Crossover from fully coherent analytical approach to Maxwell-Bloch equations

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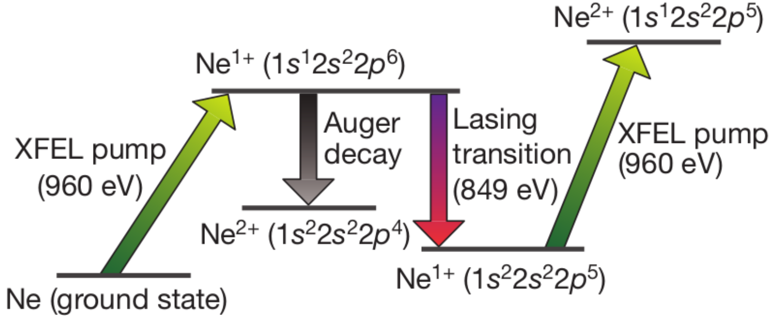
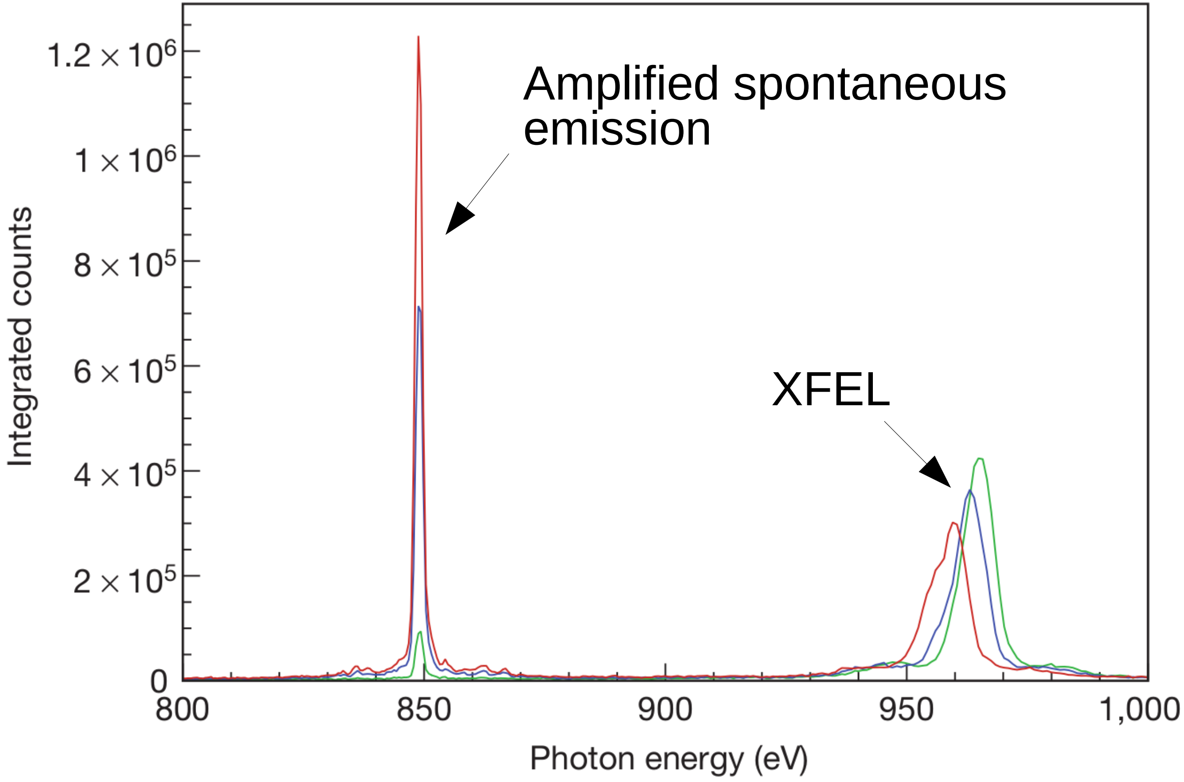
FS-TUXS

Laser-like amplified spontaneous emission





Nina Rohringer, Nature, 481(7382):488, 2012.

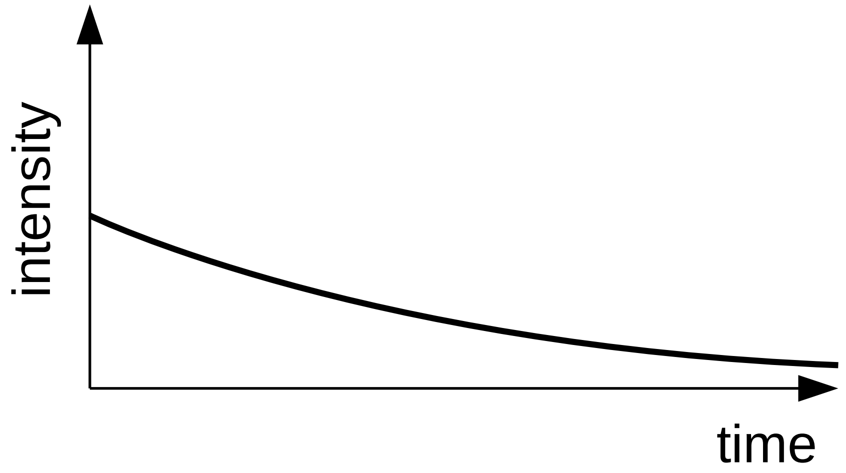
Ordinary fluorescence and superradiance

Ordinary fluorescence

Maximum intensity ~ N

Characteristic time ~ τ_{sp}

Essentially isotropic



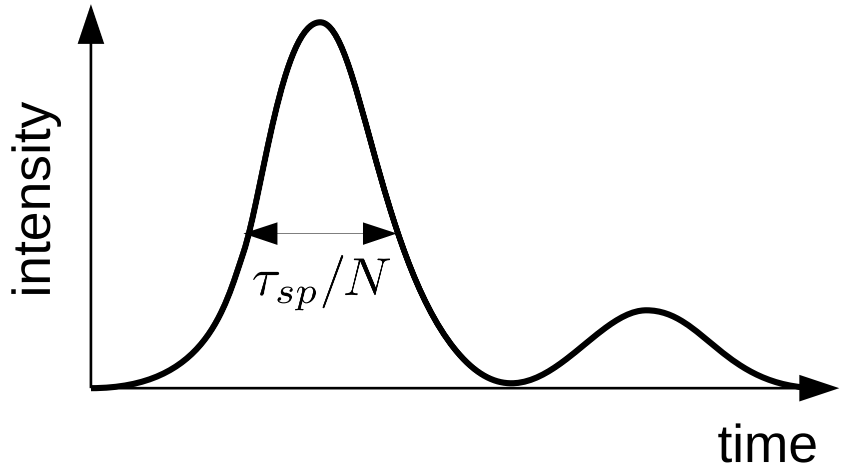
Superradiance

Maximum intensity ~ N^2

Characteristic time ~ τ_{sp}/N

Anisotropic

Delay changes from realization to realization



Correlation functions

Equal-position correlation function of field

$$K(\vec{r}, t_1, t_2) = \langle \hat{A}^{(+)}(\vec{r}, t_1)\hat{A}^{(-)}(\vec{r}, t_2)\rangle.$$

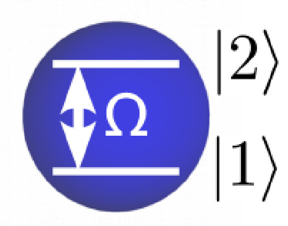
Intensity

$$I(\vec{r}, t) \sim \langle \hat{A}^{(+)}(\vec{r}, t)\hat{A}^{(-)}(\vec{r}, t)\rangle = K(\vec{r}, t, t).$$

Wiener–Khinchin theorem leads to $I(\vec{r}, \omega)$.

Equal-time correlation function of 2-level atoms

$$S(\vec{r}_1, \vec{r}_2, t) = \langle \hat{\rho}_{12}(\vec{r}_1, t)\hat{\rho}_{21}(\vec{r}_2, t)\rangle.$$



$\hat{\rho}_{12}(\vec{r}, t)$ is a density matrix element related to transition from an excited state to ground one.

2-level Maxwell Bloch equations

Classical Maxwell equation with polarization

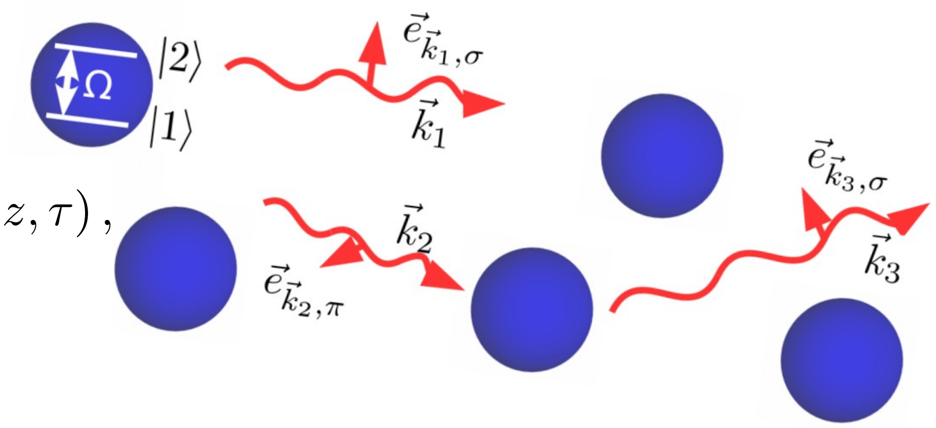
$$\frac{\partial A(z,\tau)}{\partial z} = gP(z,\tau),$$

Equations for population inversion and polarization

$$\sigma(z,\tau) = \frac{1}{2}\left(\rho_{22}(z,\tau) - \rho_{11}(z,\tau)\right),$$

$$\frac{\partial \sigma(z,\tau)}{\partial \tau} = \underbrace{-\Gamma_{sp}\left(\sigma(z,\tau) + \frac{1}{2}\right)}_{\text{decay}} - \gamma P(z,\tau) A(z,\tau),$$

$$\frac{\partial P(z,\tau)}{\partial \tau} = \underbrace{-\frac{\Gamma_{sp}}{2} P(z,\tau)}_{\text{decay}} + \gamma\sigma(z,\tau) A(z,\tau).$$



Existing theoretical approach

Maxwell – Bloch with semi-phenomenological noise source terms

O. Larroche et al, PRA 62, 043815 (2000)

$$\frac{\partial P\left(z,\tau\right)}{\partial\tau} = \ldots + \text{noise term} \quad \rightarrow \quad \langle\text{observables}\rangle$$

Incorrect description of initial stage

Different noise terms for different effects!

First principles formalism

Hamiltonian for two-levels atoms and quantized field

Heisenberg operator equations

-1D approximation
-co-moving frame
field elimination

Triple product factorization
Closed system for two-point observables

$$\frac{\partial S(z_1,z_2,\tau)}{\partial\tau} = -\Gamma_{sp}S(z_1,z_2,\tau) + \underbrace{\frac{3\Delta o}{8\pi}\Gamma_{sp}n_1\left(\sigma(z_1,\tau)\int_0^{z_1} dz' e^{-\frac{\kappa}{2}(z_1-z')}S(z',z_2,\tau) + 1\leftrightarrow 2\right)}_{\text{Stimulated emission}}$$

$$+\underbrace{\frac{3\Delta o}{8\pi}\Gamma_{sp}\left(\sigma(z_1,\tau)\rho_{ee}(z_2,\tau)e^{-\frac{\kappa}{2}(z_2-z_1)} + 1\leftrightarrow 2\right)}_{\text{Spontaneous emission}}$$

$$\frac{\partial K(z,\tau_1,\tau_2)}{\partial z} = -\kappa(z)K(z,\tau_1,\tau_2) + \underbrace{\frac{3\Delta o}{8\pi}\Gamma_{sp}n_1\left(\int_0^{\tau_1} d\tau_1' e^{-\frac{\Gamma}{2}(\tau_1-\tau_1')}\sigma(z,\tau_1')K(z,\tau_1',\tau_2) + 1\leftrightarrow 2\right)}_{\text{Stimulated emission}}$$

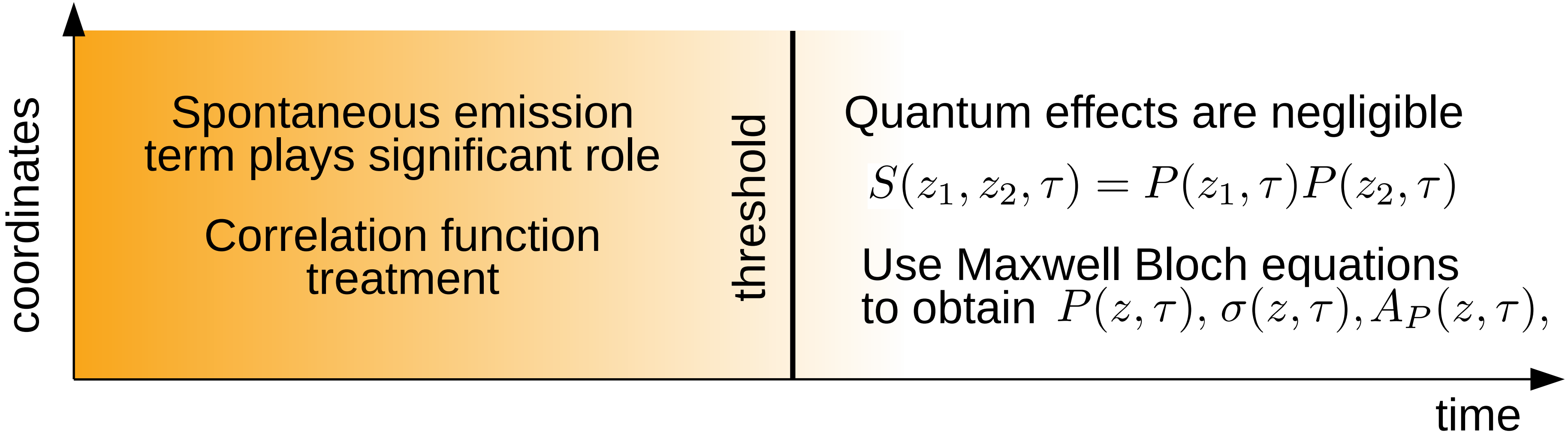
$$+\underbrace{\frac{3\Delta o}{64\pi\lambda^2}\Gamma_{sp}n_1e^{-\frac{\Gamma}{2}(\tau_1+\tau_2)}\rho_{ee}(z,0)}_{\text{Spontaneous emission}}$$

Spontaneous emission term

atomic observables

$$\frac{\partial S(z_1,z_2,\tau)}{\partial \tau} = -\Gamma_{sp} S(z_1,z_2,\tau) + \underbrace{\frac{3\Delta o}{8\pi}\Gamma_{sp} n_1 \left(\sigma(z_1,\tau) \int_0^{z_1} dz' e^{-\frac{\kappa}{2}(z_1-z')} S(z',z_2,\tau) + 1 \leftrightarrow 2 \right)}_{\text{Stimulated emission}}$$

$$+ \underbrace{\frac{3\Delta o}{8\pi}\Gamma_{sp} \left(\sigma(z_1,\tau)\rho_{ee}(z_2,\tau) e^{-\frac{\kappa}{2}(z_2-z_1)} + 1 \leftrightarrow 2 \right)}_{\text{Spontaneous emission}}$$

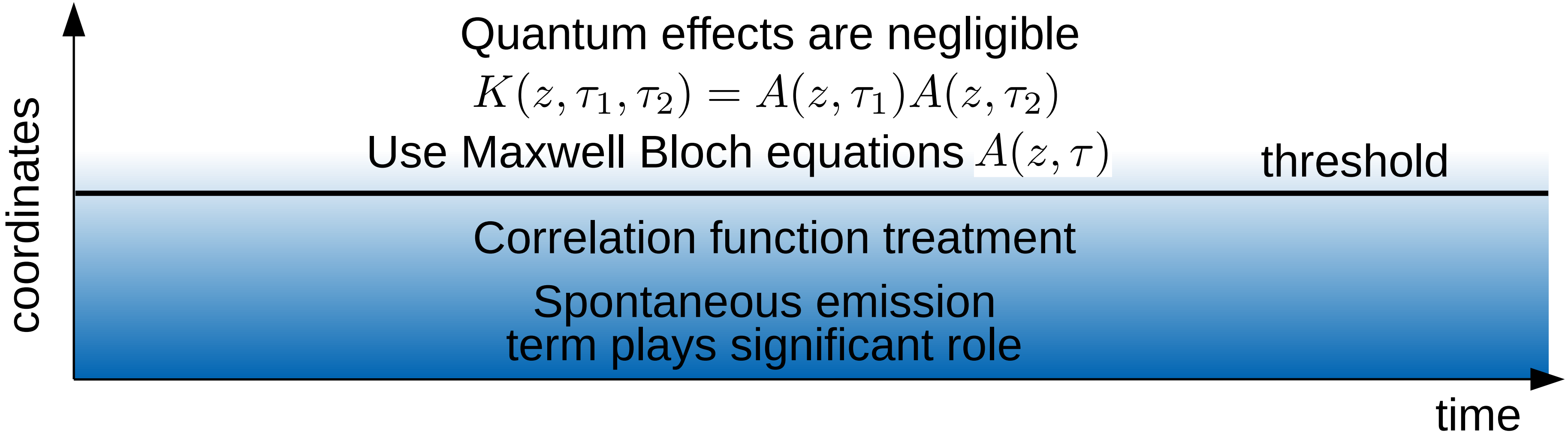


Spontaneous emission term

field

$$\frac{\partial K(z,\tau_1,\tau_2)}{\partial z} = -\kappa(z)K(z,\tau_1,\tau_2) + \underbrace{\frac{3\Delta o}{8\pi}\Gamma_{sp}n_1\left(\int_0^{\tau_1} d\tau_1' e^{-\frac{\Gamma}{2}(\tau_1-\tau_1')}\sigma(z,\tau_1')K(z,\tau_1',\tau_2) + 1 \leftrightarrow 2\right)}_{\text{Stimulated emission}}$$

$$+ \underbrace{\frac{3\Delta o}{64\pi\lambda^2}\Gamma_{sp}n_1 e^{-\frac{\Gamma}{2}(\tau_1+\tau_2)}\rho_{ee}(z,0)}_{\text{Spontaneous emission}}$$



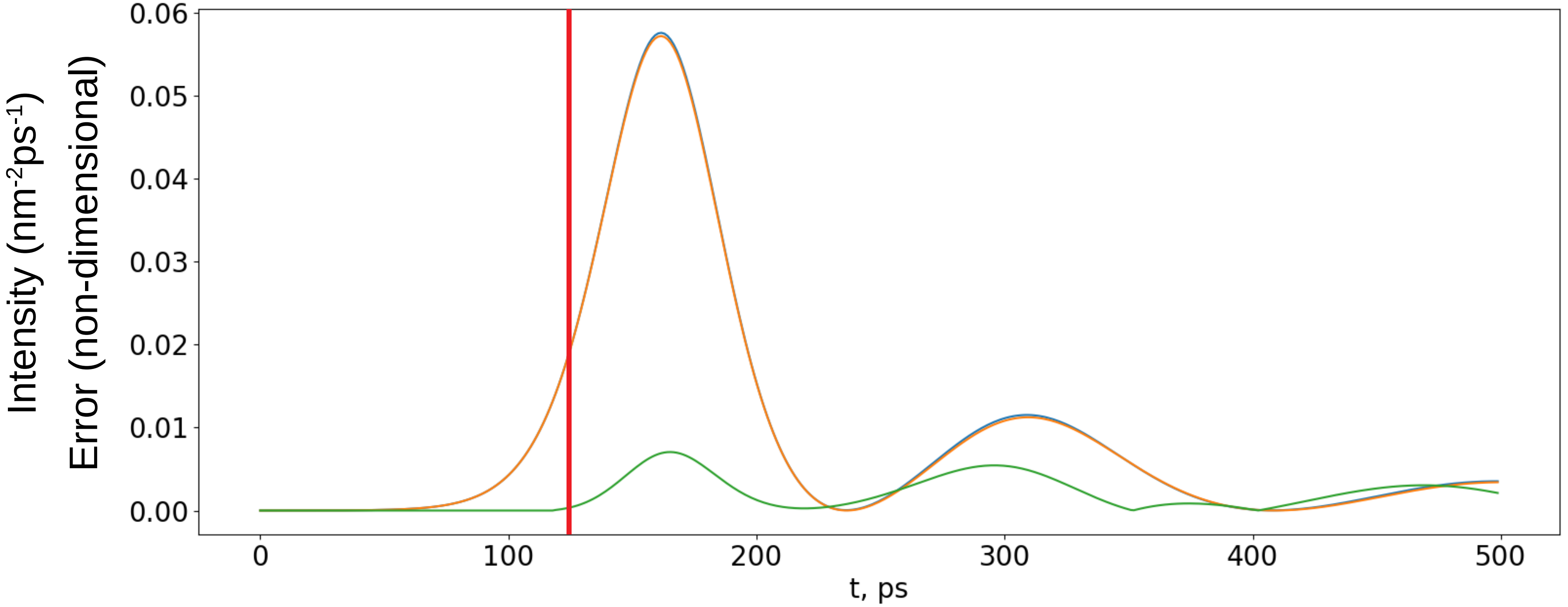
Numerical results

Intensity

$\varepsilon = 0.001$

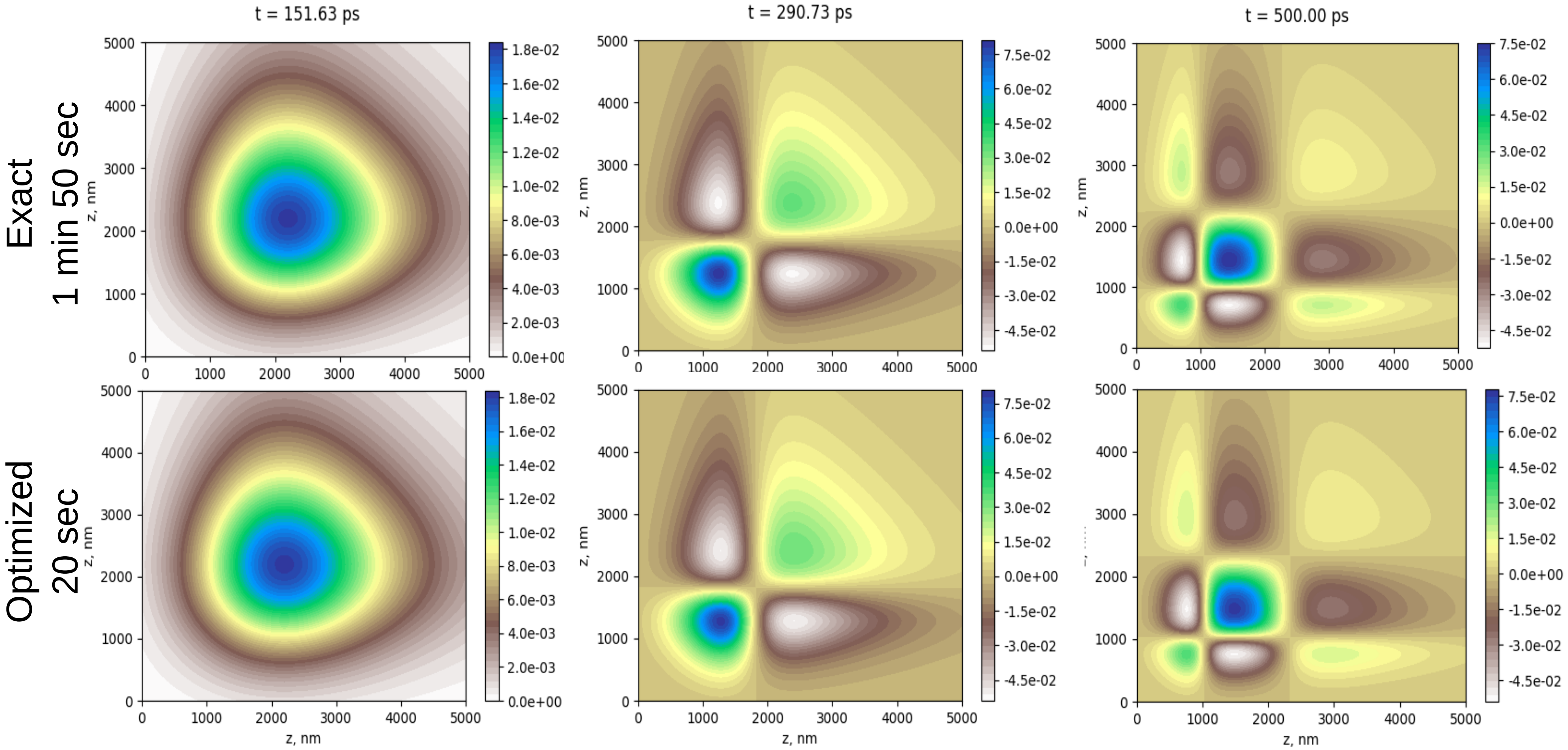
Exact
1 min 50 sec

Optimized
19 sec + 1 sec



Numerical results
Atom correlation function

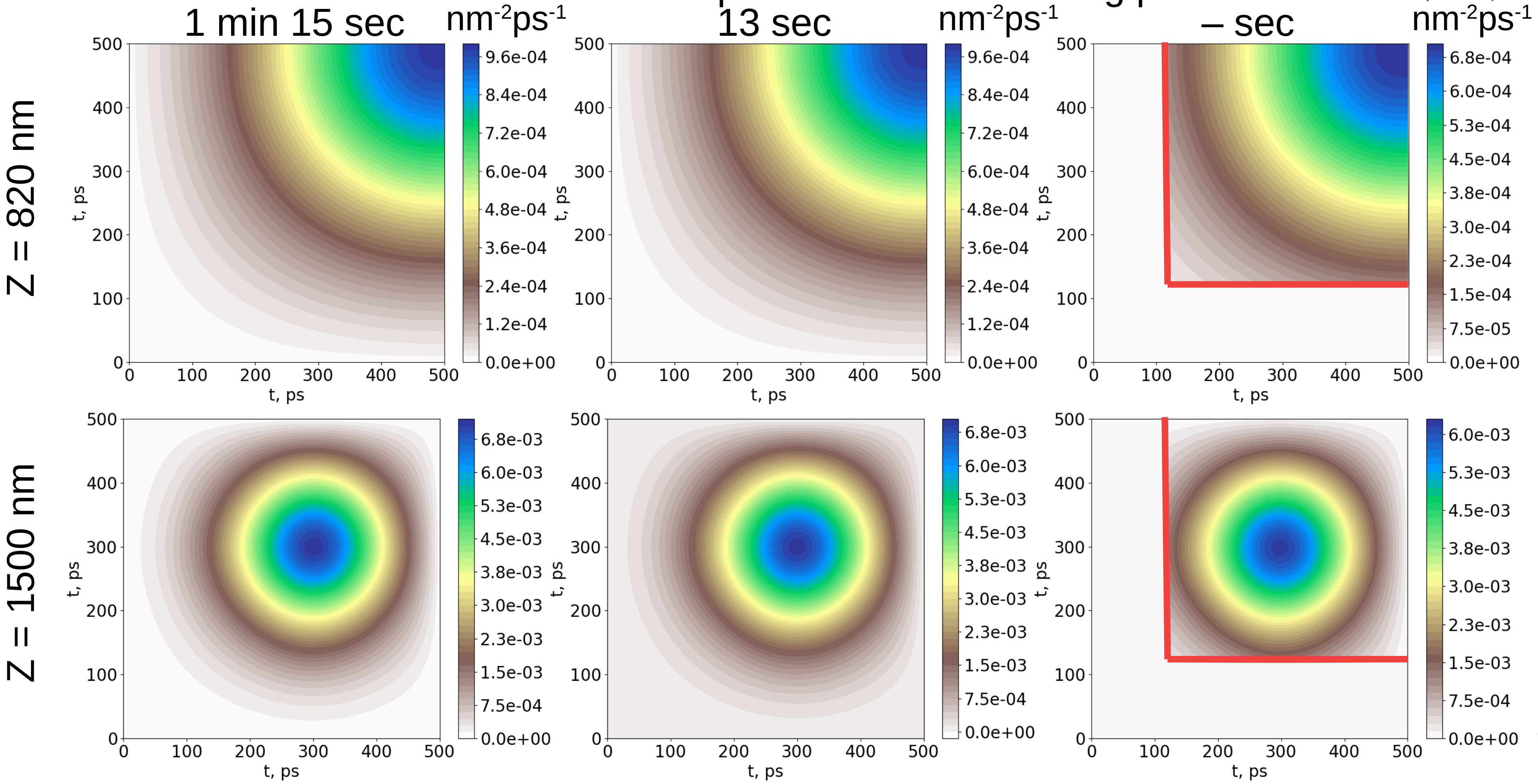
$\varepsilon = 0.001$



Numerical results
Field correlation function

$\varepsilon = 0.001$

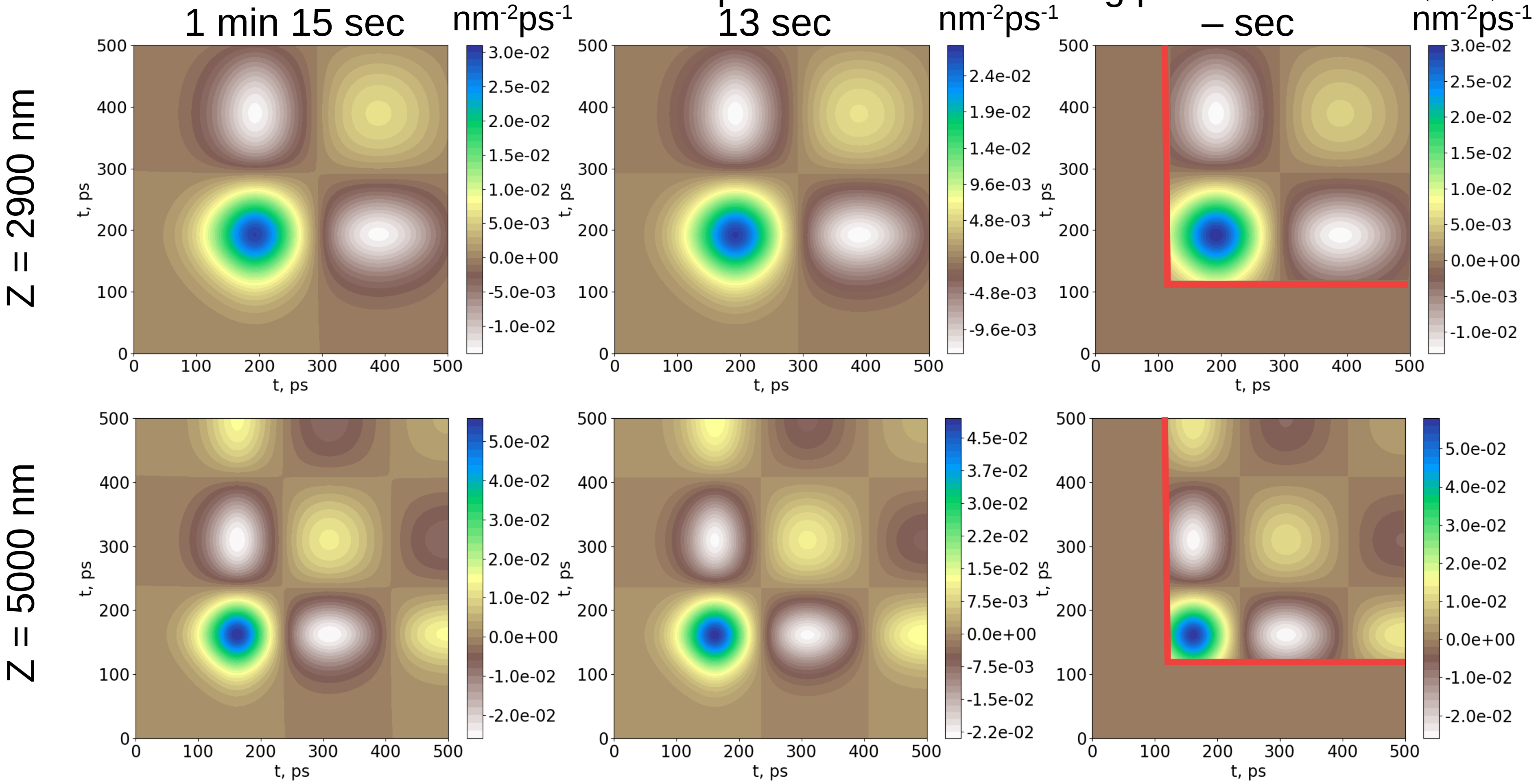
Exact	Optimized	Using pseudo field $A_P(z, \tau)$
1 min 15 sec	13 sec	– sec



Numerical results
Field correlation function

$\varepsilon = 0.001$

Exact — 1 min 15 sec | Optimized — 13 sec | Using pseudo field $A_P(z,\tau)$ – sec



Conclusions

The formalism based on correlation functions is presented.

The optimization technique based on Maxwell Bloch equations is proposed.

The code that runs optimized algorithm for 2-level system is written.

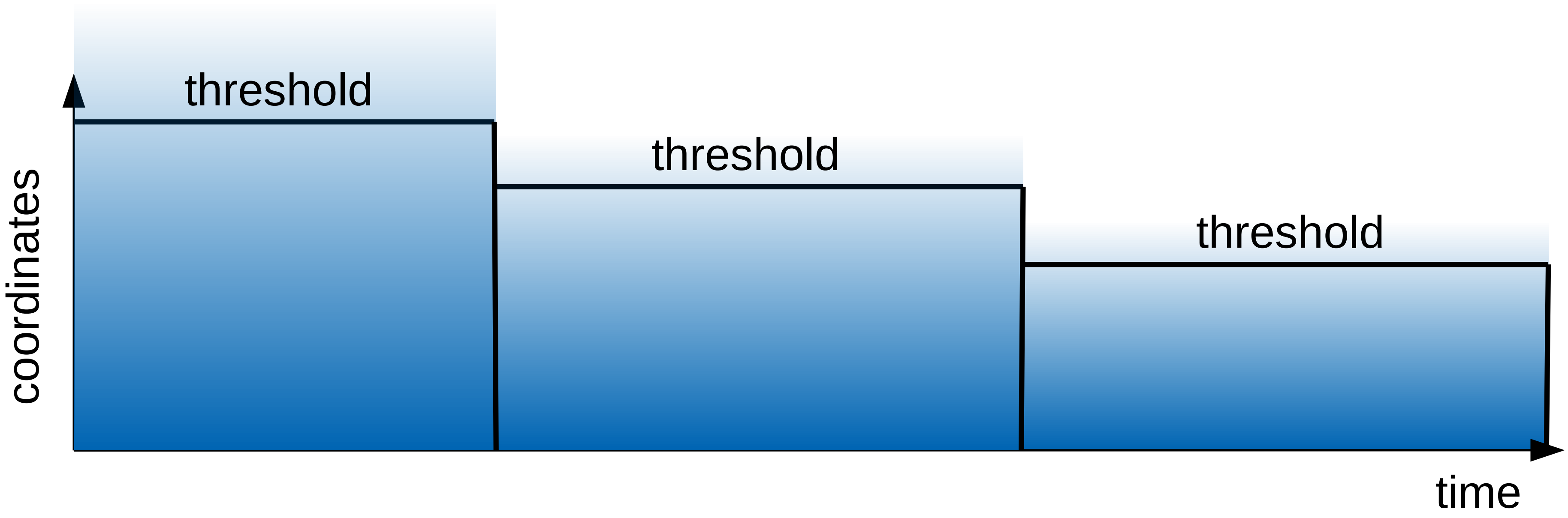
Also...

The code for n-level systems is written.

A lot of analytical results for 2-level and n-level systems are obtained.

Thank you for your attention!

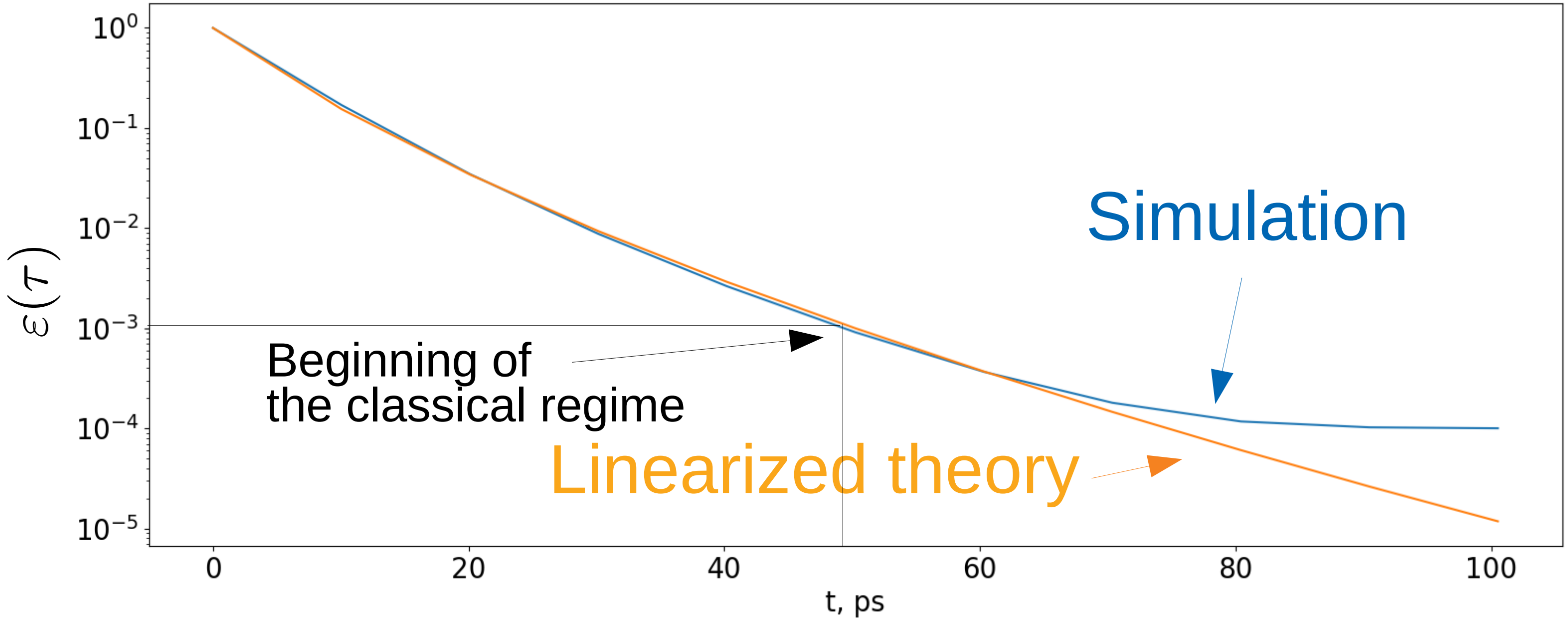
Field correlation function



Relative contribution of spontaneous emission for atomic observables

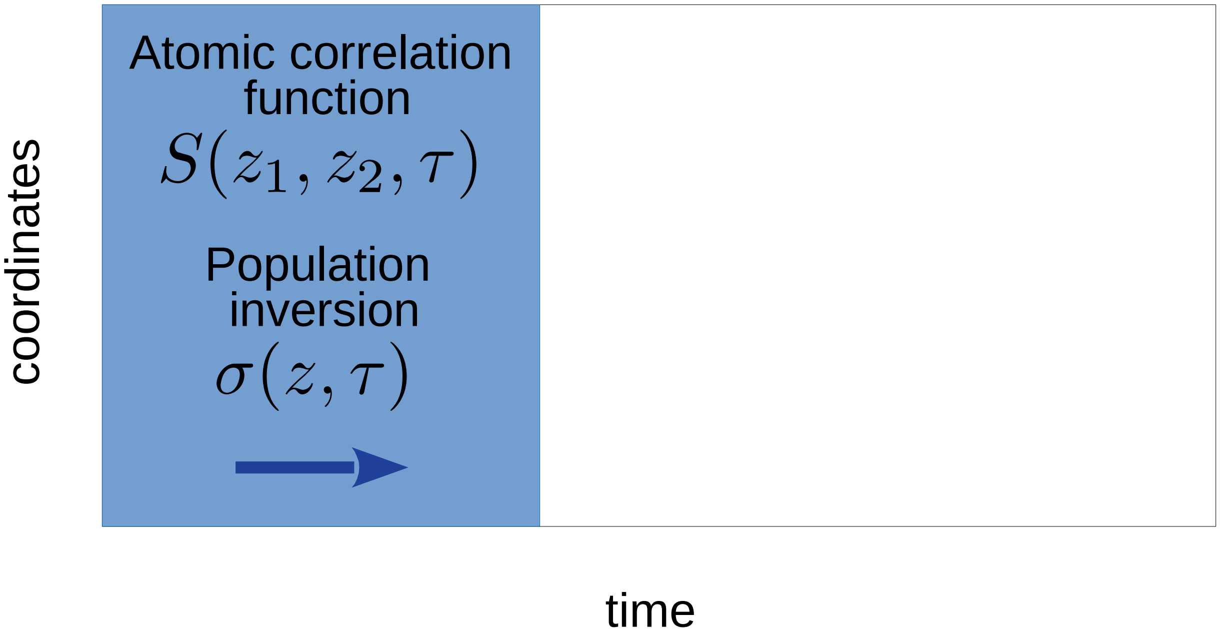
In case of linearized equations

$$\varepsilon(\tau) = \frac{\text{spontaneous emission term}}{\text{right hand side of the equation}} \approx \frac{\tau N g \Gamma_{sp}}{4 I_1^2 \left(\sqrt{\tau N g \Gamma_{sp}}\right)}$$



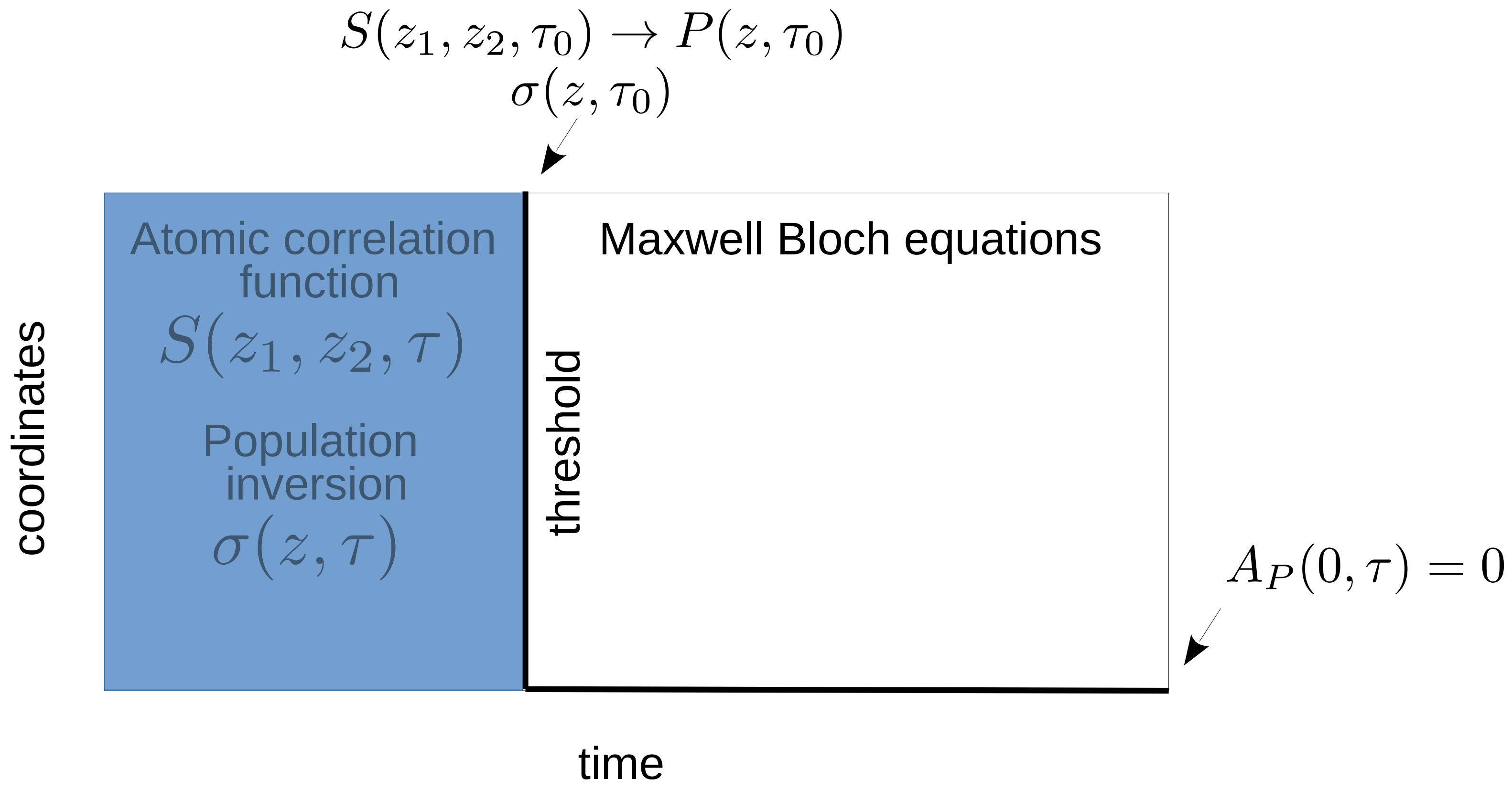
Algorithm

Step 1



Algorithm

Step 1



Algorithm

Step 2

$$S(z_1, z_2, \tau_0) \rightarrow P(z, \tau_0)$$
$$\sigma(z, \tau_0)$$

Atomic correlation function

$S(z_1, z_2, \tau)$

Population inversion

$\sigma(z, \tau)$

Maxwell Bloch equations

Polarization

$P(z, \tau)$

Pseudo field

$A_P(z, \tau)$

Population inversion

$\sigma(z, \tau)$

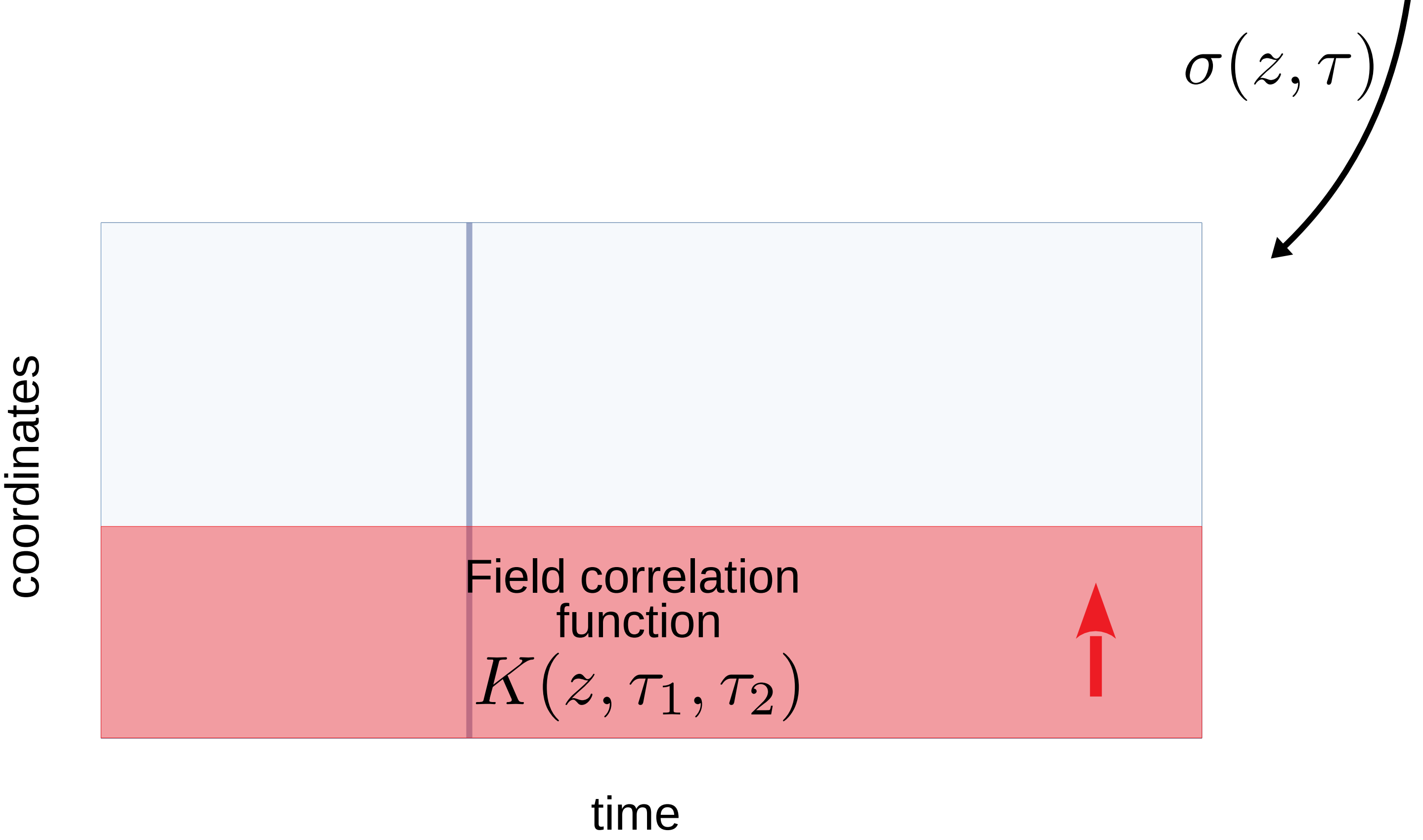
$A_P(0, \tau) = 0$

coordinates

time

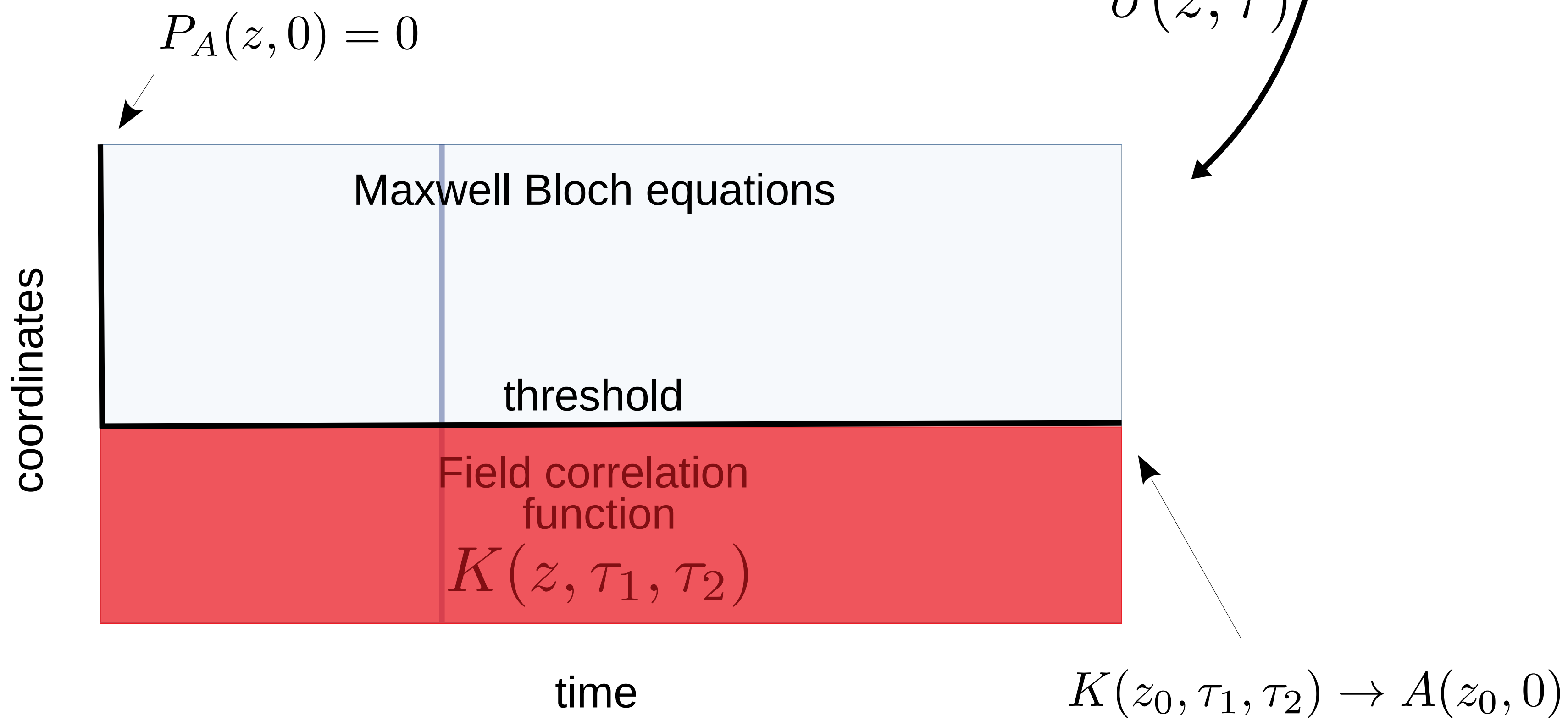
Algorithm

Step 3



Algorithm

Step 3



Algorithm

Step 4