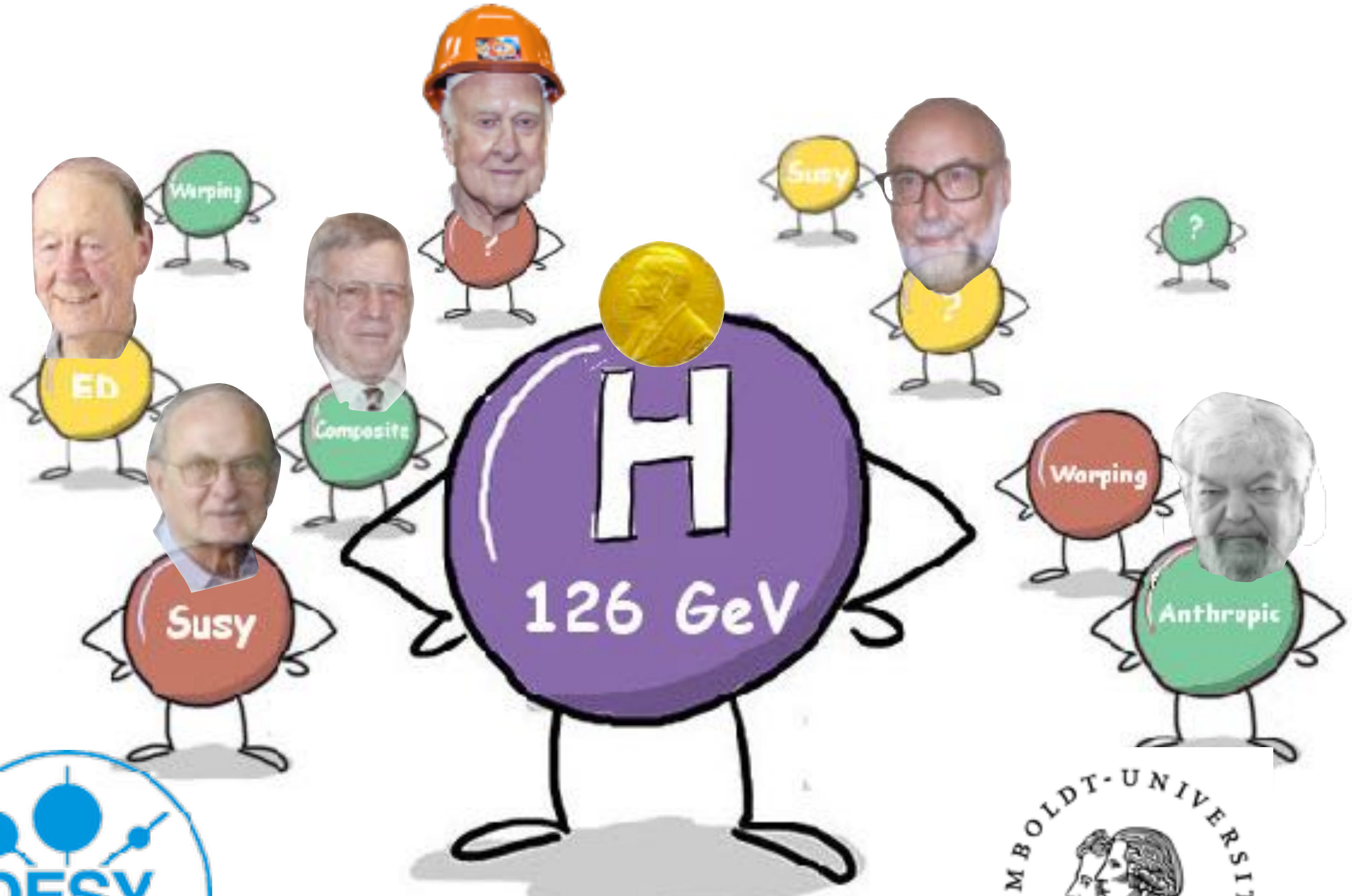
Introduction

HEP Theory

DESY summer student lectures 2018



Lectures 5+6/6

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Recap

1. The same Fermi theory describes different phenomena

2. The Fermi theory is an effective theory valid at low energy only (violation of unitarity).

3. At higher energies, the 4-fermion contact interactions open up and are truly generated by the exchange of a massive gauge boson

4. By analogy with QED, we understood that the weak interactions are associated with the invariance under non-abelian SU(2)xU(1) local symmetries

5. SU(2)xU(1) is spontaneously broken to $U(1)_{em}$ by the vacuum expectation value of the Higgs field

6. Masses for gauge bosons and fermions are generated by the Higgs vev

Homework

1. Show that a single particle state in relativistic QM leads to causality violation

2. Is pion decay mediated by weak interaction? Explain why $\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \sim 10^{-4}$

3. Check the transformation law of a non-abelian gauge field: $A_\mu \to U A_\mu U^{-1} - \frac{i}{g} U \partial_\mu U^{-1}$

4. Check the transformation law of the non-abelian gauge field strength:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \to U F_{\mu\nu} U^{-1}$$

5. Noether theorem: check that the Euler-Lagrange eqs imply that for a scalar theory invariant under U(1) continuous transformation the following current is conserved

$$\partial_\mu \left(\varphi \frac{\delta \mathcal{L}}{\delta \partial_\mu \varphi} \right) = 0$$

6. Prove the Goldstone theorem: for each global symmetry spontaneously broken, there exists a massless boson in the spectrum

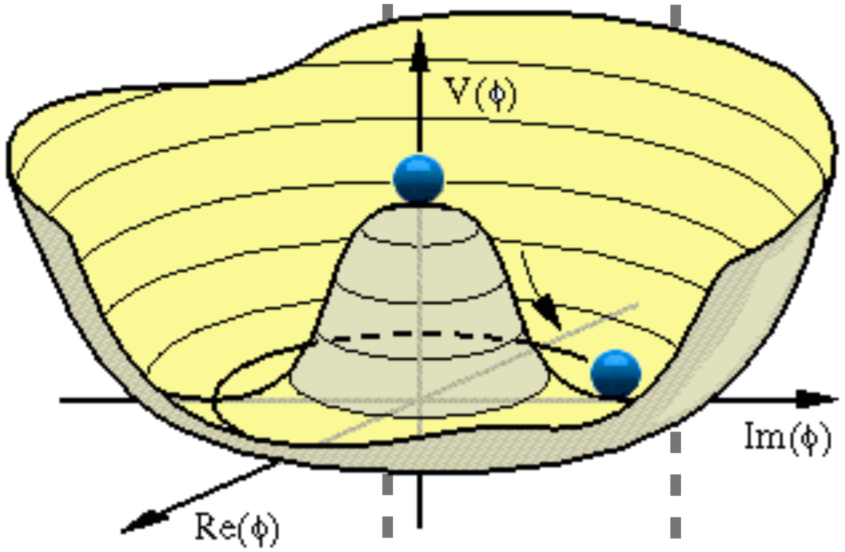
Spontaneous Symmetry Breaking

Symmetry of the Lagrangian

$$SU(2)_L \times U(1)_Y$$

Higgs Doublet

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$



Symmetry of the Vacuum

$$U(1)_{e.m.}$$

Vacuum Expectation Value

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad \text{with} \quad v \approx 246 \text{ GeV}$$

$$D_\mu H = \partial_\mu H - \frac{i}{2} \begin{pmatrix} g W^3_\mu + g' B_\mu & \sqrt{2} g W^+_\mu \\ \sqrt{2} g W^-_\mu & -g W^3_\mu + g' B_\mu \end{pmatrix} H \quad \text{with} \quad \mathrm{W}^\pm_\mu = \tfrac{1}{\sqrt{2}} \left(\mathrm{W}^1_\mu \mp \mathrm{W}^2_\mu \right)$$

$$\left| D_\mu H \right|^2 = \tfrac{1}{4} \, g^2 v^2 \, W^+_\mu W^{-\,\mu} + \tfrac{1}{8} \left(W^3_\mu \; B_\mu \right) \begin{pmatrix} g^2 v^2 & -g g' v^2 \\ -g g' v^2 & g'^2 v^2 \end{pmatrix} \begin{pmatrix} W^{3\,\mu} \\ B^\mu \end{pmatrix}$$

- Gauge boson spectrum
  - electrically charged bosons $\boxed{M_W^2 = \frac{1}{4} g^2 v^2}$
  - electrically neutral bosons $\boxed{\begin{array}{c} Z_\mu = c W^3_\mu - s B_\mu \\ \gamma_\mu = s W^3_\mu + c B_\mu \end{array}}$

Weak mixing angle

$$c = \frac{g}{\sqrt{g^2+g'^2}}$$

$$s = \frac{g'}{\sqrt{g^2+g'^2}}$$

$$\boxed{M_Z^2 = \tfrac{1}{4} (g^2 + g'^2) v^2}$$

$$\boxed{M_\gamma = 0}$$

Interactions Fermions-Gauge Bosons

Gauge invariance says:

$$\mathcal{L} = g W^3_\mu \left(\sum_i T_{3L\,i}\ \bar{\psi}_i \bar{\sigma}^\mu \psi_i \right) + g' B_\mu \left(\sum_i y_i\ \bar{\psi}_i \bar{\sigma}^\mu \psi_i \right)$$

Going to the mass eigenstate basis:

$$Z_\mu = c W^3_\mu - s B_\mu \qquad \text{with} \qquad c = \frac{g}{\sqrt{g^2+g'^2}}$$

$$\gamma_\mu = s W^3_\mu + c B_\mu \qquad\qquad s = \frac{g'}{\sqrt{g^2+g'^2}}$$

$$Q = T_{3L} + Y$$

$$\mathcal{L} = \sqrt{g^2+g'^2} Z_\mu \left(\sum_i (T_{3L\,i} - s^2 Q_i)\ \bar{\psi}_i \bar{\sigma}^\mu \psi_i \right) + \frac{gg'}{\sqrt{g^2+g'^2}} \gamma_\mu \left(\sum_i Q_i\ \bar{\psi}_i \bar{\sigma}^\mu \psi_i \right)$$

protected by U(1)$_{\text{em}}$ gauge invariance
⇒ no correction

not protected by gauge invariance
corrected by radiative corrections + new physics

electric charge

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} = sg = cg'$$

Custodial Symmetry

- Rho parameter

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_w} = \frac{\frac{1}{4}g^2 v^2}{\frac{1}{4}(g^2+g'^2)v^2 \frac{g^2}{g^2+g'^2}} = 1$$

- Consequence of an approximate global symmetry of the Higgs sector

$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$ Higgs doublet = 4 real scalar fields

$V(H) = \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2$ is invariant under the rotation of the four real components

$$SO(4) \sim SU(2)_L \times SU(2)_R$$

$$SU(2)_R$$

$$\Downarrow$$

$$SU(2)_L \Rightarrow \begin{pmatrix} i\sigma^2 H^\star & H \end{pmatrix} = \Phi$$

2x2 matrix

$$\Phi^\dagger \Phi = H^\dagger H \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

$$V(H) = \frac{\lambda}{4}\left(\mathrm{tr}\Phi^\dagger\Phi - \mathrm{v}^2\right)^2$$

explicitly invariant under $SU(2)_L \times SU(2)_R$

Custodial Symmetry

Higgs vev

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad \langle \Phi \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

unbroken symmetry in the broken phase

$$(W^1_\mu, W^2_\mu, W^3_\mu) \quad \text{transforms as a triplet}$$

$$(Z_\mu \, \gamma_\mu) \begin{pmatrix} M_Z^2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} Z^\mu \\ \gamma^\mu \end{pmatrix} = (W^3_\mu \, B_\mu) \begin{pmatrix} c^2 M_Z^2 & -csM_Z^2 \\ -csM_Z^2 & s^2 M_Z^2 \end{pmatrix} \begin{pmatrix} W^{3\,\mu} \\ B^\mu \end{pmatrix}$$

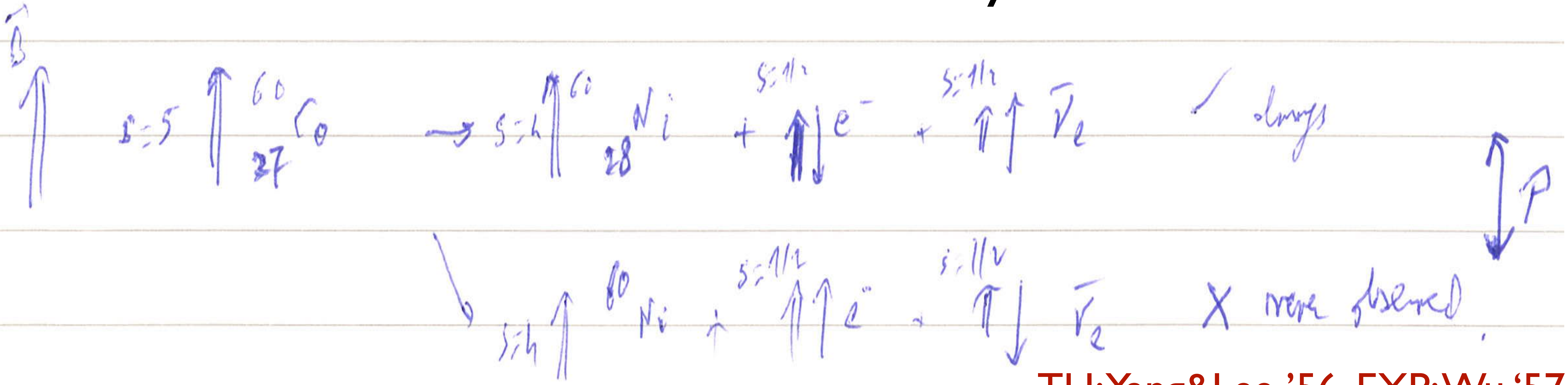
The $SU(2)_V$ symmetry imposes the same mass term for all W^i thus $c^2 M_Z^2 = M_W^2$

ρ = 1

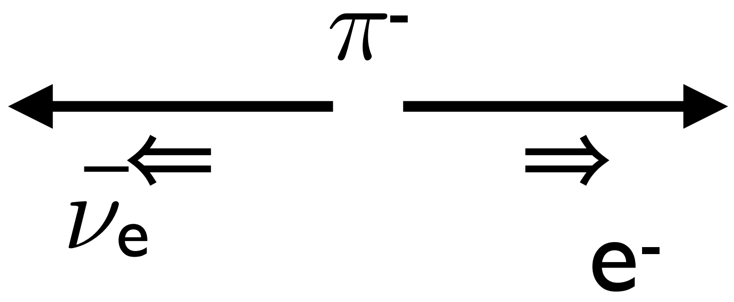
The hypercharge gauge coupling and the Yukawa couplings break the custodial SU(2)$_V$, which will generate a (small) deviation to ρ = 1 at the quantum level.

SM is a chiral theory

Weak interactions maximally violates P



TH: Yang&Lee ’56. EXP: Wu ‘57



Conservation of momentum and spin imposes to have a RH e⁻

Weak decays proceed only w/ LH e⁻
So the amplitude is prop. to m_e

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \propto \frac{m_e^2}{m_\mu^2} \sim 2 \times 10^{-5} \underset{\uparrow}{\sim} 10^{-4}_{\text{obs}}$$

Extra phase-space factor

Fermion Masses

SM is a chiral theory (≠ QED that is vector-like)

$m_e \bar{e}_L e_R + h.c.$ is not gauge invariant

The SM Lagrangian doesn't not contain fermion mass terms
fermion masses are emergent quantities
that originate from interactions with Higgs vev

$$y_{ij} \bar{f}_{L_i} H f_{R_j} = \frac{y_{ij} v}{\sqrt{2}} \bar{f}_{L_i} f_{R_j} + \frac{y_{ij}}{\sqrt{2}} h \bar{f}_{L_i} f_{R_j}$$

Fermion Masses

In SM, the Yukawa interactions are the only source of the fermion masses

$$y_{ij} \bar{f}_{L_i} H f_{R_j} = \frac{y_{ij} v}{\sqrt{2}} \bar{f}_{L_i} f_{R_j} + \frac{y_{ij}}{\sqrt{2}} h \bar{f}_{L_i} f_{R_j}$$

mass

higgs-fermion interactions

both matrices are simultaneously diagonalizable

no tree-level Flavor Changing Current induced by the Higgs

Not true anymore if the SM fermions mix with vector-like partners(*) or for non-SM Yukawa

$$y_{ij} \left(1 + c_{ij} \frac{|H|^2}{f^2}\right) \bar{f}_{L_i} H f_{R_j} = \frac{y_{ij} v}{\sqrt{2}} \left(1 + c_{ij} \frac{v^2}{2f^2}\right) \bar{f}_{L_i} f_{R_j} + \left(1 + 3c_{ij} \frac{v^2}{2f^2}\right) \frac{y_{ij}}{\sqrt{2}} h \bar{f}_{L_i} f_{R_j}$$

Look for SM forbidden Flavor Violating decays h → μτ and h → eτ

(look also at t→hc ATLAS '14)

- weak indirect constrained by flavor data (μ→ eγ): BR<10%
- ATLAS and CMS have the sensitivity to set bounds O(1%)
- ILC/CLIC/FCC-ee can certainly do much better

Blankenburg, Ellis, Isidori '12

Harnik et al '12
Davidson, Verdier '12
CMS-PAS-HIG-2014-005

(*) e.g. Buras, Grojean, Pokorski, Ziegler '11

Fermion Masses

In SM, the Yukawa interactions are the only source of the fermion masses

$$y_{ij}\bar{f}_{L_i} H f_{R_j} = \frac{y_{ij} v}{\sqrt{2}} \bar{f}_{L_i} f_{R_j} + \frac{y_{ij}}{\sqrt{2}} h \bar{f}_{L_i} f_{R_j}$$

mass

higgs-fermion interactions

both matrices are simultaneously diagonalizable

⇓ ⇓ ⇓

no tree-level Flavor Changing Current induced by the Higgs

Quark mixings

$$\mathcal{L}_{Yuk} \quad = \quad \lambda^L_{ij}(\bar{L}^i_L \phi^c) l^j_R + \lambda^U_{ij}(\bar{Q}^i_{L,\alpha}\phi) u^j_{R,\alpha} + \lambda^D_{ij}(\bar{Q}^i_{L,\alpha}\phi^c) d^j_{R,\alpha} + cc$$

$$\mathcal{L}^\dagger_L \left(\frac{v}{\sqrt{2}}\lambda^L\right)\mathcal{L}_R = \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix}$$

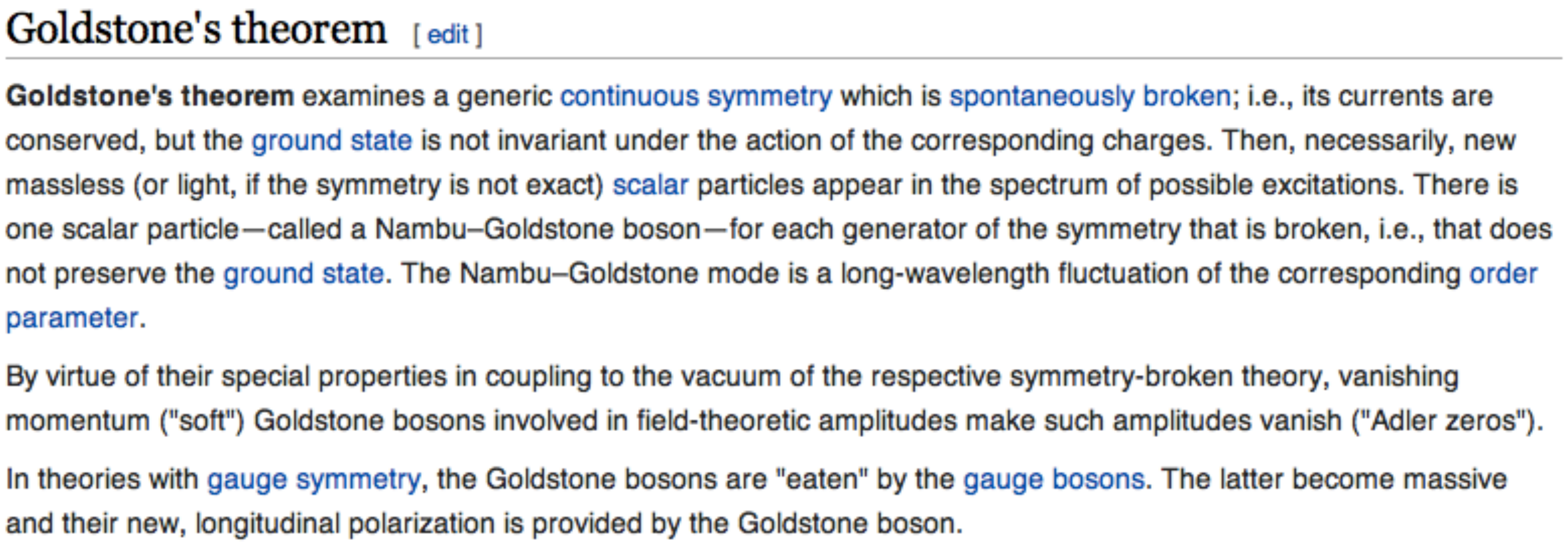
$$\mathcal{U}^\dagger_L \left(\frac{-v}{\sqrt{2}}\lambda^U\right)\mathcal{U}_R = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}$$

$$\mathcal{D}^\dagger_L \left(\frac{v}{\sqrt{2}}\lambda^D\right)\mathcal{D}_R = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_{Yuk\,quad} = & - \left(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L\right)\begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix}\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \\ & - \left(\bar{u}_{L,\alpha}, \bar{c}_{L,\alpha}, \bar{t}_{L,\alpha}\right)\begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}\begin{pmatrix} u_{R,\alpha} \\ c_{R,\alpha} \\ t_{R,\alpha} \end{pmatrix} \\ & - \left(\bar{d}_{L,\alpha}, \bar{s}_{L,\alpha}, \bar{b}_{L,\alpha}\right)\mathcal{V}^\dagger_{KM}\begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}\begin{pmatrix} d_{R,\alpha} \\ s_{R,\alpha} \\ b_{R,\alpha} \end{pmatrix} \\ & + \quad cc \end{aligned}$$

$$\mathcal{V}_{KM} = \mathcal{D}^\dagger_L \mathcal{U}_L$$

Goldstone Theorem



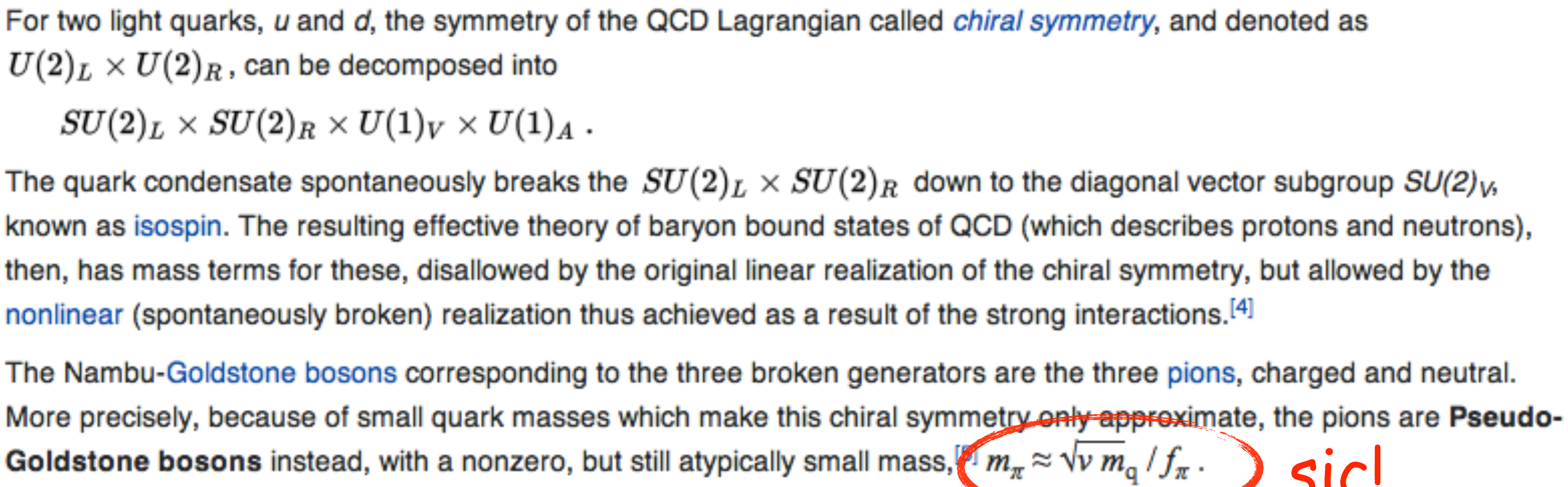
Goldstone's theorem [edit]

Goldstone's theorem examines a generic continuous symmetry which is spontaneously broken; i.e., its currents are conserved, but the ground state is not invariant under the action of the corresponding charges. Then, necessarily, new massless (or light, if the symmetry is not exact) scalar particles appear in the spectrum of possible excitations. There is one scalar particle—called a Nambu–Goldstone boson—for each generator of the symmetry that is broken, i.e., that does not preserve the ground state. The Nambu–Goldstone mode is a long-wavelength fluctuation of the corresponding order parameter.

By virtue of their special properties in coupling to the vacuum of the respective symmetry-broken theory, vanishing momentum ("soft") Goldstone bosons involved in field-theoretic amplitudes make such amplitudes vanish ("Adler zeros").

In theories with gauge symmetry, the Goldstone bosons are "eaten" by the gauge bosons. The latter become massive and their new, longitudinal polarization is provided by the Goldstone boson.

QCD example:



For two light quarks, *u* and *d*, the symmetry of the QCD Lagrangian called *chiral symmetry*, and denoted as $U(2)_L \times U(2)_R$, can be decomposed into

$$SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A \, .$$

The quark condensate spontaneously breaks the $SU(2)_L \times SU(2)_R$ down to the diagonal vector subgroup $SU(2)_V$, known as isospin. The resulting effective theory of baryon bound states of QCD (which describes protons and neutrons), then, has mass terms for these, disallowed by the original linear realization of the chiral symmetry, but allowed by the nonlinear (spontaneously broken) realization thus achieved as a result of the strong interactions.[4]

The Nambu-Goldstone bosons corresponding to the three broken generators are the three pions, charged and neutral. More precisely, because of small quark masses which make this chiral symmetry only approximate, the pions are **Pseudo-Goldstone bosons** instead, with a nonzero, but still atypically small mass,[5] $m_\pi \approx \sqrt{v\, m_q} \, / \, f_\pi$.

sic!

Goldstone Boson

$U(1)$

$$\phi \to e^{i\alpha}\phi$$

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - \lambda \left(\left|\phi^2\right|^2 - \frac{f^2}{2} \right)^2$$

$$\phi = \frac{1}{\sqrt{2}} \left(f + h(x) \right) e^{i\theta(x)/f}$$

$$h \to h$$

$$\theta \to \theta + \alpha f$$

$U(1)$ non-linearly realized

shift symmetry forbids any mass term for θ

$$\mathcal{L} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\left(\frac{f+h}{f}\right)^2 \partial_\mu \theta \partial^\mu \theta - \lambda \left(f^2 h^2 + f h^3 + \frac{1}{4} h^4 \right)$$

θ remains a massless field

== Goldstone boson ==

To each continuous global symmetry spontaneously broken corresponds a massless field

If the U(1) symmetry is gauged, the Goldstone boson is eaten and it becomes the longitudinal component of the massive gauge boson

Example of Uneaten Goldstone Bosons

$$SU(N) \to SU(N-1)$$

$$(N^2-1)-((N-1)^2-1) = 2N-1$$ Goldstone bosons

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ f \end{pmatrix} \qquad \phi = \exp\left(\frac{i}{f} \left(\begin{array}{ccc|c} -\pi_0 & & & \pi_1 \\ & \ddots & & \vdots \\ & & -\pi_0 & \pi_{N-1} \\ \hline \pi_1^\star & \dots & \pi_{N-1}^\star & (N-1)\pi_0 \end{array} \right) \right) \begin{pmatrix} 0 \\ \vdots \\ 0 \\ f \end{pmatrix}$$

$$\phi = e^{i\pi} \phi_0$$

(N-1) complex, $\vec{\pi}$, and 1 real, π_0, scalars

Let us assume that only SU(N-1) is gauged: then the Goldstone are uneaten.

$$SU(N-1)$$

$$\phi \to U_{N-1}\phi = U_{N-1} e^{i\pi} U_{N-1}^\dagger U_{N-1} \phi_0 = e^{i U_{N-1} \pi U_{N-1}^\dagger} \phi_0$$

$$\pi \to \left(\begin{array}{c|c} U_{N-1} & \\ \hline & 1 \end{array} \right) \left(\begin{array}{c|c} \pi_0 & \pi \\ \hline \pi^\dagger & \pi_0 \end{array} \right) \left(\begin{array}{c|c} U_{N-1}^\dagger & \\ \hline & 1 \end{array} \right) = \left(\begin{array}{c|c} \pi_0 & U_{N-1}\pi \\ \hline \pi^\dagger U_{N-1}^\dagger & \pi_0 \end{array} \right)$$

linear transformations

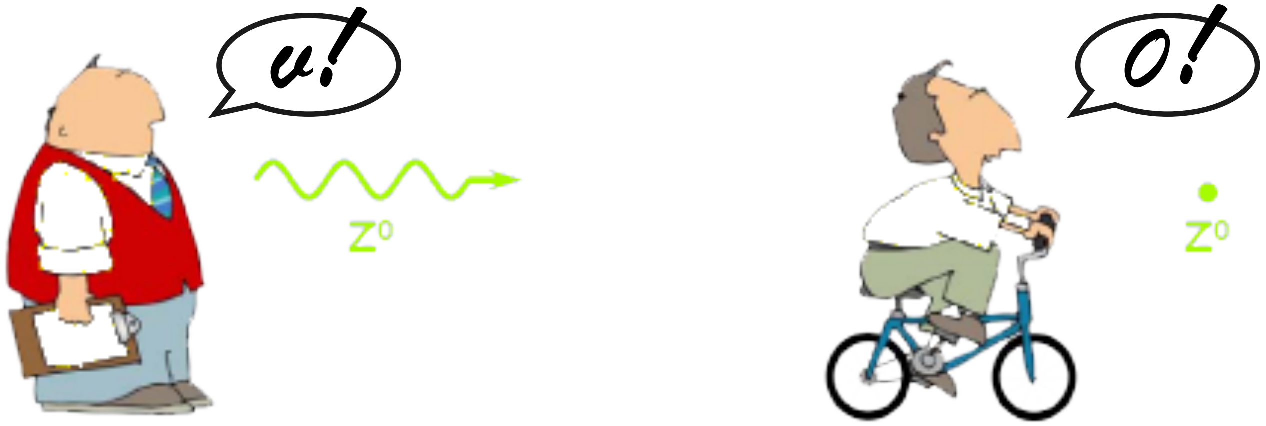
$$\frac{SU(N)}{SU(N-1)}$$

$$\phi \to \exp\left(i \left(\begin{array}{c|c} & \vec{\alpha} \\ \hline \vec{\alpha}^\dagger & \end{array} \right) \right) \exp\left(i \left(\begin{array}{c|c} & \vec{\pi} \\ \hline \vec{\pi}^\dagger & \end{array} \right) \right) \phi_0 \approx \exp\left(i \left(\begin{array}{c|c} & \vec{\pi}+\vec{\alpha} \\ \hline \vec{\pi}^\dagger + \vec{\alpha}^\dagger & \end{array} \right) \right) \phi_0$$

non-linear transformations

The longitudinal polarization of massive W, Z

a massless particle is never at rest: always possible to distinguish (and eliminate!) the longitudinal polarization



the longitudinal polarization is physical for a massive spin-1 particle

(pictures: courtesy of G. Giudice)

symmetry breaking: new phase with more degrees of freedom

$$\epsilon_{\parallel} = \left(\frac{|\vec{p}|}{M}, \frac{E}{M} \frac{\vec{p}}{|\vec{p}|} \right)$$ polarization vector grows with the energy

The longitudinal polarization of massive W, Z

a massless particle is never at rest: cl o distinguish

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3=2+1

Guralnik et al '64

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Longitudinal polarization of a massive spin 1

a massive spin 1 particle has 3 physical polarizations:

$$A_\mu = \epsilon_\mu \; e^{ik_\mu x^\mu}$$

$$\epsilon^\mu \epsilon_\mu = -1 \qquad k^\mu \epsilon_\mu = 0$$

$$k^\mu = (E, 0, 0, k)$$

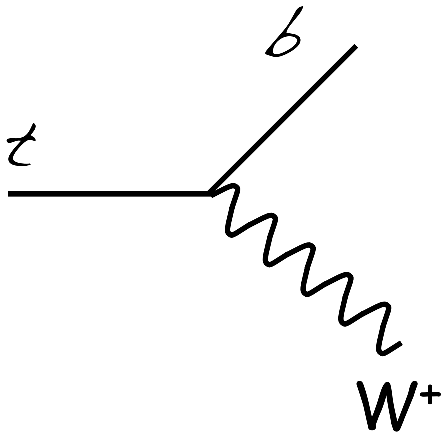
with $k_\mu k^\mu = E^2 - k^2 = M^2$

- 2 transverse: $\begin{cases} \epsilon_1^\mu = (0,1,0,0) \\ \epsilon_2^\mu = (0,0,1,0) \end{cases}$
- 1 longitudinal: $\epsilon_\parallel^\mu = (\frac{k}{M}, 0, 0, \frac{E}{M}) \approx \frac{k^\mu}{M} + \mathcal{O}(\frac{E}{M})$

(in the R-ξ gauge, the time-like polarization ($\epsilon^\mu \epsilon_\mu = 1 \quad k^\mu \epsilon_\mu = M$) is arbitrarily massive and decouple)

The BEH mechanism: “V_L=Goldstone bosons”

At high energy, the physics of the gauge bosons becomes simple



$$\Gamma(t \to bW_L) = \frac{g^2}{64\pi} \boxed{\frac{m_t^2}{m_W^2}} \frac{(m_t^2 - m_W^2)^2}{m_t^3}$$

$$\Gamma(t \to bW_T) = \frac{g^2}{64\pi} \frac{2(m_t^2 - m_W^2)^2}{m_t^3}$$

- at threshold ($m_t \sim m_W$)
  democratic decay
- at high energy ($m_t \gg m_W$)
  W_L dominates the decay

At high energy, the dominant degrees of freedom are W_L

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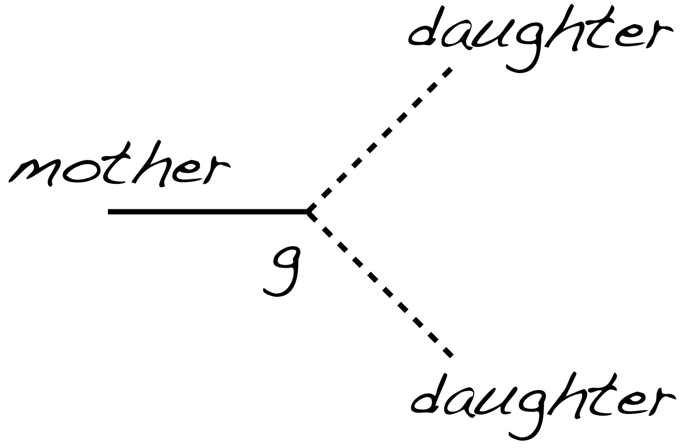
~~ why you should be stunned by this result: ~~



we expect:
(dimensional analysis)

$$\Gamma \sim g^2\, m_{\rm mother}$$

instead $\Gamma \propto m_{\rm mother}^3$ means $g \propto m$ like the Higgs couplings!

very efficient way to suck up energy from the mother particle

$$\tau \ll \tau_{\rm naive}$$

LEP already established the BEH mechanism
The pending question was: how is it realized?
Via a fundamental EW doublet? A la technicolor?
Is there a Higgs boson in addition to the 3 Goldstone bosons?

In other words, LEP established a simple description of the electroweak sector for E >> m_W.

This description is valid for $m_W \ll E \ll 4\pi v = \dfrac{8\pi m_W}{g}$

The goal of the LHC was/is to understand what comes next

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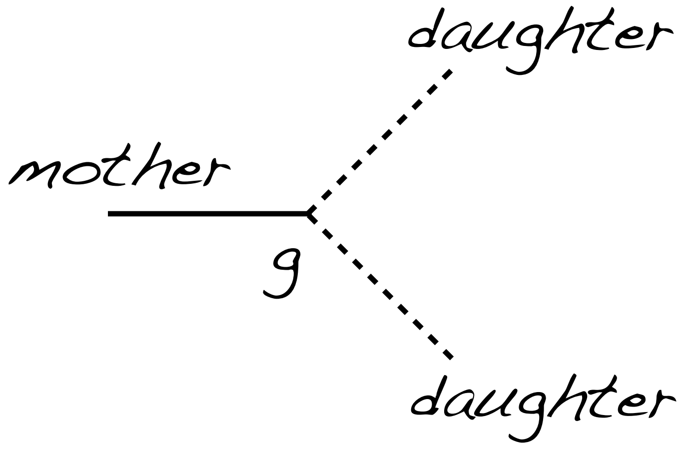

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Goldstone equivalence theorem
$W^{\pm}_L$, Z_L ≈ SO(4)/SO(3)

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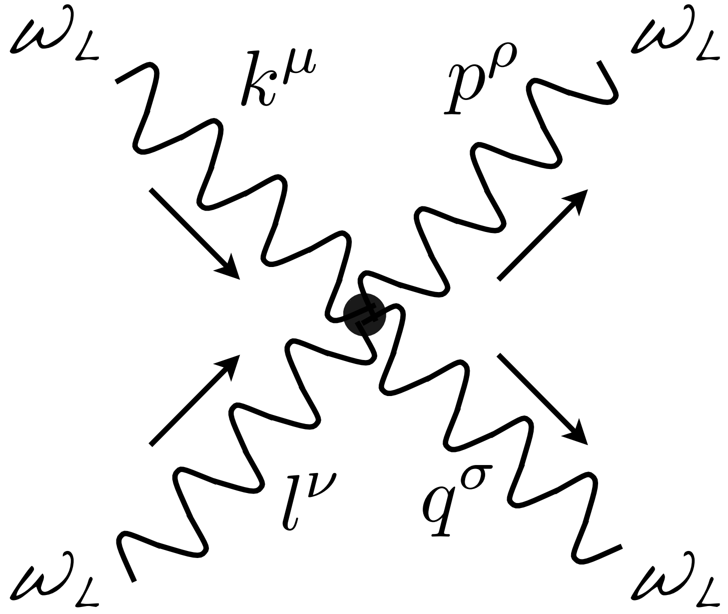
Call for extra degrees of freedom

NO LOSE THEOREM

Bad high-energy behavior for the scattering of the longitudinal polarizations

$$\mathcal{A} = \epsilon_{\|}^{\mu}(k)\epsilon_{\|}^{\nu}(l) g^2 \left(2\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho}\right) \epsilon_{\|}^{\rho}(p)\epsilon_{\|}^{\sigma}(q)$$

$$\mathcal{A} = g^2 \frac{E^4}{4M_W^4}$$



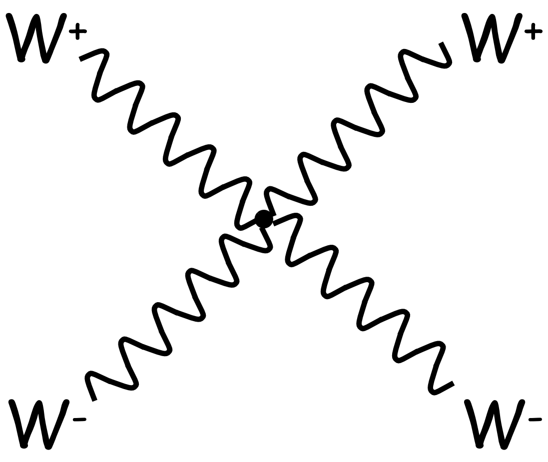
violations of perturbative unitarity around E ~ M/√g (actually M/g)

Extra degrees of freedom are needed to have a good description of the W and Z masses at higher energies

numerically: E ~ 3 TeV ⬅ the LHC was sure to discover something!

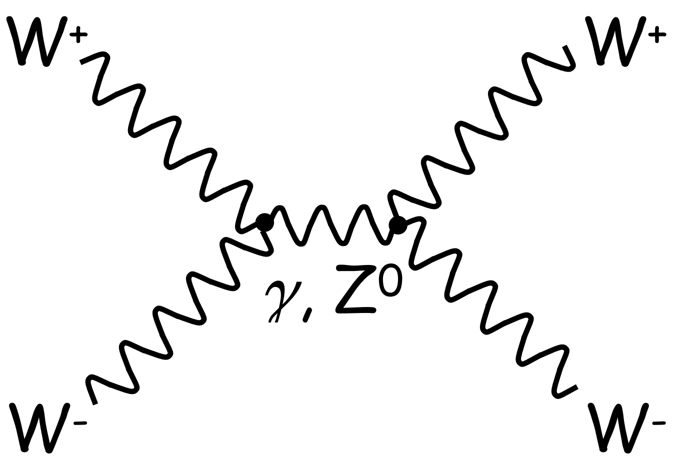
$M_W/\sqrt{(g/4\pi)}\sim 500\text{GeV}$ or $M_W/(g/4\pi)\sim 3\text{TeV}$?

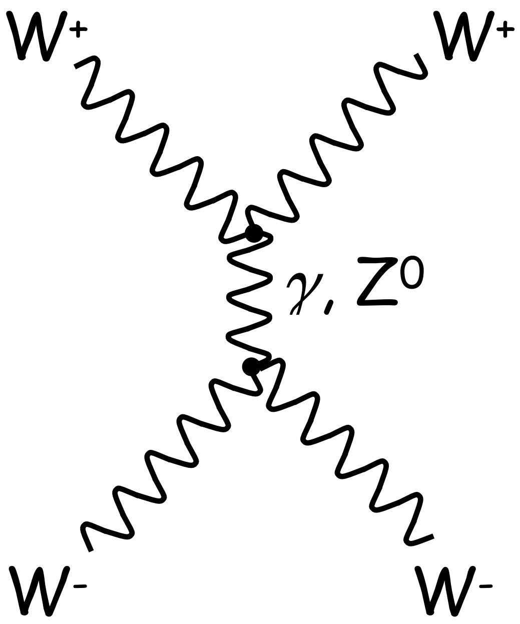
Lewellyn Smith ‘73
Dicus, Mathur ‘73
Cornwall, Levin, Tiktopoulos ’73



$$\mathcal{A} = g^2 \left(\frac{E}{M_W}\right)^4$$

+





+

$$\mathcal{A} = -g^2 \left(\frac{E}{M_W}\right)^4$$

$$\mathcal{A} = g^2 \left(\frac{E}{M_W}\right)^2$$

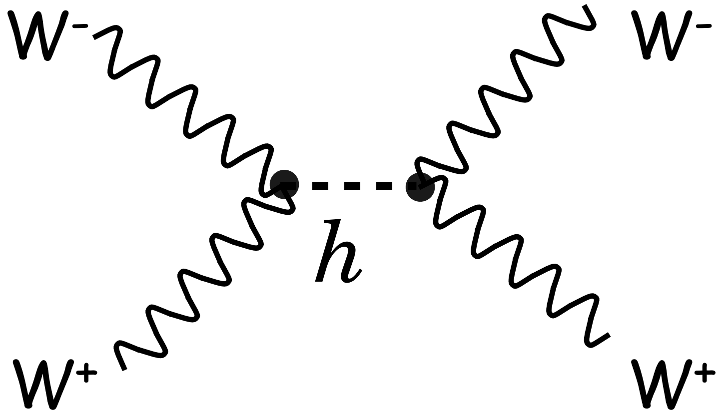
impossible to further cancel the amplitude without introducing new degrees of freedom

What is the SM Higgs?

A single scalar degree of freedom that couples to the mass of the particles

$$\mathcal{L}_{\text{EWSB}} = m_W^2 W_\mu^+ W_\mu^+ \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2}\right) - m_\psi \bar{\psi}_L \psi_R \left(1 + c\frac{h}{v}\right)$$

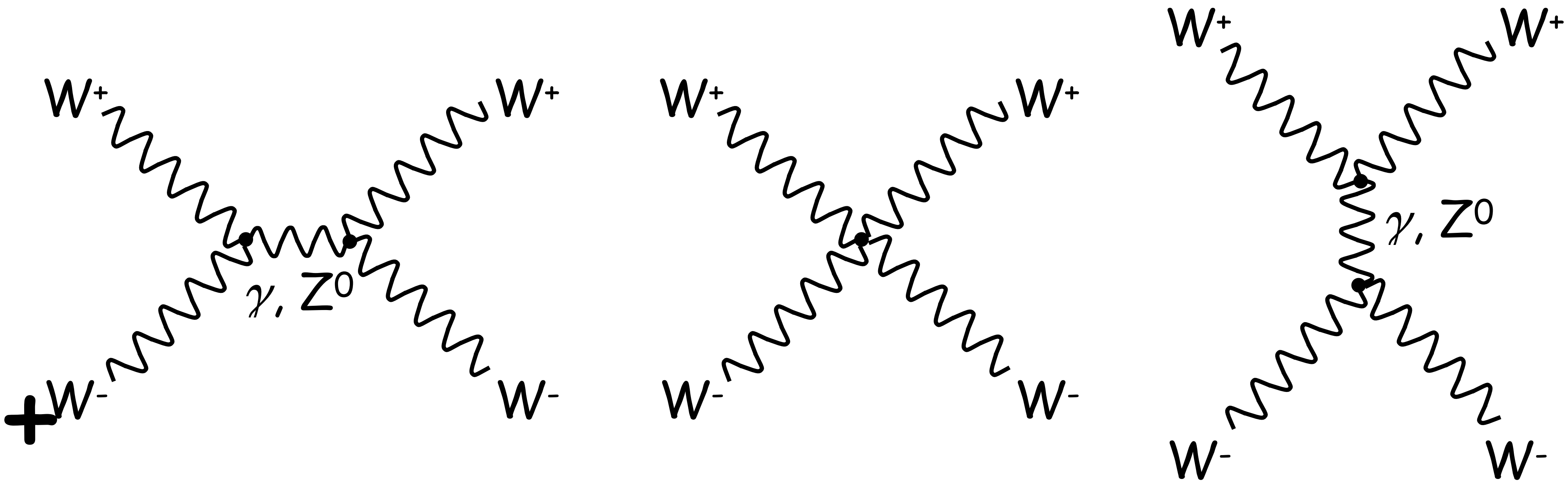
'a', 'b' and 'c' are arbitrary free couplings



$$\mathcal{A} = \frac{1}{v^2}\left(s - \frac{a^2 s^2}{s - m_h^2}\right)$$

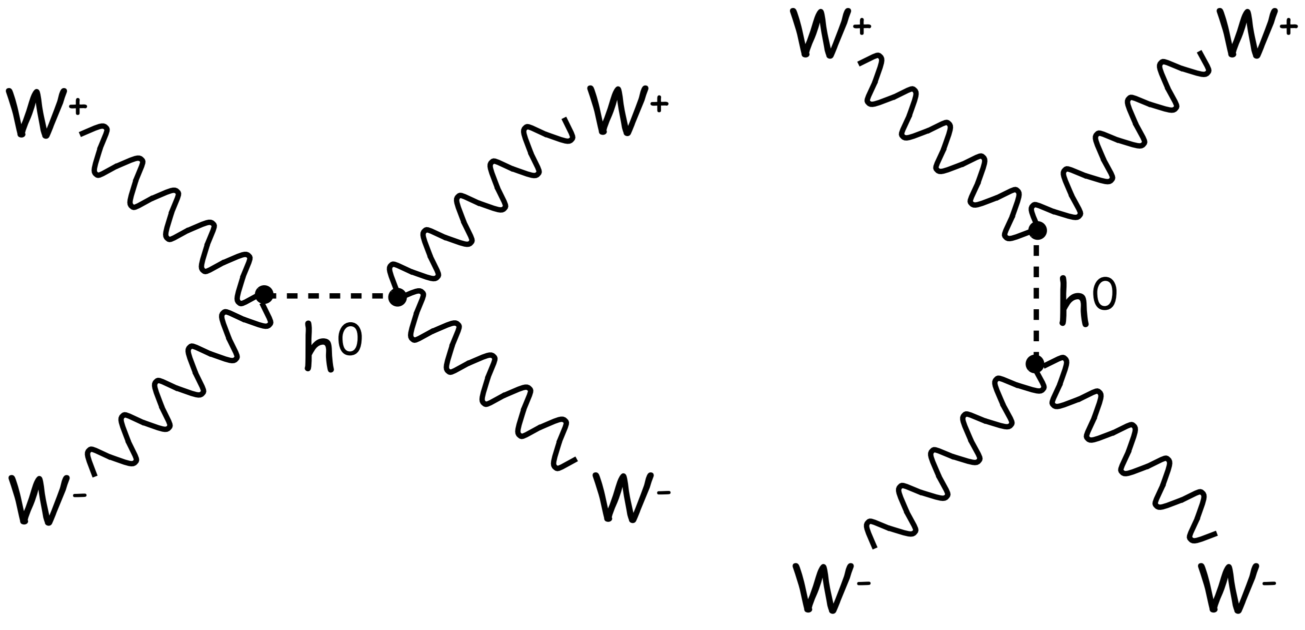
growth cancelled for
a = 1
restoration of
perturbative unitarity

What is the SM Higgs?



$$\mathcal{A} = g^2 \left(\frac{E}{M_W} \right)^2$$

+



$$\mathcal{A} = -g^2 \left(\frac{E}{M_W} \right)^2$$

$$\mathcal{A} = g^2 \left(\frac{M_H}{2M_W} \right)^2$$

The Higgs boson unitarizes the W scattering
(if its mass is below ~ 1 TeV)

Lee, Quigg, Thacker '77

What is the Higgs the name of?

A single scalar degree of freedom that couples to the mass of the particles

$$\mathcal{L}_{\rm EWSB} = m_W^2 W_\mu^+ W_\mu^+ \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2}\right) - m_\psi \bar{\psi}_L \psi_R \left(1 + c\frac{h}{v}\right)$$

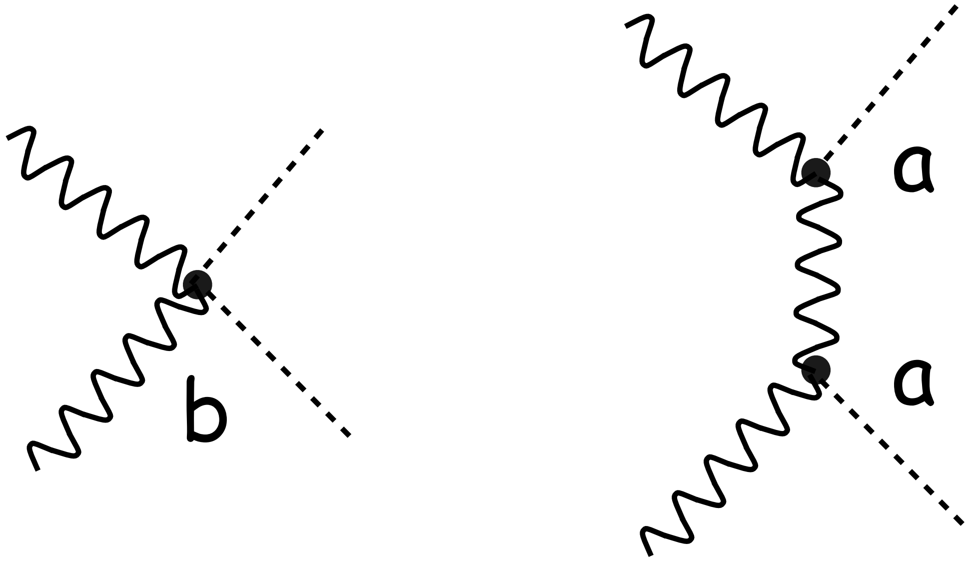
'a', 'b' and 'c' are arbitrary free couplings

For a=1: perturbative unitarity in elastic channels WW → WW

For b = a^2: perturbative unitarity in inelastic channels WW → hh

Cornwall, Levin, Tiktopoulos '73

Contino, Grojean, Moretti, Piccinini, Rattazzi '10



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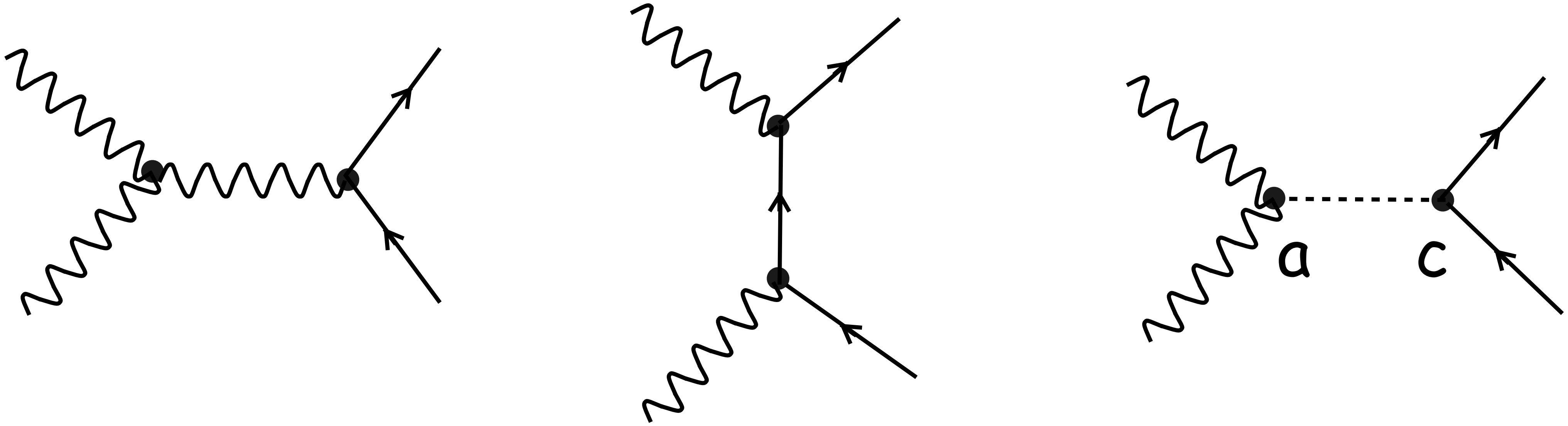
For ac=1: perturbative unitarity in inelastic WW → $\psi\,\psi$

Cornwall, Levin, Tiktopoulos '73

Contino, Grojean, Moretti, Piccinini, Rattazzi '10



What is the Higgs the name of?

A single scalar degree of freedom that couples to the mass of the particles

$$\mathcal{L}_{\mathrm{EWSB}} = m_W^2 W_\mu^+ W_\mu^+ \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2}\right) - m_\psi \bar{\psi}_L \psi_R \left(1 + c\frac{h}{v}\right)$$

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For a=1: perturbative unitarity in elastic channels WW → WW

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For ac=1: perturbative unitarity in inelastic WW → $\psi\,\psi$

Cornwall, Levin, Tiktopoulos '73

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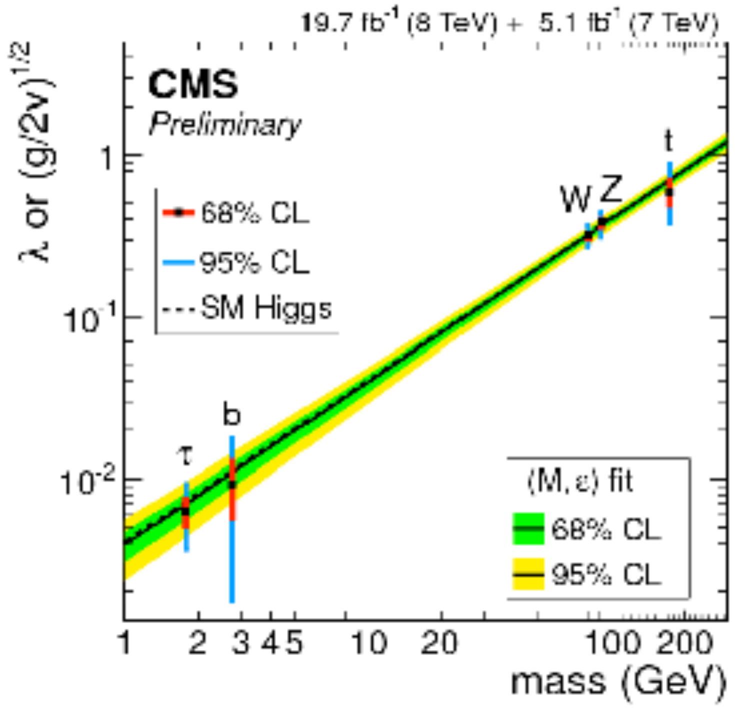


CMS PAS-HIG-14-009

Higgs couplings
are proportional
to the masses of the particles

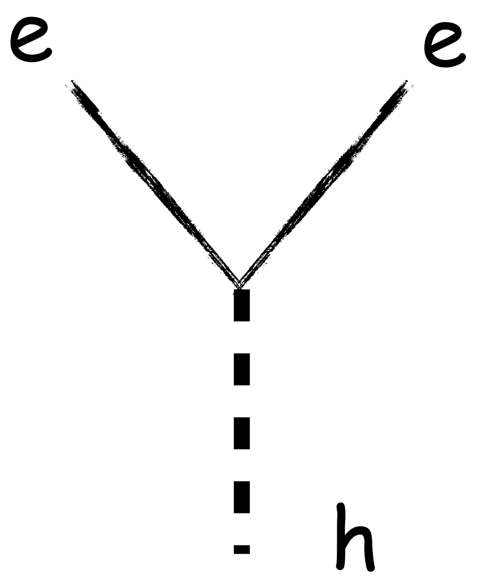
$$\lambda_\psi \propto \frac{m_\psi}{v}\,, \qquad \lambda_V^2 \equiv \frac{g_{VVh}}{2v} \propto \frac{m_V^2}{v^2}$$

Higgs boson at the LHC

producing a Higgs boson is a rare phenomenon
since its interactions with particles are proportional to masses
and ordinary matter is made of light elementary particles

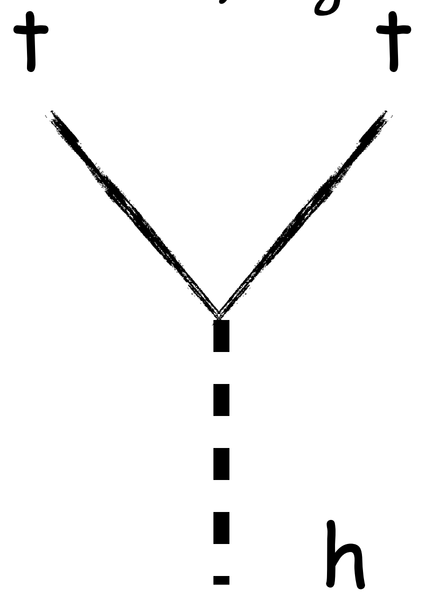
NB: the proton is not an elementary particle,
its mass doesn't measure its interaction with the Higgs substance

From electrons



probability ~ 10^{-11}

From top quarks



probability ~ 1

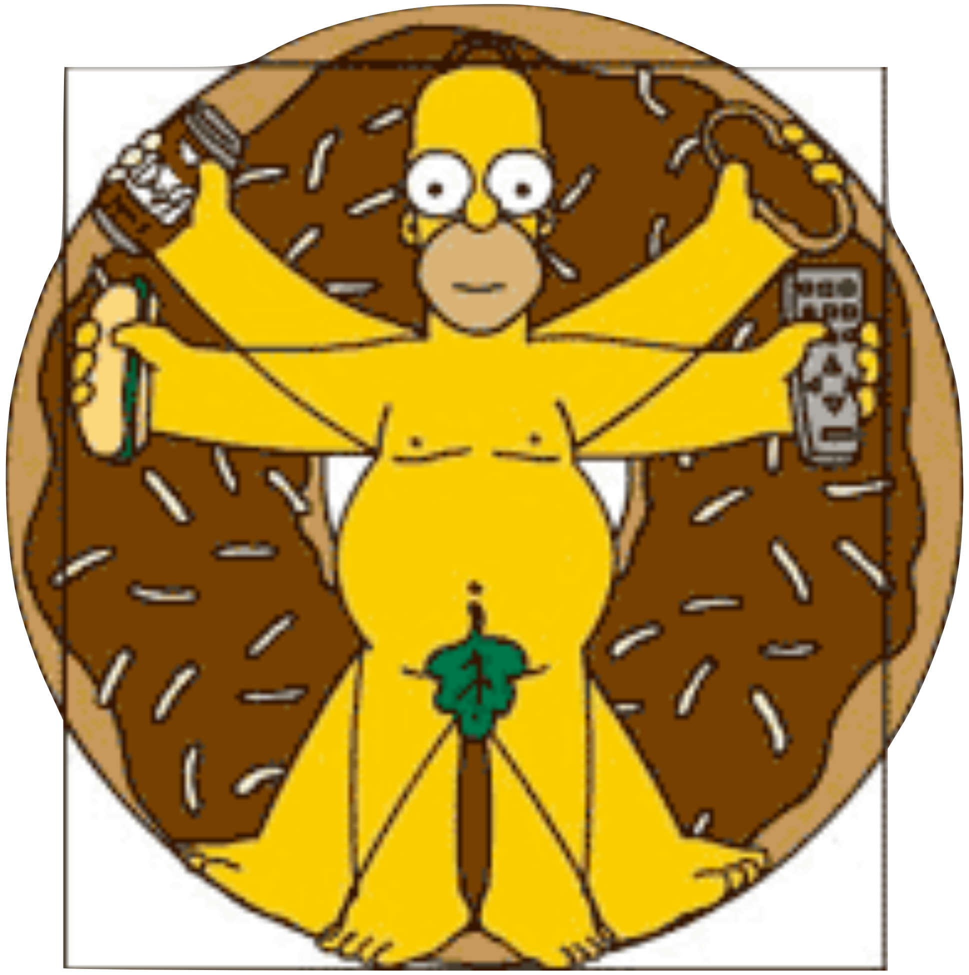
but no top quark at our disposal

Higgs boson at the LHC

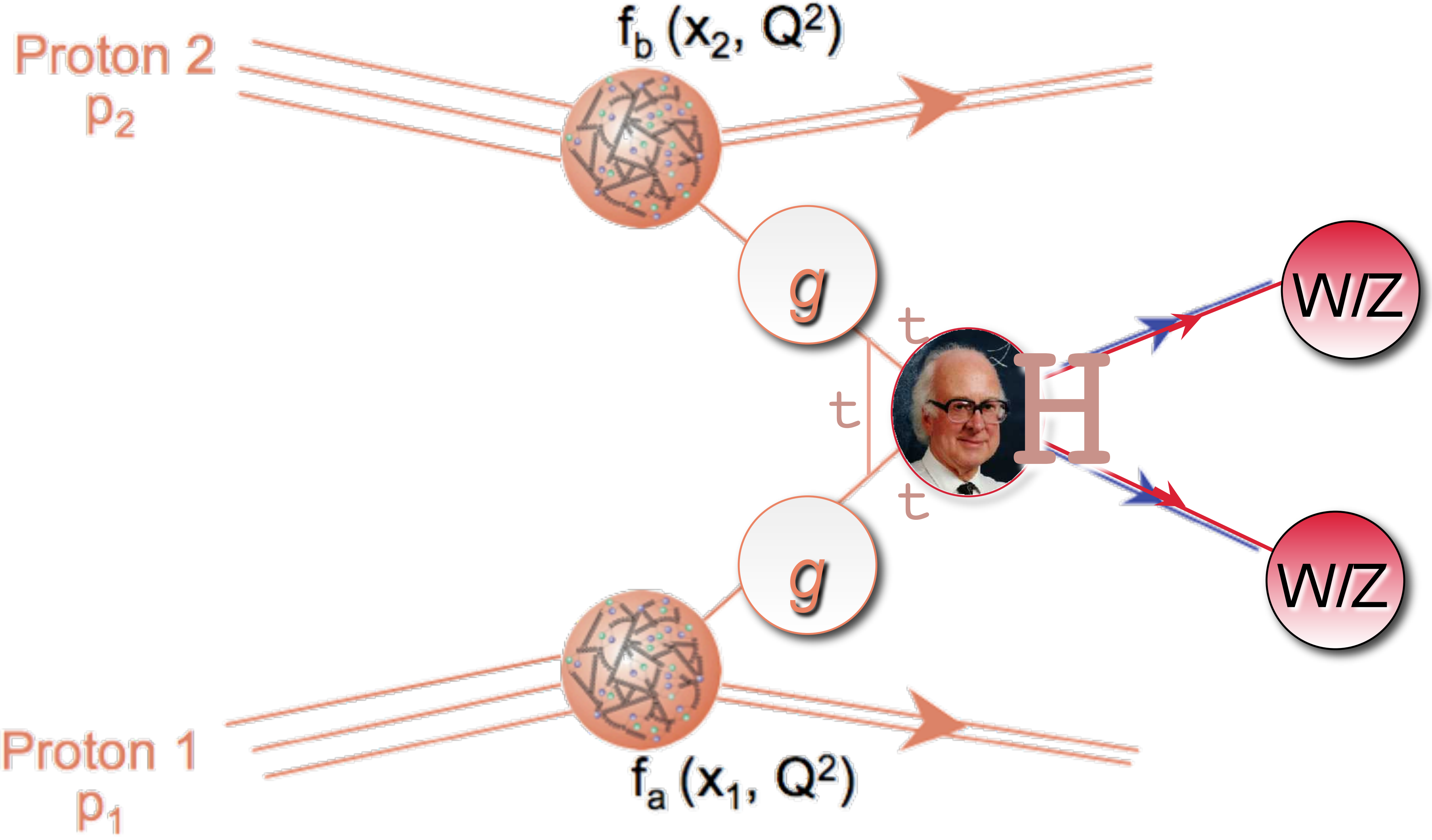
Difficult task

Homer Simpson's principle of life:

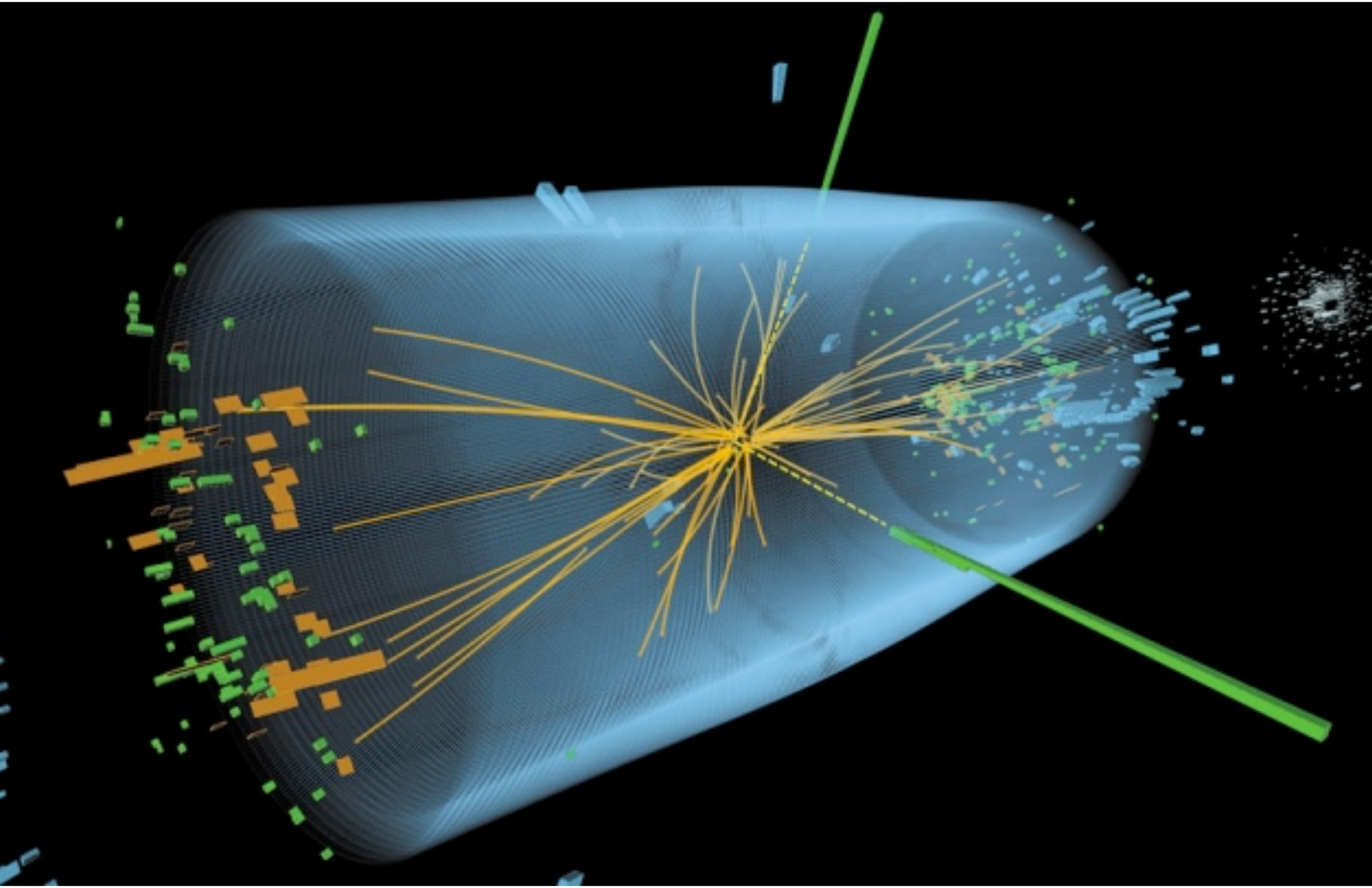
If something's hard to do, is it worth doing?



Higgs boson at the LHC



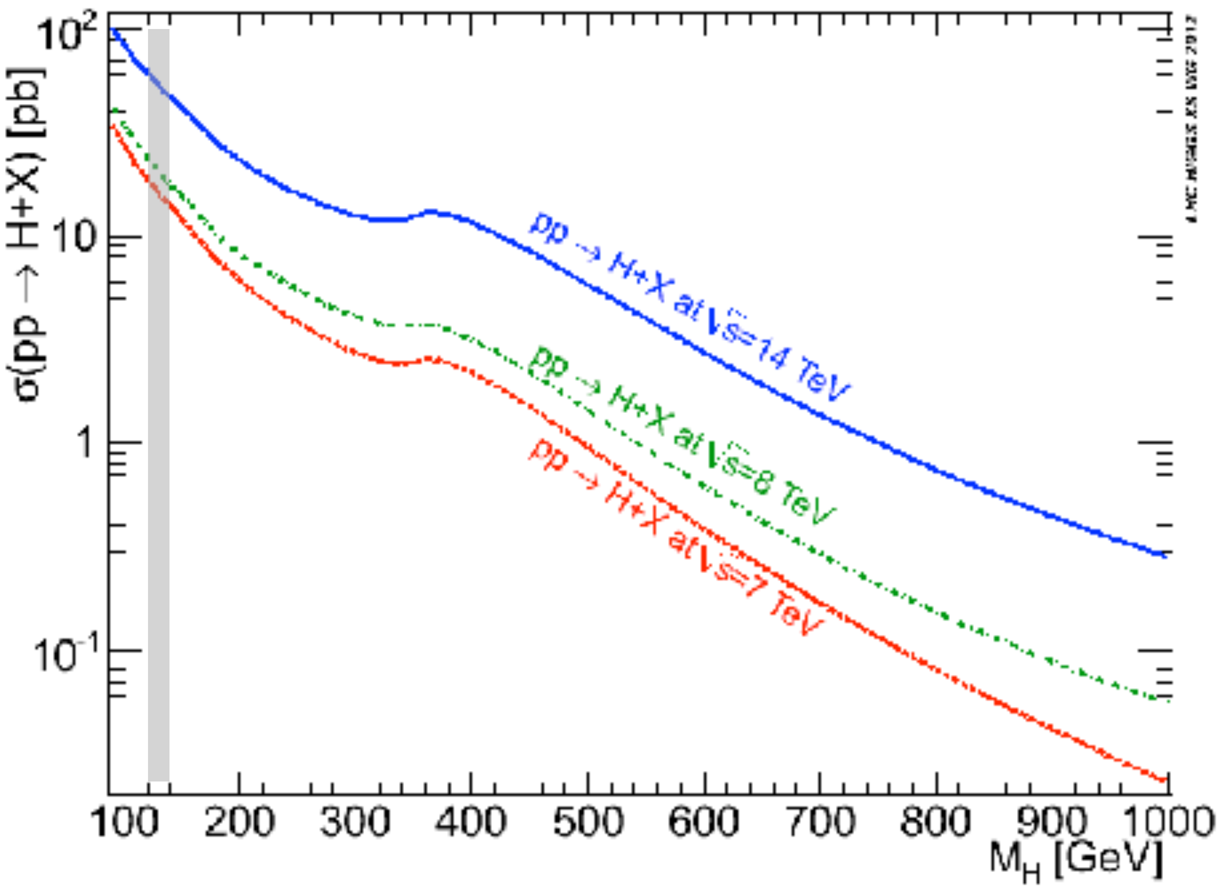
Higgs boson at the LHC



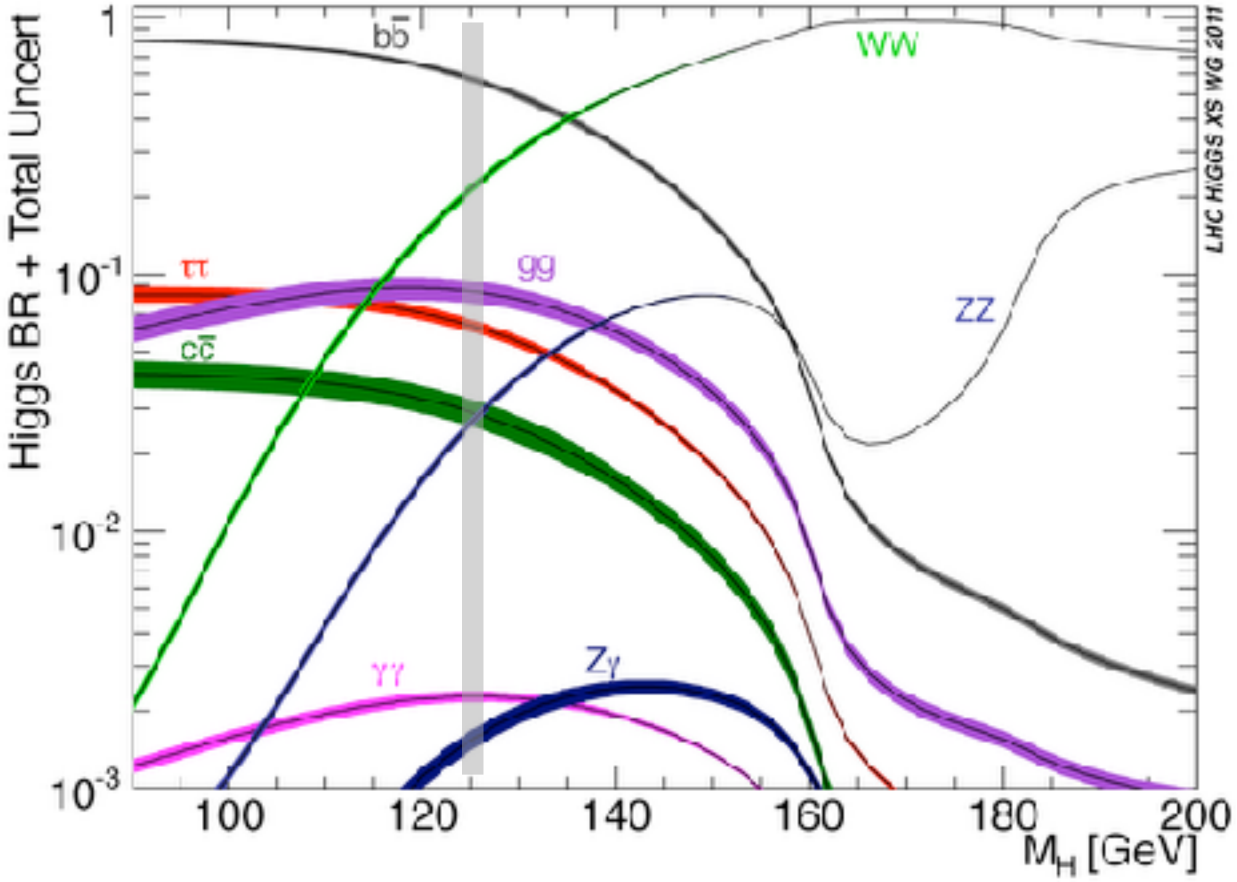
Higgs boson at the LHC

$\sigma \sim$ 10 pb ⇔ 10^5 events for L=10 fb^{-1}

Higgs production



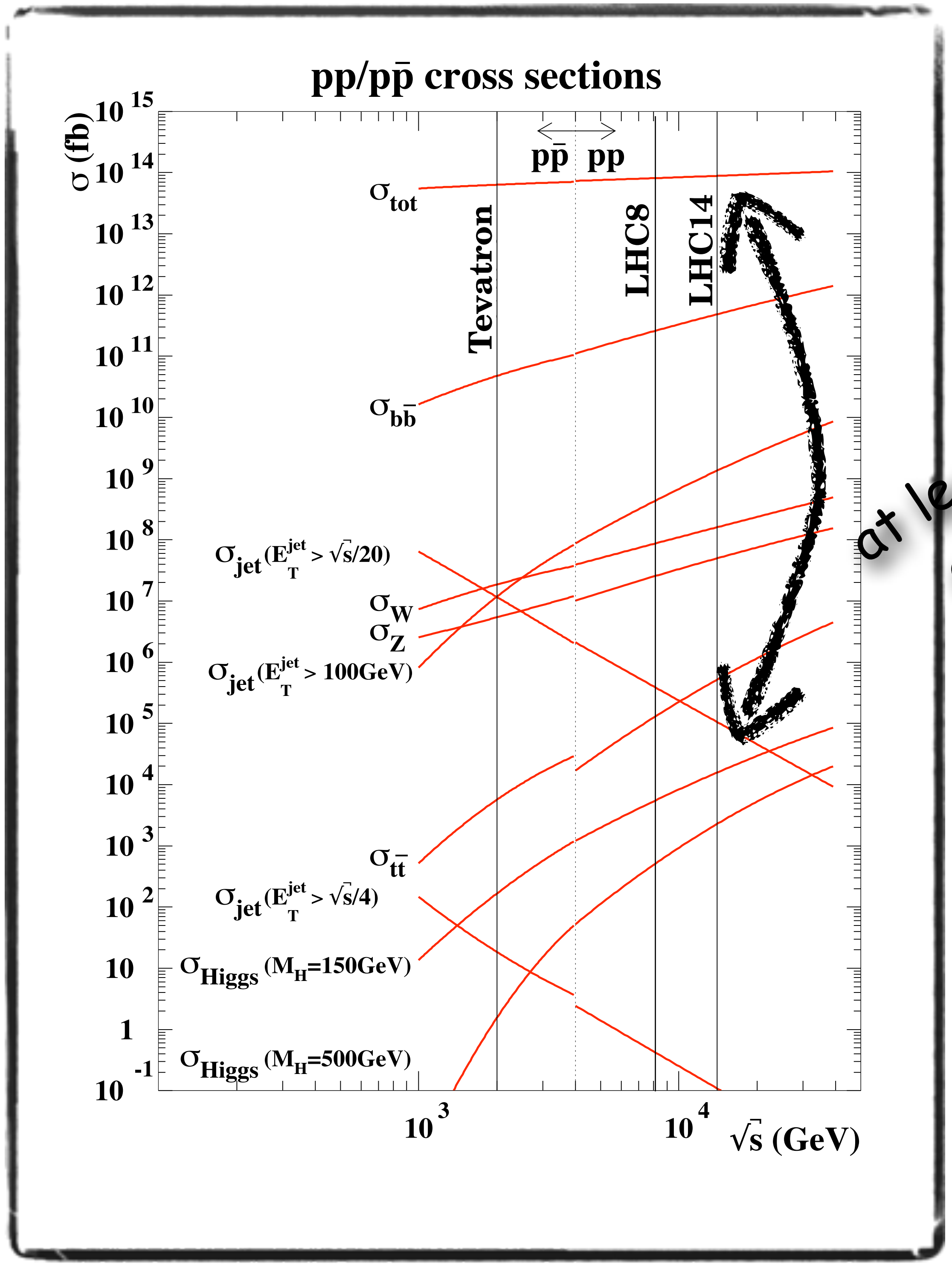
Higgs decay



The LHC has produced 10^5 Higgs bosons out of 10^{16} pp collisions

SM Higgs @ LHC

The production of a Higgs is wiped out by QCD background



only 1 out of 100 billions events are "interesting"

(for comparison, Shakespeare's 43 works contain only 884,429 words in total)

furthermore many of the background events furiously look like signal events

SM Higgs @ LHC

The production of a Higgs is wiped out by QCD background

only 1 out of 100 billions events are “interesting”

(for comparison, Shakespeare’s 43 works contain only 884,429 words in total)

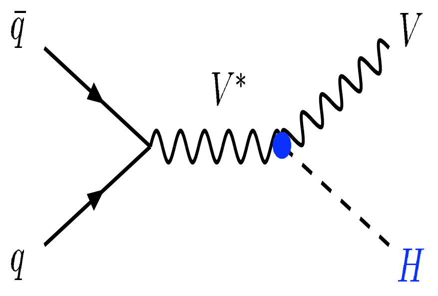
furthermore many of the background events furiously look like signal events

... like finding the paper you are looking for in (10^8 copies of) John Ellis’ office

(SM) Higgs Production @ the LHC

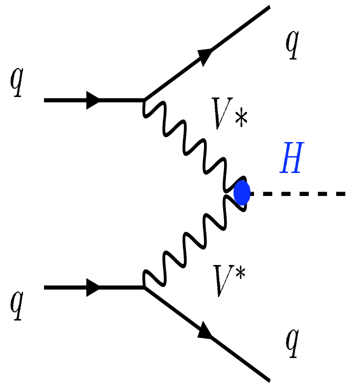
more about Higgs physics
see A. Raspereza's lectures

Higgs-strahlung



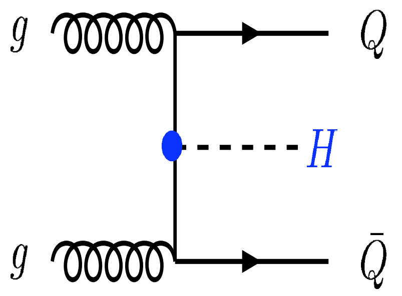
∝ 1/s: Tevatron, LHC

Vector boson fusion

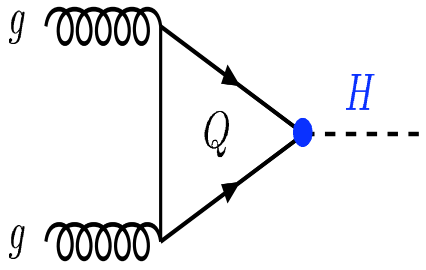


forward jet tagging
central jet veto
small hadronic activity

QQ associated production

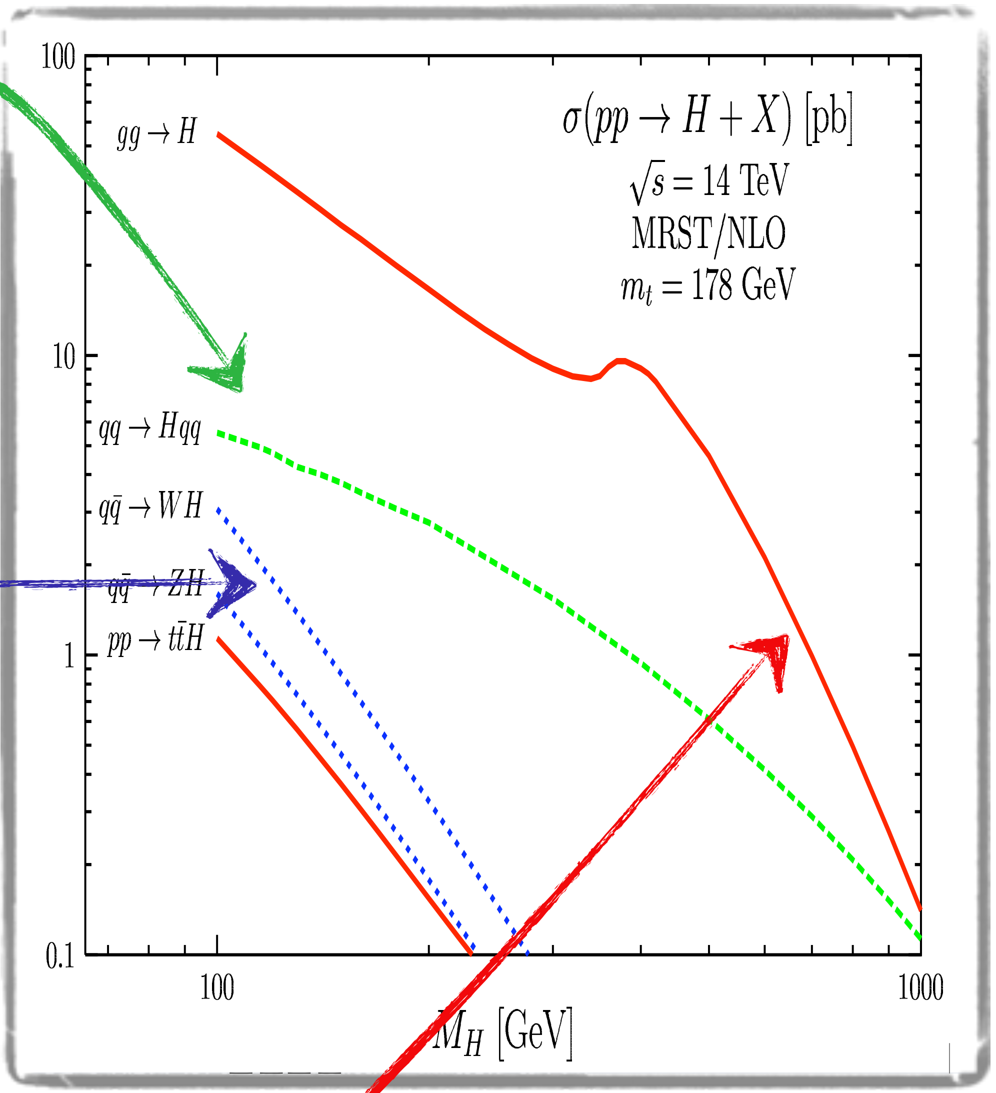


Gluon fusion



single final state
large NLO enhancement

LHC



Appendix III

SM Lagrangian

SM gauge symmetries explicitly

Gauge Group $U(1)_Y$ (abelian)

$$\psi' = e^{+iY\alpha_Y}\psi,$$

$$B'_\mu = B_\mu - \frac{1}{g'}\partial_\mu \alpha_Y$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu \psi_R = (\partial_\mu + i\, g'\, Y\, B_\mu)\, \psi_R$$

Gauge Group $SU(2)_L$ acts on the two components of a doublet Ψ_L =(u_L,d_L) or (ν_L ,e_L)

$$\Psi_L \to e^{-iT^a\alpha^a}\psi_L \qquad U = e^{-iT^a\alpha^a} \qquad T^a = \sigma^a/2$$

Pauli matrices

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^b_\mu W^c_\nu, \quad a = 1, \dots, 3$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = -i\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$D_\mu \psi_L = (\partial_\mu - i\, g\, W^a_\mu T^a)\, \psi_L$$

Gauge Group $SU(3)_c$ q=(q_1,q_2,q_3) (the three color degrees of freedom)

$$q \to e^{-iT^a\alpha^a} q \qquad U = e^{-iT^a\alpha^a} \qquad \left[T^a, T^b\right] = i f^{abc} T^c$$

(3×3) Gell-Man matrices

$$G^a_\mu T^a \to U G^a_\mu T^a U^{-1} - \frac{i}{g}\partial_\mu U U^{-1}$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu, \quad a = 1, \dots, 8$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$D_\mu q = (\partial_\mu - i\, g\, G^a_\mu T^a)\, q$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

SM gauge symmetries explicitly

Gauge Group $U(1)_Y$ (abelian)

$$\psi' = e^{+iY\alpha_Y}\psi,$$

$$B'_\mu = B_\mu - \frac{1}{g'}\partial_\mu\alpha_Y$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu\psi_R = \left(\partial_\mu + i\,g'\,Y\,B_\mu\right)\psi_R$$

all Standard Model fermions
carry U(1) charge

Gauge Group $SU(2)_L$

$$\Psi_L \rightarrow e^{-iT^a\alpha^a}\psi_L \qquad U = e^{-iT^a\alpha^a}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^b_\mu W^c_\nu, \quad a = 1,\ldots,3$$

$$D_\mu\psi_L = \left(\partial_\mu - i\,g\,W^a_\mu T^a\right)\psi_L$$

Ψ_L =(u_L,d_L) or (ν_L ,e_L)

only left-handed fermions charged under it
-> chiral interactions

Gauge Group $SU(3)_c$

$$q \rightarrow e^{-iT^a\alpha^a}q \qquad U = e^{-iT^a\alpha^a}$$

$$G^a_\mu T^a \rightarrow UG^a_\mu T^a U^{-1} - \frac{i}{g}\partial_\mu U U^{-1}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + gf^{abc}G^b_\mu G^c_\nu, \quad a = 1,\ldots,8$$

$$D_\mu q = \left(\partial_\mu - i\,g\,G^a_\mu T^a\right)q$$

q=(q_1,q_2,q_3)

all quarks transform under QCD
-> vector-like interactions

The SM particle content

Field	$SU(3)$	$SU(2)_L$	T^3	$\frac{Y}{2}$	$Q = T^3 + \frac{Y}{2}$
g^a_μ (gluons)	$\mathbf{8}$	$\mathbf{1}$	0	0	0
$(W^\pm_\mu, W^0_\mu)$	$\mathbf{1}$	$\mathbf{3}$	$(\pm 1, 0)$	0	$(\pm 1, 0)$
B^0_μ	$\mathbf{1}$	$\mathbf{1}$	0	0	0
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
u_R	$\mathbf{3}$	$\mathbf{1}$	0	$\frac{2}{3}$	$\frac{2}{3}$
d_R	$\mathbf{3}$	$\mathbf{1}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$E_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
e_R	$\mathbf{1}$	$\mathbf{1}$	0	-1	-1
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$	$\frac{1}{2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

The SM Lagrangian

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

describe massless gauge bosons

$$\mathcal{L}_{\text{Fermion}} = \sum_{\text{quarks}} i\bar{q}\gamma^\mu D_\mu q + \sum_{\psi_L} i\overline{\psi_L}\gamma^\mu D_\mu \psi_L + \sum_{\psi_R} i\overline{\psi_R}\gamma^\mu D_\mu \psi_R$$

describe massless fermions and their interactions with gauge bosons

$$D_\mu \psi_R = \left[\partial_\mu + i g' Y B_\mu\right]\psi_R$$

only left-handed fermions

all fermions carrying a U(1)$_Y$ charge i.e. all Standard Model fermions

$$\mathcal{L}_{\text{Higgs}} = \left(D_\mu \Phi\right)^\dagger D_\mu \Phi + \mu^2 \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi\right)^2 \longrightarrow$$

gives mass to EW gauge bosons

$$\frac{1}{2}M_Z^2 Z_\mu Z^\mu + M_W^2 W_\mu^+ W^{-\mu}$$

$$D_\mu \Phi = \left[\partial_\mu - i\frac{g}{\sqrt{2}}\left(\tau^+ W_\mu^+ + \tau^- W_\mu^-\right) - i\frac{g}{2}\tau_3 W_\mu^3 + i\frac{g'}{2}B_\mu\right]\Phi$$

: covariant derivative of the Higgs

H charged under SU(2) ×U(1)$_Y$

$$\mathcal{L}_{\text{Yukawa}} = -Y_l\,\overline{L}\,\Phi\,\ell_R - Y_d\,\overline{Q}\,\Phi\,d_R - Y_u\,\overline{Q}\,\widetilde{\Phi}\,u_R + \text{h.c.} \longrightarrow$$

gives mass to fermions

responsible for electroweak symmetry breaking!

$$SU(3) \times SU(2)_L \times U(1)_Y \longrightarrow SU(3) \times U(1)_{em}$$

8 massless gluons

3 massive gauge bosons W^+ W^- Z_0

8 massless gluons

1 massless photon γ

remaining unbroken symmetry

The W and Z bosons interact with the Higgs medium, the γ doesn't.

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$SU(3)_c$ $SU(2)_L$ $U(1)_Y$

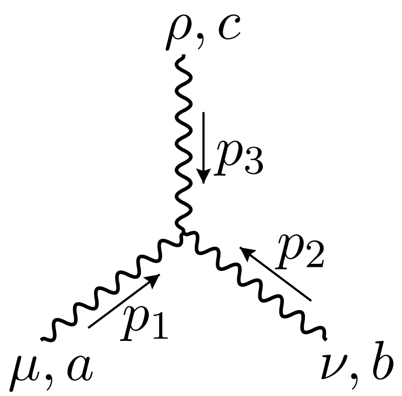
$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu \qquad W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

in mass eigenstate basis

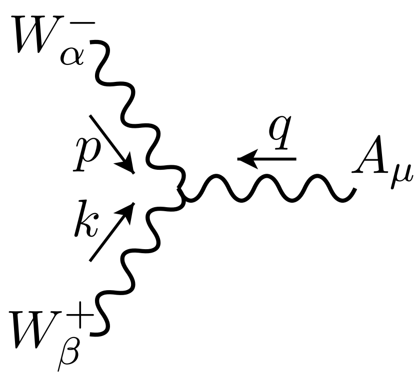
$$W^\pm_\mu = \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}} \qquad Z_\mu = W^3_\mu \cos\theta_W + B_\mu \sin\theta_W$$

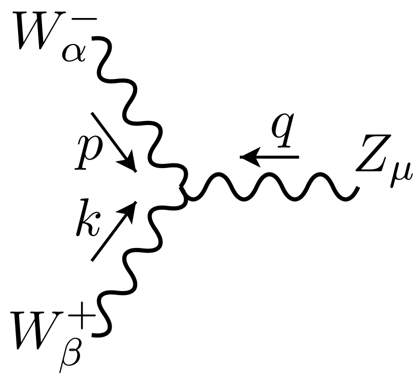
$$A_\mu = -W^3_\mu \sin\theta_W + B_\mu \cos\theta_W$$

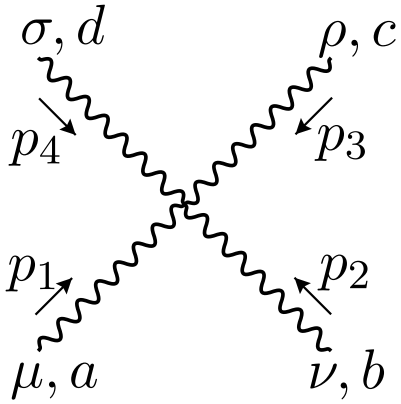
$$\cos\theta_W = g/\sqrt{g^2+g'^2} \qquad \sin\theta_W = g'/\sqrt{g^2+g'^2}$$



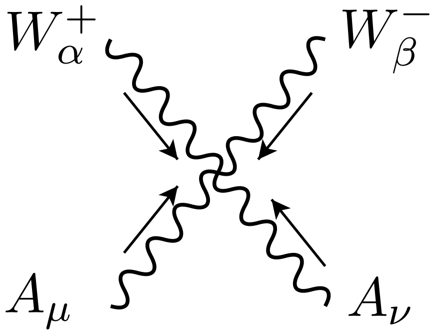
three gauge boson vertex

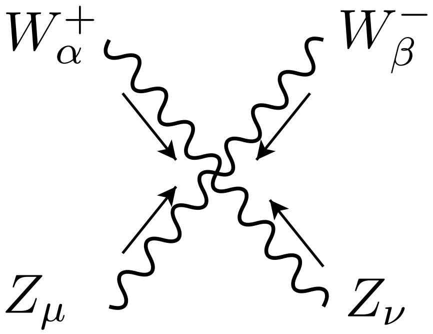


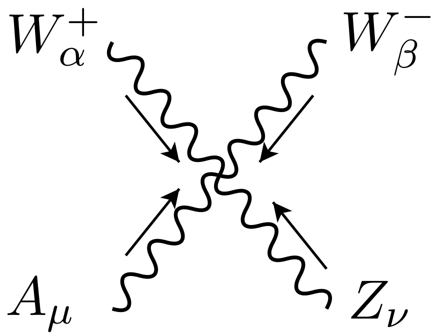


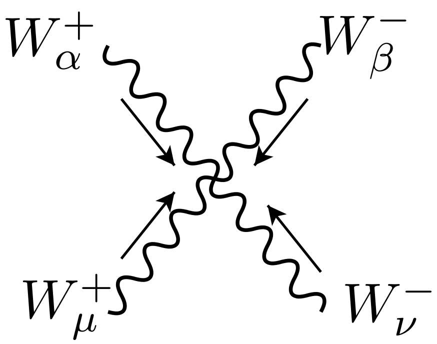


four gauge boson vertex





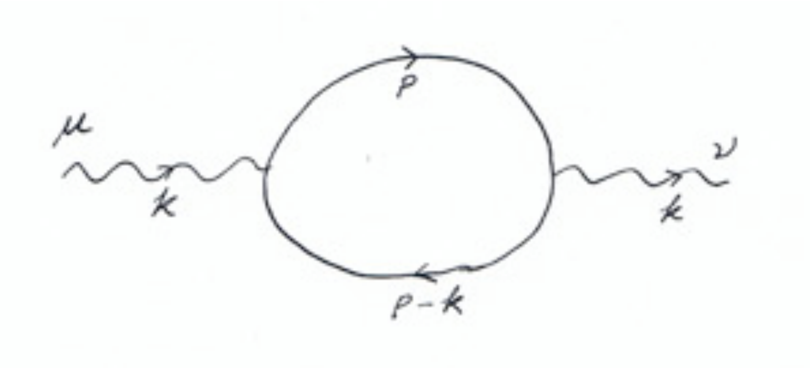




no such interactions for photon!

Renormalization

particles created from the vacuum screen the electric charge

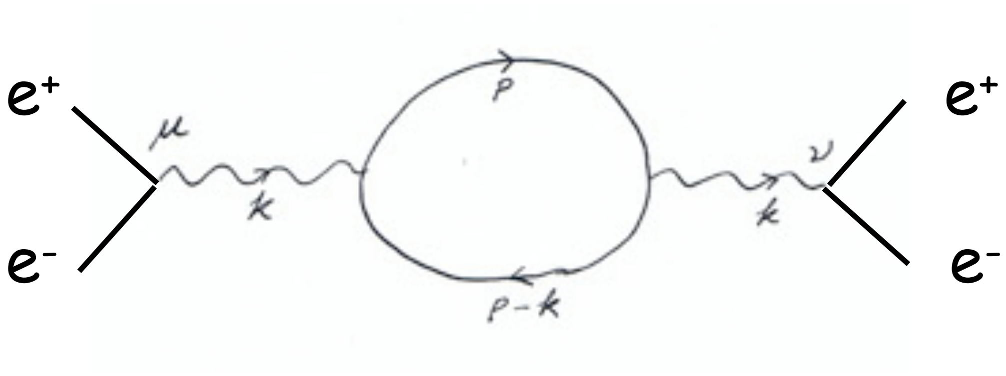


$$-e^2\int\frac{d^4p}{(2\pi)^4}\mathrm{TR}\left(\gamma_\mu\frac{1}{\not p-\not k-m}\gamma_\nu\frac{1}{\not p-m}\right)$$

$$=-\frac{e^2}{12\pi^2}\left(\log\frac{\Lambda^2}{m^2}-\log\frac{-k^2}{m^2}\right)$$

Renormalization

particles created from the vacuum screen the electric charge



$$e^2\,(1\ -e^2\int \frac{d^4p}{(2\pi)^4}\mathrm{TR}\left(\gamma_\mu\frac{1}{\not{p}-\not{k}-m}\gamma_\nu\frac{1}{\not{p}-m}\right)\ )$$

$$e^2\,(1\ -\frac{e^2}{12\pi^2}\left(\log\frac{\Lambda^2}{m^2}-\log\frac{-k^2}{m^2}\right))$$ at least for $k^2 >> m^2$

absorb the divergent piece into a renormalized charge

$$e_R^2 = e^2\left(1-\frac{e^2}{12\pi^2}\log\frac{\Lambda^2}{m^2}\right)$$

express result in term of renormalized charge

$$e_R^2(k^2)\left(1-\frac{e_R^2(k^2=0)}{12\pi^2}\log\frac{-k^2}{m^2}\right)$$

the electric charge now depends on the energy of the electrons
Ward identity: the dependence with the energy is the same for all particles (electrons, muons, quarks...)
it is universal physical effect associated to the gauge symmetry

Evolution of coupling constants

Classical physics: the forces depend on distances

Quantum physics : the charges depend on distances

QED: virtual particles screen
the electric charge: α ↘ when d ↗

QCD: virtual particles (quarks and
gluons) screen the strong charge:
α_s ↗ when d ↗

'asymptotic freedom'

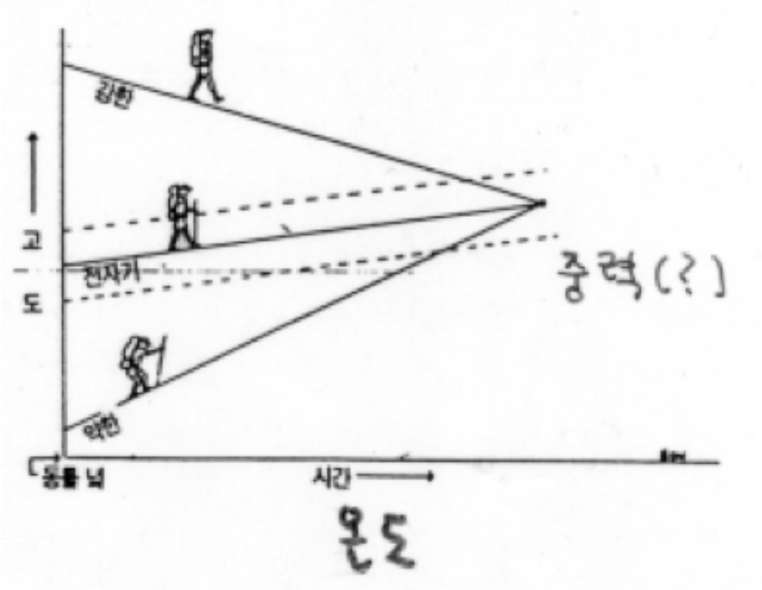
more about QCD/jets
see M. Diehl's lectures

$$\frac{\partial \alpha_s}{\partial \log E} = \beta(\alpha_s) = \frac{\alpha_s^2}{\pi}\left(-\frac{11N_c}{6}+\frac{N_f}{3}\right)$$

SM β fcts

g, g′ and g_s are different but it is a low energy artifact!

$$\boxed{\beta = \frac{dg}{d\log\mu} = -\frac{1}{16\pi^2} b g^3 + \ldots}$$



$$\boxed{\frac{1}{g^2(Q)} = \frac{1}{g^2(Q_0)} + \frac{b}{16\pi^2}\ln\frac{Q^2}{Q_0^2}}$$

$$b = \frac{11}{3} T_2(\texttt{spin-1}) - \frac{2}{3} T_2(\texttt{chiral spin-1/2}) - \frac{1}{3} T_2(\texttt{complex spin-0})$$

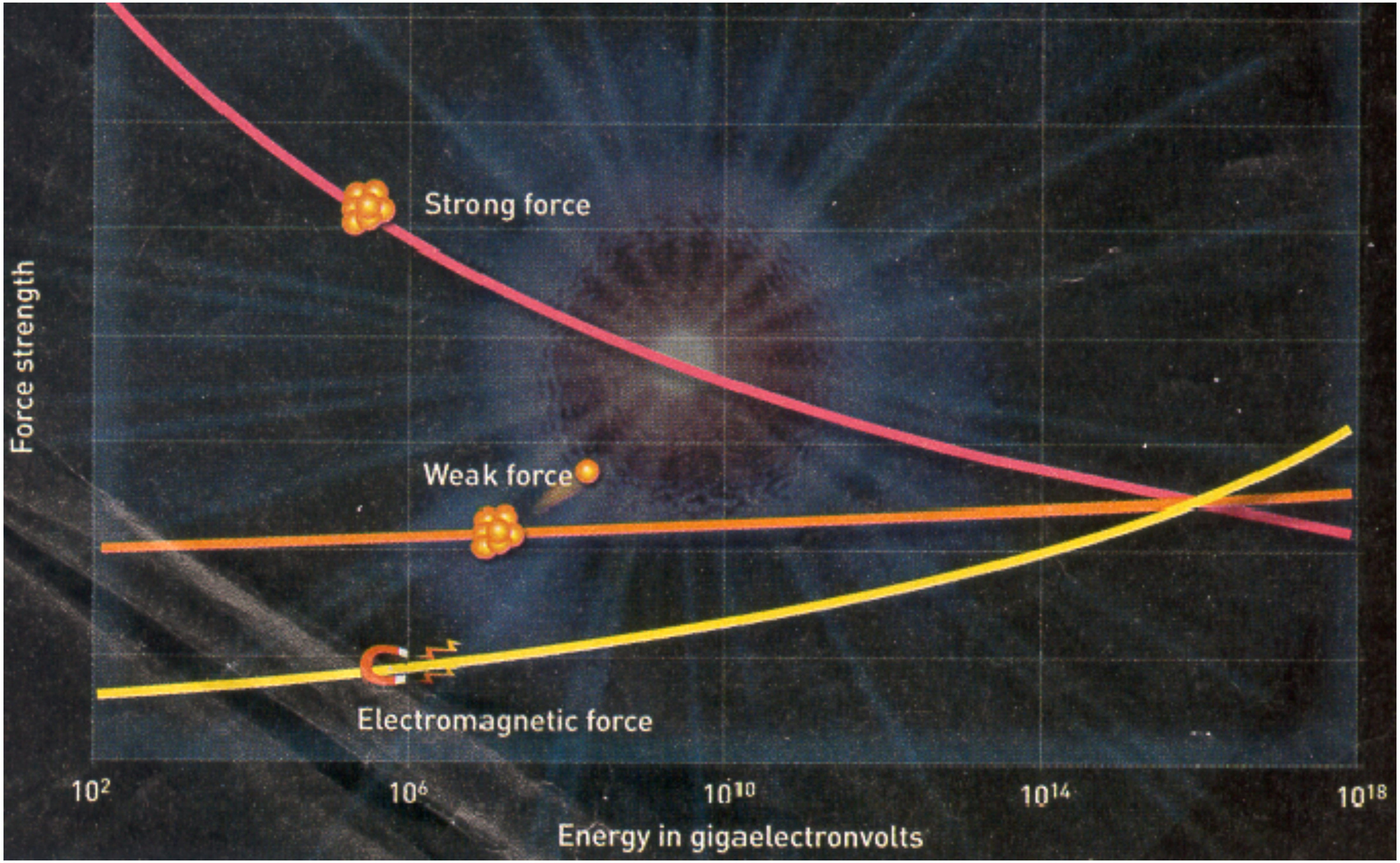
$$\mathrm{Tr}\left(T^a(R) T^b(R)\right) = T_2(R)\delta^{ab} \qquad T_2(\texttt{fund}) = \frac{1}{2} \qquad T_2(\texttt{adj}) = N$$

$$b_{SU(3)} = \overset{g}{\frac{11}{3} \times 3} - \frac{2}{3}\left(\overset{Q_L}{\frac{1}{2} \times 2 \times 3} + \overset{u_R^c}{\frac{1}{2} \times 1 \times 3} + \overset{d_R^c}{\frac{1}{2} \times 1 \times 3}\right) = 7$$

$$b_{SU(2)} = \overset{W^\pm, Z}{\frac{11}{3} \times 2} - \frac{2}{3}\left(\overset{Q_L}{\frac{1}{2} \times 3 \times 3} + \overset{L}{\frac{1}{2} \times 1 \times 3}\right) - \overset{H}{\frac{1}{3} \times \frac{1}{2}} = \frac{19}{6}$$

$$b_Y = -\frac{2}{3}\left(\overset{Q_L}{\left(\frac{1}{6}\right)^2 3 \times 2 \times 3} + \overset{u_R^c}{\left(-\frac{2}{3}\right)^2 3 \times 3} + \overset{d_R^c}{\left(\frac{1}{3}\right)^2 3 \times 3} + \overset{L}{\left(-\frac{1}{2}\right)^2 2 \times 3} + \overset{e_R^c}{(1)^2 \times 3}\right) - \overset{H}{\frac{1}{3}\left(\frac{1}{2}\right)^2 \times 2} = -\frac{41}{6}$$

Grand Unified Theories



A single form of matter
A single fundamental interaction

SU(5) GUT: Gauge Group Structure

SU(3)cxSU(2)LxU(1)Y: SM Matter Content

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} = (3,2)_{1/6}, \quad u_R^c = (\bar{3},1)_{-2/3}, \quad d_R^c = (\bar{3},1)_{1/3}, \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = (1,2)_{-1/2}, \quad e_R^c = (1,1)_1$$

How can you ever remember all these numbers?

SU(3)cxSU(2)LxU(1)Y ⊂ SU(5)

SU(5)
Adjoint rep.

$$\left(\begin{array}{c|c} SU(2) & \\ \hline & SU(3) \end{array}\right)$$

$$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$$

additional U(1) factor that commutes with SU(3)xSU(2)

$$T^{12} = \sqrt{\frac{3}{5}} \left(\begin{array}{cc|ccc} 1/2 & & & & \\ & 1/2 & & & \\ \hline & & -1/3 & & \\ & & & -1/3 & \\ & & & & -1/3 \end{array}\right)$$

$$\boxed{\begin{array}{c} \bar{5} = (1,2)_{-\frac{1}{2}\sqrt{\frac{3}{5}}} + (\bar{3},1)_{\frac{1}{3}\sqrt{\frac{3}{5}}} \\ \bar{5} = L + d_R^c \end{array}}$$

$$\boxed{\begin{array}{c} 10 = (5\times 5)_A = (\bar{3},1)_{-\frac{2}{3}\sqrt{\frac{3}{5}}} + (3,2)_{\frac{1}{6}\sqrt{\frac{3}{5}}} + (1,1)_{\sqrt{\frac{3}{5}}} \\ 10 = u_R^c + Q_L + e_R^c \end{array}}$$

$$T^{12} = \sqrt{\frac{3}{5}} Y \qquad g_5 \sqrt{\frac{3}{5}} = g' \qquad g_5 = g = g_s$$

$$g_5 T^{12} = g' Y$$

$\sin^2\theta_W = \dfrac{3}{8}$ @ MGUT

SU(5) GUT: Gauge Group Structure

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How can you even

the SM matter fits nicely into representations of SU(5), even more nicely into SO(10) unification baryon-lepton

Ac

Tr(

$$\begin{pmatrix} & \\ & \begin{matrix} -1/3 & & \\ & -1/3 & \\ & & -1/3 \end{matrix} \end{pmatrix}$$

$$= (1,2)_{-\frac{1}{2}\sqrt{\frac{3}{5}}} + (\bar{3},1)_{\frac{1}{3}\sqrt{\frac{3}{5}}}$$

$$\bar{5} = L + d_R^c$$

$$10 = (5\times 5)_A = (\bar{3},1)_{-\frac{2}{3}\sqrt{\frac{3}{5}}} + (3,2)_{\frac{1}{6}\sqrt{\frac{3}{5}}} + (1,1)_{\sqrt{\frac{3}{5}}}$$

$$10 = u_R^c + Q_L + e_R^c$$

$$T^{12} = \sqrt{\frac{3}{5}}Y \qquad g_5\sqrt{\frac{3}{5}} = g' \qquad g_5 = g = g_s$$

$$g_5 T^{12} = g' Y$$

$\sin^2\theta_W = \frac{3}{8}$ @ MGUT

SU(5) GUT: low energy consistency condition

more about GUT, see F. Bruemmer's lectures

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{GUT}} - \frac{b_i}{4\pi} \ln \frac{M_{GUT}^2}{M_Z^2} \qquad i = SU(3), SU(2), U(1)$$

$\alpha_3(M_Z), \alpha_2(M_Z), \alpha_1(M_Z)$ ← experimental inputs

b_3, b_2, b_1 ← predicted by the matter content

3 equations & 2 unknowns (α_{GUT}, M_{GUT})

one consistency relation for unification

$$\epsilon_{ijk} \frac{b_j - b_k}{\alpha_i(M_Z)} = 0 \quad \Rightarrow \quad \boxed{\sin^2\theta_W = \frac{3(b_3-b_2)}{8b_3-3b_2-5b_1} + \frac{5(b_2-b_1)}{8b_3-3b_2-5b_1} \frac{\alpha_{em}(M_Z)}{\alpha_s(M_Z)}}$$

$\alpha_{em}(M_Z) \approx \frac{1}{128} \qquad \alpha_s(M_Z) \approx 0.1184 \pm 0.0007$

$$\Downarrow$$

$$\sin^2\theta_W \approx 0.207$$

not so bad...

SU(5) GUT: low energy consistency condition

more about GUT, see F. Bruemmer's lectures

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{GUT}} - \frac{b_i}{4\pi} \ln \frac{M_{GUT}^2}{M_Z^2} \qquad i = SU(3), SU(2), U(1)$$

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b_3, b_2, b_1 ← predicted by the matter content

3 equations & 2 unknowns (α_{GUT}, M_{GUT})

one consistency relation for unification

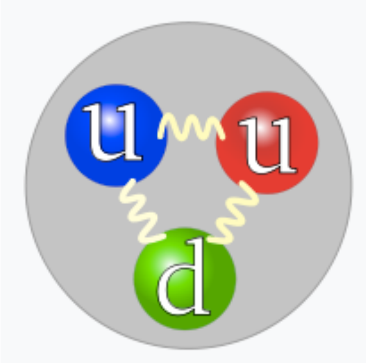
$$M_{GUT} = M_Z \exp\left(2\pi \frac{3\alpha_s(M_Z) - 8\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)}\right) \approx 7 \times 10^{14} \text{ GeV}$$

$$\alpha_{GUT}^{-1} = \frac{3b_3\alpha_s(M_Z) - (5b_1 + 3b_2)\alpha_{em}(M_Z)}{(8b_3 - 3b_2 - 5b_1)\alpha_s(M_Z)\alpha_{em}(M_Z)} \approx 41.5$$

self-consistent computation:

- $M_{GUT} < M_{Pl}$ safe to neglect quantum gravity effects
- $\alpha_{GUT} \ll 1$ perturbative computation

Proton Decay

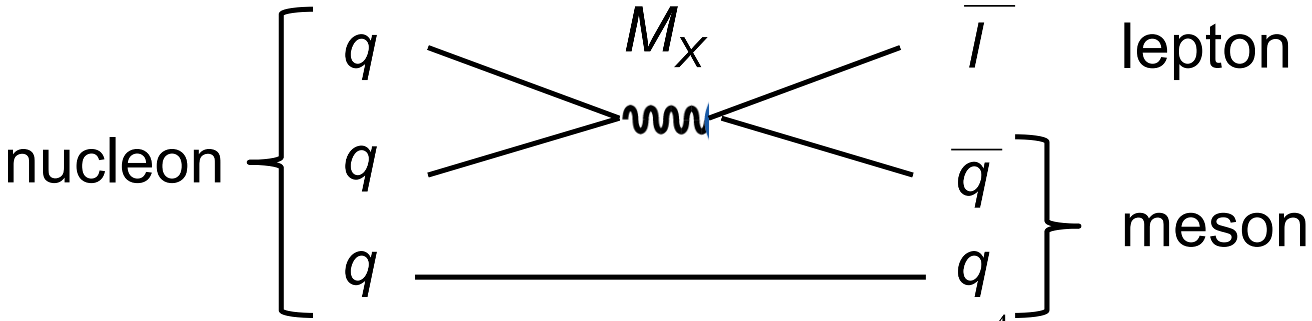


938.2720813(58) MeV

why is the proton stable?

electric charge conservation?

baryon number conservation?



in GUT, “matter” is unstable decay of proton mediated by new SU(5)/SO(10) gauge bosons

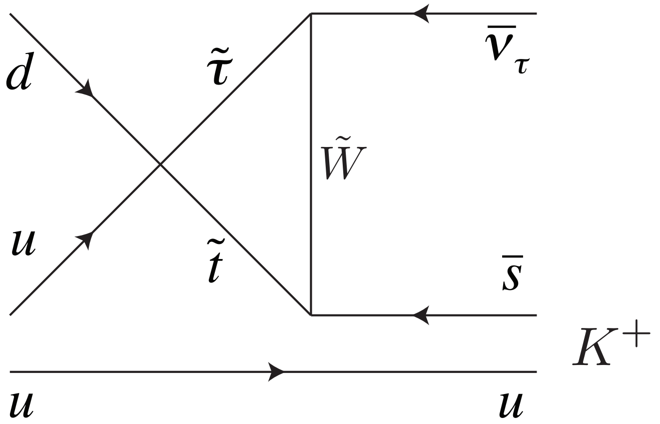
GUT: $\tau_p(p \to e^+\pi^0) = \left(\frac{M_X}{10^{15}\text{ GeV}}\right)^4 10^{31-32}$ yr

Exp: $\tau_p(p \to e^+\pi^0) > 8.2 \times 10^{33}$ yr

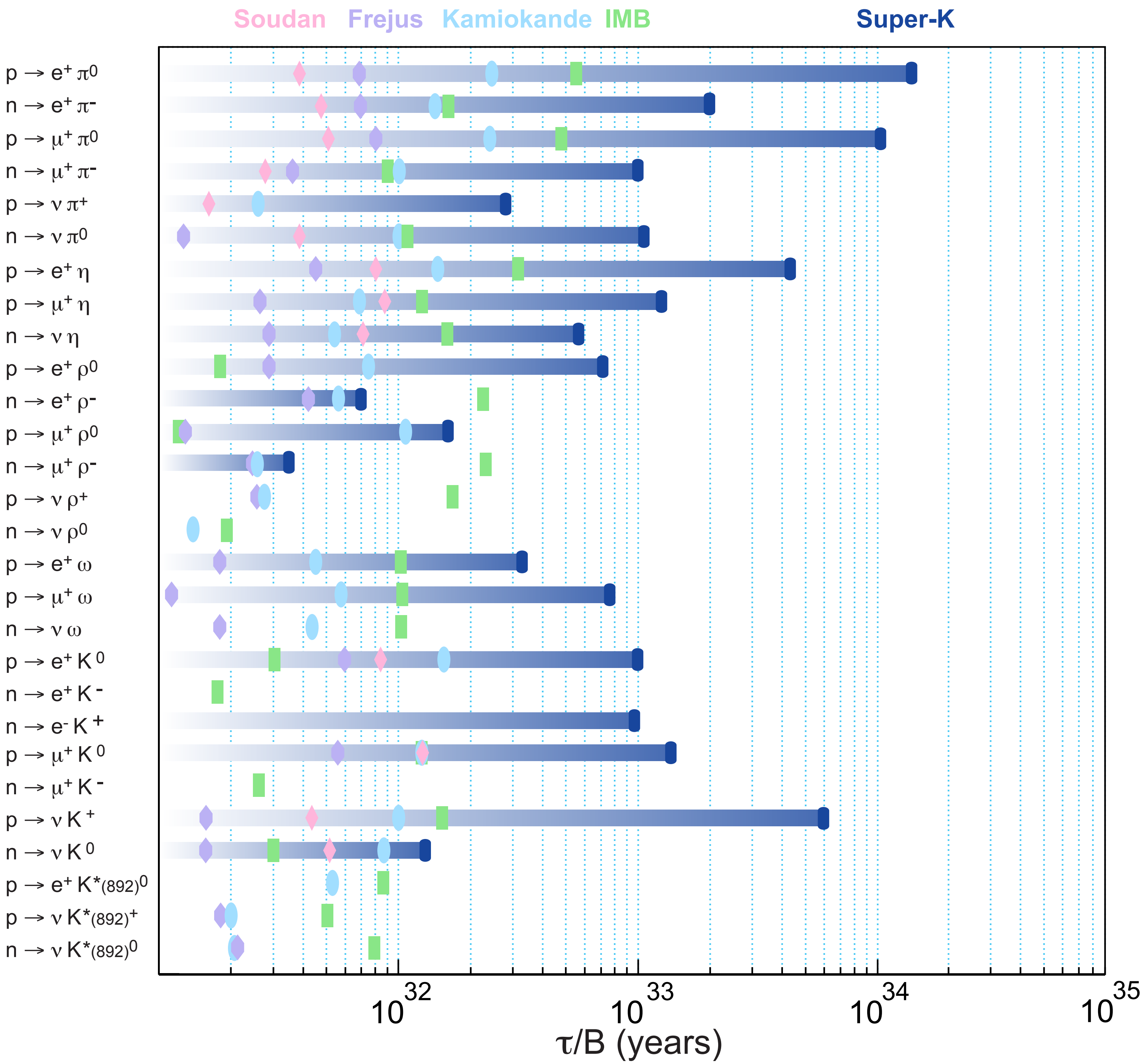
(G. Giudice SSLP’15)

other decay mode:

$$p^+ \to K^+\overline{\nu}$$



Proton Decay



Babu et al '13