Introduction to Accelerator Physics

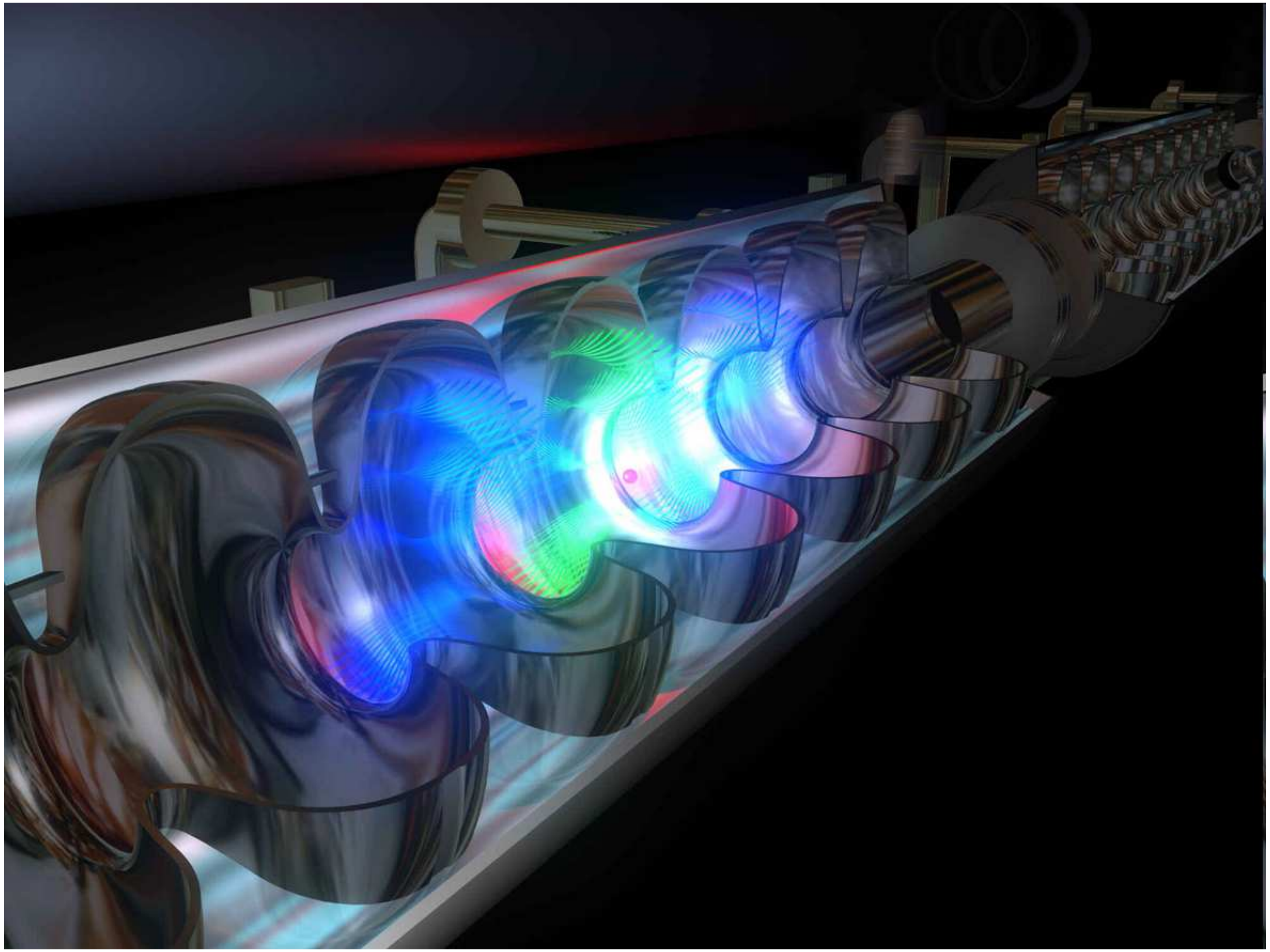
Part 3

Pedro Castro / Accelerator Physics Group (MPY)
Hamburg, 24th July 2018

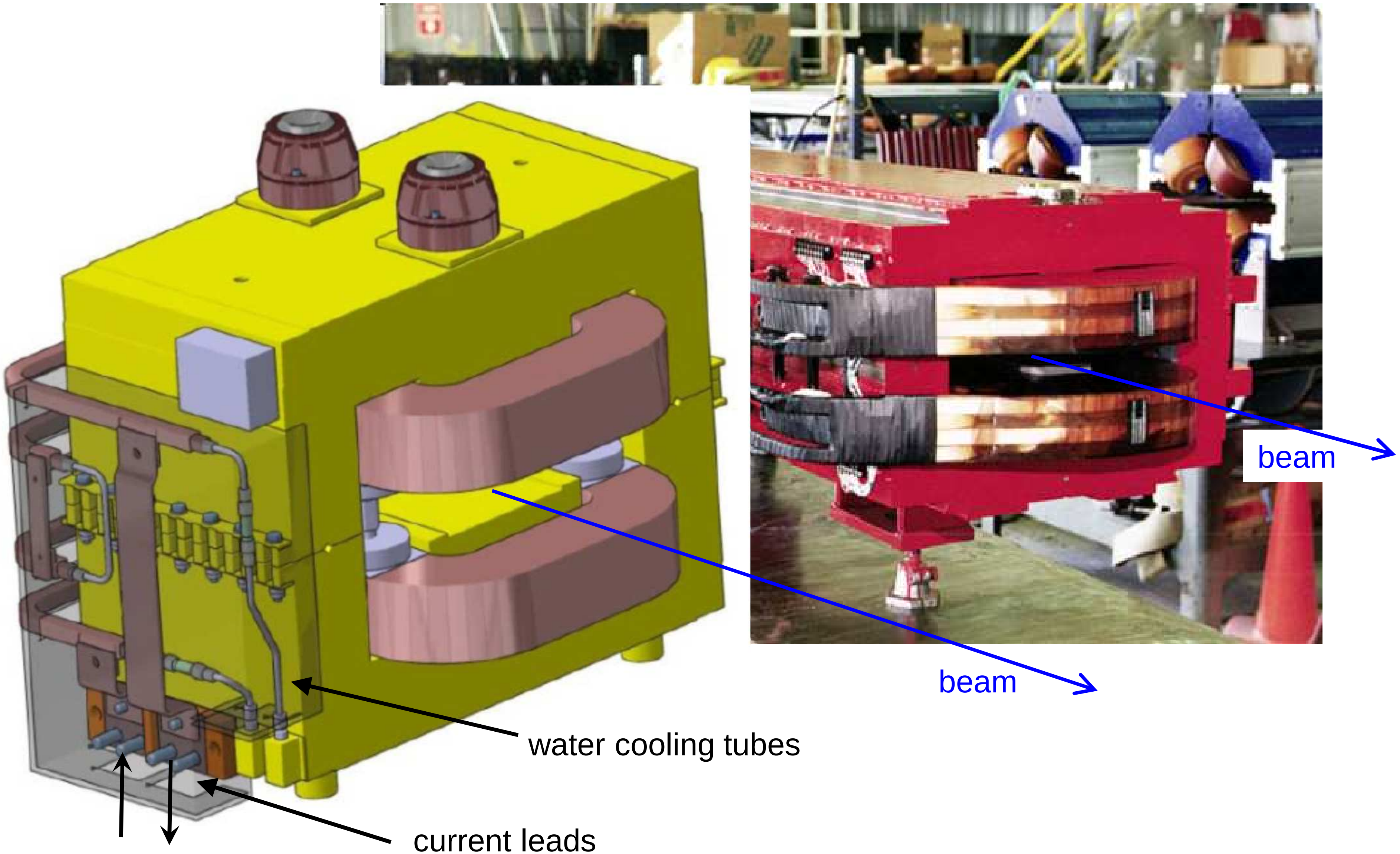
HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

DESY.

How electromagnetic fields accelerate particles



Synchrotrons: dipoles



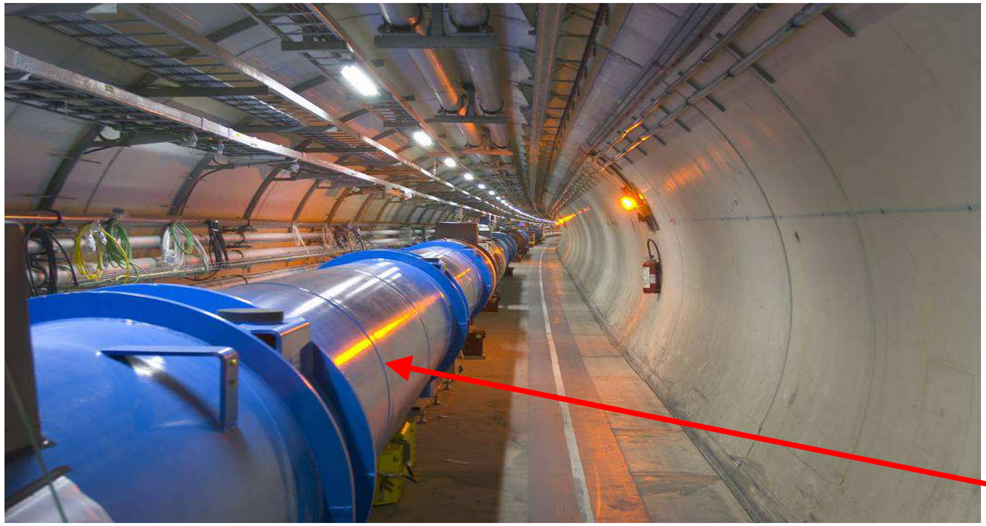
beam

beam

water cooling tubes

current leads

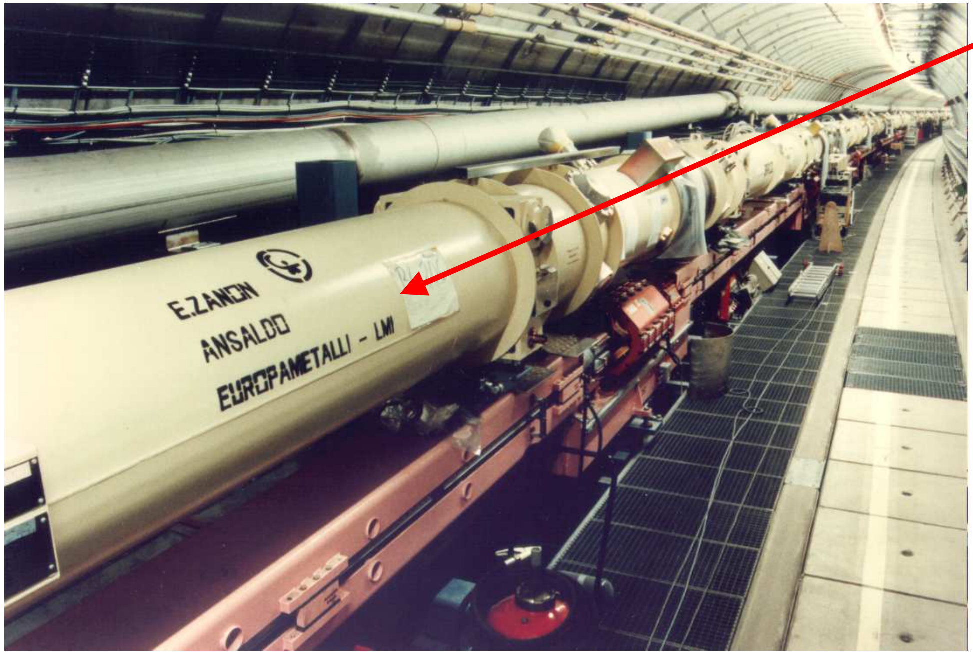
Why we need superconducting magnets



LHC: Large Hadron Collider
at CERN

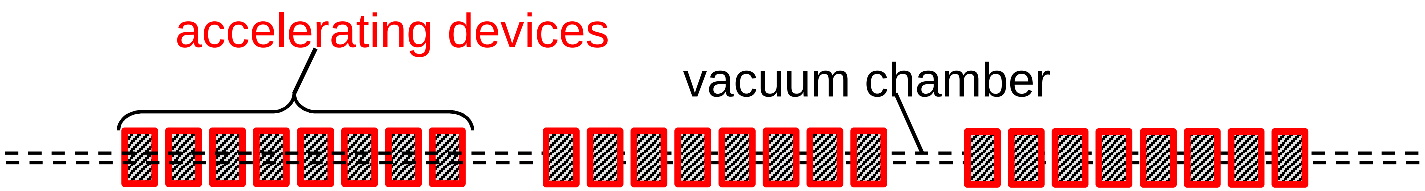
p: 7 TeV

superconducting magnets

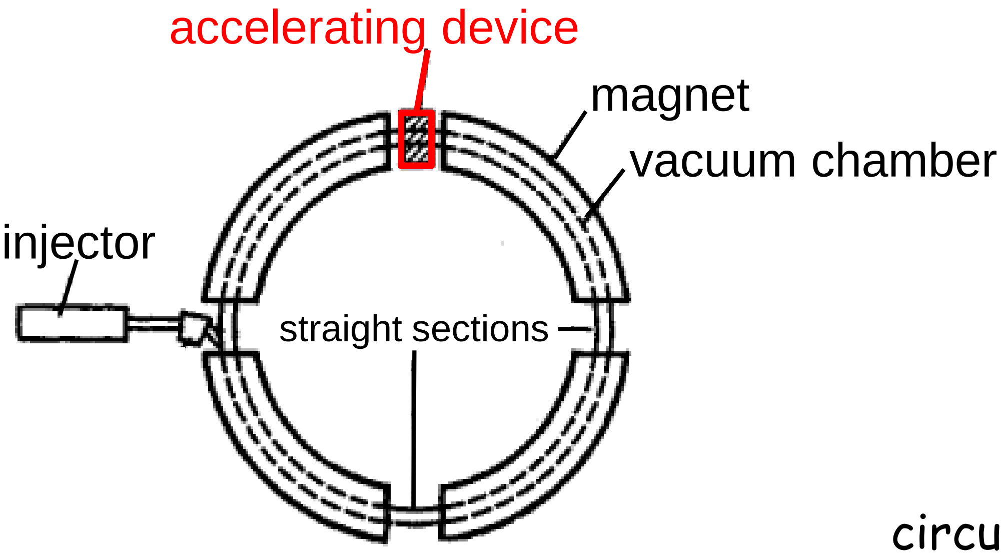


HERA: Hadron-Electron Ring Accelerator
at DESY

p: 920 GeV
e- or e+: 27.5 GeV



linear accelerator (linac)

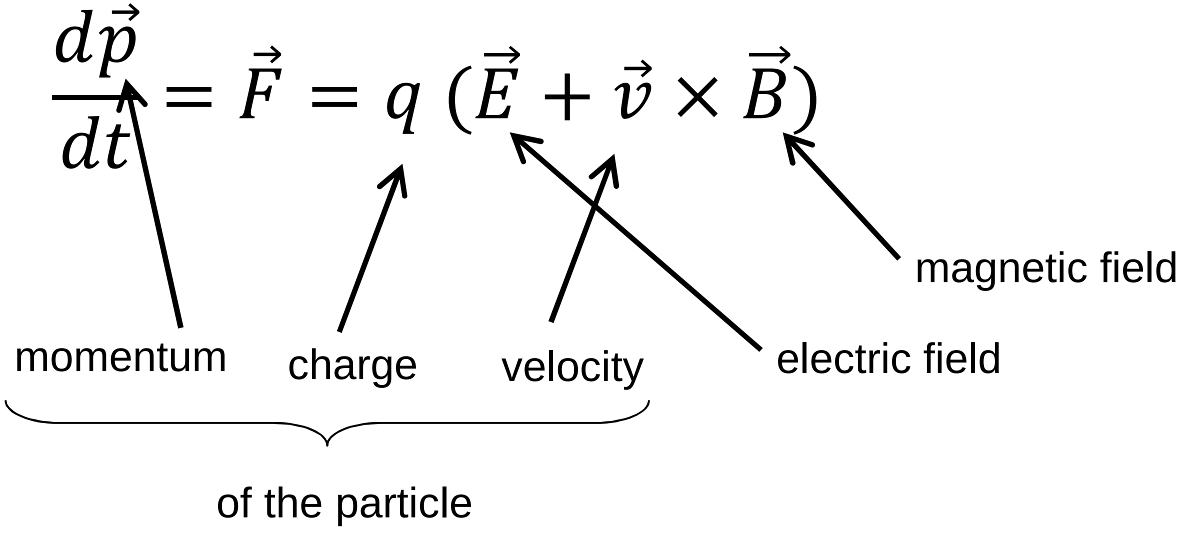


circular accelerator: synchrotron

Motion in electric and magnetic fields

Equation of motion under Lorentz Force

$$\frac{d\vec{p}}{dt} = \vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B})$$



momentum

charge

velocity

electric field

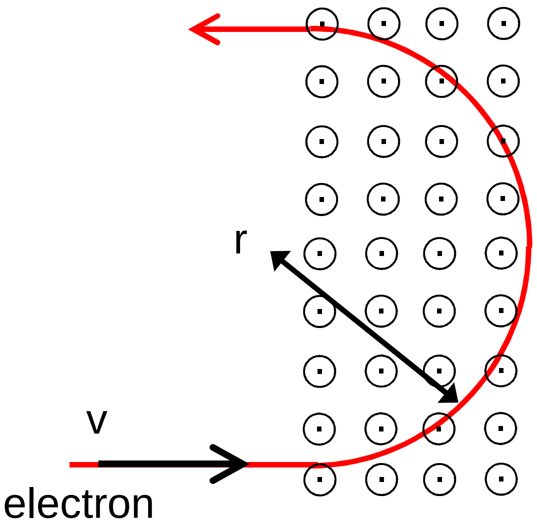
magnetic field

of the particle

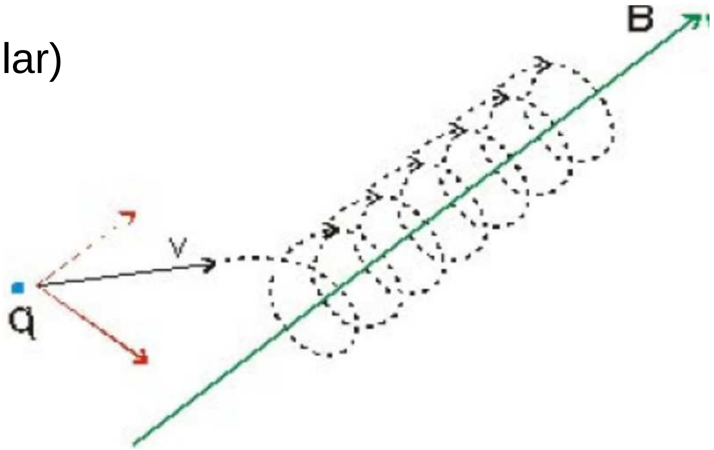
Motion in magnetic fields

if the electric field is zero ($\vec{E} = 0$), then

$$\vec{F} = \frac{d\vec{p}}{dt} = q \cdot \vec{v} \times \vec{B} \quad \rightarrow \quad \vec{F} \perp \vec{v}$$



B (perpendicular)



Magnetic fields <u>do not change</u> the particles energy, only electric fields do !

Motion in magnetic fields

if the electric field is zero (E=0), then

$$\vec{F} = \frac{d\vec{p}}{dt} = q \cdot \vec{v} \times \vec{B}$$

$$E^2 = \vec{p}^{\,2} c^2 + E_0^2$$

energy-momentum relation in special relativity

total energy

momentum

energy at rest

Motion in magnetic fields

if the electric field is zero (E=0), then

$$\vec{F} = \frac{d\vec{p}}{dt} = q \cdot \vec{v} \times \vec{B}$$

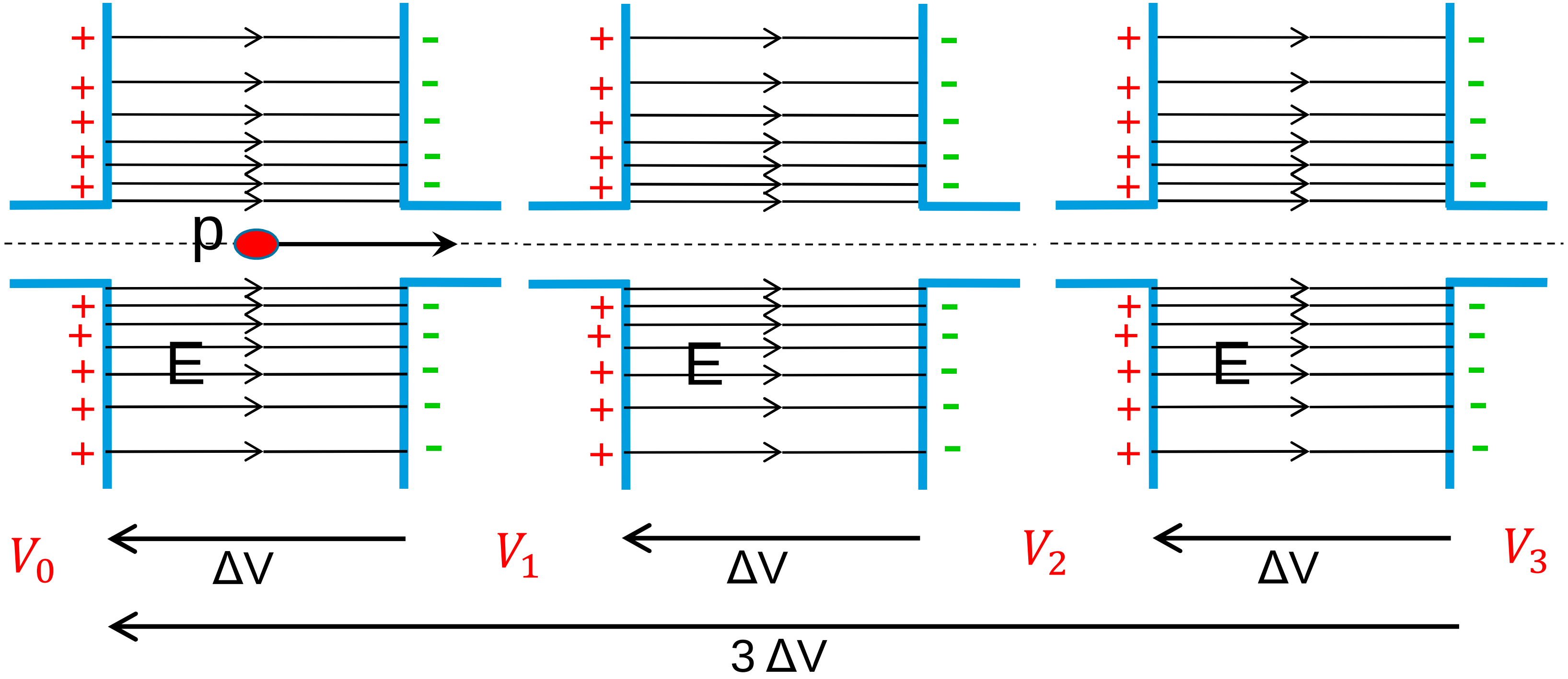
$$E^2 = \vec{p}^2 c^2 + E_0^2$$

$$E\frac{dE}{dt} = c^2 \vec{p}\frac{d\vec{p}}{dt} = c^2 q \vec{p}\left(\vec{v} \times \vec{B}\right) = c^2 q |\vec{p}| \left|\vec{v} \times \vec{B}\right| \cos \emptyset = 0$$

since $\vec{v} \times \vec{B} \quad \perp \quad \vec{v}$ ➔ $\emptyset = 90°$

Magnetic fields do not change the particles energy, only electric fields do !

acceleration with DC electric fields

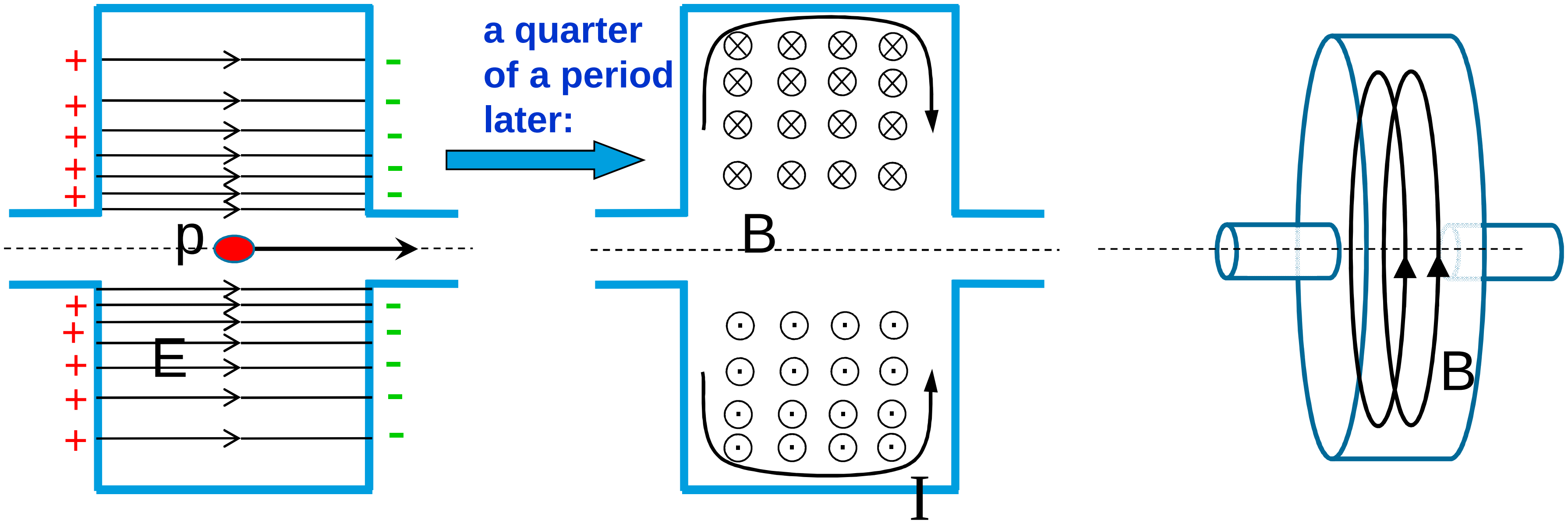


In general:

- Static magnetic fields → to guide (bend + focus) particle beams

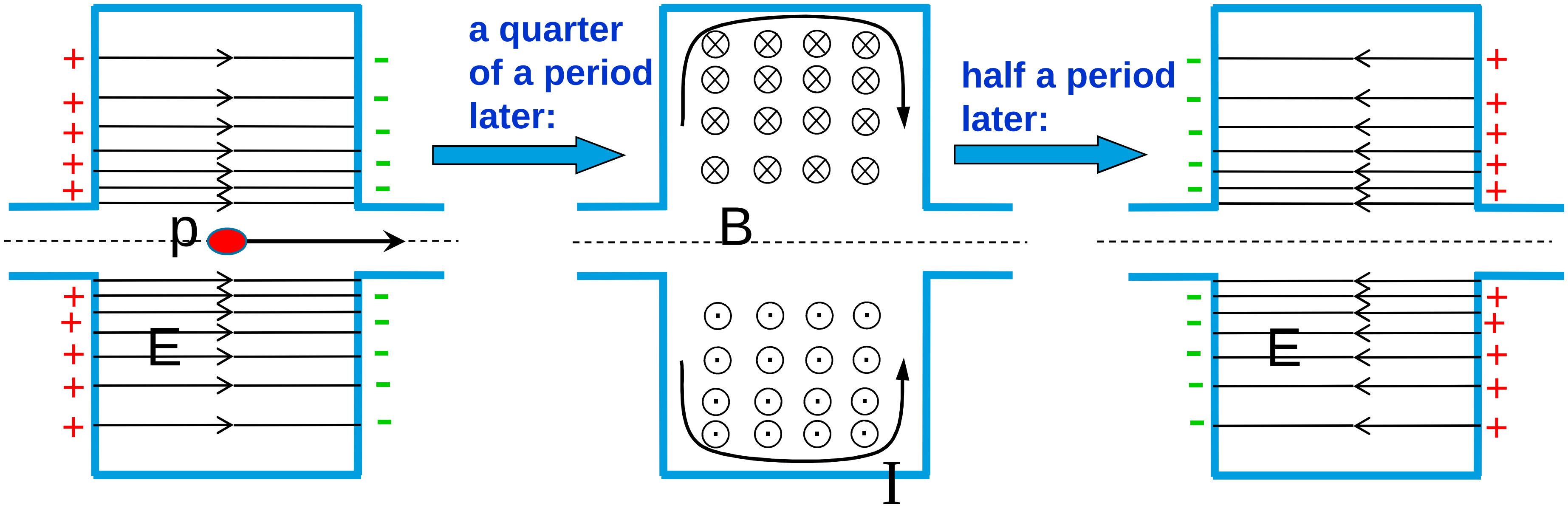
- Static electric fields → accelerate particle beams (low energy)
- Radio-frequency EM fields → accelerate particle beams (high E)

RF cavity basics: the pill box cavity



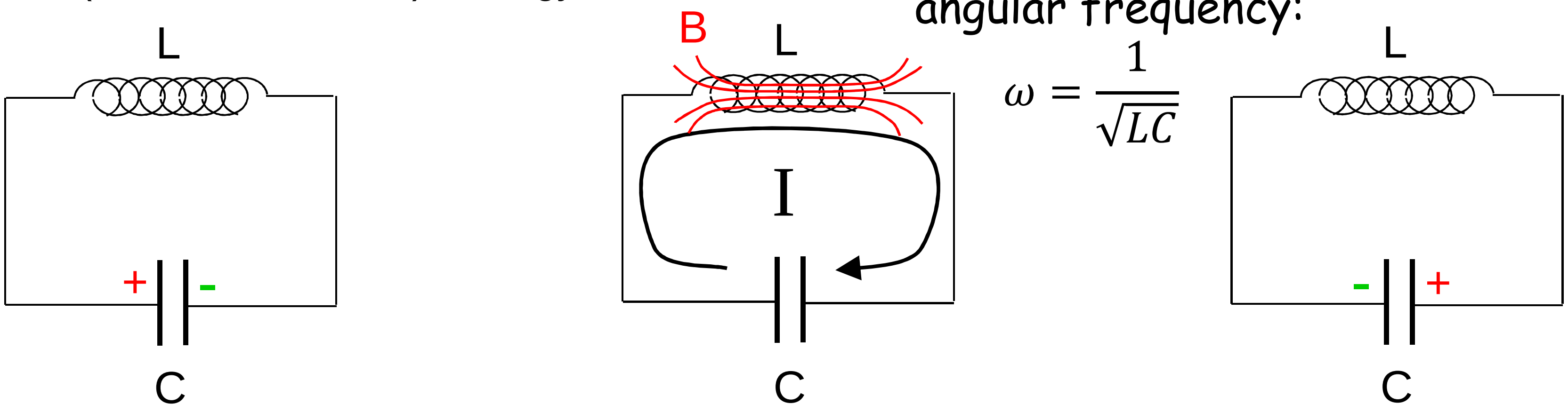
a quarter of a period later:

RF cavity basics: the pill box cavity


a quarter of a period later:

half a period later:

LC circuit (or resonant circuit) analogy:



angular frequency:

$$\omega = \frac{1}{\sqrt{LC}}$$

Maxwell's equations

(differential formulation in SI units)

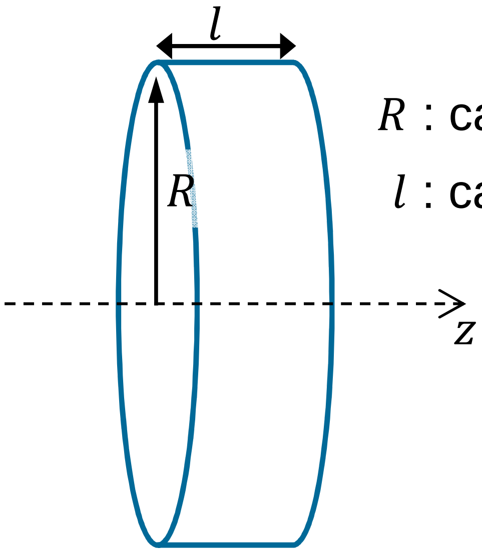
$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

+ boundary conditions



R : cavity radius

l : cavity length

TM modes
(transverse magnetic modes)

set of solutions with $B_z = 0$ (that is, $\vec{B}$ is transverse)

~~set of solutions with $E_z = 0$ (that is, $\vec{E}$ is transverse)~~

~~**TE modes**~~
~~**(transverse electric modes)**~~

Maxwell's equations

(differential formulation in SI units)

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

+ boundary conditions

R : cavity radius

l : cavity length

set of solutions with $B_z = 0$ (that is, $\vec{B}$ is transverse)

$$\begin{cases} E_z = E_0 J_m\left(x_{mn}\frac{r}{R}\right)\cos m\theta\cos\left(\frac{p\pi}{l}z\right)e^{j\omega t} \\ E_r = -\frac{p\pi}{l}\frac{R}{x_{mn}}E_0 J'_m\left(x_{mn}\frac{r}{R}\right)\cos m\theta\sin\left(\frac{p\pi}{l}z\right)e^{j\omega t} \\ E_\theta = -\frac{p\pi}{l}\frac{mR^2}{x_{mn}{}^2 r}E_0 J_m\left(x_{mn}\frac{r}{R}\right)\sin m\theta\sin\left(\frac{p\pi}{l}z\right)e^{j\omega t} \\ B_z = 0 \\ B_r = -j\omega\frac{mR^2}{x_{mn}{}^2 r c^2}E_0 J_m\left(x_{mn}\frac{r}{R}\right)\sin m\theta\cos\left(\frac{p\pi}{l}z\right)e^{j\omega t} \\ B_\theta = -j\omega\frac{R}{x_{mn}c^2}E_0 J'_m\left(x_{mn}\frac{r}{R}\right)\cos m\theta\cos\left(\frac{p\pi}{l}z\right)e^{j\omega t} \end{cases}$$

indices:

$m = 0,1,2,\ldots$: number of full period variations in θ of the fields

$n = 1,2,\ldots$: number of zeros of the axial field component in $\vec{r}$

$p = 0,1,2,\ldots$: number of half period variations in z of the fields

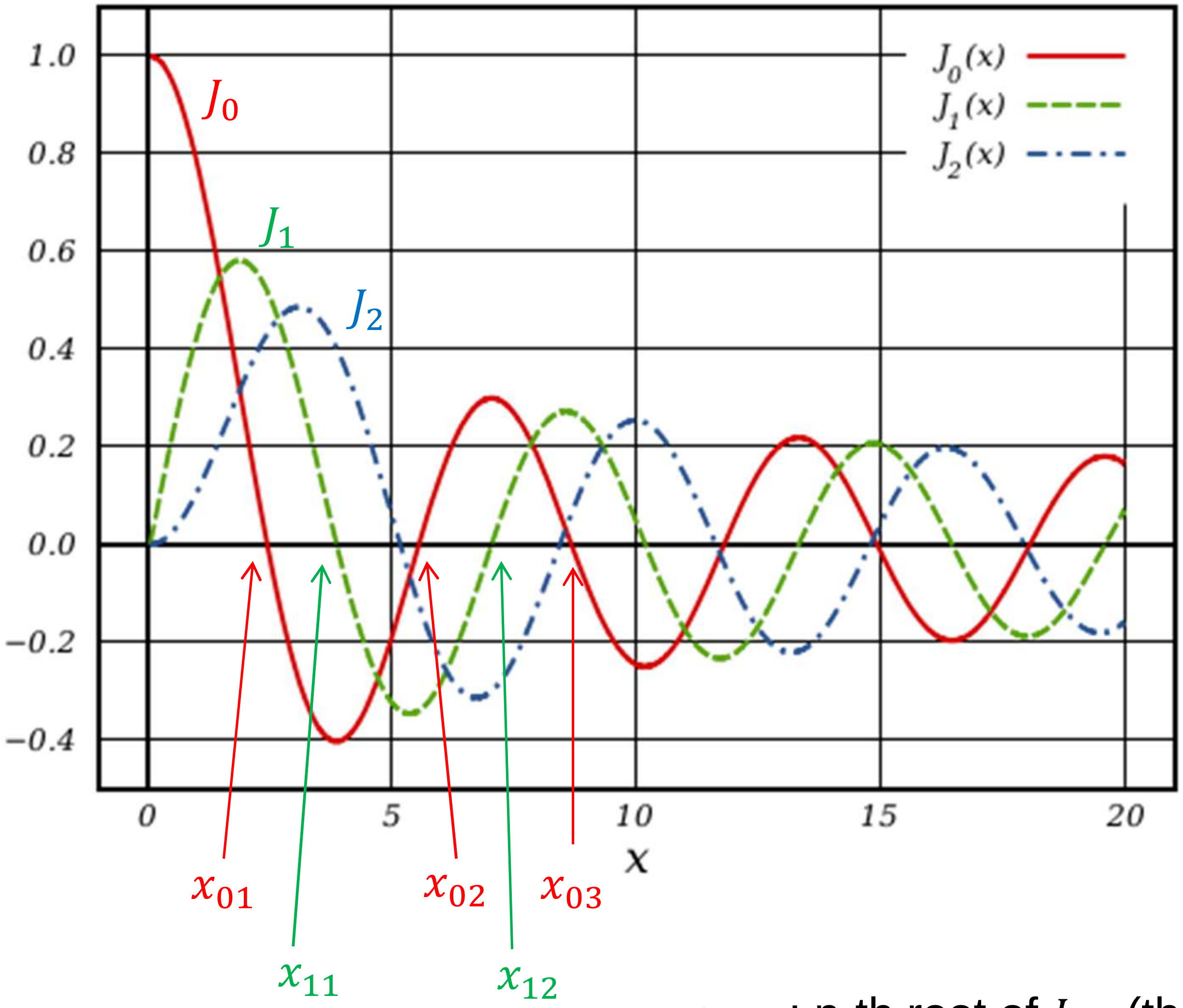
J_m : Bessel´s functions

J'_m : derivative of the Bessel´s functions

x_{mn} : n-th root of J_m (that is, $J_m(x_{mn}) = 0$)

angular frequency : $\omega = c\sqrt{\left(\frac{x_{mn}}{R}\right)^2 + \left(\frac{p\pi}{l}\right)^2}$

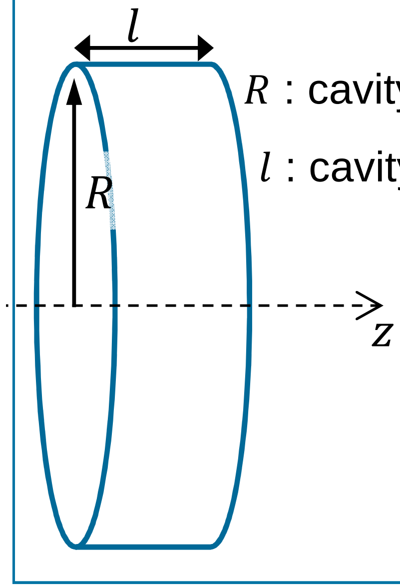
J_m : Bessel´s functions



m	x_{m1}	x_{m2}	x_{m3}
0	2.405	5.520	8.654
1	3.832	7.016	10.173
2	5.136	8.417	11.620

x_{mn} : n-th root of J_m (that is, $J_m(x_{mn}) = 0$)

boundary conditions



R : cavity radius

l : cavity length

fundamental solution with $B_z = 0$ (that is, $\vec{B}$ is transverse)

$$E_z = E_0 J_0\left(x_{01}\frac{r}{R}\right) e^{j\omega t}$$

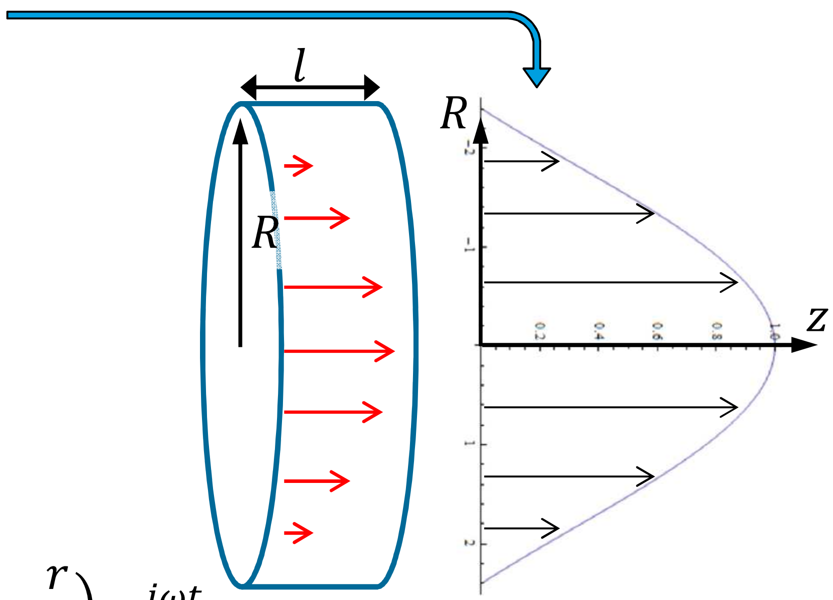
$$E_r = 0$$

$$E_\theta = 0$$

$$B_z = 0$$

$$B_r = 0$$

$$B_\theta = j\omega\,\frac{R}{x_{01}c^2}\,E_0 J_1\left(x_{01}\frac{r}{R}\right) e^{j\omega t}$$



$m = 0$: rotation symmetry of the fields

$n = 1$: no zeros of the axial field component in $\vec{r}$

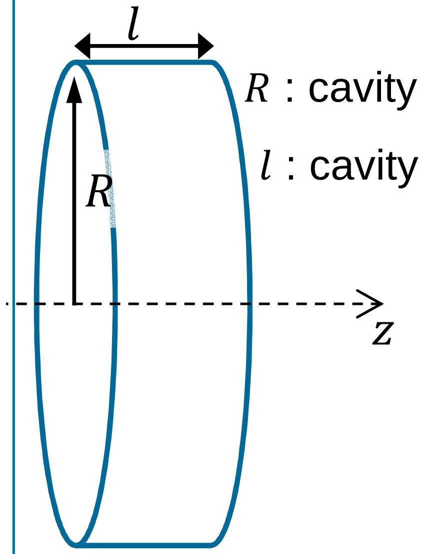
$p = 0$: no variation in z of the fields

J_m : Bessel´s functions

J'_m : derivative of the Bessel´s functions

angular frequency : $\omega = c\,\dfrac{x_{01}}{R}$ $x_{01} = 2.405$

boundary conditions



R : cavity radius

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fundamental solution with $B_z = 0$ (that is, $\vec{B}$ is transverse)

$$E_z = E_0 J_0\left(x_{01}\frac{r}{R}\right) e^{j\omega t}$$

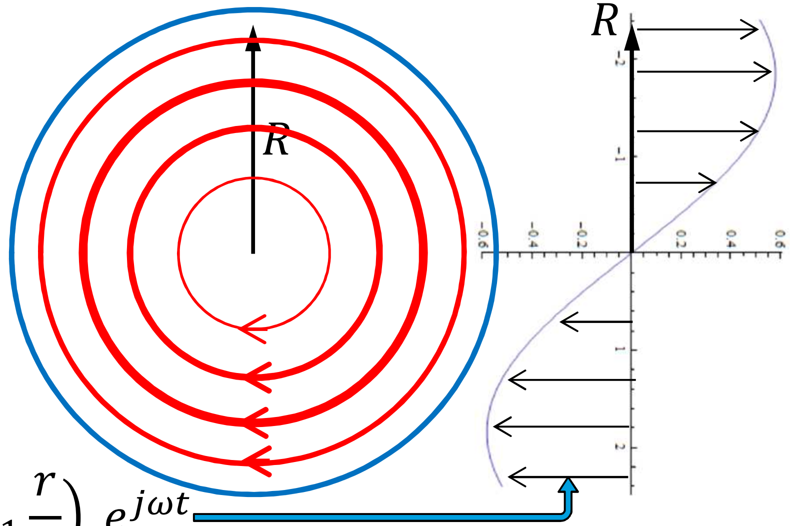
$$E_r = 0$$

$$E_\theta = 0$$

$$B_z = 0$$

$$B_r = 0$$

$$B_\theta = j\omega\,\frac{R}{x_{01}c^2}\,E_0 J_1\left(x_{01}\frac{r}{R}\right) e^{j\omega t}$$



$m = 0$: rotation symmetry of the fields

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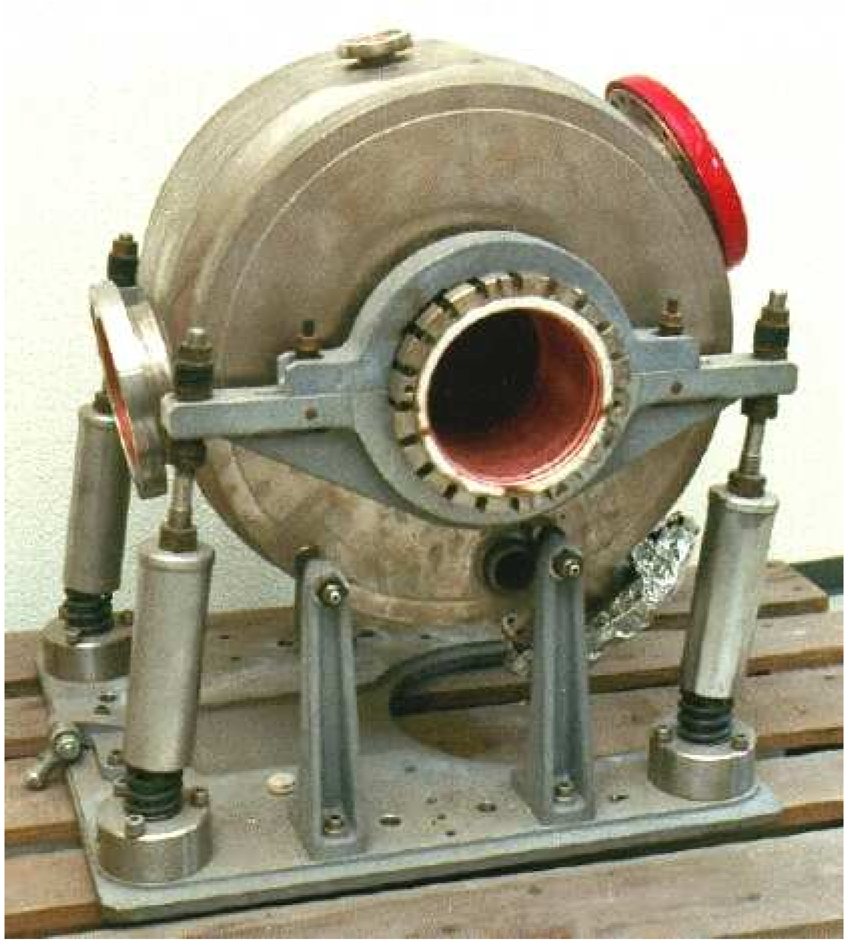
J_m : Bessel´s functions

J'_m : derivative of the Bessel´s functions

angular frequency : $\omega = c\dfrac{x_{01}}{R}$ $\quad x_{01} = 2.405$

Examples of pill box cavities

DESY cavity (pill box)

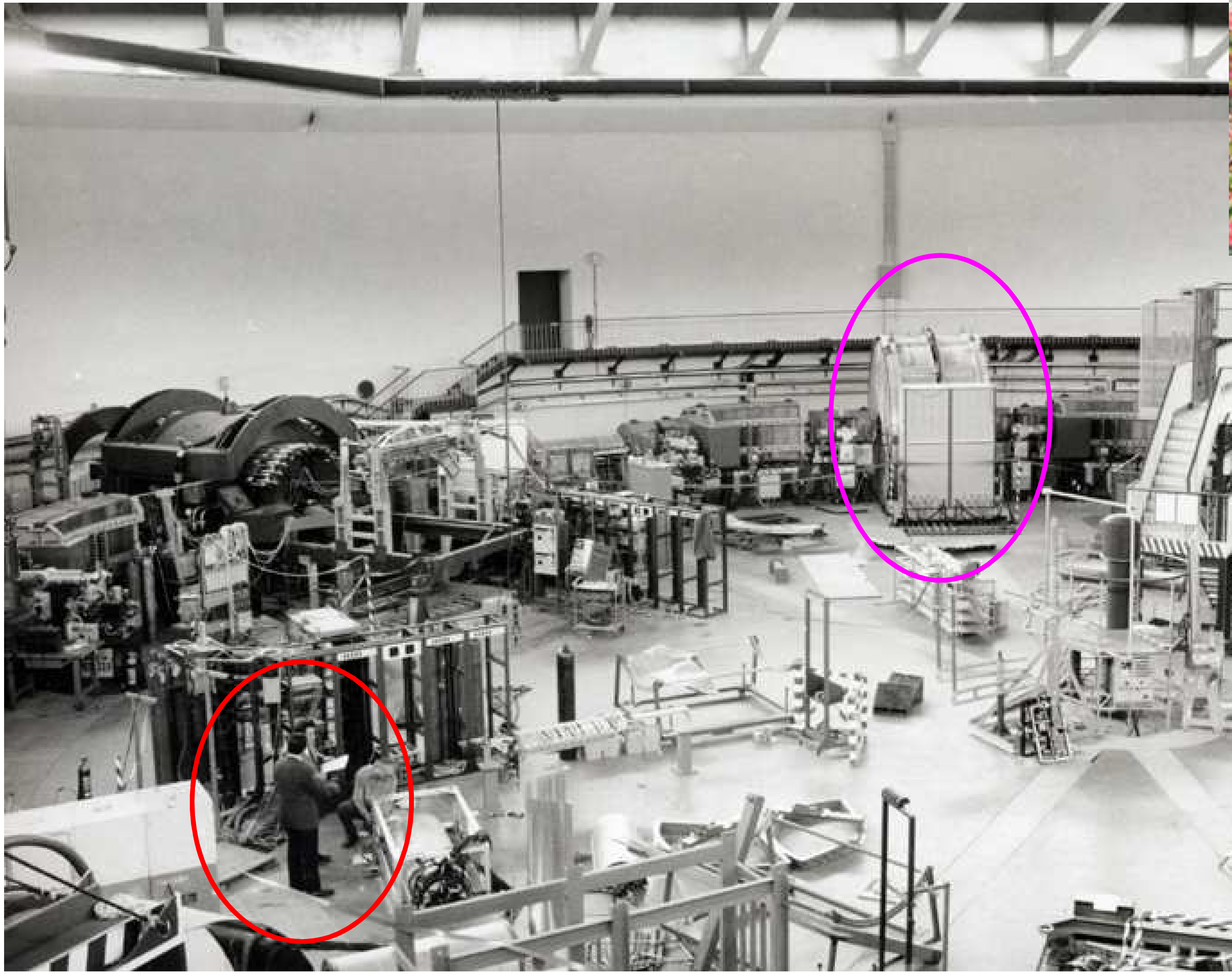


ADONE cavity 51 MHz (pill box)
Frascati lab, Italy

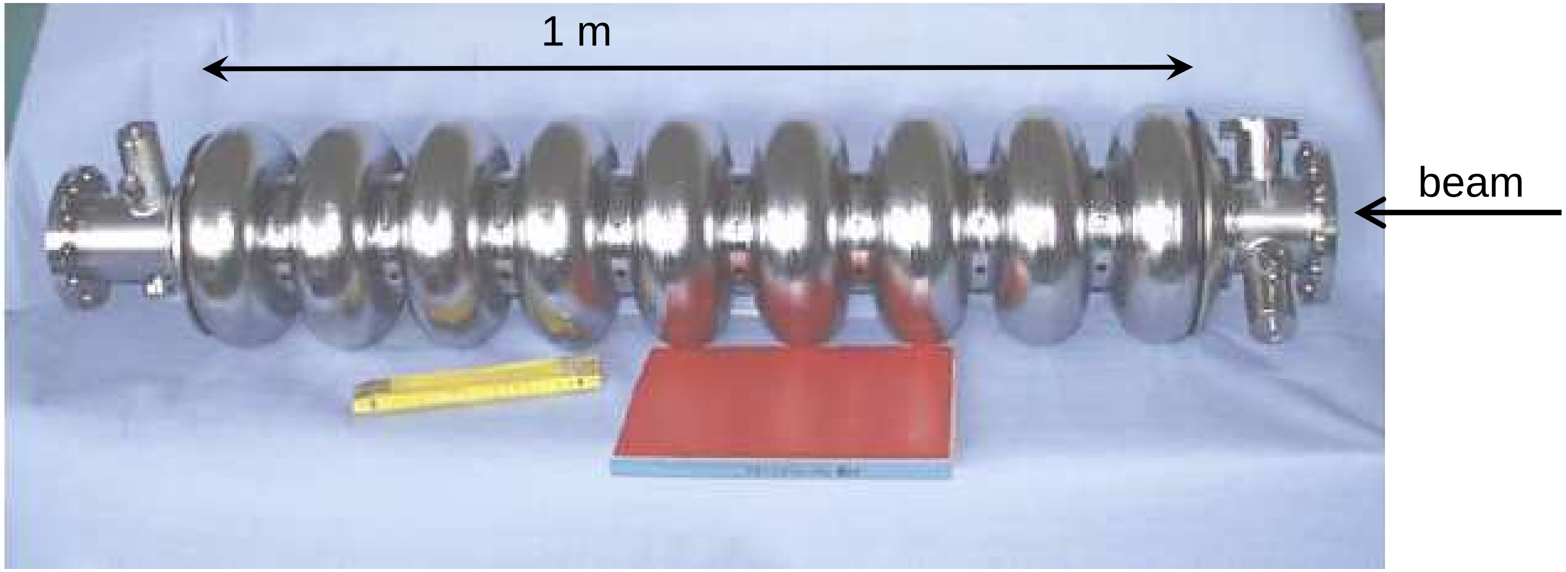
Examples of pill box cavities

ADONE cavity 51 MHz (pill box)
Frascati lab, Italy

ADONE in 1963, Laboratori Nazionali di Frascati, Italy



Superconducting cavity used at DESY



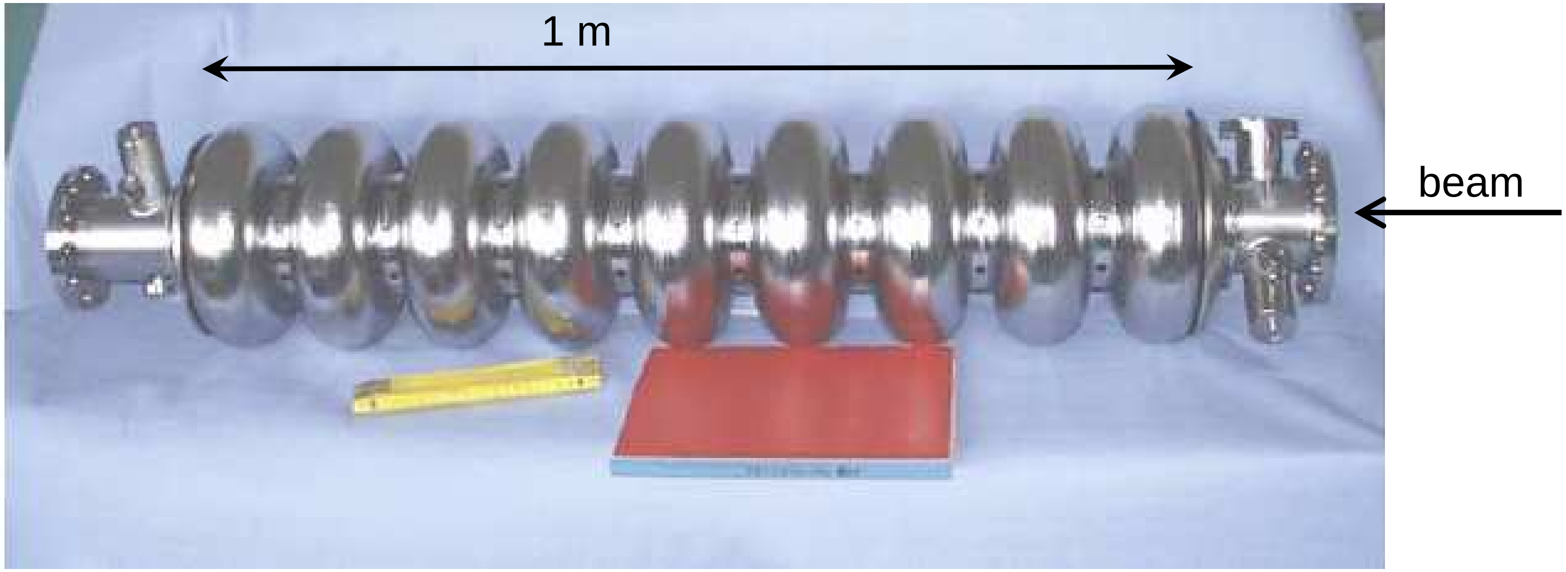
material: pure Niobium

operating temperature: 2 K

accelerating field gradient: up to 35 MV/m

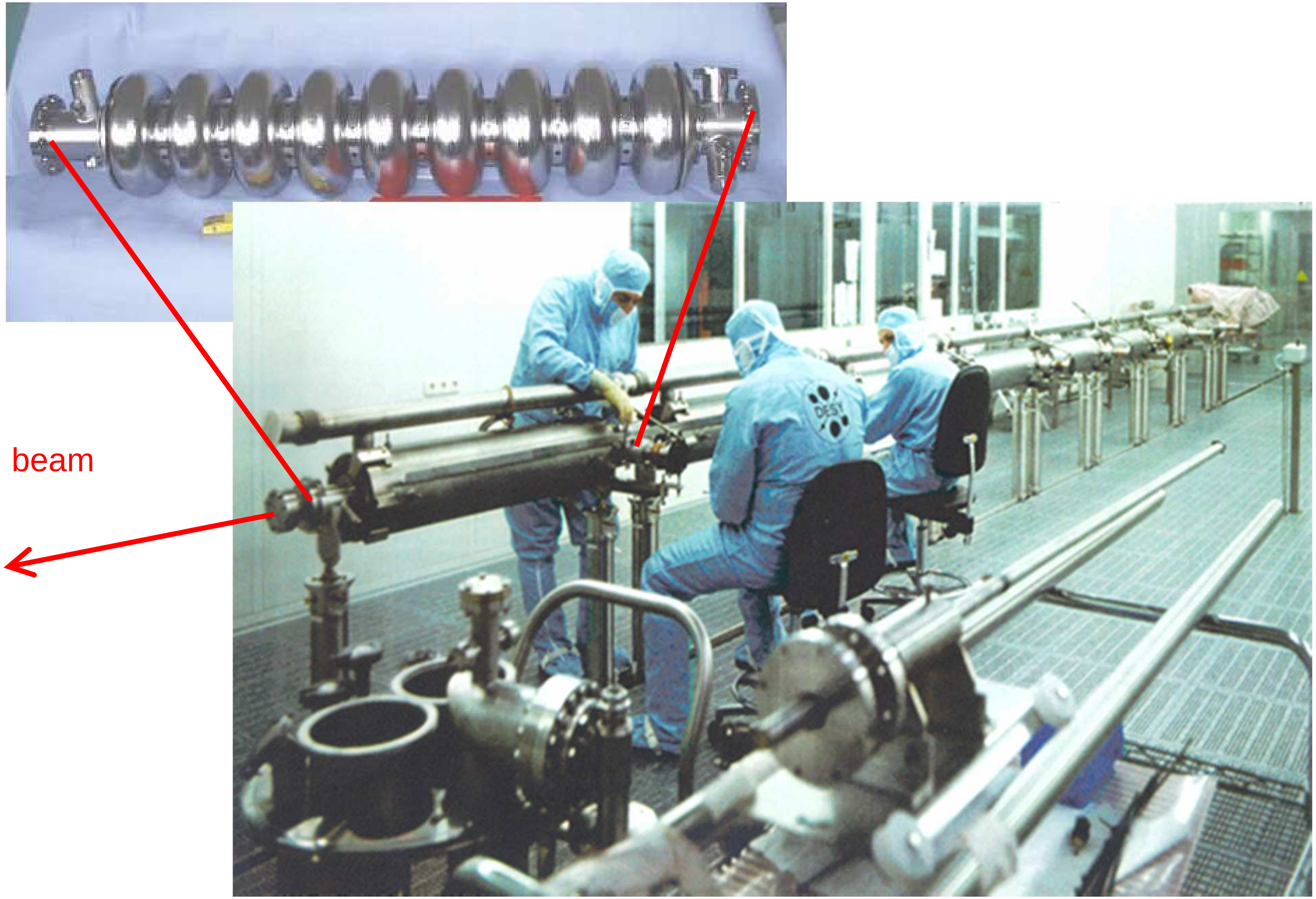
Superconducting cavity used at DESY

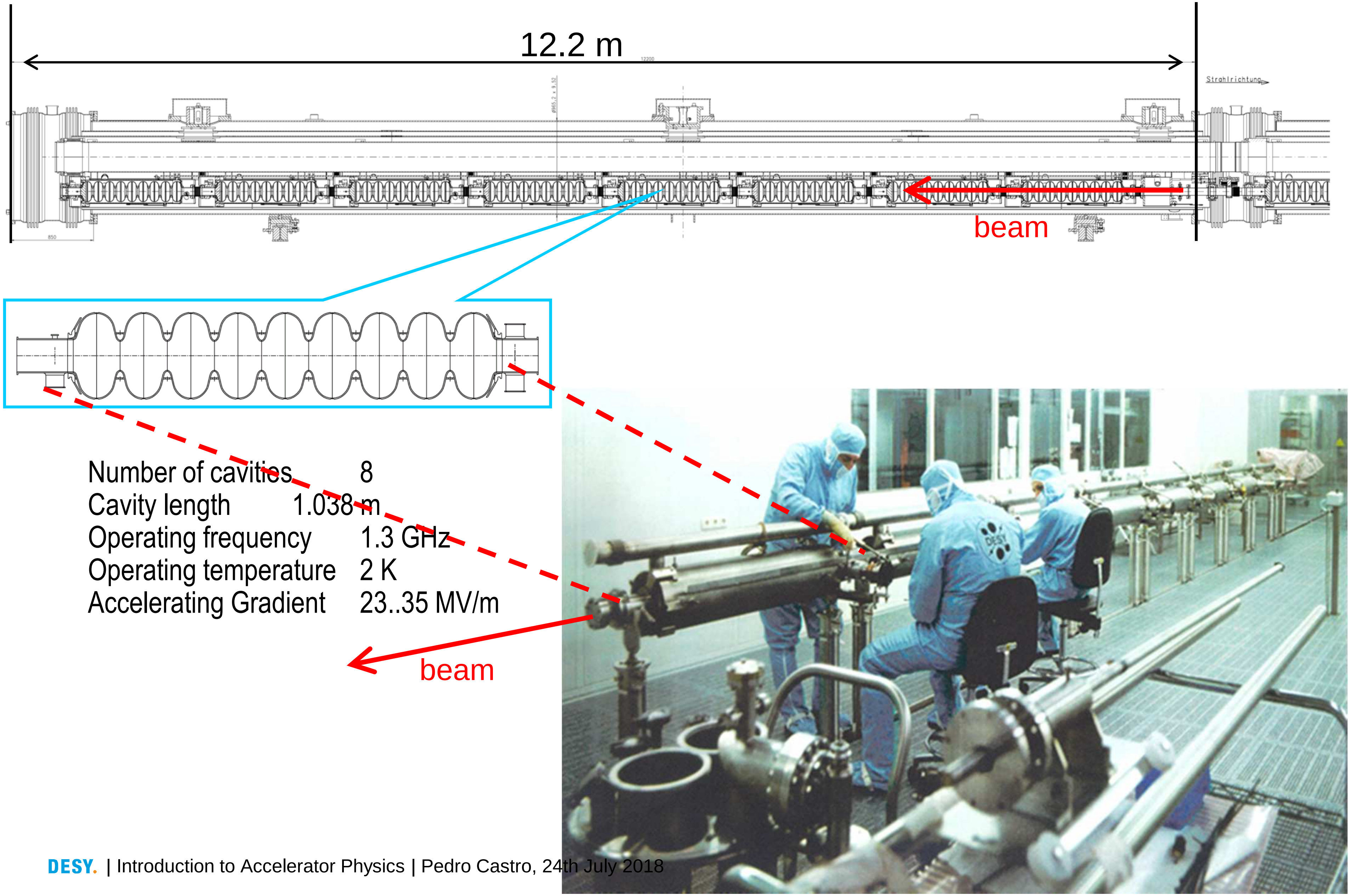
Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)



Free-electron LASer in Hamburg	0.3 km	DESY	2004- ?	e-	1.2 GeV
European X-ray Free-Electron Laser	3 km	DESY	2016- ?	e-	17.5 GeV
International Linear Collider	30 km	?	?	e-/e+	2x250 GeV

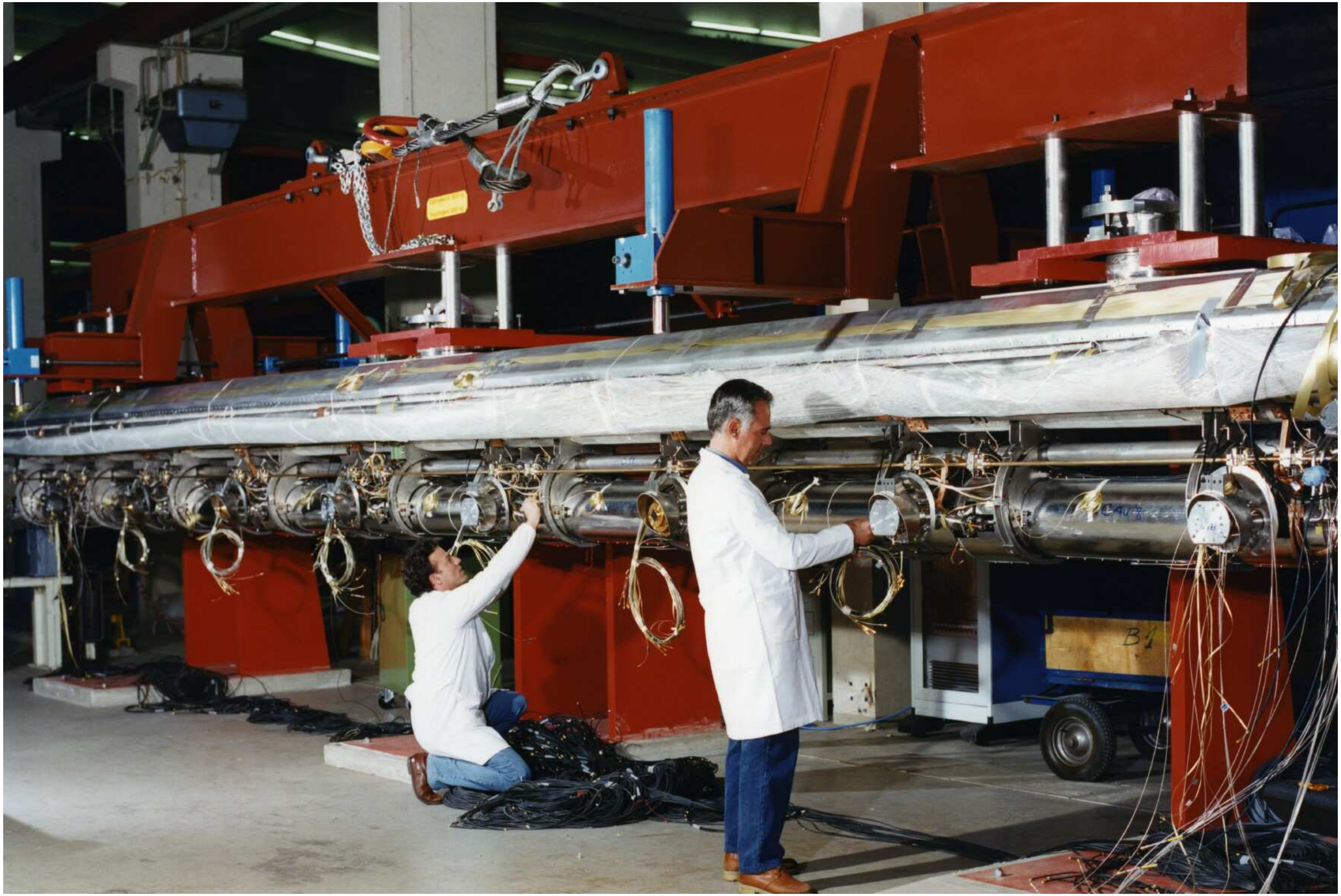
Cavities inside a cryostat



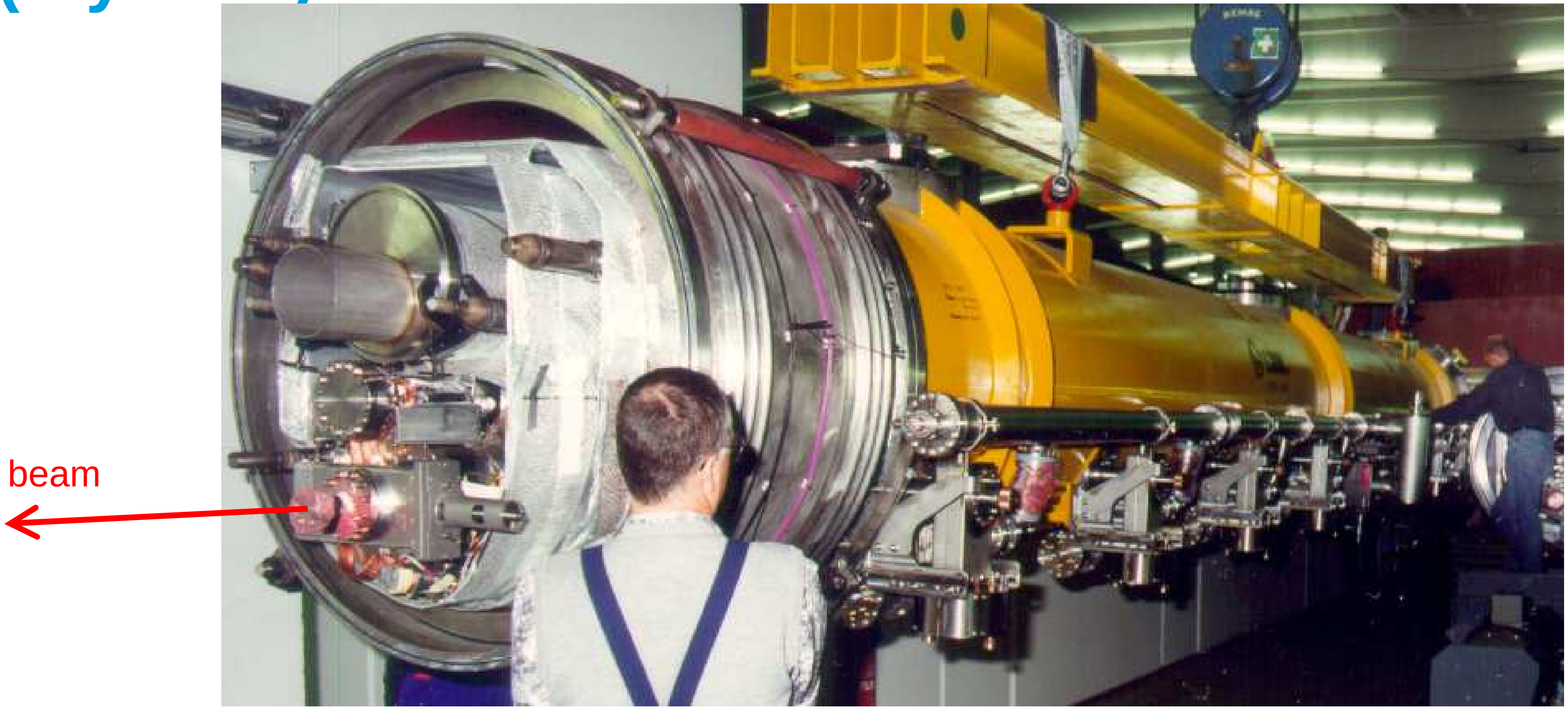


Number of cavities	8
Cavity length	1.038 m
Operating frequency	1.3 GHz
Operating temperature	2 K
Accelerating Gradient	23..35 MV/m

Cavities inside a cryostat

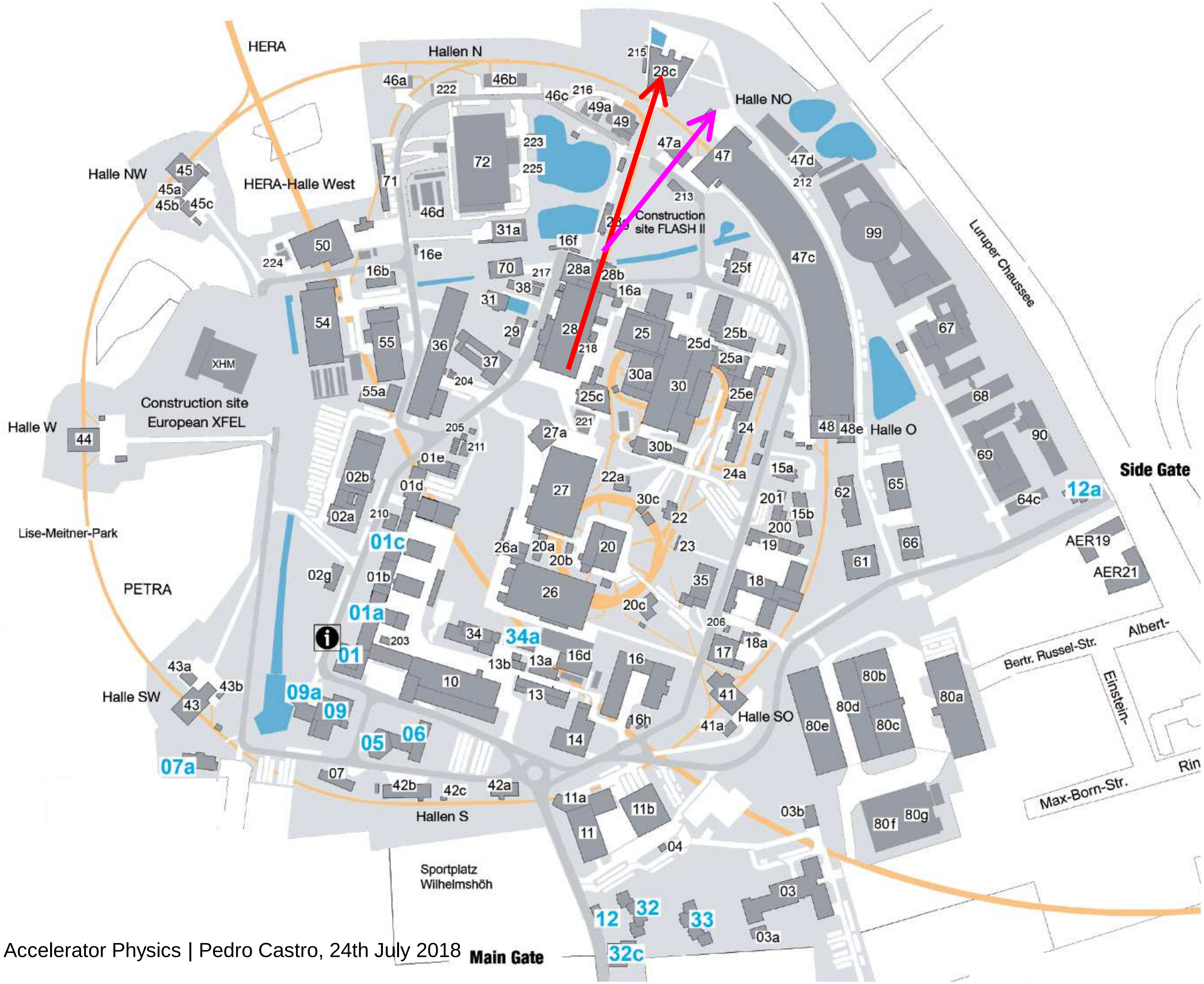


Cavities inside an accelerator module (cryostat)

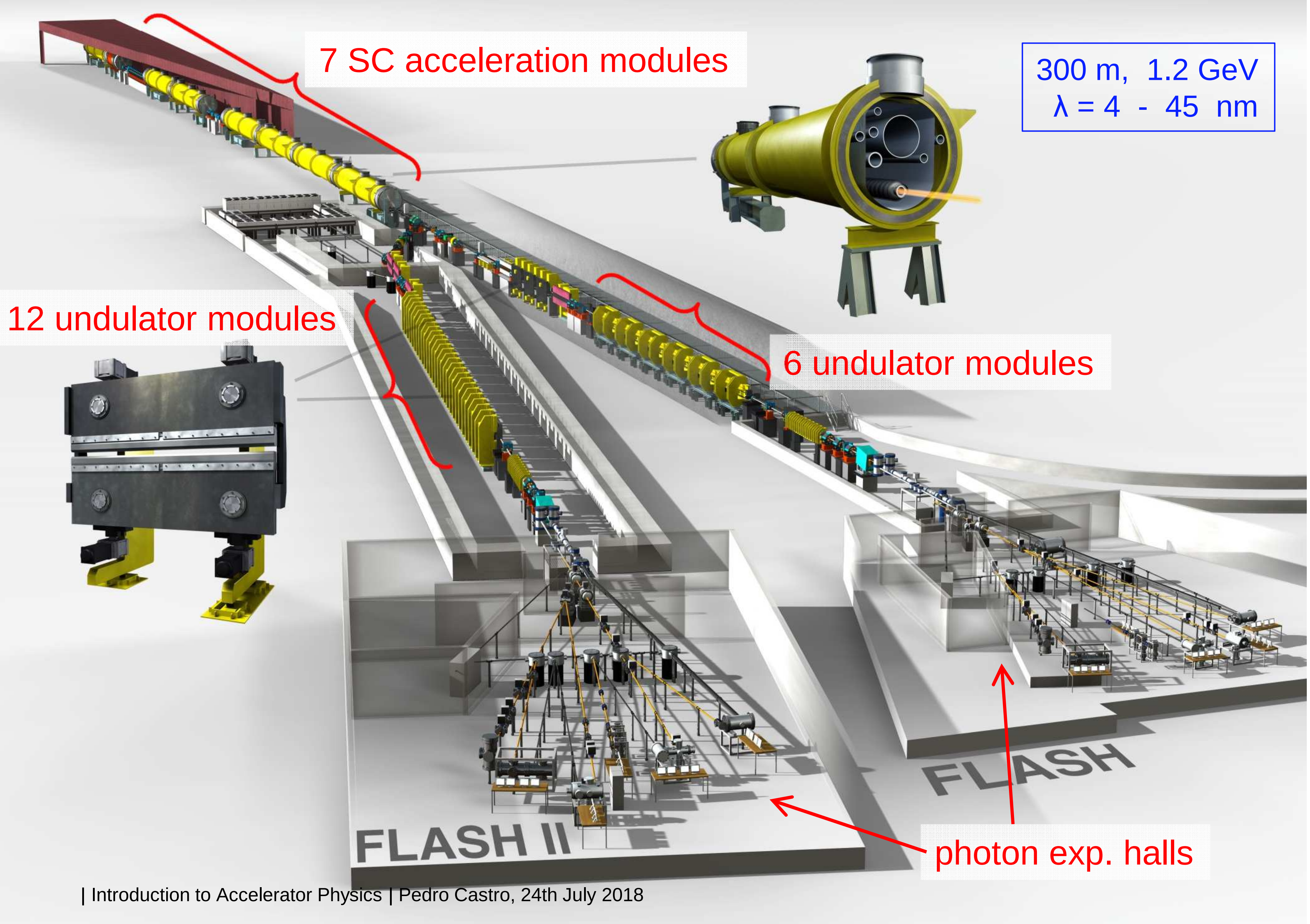


module installation in FLASH (2004)

Free-electron LASer in Hamburg (FLASH)



Free-electron LASer in Hamburg (FLASH)



300 m, 1.2 GeV
λ = 4 - 45 nm

7 SC acceleration modules

12 undulator modules

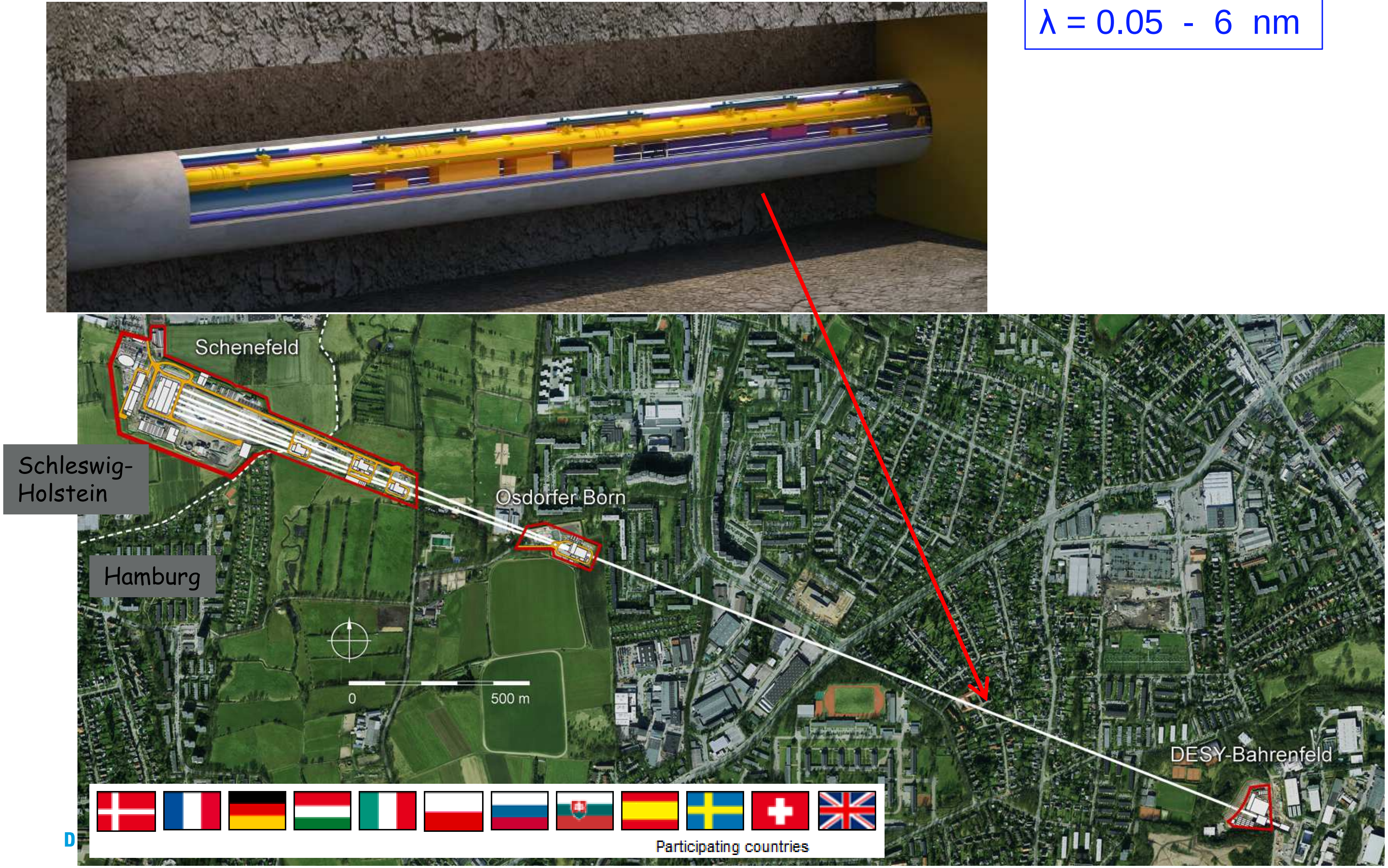
6 undulator modules

photon exp. halls

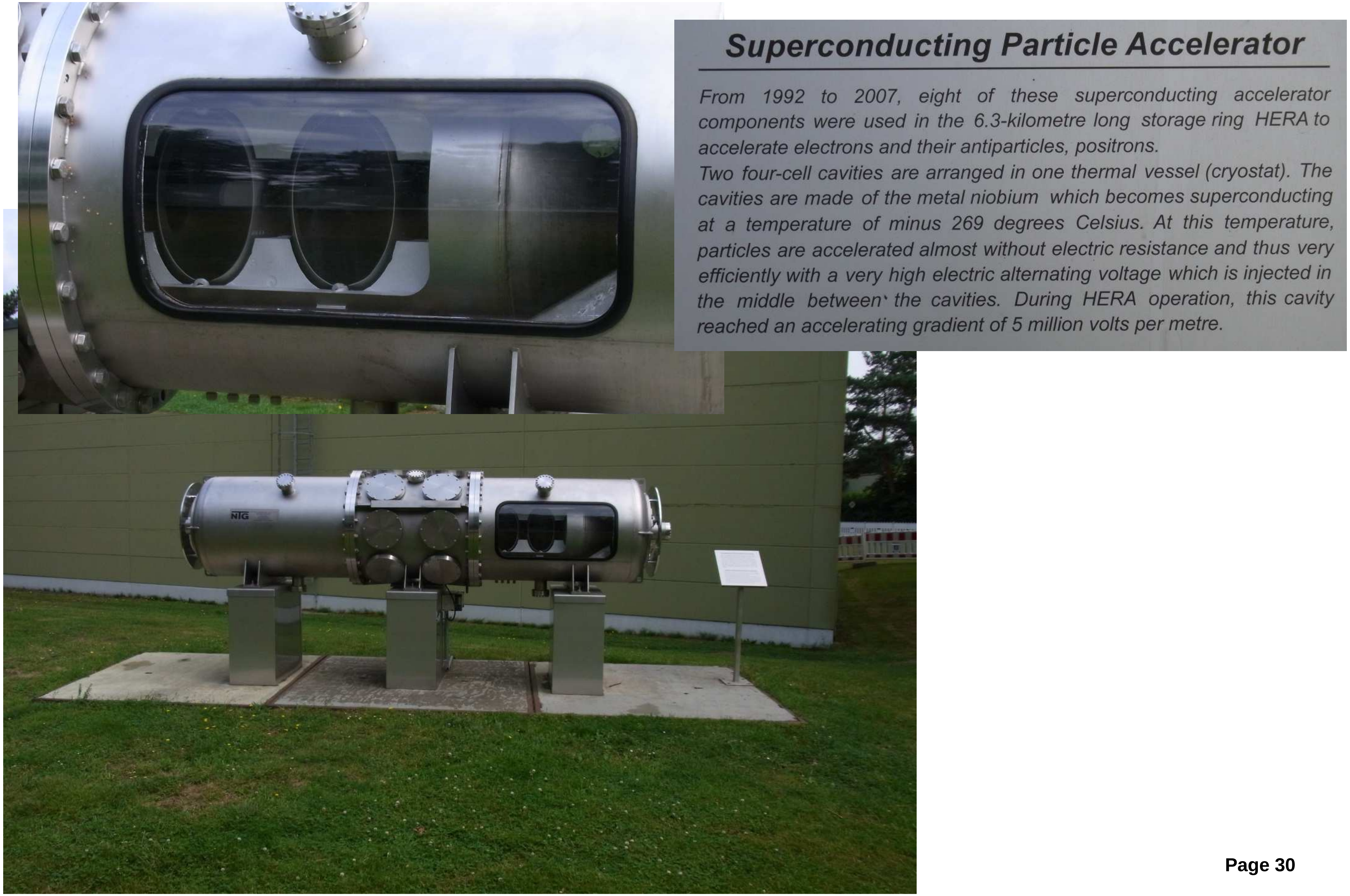
100 accelerator modules (cryostats) in XFEL

European X-ray Free-Electron Laser (XFEL) (3 km, 17.5 GeV)

λ = 0.05 - 6 nm



Superconducting cavities at HERA



Superconducting Particle Accelerator

From 1992 to 2007, eight of these superconducting accelerator components were used in the 6.3-kilometre long storage ring HERA to accelerate electrons and their antiparticles, positrons.

Two four-cell cavities are arranged in one thermal vessel (cryostat). The cavities are made of the metal niobium which becomes superconducting at a temperature of minus 269 degrees Celsius. At this temperature, particles are accelerated almost without electric resistance and thus very efficiently with a very high electric alternating voltage which is injected in the middle between the cavities. During HERA operation, this cavity reached an accelerating gradient of 5 million volts per metre.

Other accelerators using superconducting cavities

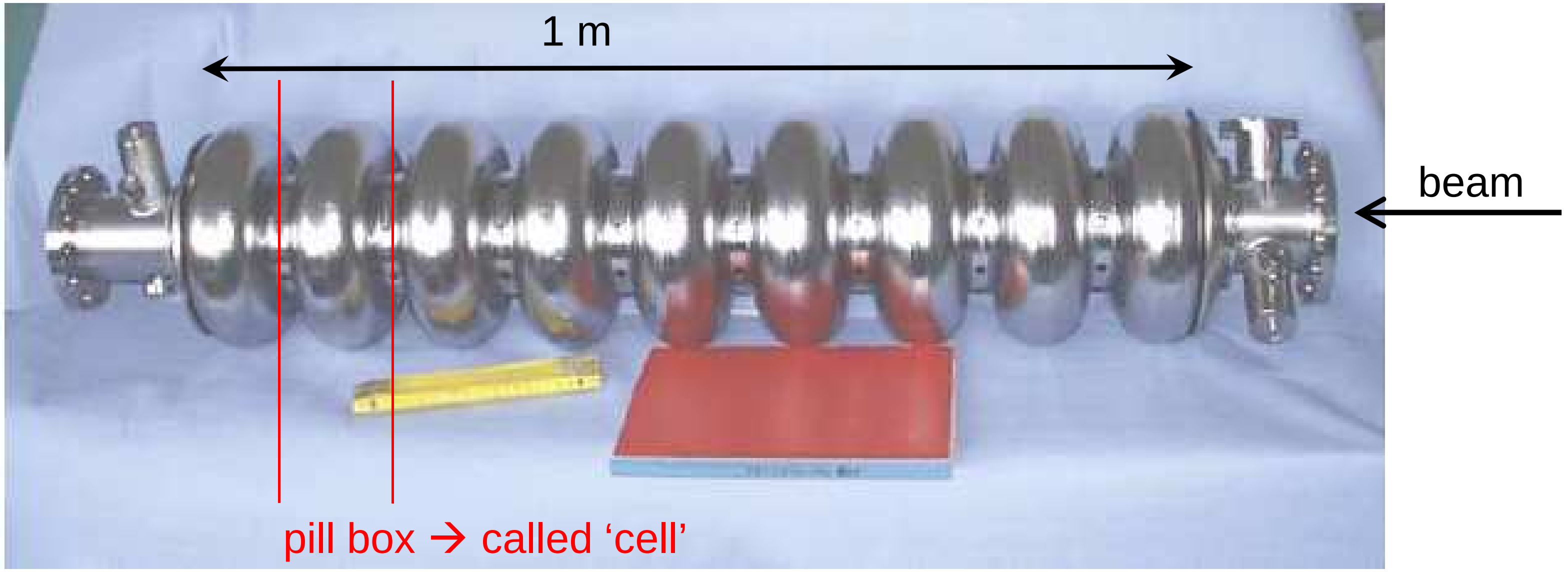
- 5 de-commissioned
- 11 in operation
- 4 in construction
- 9 in design phase

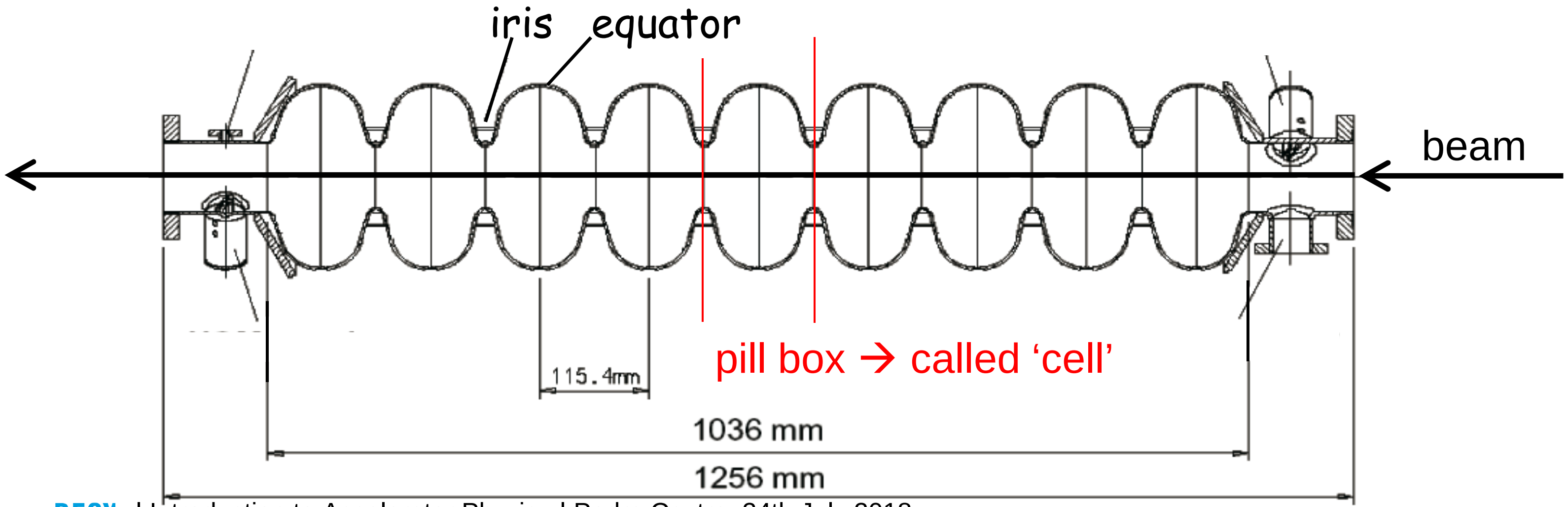
Total = 29

full list: http://tesla-new.desy.de/srf_accelerators

Superconducting cavity used in FLASH and in XFEL

Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)



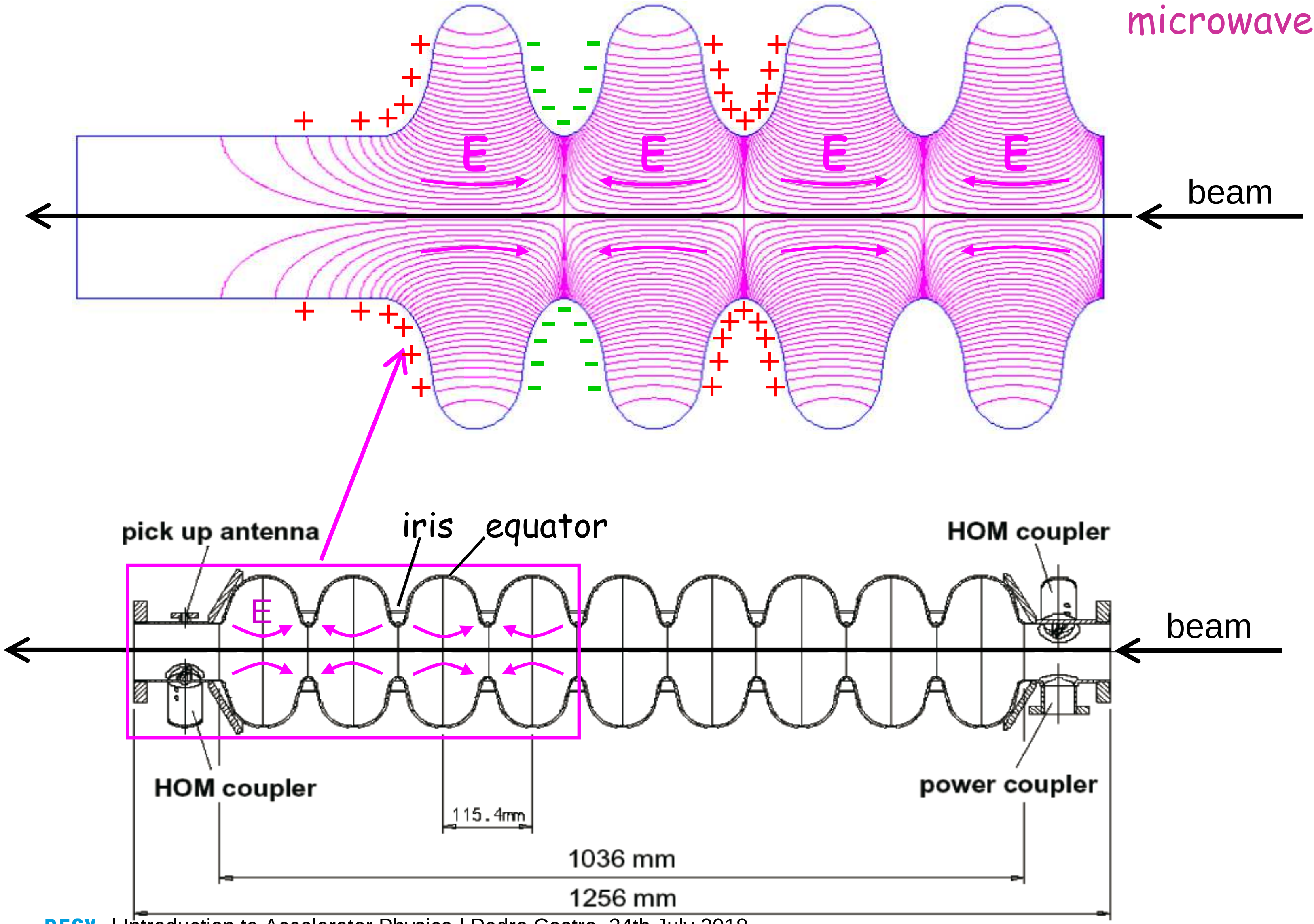


Accelerating field map

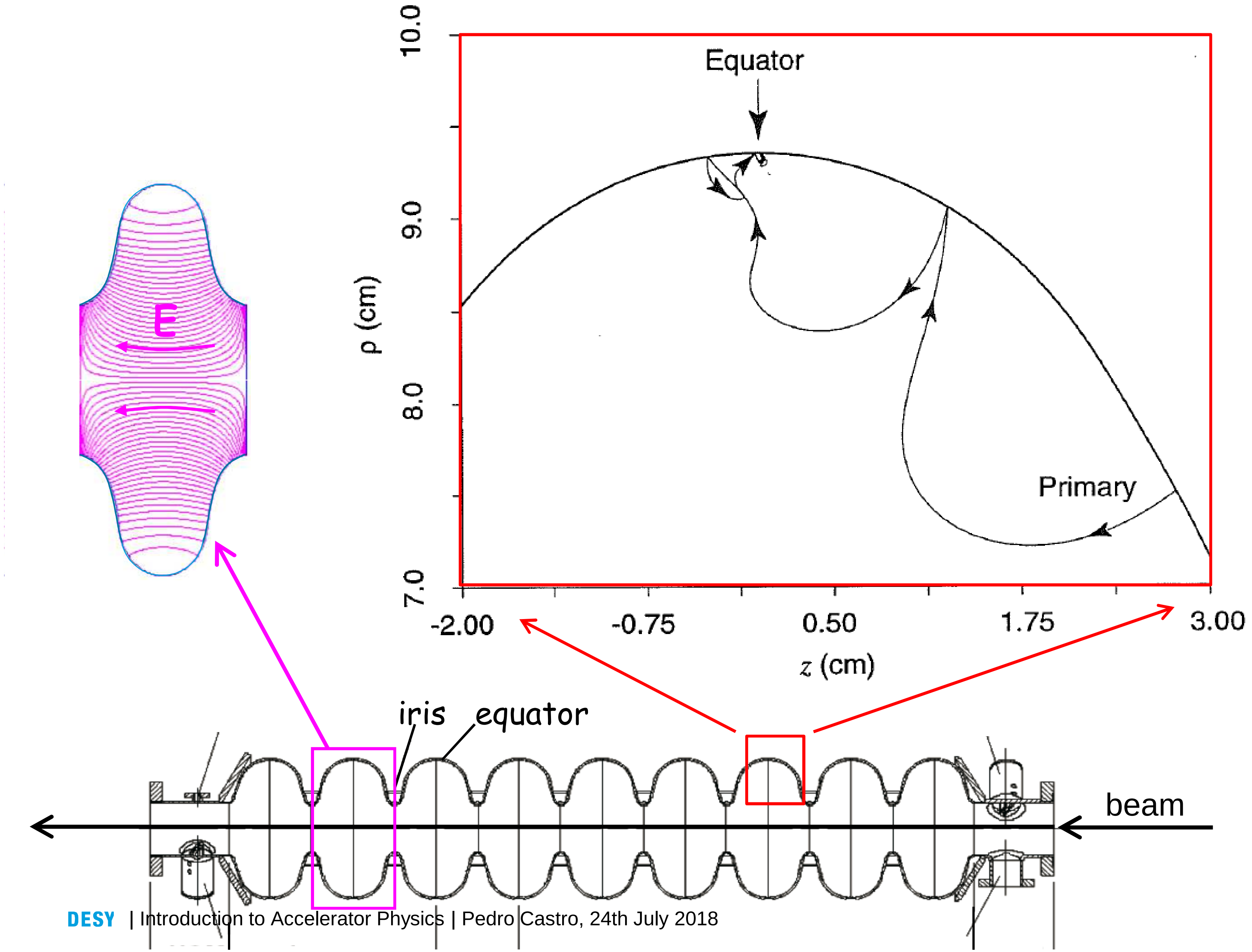
Simulation of the fundamental mode: electric field lines

$f_{RF} = 1.3\ \mathrm{GHz}$

microwaves: (L-band)



Multipacting mitigation in superconducting cavities



1) Why this shape? ……………………………………………… to reduce/avoid multipacting

2) How to feed $\vec{E}$ in?

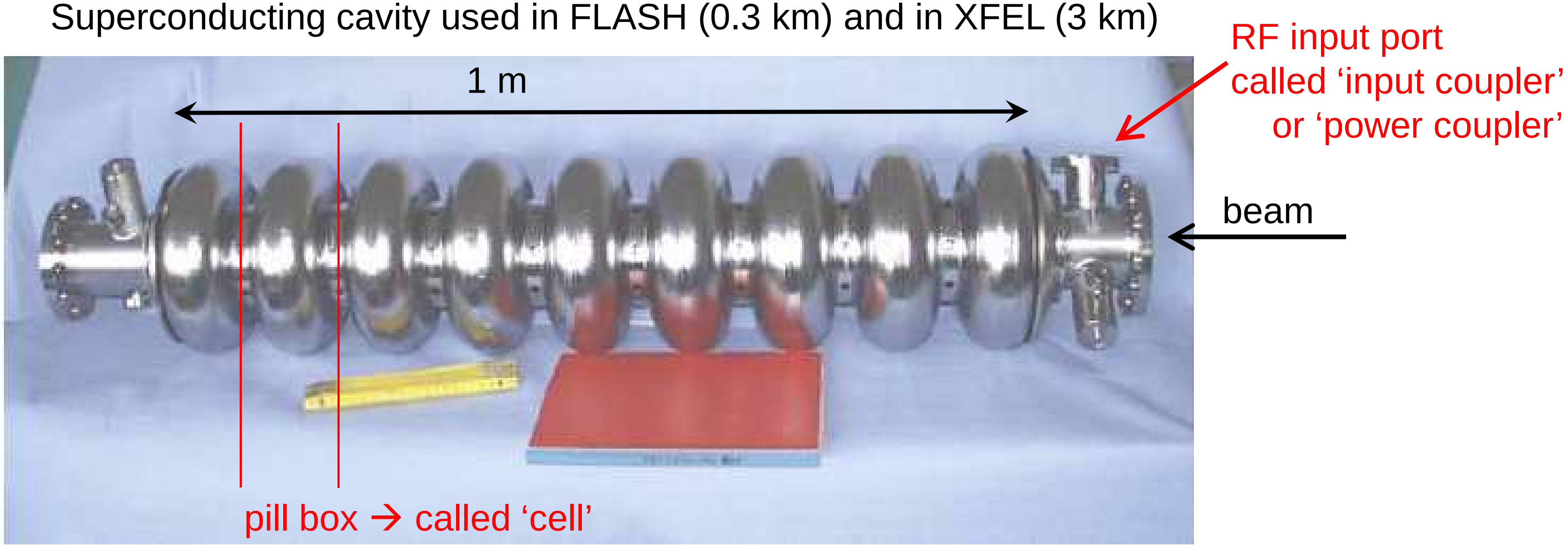
3)

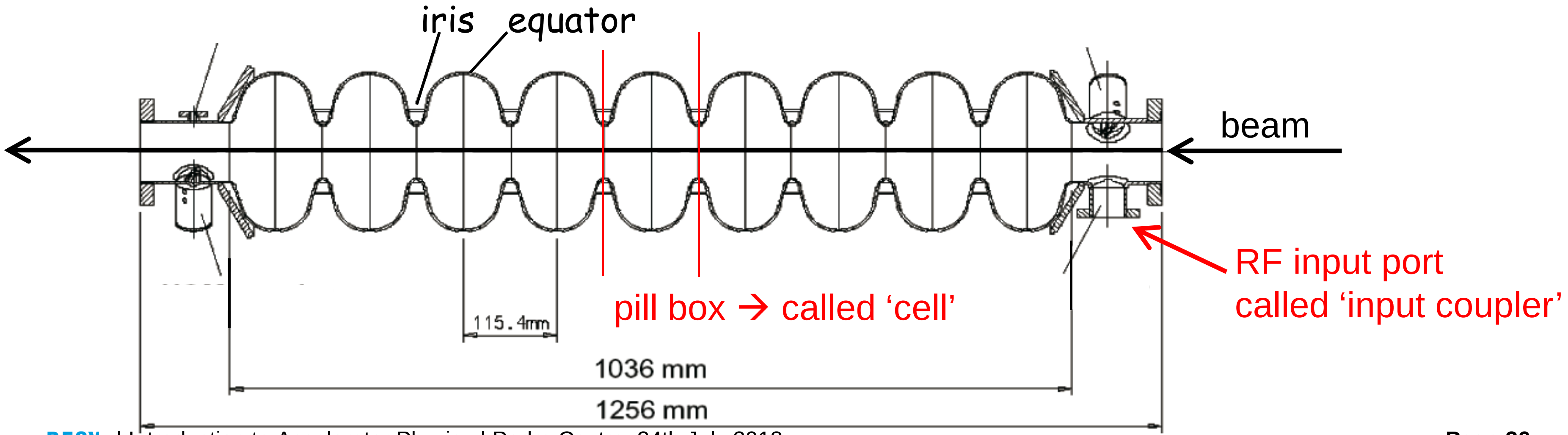
4)

5)

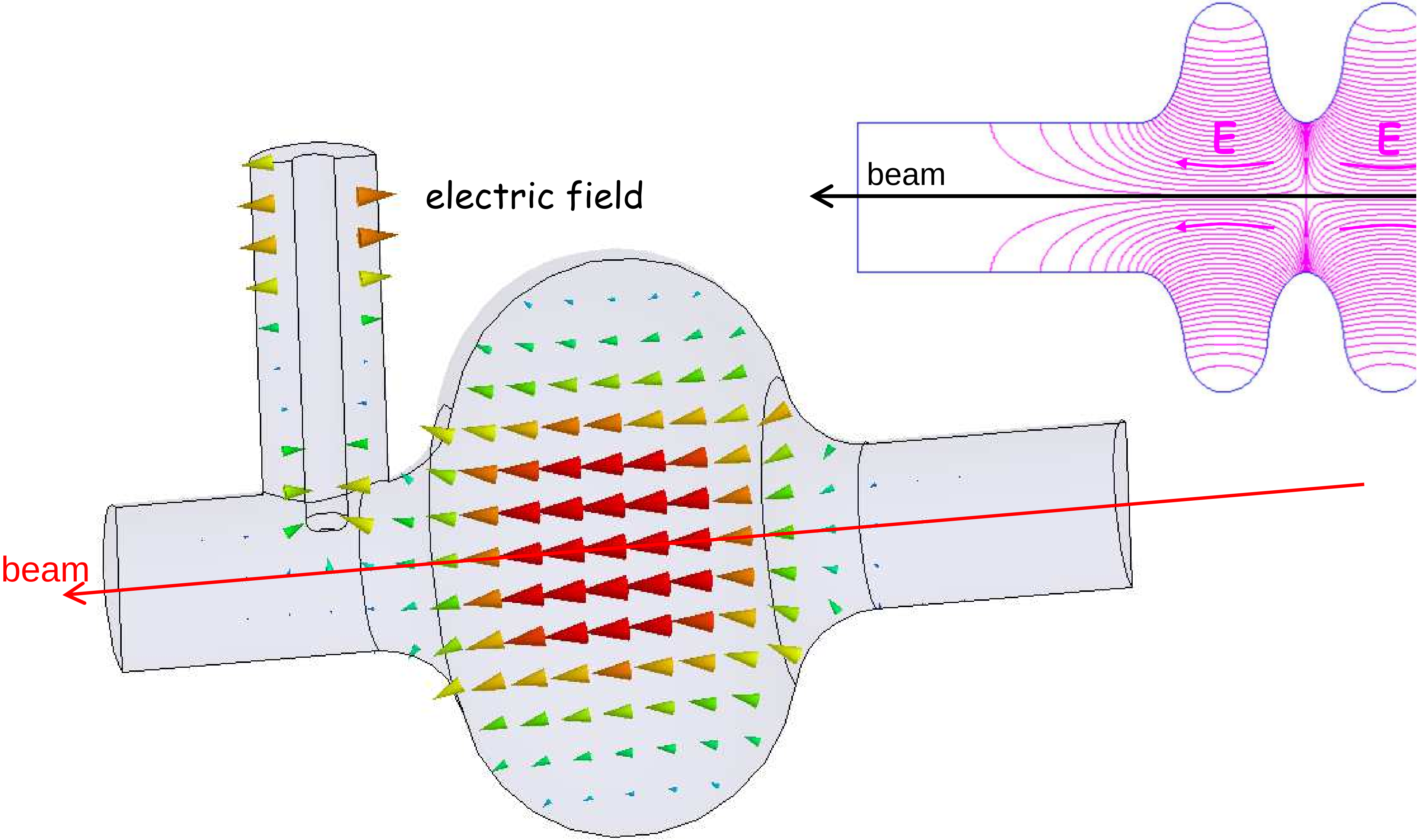
Superconducting cavity used in FLASH and in XFEL

Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)





Fundamental mode coupler (input coupler)



1) Why this shape? ………………………………………………… to reduce/avoid multipacting

2) How to feed $\vec{E}$ in? ……………………………………………… with input couplers

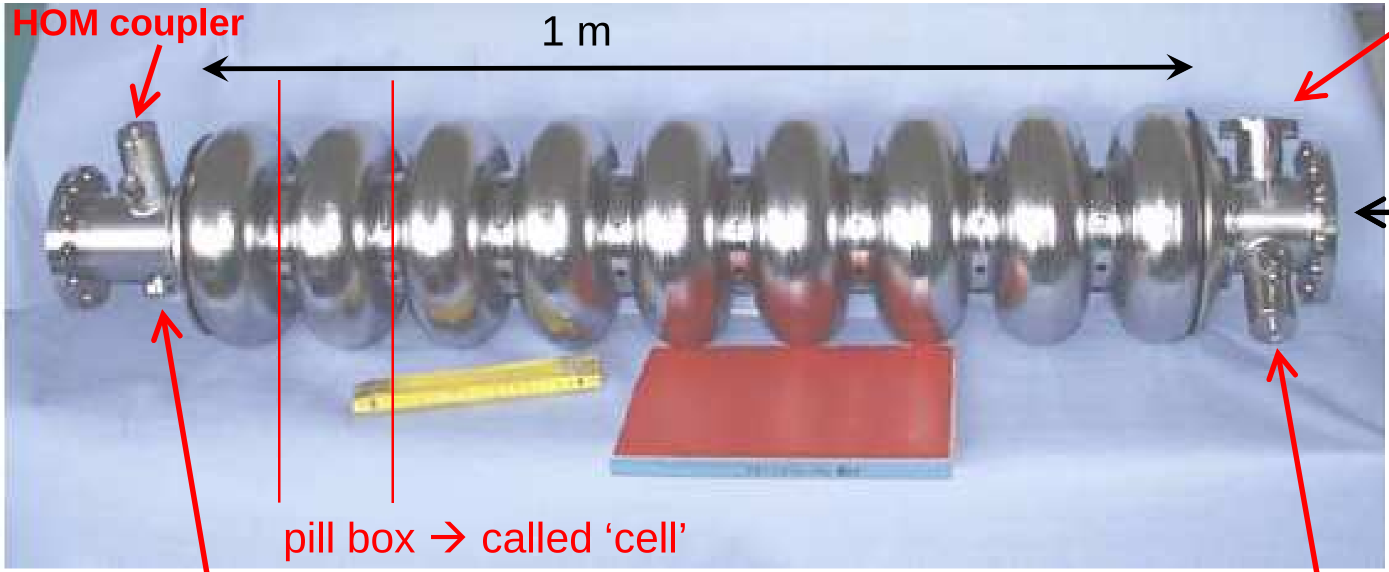
3) How to measure $\vec{E}$?

4)

5)

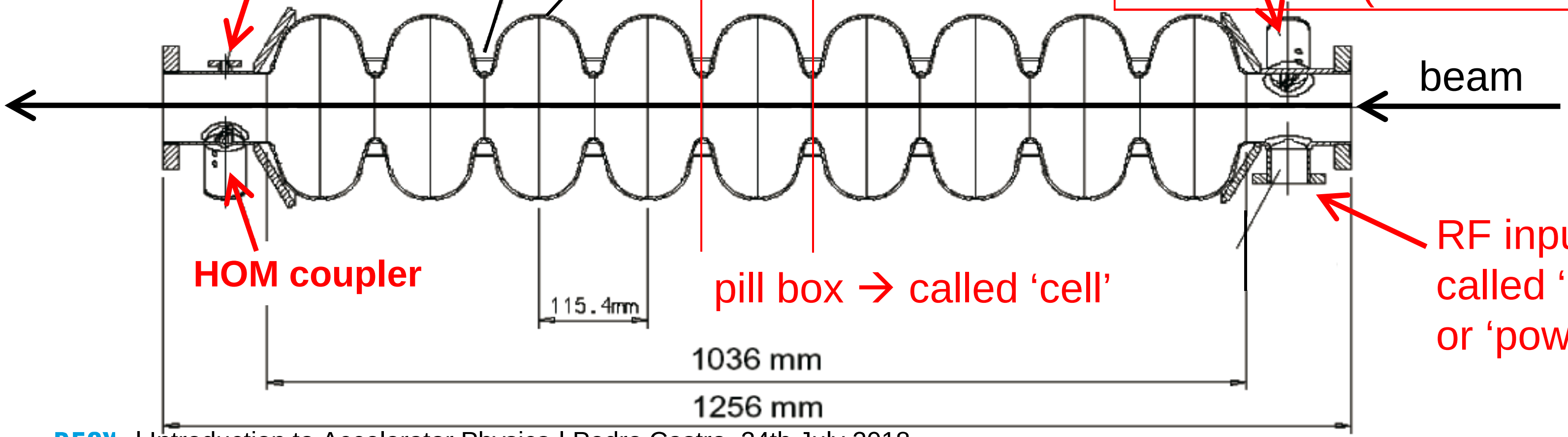
Superconducting cavity used in FLASH and in XFEL

Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)



RF input port
called 'input coupler'
or 'power coupler'

beam



RF input port
called 'input coupler'
or 'power coupler'

1) Why this shape? ……………………………………………… to reduce/avoid multipacting

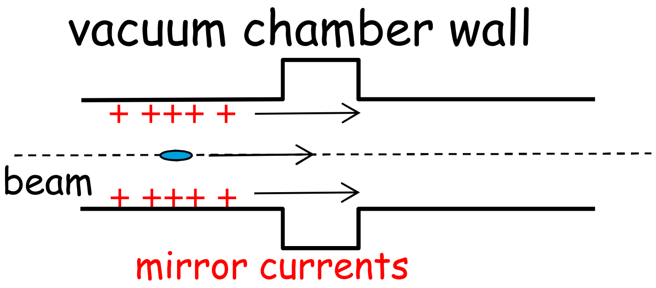
2) How to feed $\vec{E}$ in? …………………………………………… with input couplers

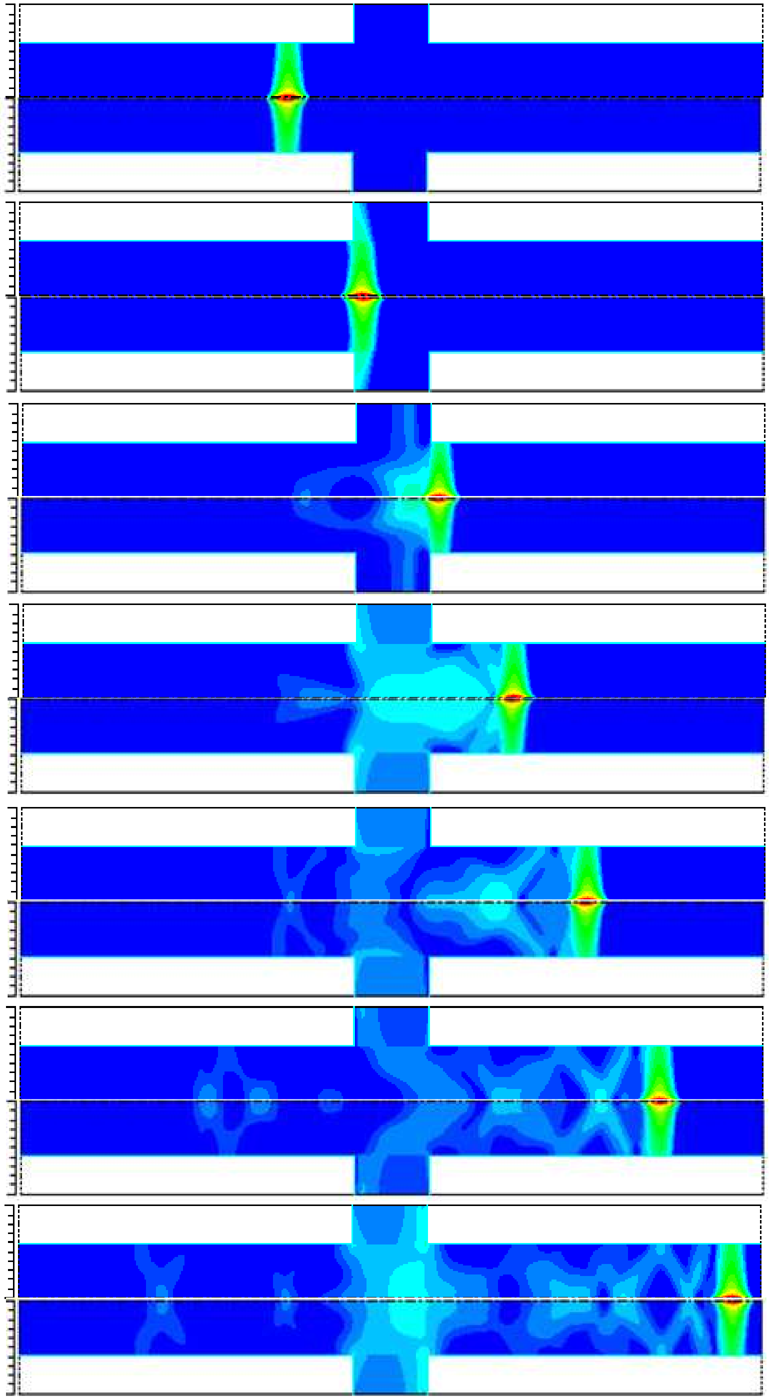
3) How to measure $\vec{E}$? …………………………………………… with pick up antennas

4) What are HOM couplers for?

5)

Wakefields





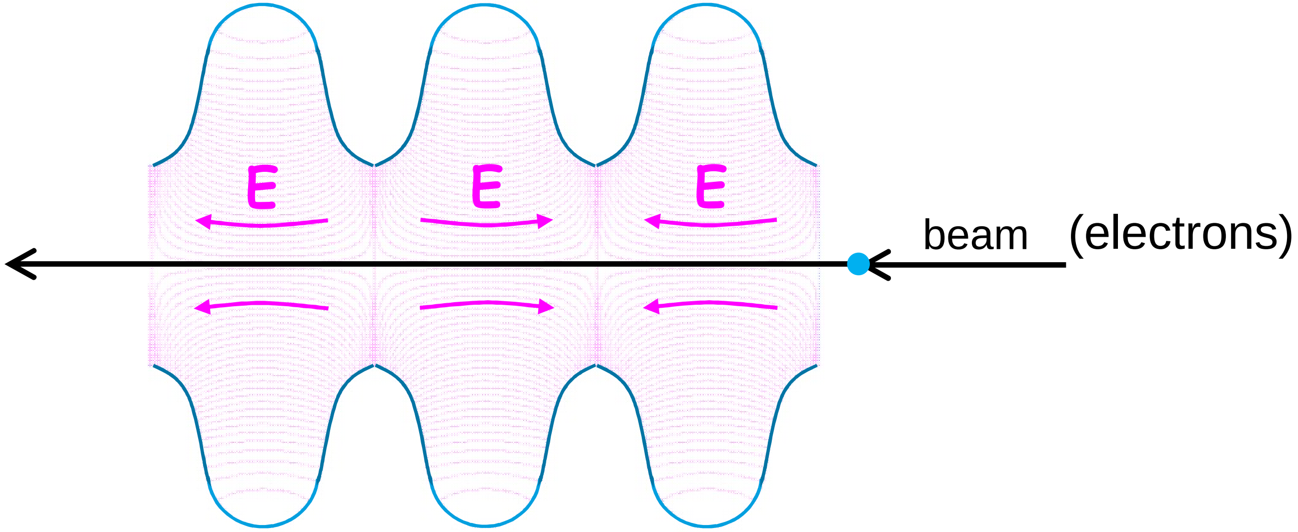
1) Why this shape? ……………………………………………… to reduce/avoid multipacting

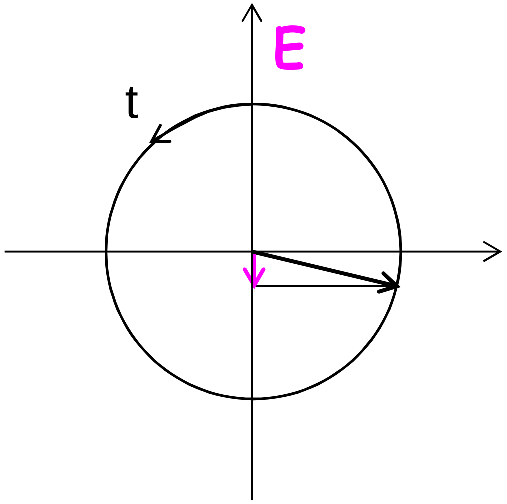
2) How to feed $\vec{E}$ in? …………………………………………… with input couplers

3) How to measure $\vec{E}$? …………………………………………… with pick up antennas

4) What are HOM couplers for? …………………………… to reduce HOM (wakefields)

5) Is there a net acceleration?


E
E
E
beam (electrons)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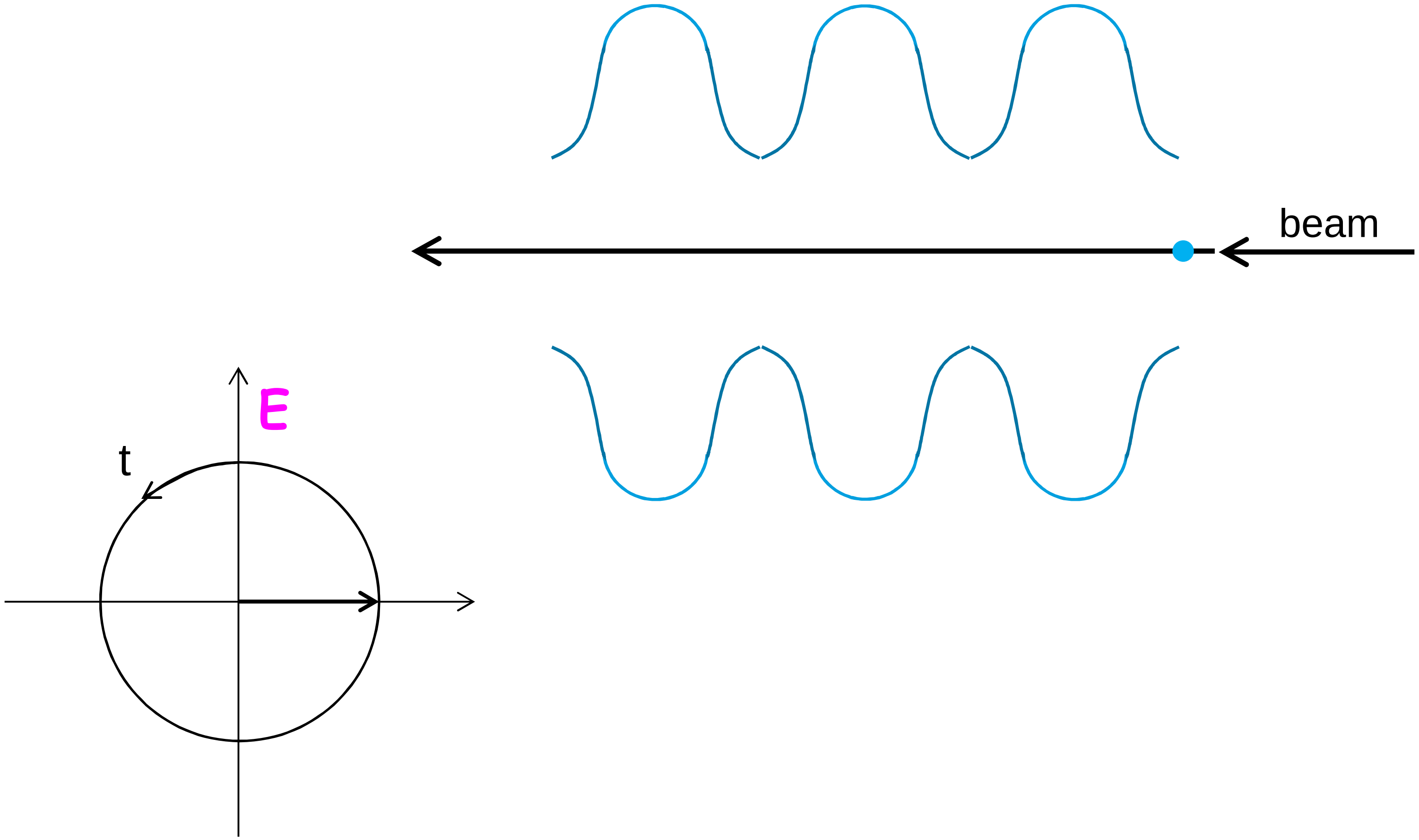
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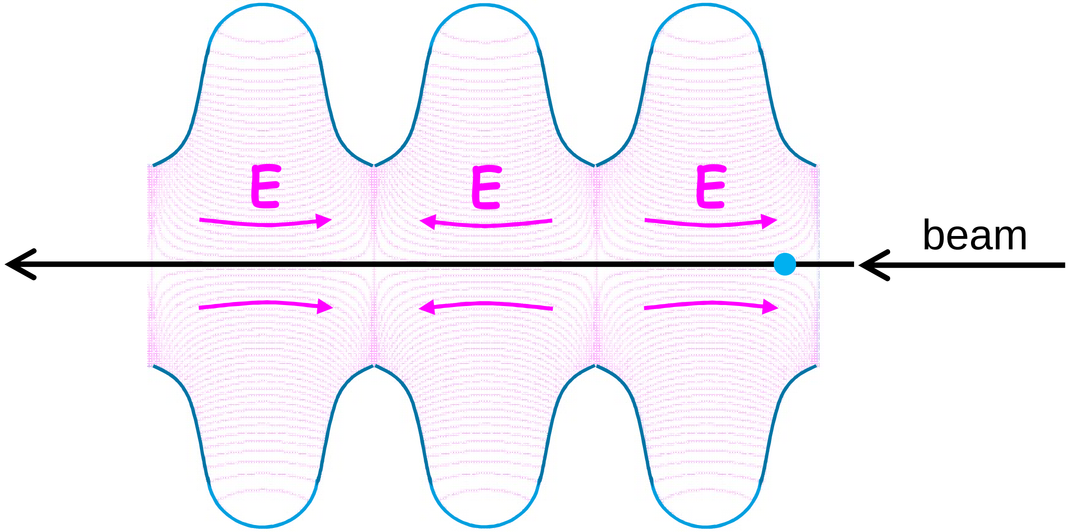
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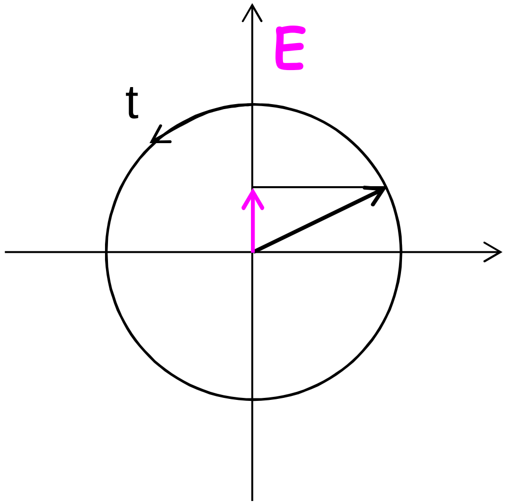
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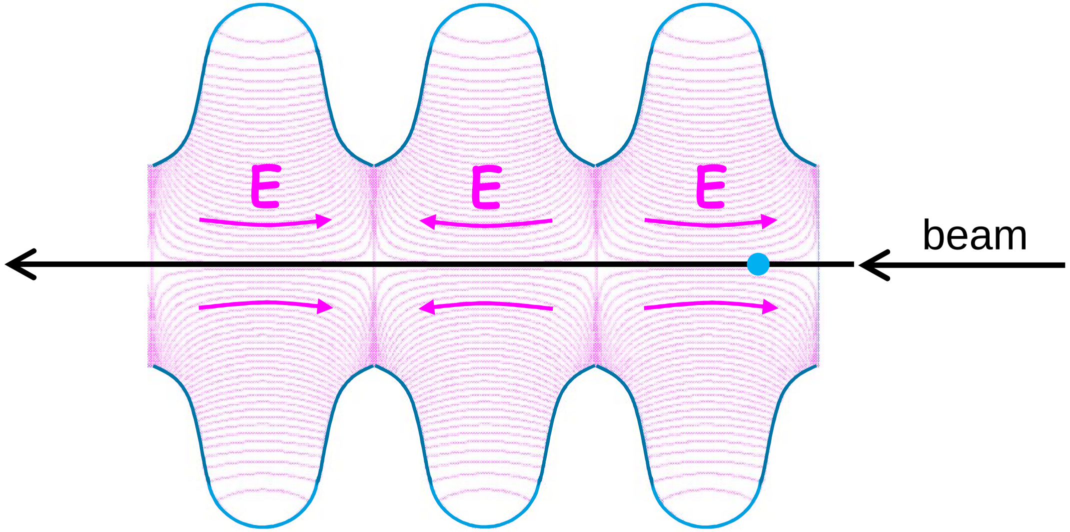
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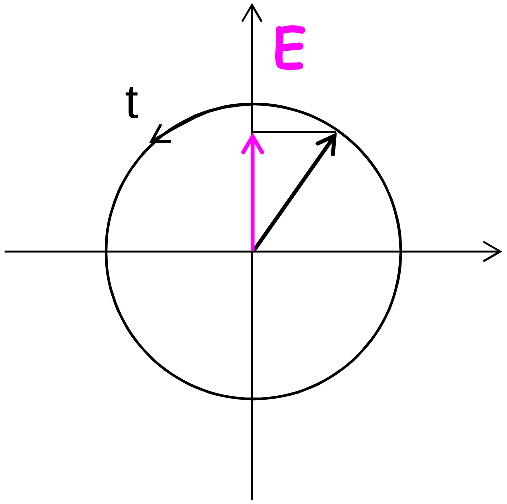
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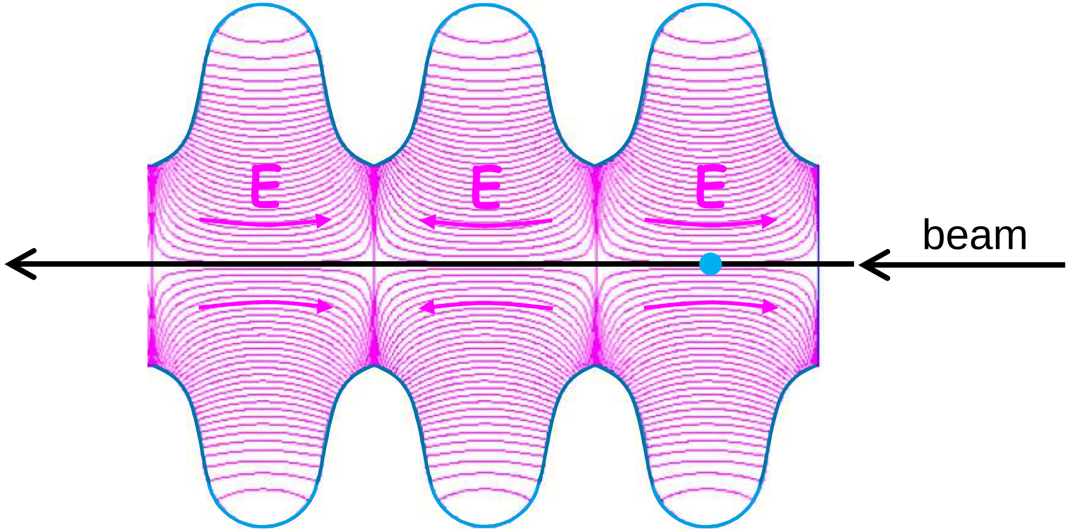
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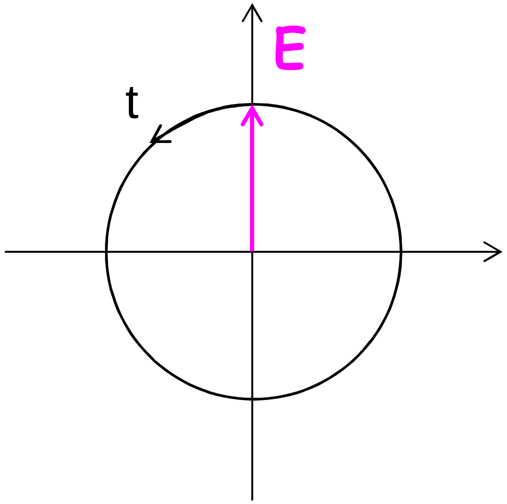
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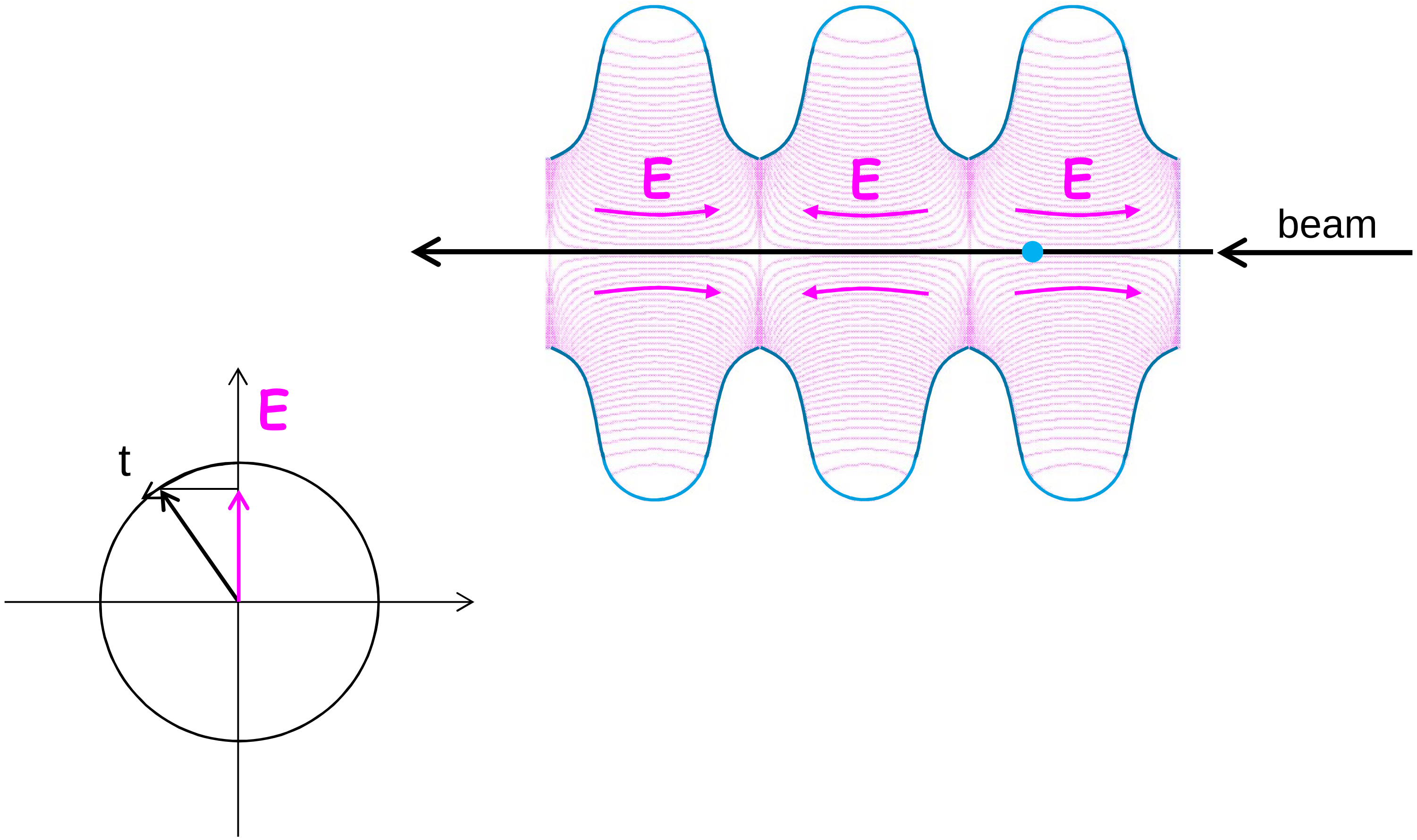
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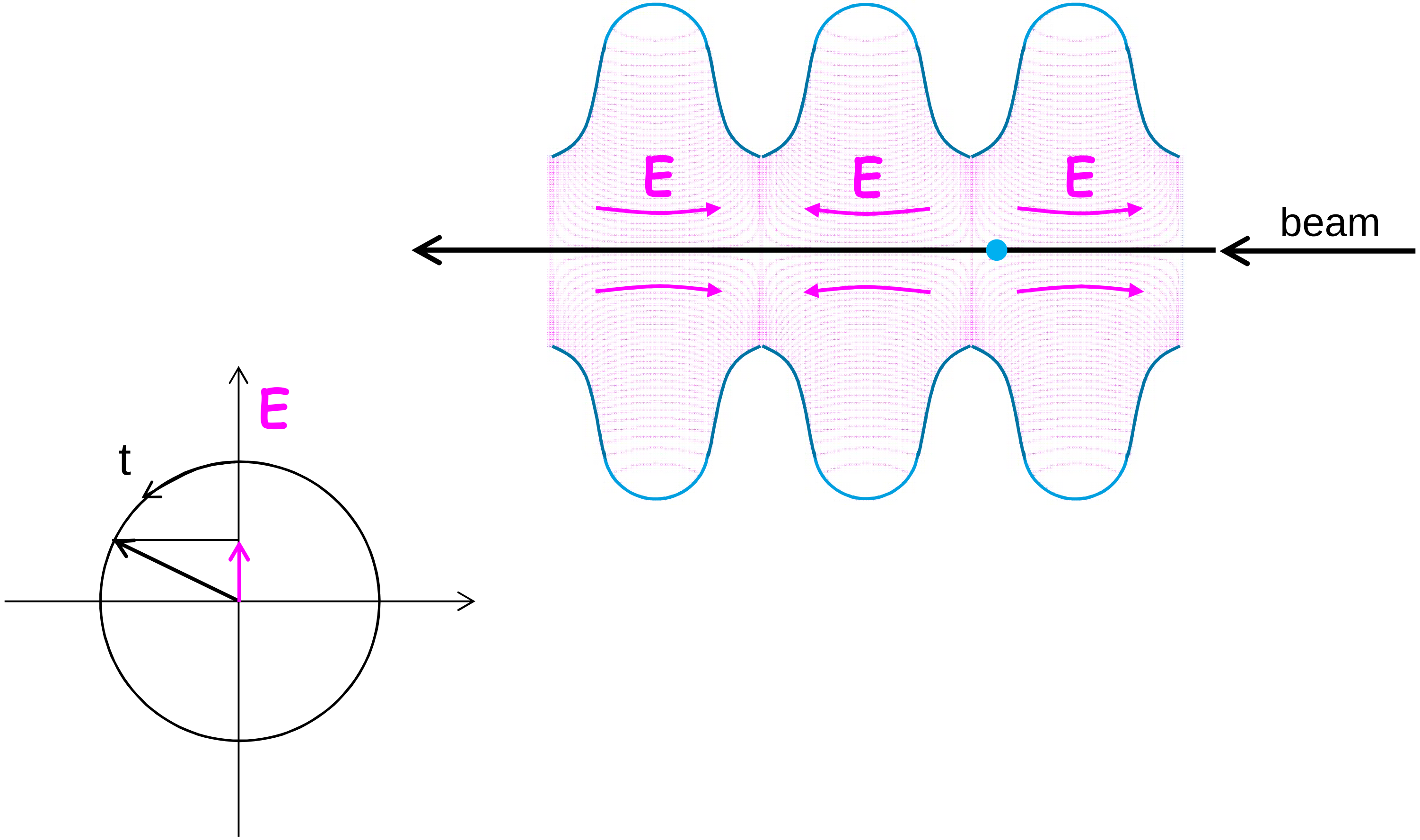
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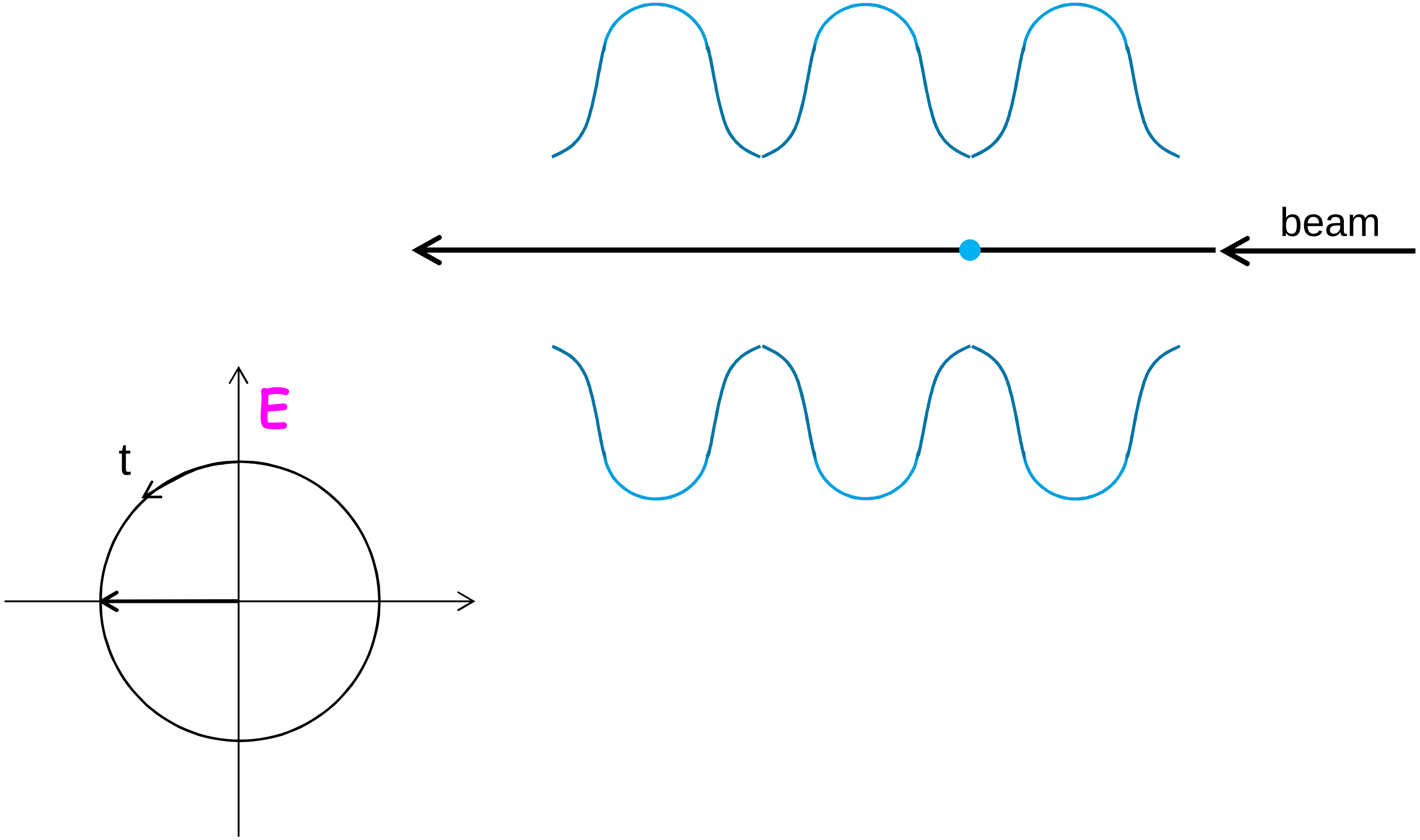
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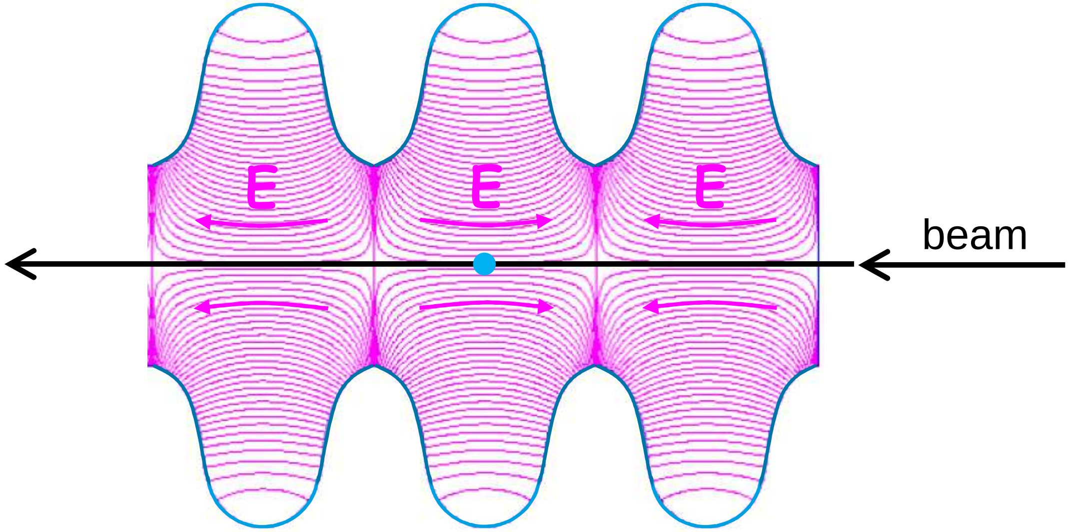
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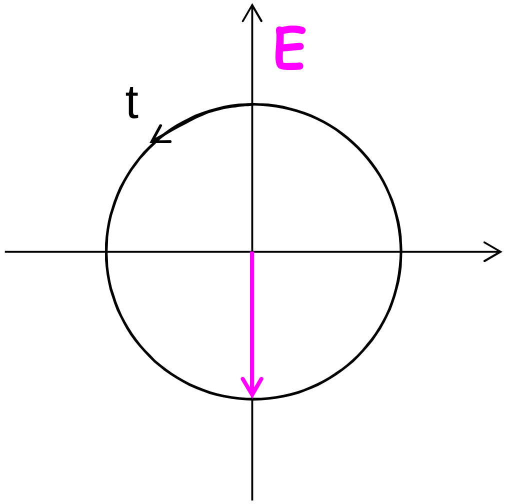
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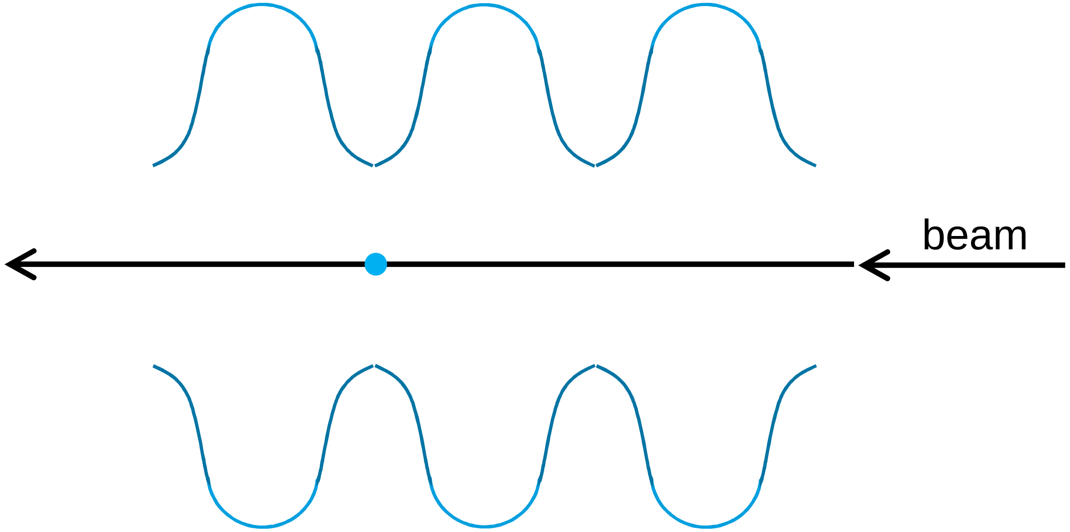
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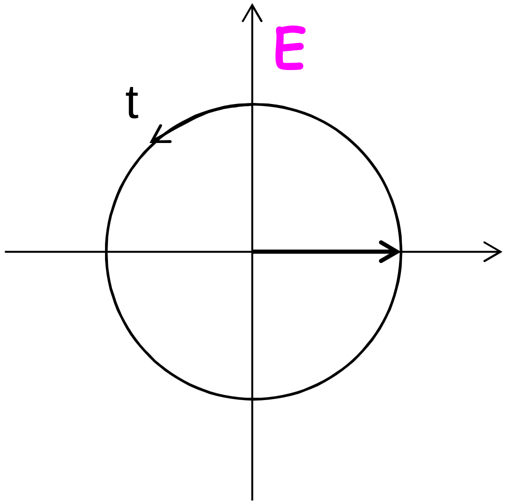
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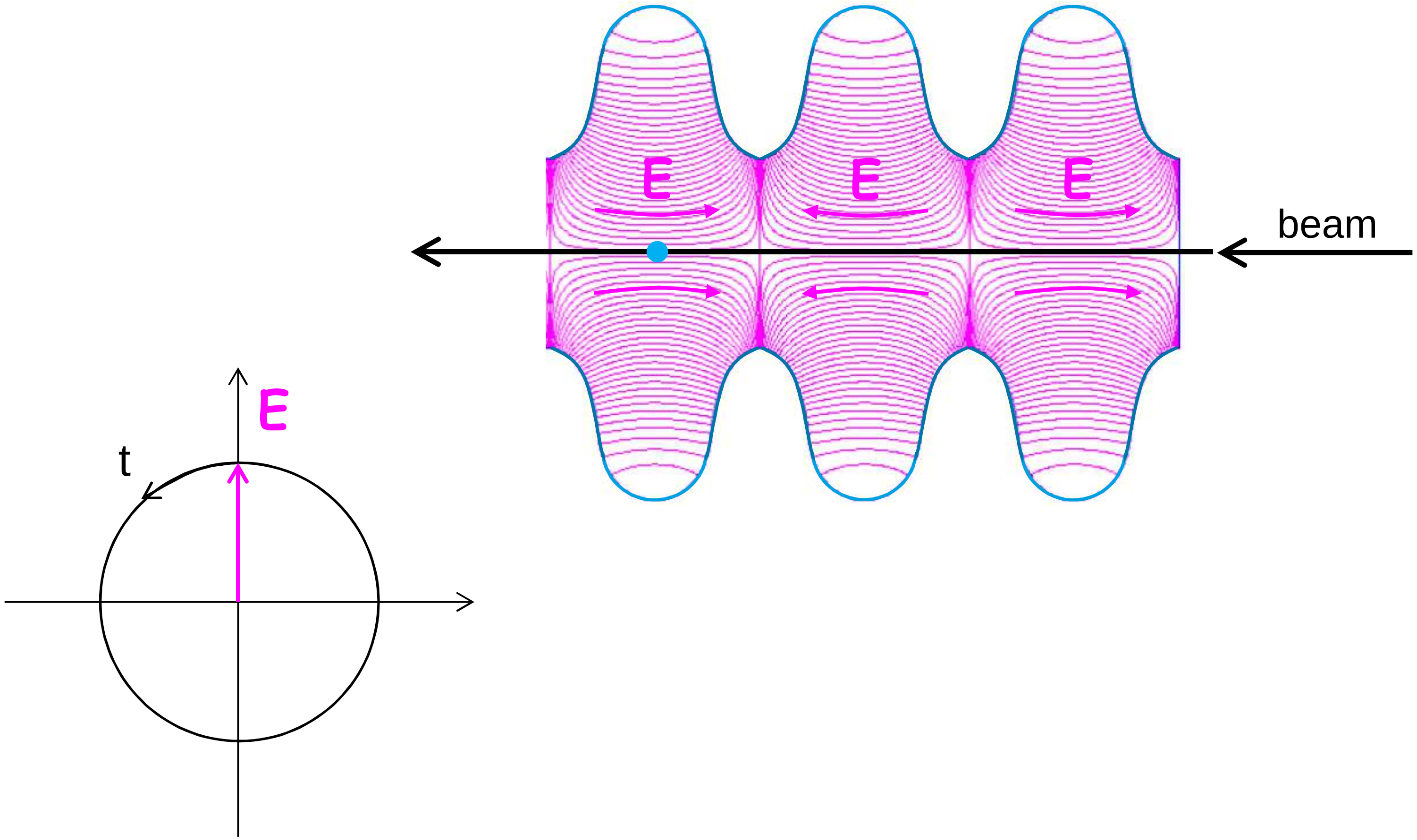
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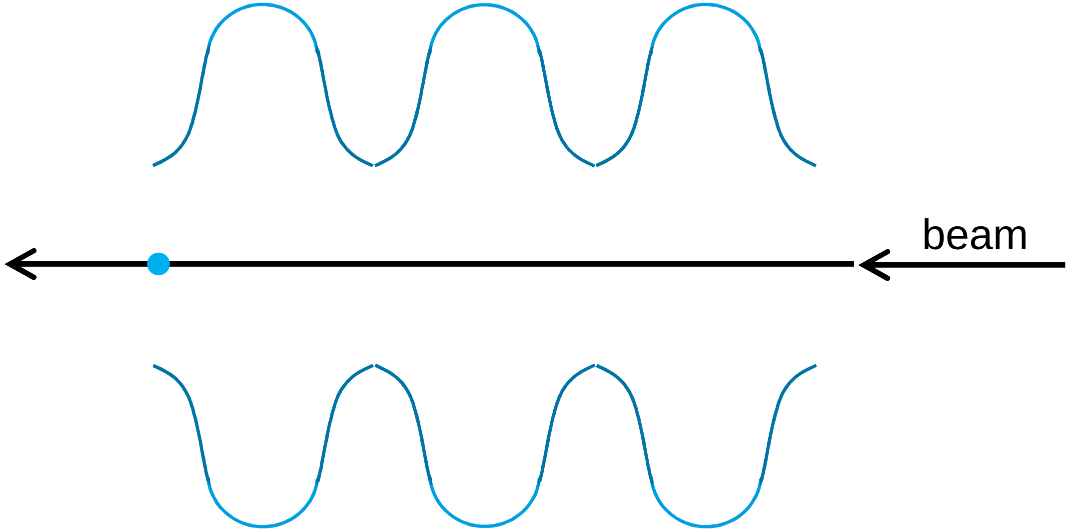
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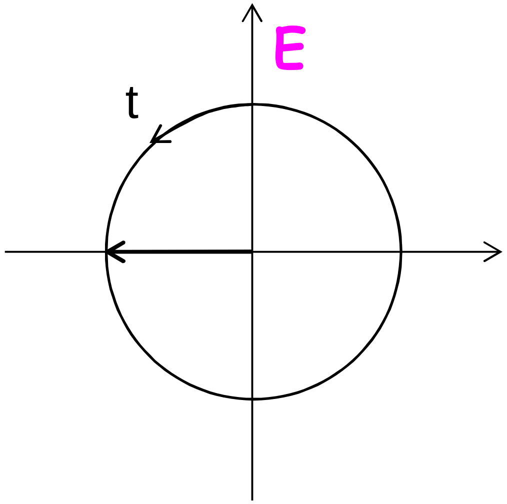
E
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1) Why this shape? …………………………………………… to reduce/avoid multipacting

2) How to feed $\vec{E}$ in? ………………………………………… with input couplers

3) How to measure $\vec{E}$? ………………………………………… with pick up antennas

4) What are HOM couplers for? …………………………… to reduce HOM (wakefields)

5) Is there a net acceleration? ……………………………… timing is the key

Summing-up of this part

Particle acceleration using radio-frequency fields:

basic cavity:
pill box

- analogy to an LC circuit
- infinite number of solutions for $\vec{E}$ and $\vec{B}$
- eq. for the fundamental solution for $\vec{E}$ and $\vec{B}$

superconducting
cavity

- multipacting mitigation
- RF couplers and antennas
- wakefields and HOMs
- FLASH and XFEL

Contact

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