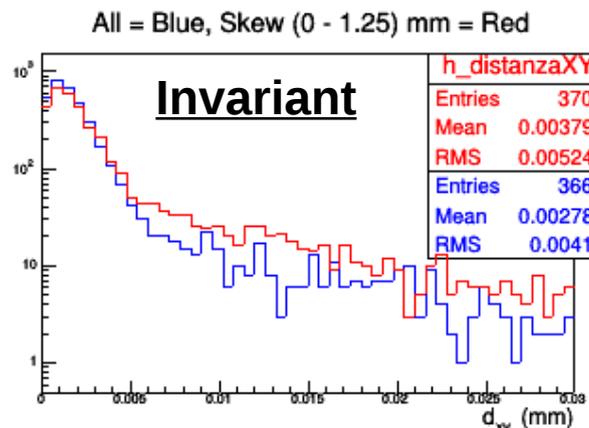
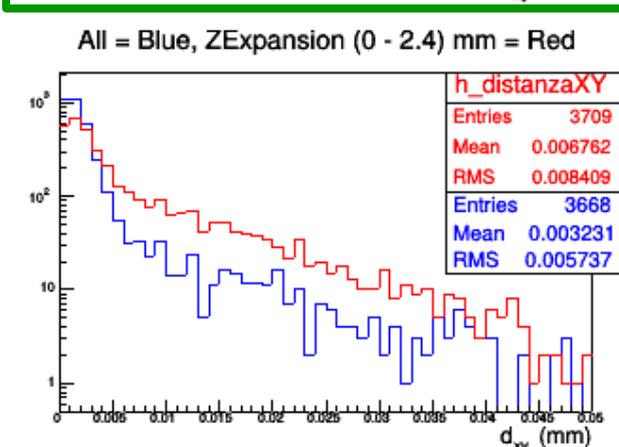
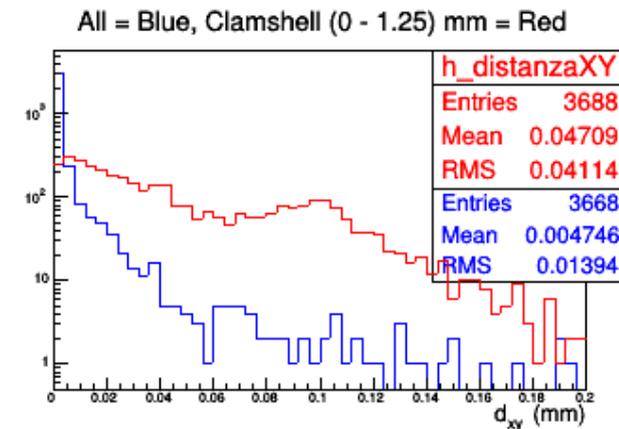
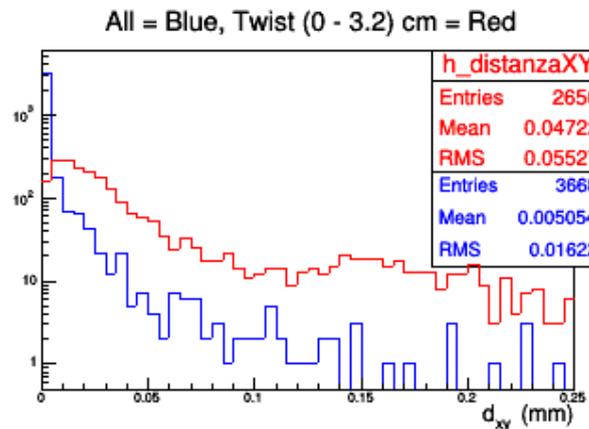
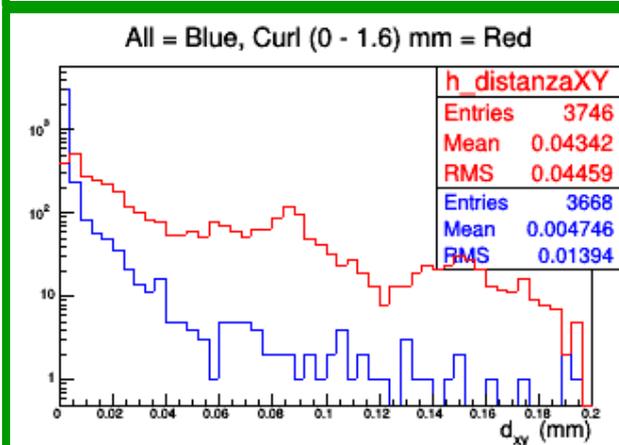
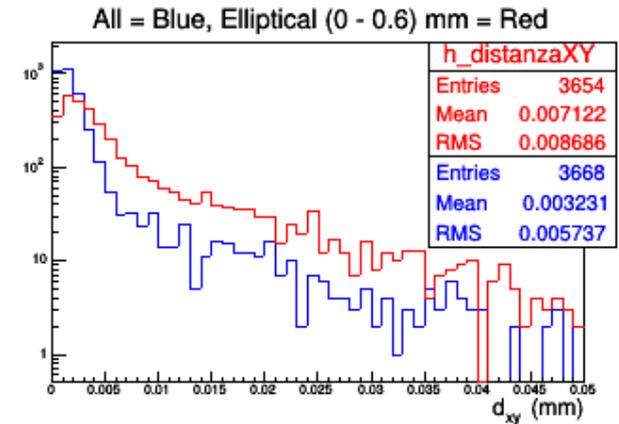
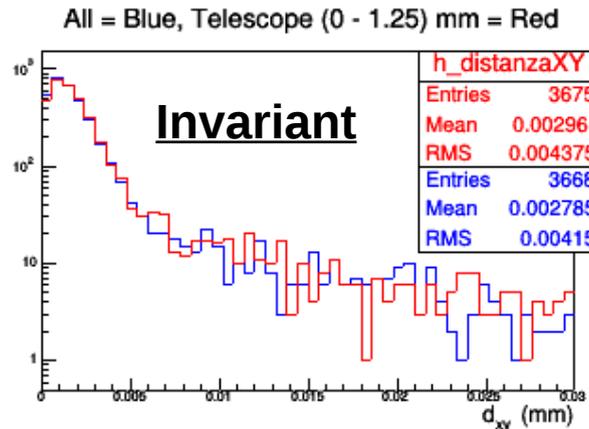
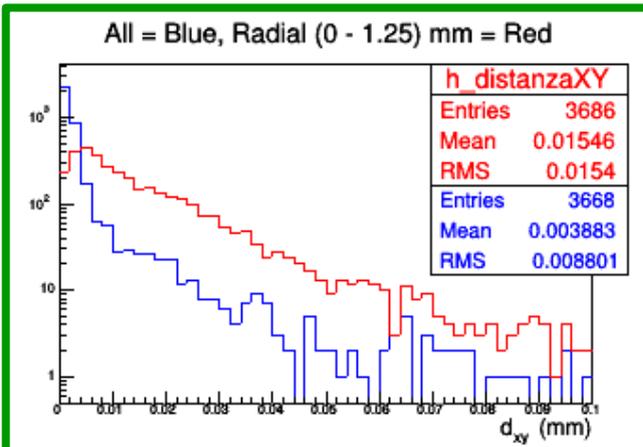
$\cos\theta_{yz}$ comparison for VXD ideally **aligned** or realistically **misaligned** according to each of the weak modes.



Unambiguously Z-Expansion: Z-Expansion $\cos\theta_{yz}$ distribution is very symmetrical and the mean is ~ -0.0001 .

Examples

d_{xy} comparison for VXD ideally **aligned** or realistically **misaligned** according to each of the weak modes.



Event generation, simulation and reconstruction

- Generation: $e^+e^- \rightarrow \mu^+\mu^-$ events using KKMC, which is the default generator for this kind of events (no resonances or interference of resonances with continuum).
- Simulation: it performs interaction with the detector, taking into account all the different materials involved and every geometry, and stores energy losses in the sensitive volumes.
Hits coming from the event are created, together with trajectories and secondary particles: **simulated information will be used as “truth” information in this analysis.**
- Reconstruction:
 - *Hit reconstruction*;
 - *Track representation*: particle propagation and extrapolation taking into account the detector geometry and the magnetic field;
 - *Track fitting algorithm*: fit of the track, given the reconstructed hits and track representation.

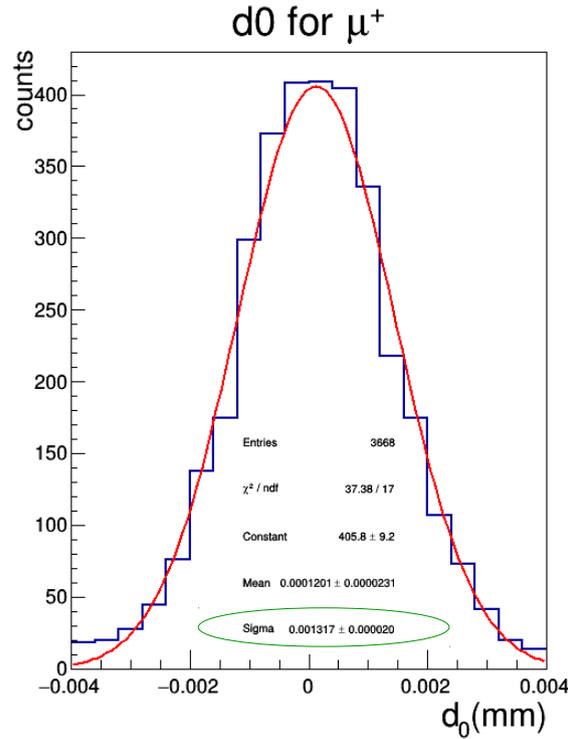
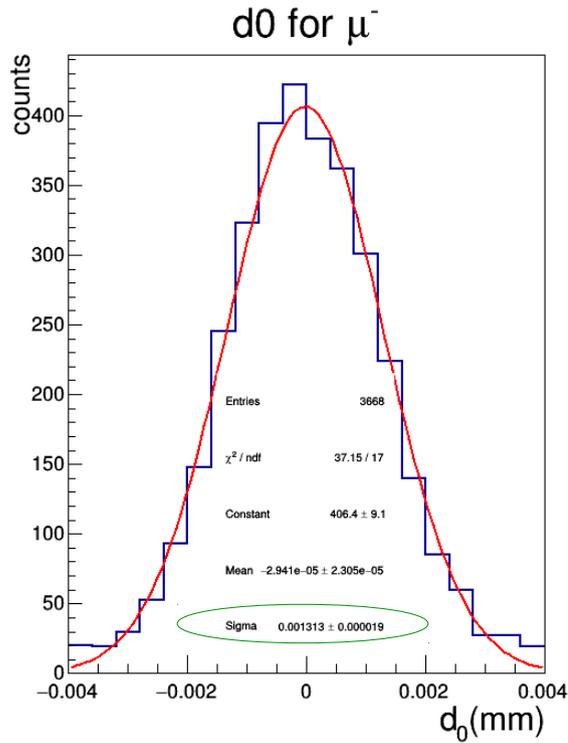
First step: charge dependency of reconstructed track parameters

Generate few thousands $e^+e^- \rightarrow \mu^+\mu^-$ signal-only events and feed them to the full Belle II simulation and event reconstruction.

Using a custom-made analysis module I restrict to those events where only 2 tracks are reconstructed and I extrapolate information from the store array whose elements are objects containing the values of the track parameters obtained performing the track fit.

This part of the work is focused on spotting possible differences between the reconstructed track parameters of the μ^+ and μ^- .

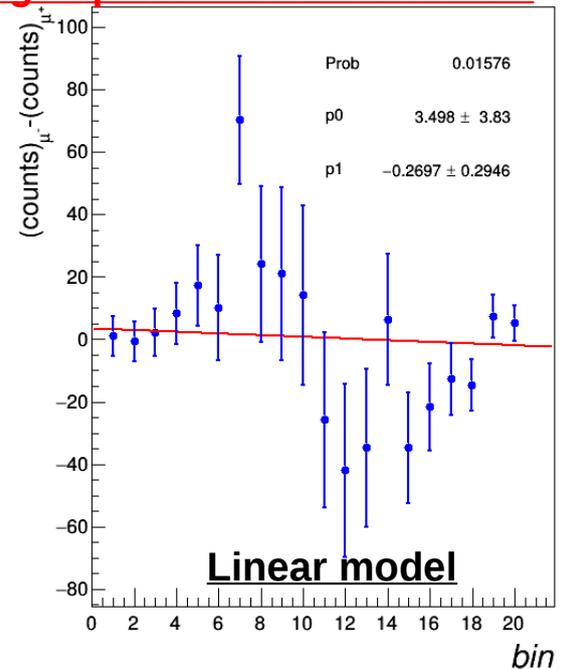
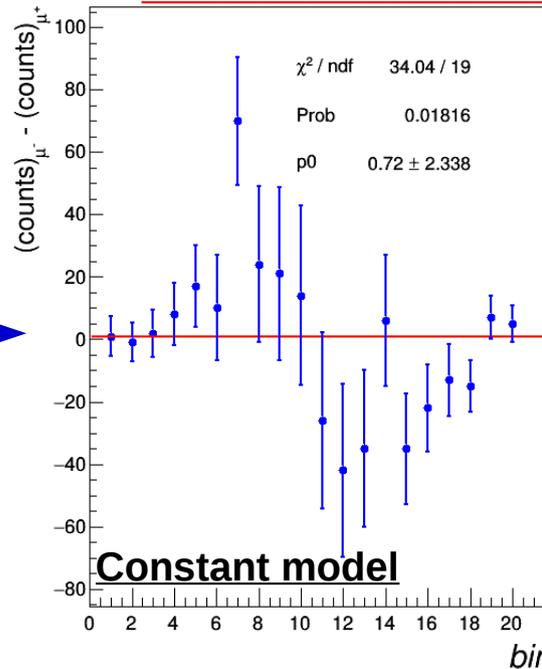
Transverse impact parameter



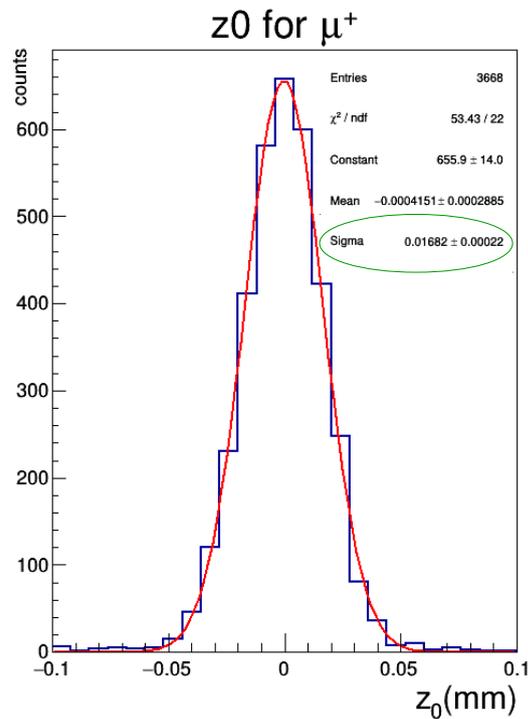
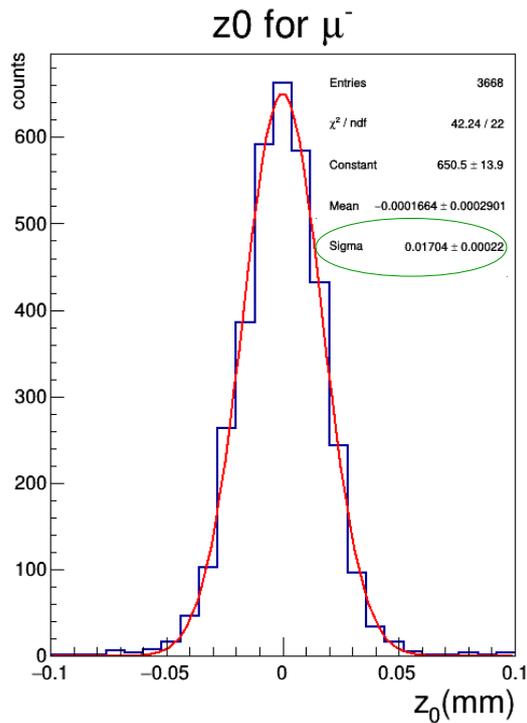
d0 resolution is O(10) of μm as expected.

The difference between the χ^2 is 1.42, so the two models are statistically equivalent but both fail to fit properly the data, which seem to show a **shift** between μ^+ and μ^- distributions.

Difference between charge-specific d0 distributions



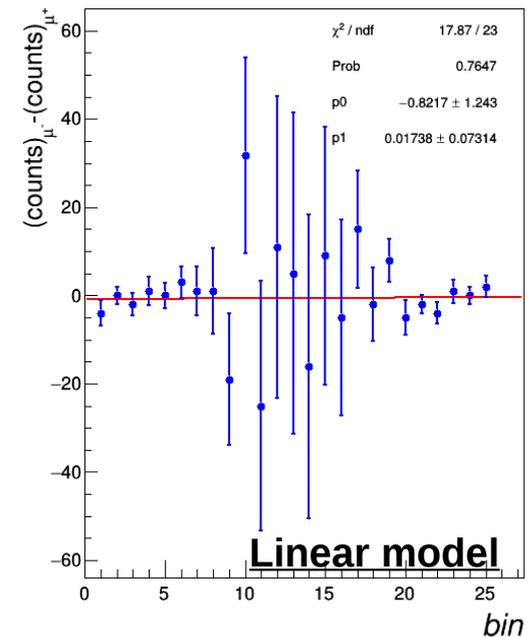
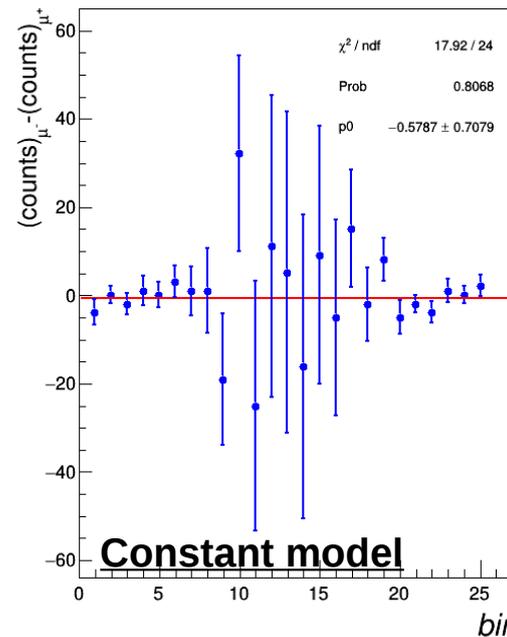
Longitudinal displacement



Longitudinal displacement resolution is, for both muons, $\sigma_{z_0} \sim 18\mu\text{m}$ as expected.

The difference between the χ^2 is compatible with having added a degree of freedom: the value of the fit parameter $p_0 = -0.53 \pm 1.03$ is compatible with 0.

Difference between charge-specific d0 distributions



Minimum distance between the reconstructed tracks

A convenient inclusive way to summarize the whole **reconstructed track** information into a single distribution is to construct the minimum signed distance between the two straight lines which best approximate the tracks near the origin.

Neglecting curvature ω (**linear approximation**) I can write the two tracks, corresponding to the positive and the negative muon, as

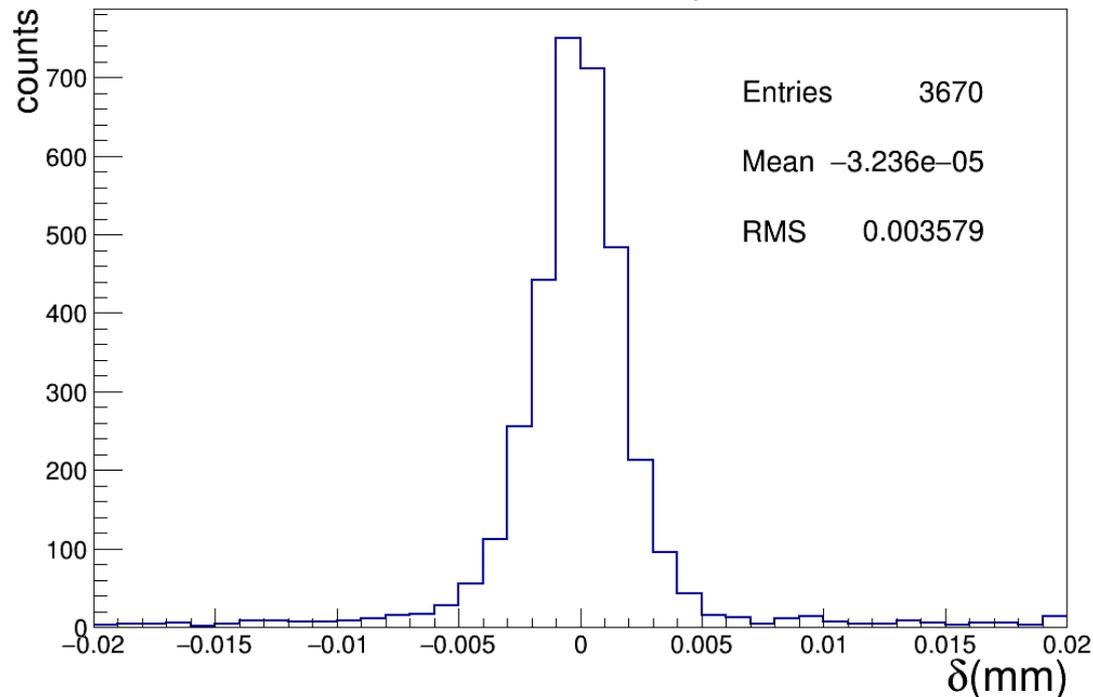
$$\mu^+ \rightarrow d_0^{(+)} \hat{t}^{(+)} + z_0^{(+)} \hat{z} + \lambda^{(+)} \bar{p}_0^{(+)}$$

$$\mu^- \rightarrow d_0^{(-)} \hat{t}^{(-)} + z_0^{(-)} \hat{z} + \lambda^{(-)} \bar{p}_0^{(-)}$$

and the **distance** between the tracks is

$$\delta = \frac{\bar{p}_0^{(-)} \times \bar{p}_0^{(+)}}{|\bar{p}_0^{(-)} \times \bar{p}_0^{(+)}|} \left[d_0^{(+)} \hat{t}^{(+)} - d_0^{(-)} \hat{t}^{(-)} + (z_0^{(+)} - z_0^{(-)}) \hat{z} \right]$$

Distr. of delta for each pair of tracks



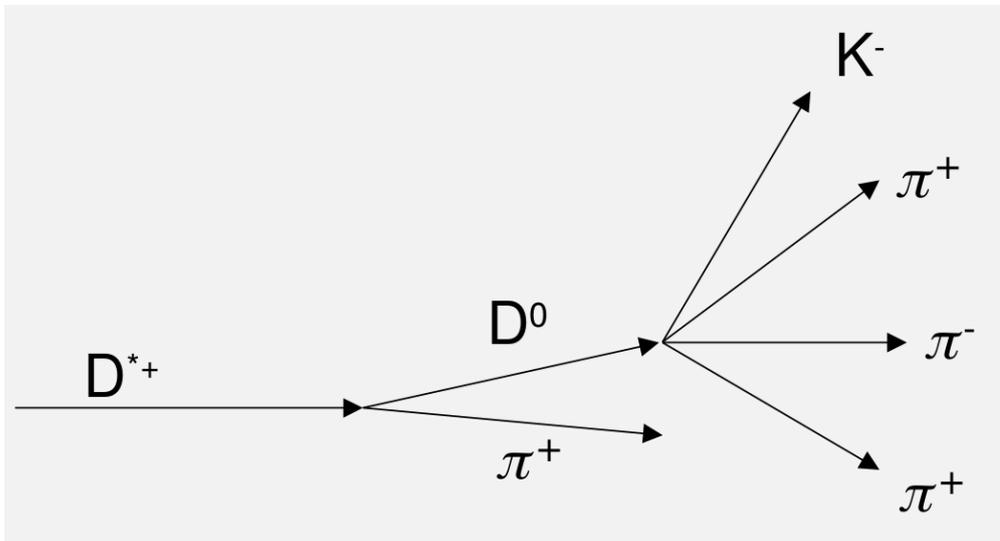
Track-finding efficiency studies: overview.

In order to estimate the track-finding efficiency, $D^{*+} \rightarrow D^0 \pi^+$ decay is used.

There are sufficient kinematic constraints to detect this decay even if one of the tracks from D^0 is missed.

Then one can process full and partial reconstruction (one track is missed) of this decay.

After the estimation of the number of candidates for each of reconstructions, it is possible to calculate the track-finding efficiency of the missed particle:



$$M(D^{*\pm}) = 2010.26 \pm 0.07 \text{ MeV}$$

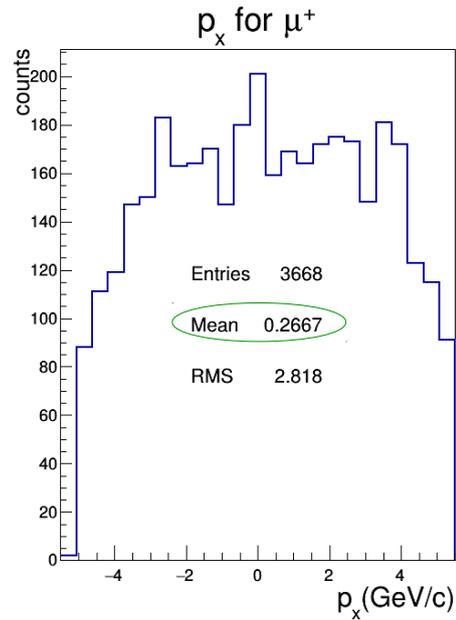
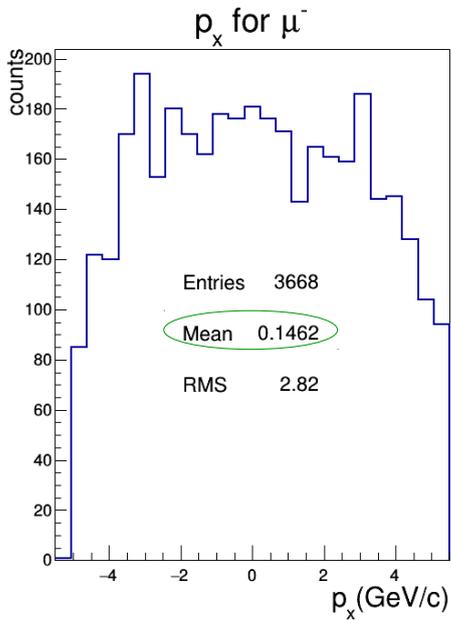
$$M(D^0) = 1864.84 \pm 0.07 \text{ MeV}$$

$$M(\pi^\pm) = 139.57018 \pm 0.00035 \text{ MeV}$$

$$Q < 20 \text{ MeV}$$

$$\epsilon(\text{charged track}) = \frac{N_{full}(D^{*+} \rightarrow D^0 \pi^+)}{N_{part.}(D^{*+} \rightarrow D^0 \pi^+)}$$

px: momentum in the plane of the ring

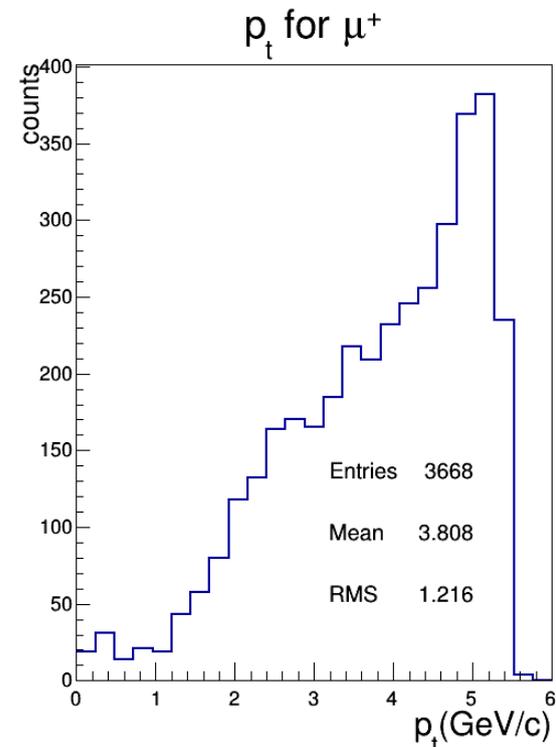
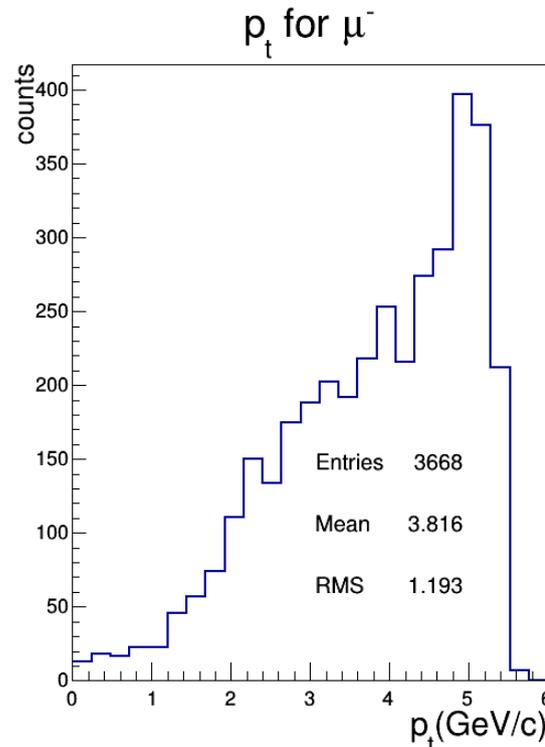


Both the distributions have **nonzero positive mean** and this may suggest that interactions push particles in the x direction outside of the accelerator tunnel.

This is consistent with an expected **crossing angle of 83 mrad** between the beams.

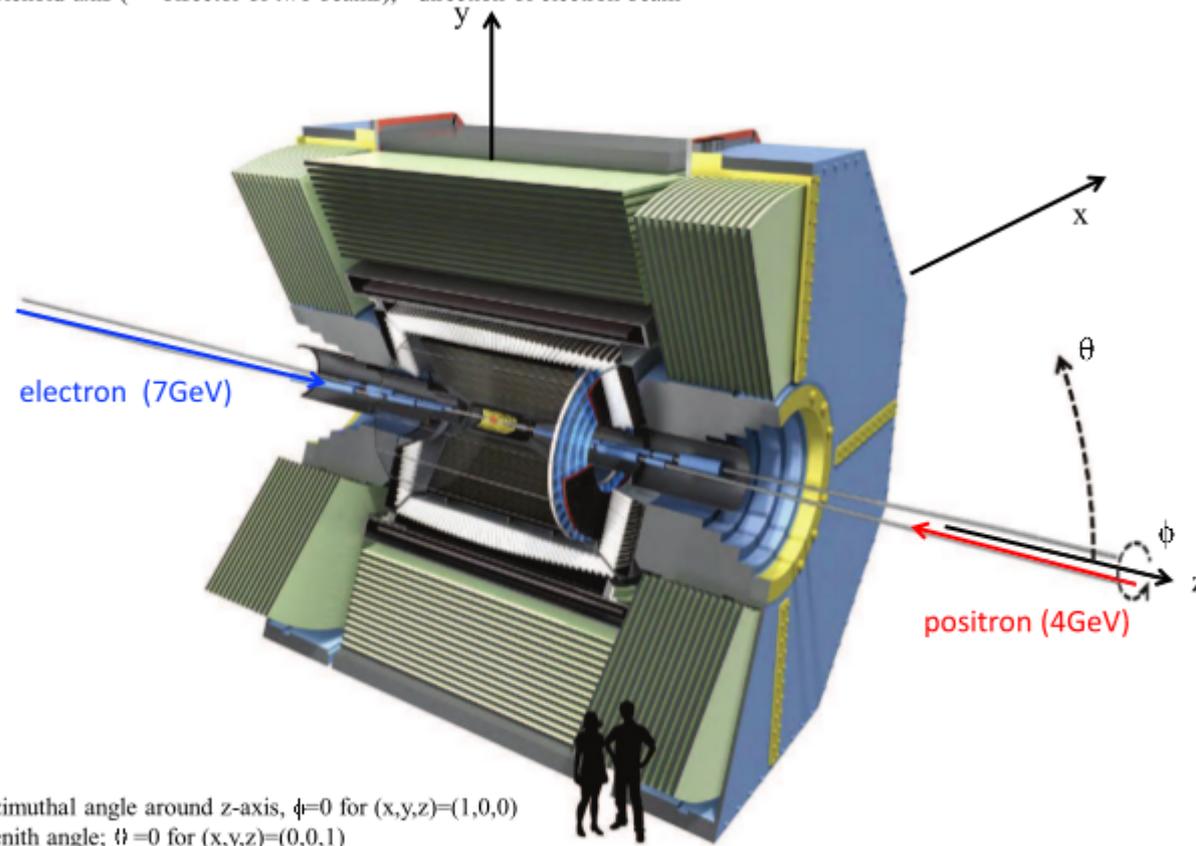
Transverse momentum

Both the histograms show a peak at ~ 5 GeV/c, which (for muons) is consistent with a $\Upsilon(4s)$ resonance at $\sqrt{s} = 10.58$ GeV.



Coordinate system of the Belle II software.

x: Horizontal outward of tunnel
y: Vertical upward
z: Solenoid axis (== bisector of two beams); \sim direction of electron beam



ϕ : azimuthal angle around z-axis, $\phi=0$ for $(x,y,z)=(1,0,0)$
 θ : zenith angle; $\theta=0$ for $(x,y,z)=(0,0,1)$

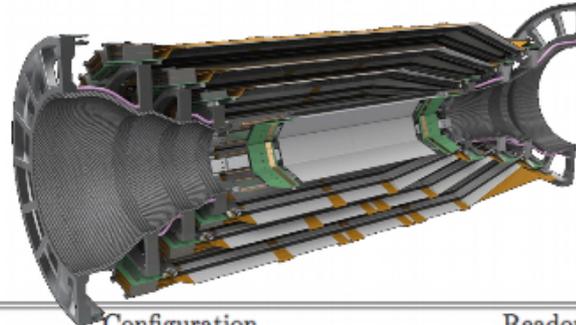
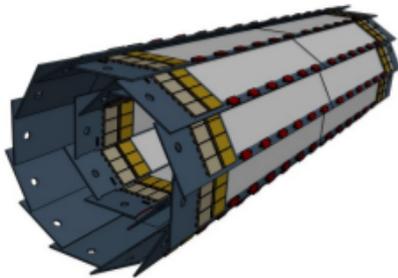
Coordinate system of Belle II software. x axis is horizontal and toward outside of the accelerator tunnel, which is roughly northeast. y is vertical upward. z is the Belle solenoid axis, which is bisector of two beams; roughly toward the direction of electron beam. ϕ is azimuthal angle around z -axis. $\phi = 0$ is defined for $(x, y, z) = (1, 0, 0)$. θ is zenith angle with respect to z -axis. $\theta = 0$ is defined for $(x, y, z) = (0, 0, 1)$

Detectors

Tracking Hardware



- Pixel (PXD), Strip (SVD), and drift chamber (CDC)



Component	Type	Configuration	Readout	Performance
Beam pipe	Beryllium double-wall	Cylindrical, inner radius 10 mm, 10 μm Au, 0.6 mm Be, 1 mm coolant (paraffin), 0.4 mm Be		
PXD	Silicon pixel (DEPFET)	Sensor size: 15 \times 100 (120) mm ² pixel size: 50 \times 50 (75) μm^2 2 layers: 8 (12) sensors	10 M	impact parameter resolution $\sigma_{z_0} \sim 20 \mu\text{m}$ (PXD and SVD)
SVD	Double sided Silicon strip	Sensors: rectangular and trapezoidal Strip pitch: 50(p)/160(n) - 75(p)/240(n) μm 4 layers: 16/30/56/85 sensors	245 k	
CDC	Small cell drift chamber	56 layers, 32 axial, 24 stereo r = 16 - 112 cm - 83 \leq z \leq 159 cm	14 k	$\sigma_{r\phi} = 100 \mu\text{m}, \sigma_z = 2 \text{ mm}$ $\sigma_{p_t}/p_t = \sqrt{(0.2\%p_t)^2 + (0.3\%/\beta)^2}$ $\sigma_{p_t}/p_t = \sqrt{(0.1\%p_t)^2 + (0.3\%/\beta)^2}$ (with SVD)

Built

Side view of the vertex detector



$e^+e^- \rightarrow \mu^+\mu^-$ differential cross section

$$d\sigma = \frac{e^4}{64\pi^2 s} \frac{\sqrt{1 - \frac{4m_\mu^2}{s}}}{\sqrt{1 - \frac{4m_e^2}{s}}} \left(1 + \frac{4}{s} (m_e^2 + m_\mu^2) + \left(1 - \frac{4m_e^2}{s} \right) \left(1 - \frac{4m_\mu^2}{s} \right) \cos^2 \theta \right) d\Omega$$

If the energy is much larger than either mass, $m_e < m_\mu \ll \sqrt{s} < 90 \text{ GeV}$, then this is approximately,

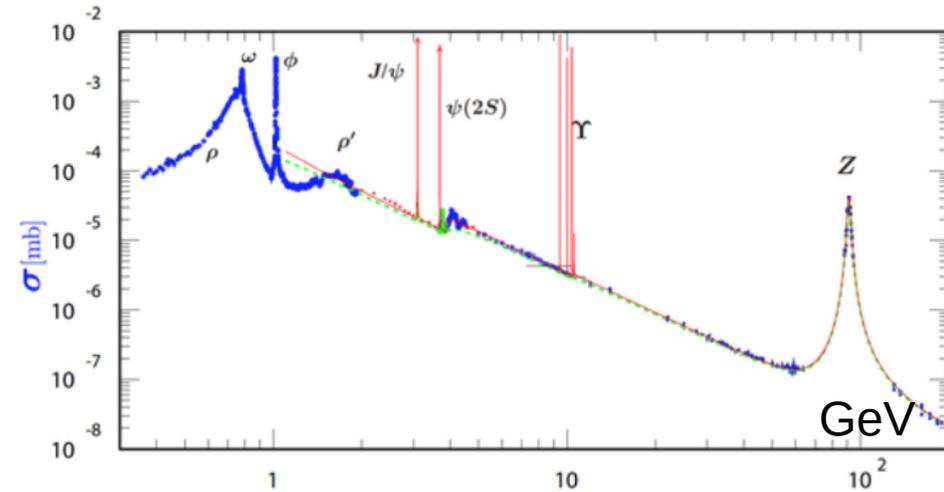
$$d\sigma = \frac{e^4}{64\pi^2 s} (1 + \cos^2 \theta) d\Omega$$

Writing e^2 in terms of the fine structure constant,

$$e^2 = \frac{e^2}{\hbar c} = 4\pi\alpha$$

this is

$$d\sigma = \frac{\alpha^2}{4s} (1 + \cos^2 \theta) d\Omega$$



$e^+e^- \rightarrow \text{hadrons}$ differential cross section

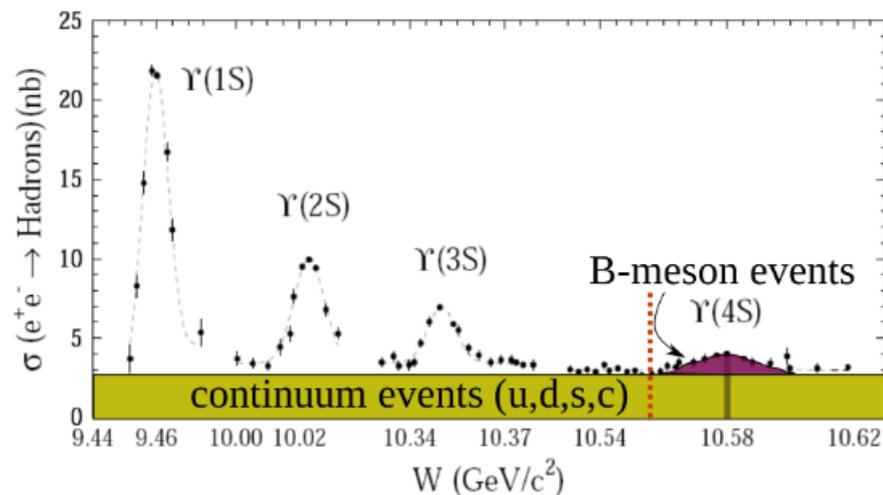


Figure 2.1.: e^+e^- cross section in the $\Upsilon(1S) - \Upsilon(4S)$ region. The red dashed line marks the kinematic threshold for the production of $B\bar{B}$ pairs [14].

Millipede II

This algorithm is based on global linear χ^2 minimization with constraints.
This method relies on linearization of the normalized residuals z_{ij} in the χ^2 function:

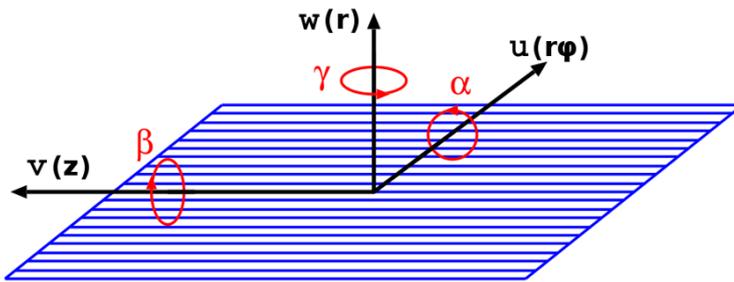
$$\chi^2(\bar{\tau}, \bar{a}) = \sum_j^{\text{tracks}} \sum_i^{\text{hits}} z_{ij}^2(\bar{a}, \bar{\tau}_j) \simeq \sum_j^{\text{tracks}} \sum_i^{\text{hits}} \frac{1}{\sigma_{ij}^2} \left(r_{ij}(\bar{\tau}_j^0, \bar{a}^0) + \frac{\partial r_{ij}}{\partial \bar{a}} \delta \bar{a} + \frac{\partial r_{ij}}{\partial \bar{\tau}_j} \delta \bar{\tau}_j \right)^2$$

Where: - $\bar{\tau}_j^0, \bar{a}^0$ are initial parameters;
- $\delta \bar{\tau}_j, \delta \bar{a}$ are small corrections.

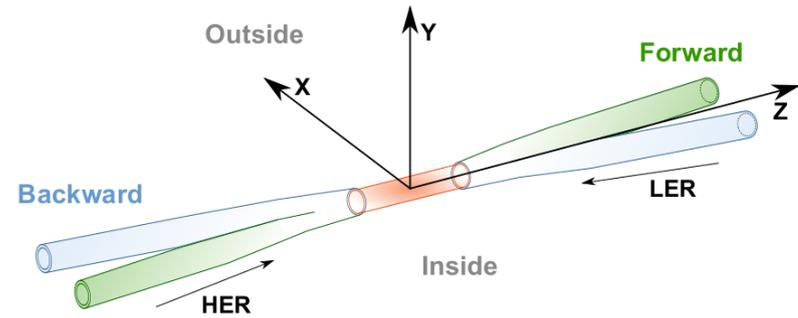
Separation of alignment parameters \bar{a} from track parameters $\bar{\tau}$ allows to simplify the problem of minimization by means of block matrix algebra and solve a reduced problem using matrix inversion.

Coordinates systems

Local



Global



$$\bar{q} = (u, v, w) \longrightarrow \bar{r} = (x, y, z)$$
$$\bar{r} = R^T \Delta R (\bar{q} + \Delta \bar{q}) + \bar{r}_0$$

With: - R rotation matrix

- $\Delta q = (\Delta u, \Delta v, \Delta w)$

- $\Delta R = R_\alpha R_\beta R_\gamma$