

Introduction to Accelerator Physics

Part 1

Pedro Castro / Accelerator Physics Group (MPY)
Hamburg, 23rd July 2018



	length	lab	run	particles	energy	dipole field
PETRA	2.3 km	DESY	1978-1986	e-/e+	2x19 GeV	0.33 T
PETRA II	2.3 km	DESY	1987-2007	e- or e+ p	12 GeV 40 GeV	0.21 T 0.7 T
PETRA III	2.3 km	DESY	2009- ?	e-	6 GeV	0.10 T
HERA	6.3 km	DESY	1992-2007	e- or e+ p	27.5 GeV 920 GeV	0.274 T 5 T
LEP	27 km	CERN	1989-2000	e-/e+	2x105 GeV	0.135 T
LHC	27 km	CERN	2010- ?	p/p	2x7000 GeV	8.3 T
FLASH	0.3 km	DESY	2004- ?	e-	1.2 GeV	
XFEL	3 km	DESY	2016- ?	e-	17.5 GeV	
ILC	30 km	?	?	e-/e+	2x250 GeV	

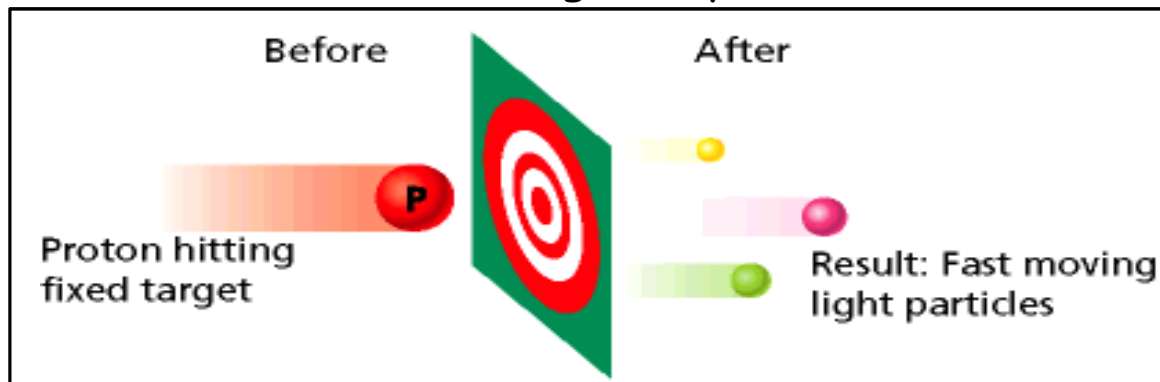
DESY

CERN

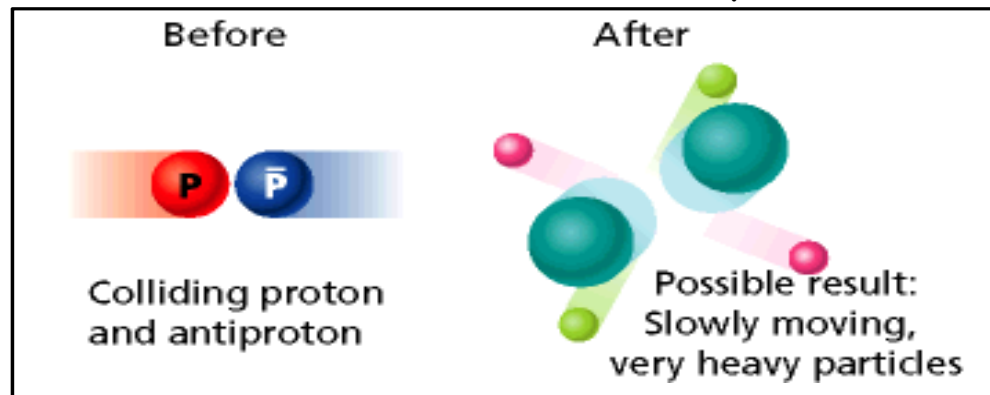
Applications of Accelerators (1)

Particle colliders for High Energy Physics (HEP) experiments

Fixed target experiments



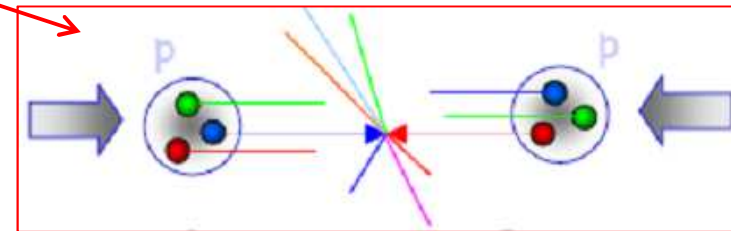
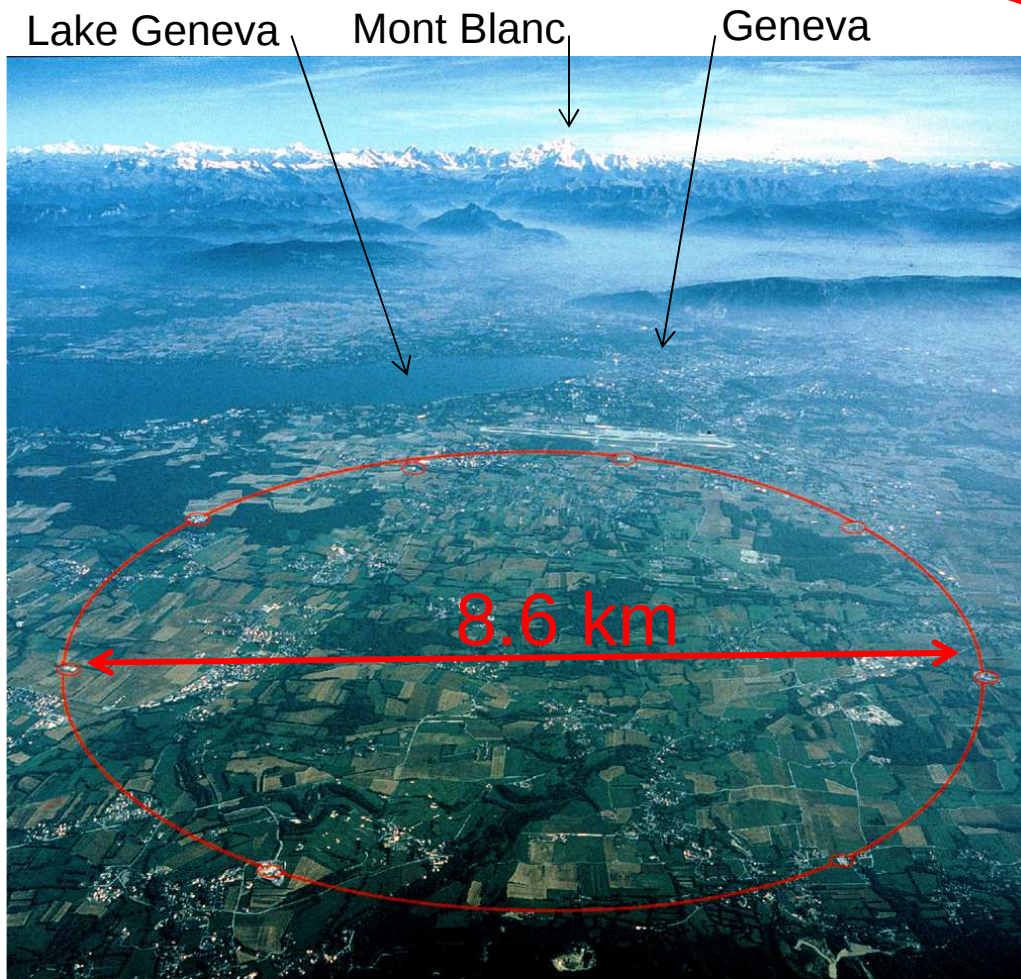
Two beams collider experiments



Applications of Accelerators (1)

Particle colliders for High Energy Physics experiments

Example: the Large Hadron Collider (LHC) at CERN



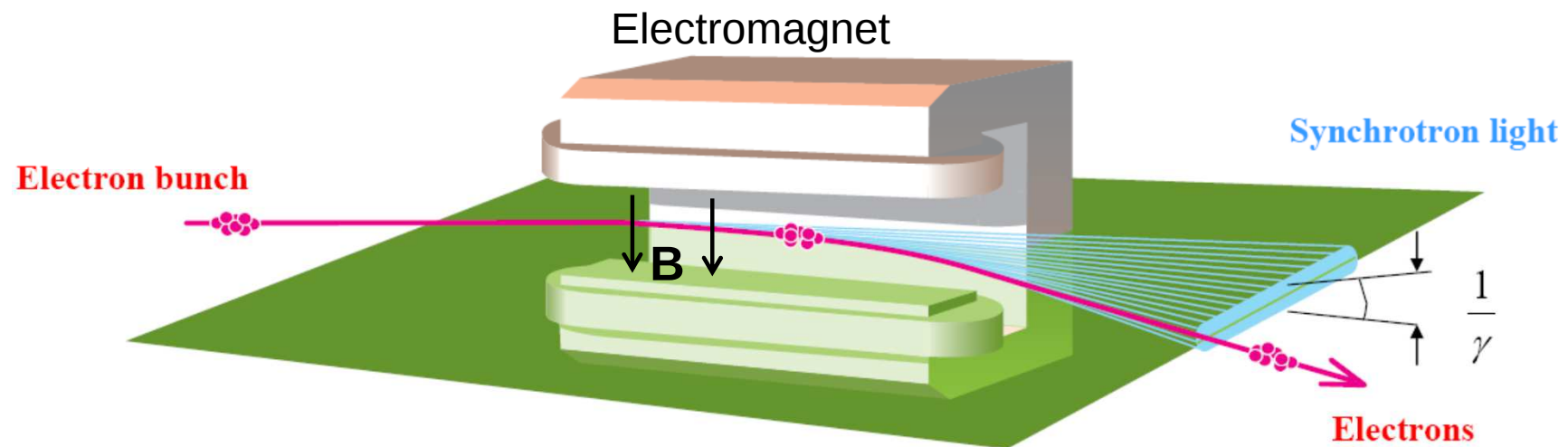
built between 2001 and 2009
Higgs discovery: July 2012



superconducting magnets
(inside a cryostat)

Applications of Accelerators (2)

Light sources for biology, physics, chemistry... experiments

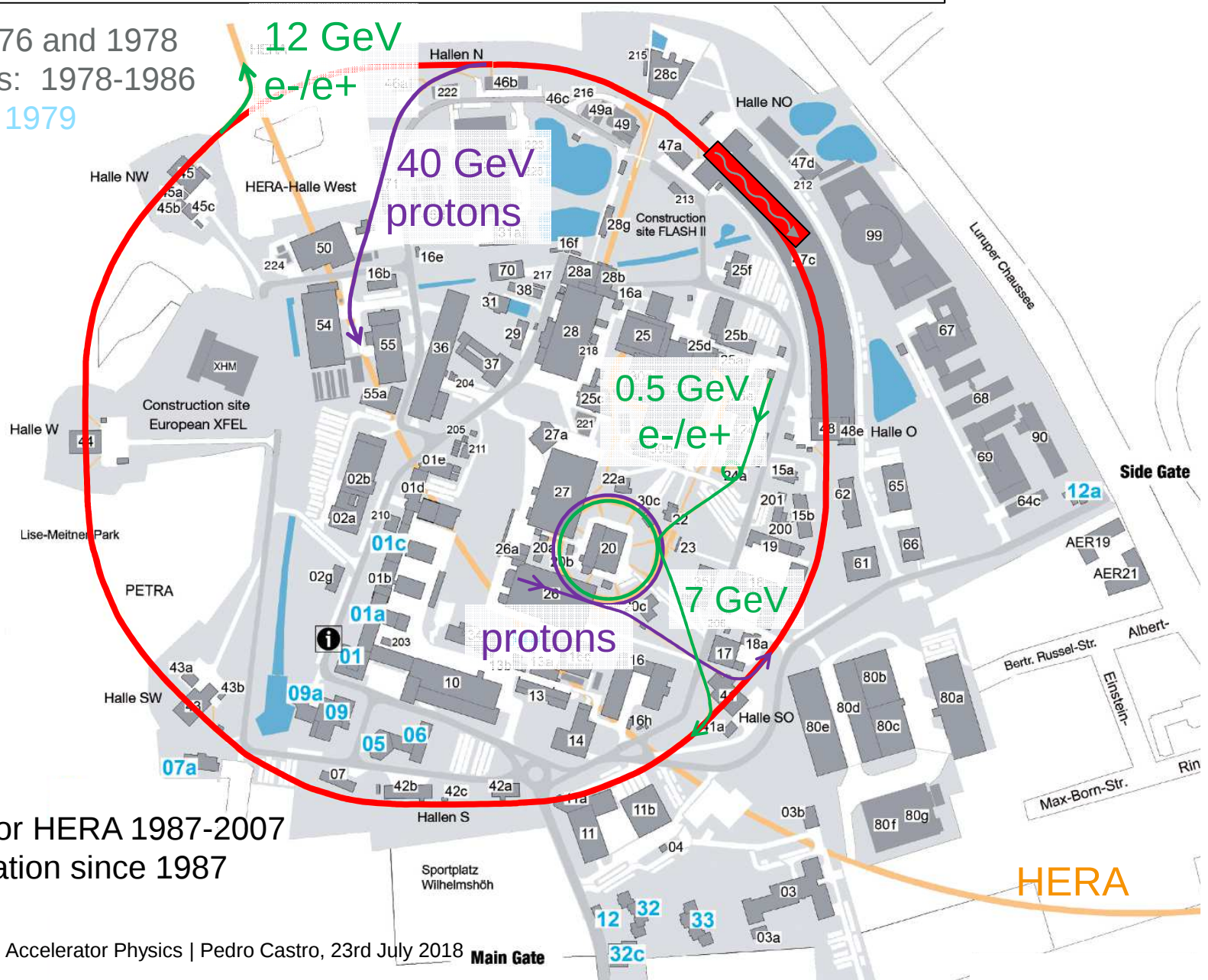


- structural analysis of crystalline materials
- X-ray crystallography (of proteins)
- X-ray microscopy
- X-ray absorption (or emission) spectroscopy
- ...

Example: Positron-Elektron-Tandem-Ring-Anlage (PETRA) 'positron-electron tandem ring accelerator' at DESY

built between 1976 and 1978
HEP experiments: 1978-1986
gluon discovery: 1979

2.3 km long



pre-accelerator for HERA 1987-2007
synchrotron radiation since 1987

Example: Positron-Elektron-Tandem-Ring-Anlage (PETRA) 'positron-electron tandem ring accelerator' at DESY

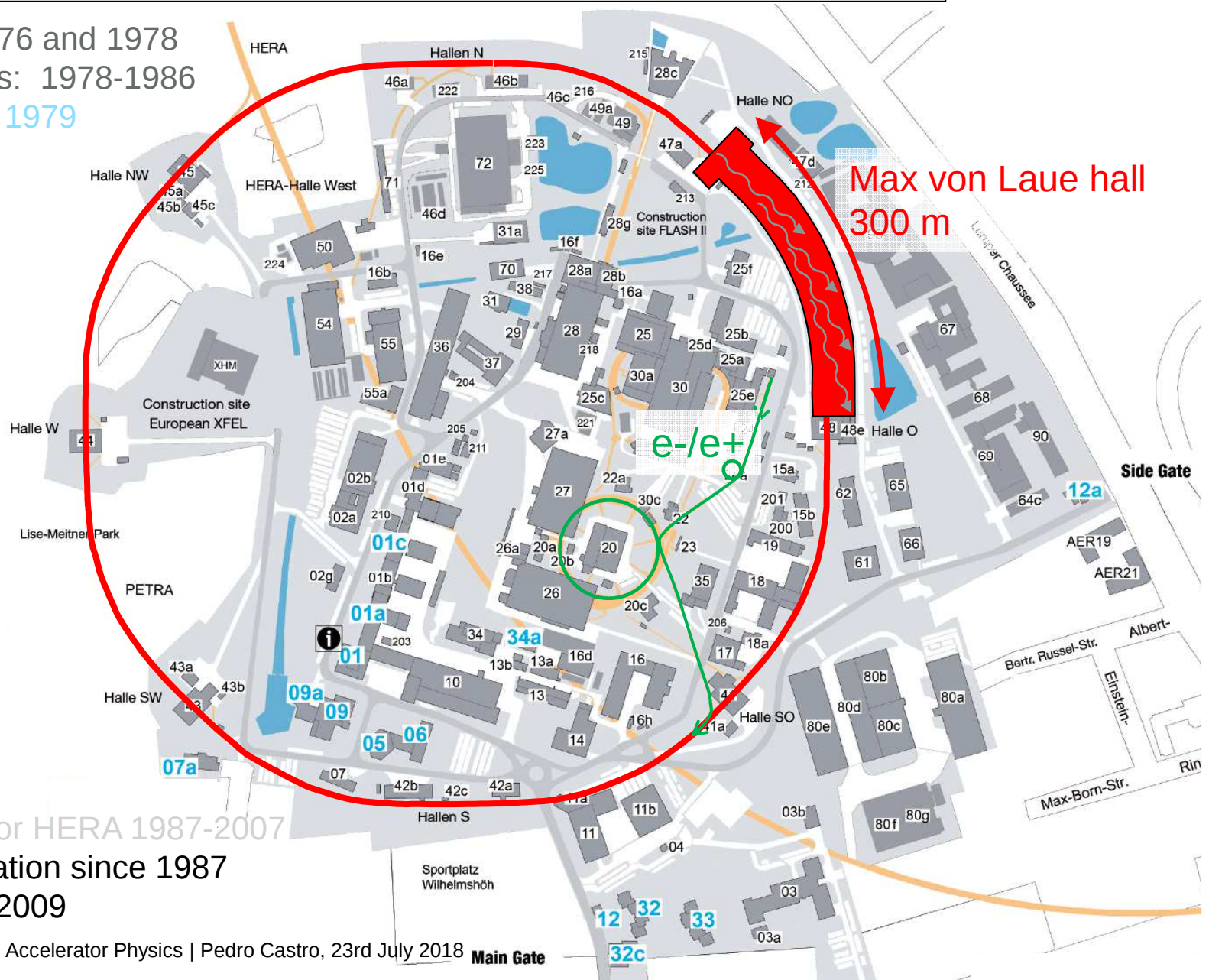
built between 1976 and 1978
HEP experiments: 1978-1986
gluon discovery: 1979

2.3 km long

Max von Laue hall
300 m

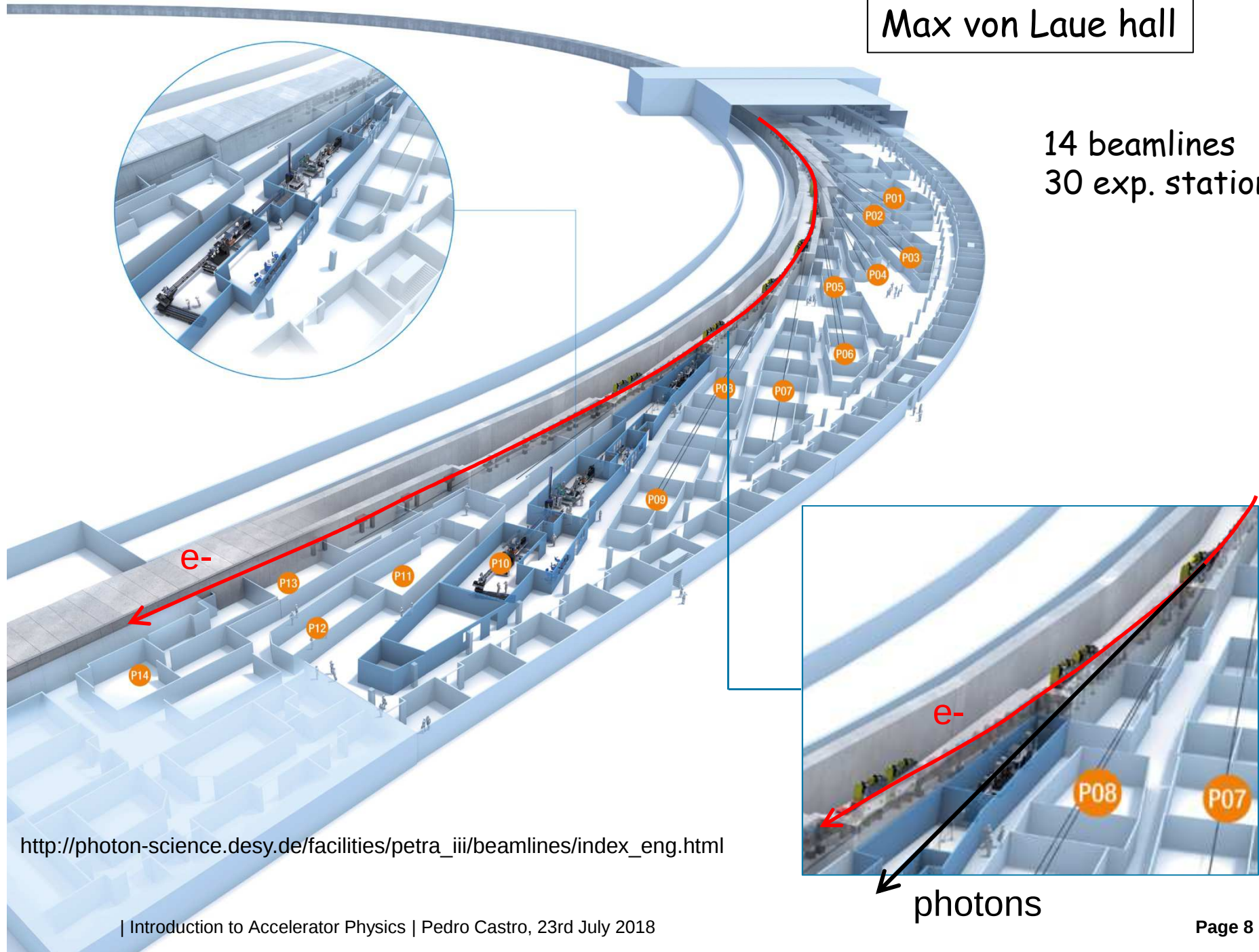
e^-/e^+

pre-accelerator for HERA 1987-2007
synchrotron radiation since 1987
PETRA III since 2009



Max von Laue hall

14 beamlines
30 exp. stations

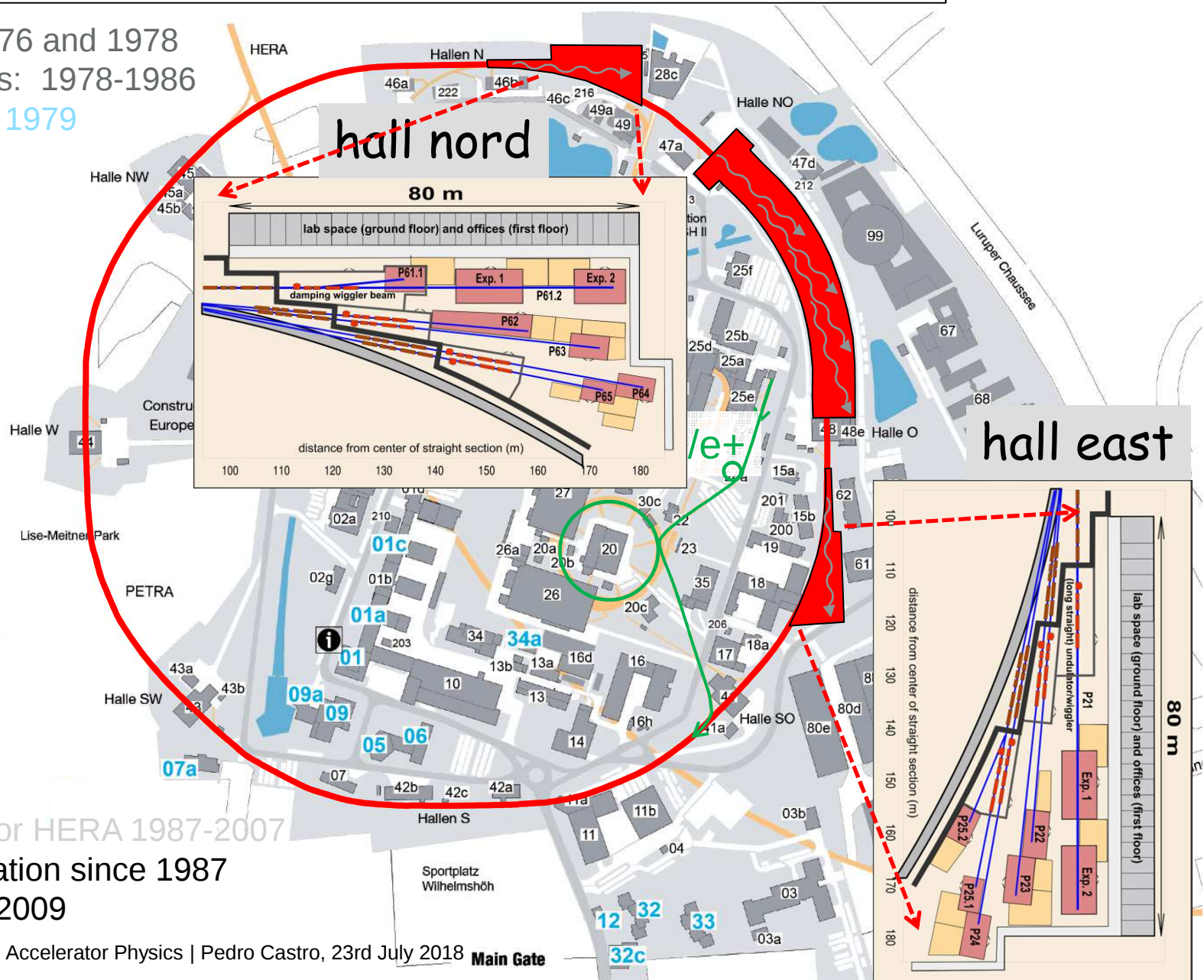


http://photon-science.desy.de/facilities/petra_iii/beamlines/index_eng.html

Example: Positron-Elektron-Tandem-Ring-Anlage (PETRA) 'positron-electron tandem ring accelerator' at DESY

built between 1976 and 1978
HEP experiments: 1978-1986
gluon discovery: 1979

2.3 km long



pre-accelerator for HERA 1987-2007
synchrotron radiation since 1987
PETRA III since 2009

Applications of Accelerators (3)

Accelerators in medicine

For radioisotope production

proton beam + stable isotope $\xrightarrow{\text{transmutation}}$ radioactive isotope

For radiotherapy and radiosurgery:

- x-rays and gamma-rays
- ions (from protons to atoms with atomic number up to 18, Argon)
- neutrons

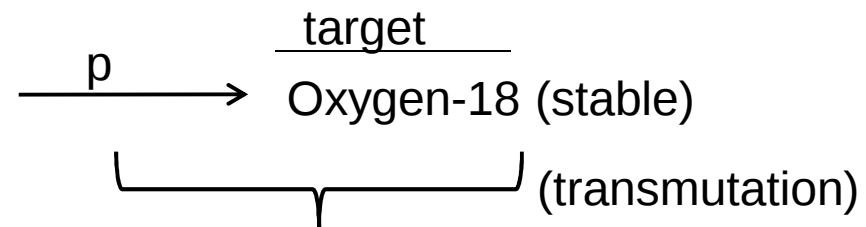
Applications of Accelerators (3)

Accelerators in medicine

For radioisotope production

For example:

18 MeV proton accelerator



Fluorine-18 (half-life time = 110 min.)

97% of decays

Oxygen-18 + positron

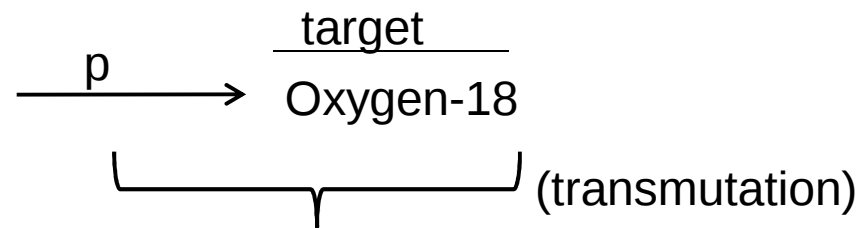
Applications of Accelerators (3)

Accelerators in medicine

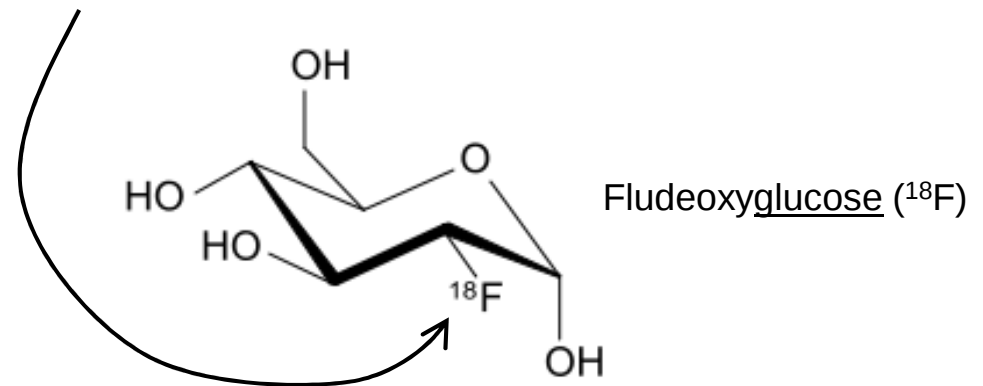
For radioisotope production

For example:

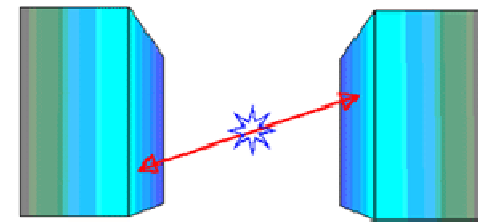
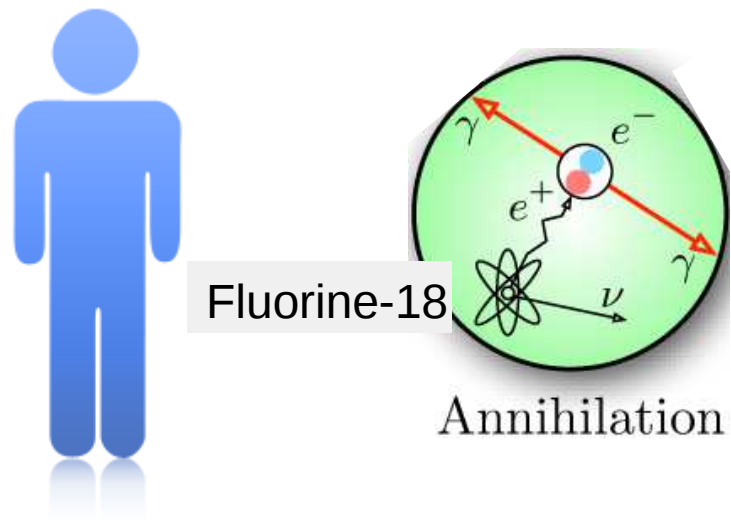
18 MeV proton accelerator



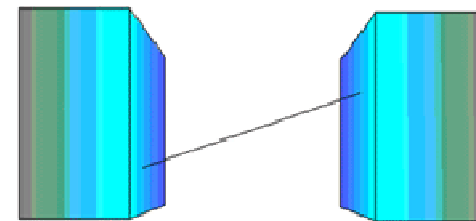
Fluorine-18 (half-life time = 110 min.)



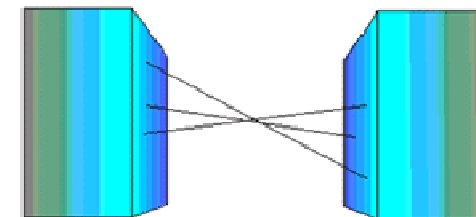
Applications of Accelerators (3)



Detection

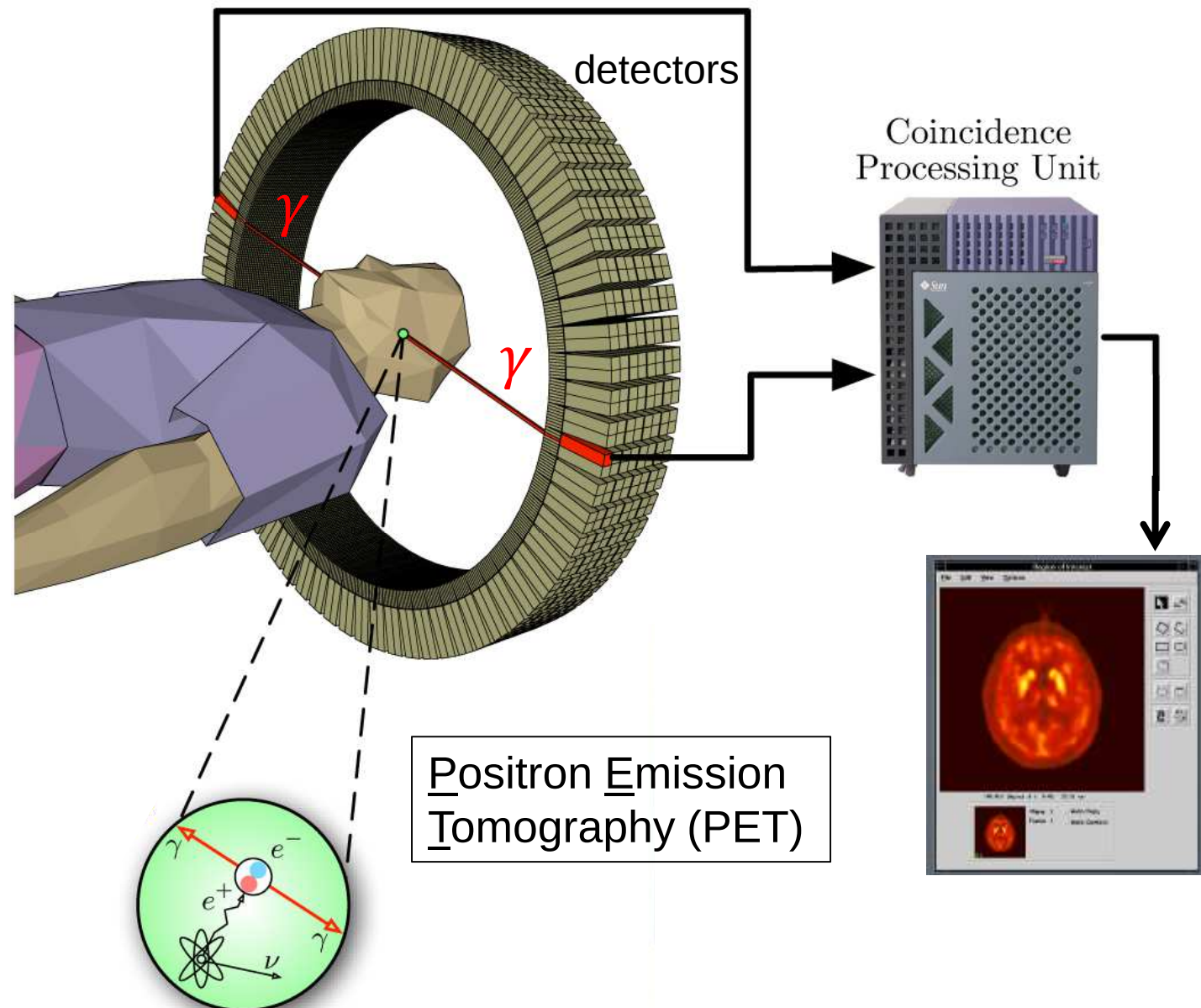


Reconstruction

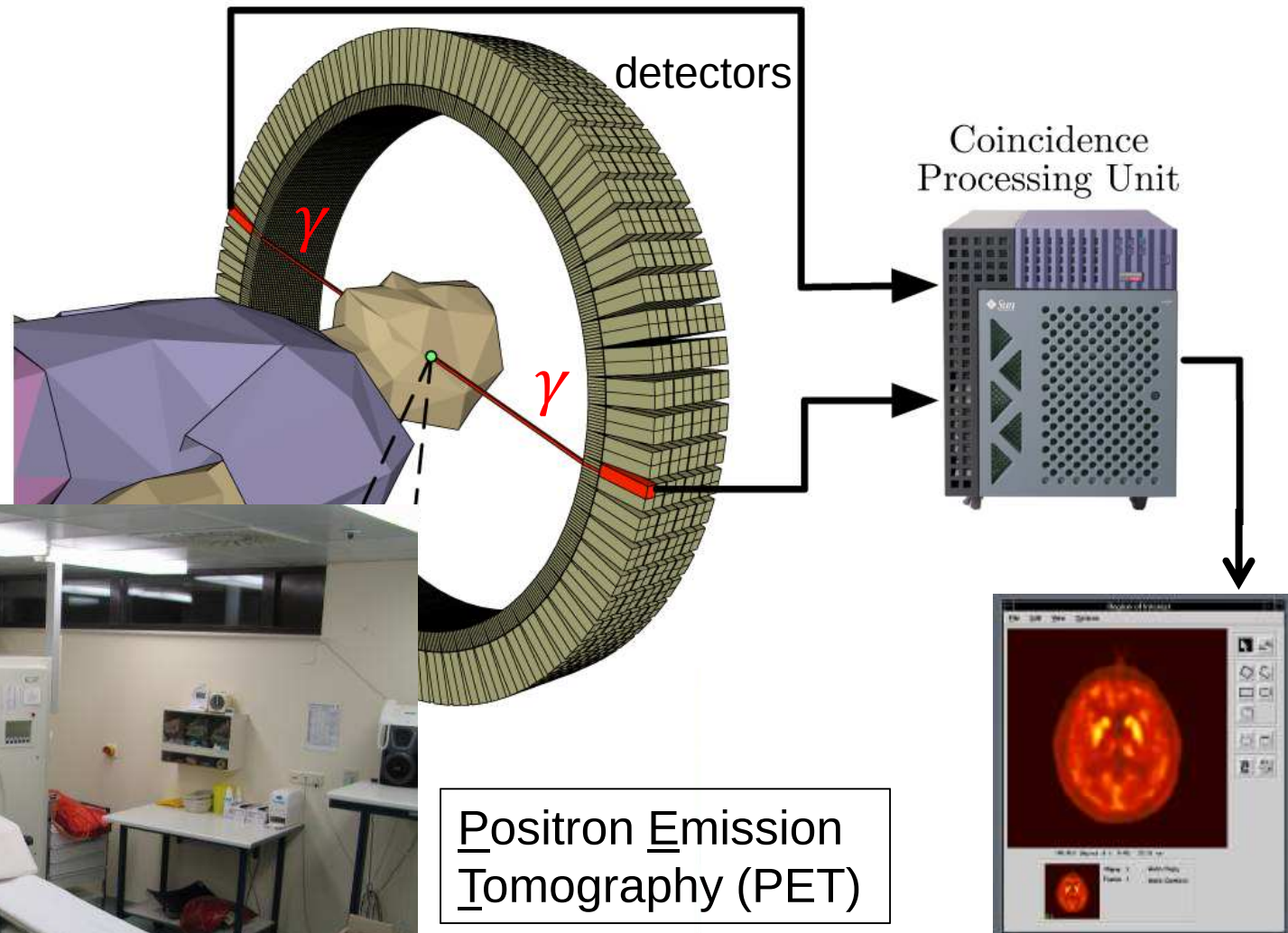


Particle Location

Applications of Accelerators (3)



Applications of Accelerators (3)



Positron Emission
Tomography (PET)



Applications of Accelerators (4)

For industrial applications:

Application	
Ion implantation	~ 9500
Electron cutting and welding	~ 4500
Electron beam and x-ray irradiators	~ 2000
Ion beam analysis (including AMS)	~ 200
Radioisotope production (including PET)	~ 900
Nondestructive testing (including security)	~ 650
Neutron generators (including sealed tubes)	~ 1000

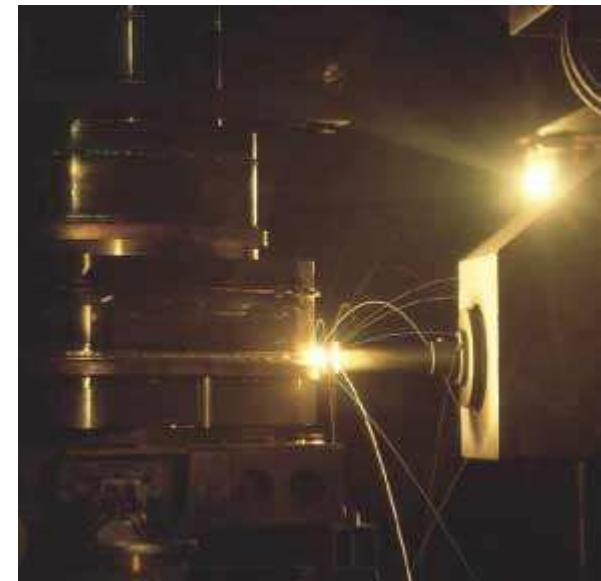
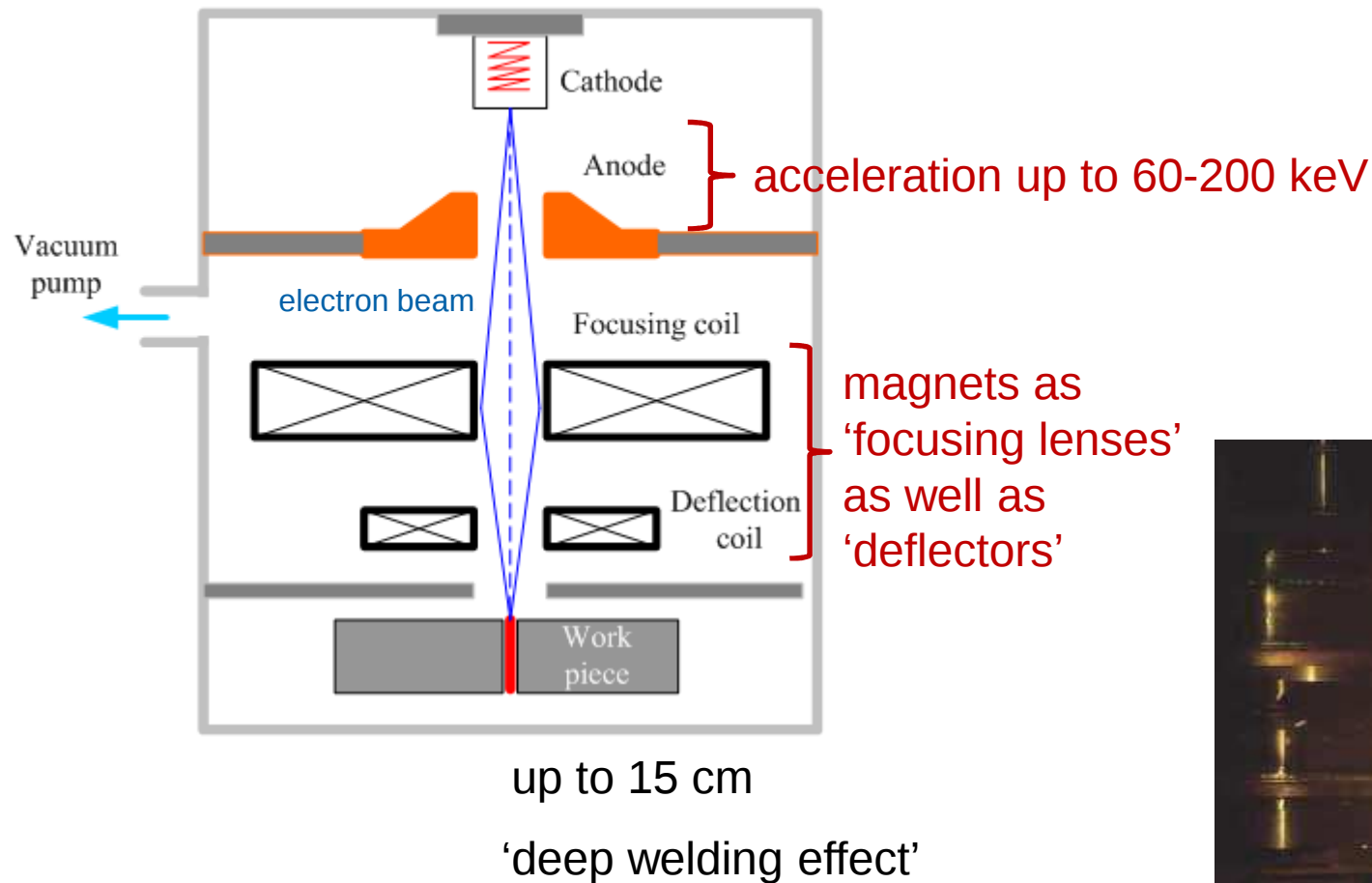
approx. numbers from 2007 (worldwide)

with energies up to 15 MeV

Applications of Accelerators (4)

For industrial applications:

an example: electron beam welding



Worldwide ...

- > About 120 accelerators for research in “nuclear and particle physics”
- > About 70 electron storage rings and electron linear accelerators used as light sources (so-called ‘synchrotron radiation sources’)

-
- > More than 7,000 accelerators for medicine
radiotherapy (>7,500), radioisotope production (200)
 - > More than 18,000 industrial accelerators
ion implantation (>9,000) , electron cutting and welding (>4,000) ...

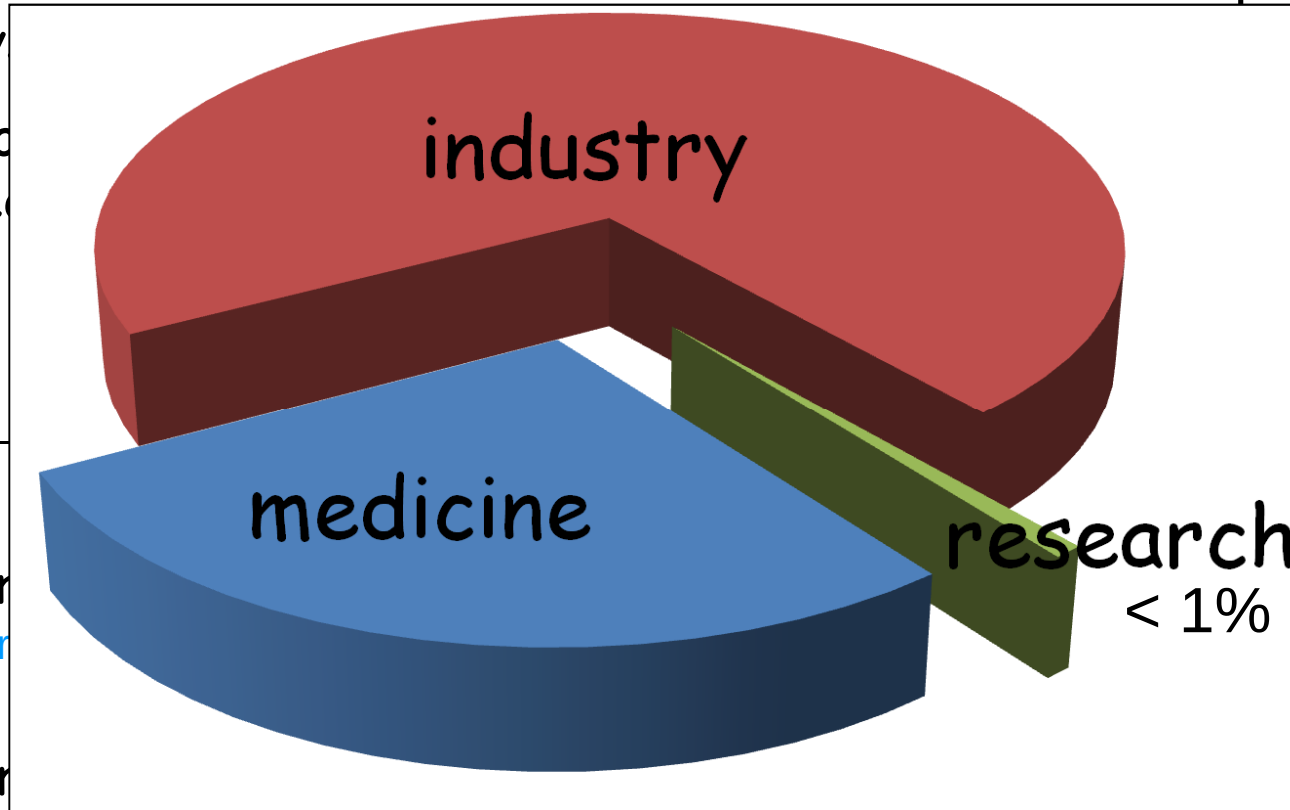
Worldwide ...

> About 120 accelerators for research in "nuclear and particle physics"

> About 100 accelerators for medical use

> More than 100 accelerators for industrial use

> More than 100 accelerators for research in "nuclear and particle physics"



accelerators
(sources')

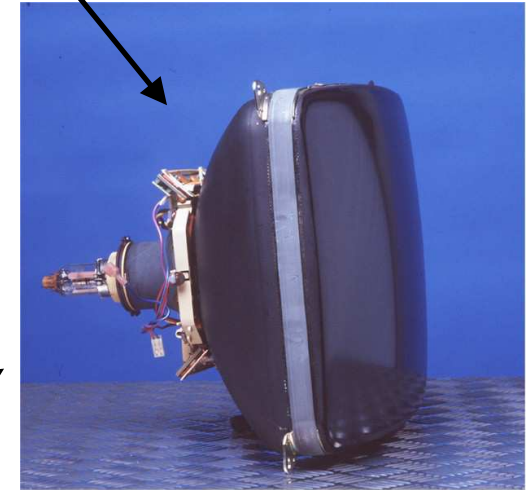
ion implantation (>9,000) , electron cutting and welding (>4,000) ...

Applications of Accelerators (5)

Many millions of television sets, oscilloscopes using CRTs (Cathode Ray Tube)



TV

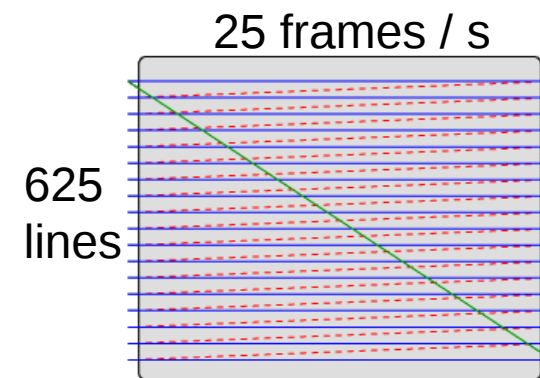
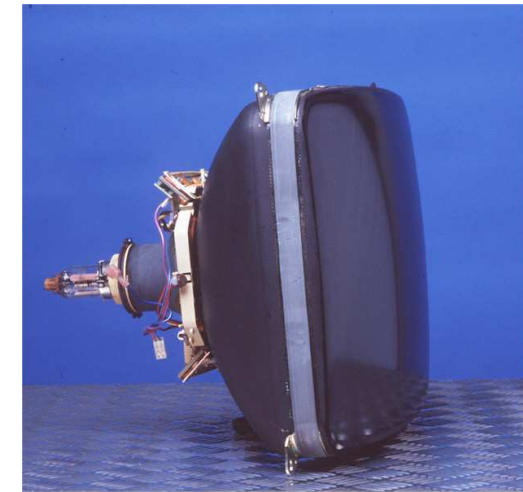
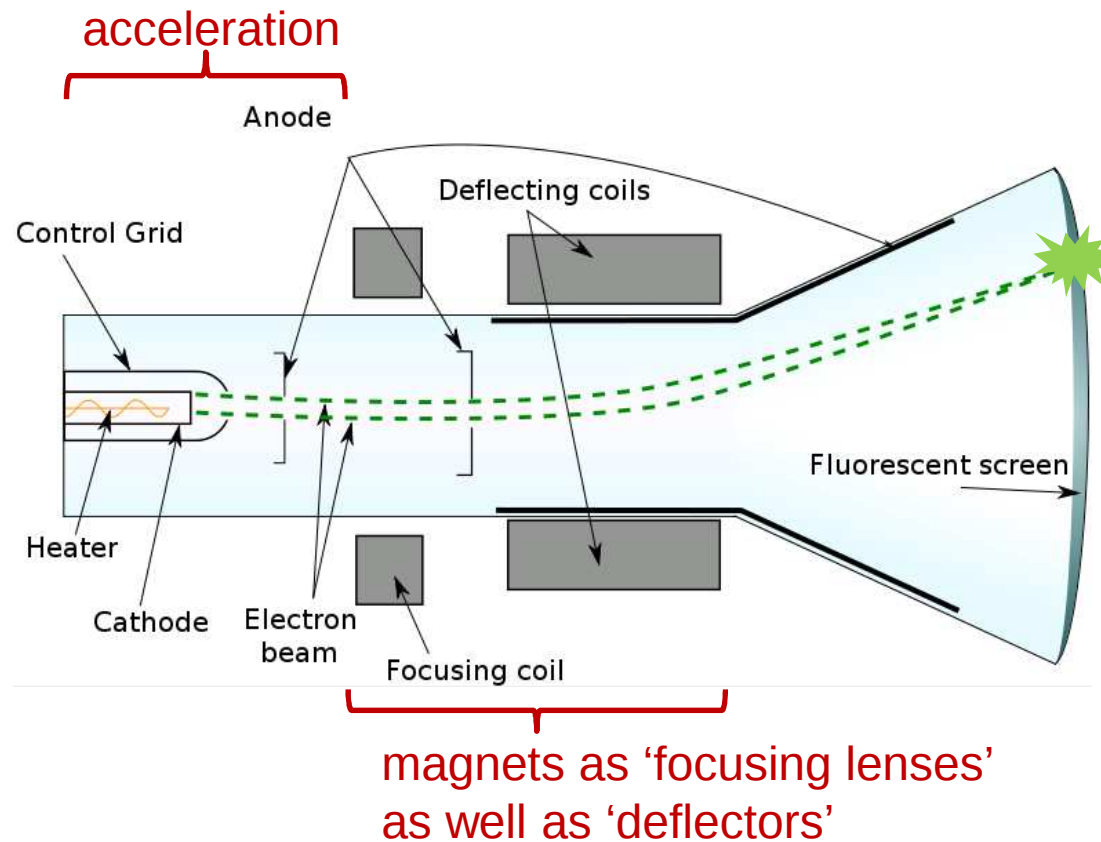


oscilloscope



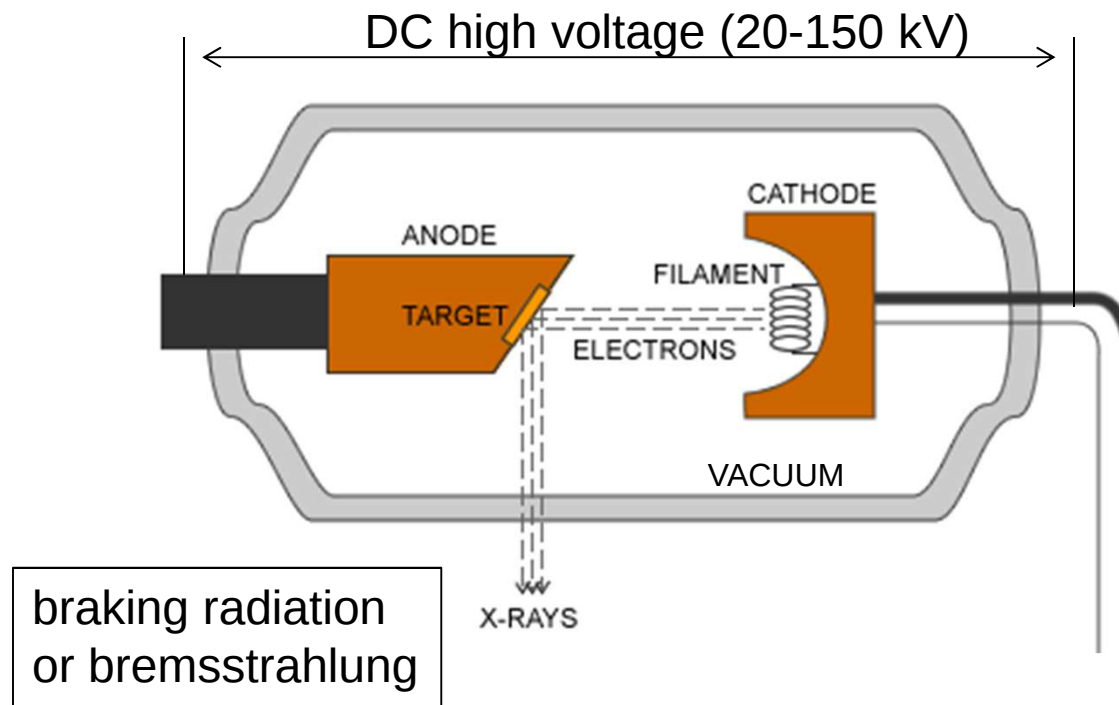
Applications of Accelerators (5)

Many millions of television sets, oscilloscopes using CRTs (Cathode Ray Tube)



Applications of Accelerators (6)

X-ray tubes



Working with accelerators in the control room ...

The case begins...

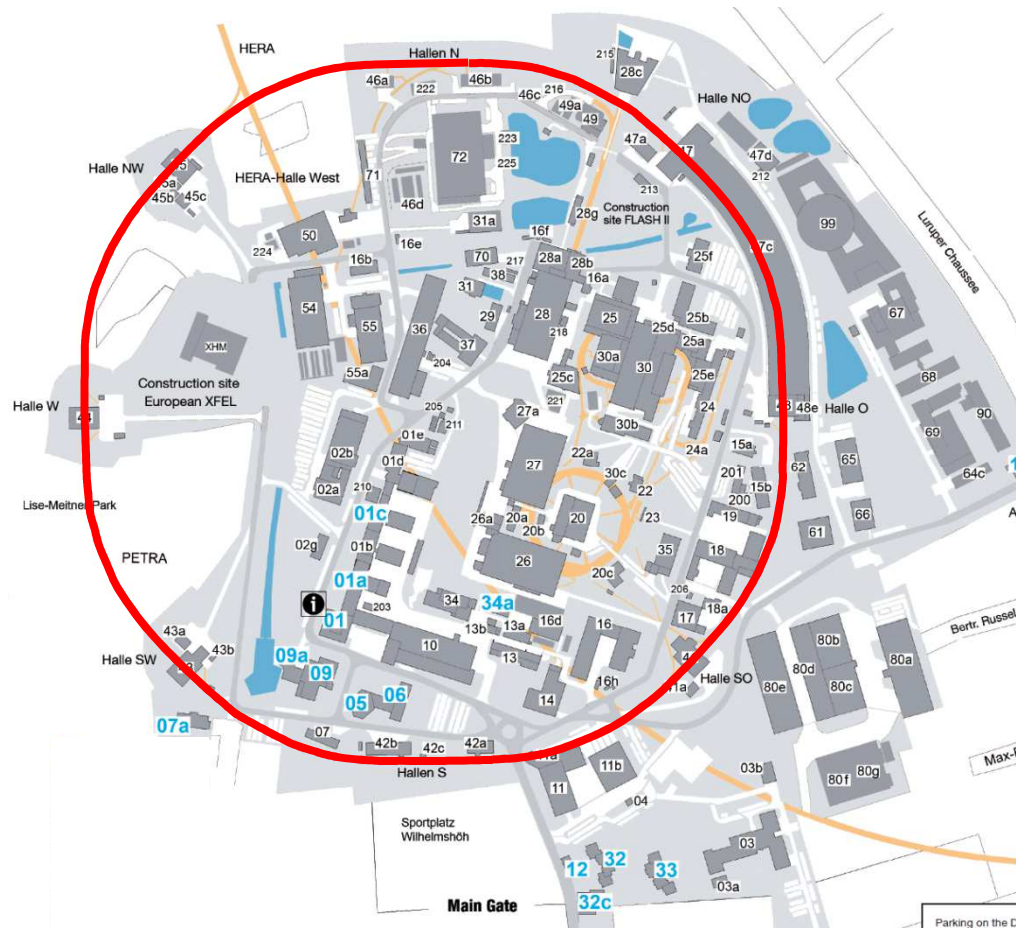
Accelerator Control Room
Hamburg, DESY

Sat. 12th June 2010

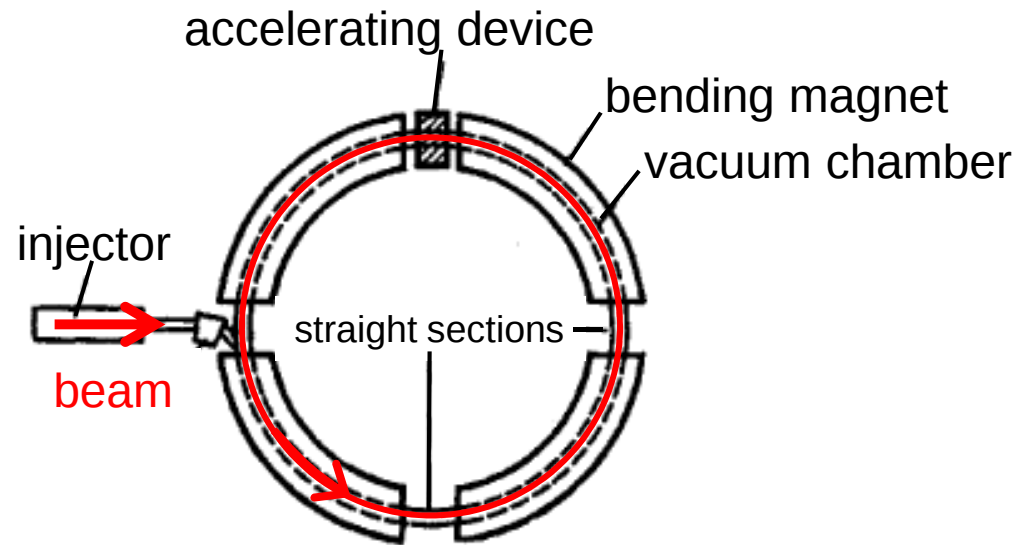
2 o'clock a.m.

PETRA runs with a beam
current of 75 mA

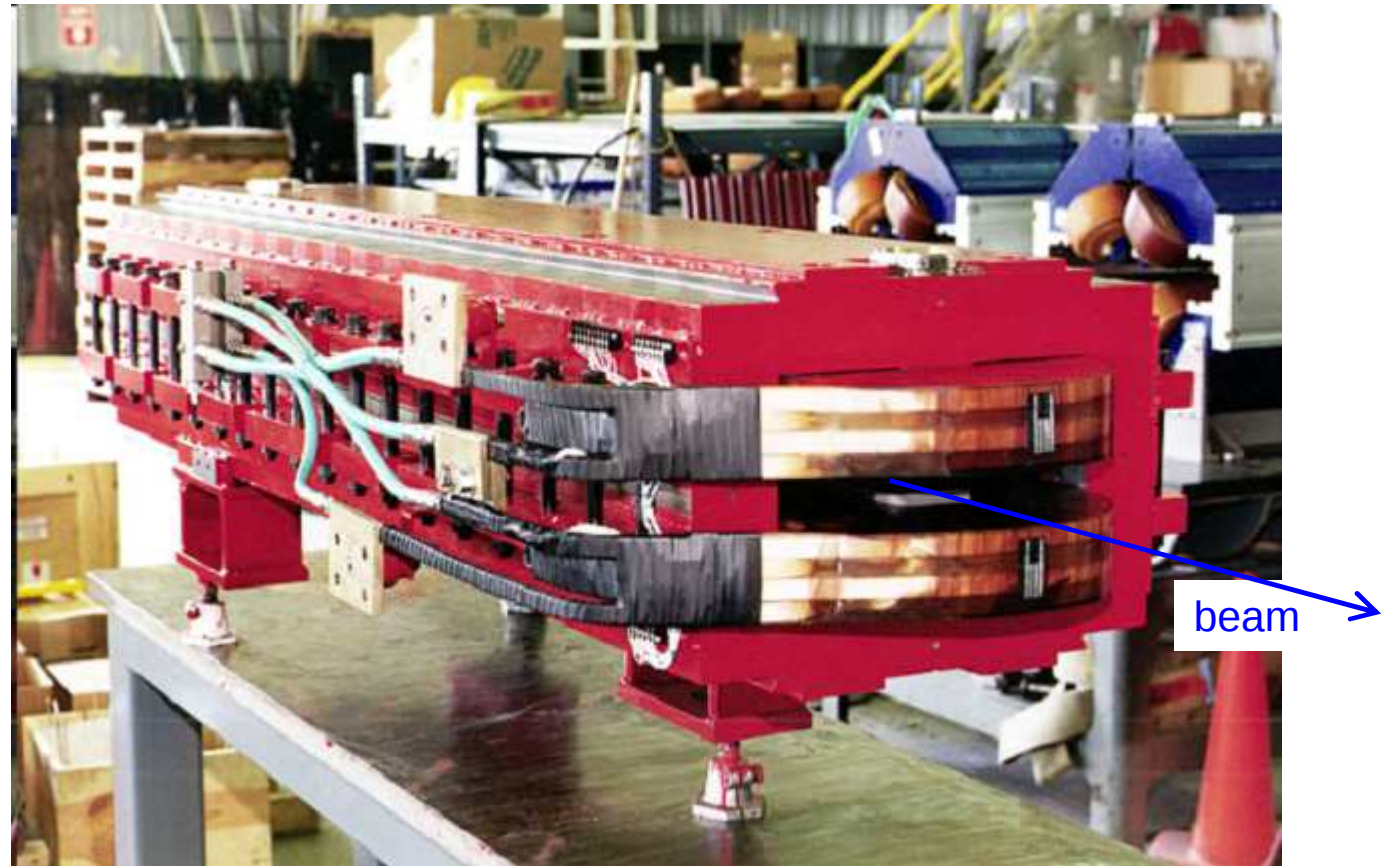
02:24 a.m.: beam lost



Circular accelerators: the synchrotron

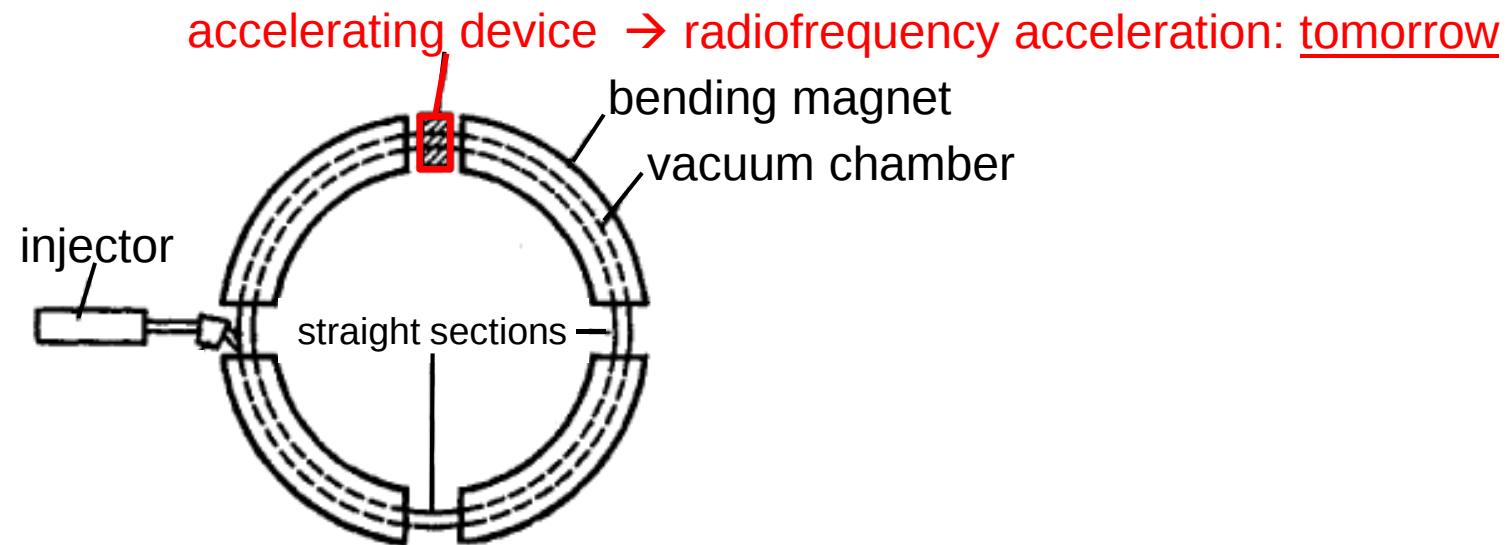


Dipole magnet

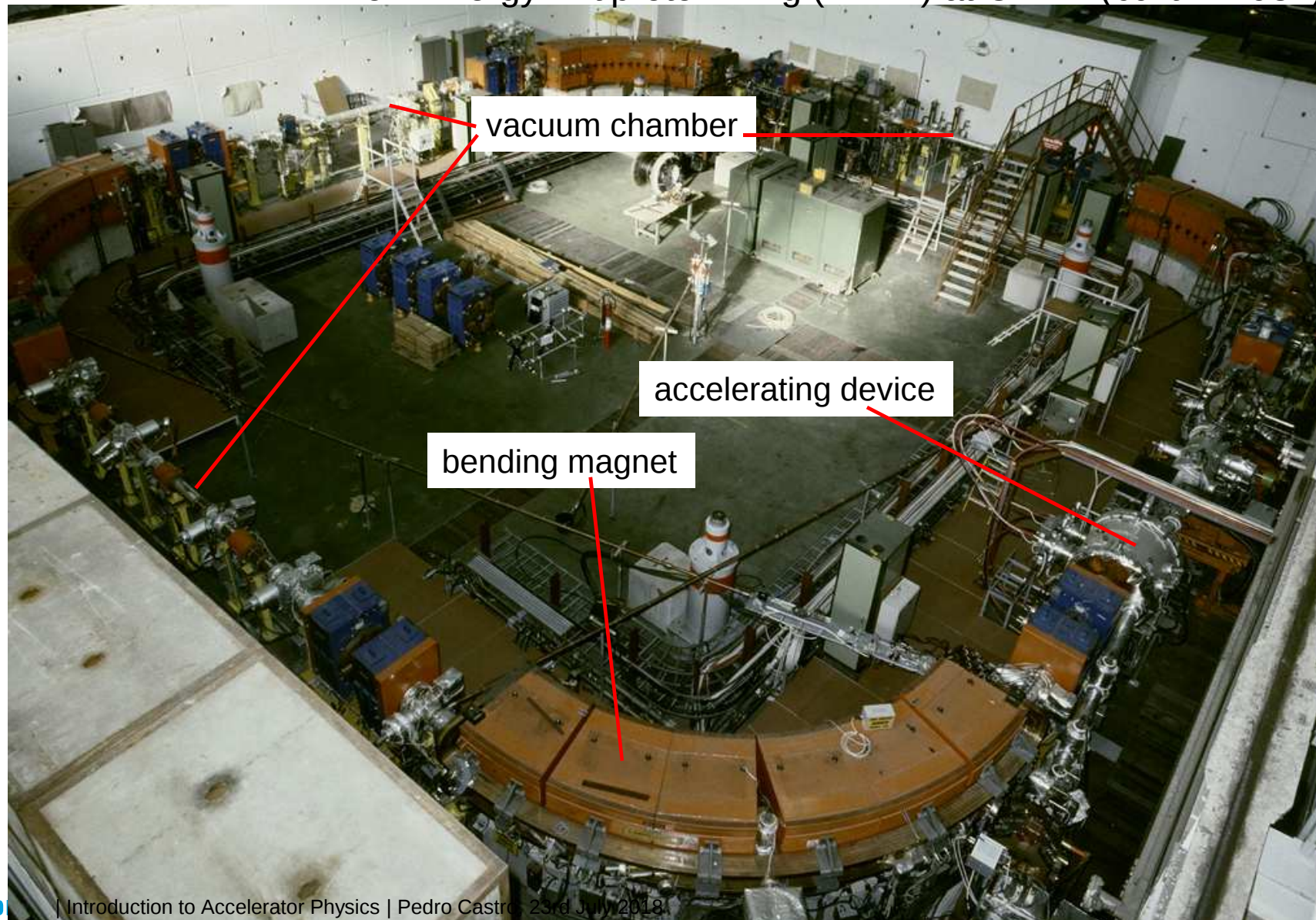


→ dipole magnets: tomorrow

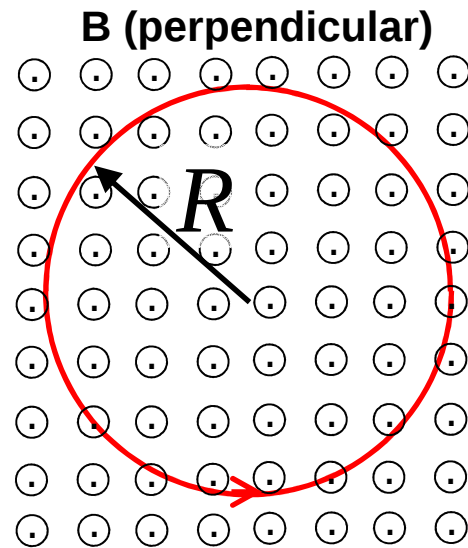
Circular accelerators: the synchrotron



Low Energy Antiproton Ring (LEAR) at CERN (built in 1982)



Circular accelerators: the synchrotron



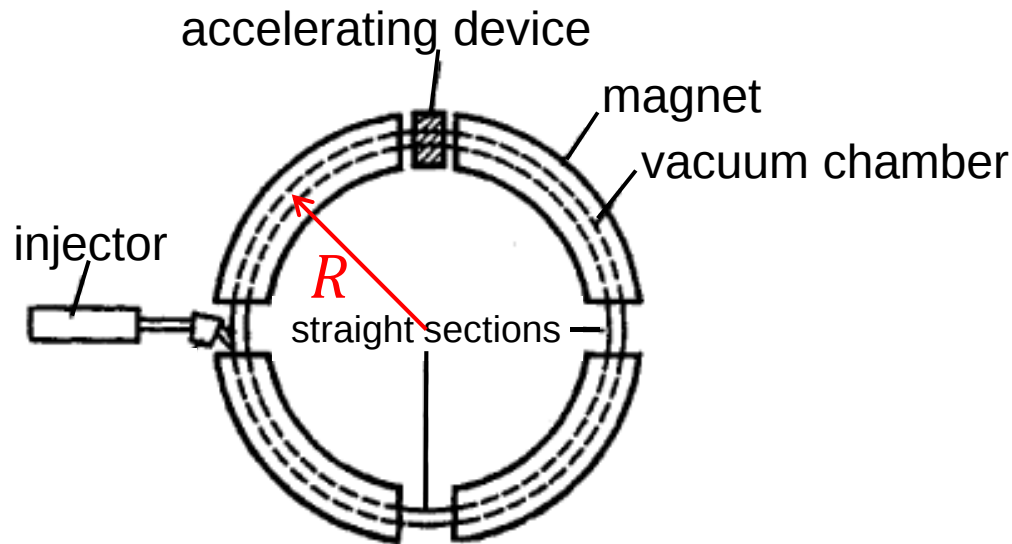
$$\vec{F} = \frac{d\vec{p}}{dt} = q\vec{v} \times \vec{B}$$

momentum charge velocity magnetic field

of the particle

$$\left. \begin{array}{l} \vec{B} \perp \vec{v} \rightarrow F = qvB \\ \vec{F} \perp \vec{v} \rightarrow F = m \frac{v^2}{R} \\ \text{(circular motion)} \end{array} \right\} qB = \frac{mv}{R} \rightarrow R = \frac{mv}{qB}$$

Circular accelerators: the synchrotron



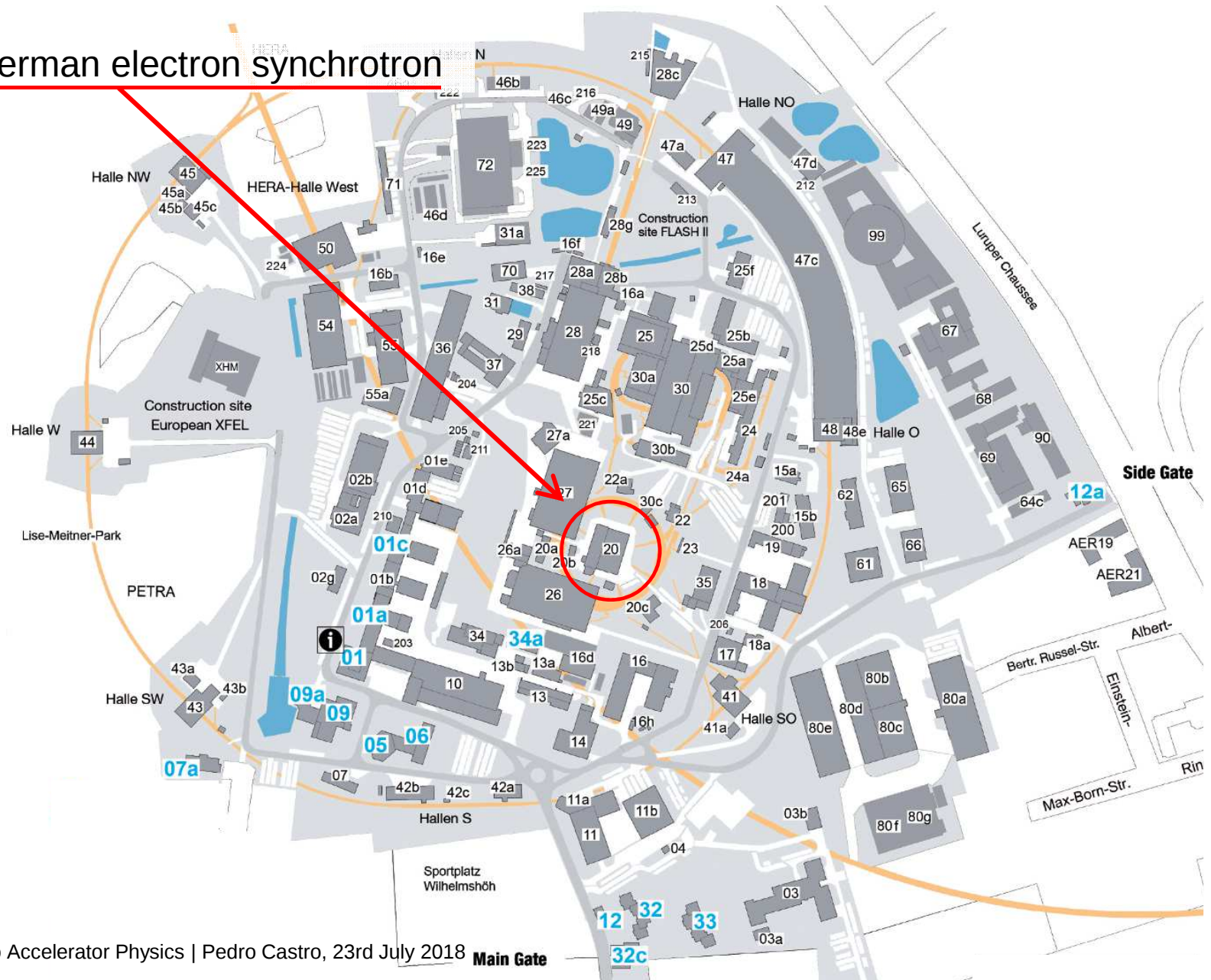
$$\left. \begin{aligned} \vec{B} \perp \vec{v} &\rightarrow F = qvB \\ \vec{F} \perp \vec{v} &\rightarrow F = m \frac{v^2}{R} \\ &\text{(circular motion)} \end{aligned} \right\}$$

$$qB = \frac{mv}{R} \rightarrow R = \frac{mv}{qB} = \text{constant}$$

→ increase B **synchronously**
with $p = mv$ of particle

DESY (Deutsches Elektronen Synchrotron)

DESY: German electron synchrotron



DESY (Deutsches Elektronen Synchrotron)

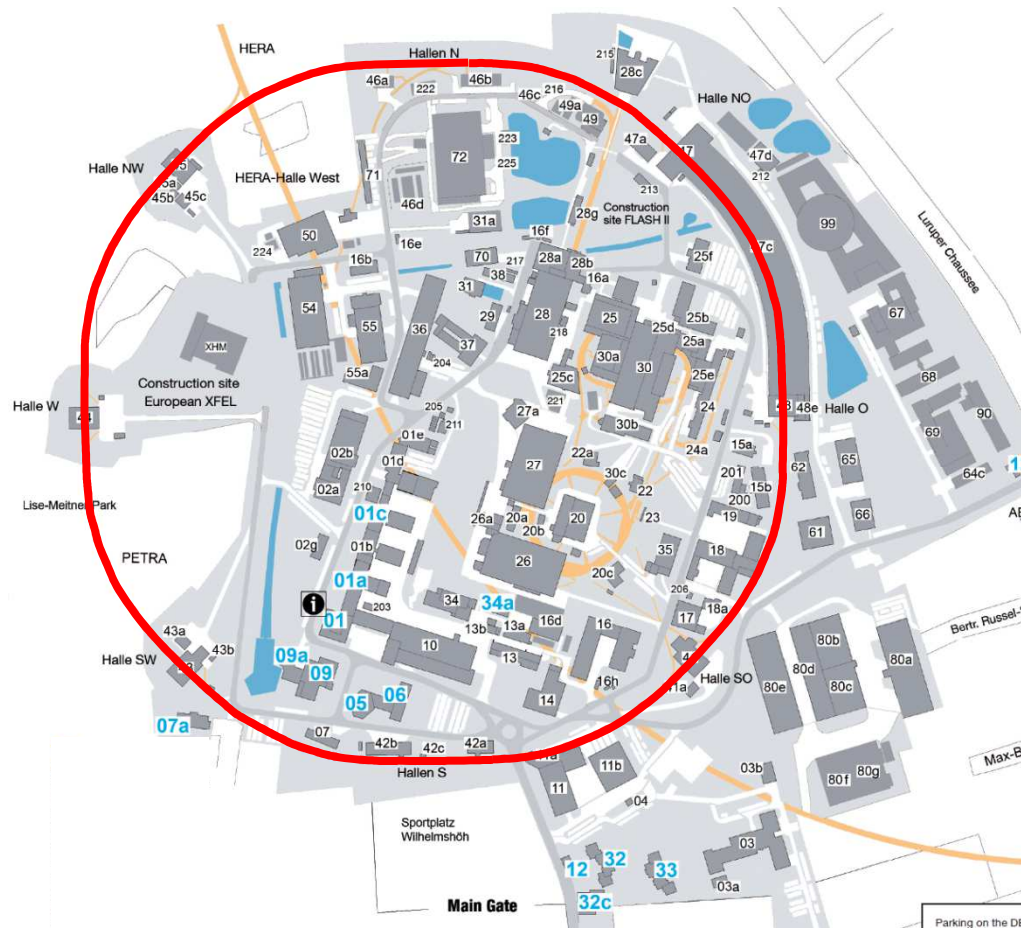
DESY: German electron synchrotron, 1964, 7.4 GeV



back to the case...

Hamburg, DESY
Sat. 12th June 2010
2 o'clock a.m.
PETRA runs with 75 mA

02:24 a.m.: beam lost



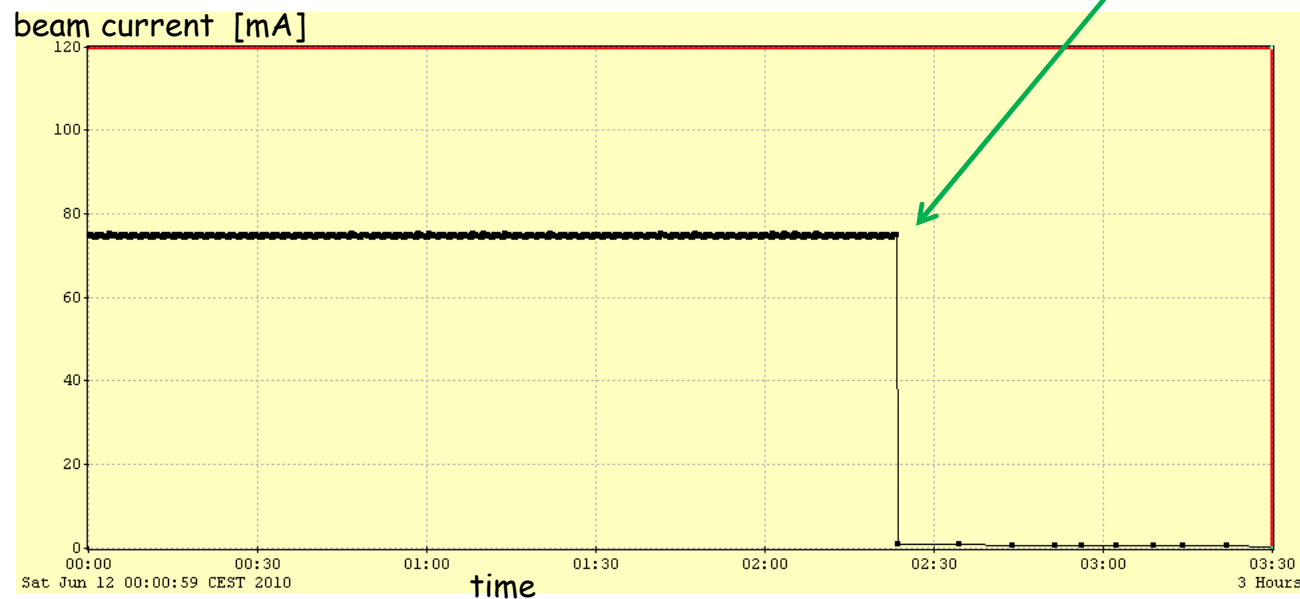
02:24 a.m.: beam lost

The Main Accelerator Control Room



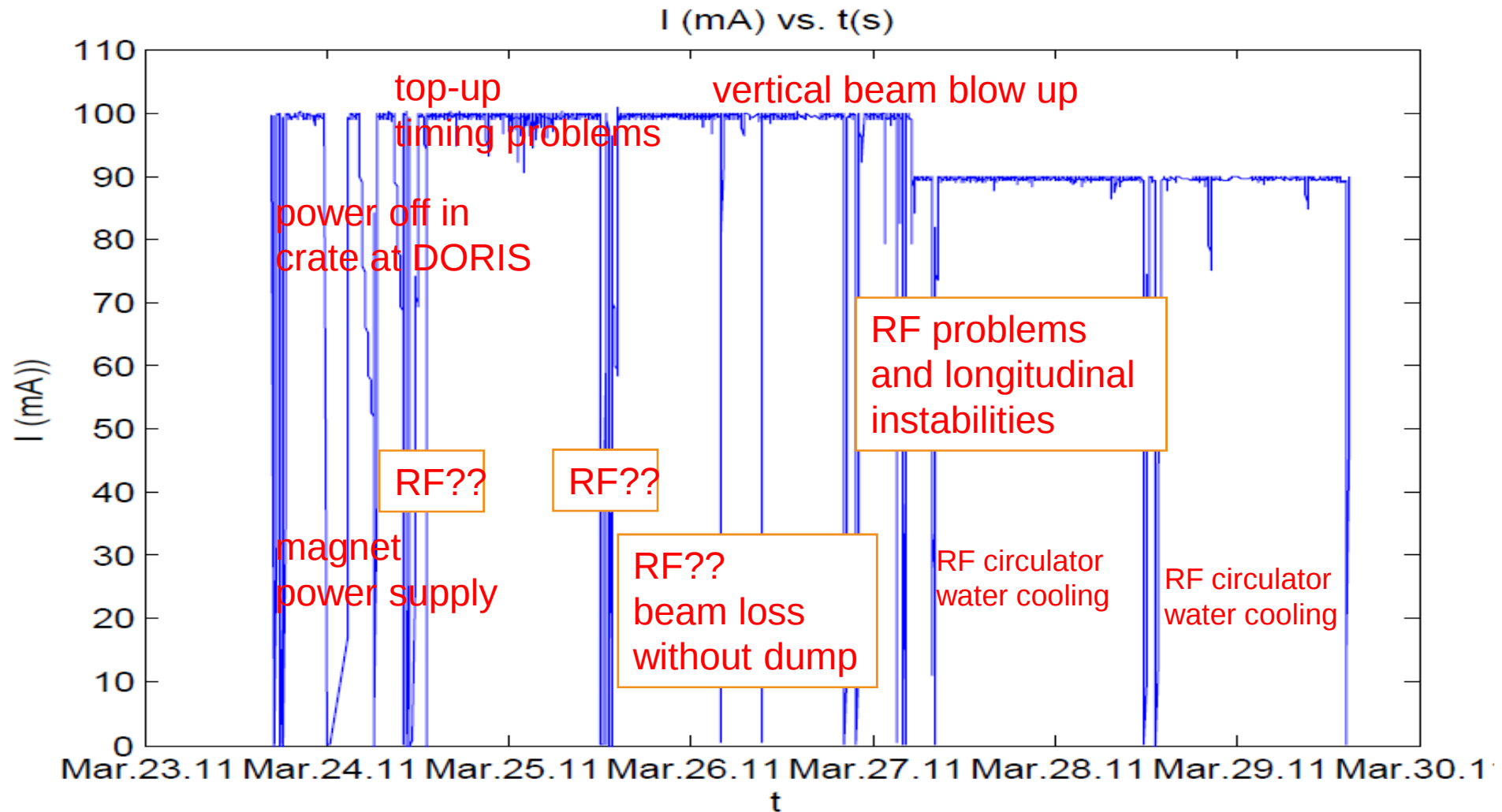
Hamburg, DESY
Sat. 12th June 2010

02:24 a.m.: beam lost



One example of PETRA run over 7 days

Run number 4: 60 Bunches; 23rd – 30th March, 2011



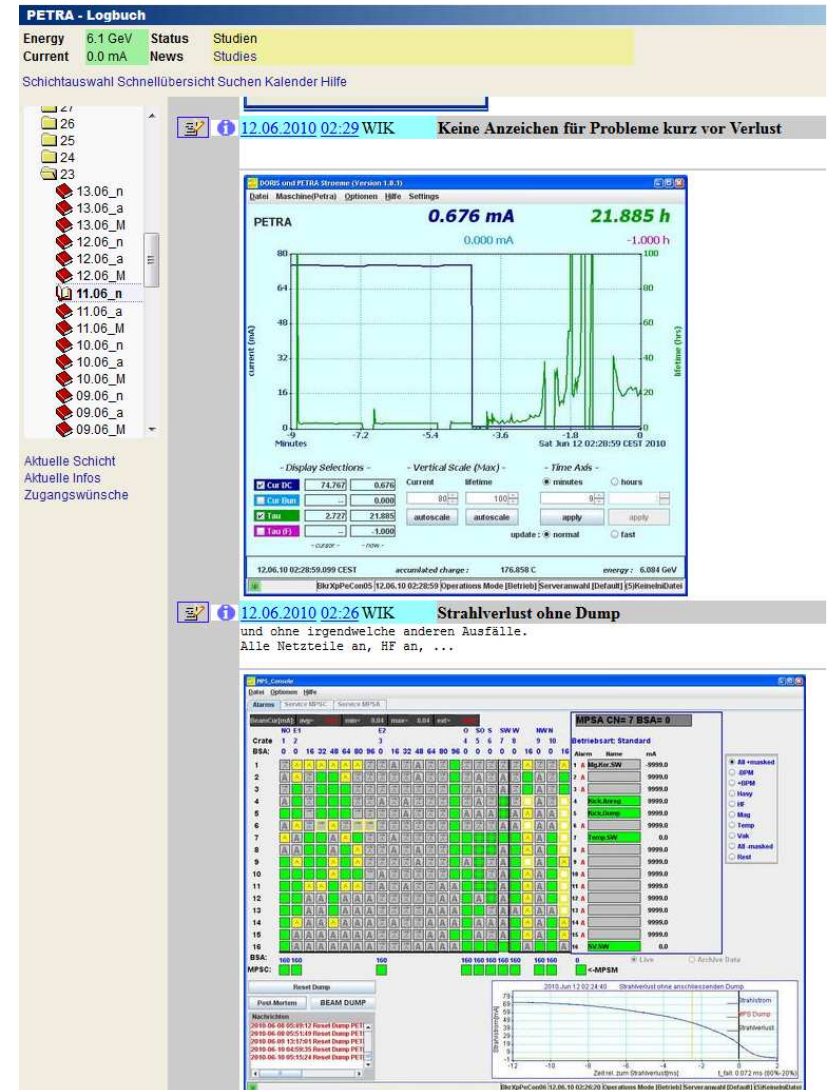
Source: K. Balewski (MAC report 2011)

Beam lost at 02:24 a.m.

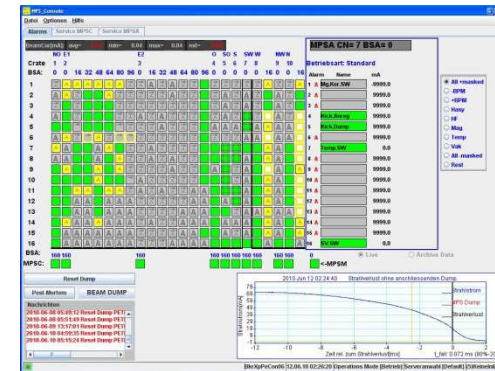
The link to the electronic logbook:

http://ttfinfo.desy.de/petra/show.jsp?dir=/2010/23/11.06_n&pos=2010-06-12T02:26:3

What to do?

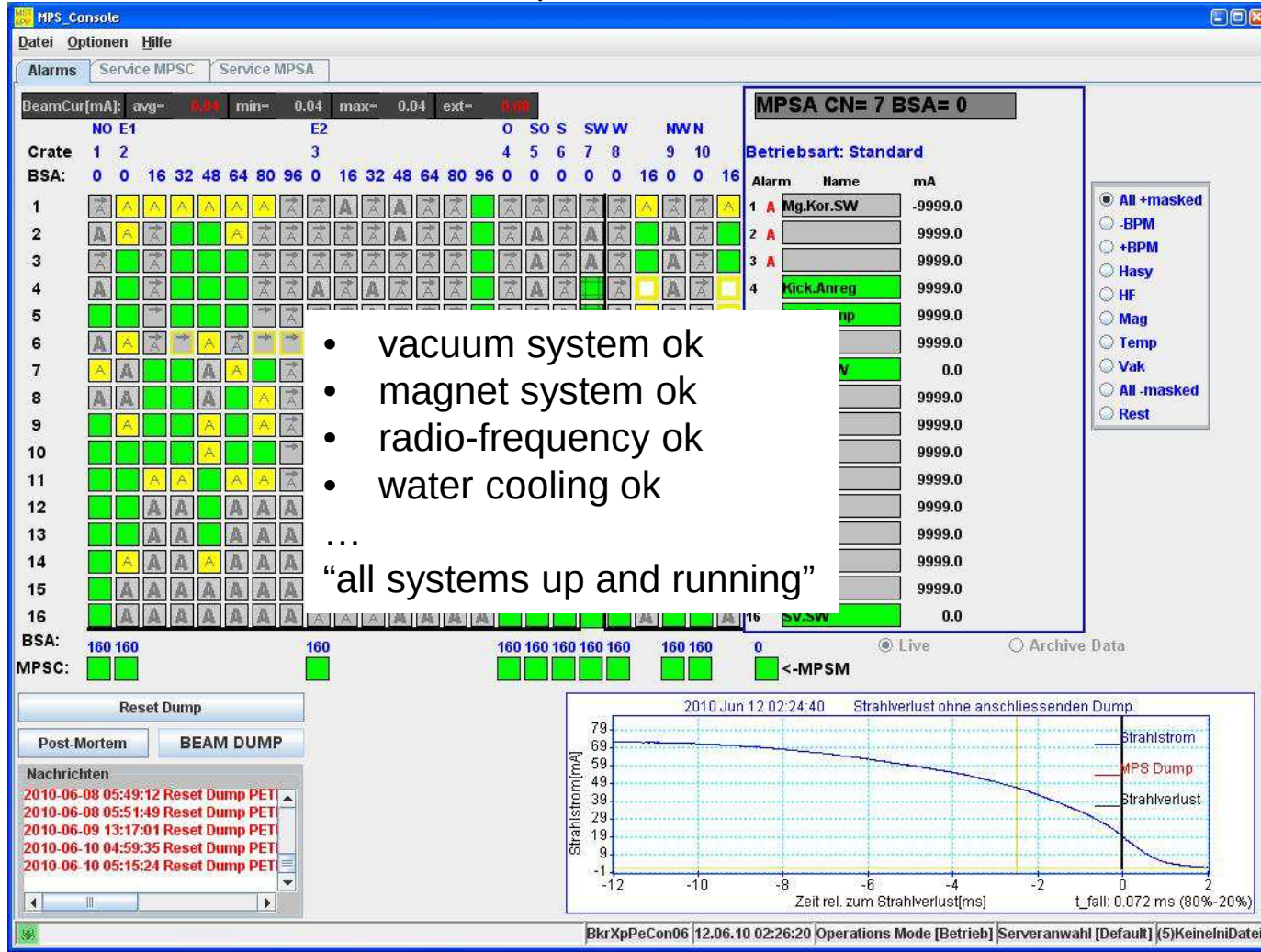


The diagram shows a site map of the European XFEL facility. A red circle highlights the central area, which is the focus of the 'status and alarm signals' section. Arrows point from this central area to the 'MPS' (Monitoring and Protection System) section. Another arrow points from the 'Main gate' area to the 'stop beam' section.



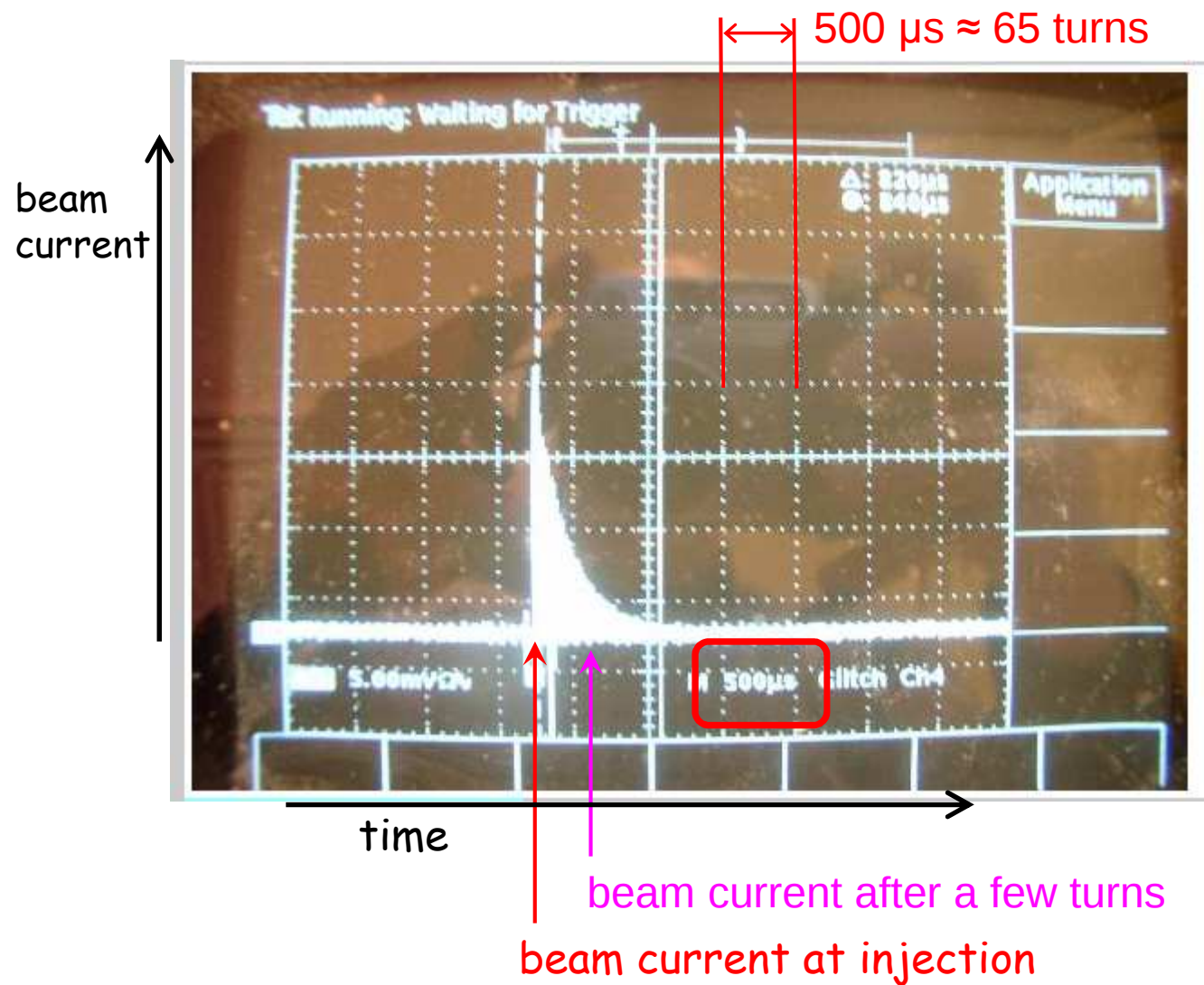
Alarm overview: the Machine Protection System

The Machine Protection System status from 12th June 2010 at 02:26



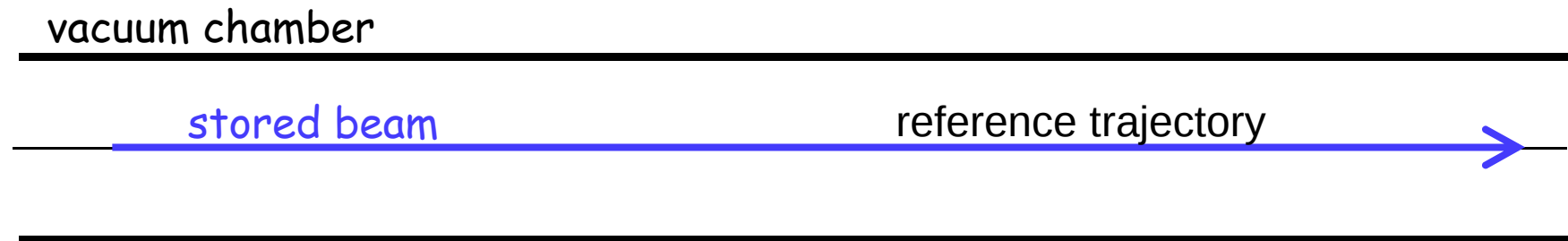
12th June 2010 02:26

Electrons can be injected but cannot be stored !

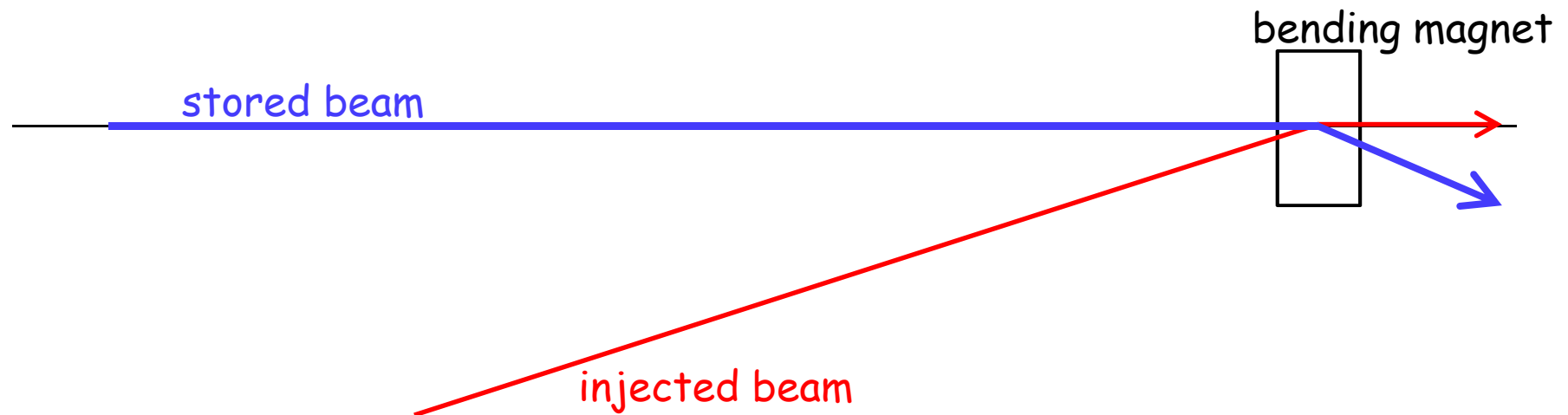


injection problem?

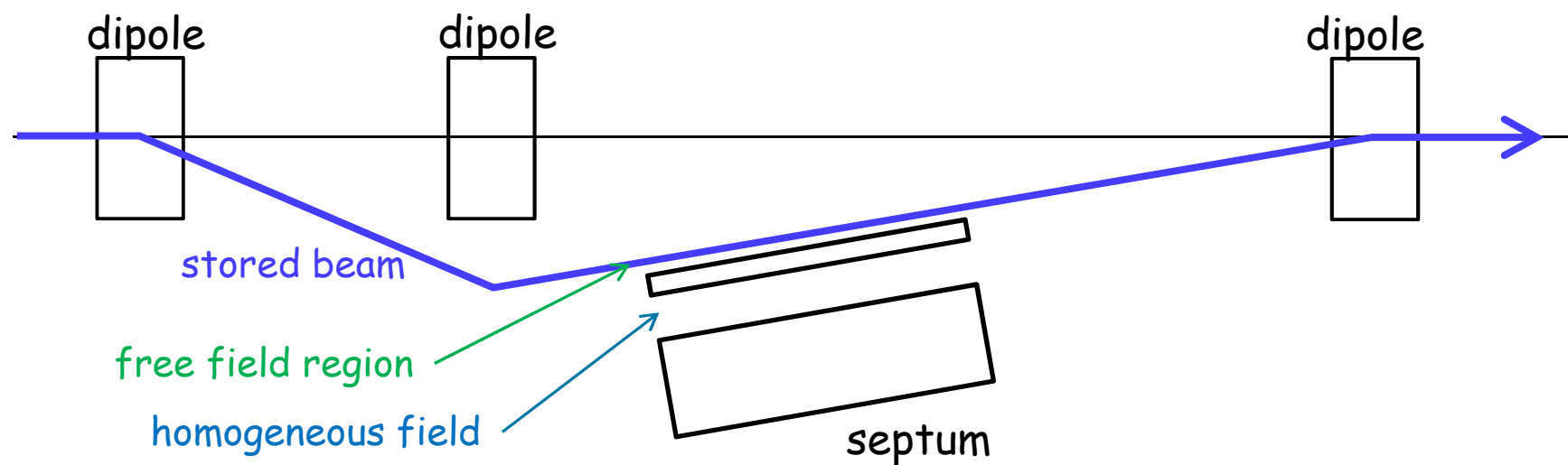
Next suspect: injection



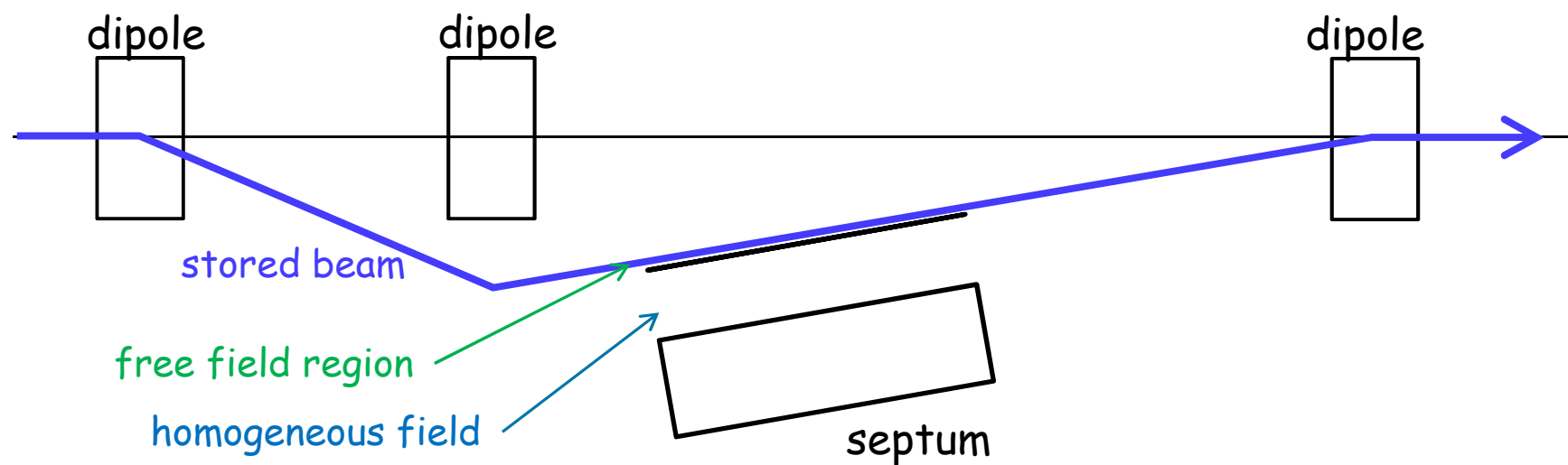
Next suspect: injection



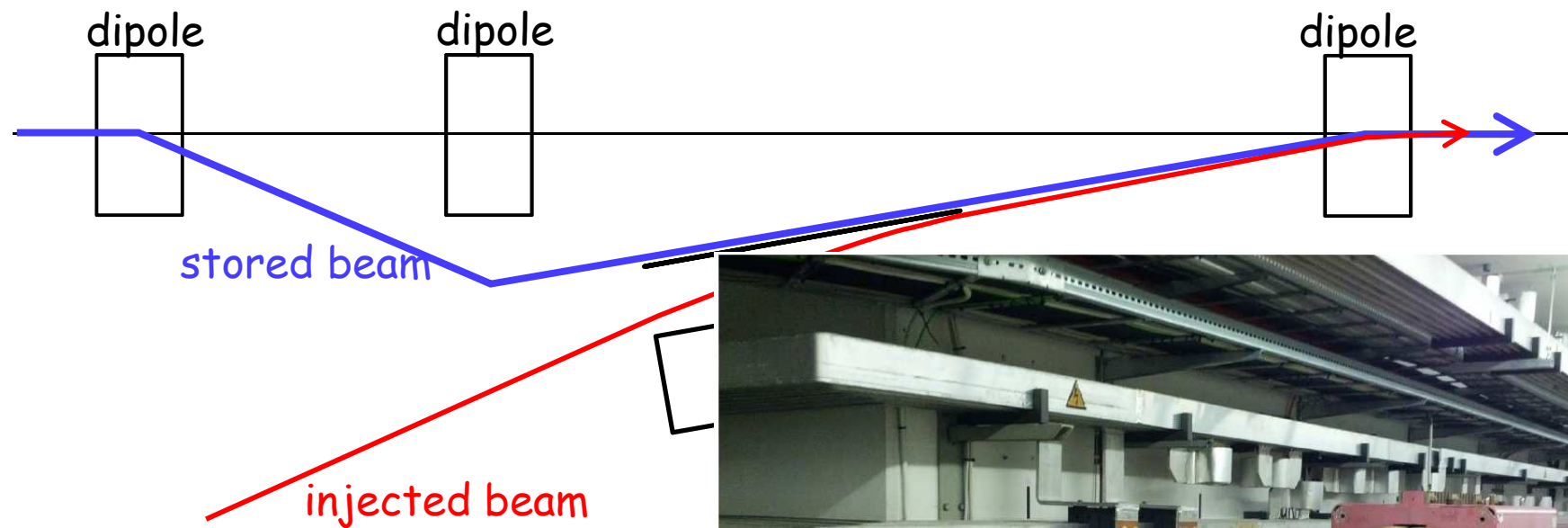
Next suspect: injection



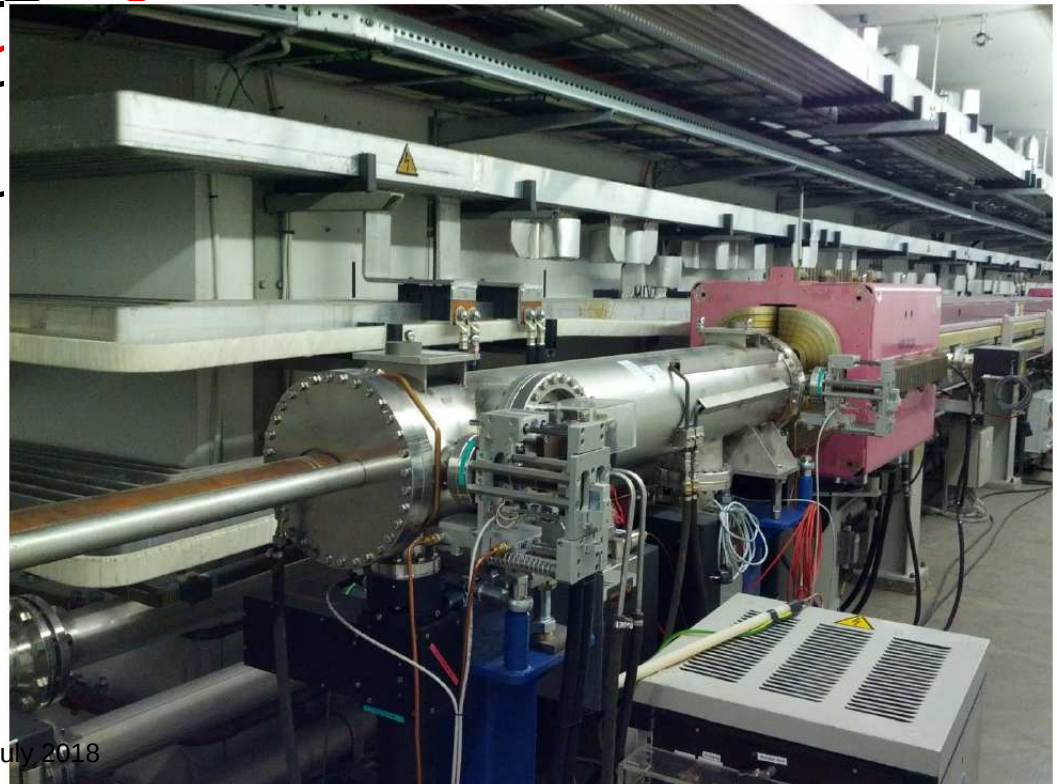
Next suspect: injection



Next suspect: injection + accumulation

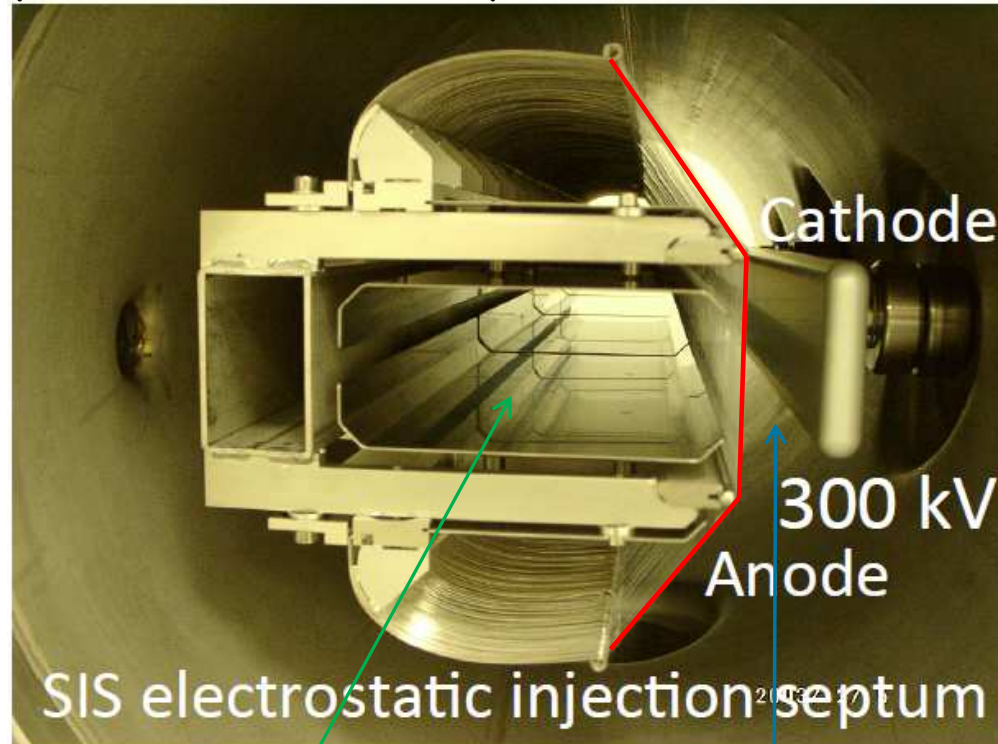


PETRA
septum



Next suspect: injection + accumulation

septum at the Proton Synchrotron Booster (PSB) at CERN

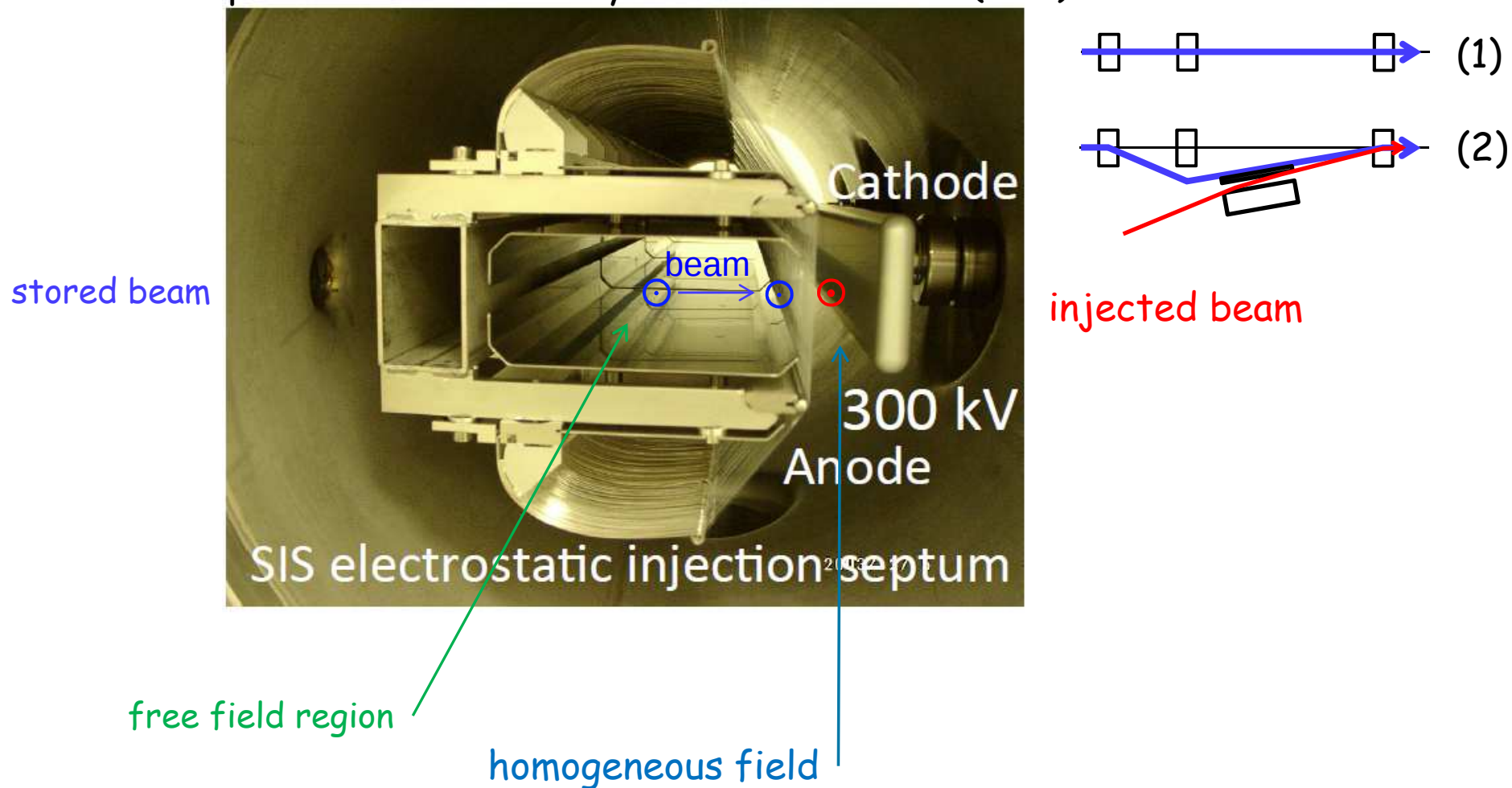


free field region

homogeneous field

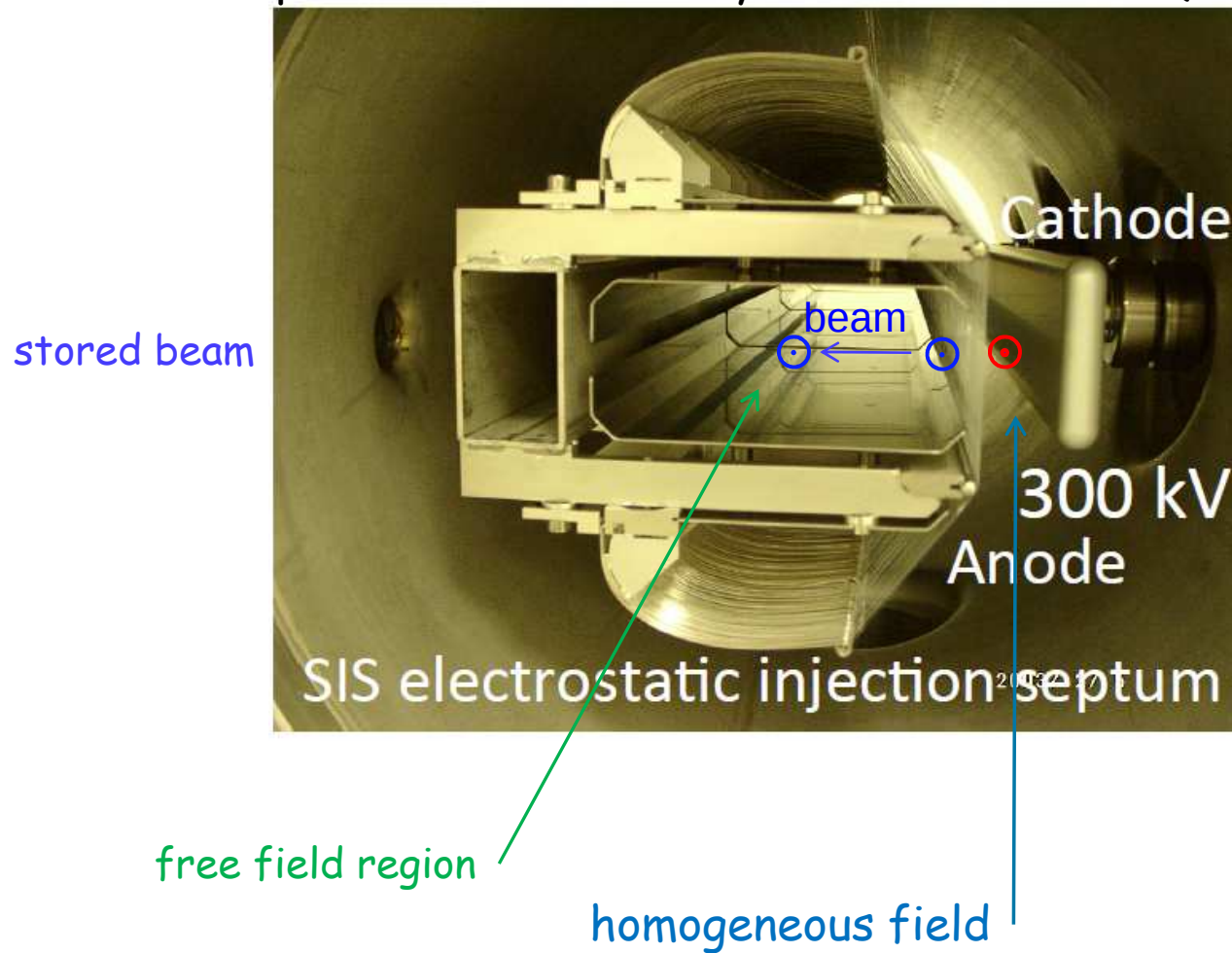
Next suspect: injection + accumulation

septum at the Proton Synchrotron Booster (PSB) at CERN



Next suspect: injection + accumulation

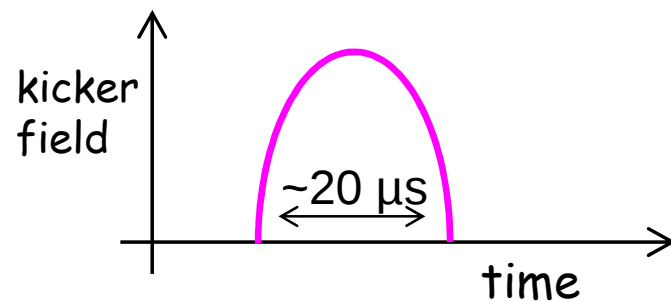
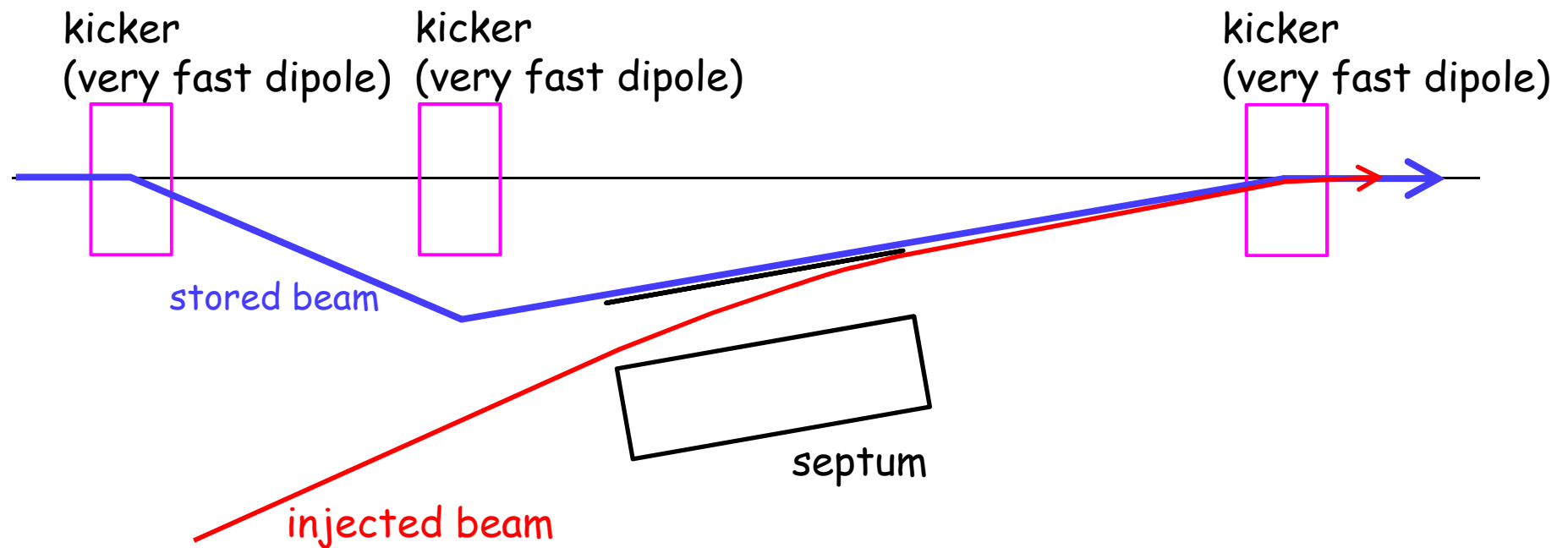
septum at the Proton Synchrotron Booster (PSB) at CERN



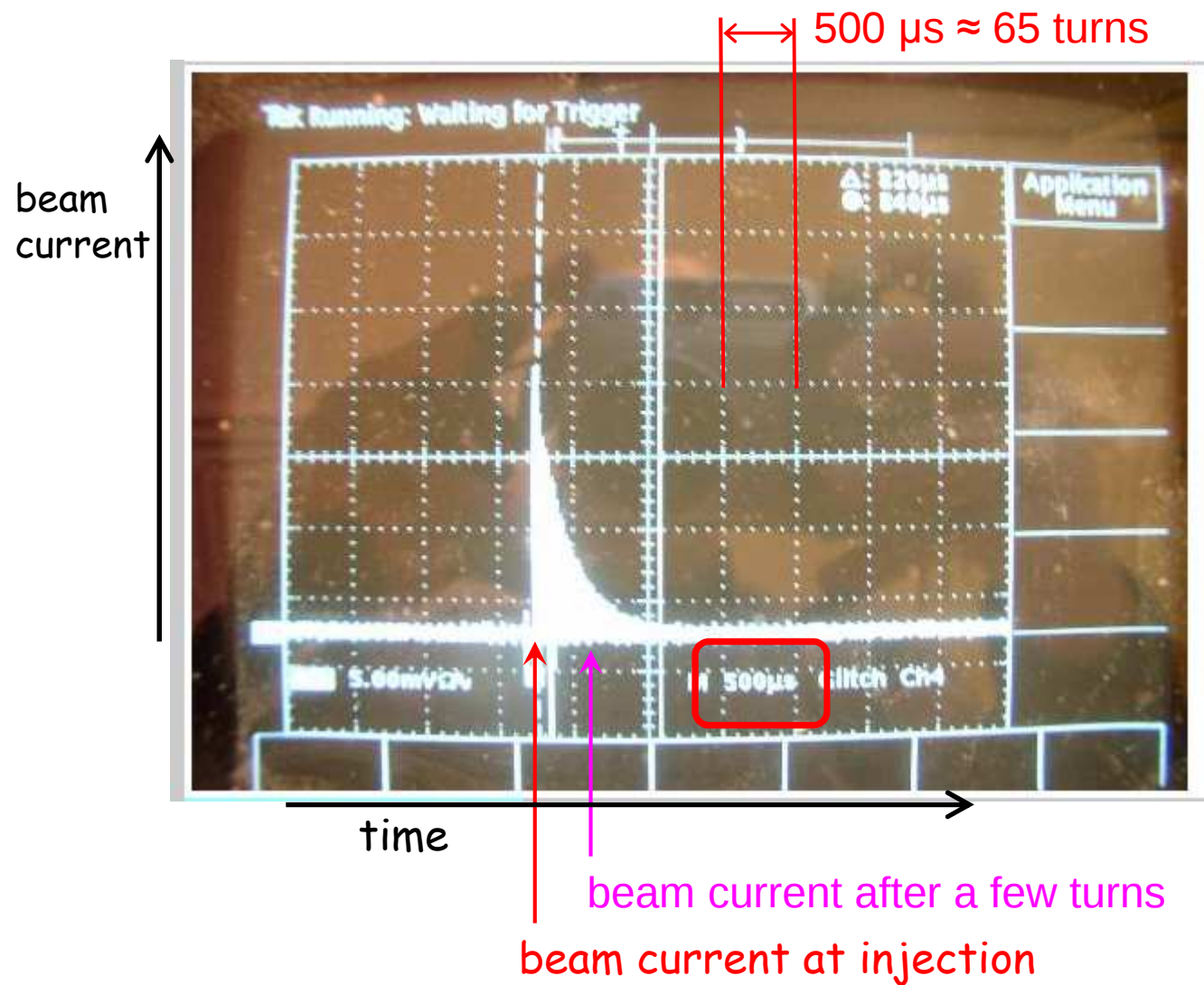
injected beam



Next suspect: injection + accumulation



Electrons can be injected but cannot be stored !



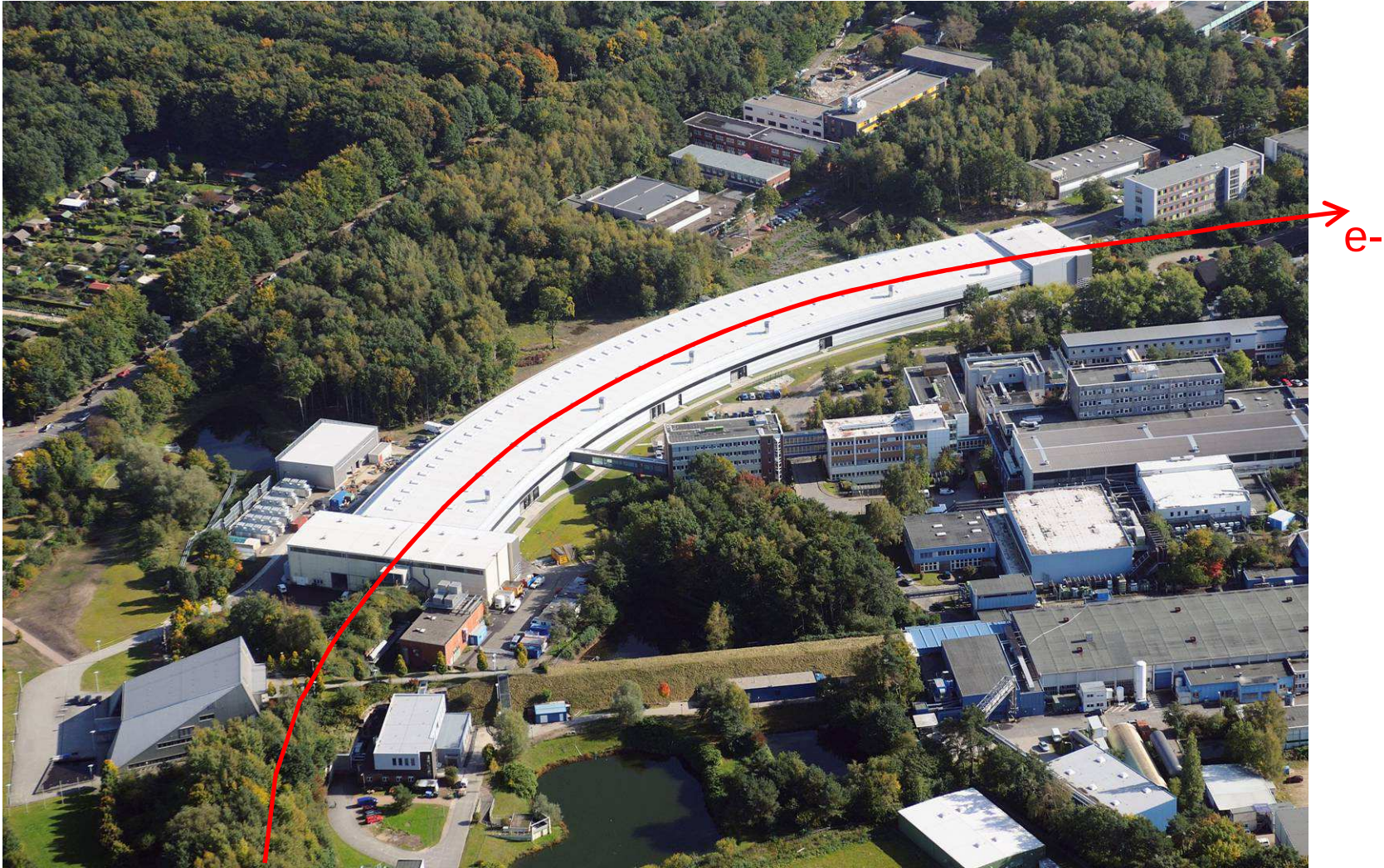
~~injection problem?~~

Next suspect: a problem with vacuum chamber

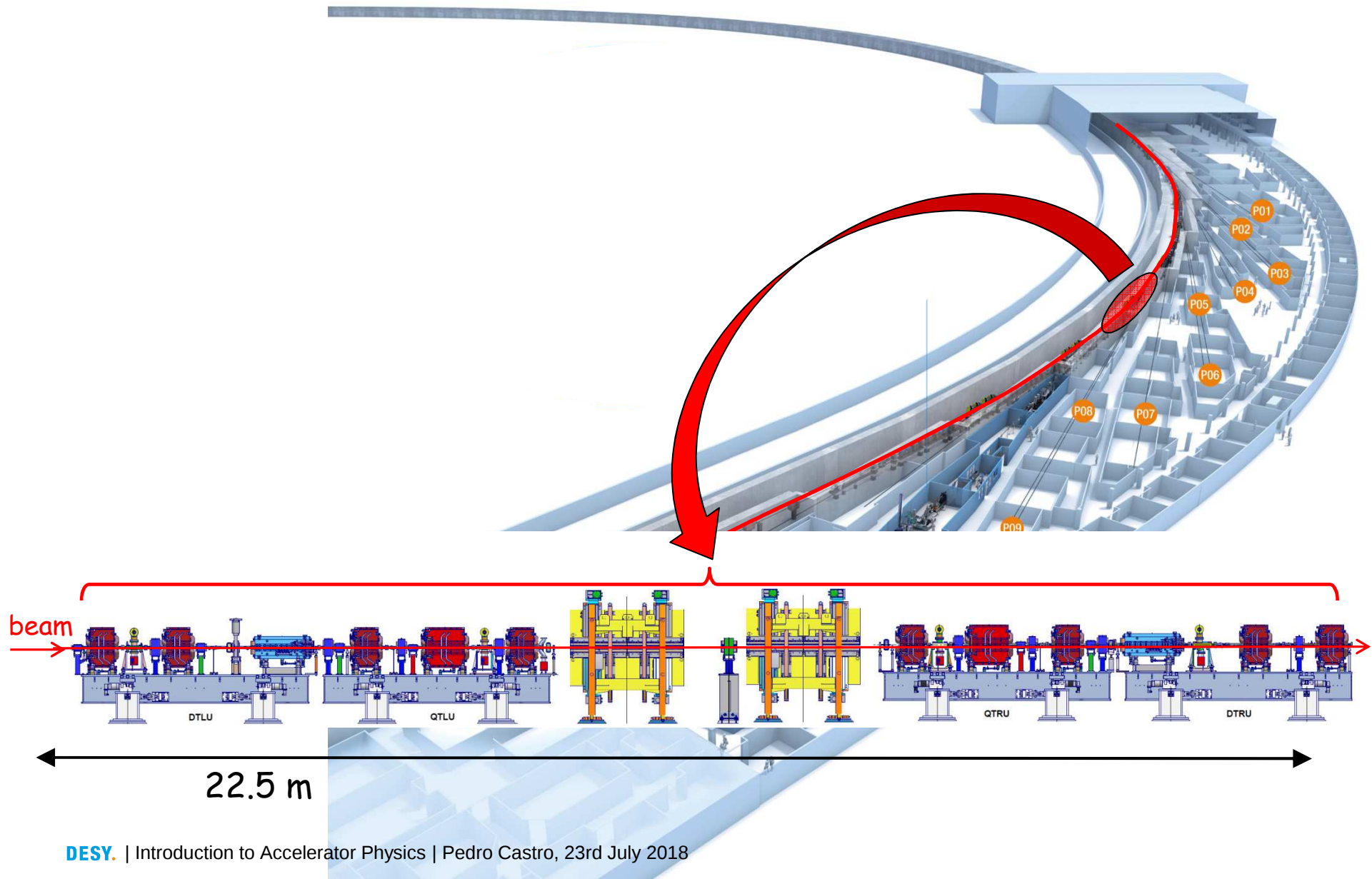
Hamburg, DESY
Sat. 12th June 2010

02:24 a.m.: beam lost
07:00 a.m.: visual inspection
in accelerator

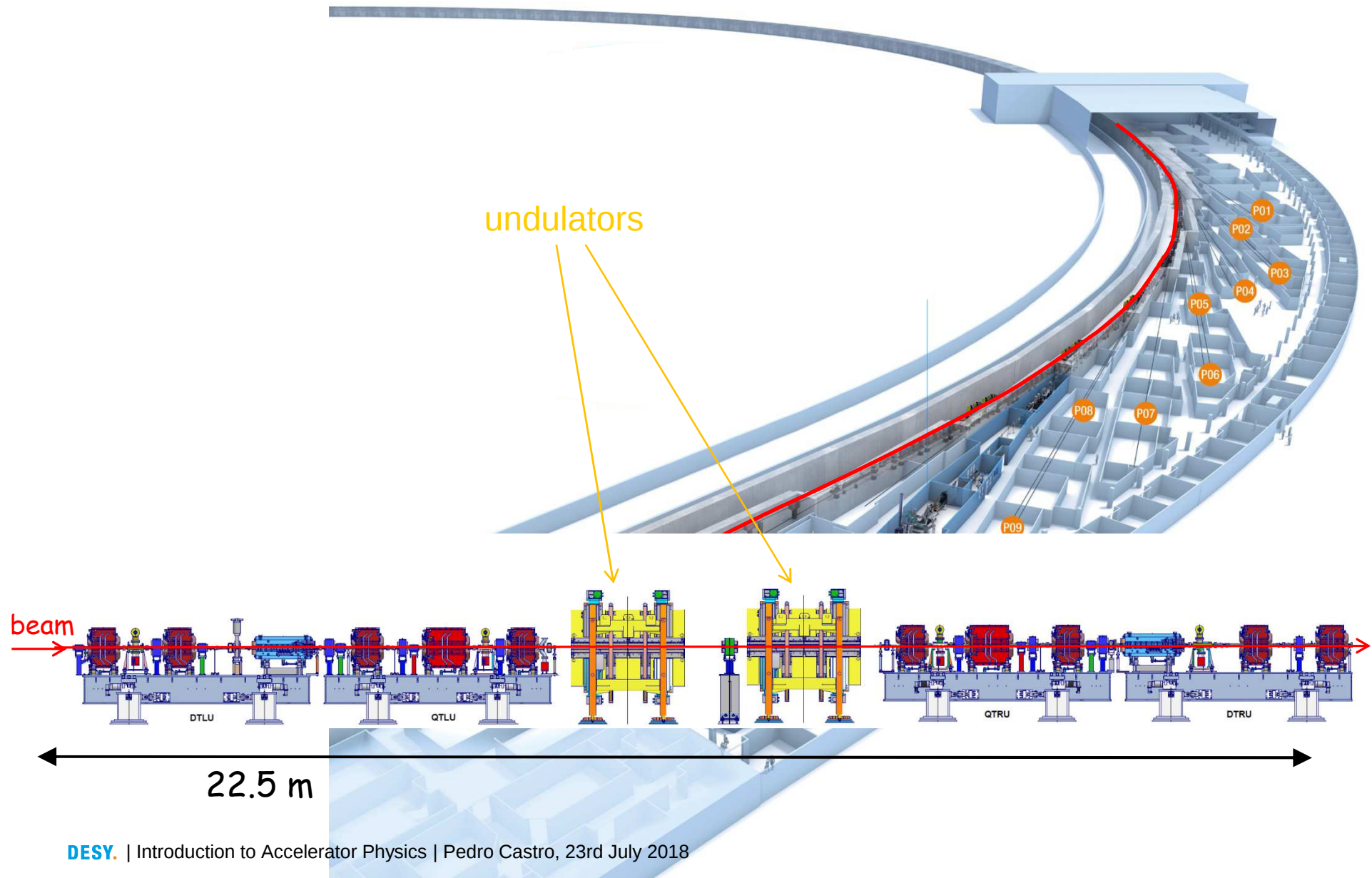
Next suspect: the new octant in 'Max von Laue hall'



Next suspect: the new octant in 'Max von Laue hall'

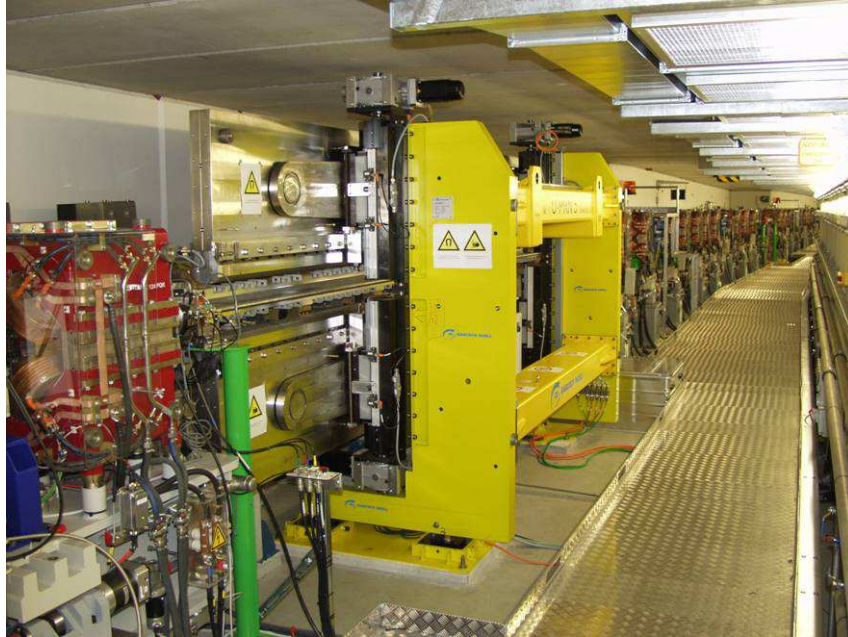


Next suspect: the new octant in 'Max von Laue hall'



Next suspect: the new octant in 'Max von Laue hall'

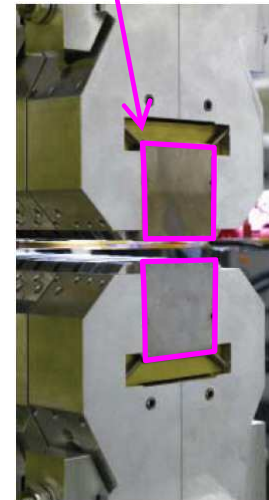
Undulator PU 10



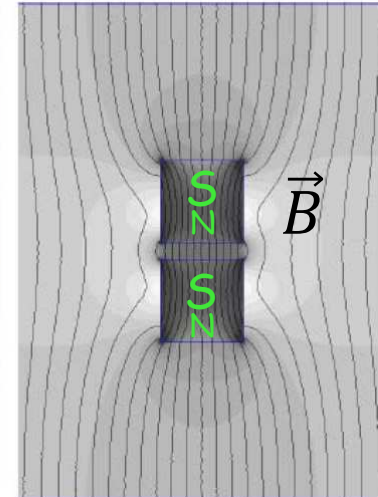
Hamburg, DESY
Sat. 12th June 2010

02:24 a.m.: beam lost
07:00 a.m.: visual inspection
in new octant

permanent
magnets



undulator field lines



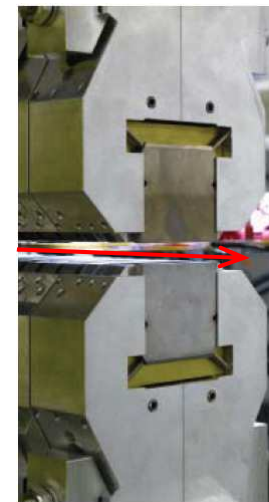
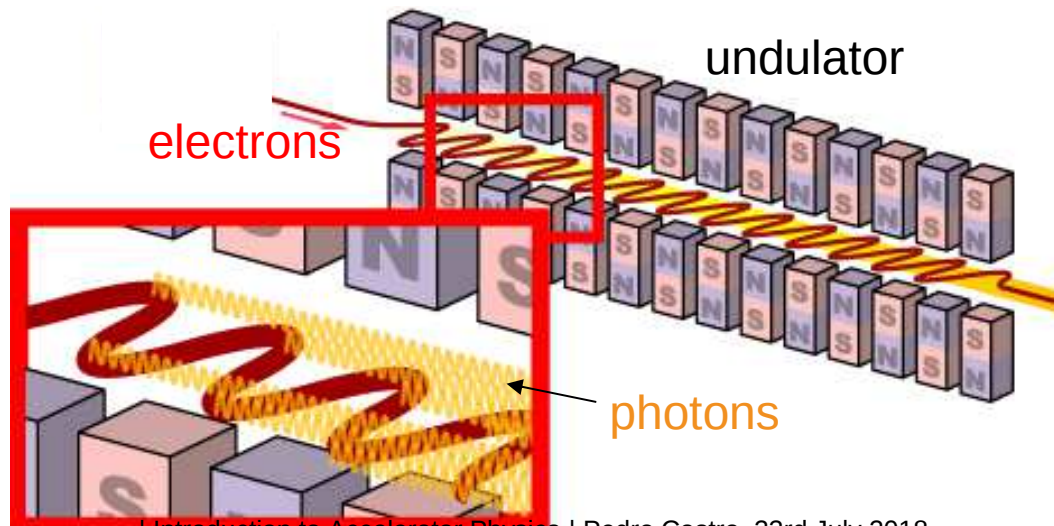
Next suspect: the new octant in 'Max von Laue hall'

Undulator PU 10

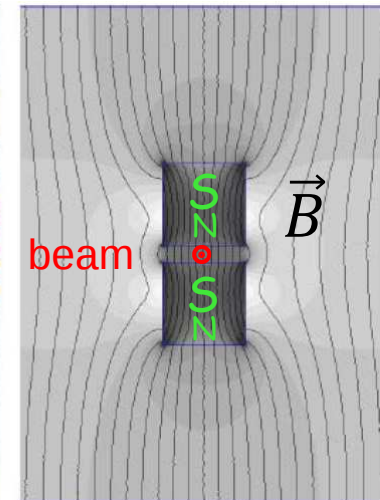


Hamburg, DESY
Sat. 12th June 2010

02:24 a.m.: beam lost
07:00 a.m.: visual inspection
in new octant



undulator field lines



Next suspect: the new octant in 'Max von Laue hall'

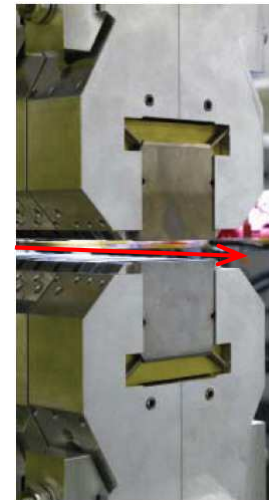
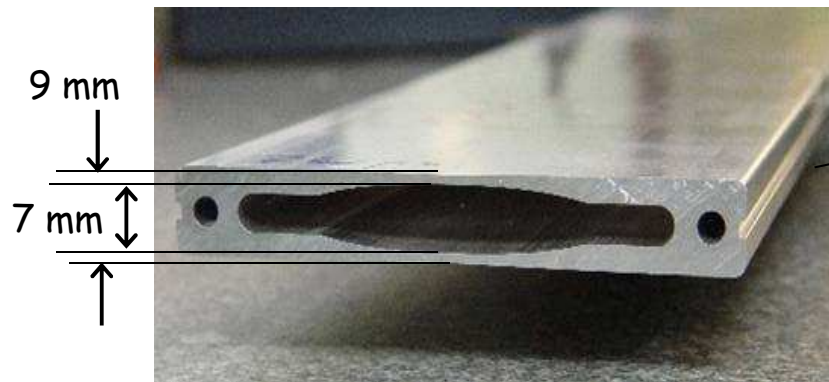
Undulator PU 10



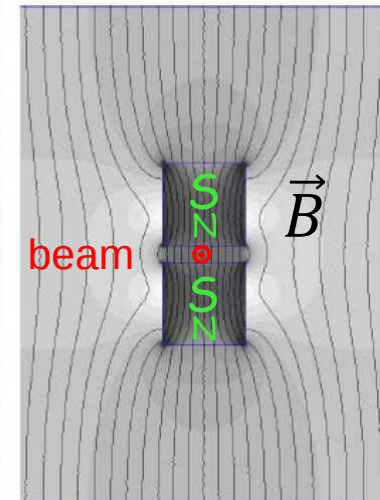
Hamburg, DESY
Sat. 12th June 2010

02:24 a.m.: beam lost
07:00 a.m.: visual inspection
in new octant

very flat undulator vacuum chambers



undulator field lines

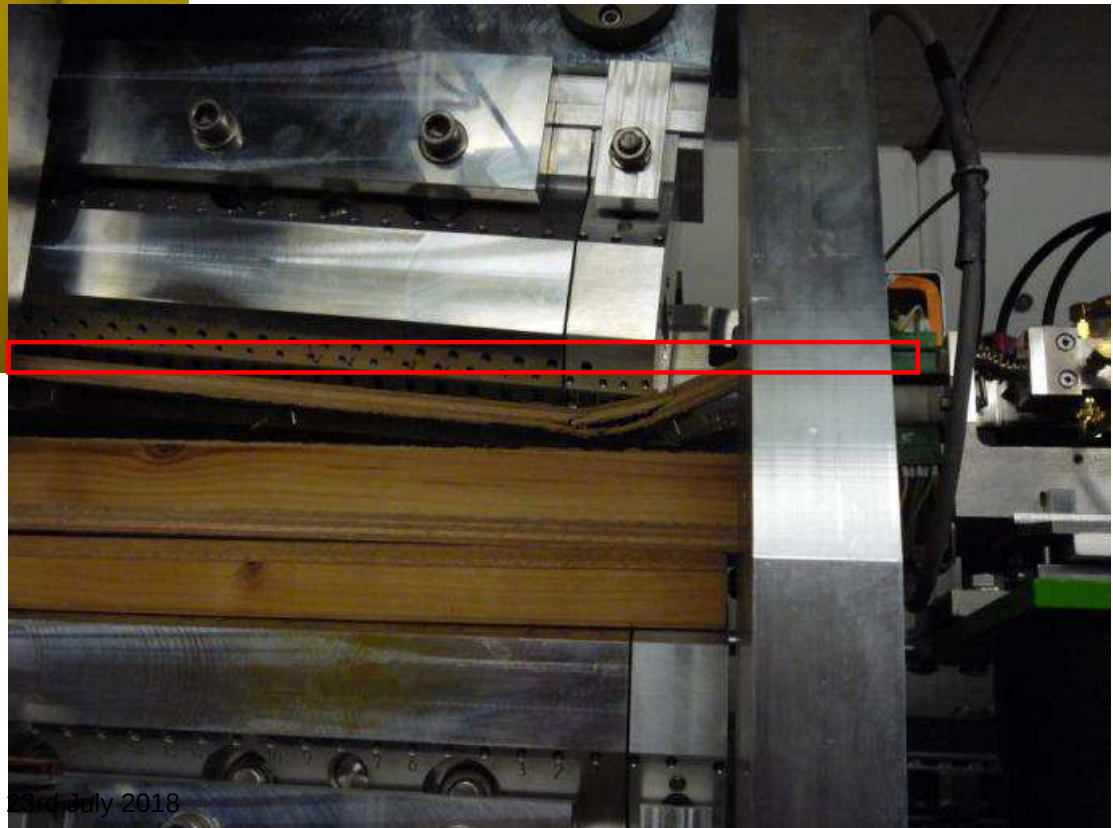


Next suspect: the new octant in 'Max von Laue hall'

a couple of months earlier...



vacuum chamber



No findings in visual inspection

The electronic logbook:

12.06.2010 07:52	Sonstiges	Kuehl, Vogt, Keil	Optische Inspektion des neuen Achtels, keine Auffälligkeiten
Naja, bis auf den BPM nach Undulator PU03 dort haben wir 6 $\mu\text{Sv/h}$ gemessen, alle anderen < 1 $\mu\text{Sv/h}$.			
12.06.2010 07:02	Sonstiges	has	Frühschicht: Kühl, Schulz, Hansen, Wierzcholek
Schichtbeginn kein gespeicherter Strahl. Nur ca. 1000 Umläufe, keine Ausfälle			

citation from the logbook: "Visual inspection of new octant: no findings"

citation from the logbook: "What we have tried so far: ..."

time
of
entries

12.06.2010 10:34 Sonstiges Kuehl, Vogt, Keil Was haben wir alles versucht:

- Optische Inspektion des neuen Achtels (nichts gefunden). Nur BPM nach Undulator PU03 zeigt 6 $\mu\text{Sv/h}$ während im Rest immer Werte unter 1 $\mu\text{Sv/h}$ gemessen werden.
- Sender-Untersuchungen:

◦ Sender beide aus	=	100 μs Strahl
◦ Sender SL aus SR ein (9 MV)	=	700 μs Strahl
◦ Sender SR aus SL ein (9 MV)	=	700 μs Strahl
◦ Beide Sender ein	=	700 μs Strahl
◦ Sender SR um 180 Grad verstellt (Gegenphase)	=	ca. 100 μs Strahl
- 500 MHz-Frequenz kontrolliert; Synchronisation kontrolliert; Orbit liegt auf dem ersten Turn mittig (damit sollte Energie stimmen). Turn-By-Turn Daten zeigen, daß Energieanpassung stimmt
- First-Turn hat nicht unübliche Amplituden (H: 5 mm, V: 2mm); horizontale Tune stimmt; vertikaler Tune ist nicht zu messen
- Einzelne Spulen vertikal und horizontal mit Phasenvorschub gedreht und die Apertur ausgeleuchtet. Es ist damit keine Vermessung zu erreichen; nach beiden Richtungen wird die Injektion schlechter (d.h. noch weniger Turns).
- 3er Beule im Norden und Westen über die Wigglerstrecken (H + V), jeweils mit Phasenverschiebung. Keine Verbesserung.
- 3er Beulen über jeweils einen halben Ring (H + V), jeweils mit Phasenverschiebung. Keine Verbesserung.
- Alle Ventile geschlossen und wieder geöffnet. Hilft nichts.
- Schirm hinter Septum rein und raus gefahren.
- Mit den letzten Spulen im Transportweg (V) sowie IME und Septum gewedelt: man kann damit die Injektion nur noch schlechter machen
- On Axis Injektion aufgesetzt (Kicker 3/Septum durchgefahren)
- Kollimatoren/Scraper rausgefahren: Keine Verbesserung
- Tune kreise gedreht: Keine Verbesserung
- Trans. Feedbacks und long. Feedback ein/aus: Keine Verbesserung

12.06.2010 07:52 Sonstiges Kuehl, Vogt, Keil Optische Inspektion des neuen Achtels, keine Auffälligkeiten

Naja, bis auf den BPM nach Undulator PU03 dort haben wir 6 $\mu\text{Sv/h}$ gemessen, alle anderen < 1 $\mu\text{Sv/h}$.

12.06.2010 07:02 Sonstiges has Frühschicht: Kühl, Schulz, Hansen, Wierzcholek

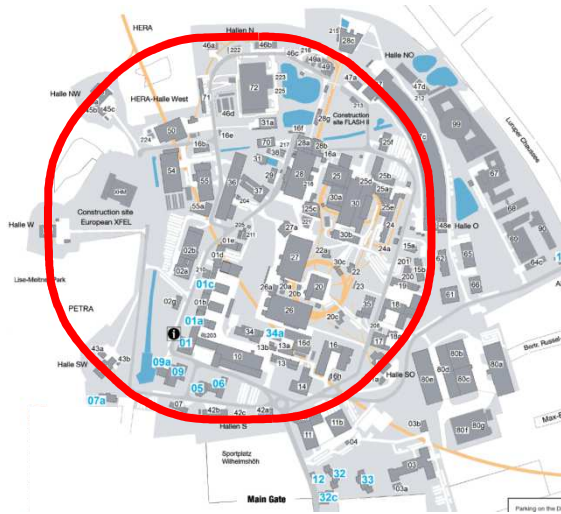
Schichtbeginn kein gespeicherter Strahl. Nur ca. 1000 Umläufe, keine Ausfälle

citation from the logbook: "Visual inspection of new octant: no findings"

**...when you have eliminated the impossible,
whatever remains, *however improbable*, must be the truth**

Sherlock Holmes, [The Sign of the Four](#)
Sir Arthur Conan Doyle

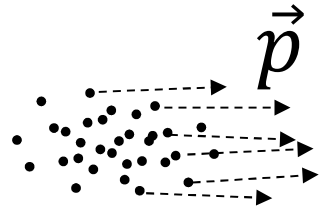
Next suspect: an aperture problem



Hamburg, DESY
Sat. 12th June 2010

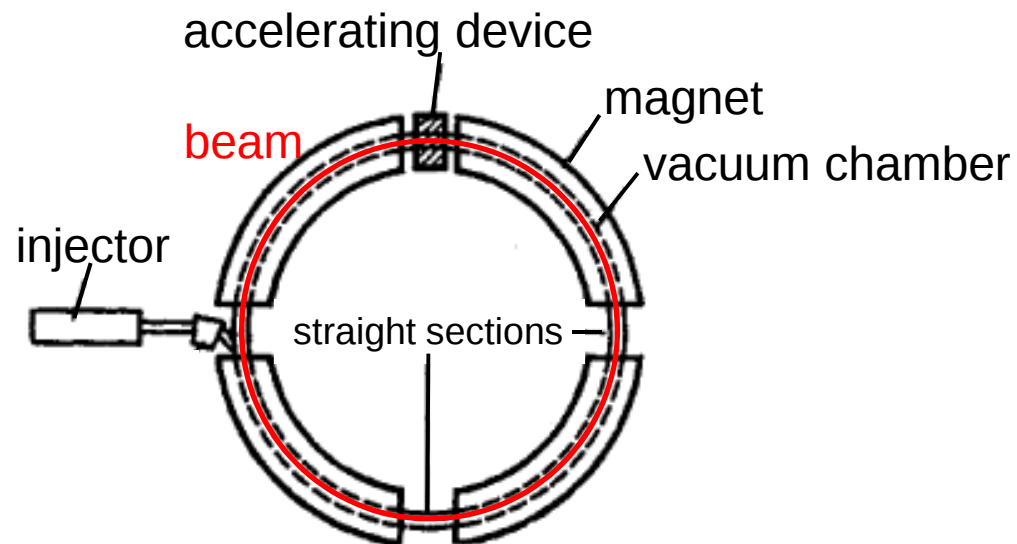
```
02:24 a.m.: beam lost
07:00 a.m.: visual inspection
              in new octant
11:52 a.m.: start aperture scan
```

Need of focusing



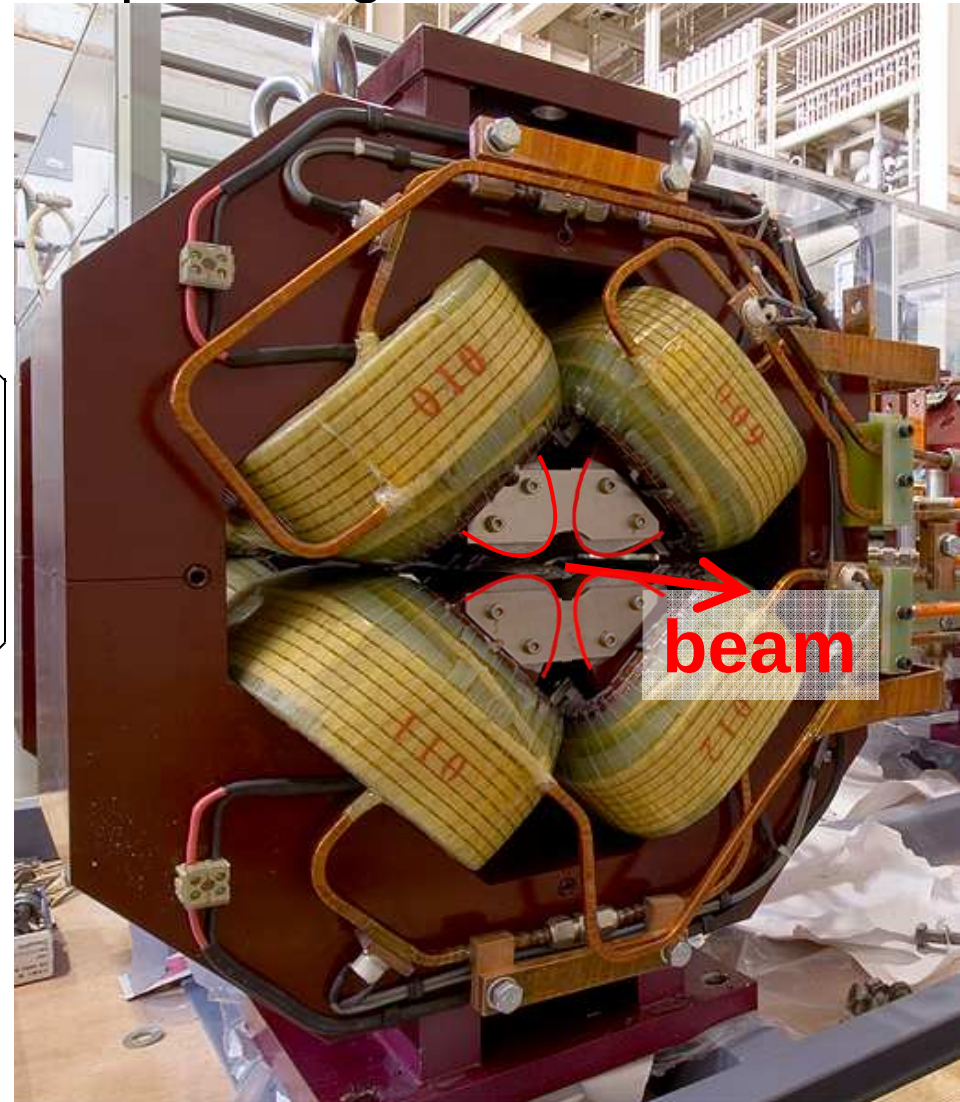
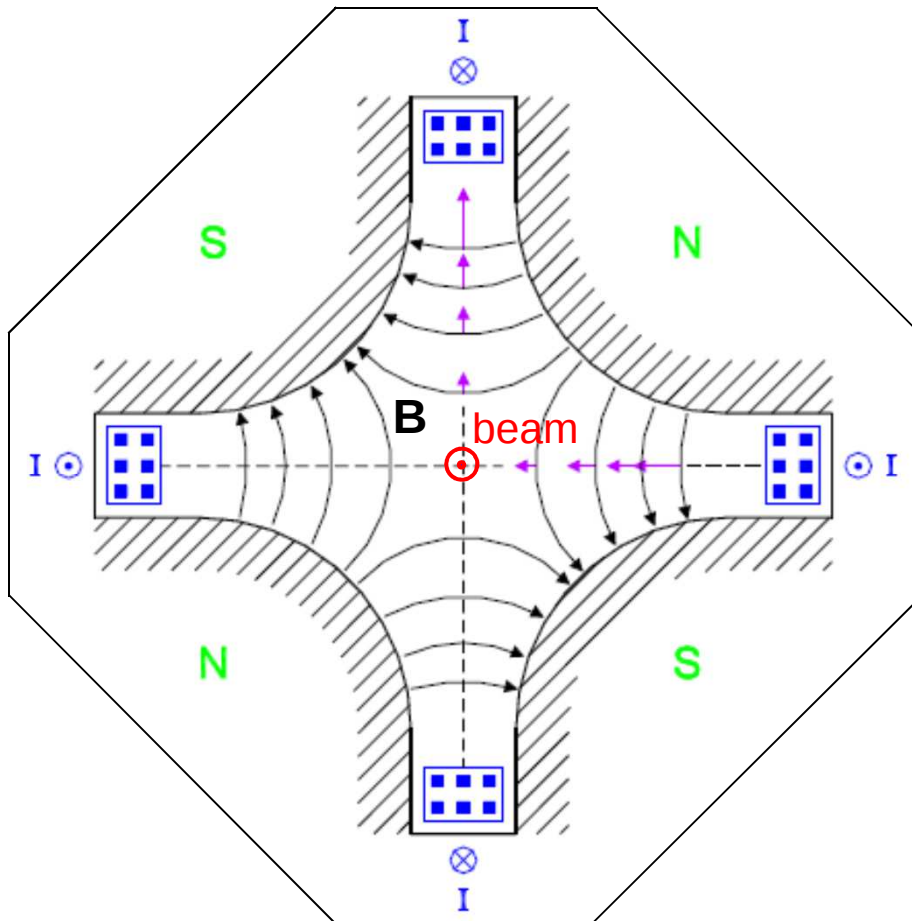
beam / bunch

we need to focus the beam !

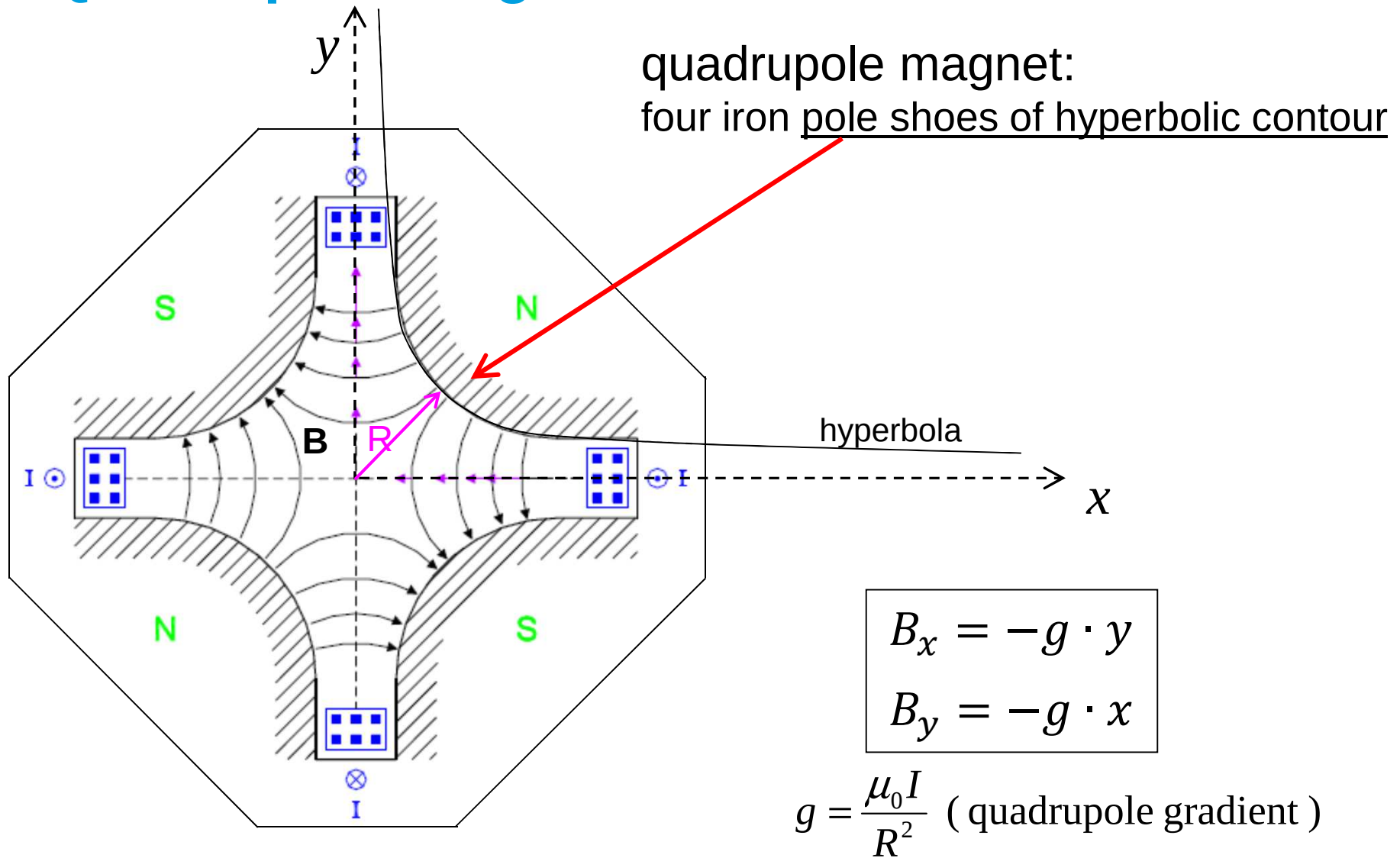


Quadrupole magnets

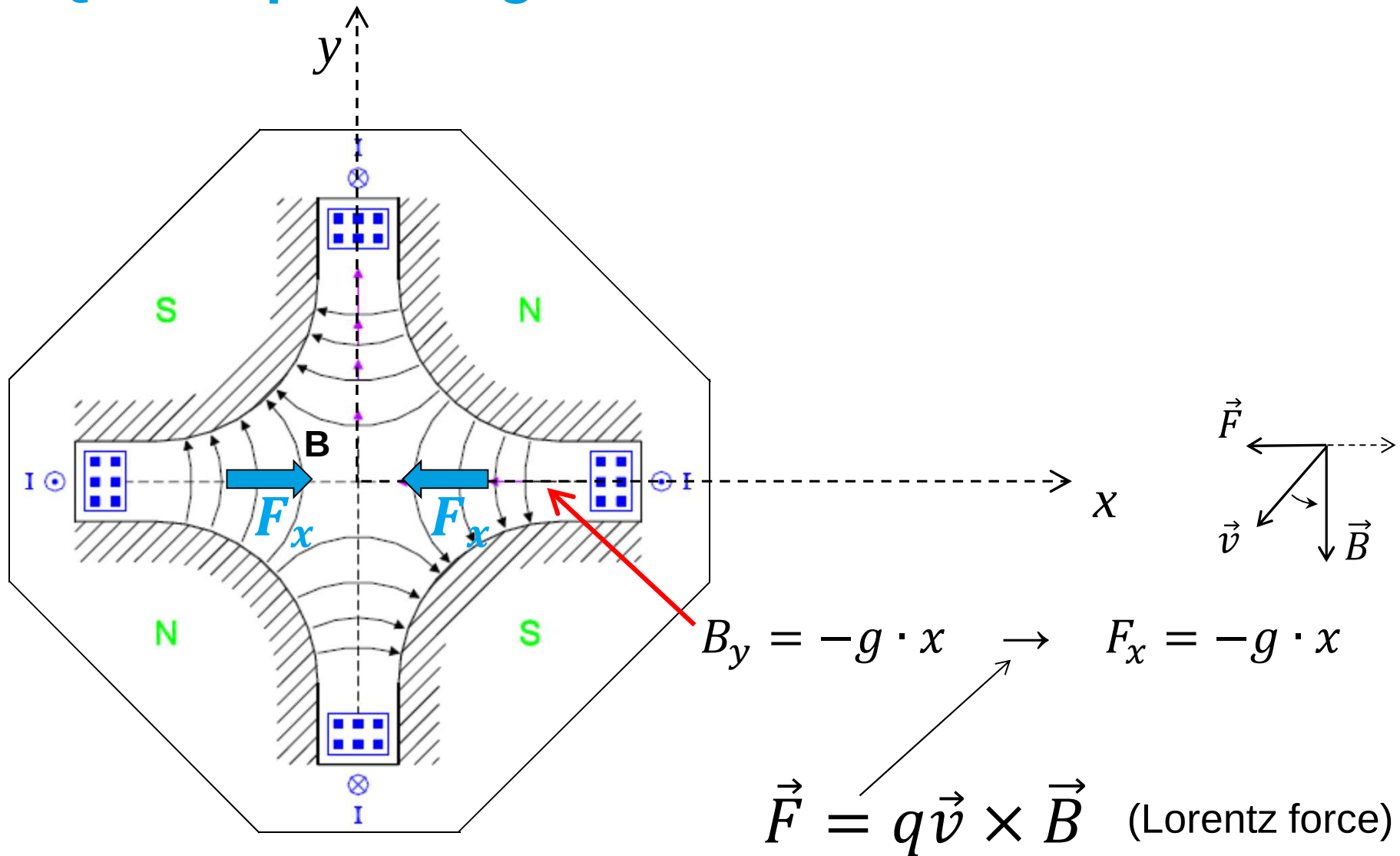
quadrupole magnet:



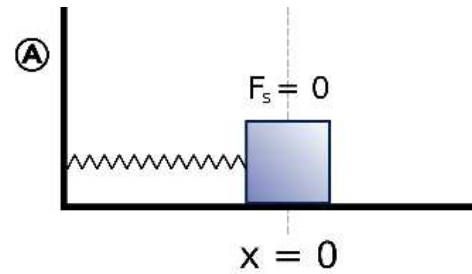
Quadrupole magnets



Quadrupole magnets

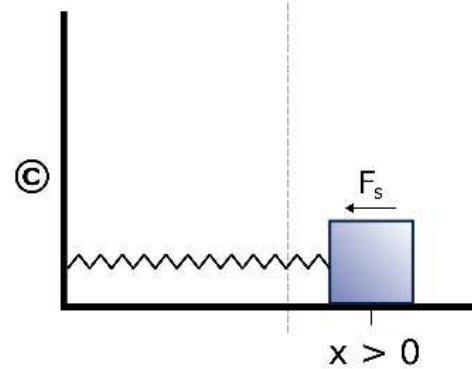
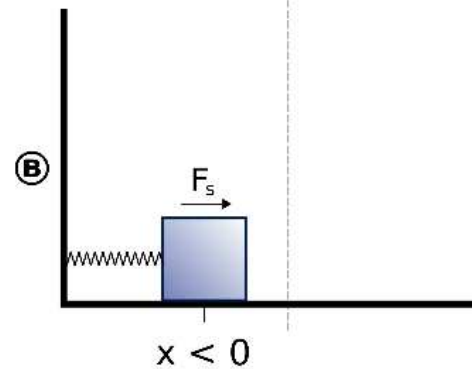


Classical mechanics: harmonic oscillator

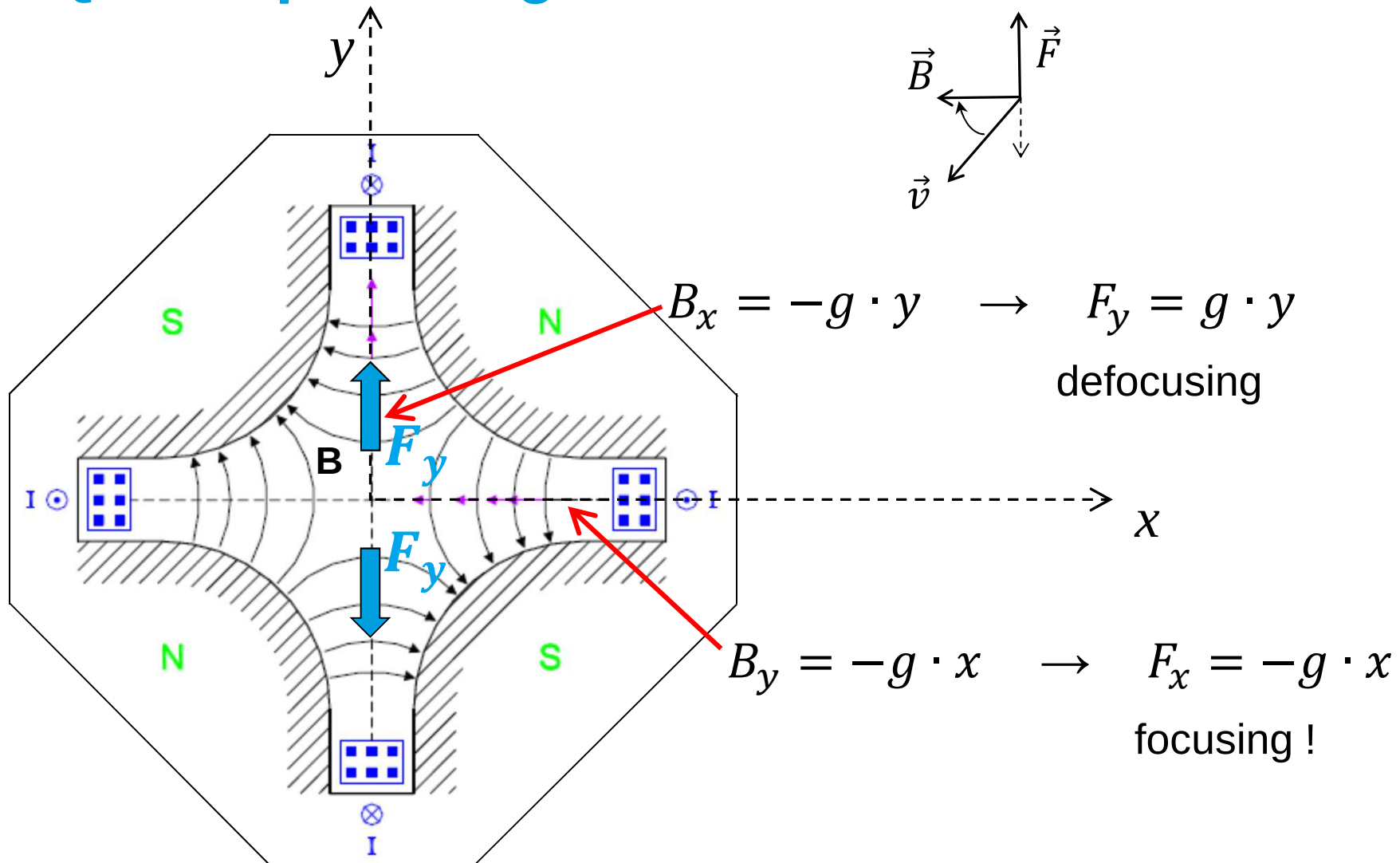


restoring force:

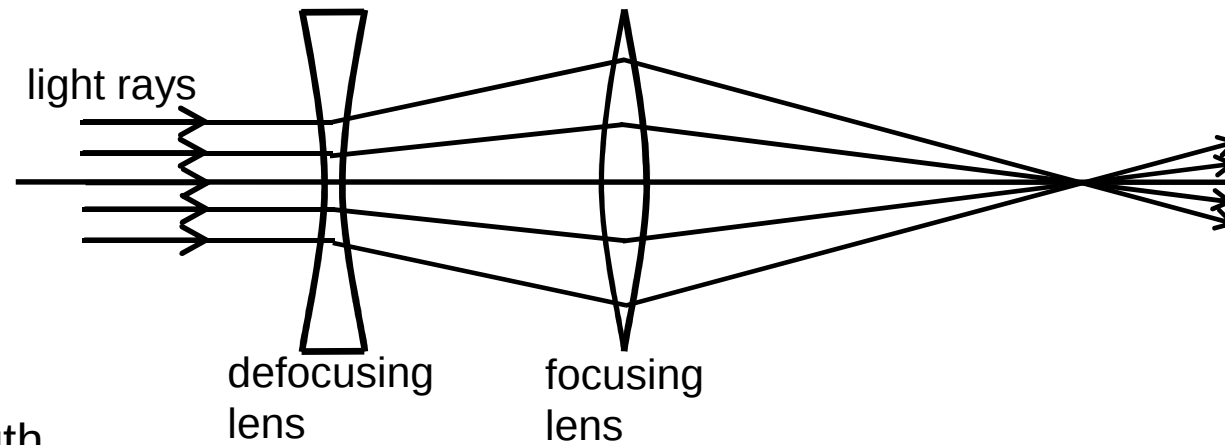
$$F = -kx$$



Quadrupole magnets



In light optics...



f : focal length

f^* : system focal length

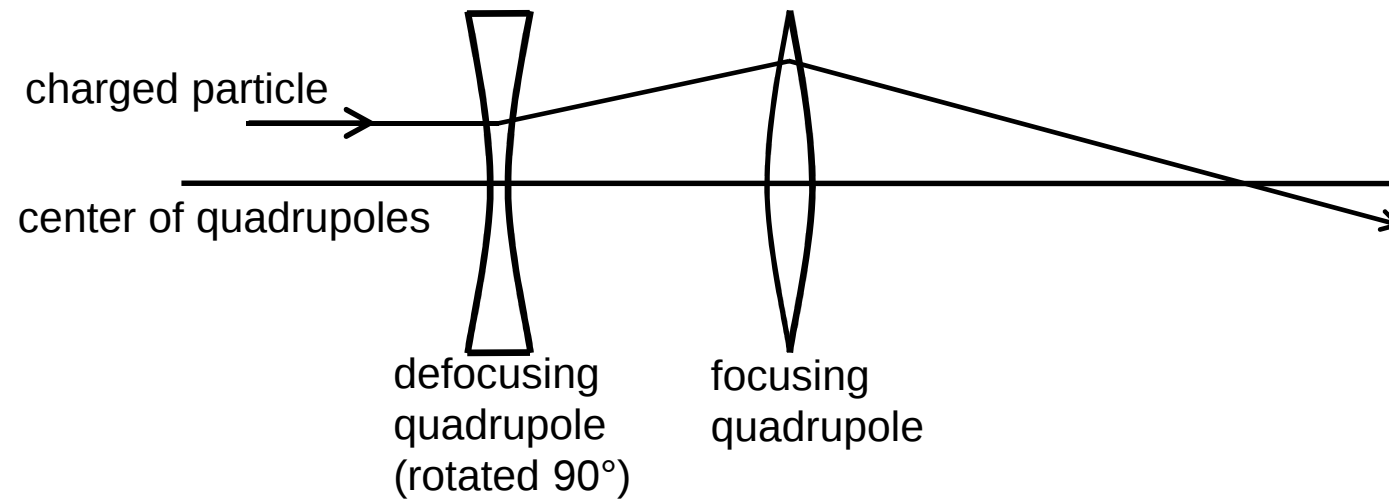
$$\frac{1}{f^*} = \frac{1}{f_D} + \frac{1}{f_F} - \frac{d}{f_D f_F} \quad (\text{light optics})$$

if $f_D = -f_F = f$

$$\frac{1}{f^*} = \frac{d}{f^2} > 0$$

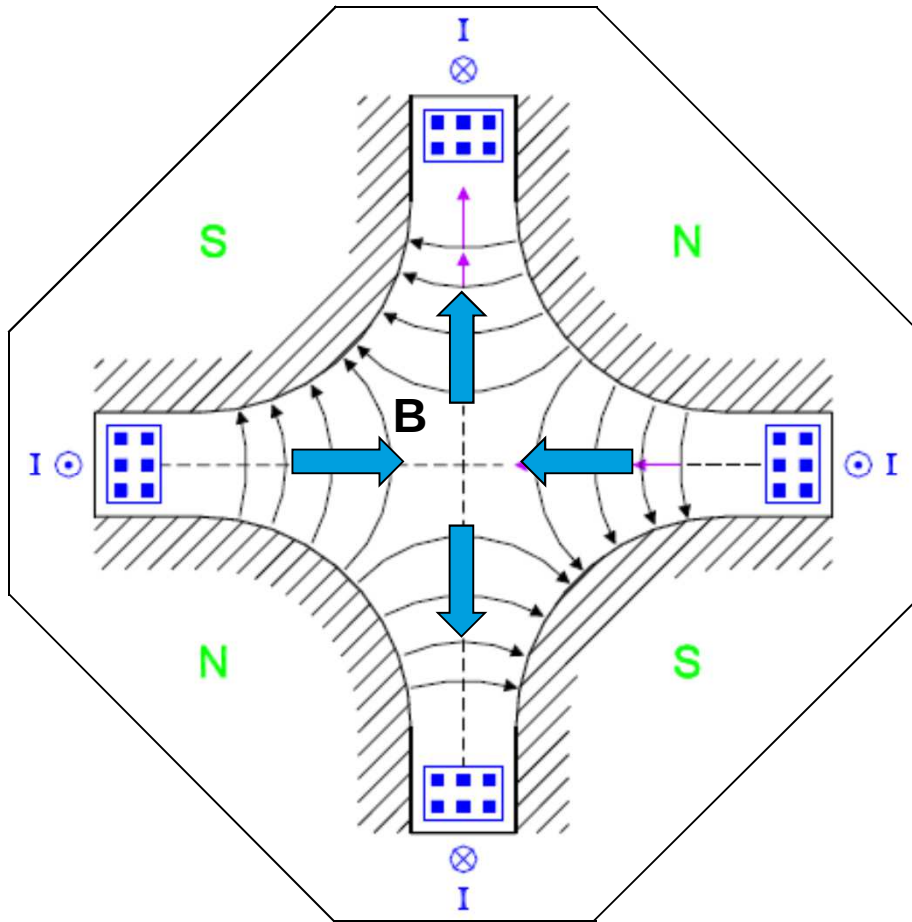
Quadrupole magnets

QD + QF = net focusing effect:

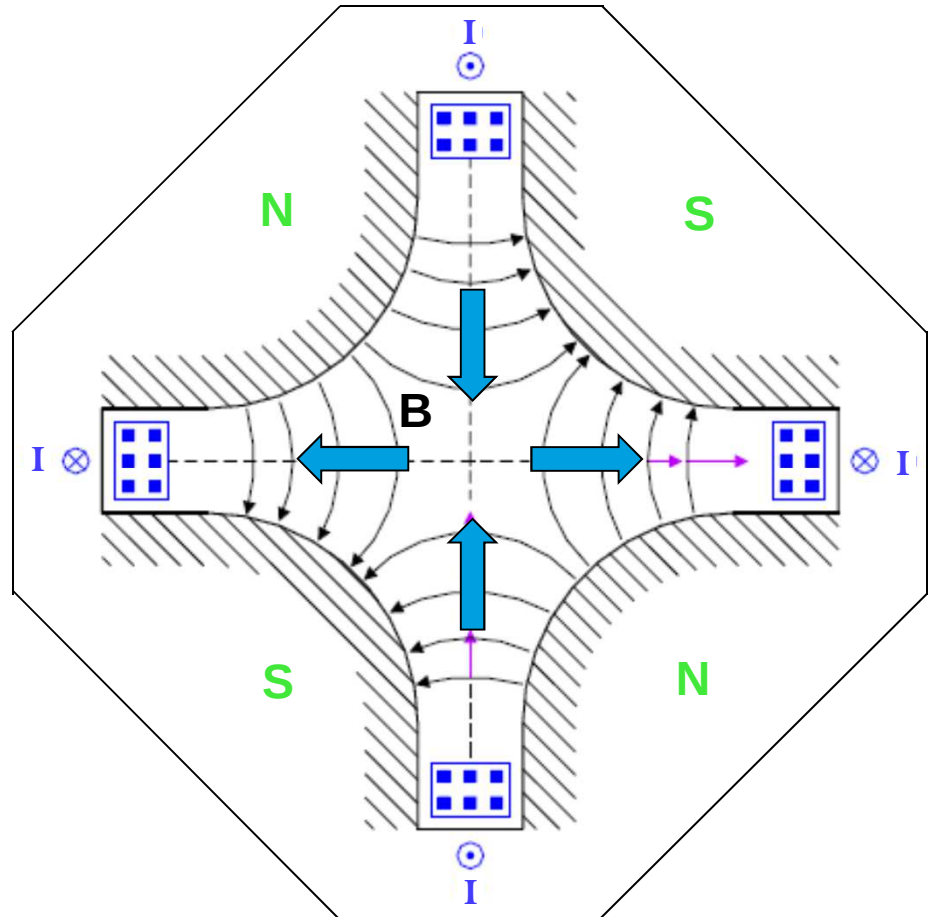


Quadrupole magnets

vertical defocusing
horizontal focusing



vertical focusing
horizontal defocusing

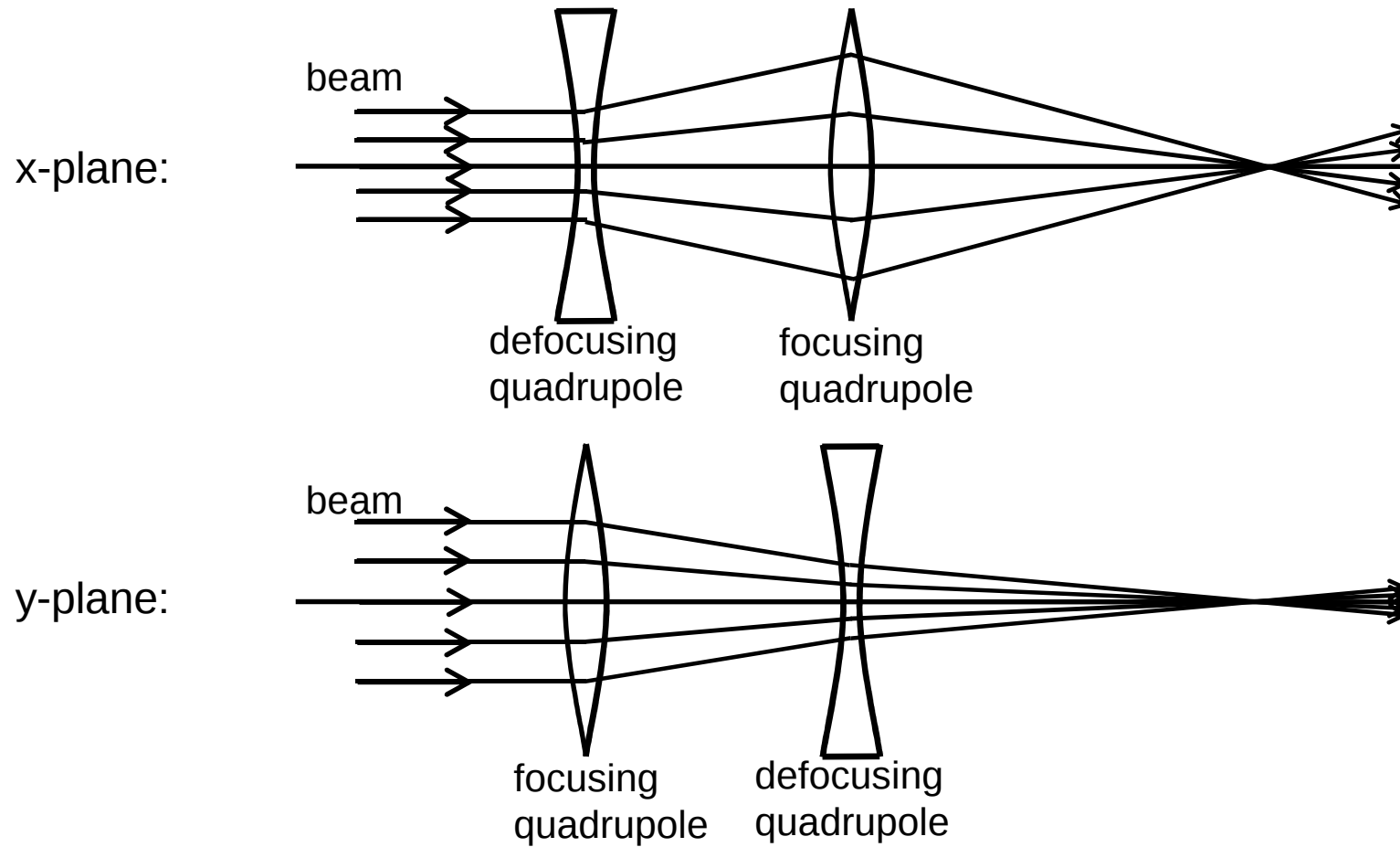


rotated 90°

$$I = -I$$

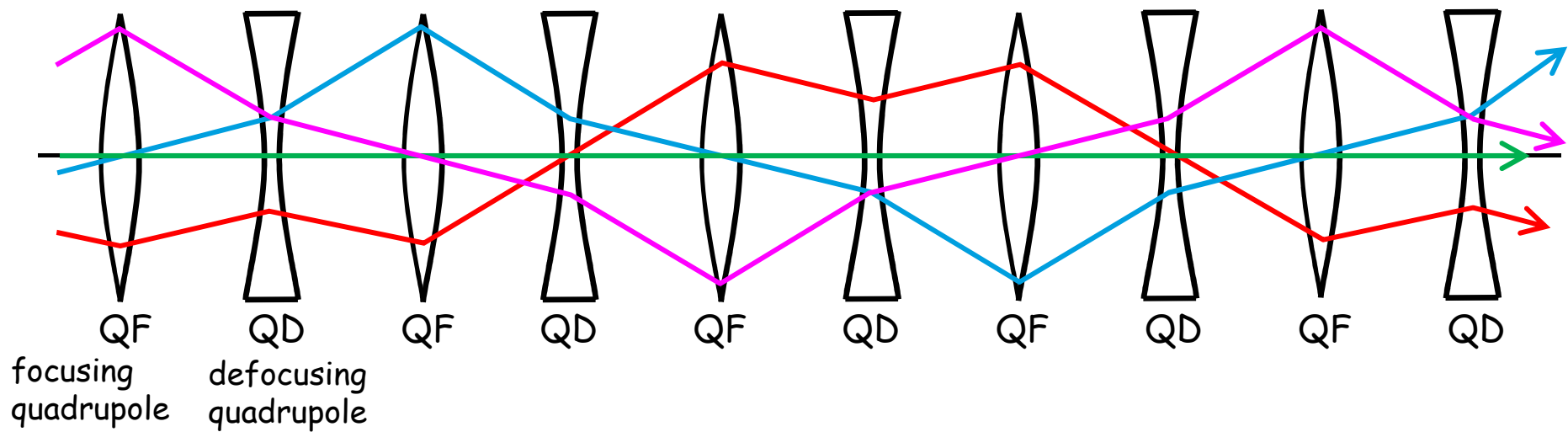
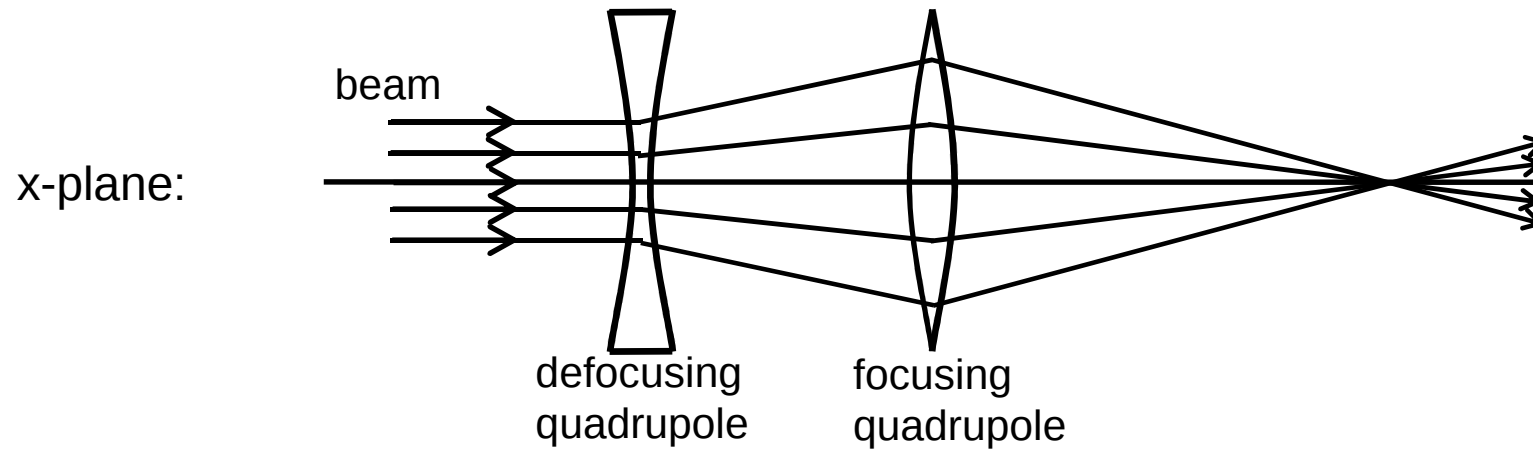
Quadrupole magnets

QD + QF = net focusing effect:

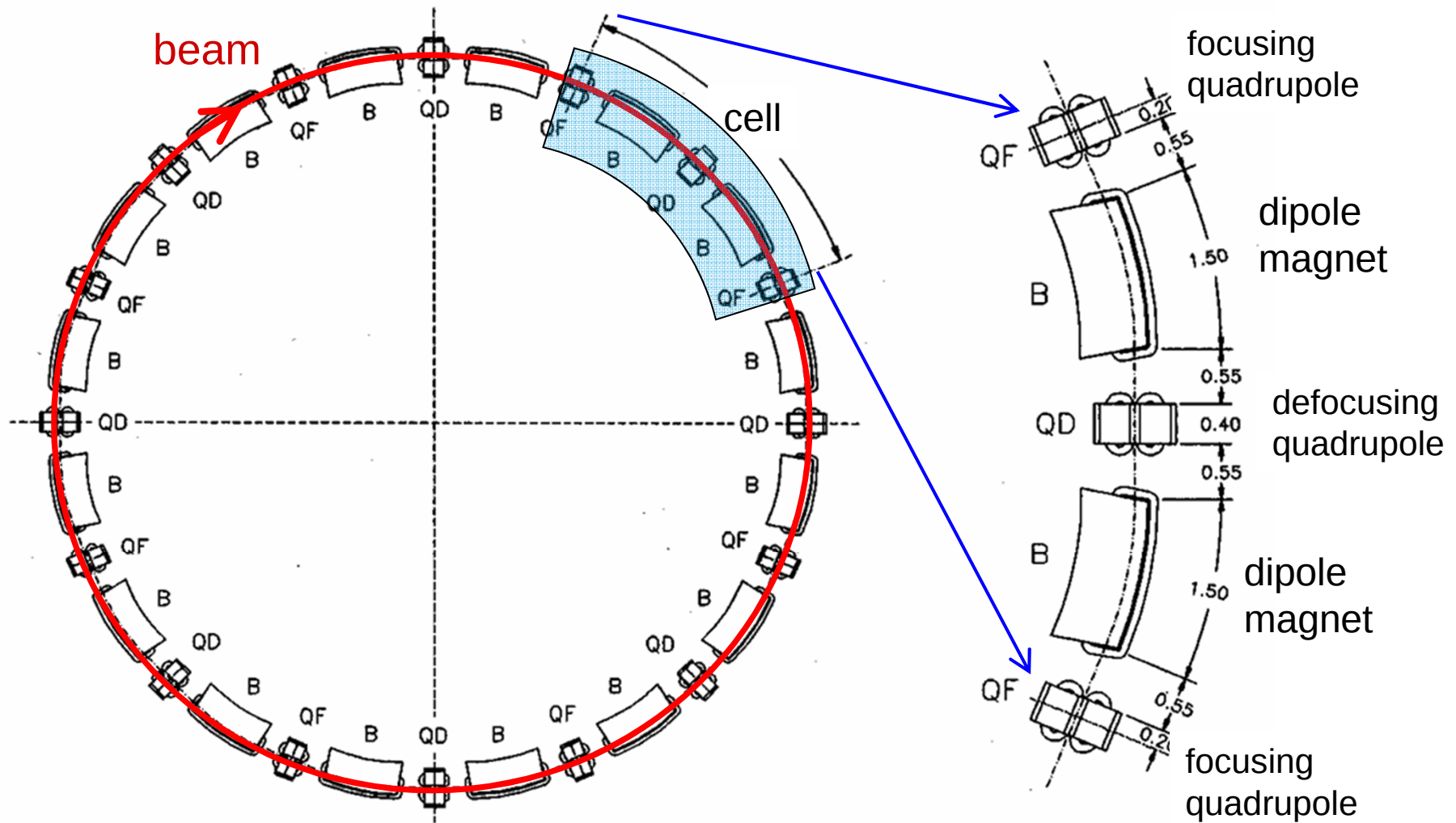


Quadrupole magnets

QD + QF = net focusing effect:

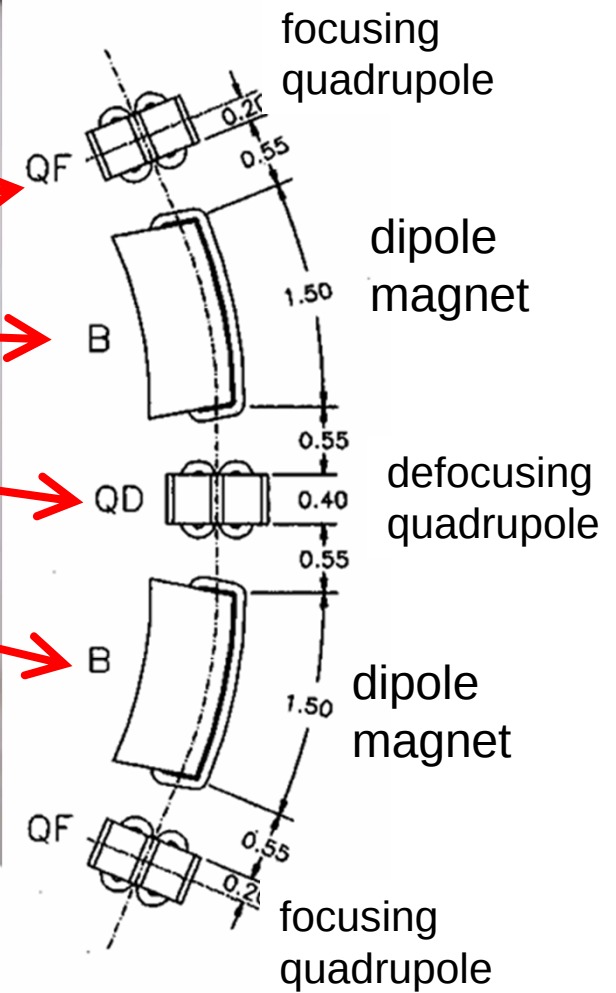


Circular accelerator

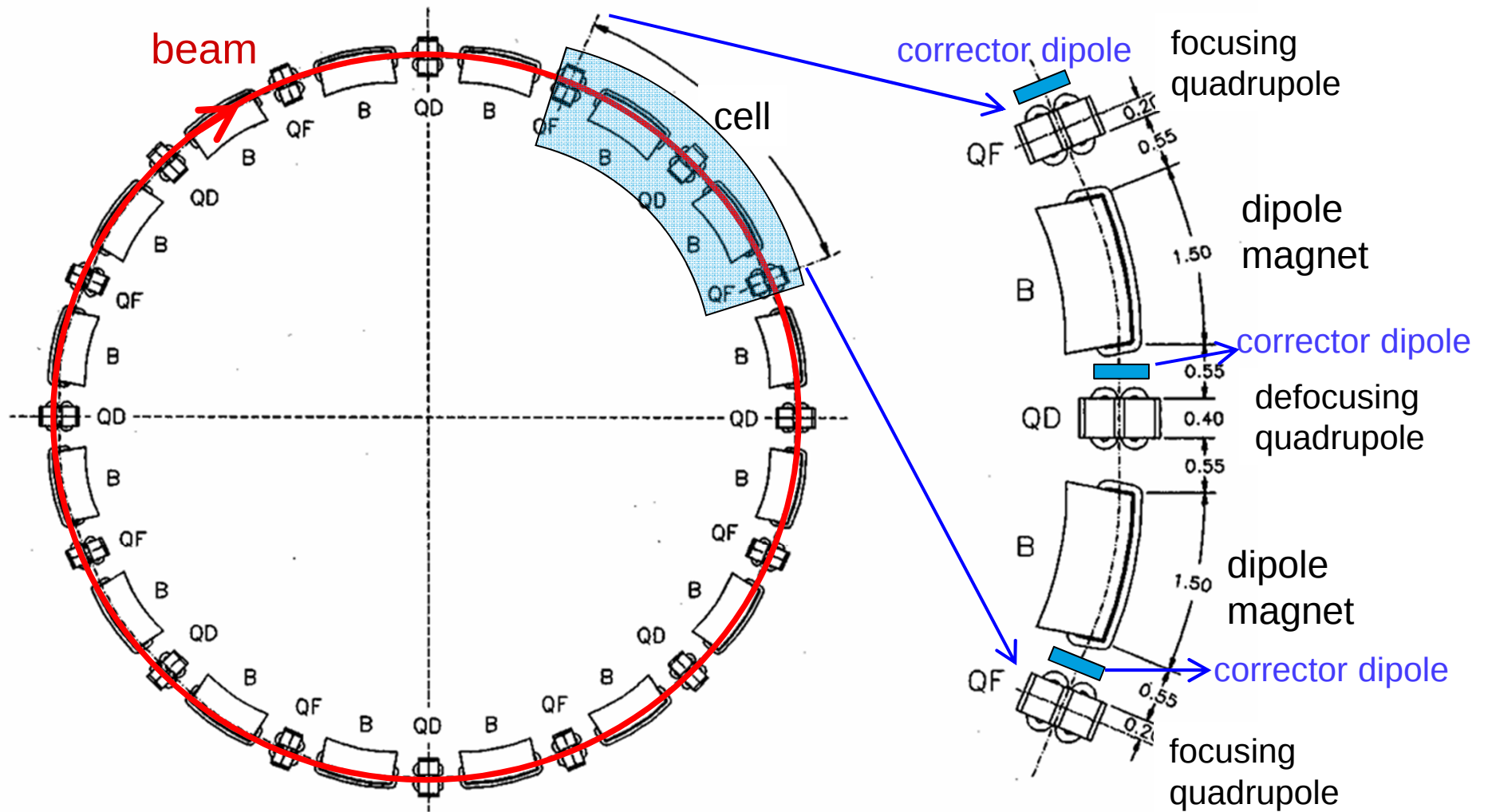


Circular accelerator

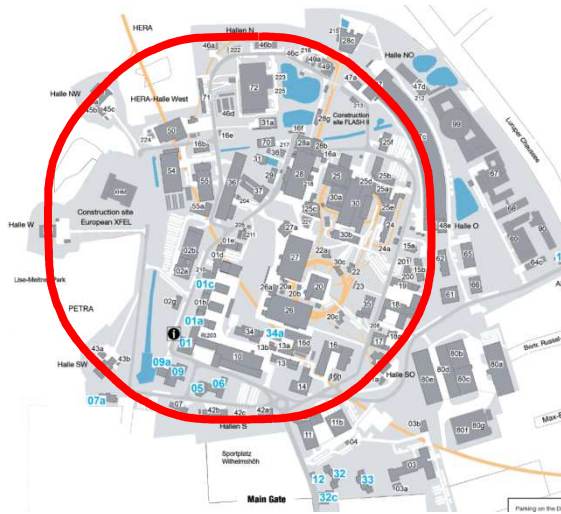
PETRA



Circular accelerator

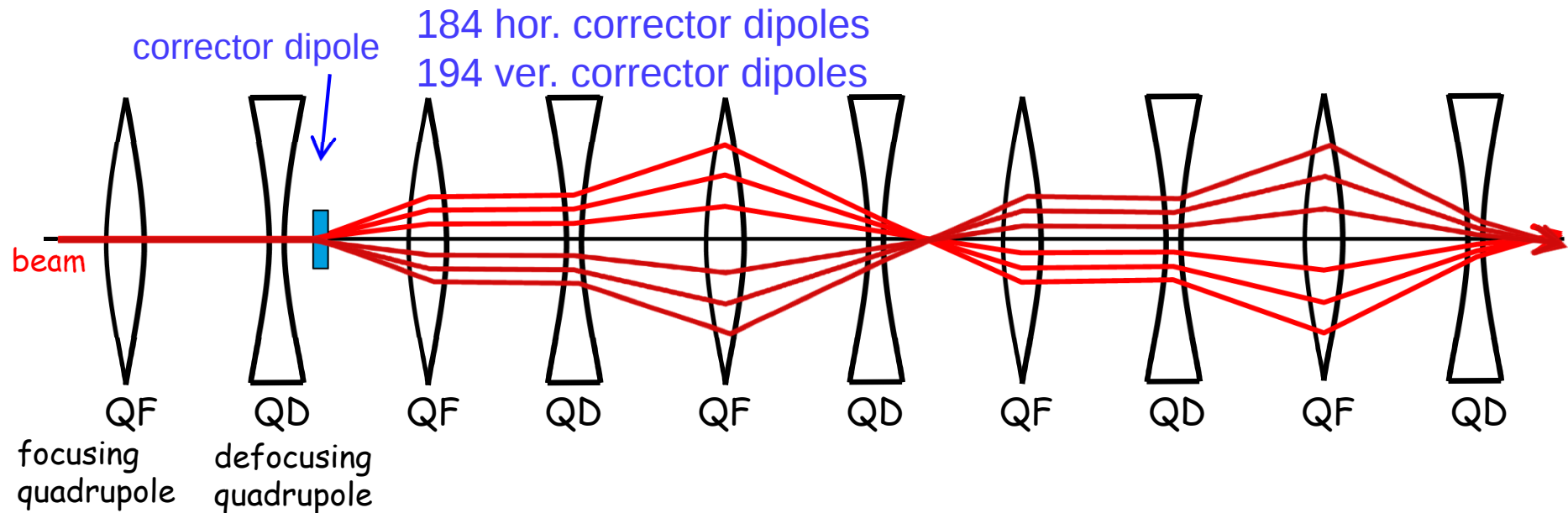


Next suspect: an aperture problem

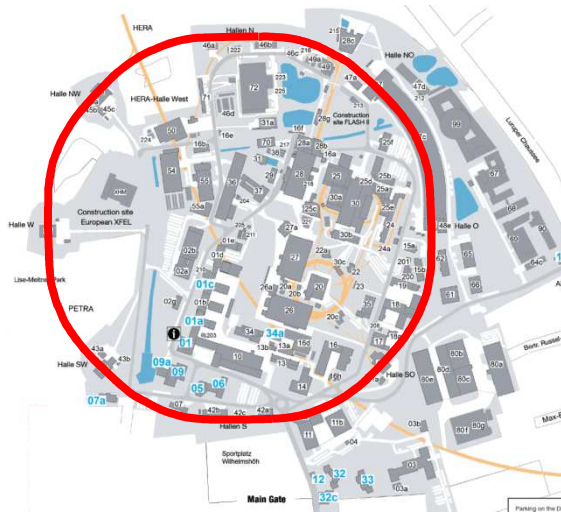


Hamburg, DESY
Sat. 12th June 2010

02:24 a.m.: beam lost
07:00 a.m.: visual inspection
in new octant
11:52 a.m.: start aperture scan

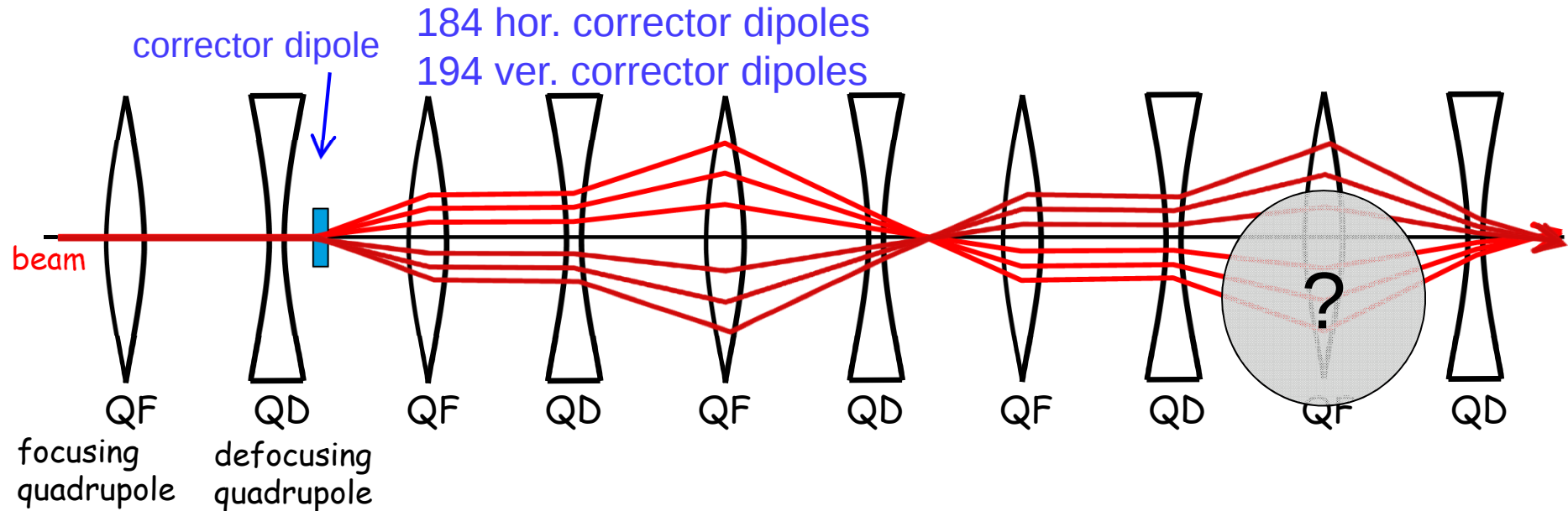


Next suspect: an aperture problem

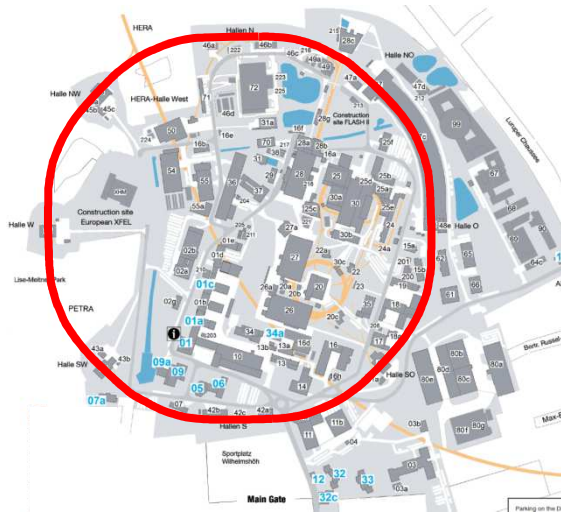


Hamburg, DESY
Sat. 12th June 2010

02:24 a.m.: beam lost
07:00 a.m.: visual inspection
in new octant
11:52 a.m.: start aperture scan



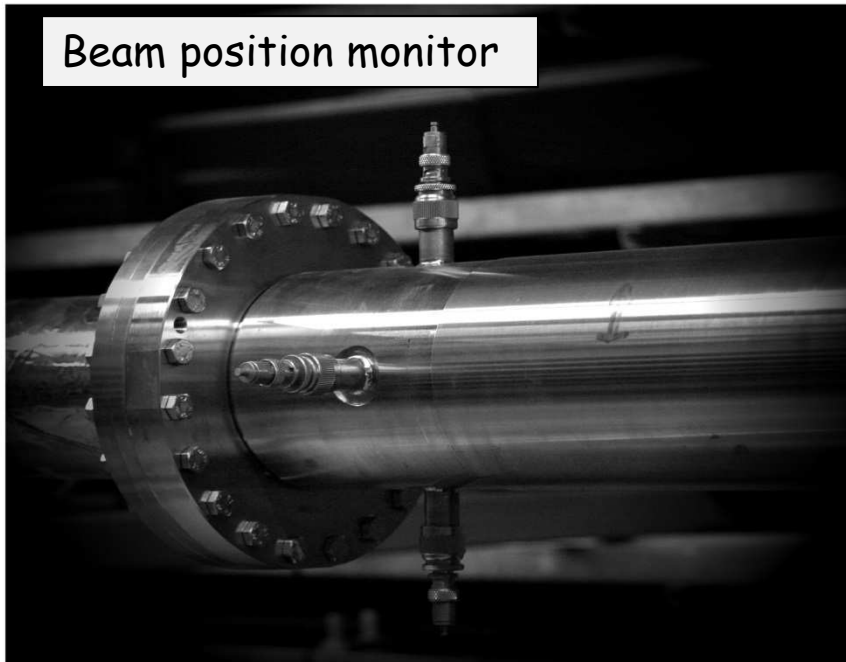
First useful hint: aperture problem



Hamburg, DESY
Sat. 12th June 2010

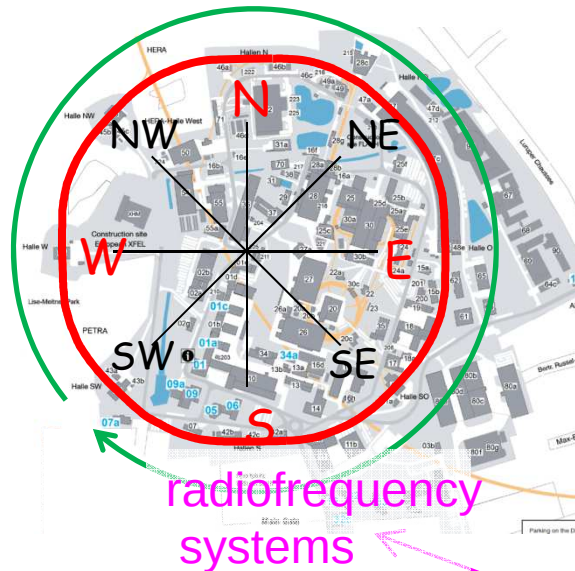
02:24 a.m.: beam lost
07:00 a.m.: visual inspection
in new octant
11:52 a.m.: start aperture scan
13:20 a.m.: beam stored

Beam position monitor



244 beam position monitors

First useful hint: horizontal aperture problem



Hamburg, DESY
Sat. 12th June 2010

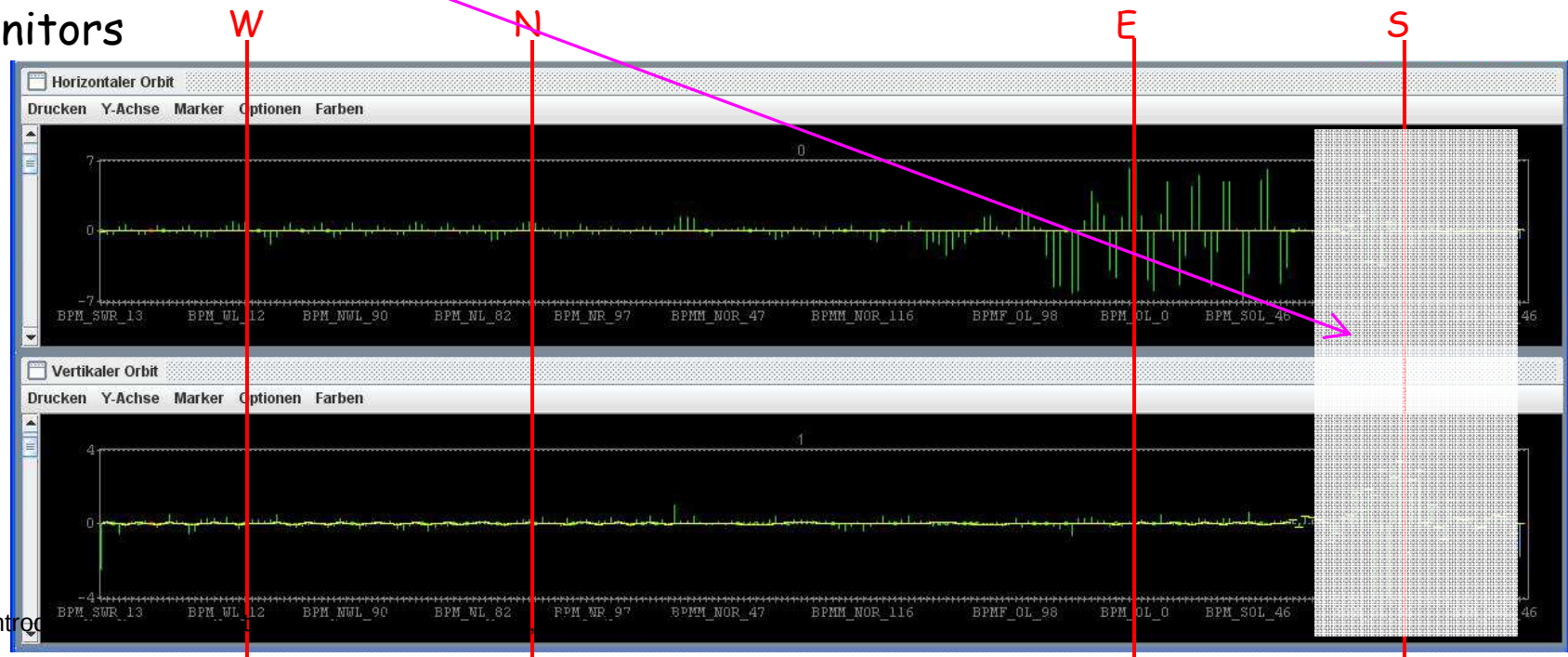
02:24 a.m.: beam lost
07:00 a.m.: visual inspection
in new octant
11:52 a.m.: start aperture scan
13:20 a.m.: beam stored

244 monitors

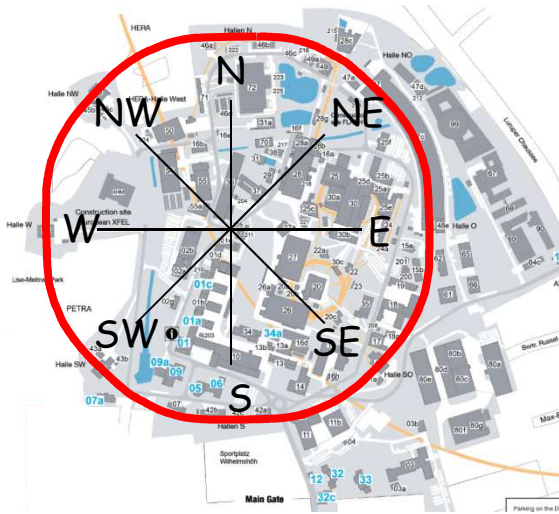
horizontal
beam pos.
[mm]

vertical
beam pos.
[mm]

DESY. | Intro

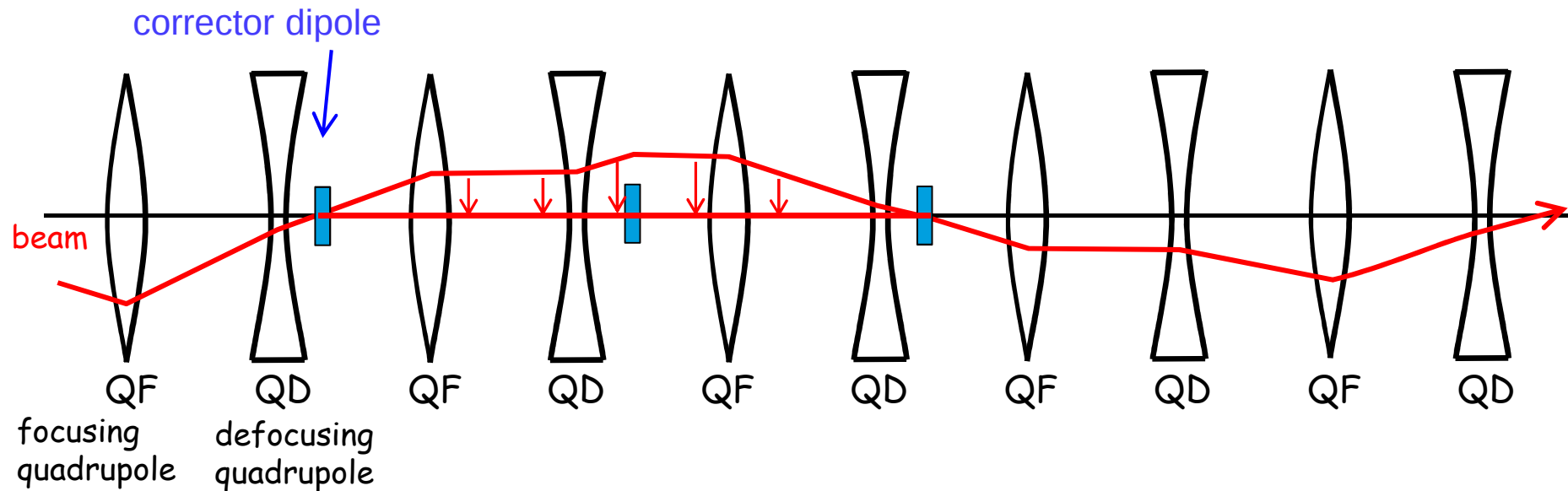


First useful hint: horizontal aperture problem

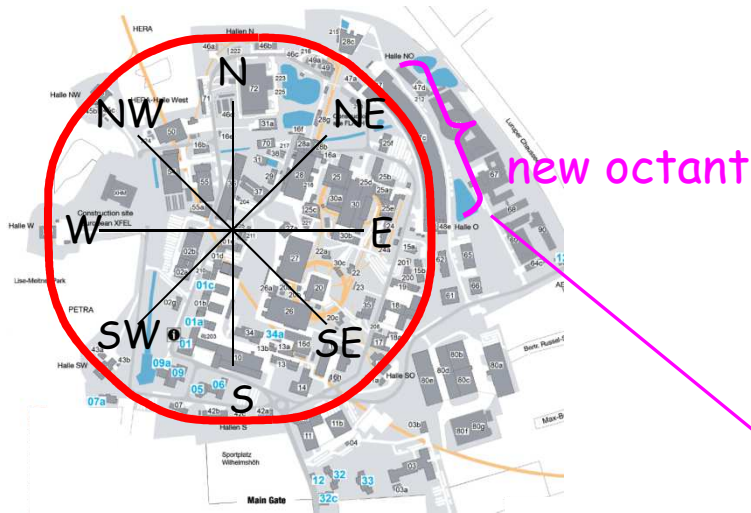


Hamburg, DESY
Sat. 12th June 2010

```
02:24 a.m.: beam lost
07:00 a.m.: visual inspection
              in new octant
11:52 a.m.: start aperture scan
13:20 a.m.: beam stored
```



First useful hint: horizontal aperture problem



Hamburg, DESY
Sat. 12th June 2010

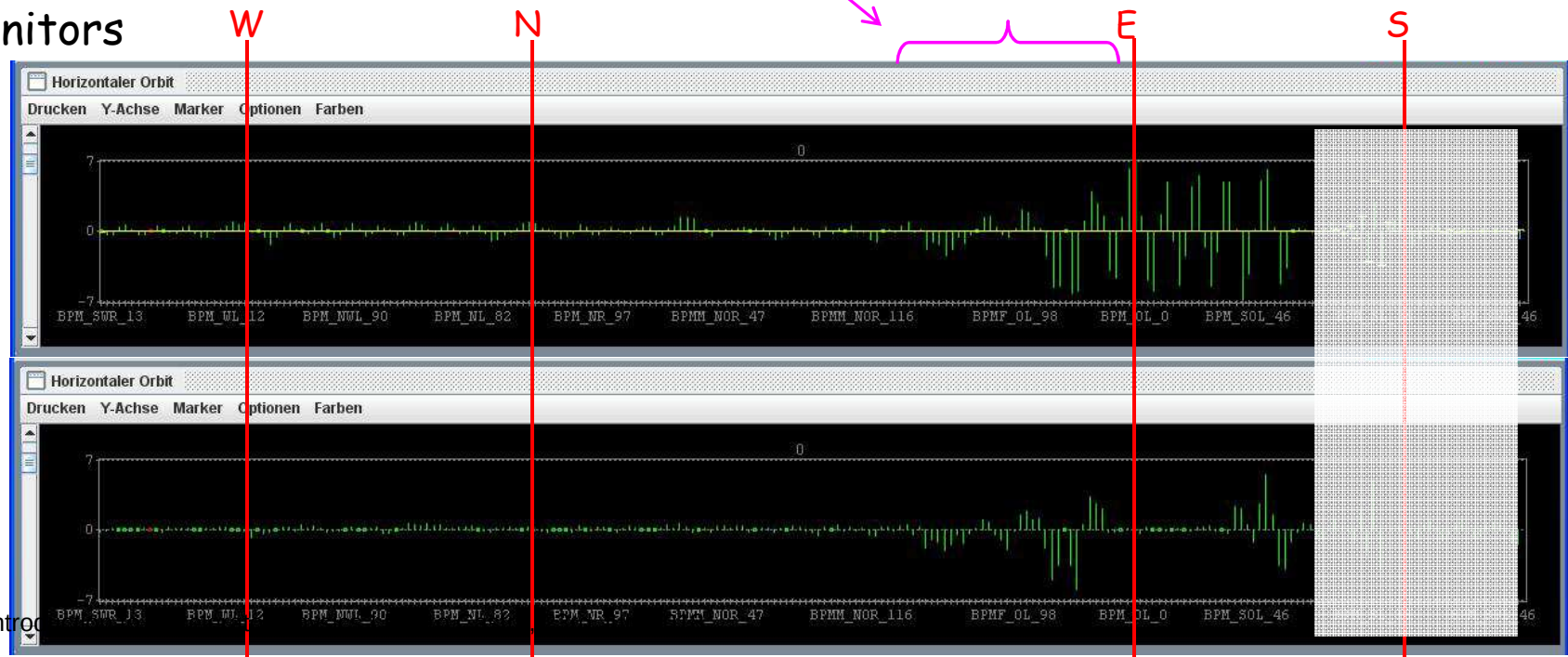
02:24 a.m.: beam lost
07:00 a.m.: visual inspection
in new octant
11:52 a.m.: start aperture scan
13:20 a.m.: beam stored

244 monitors

horizontal
beam pos.
[mm]

after
'flattening'
the orbit

DESY | Intro



horizontal aperture problem in the new octant

citation from the logbook: "the problem is at the end of the new octant"

PETRA Samstag 12. Juni 2010 Morgen

new

12.06.2010 13:56 KB, JK Das Problem ist am Ende des neuen Achtels

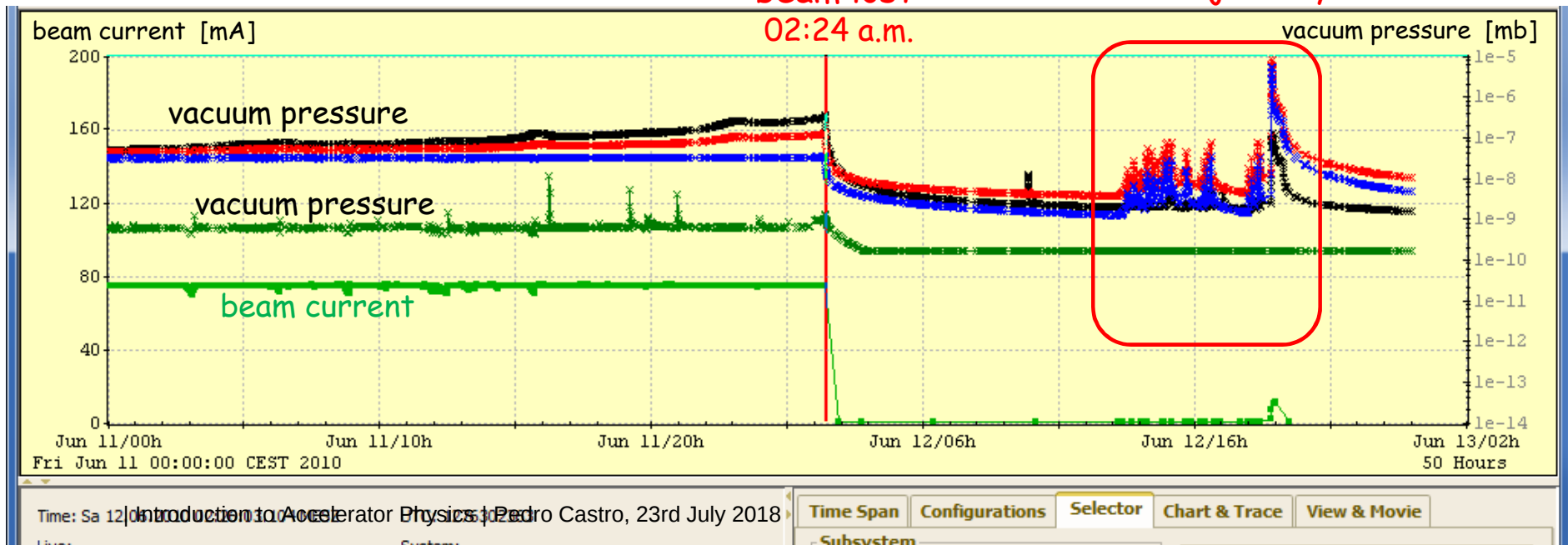
Wir korrigieren abschnittsweise den horizontalen Orbit auf den goldenen Orbit und überprüfen nach jeder Korrektur, ob sich noch Strahl injizieren lässt. Der Injektionsbereich SOR wird dabei ausgelassen.

Es bleibt eine symmetrische horizontale Beule in Zelle Nr. 8. Wird diese Beule herauskorrigiert, ist der Strahl weg und das bekannte Muster des exponentiellen Strahlverlusts ist wieder da. Das Problem liegt daher horizontal in Zelle 8.

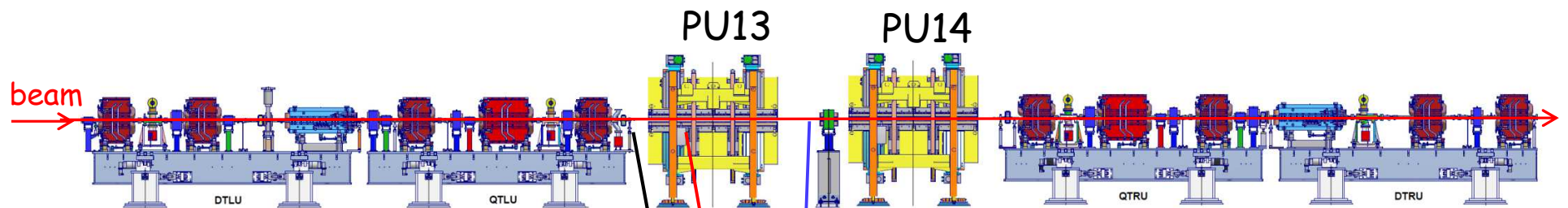
second hint: vacuum pressure raise in the new octant

beam lost
02:24 a.m.

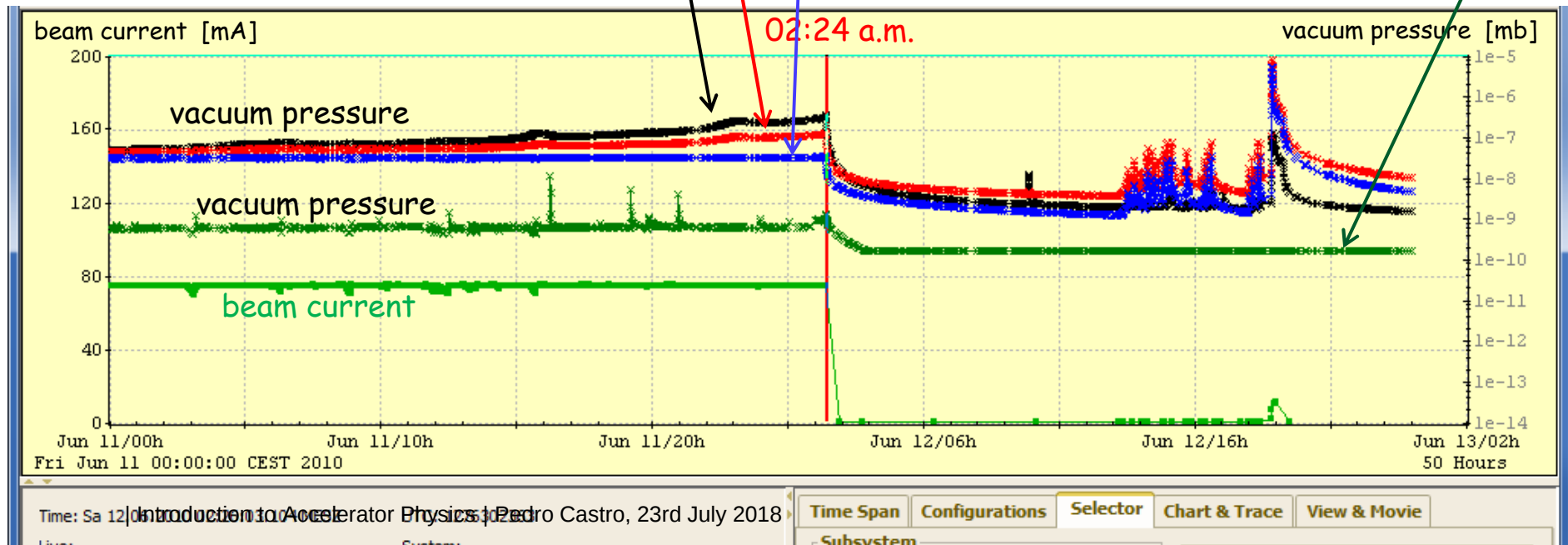
aperture scan
+ trajectory corrections



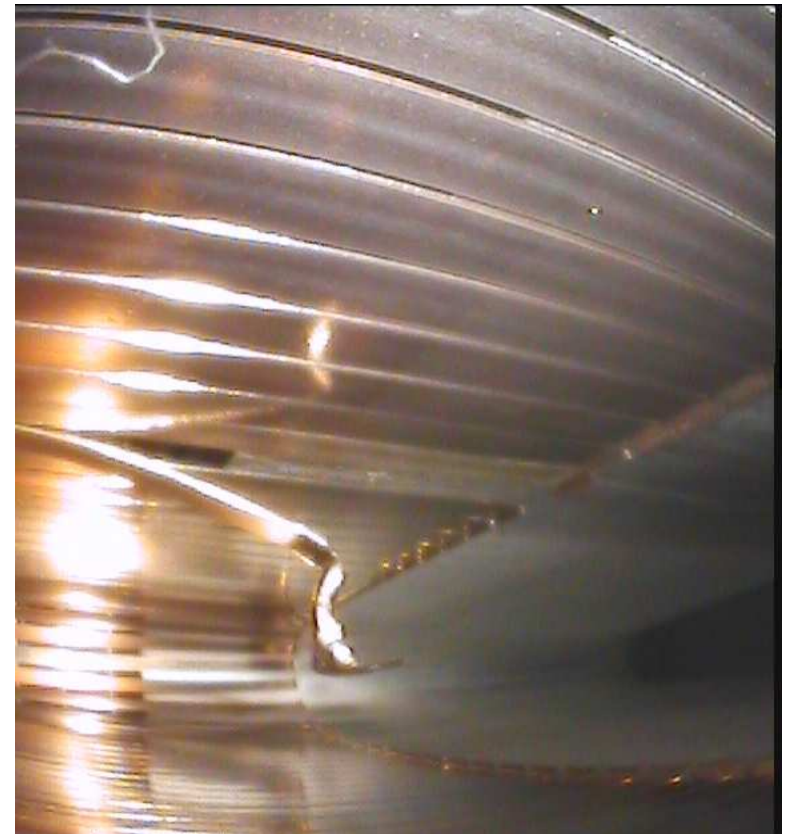
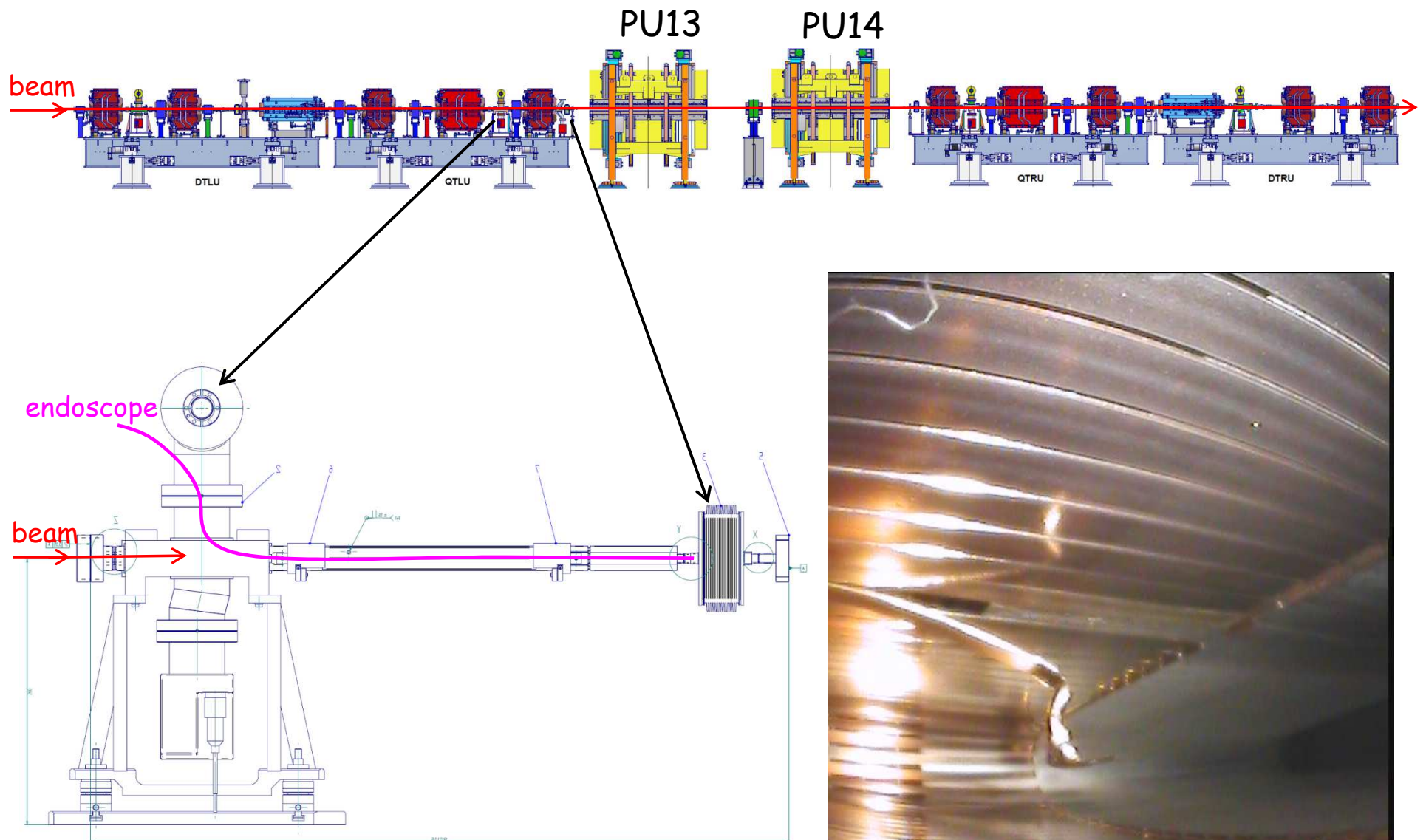
horizontal aperture problem in the new octant



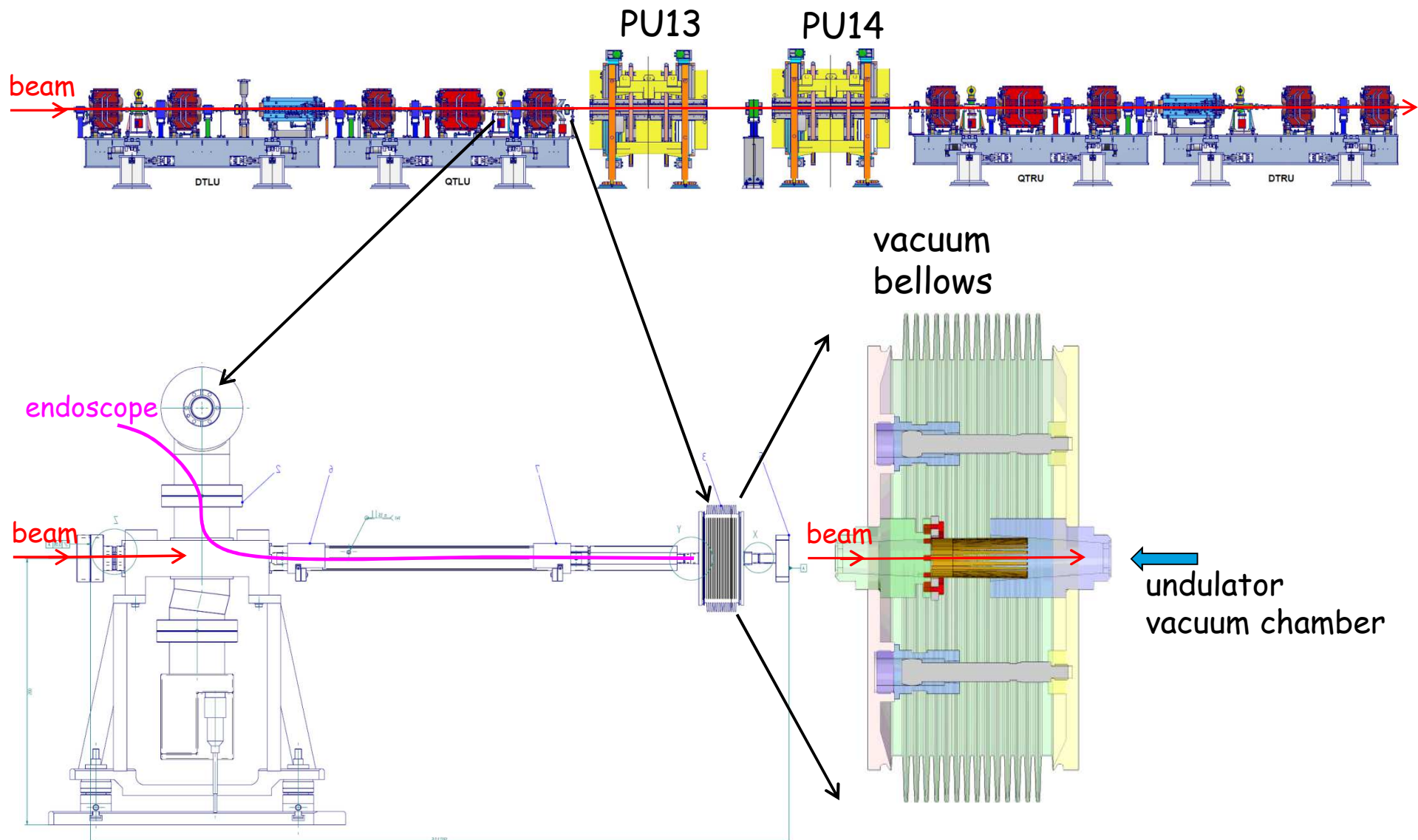
second hint: vacuum pressure raise in the new octant



visual inspection inside the vacuum chamber...



visual inspection inside the vacuum chamber...



visual inspection inside the vacuum chamber...

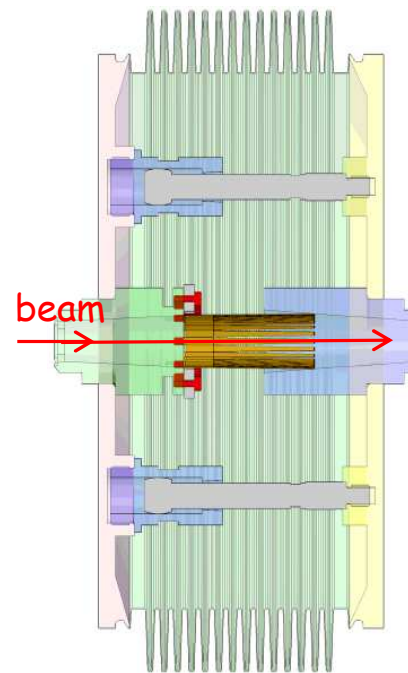
an example of vacuum bellows



dipole magnet
vacuum chamber

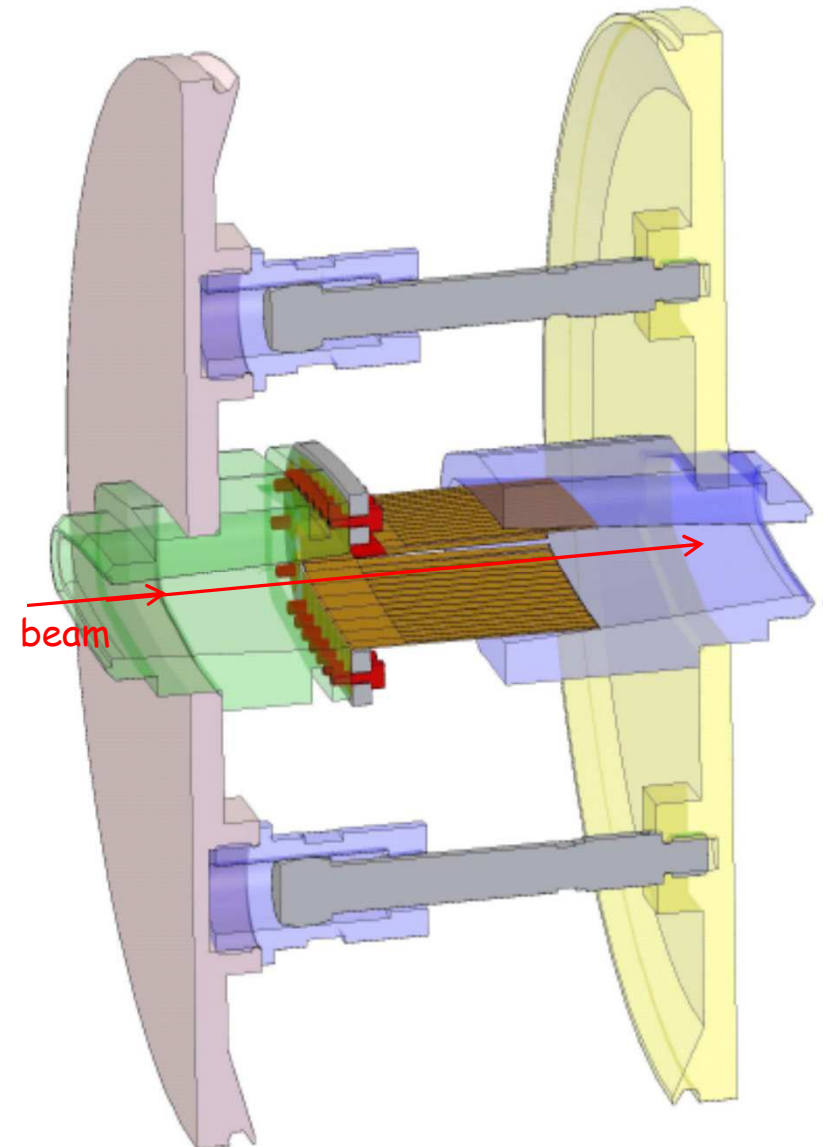
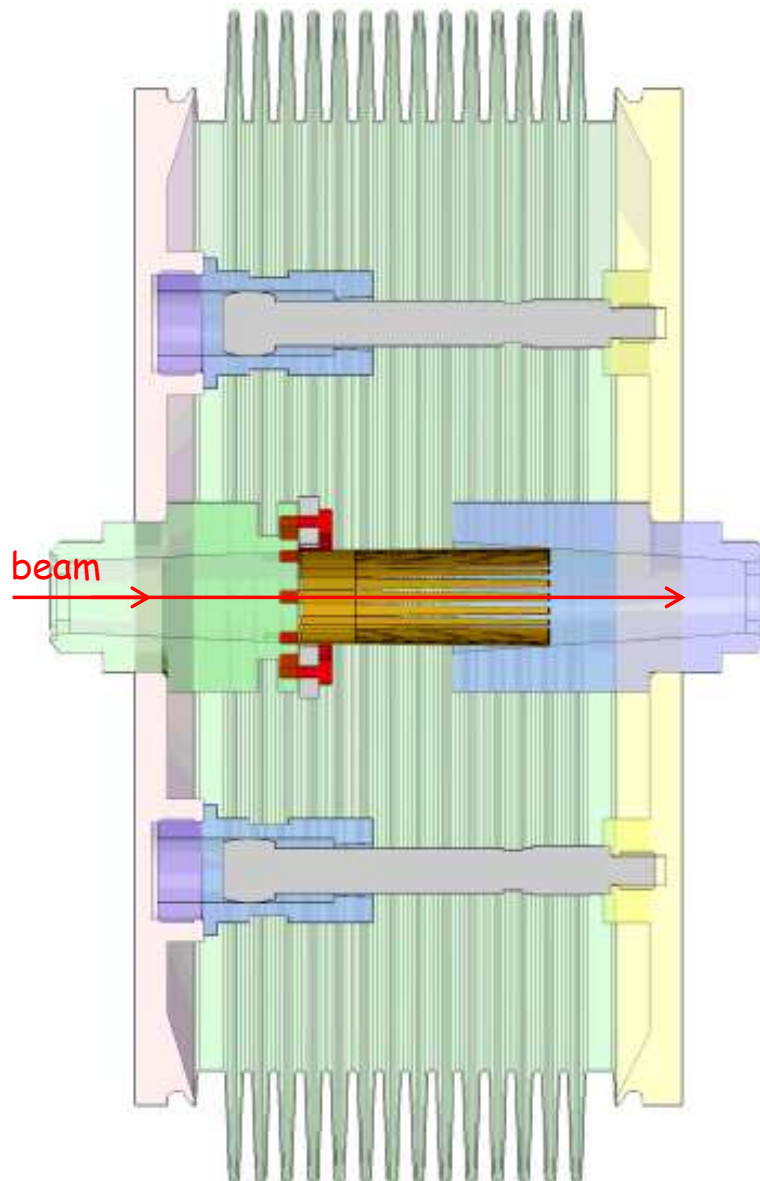


vacuum
bellows

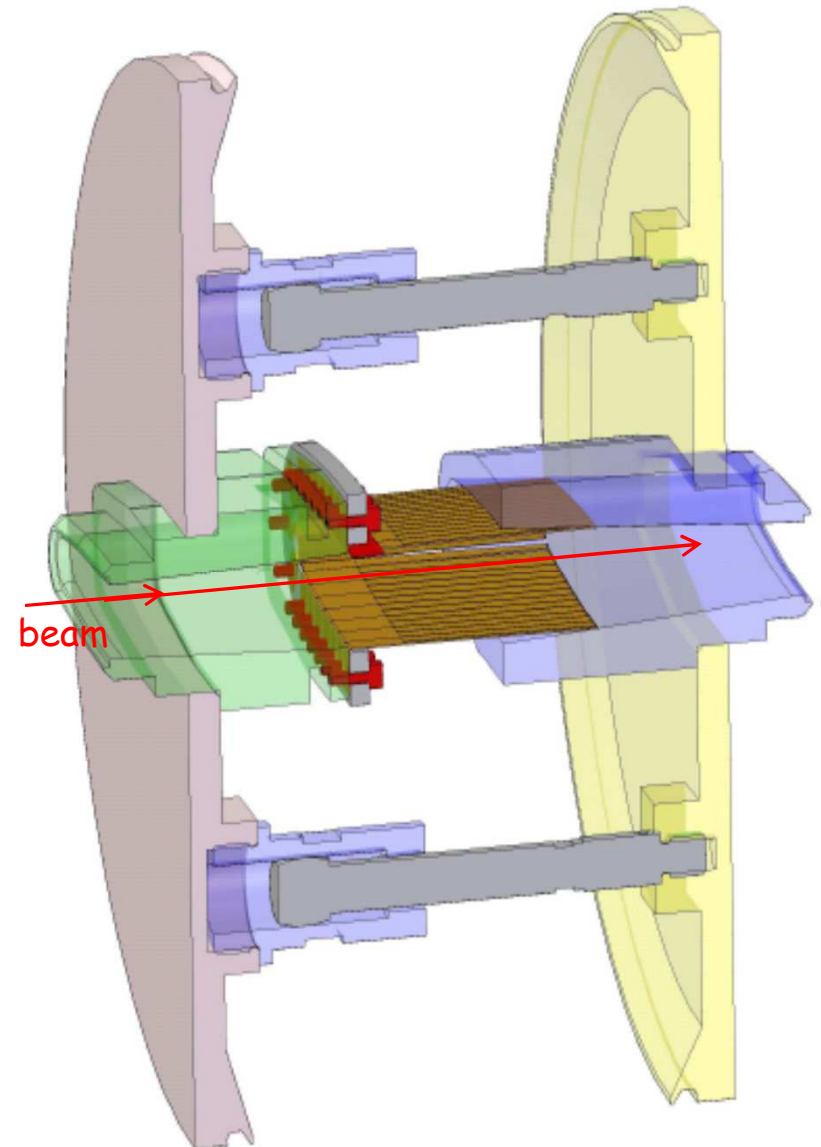


undulator
vacuum chamber

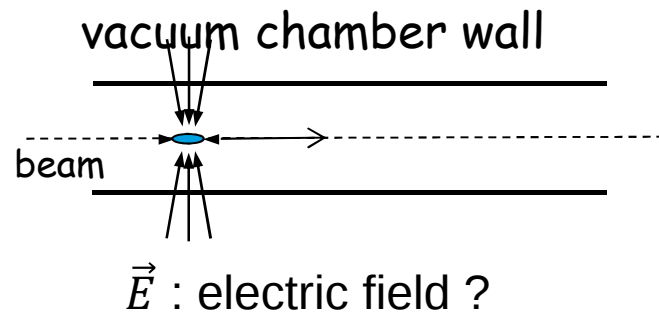
the problem was found: RF fingers



the problem was found: RF fingers

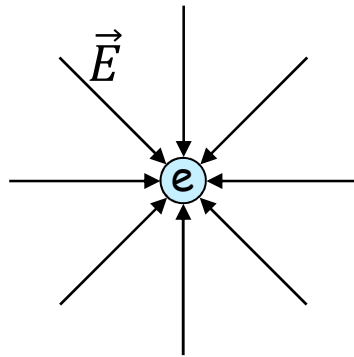


RF fingers and wakefields



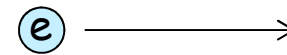
electric field of a charged particle

$$v = 0$$

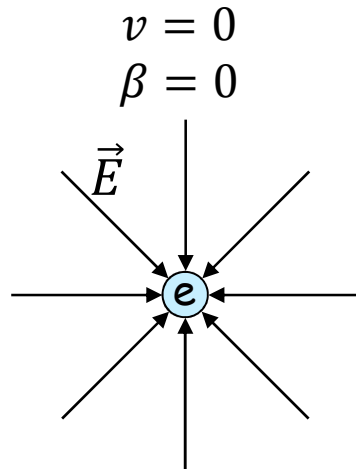


$$v = \beta c$$

$$\vec{E} \quad ?$$



electric field of a relativistic particle



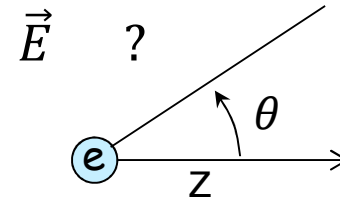
$$\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{1}{r^2} \frac{\vec{r}}{r}$$

$\gamma = 1$
 $\beta = 0$

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{(1 - \beta^2)}{(1 - \beta^2 \sin^2 \theta)^{3/2}} \frac{\vec{r}}{r^2}$$

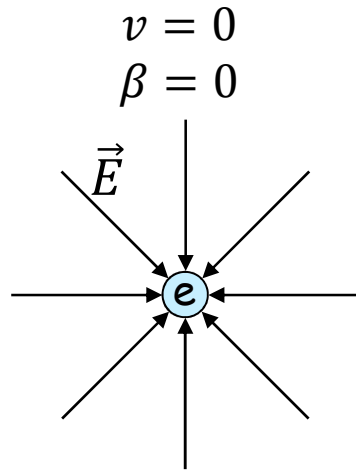
$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$v = \beta c$



$$\left\{ \begin{array}{l} E_z(\theta = 0) = \frac{q}{4\pi\epsilon_0} \frac{1}{\gamma^2 r^2} \frac{\vec{r}}{r} \\ E_r\left(\theta = \frac{\pi}{2}\right) = \frac{q}{4\pi\epsilon_0} \frac{\gamma}{r^2} \frac{\vec{r}}{r} \end{array} \right.$$

electric field of a relativistic particle

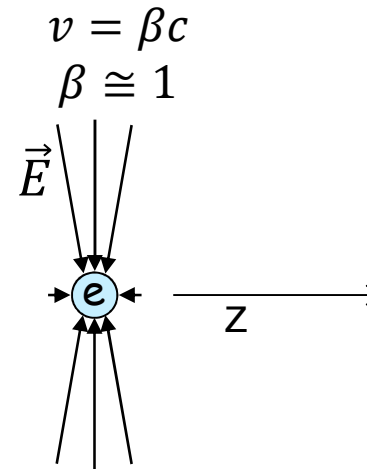


$$\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{1}{r^2} \frac{\vec{r}}{r}$$

$\gamma = 1$
 $\beta = 0$

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{(1 - \beta^2)}{(1 - \beta^2 \sin^2 \theta)^{3/2}} \frac{\vec{r}}{r^2}$$

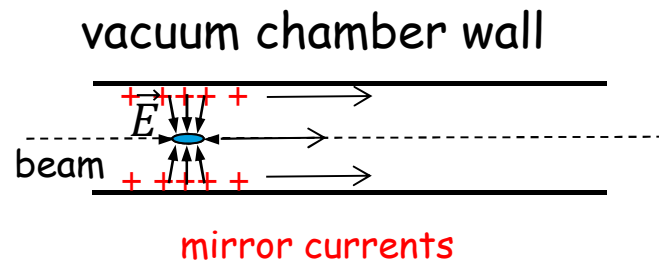
$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$



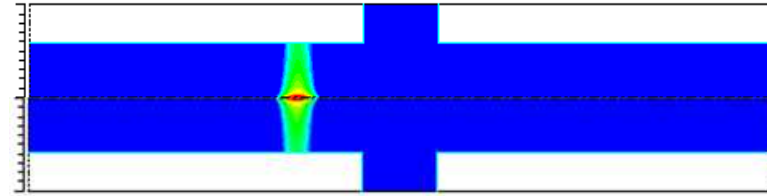
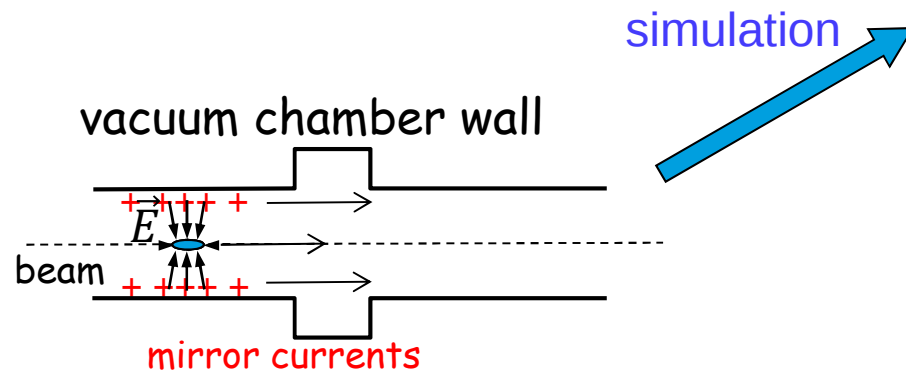
$\gamma \gg 1$
 $\beta \cong 1$

$$\left\{ \begin{array}{l} E_z(\theta = 0) = \frac{q}{4\pi\epsilon_0} \frac{1}{\gamma^2 r^2} \frac{\vec{r}}{r} \xrightarrow{\gamma \rightarrow \infty} 0 \\ E_r\left(\theta = \frac{\pi}{2}\right) = \frac{q}{4\pi\epsilon_0} \frac{\gamma}{r^2} \frac{\vec{r}}{r} \xrightarrow{\gamma \rightarrow \infty} \infty \end{array} \right.$$

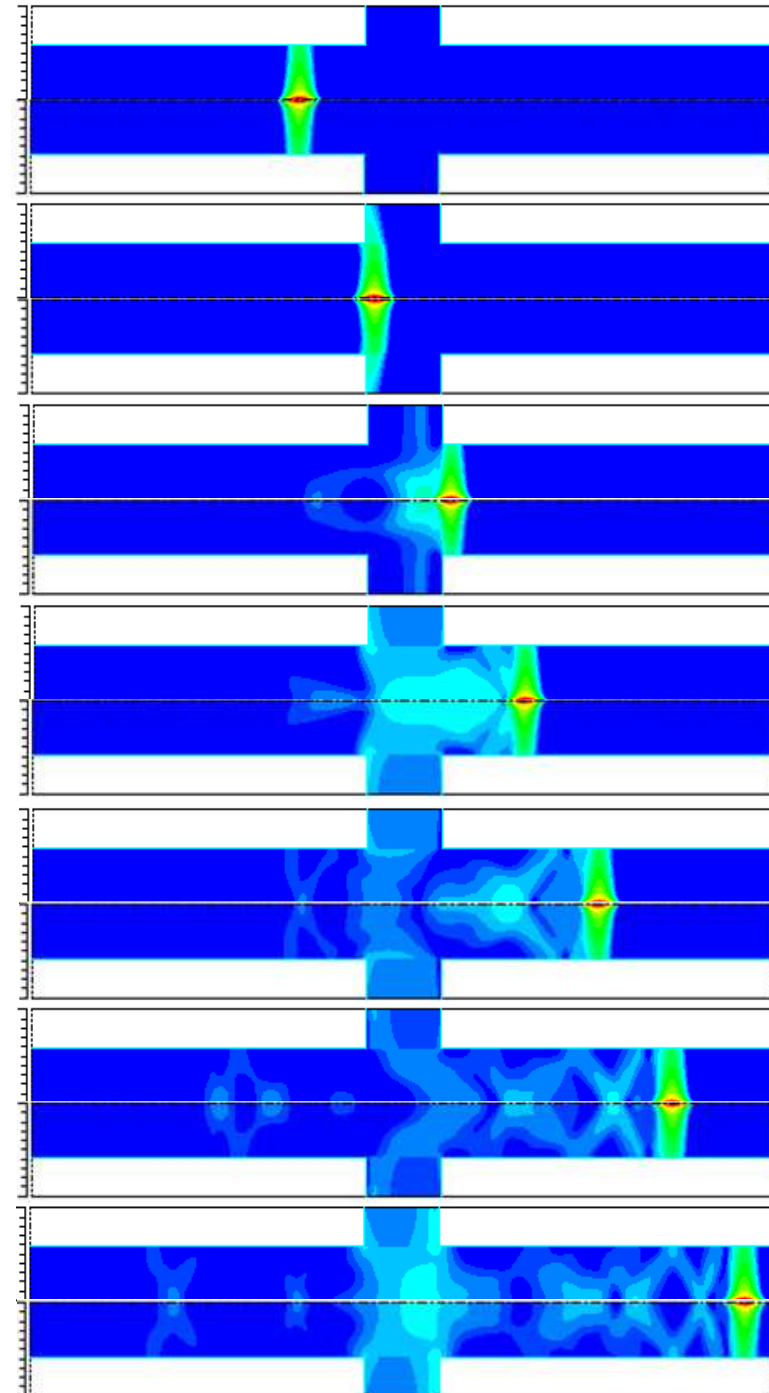
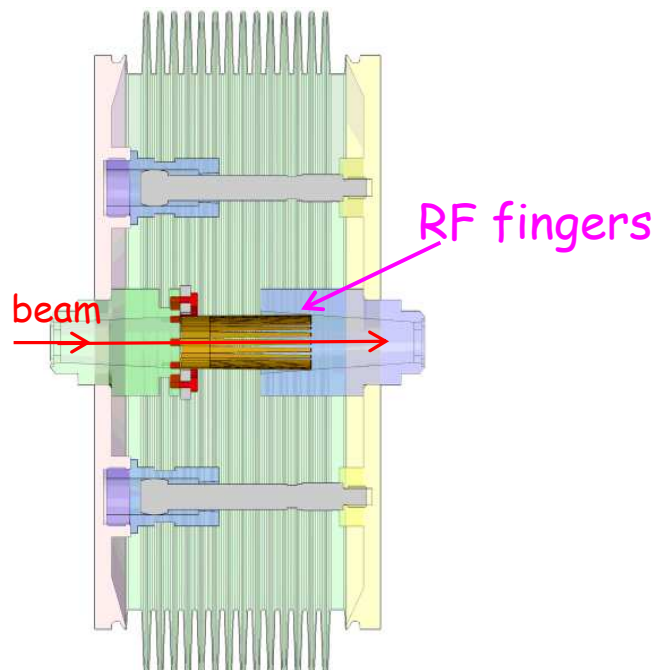
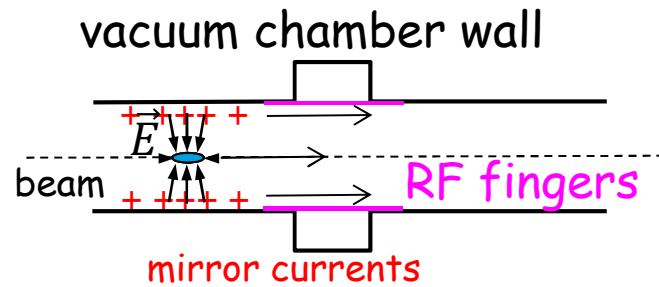
RF fingers and wakefields



RF fingers and wakefields

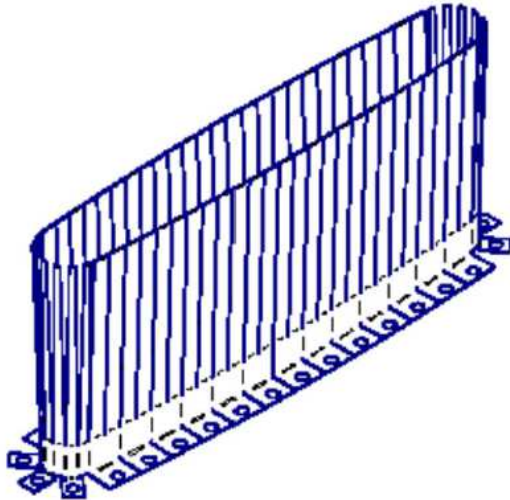


RF fingers and wakefields



RF fingers: improvements done

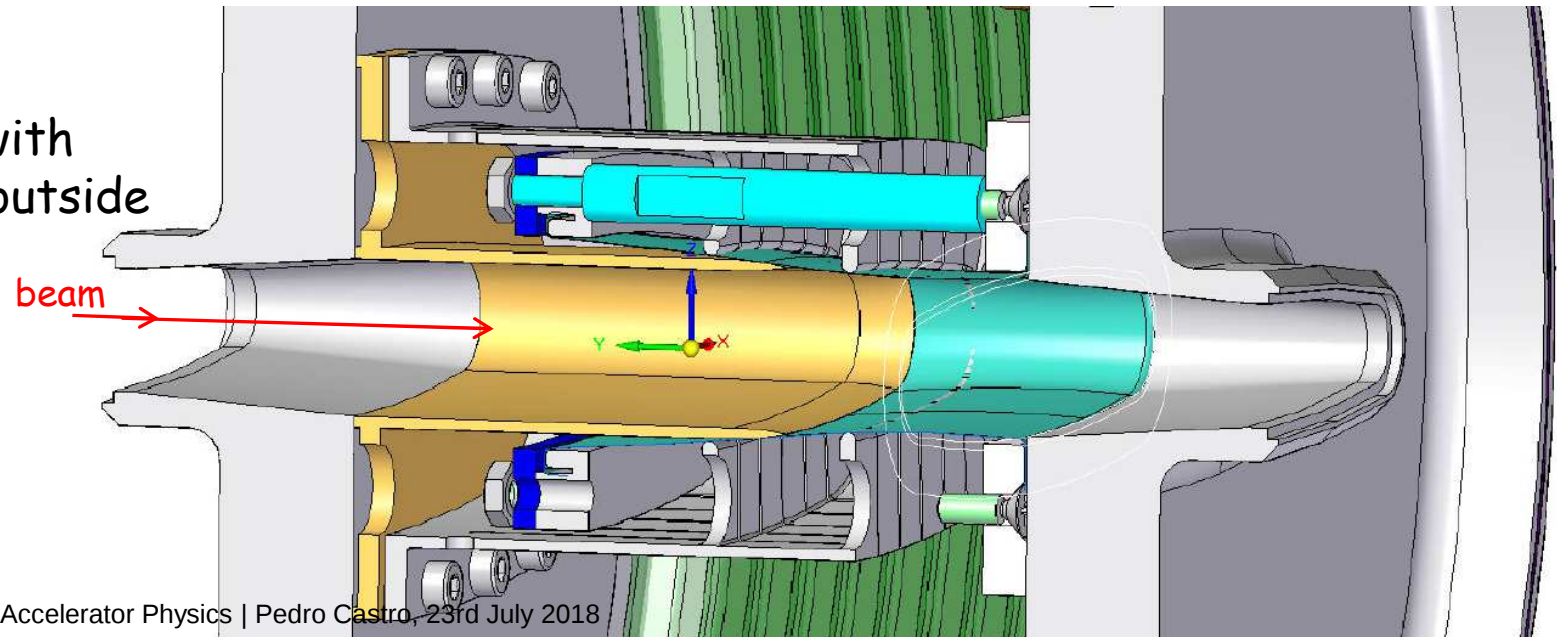
old RF fingers were tilted outwards by 2 degrees



new RF fingers have stronger tilt, more tension

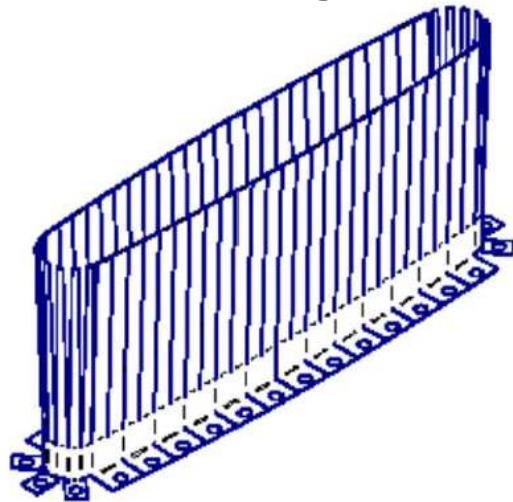


new design with
RF fingers outside



RF fingers: improvements done

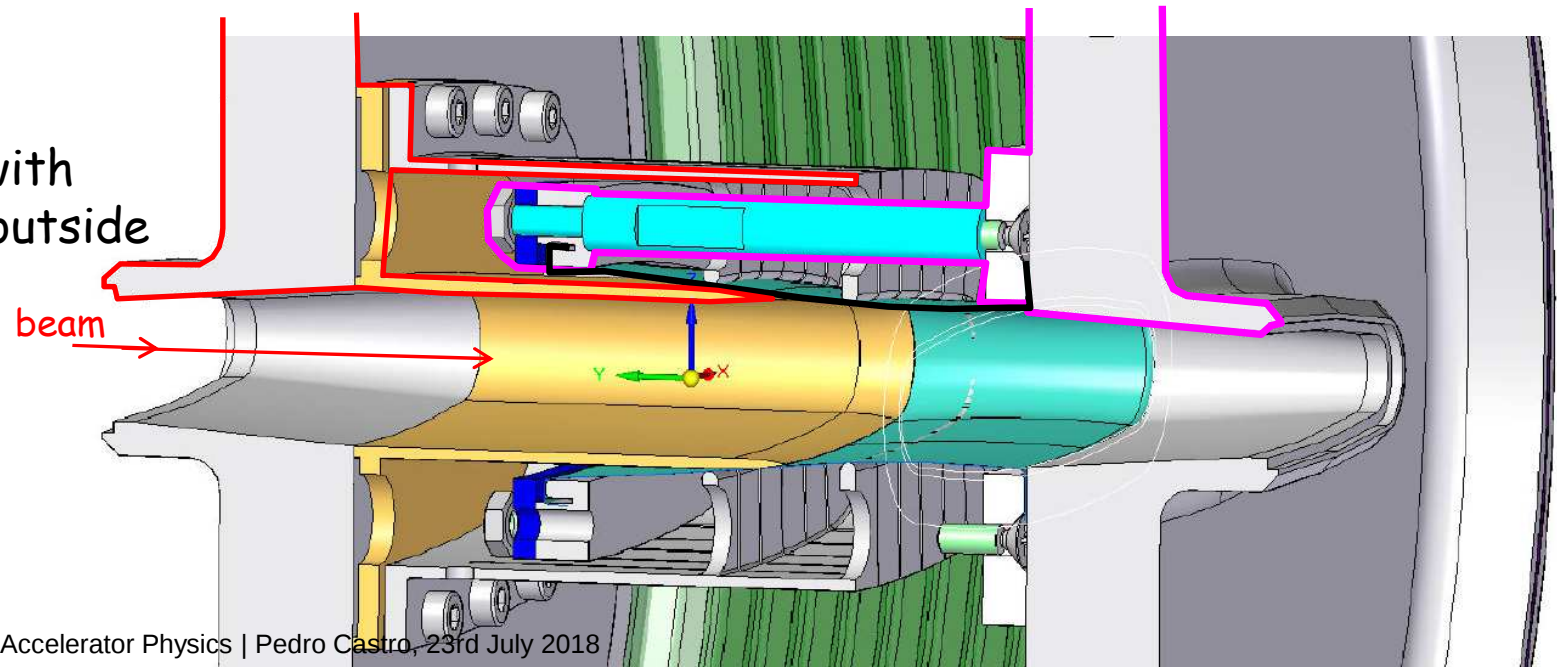
old RF fingers were tilted outwards by 2 degrees



new RF fingers have stronger tilt, more tension

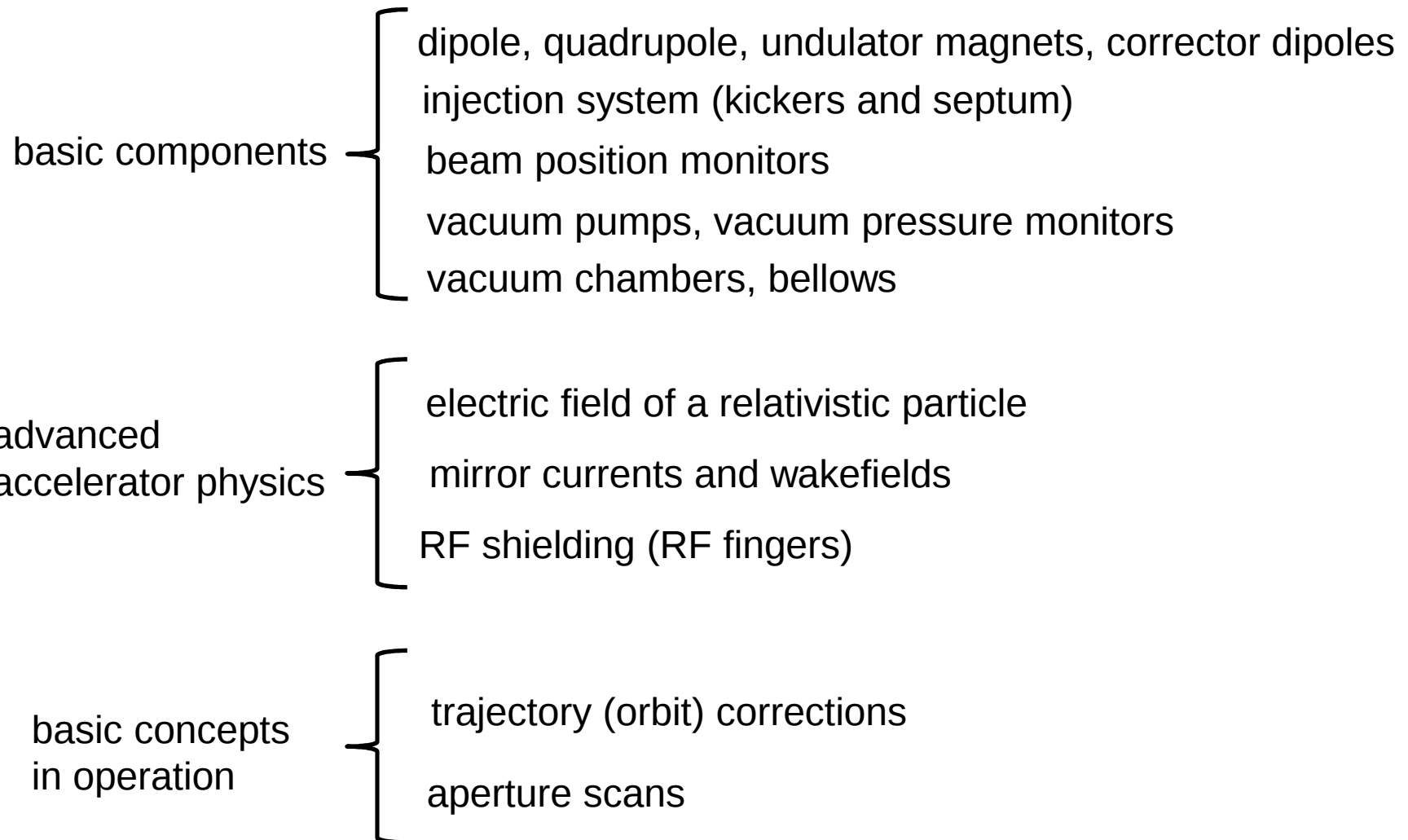
new design with
RF fingers outside

beam



Summing-up of this part

Circular accelerators: the synchrotron



Contact

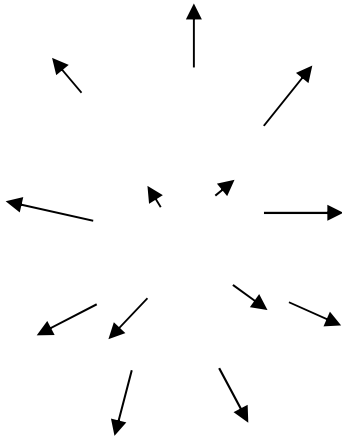
DESY. Deutsches
Elektronen-Synchrotron

www.desy.de

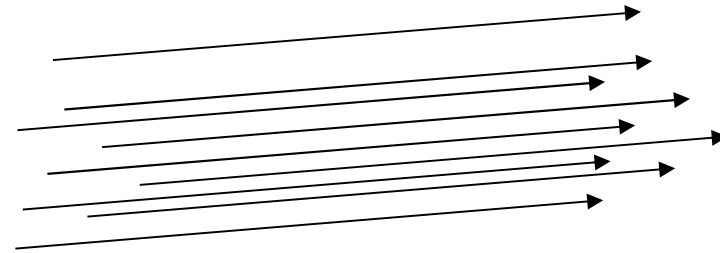
Pedro Castro
MPY
pedro.castro@desy.de

Relativity helps against Coulomb repulsion

a bunch of electrons at rest



a high-relativistic bunch of electrons



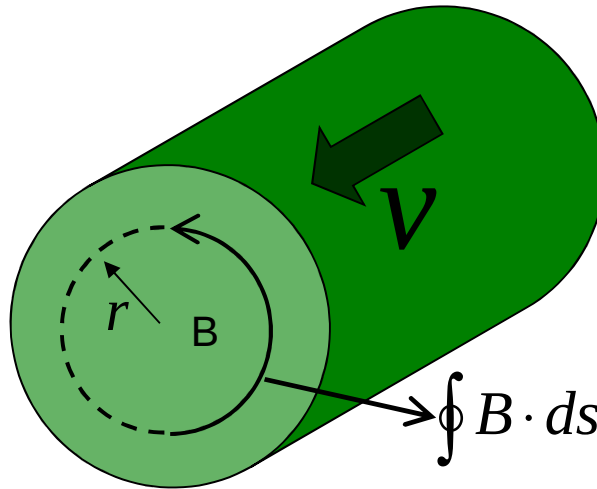
Magnetic field of a charged-particle beam

ρ = uniform charge density

Ampere's law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

current through the circle

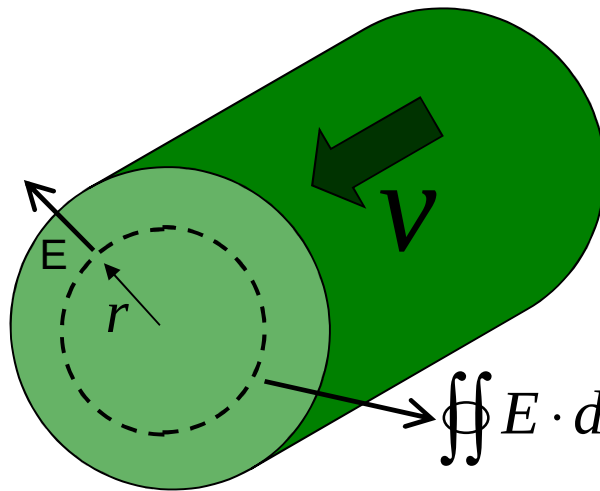


$$\oint B \cdot ds = 2\pi r B = \mu_0 \pi r^2 v \rho \rightarrow B = \frac{\mu_0 v \rho}{2} r$$

$$\left\{ \begin{array}{l} B_x = -\frac{\mu_0 v \rho}{2} r \sin \theta \\ B_y = \frac{\mu_0 v \rho}{2} r \cos \theta \end{array} \right.$$

Electric field of a charged-particle beam

ρ = uniform charge density



Gauss's law:

$$\oiint \vec{E} \cdot d\vec{a} = \frac{Q}{\epsilon_0}$$

charge inside
the volume

$$\oiint \vec{E} \cdot d\vec{a} = E 2\pi r l = \frac{\pi r^2 l \rho}{\epsilon_0} \rightarrow E = \frac{\rho}{2\epsilon_0} r$$

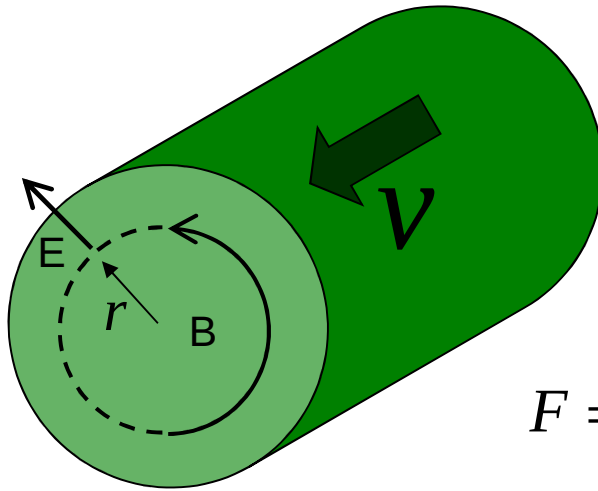
A 2D vector diagram showing a vector $\vec{r}$ at an angle θ to the horizontal axis. A green vector $\vec{E}$ points along the same direction as $\vec{r}$.

$$\left\{ \begin{array}{l} E_x = \frac{\rho}{2\epsilon_0} r \cos \theta \\ E_y = \frac{\rho}{2\epsilon_0} r \sin \theta \end{array} \right.$$

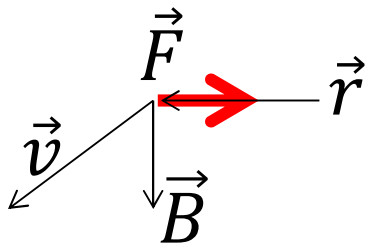
Electric and magnetic forces

$\rho = \text{uniform charge density}$

$$\vec{F} = \frac{d\vec{p}}{dt} = q (\vec{E} + \vec{v} \times \vec{B})$$



$$F = q(E - vB) = q \left(\frac{\rho}{2\epsilon_0} r - v \frac{\mu_0 v \rho}{2} r \right)$$



$$F = \frac{q \rho r}{2\epsilon_0} (1 - \epsilon_0 \mu_0 v^2) = \frac{q \rho r}{2\epsilon_0} \left(1 - \frac{v^2}{c^2} \right)$$

$$F = \frac{q \rho r}{2\epsilon_0 \gamma^2}$$