Collinear Unpolarised Fragmentation Functions:

a general overview and an (incomplete) collection of recent results

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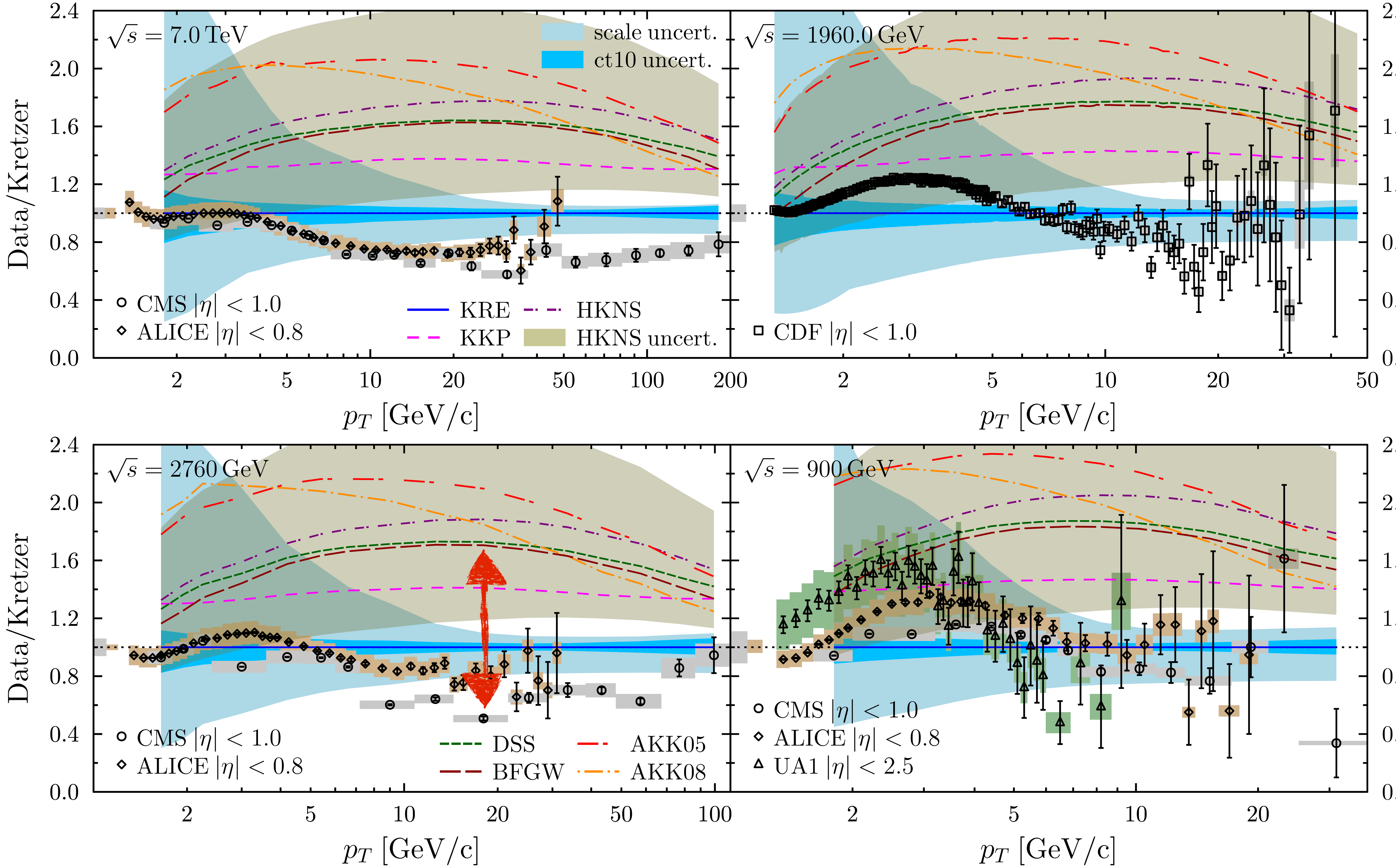
July 5, 2018, DESY Hamburg

Interesting facts

Why bother about fragmentation functions

Comparison data/theory for inclusive **charged-hadron p_T spectra**:



d'Enterria *et al.* [ArXiv:1311.1415]

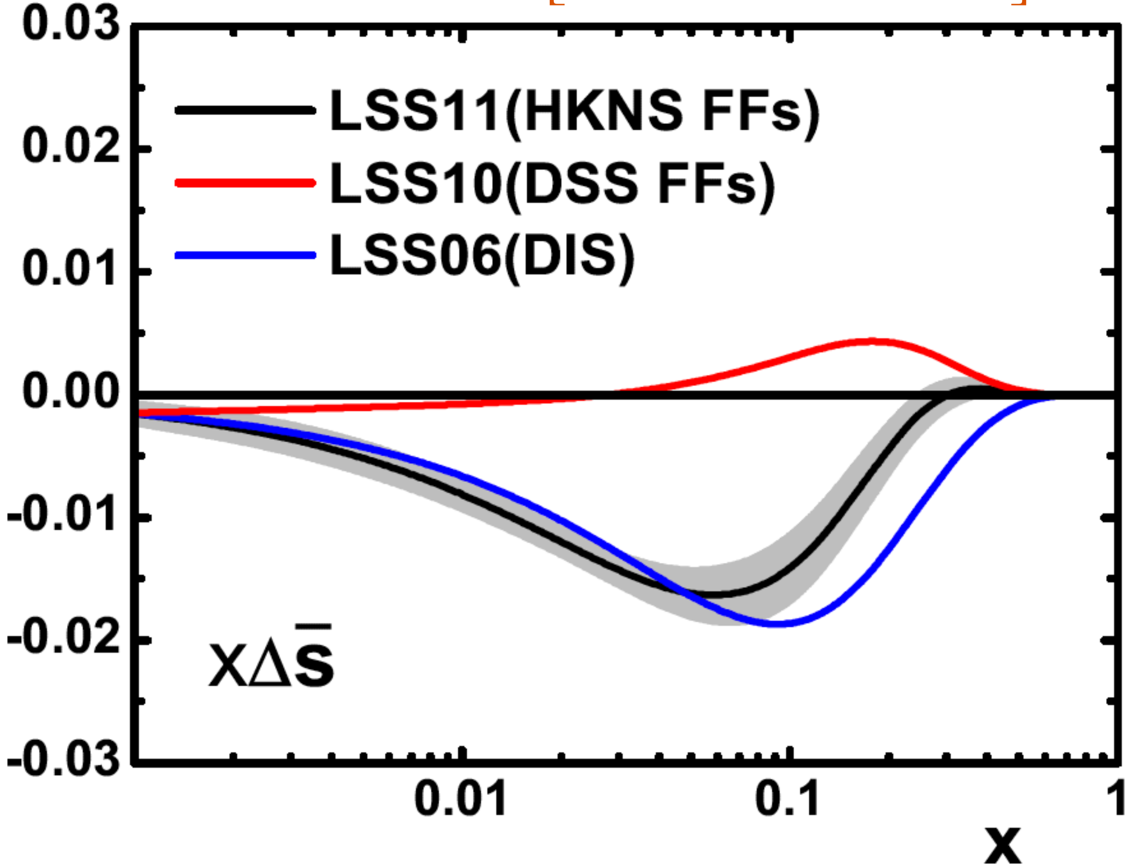
Large energy data tend to be **overshot** by predictions obtained with most of the current FF sets ⇒ **too hard gluon** FF at large z?

Interesting facts

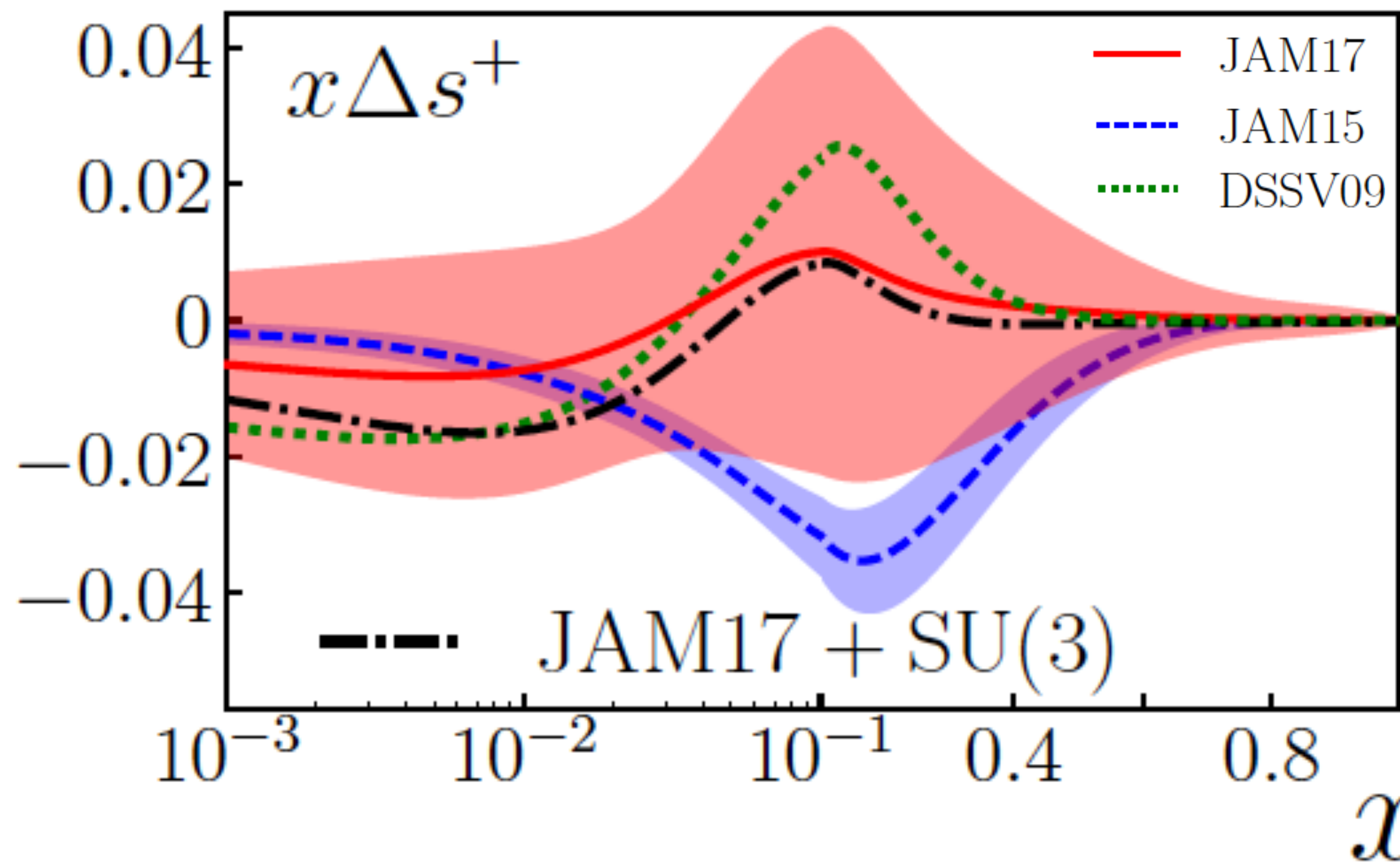
Why bother about fragmentation functions

Extraction of the **longitudinally polarised parton distribution functions**:

Laeder *et al.* [ArXiv:1103.5979]



J. J. Ethier *et al.* [ArXiv:1705.05889]



- In the presence of semi-inclusive DIS (SIDIS) data the **strange quark distribution** is very sensitive to the FF set used in the analysis.

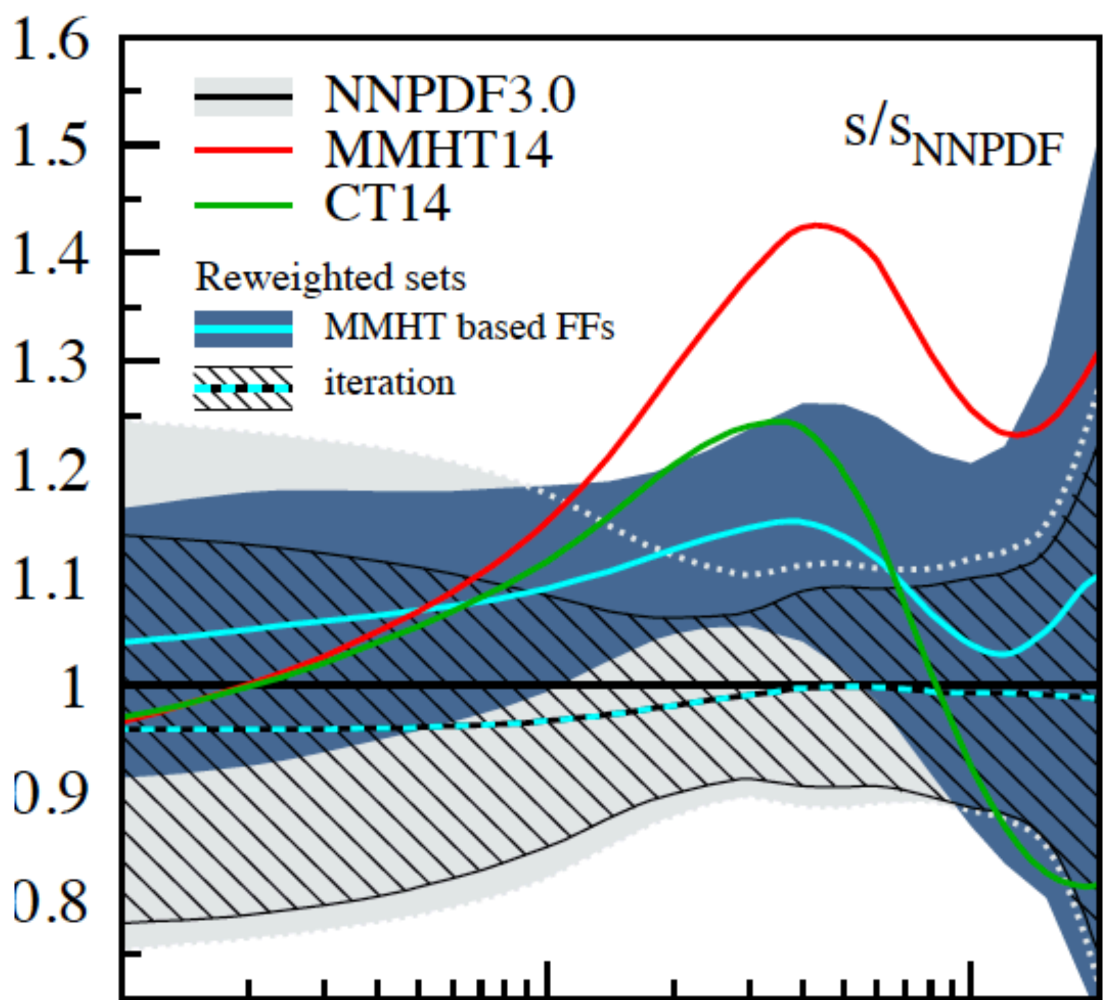
- Even fitting PDFs and FFs simultaneously does no lead to a definitive answer.

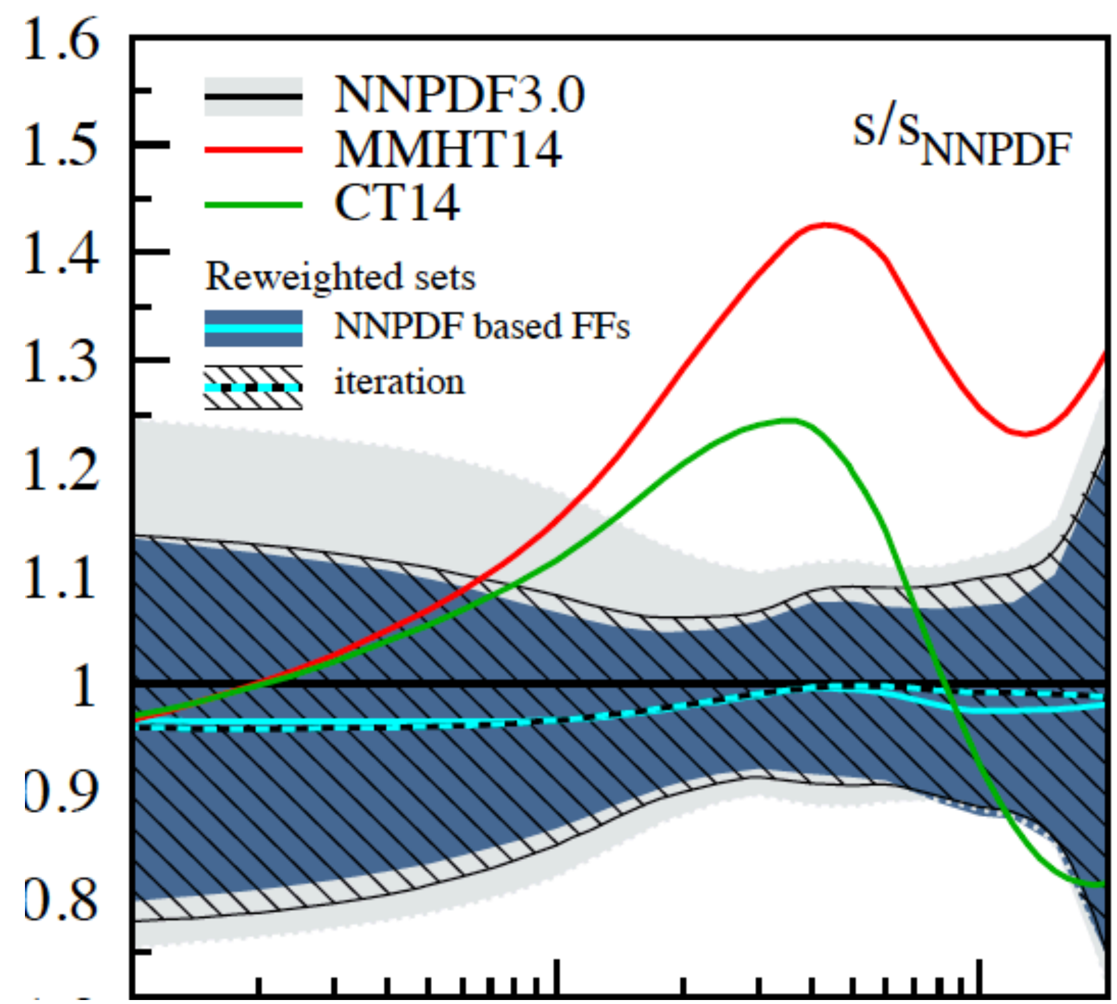
Interesting facts

Why bother about fragmentation functions

- **SIDIS** multiplicities depend on **PDFs** and thus the precise data from HERMES and COMPASS should in principle help constrain PDFs (noticeably the **strange** PDFs if a kaon is produced in the final state).

- On the other hand, SIDIS multiplicities also depend on **FFs** whose determination in turn often depends on PDFs.





Borsa *et al.* [ArXiv:1708.01630]

- A **combined** extraction of PDFs and FFs is a way to overcome this limitation.

Everything starts from…

The **collinear factorisation theorem** (assumed to work):

$$d\sigma_{\rm had} = \boxed{W_{\{i\}}} \otimes \boxed{\mathcal{L}_{\{i\}}} d\Phi$$

Hard cross sections:

- process dependent,
- high-energy dominated,
- computable in perturbation theory.

PDFs and/or FFs:

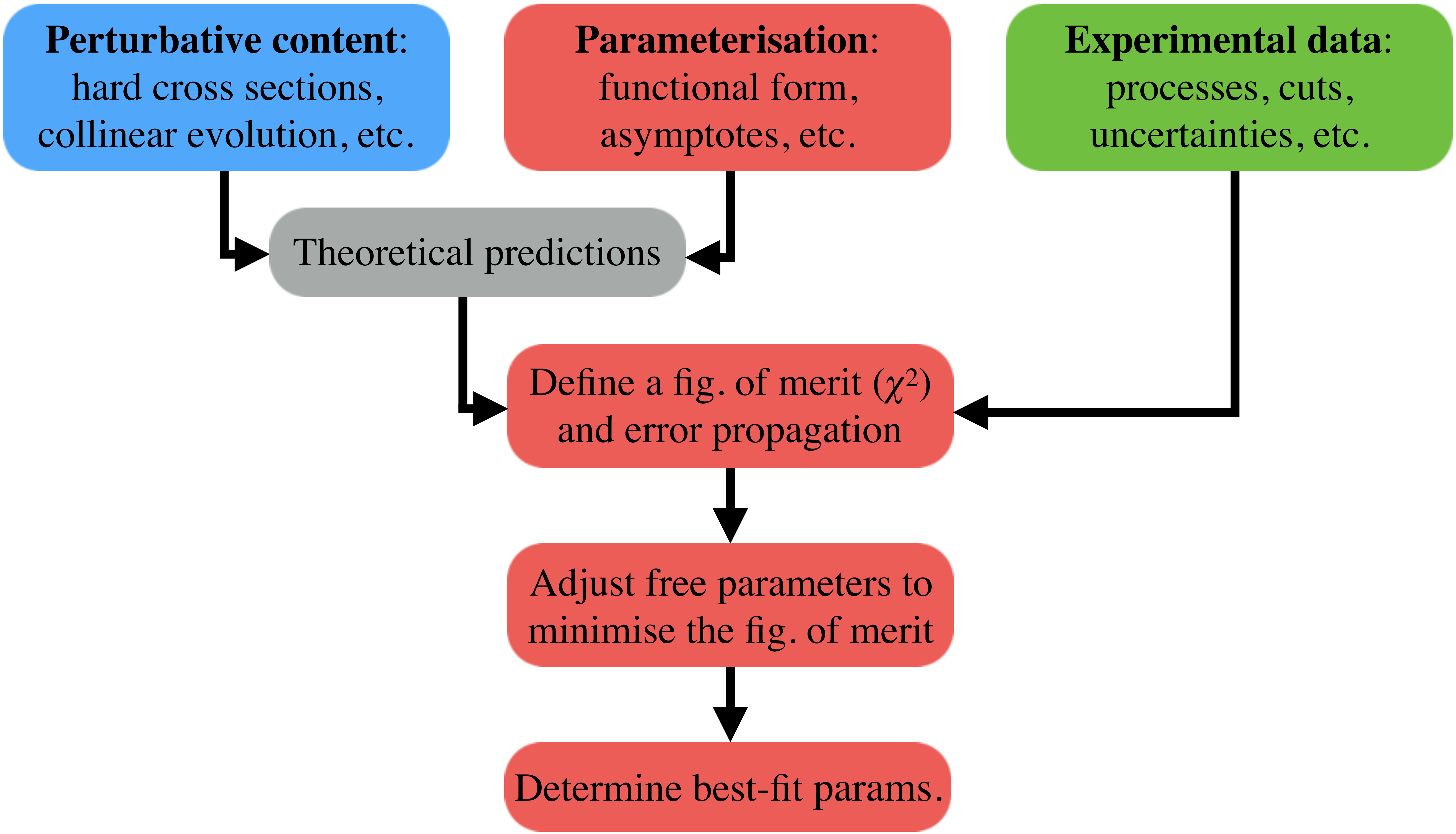
- universal (given the hadronic species),
- low-energy dominated,
- perturbation theory inapplicable.

How do we determine PDFs and FFs?

⇓

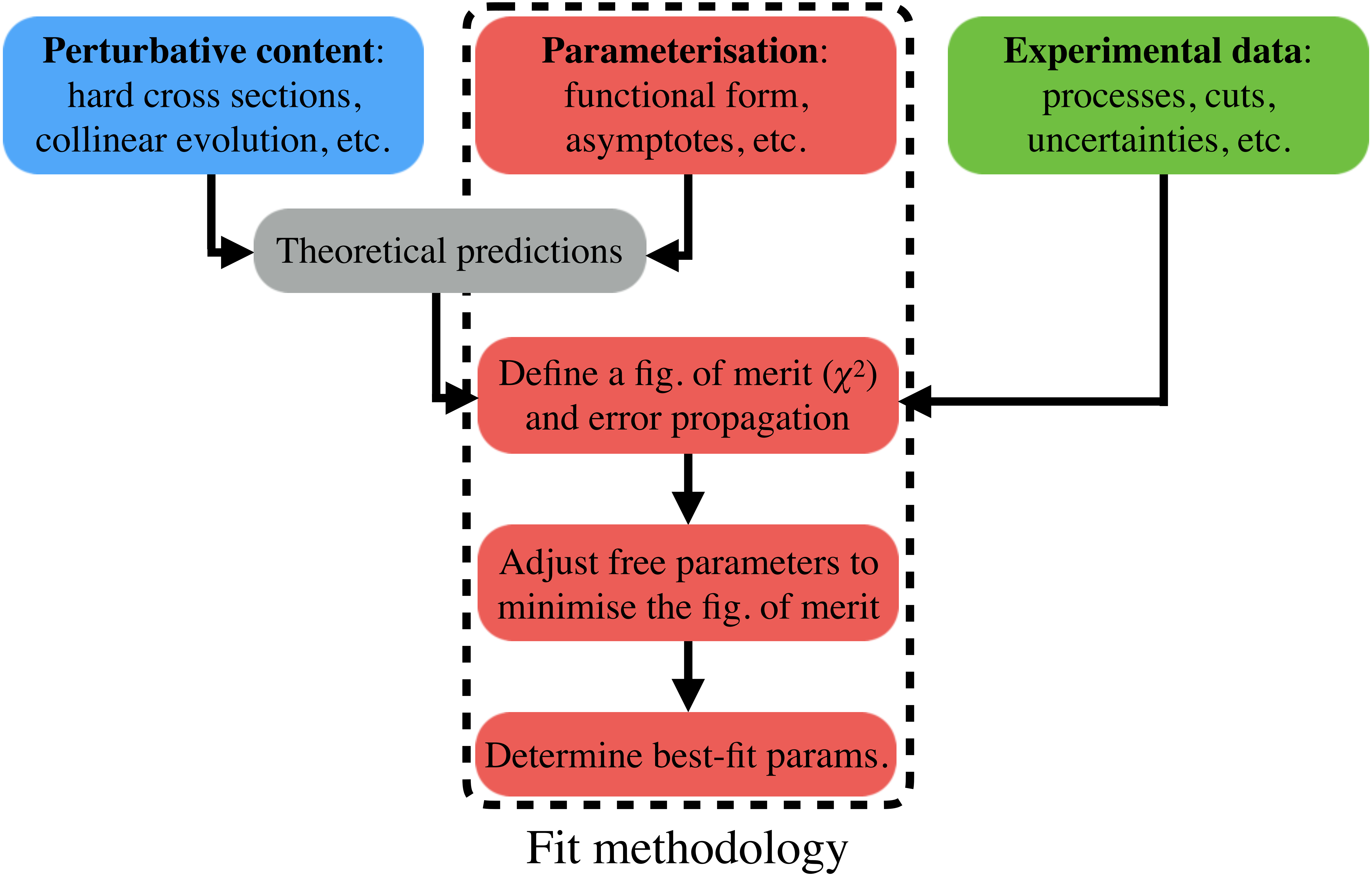
Currently, the most accurate and reliable way is through **fits to data**.

The general strategy



Each box requires a choice. **Different choices** lead to **different determinations**.

The general strategy



Fit methodologies

Parameterisation: the "standard" approach

- Distributions are parametrised by means of the functional form:

$$f_i(x) = A_i\, x^{\alpha_i}(1-x)^{\beta_i} P_i(x)$$

with:

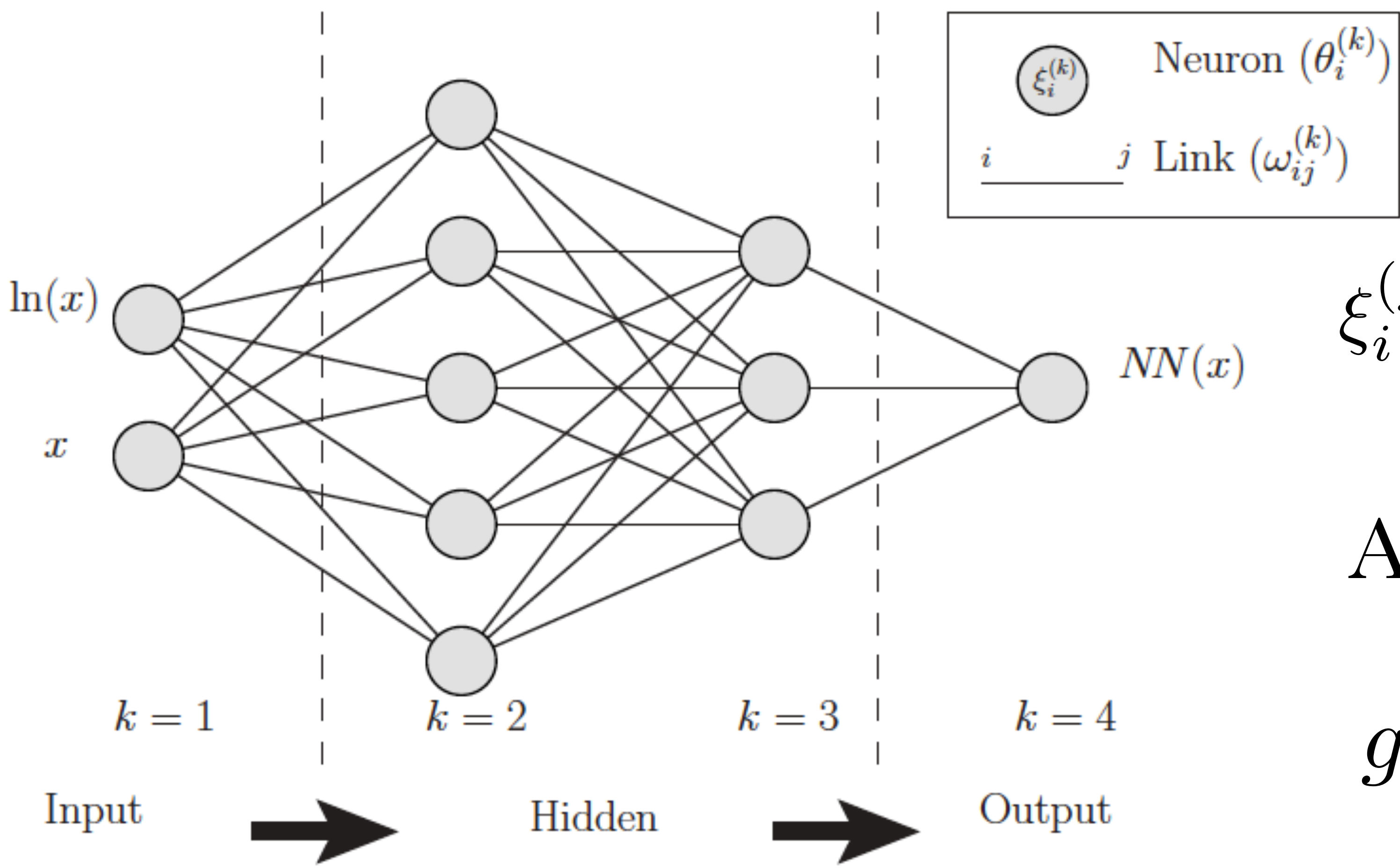
$$P_i(x) = \begin{cases} 1 \\ 1+\gamma_i x \\ 1+\gamma_i x + \delta_i \sqrt{x} \\ \dots \end{cases}$$

- **O(3-5) free parameters** for each distribution.
- **Asymptotic behaviour** defined by the exponents $\boldsymbol{\alpha_i}$ and $\boldsymbol{\beta_i}$.
- Easy to transform analytically in **Mellin space**.
- **Easy to handle** in a fit thanks to its simplicity.
- Potential **source of bias**.

Fit methodologies

Parameterisation: neural networks (in NNFF)

- Distributions are parametrised in terms of NNs with arch. (2-5-3-1):



$$\xi_i^{(j)} = g\left(\sum_k^{\text{(j-1)th layer}} \xi_k^{(j-1)} \omega_{ki}^{(j)} - \theta_i^{(j)} \right)$$

Activation function:

$$g(x) = \text{sign}(x)\ln(|x|+1)$$

- Each NN has **37 free** parameters each.
- Distributions are expressed as $f_i(x) = \text{NN}_i(x) - \text{NN}_i(1)$
  - The $\text{NN}_i(1)$ term ensures that $f_i(x) \xrightarrow[x\to 1]{} 0$
- NNs **flexible** and thus limit biases but **harder to handle**.

Fit methodologies

Figure of merit: the χ^2 definition

- A crucial aspect in the determination of PDFs/FFs is the definition of the **figure of merit** to be minimised/maximised.

- A popular choice is the χ^2 but **many variants** are possible:
  - No correlation, no normalisation unc.: $$\chi^2 = \sum_{i=1}^{N_{\rm dat}} \frac{(T_i - D_i)^2}{\sigma_i^2}$$
  - Only normalisation uncertainty: $$\chi^2 = \sum_{j=1}^{N_{\rm exp}} \left[\left(\frac{1-\mathcal{N}_j}{\delta\mathcal{N}_j} \right) + \sum_{i=1}^{N_{\rm dat}^j} \frac{(\mathcal{N}_j T_i - D_i)^2}{\sigma_i^2} \right]$$
  - Nuisance parameters: $$\chi^2 = \sum_i \frac{\left[T_i \left(1 - \sum_j \gamma_j^i b_j \right) - D_i \right]^2}{\delta_{i,\rm unc}^2 T_i^2 + \delta_{i,\rm stat}^2 D_i T_i} + \sum_j b_j^2$$
  - Covariance matrix: $$\chi^2 = \sum_{ij} (T_i - D_i) \sigma_{ij}^{-2} (T_j - D_j)$$

- Due to the **D'Agostini bias**, a sound treatment of normalisation uncertainties may require particular care (*e.g.* the t_0 prescription).

Fit methodologies

Error propagation

- A faithful determination implies an estimate of the **uncertainty** on FFs/PDFs propagating from the **experimental** dataset.

1. **Hessian** method: the χ^2 is **expanded** around its minimum $\mathbf{a}_0$:

$$\chi^2(\{\mathbf{a}\}) \simeq \chi^2(\{\mathbf{a}_0\}) + \underbrace{\frac{1}{2}\left.\frac{\partial\chi^2}{\partial a_i \partial a_j}\right|_{\mathbf{a}_0}}_{H_{ij}} (a_i - a_{0i})(a_j - a_{0j})$$

   The Hessian matrix H_{ij} is **diagonalised** and an uncertainty along each eigenvector is defined as $\Delta\chi^2 = 1$ (sometimes a **tolerance** is introduced).

2. **Monte Carlo** sampling: artificial **replicas** of the dataset generated as:

$$D_i^{(k)} = D_i + r_i^{(k)}\sigma_i \,, \qquad \begin{array}{l} k = 1, \dots, N_{\rm rep} \\ i = 1, \dots, N_{\rm dat} \end{array}$$

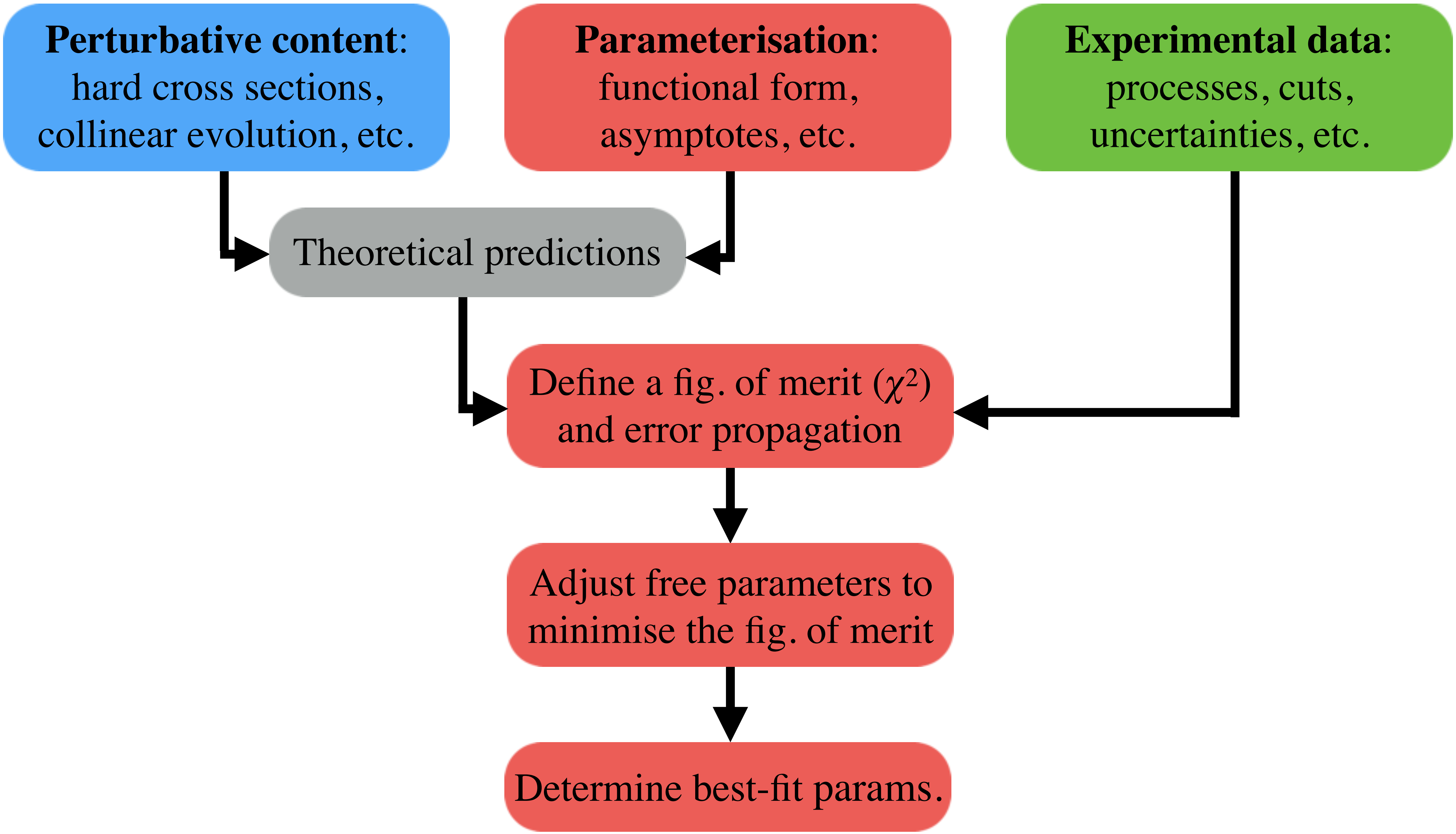
   $r_i^{(k)}$ is a *normally distributed* and *univariate* random number. A fit is performed to each replica to produce $N_{\rm rep}$ sets of distributions $\{f_k\}$, such that:

$$\langle \mathcal{O} \rangle = \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} \mathcal{O}[f_k] \quad \text{and} \quad \sigma_{\mathcal{O}} = \sqrt{\langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2}$$

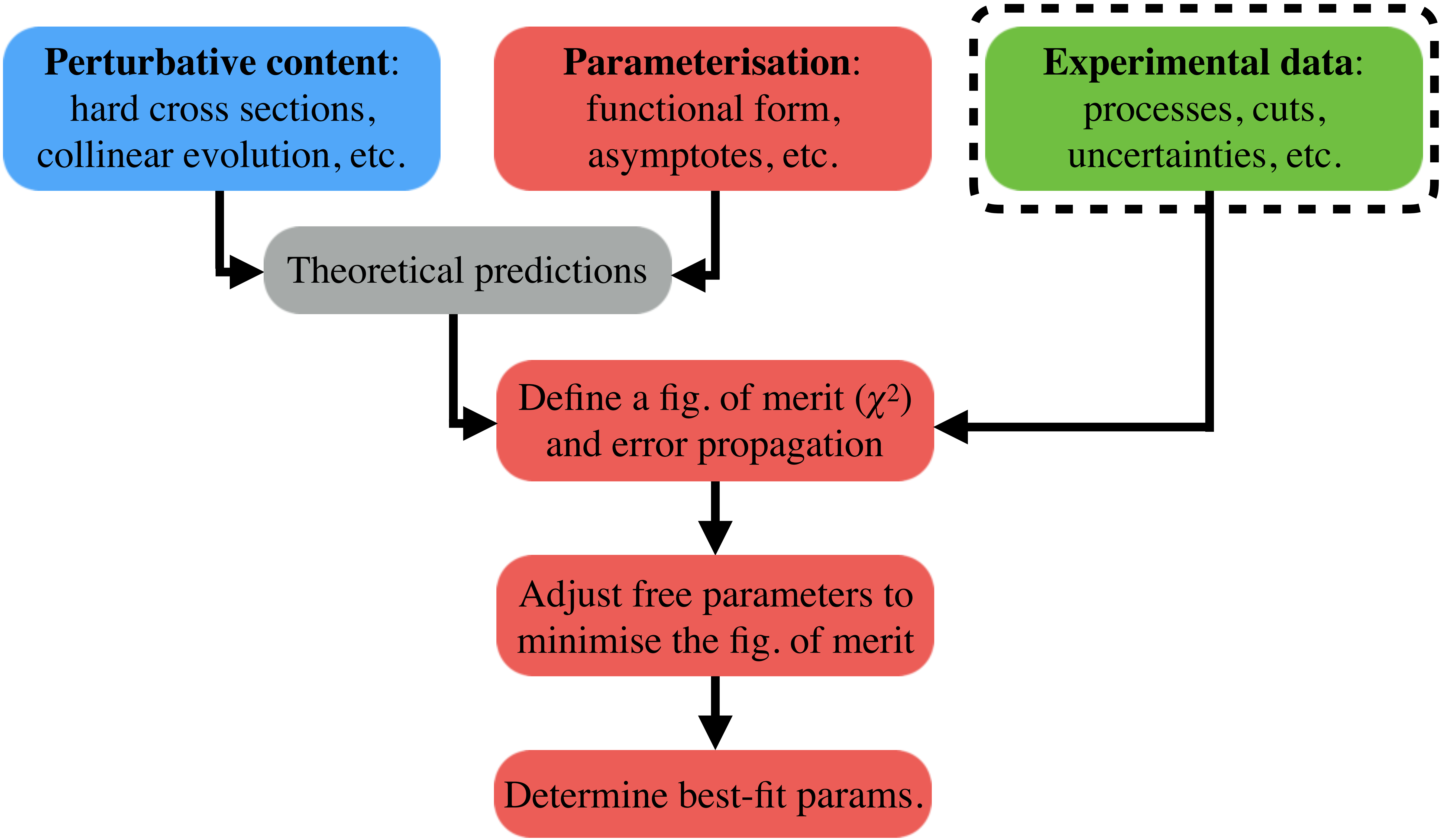
Fit methodologies
Minimisation and stopping

- Simple parameterisations (**O(20) free parameters**) are usually fitted using **MINUIT** (or similar):
  - the absolute minimum of the χ^2 is found *deterministically* by computing (numerically or analytically) the first derivative and moving **downhill**.
- A NN parameterisation (**O(200) free parameters**) generates a too complex parameter space to be treated deterministically:
  - a **genetic algorithm** is typically used to explore the parameter space,
  - this avoids getting trapped into **local minima** of the χ^2.
- The extreme flexibility of NNs may cause **overfitting**, *i.e.* statistical fluctuations of the data sample may be unwillingly fitted:
  - the **cross-validation** method allows one to overcome this problem.
- More refined algorithms based on **machine-learning** techniques are currently being explored (*e.g.* CMA-ES).

The general strategy

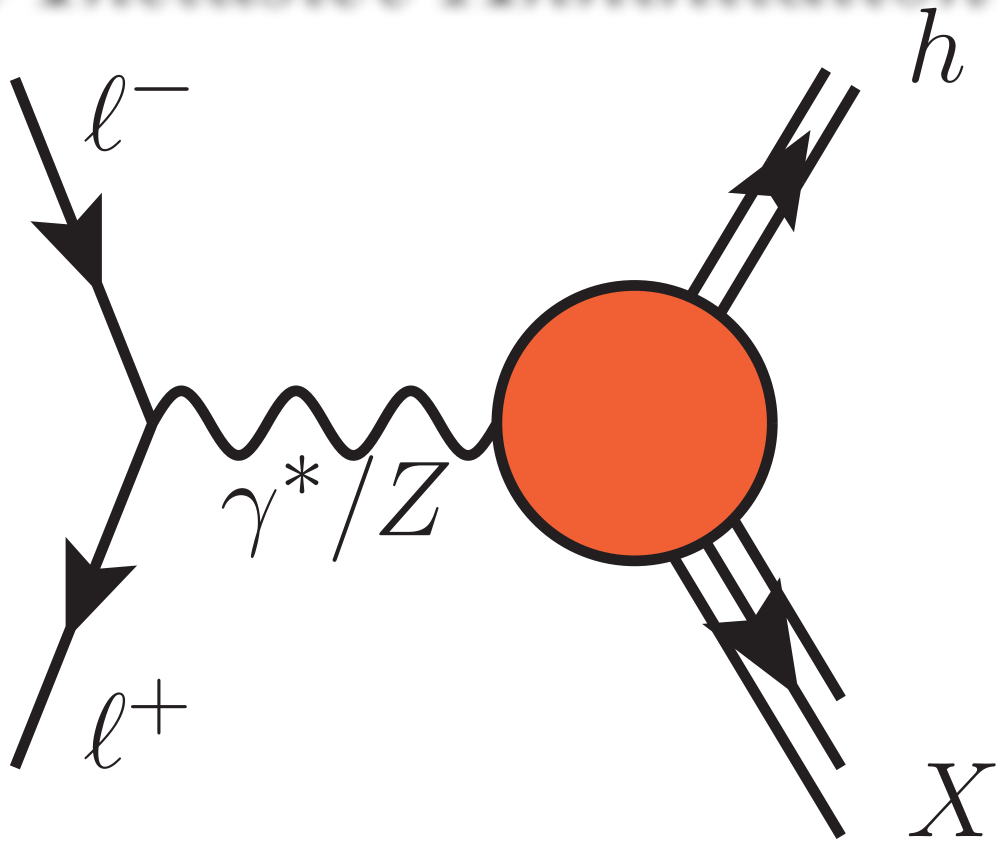


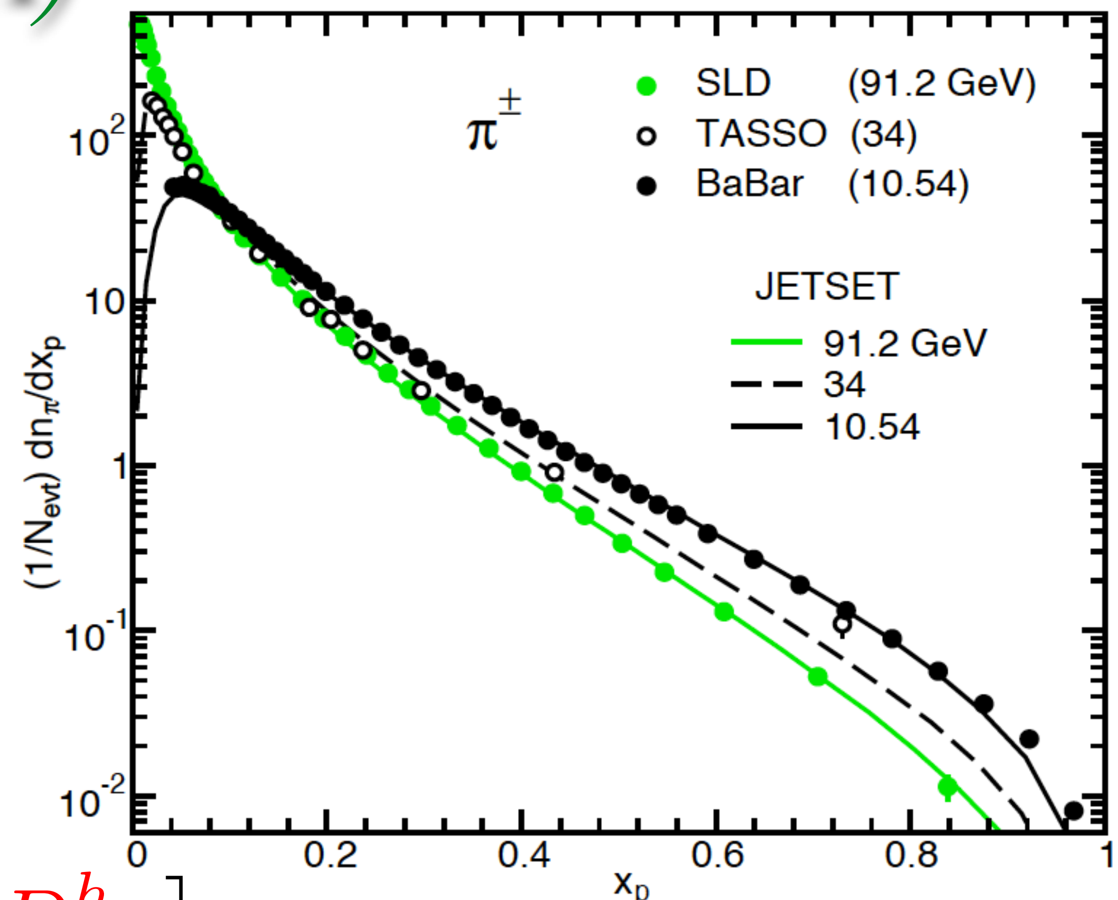
The general strategy



Experimental data

Single Inclusive Annihilation (SIA)





$$\frac{d\sigma^h}{dz} = \hat{\sigma}_0^h \left[C_q \otimes D_\Sigma^h + C_g \otimes D_g^h + C_{\rm NS} \otimes D_{\rm NS}^h \right]$$

- ✅ **Clean** channel: only FFs involved,
- ✅ **higher-order** corrections to NNLO,
- ✅ **precise data** available (BELLE/BABAR).
- ❌ **No flavour separation**,
  - ✅ tagged data for heavy-quark FFs.
- ❌ gluon distribution **suppressed** by α_s.

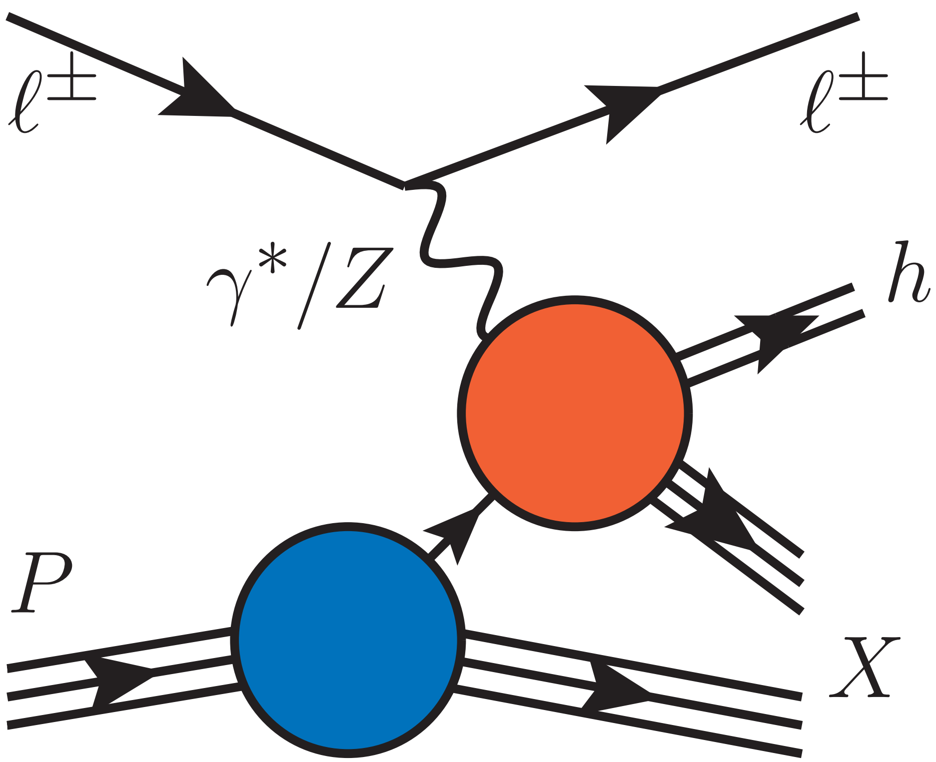
$$D_\Sigma^h = \sum_q \left(D_q^h + D_{\bar{q}}^h \right) = \sum_q D_{q^+}^h$$

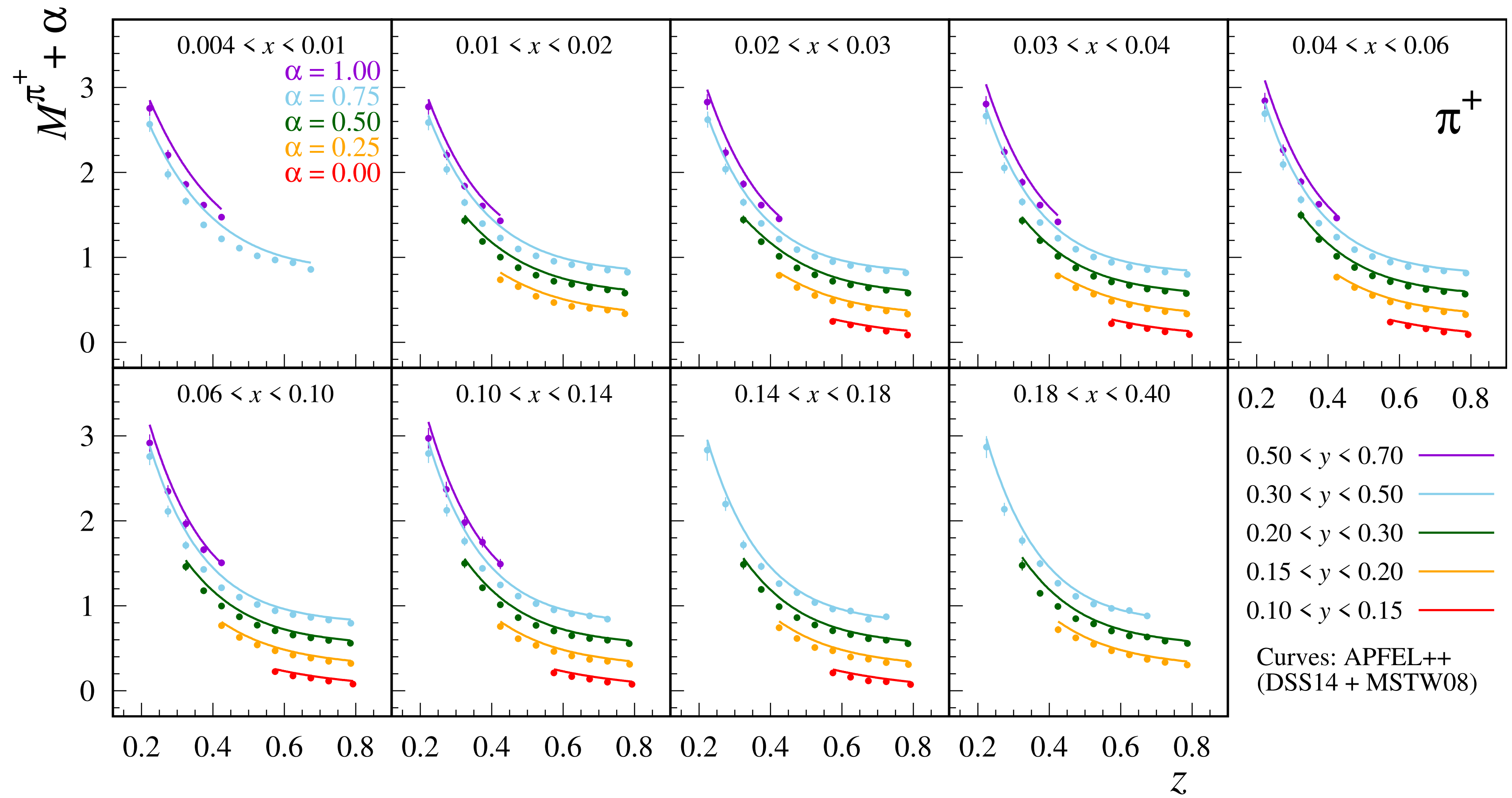
$$D_{\rm NS}^h = \sum_q \left(\frac{\hat{e}_q^2}{\langle \hat{e}_q^2 \rangle} - 1 \right) D_{q^+}^h$$

$$C_q, C_{\rm NS} \propto \mathcal{O}(1) \quad \text{while} \quad C_g \propto \mathcal{O}(\alpha_s)$$

Experimental data

Semi Inclusive Deep Inelastic Scattering (SIDIS)



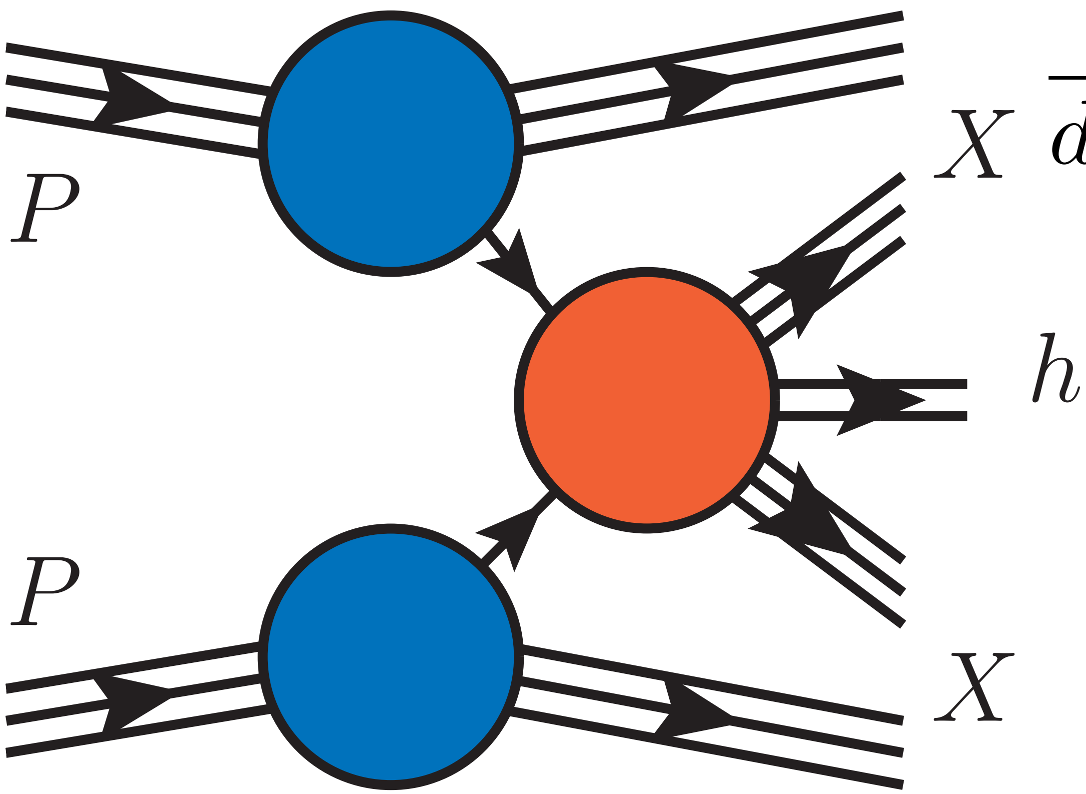


$$\frac{d\sigma^h}{dxdydz} = \hat{\sigma}_0^h \sum_{q,\bar{q}} e_q^2 \left[f_q \otimes C_{qq} \otimes D_q^h + f_g \otimes C_{gq} \otimes D_q^h + f_q \otimes C_{qg} \otimes D_g^h \right]$$

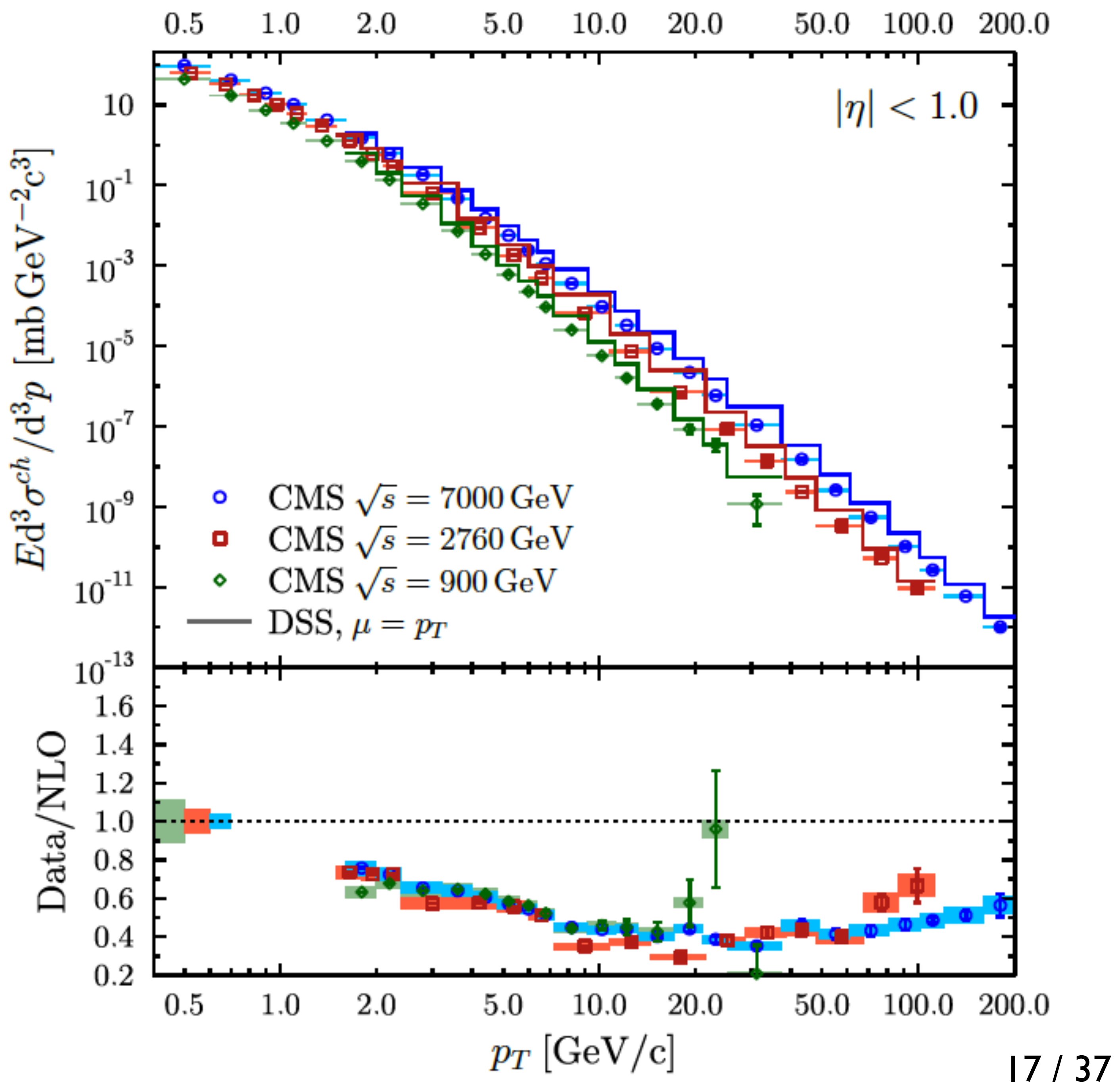
- ✅ handle on **flavour separation**,
- ✅ **precise data** available (HERMES/COMPASS).
- ❌ Involves both **FFs** and **PDFs**,
- ❌ Fully known so far up to $O(\alpha_s)$, *i.e.* NLO.

Experimental data

Hadroproduction in proton-proton collisions (pp)

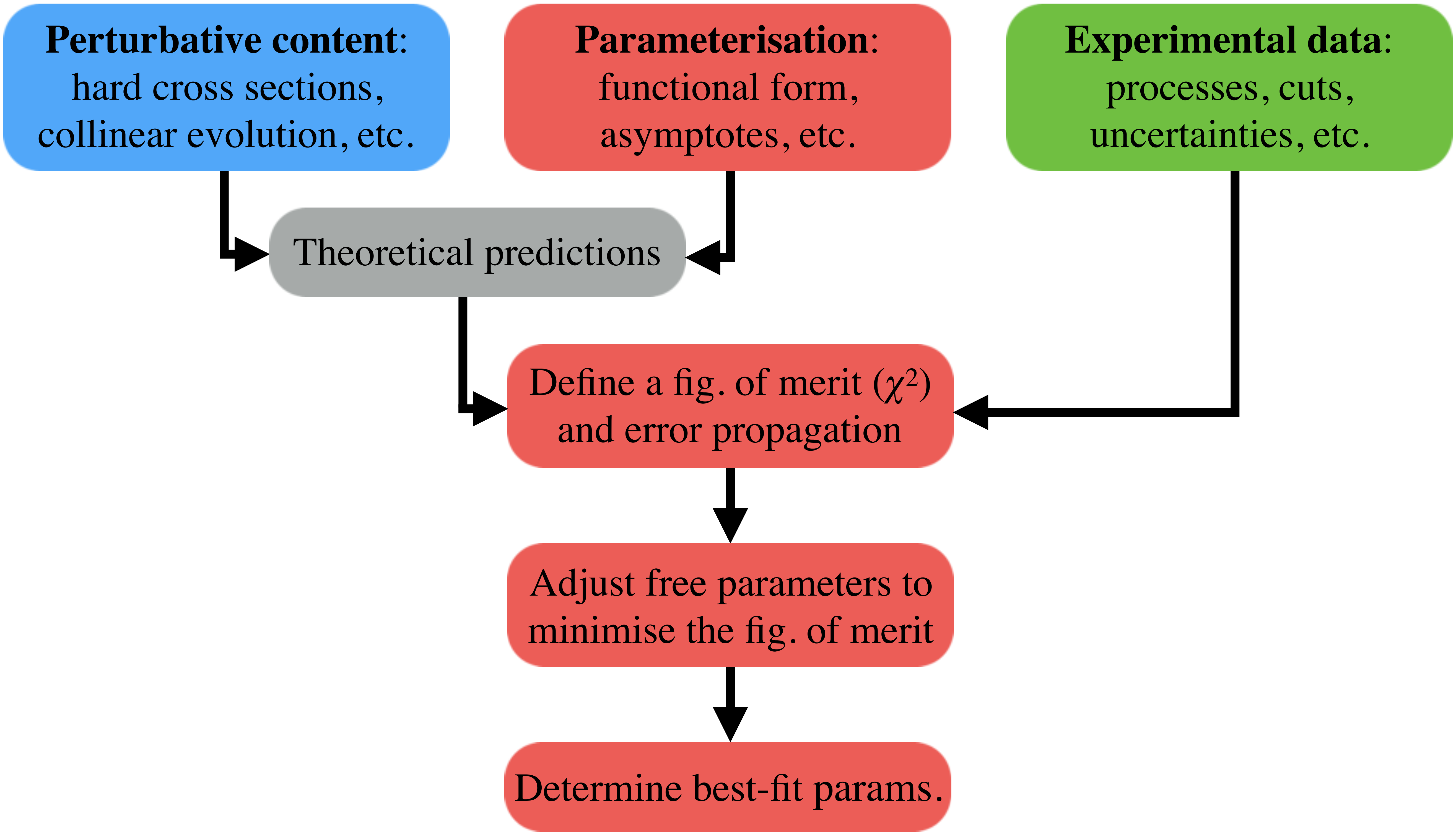


$$\frac{d\sigma^h}{dp_T d\eta} = \sum_{ijk} f_i^{(1)} \otimes f_j^{(2)} \otimes \frac{d\hat{\sigma}_{ijk}}{dp_T d\eta} \otimes D_k^h$$

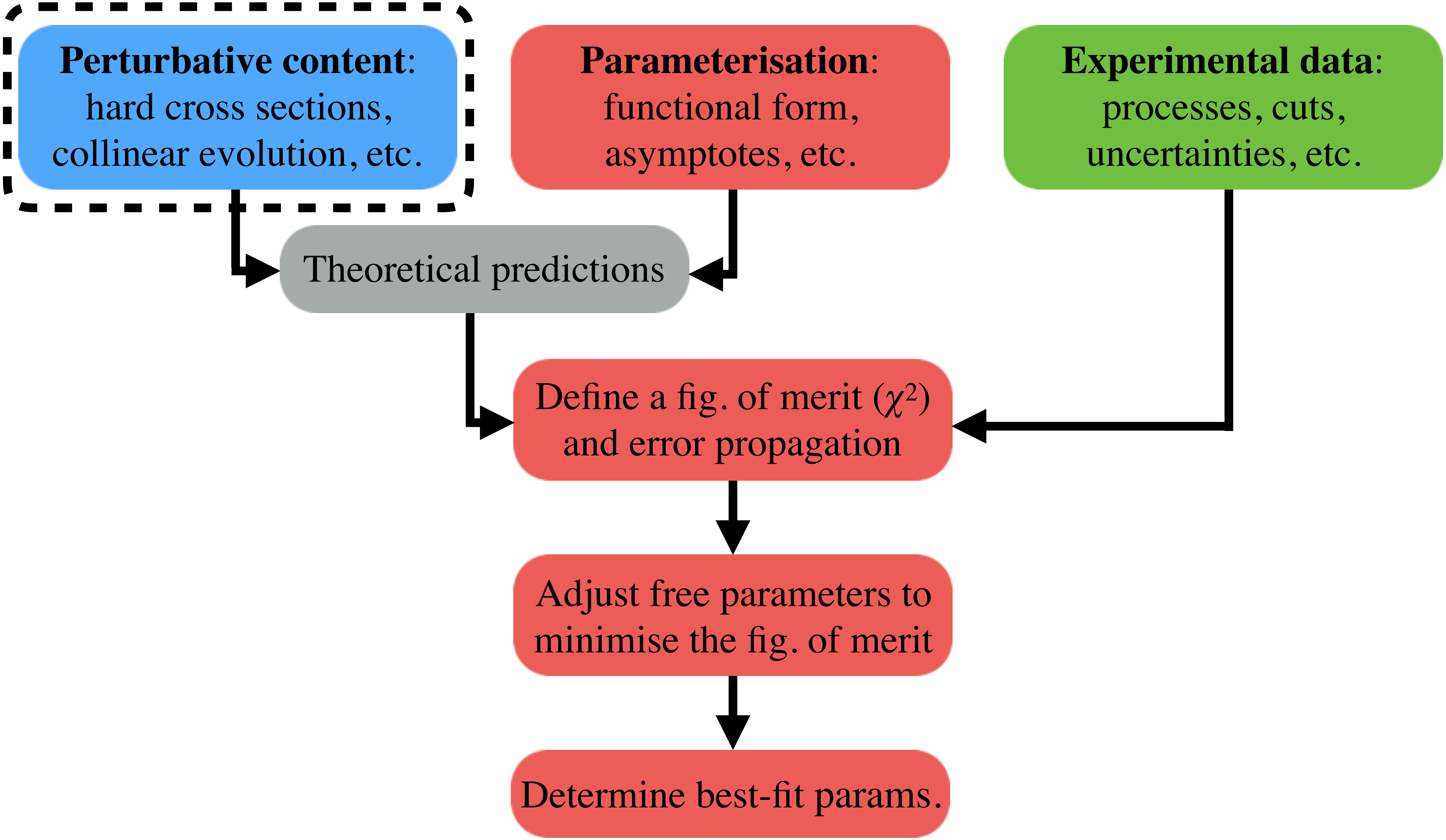


- ✅ Direct sensitivity to the **gluon FF**,
- ✅ **Scale scan** ($\mu_F \propto p_T$),
- ✅ Precise data from LHC/Tevatron.
- ❌ Involves both **FFs** and **PDFs**,
- ❌ Known so far up to NLO,
  - ❌ large scale variations at low p_T,
- ❌ cumbersome to compute.

The general strategy



The general strategy



Perturbative content

DGLAP evolution

- FFs obey the standard collinear DGLAP evolution equations:

$$\mu^2 \frac{\partial}{\partial \mu^2} D^h_{\rm NS} = P_{\rm NS} \otimes D^h_{\rm NS}$$

$$\mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} D^h_\Sigma \\ D^h_g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} D^h_\Sigma \\ D^h_g \end{pmatrix}$$

- Time-like splitting functions fully known up to NNLO.
  A. Mitov and S. O. Moch [hep-ph/0604160], M. Gluck, E. Reya, and A. Vogt [Phys.Rev. D48 (1993)]
- Numerical implementation in the **APFEL** code:
  V. Bertone, C. Carrazza, J. Rojo [arXiv:1310.1394]
  - careful benchmark against in the the N-space **MELA** code,
    V. Bertone, S. Carrazza, E. R. Nocera [arXiv:1501.00494]
  - perfect agreement with **QCDNUM** (after a correction of a bug in the latter).
    M. Botje [arXiv:1602.08383]

Perturbative content

Hard cross sections

- **Single-inclusive annihilation**:
  - currently known up to $O(\alpha_s^2)$, *i.e.* NNLO, in the **zero-mass** scheme.
    A. Mitov and S. O. Moch [hep-ph/0604160]
  - **Mass corrections** known up to to $O(\alpha_s)$,
    T. Kneesch *et al.* [desy-thesis-10-049]
  - **Small-*z* resummation** corrections up to NNLL,
    D. Anderle *et al.* [arXiv:1611.03371]
  - **Hadron-mass** corrections (particularly relevant for kaons and protons).
    e.g. A. Accardi *et al.* [arXiv:1411.3649]
  - **Threshold (large-*z*) resummation** corrections up to N^3LL.
    S. O. Moch and A. Vogt [arXiv:0908.2746]
- **Semi-Inclusive Deep-Inelastic-Scattering**:
  - currently fully know up to $O(\alpha_s)$, *i.e.* NLO, in the **zero-mass** scheme.
  - partial knowledge of the $O(\alpha_s^2)$ corrections to F_L.
    D. Anderle *et al.* [arXiv:1612.01293]
  - **Threshold resummation** corrections to NLL.
    e.g. D. Anderle *et al.* [arXiv:1304.1373]
- **Hadroproduction in proton-proton collisions**:
  - currently fully know up to $O(\alpha_s^3)$, *i.e.* NLO, in the **zero-mass** scheme.
    P. Aurenche *et al.* [hep-ph/9910252]

A selection of recent results

Most recent determinations

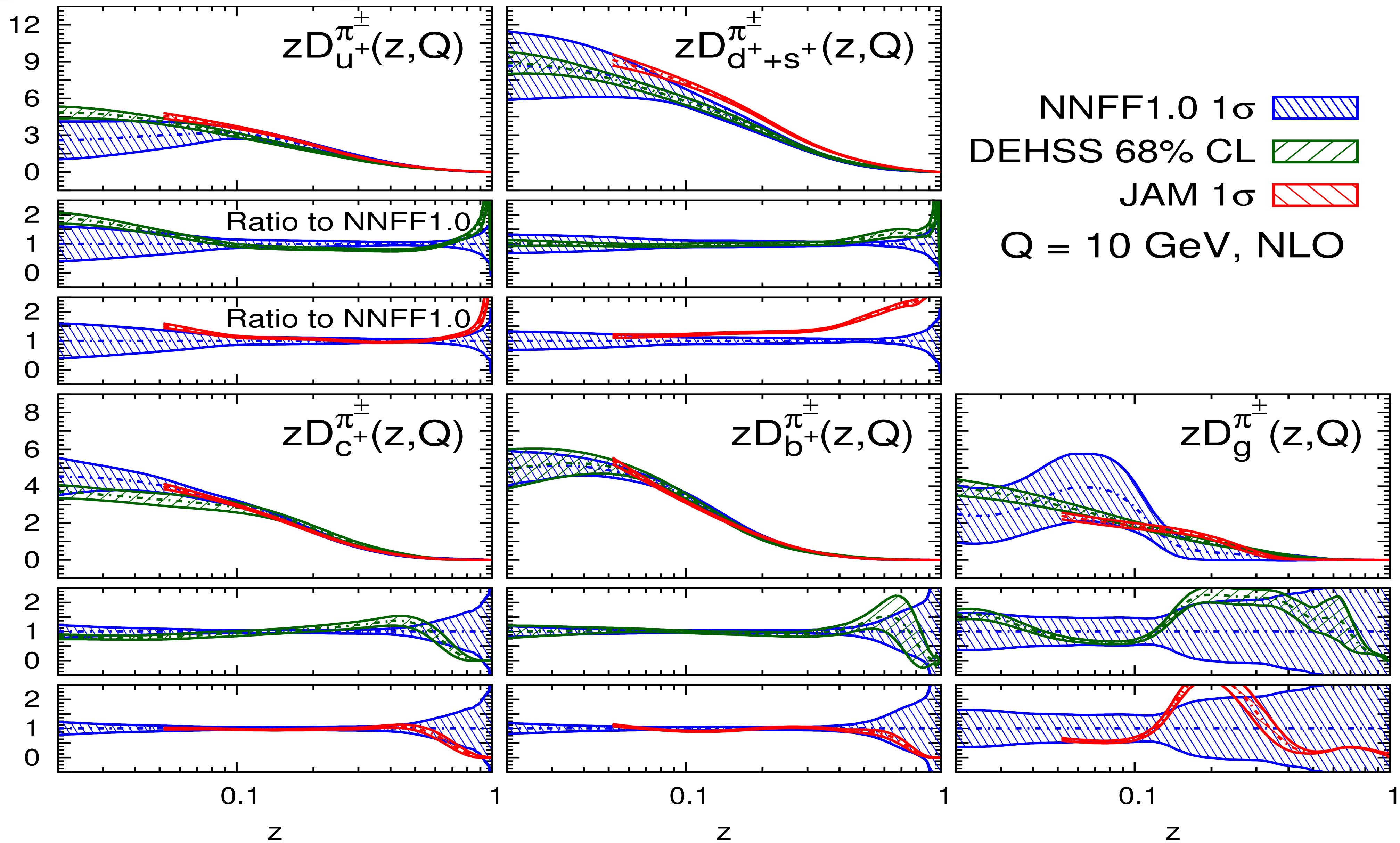
"Global" fits: Overview

	DEHSS [arXiv:1410.6027]...	HKKS [arXiv:1608.04067]	JAM [arXiv:1609.00899]	NNFF [arXiv:1706.07049]
Parameterisation	standard	standard	standard	neural nets
Figure of merit	χ^2, only norm.	χ^2, only norm.	χ^2, nuisance pars.	χ^2, cov. matrix
Error propagation	Hessian	Hessian	Monte Carlo	Monte Carlo
Minimisation	MINUIT	MINUIT	MINPACK	GA/CMA-ES
Dataset	SIA, SIDIS, pp	SIA	SIA	SIA, pp
Hadronic species	$\pi^\pm$, $K^\pm$, $p/\overline{p}$, $h^\pm$	$\pi^\pm$, $K^\pm$	$\pi^\pm$, $K^\pm$	$\pi^\pm$, $K^\pm$, $p/\overline{p}$, $h^\pm$
Perturbative orders	LO, NLO	NLO	NLO	LO, NLO, NNLO
Mass scheme	ZM-VFNS	ZM-VFNS	ZM-VFNS	ZM-VFNS

- **Many more** on the market (see *e.g.* http://lapth.cnrs.fr/ffgenerator/ and http://www2.pv.infn.it/~radici/FFdatabase/).
- More hadronic species available (D^*, Λ, etc.).
- HKKS set not publicly available.

Most recent determinations

Comparison for pions

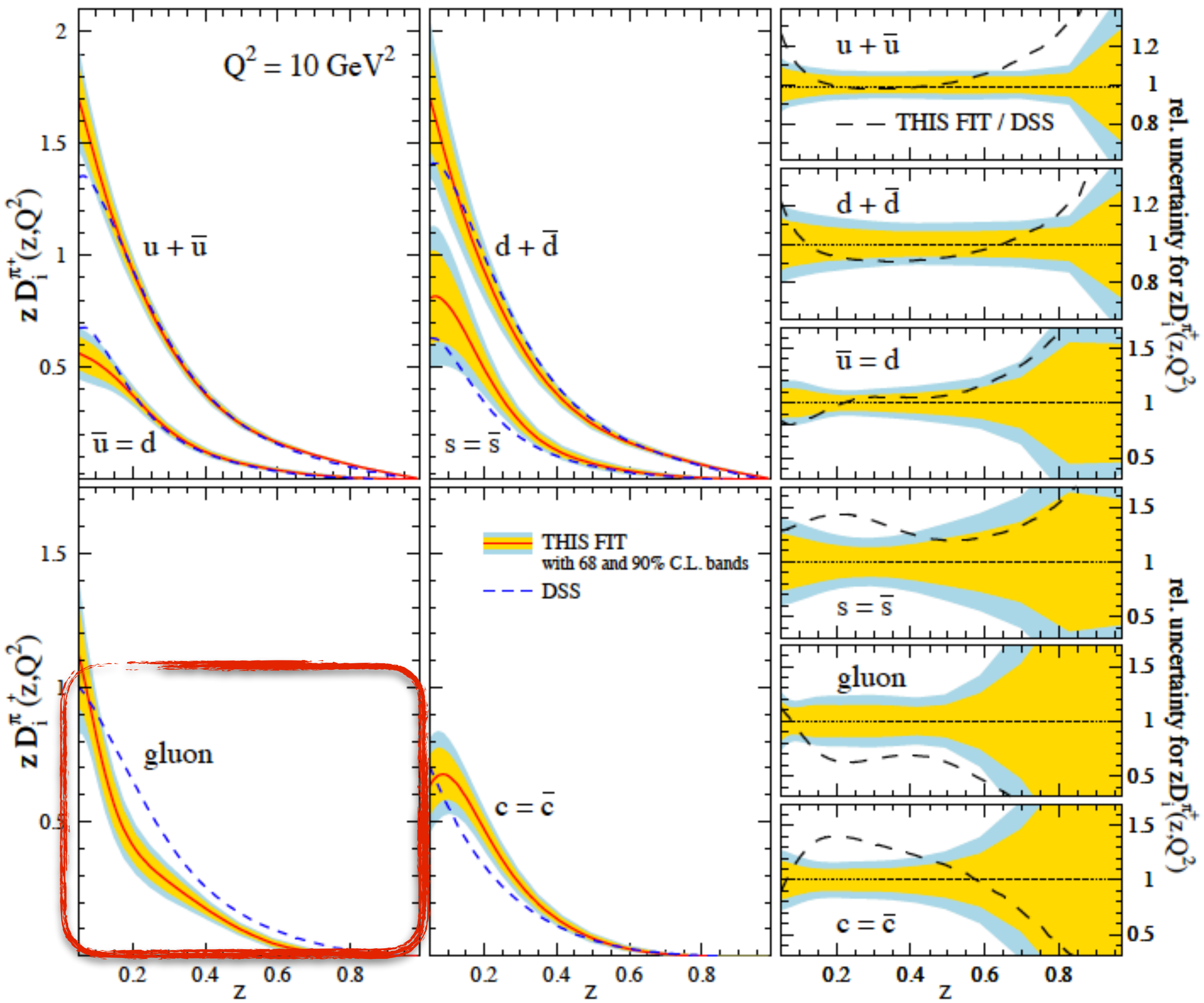


- Fair agreement between **NNFF1.0** (**red**) and **DEHSS** (**green**):
  - differences u^+ and gluon distributions at the **one-σ level**.
- More sizeable differences between **NNFF1.0** and **JAM** (**blue**):
  - the $d^+ + s^+$ and gluon distributions **well beyond one-σ**,
  - generally **smaller uncertainties** of the JAM (despite similar dataset).

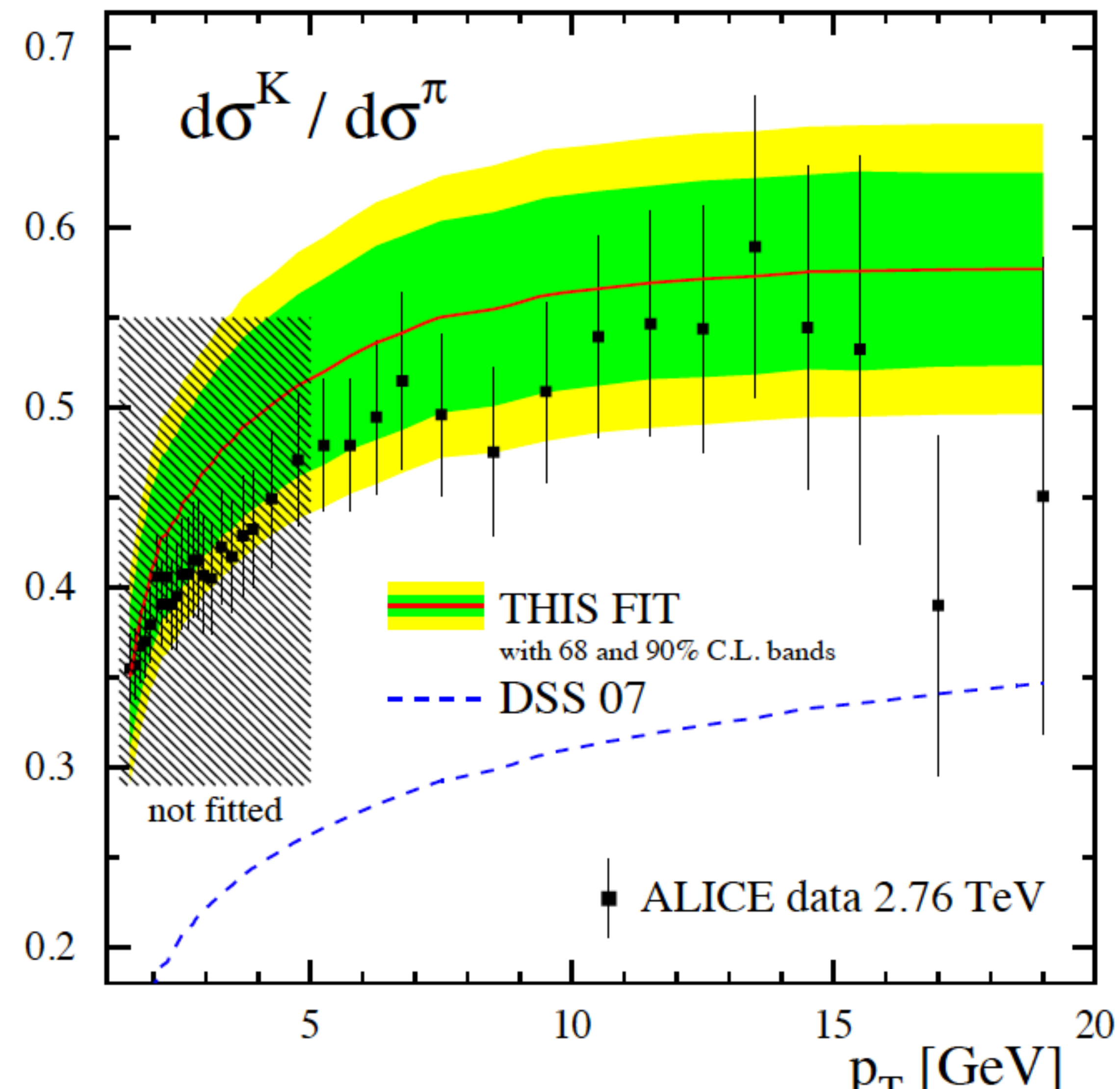
Recent results

The impact of LHC data on FFs

The DSS group included for the first time ALICE data for π^0 production at 7 TeV and K/π ratio data at 2.76 TeV in two separate analyses for **pion** (lefts) and **kaon** (right) FFs.



de Florian *et al.* [ArXiv:1702.06353]



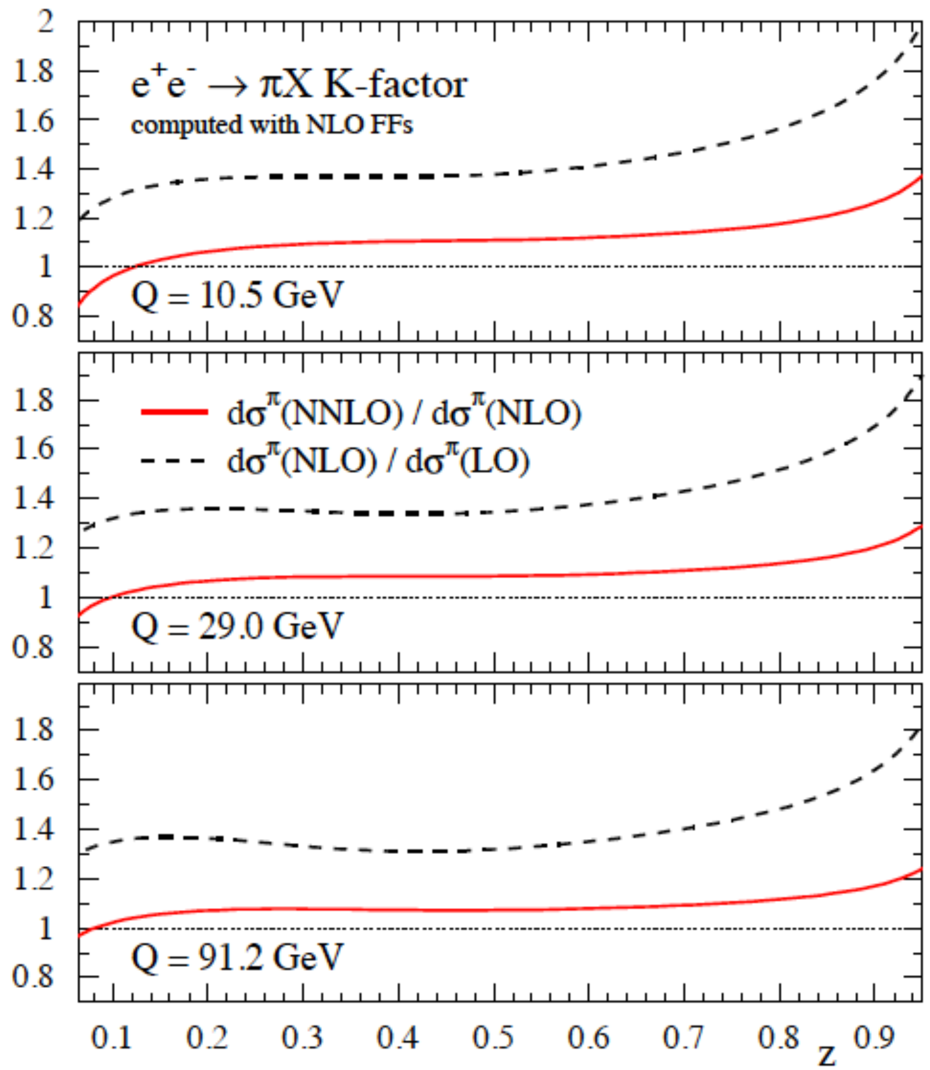
de Florian *et al.* [ArXiv:1410.6027]

- **Poor description** before inclusion in the fit (**DSS07**, blue curves).
- The ALICE data for **pions** caused:
  - a strong **suppression** of the **gluon distribution**,
- Larger and more general difference for **kaons**.

Recent results

The first determination of FFs at NNLO

NNLO corrections to the DGLAP evolution and the hard cross sections were for the first time included in an analysis based on **SIA data** only (not available for other processes)



experiment	data type	# data in fit	χ^2 LO	NLO	NNLO
SLD [40]	incl.	23	15.0	14.8	15.5
	uds tag	14	9.7	18.7	18.8
	c tag	14	10.4	21.0	20.4
	b tag	14	5.9	7.1	8.4
ALEPH [41]	incl.	17	19.2	12.8	12.6
DELPHI [42]	incl.	15	7.4	9.0	9.9
	uds tag	15	8.3	3.8	4.3
	b tag	15	8.5	4.5	4.0
OPAL [43]	incl.	13	8.9	4.9	4.8
TPC [44]	incl.	13	5.3	6.0	6.9
	uds tag	6	1.9	2.1	1.7
	c tag	6	4.0	4.5	4.1
	b tag	6	8.6	8.8	8.6
BABAR [10]	incl.	41	108.7	54.3	37.1
BELLE [9]	incl.	76	11.8	10.9	11.0
NORM. SHIFTS			7.4	6.8	7.1
TOTAL:		288	241.0	190.0	175.2

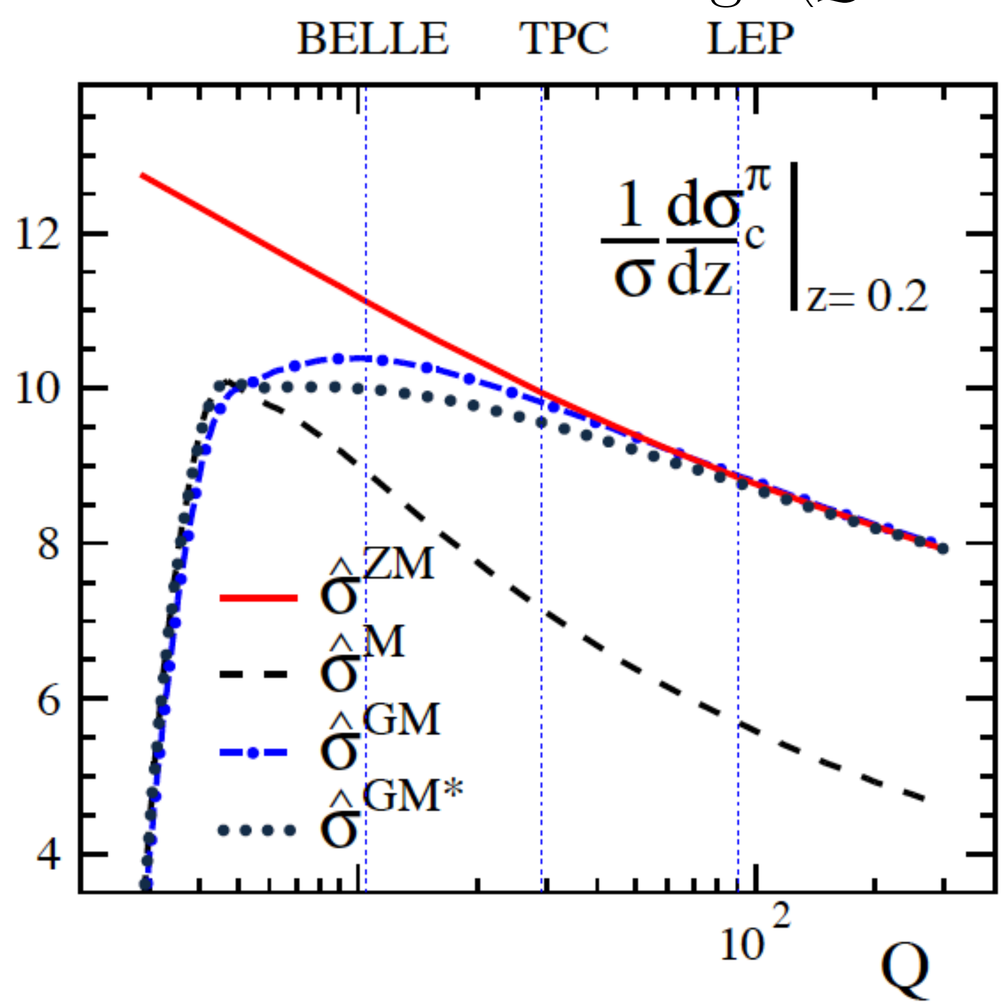
Anderle *et al.* [ArXiv:1510.05845]

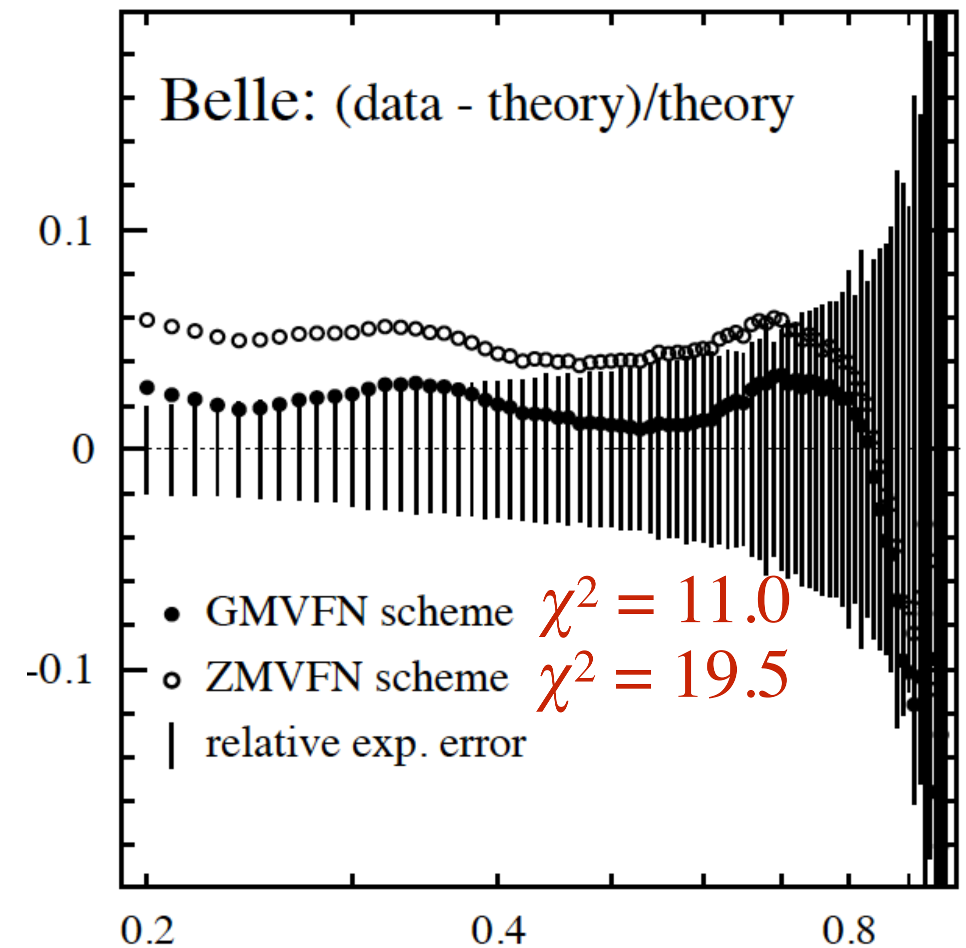
- **Large** NNLO/NLO **K-factors** (O(10%)) in the **B-factory** region ($Q \simeq 10.5$ GeV),
- Marked improvement of the global χ^2 upon inclusion of higher-order corrections:
  - mostly driven by **BABAR** when moving from NLO to NNLO.

Recent results

Impact of mass corrections on FFs

Heavy-quark mass corrections relevant for $Q \gtrsim m_h$. The precision of the B-factories data and the kinematic coverage ($Q \simeq 2m_b$) is such that these corrections are significant.





Epele et al. [ArXiv:1604.08427]

- Mass corrections known to $O(\alpha_s)$ for **SIA**.
- Use of **FONLL** to smoothly interpolate between massive and massless scheme:
  - appropriate both for B-factories ($Q \simeq 10.5$ GeV) and LEP ($Q = M_Z$) data.
- Marked improvement of the χ^2 of **BELLE** (and BABAR).

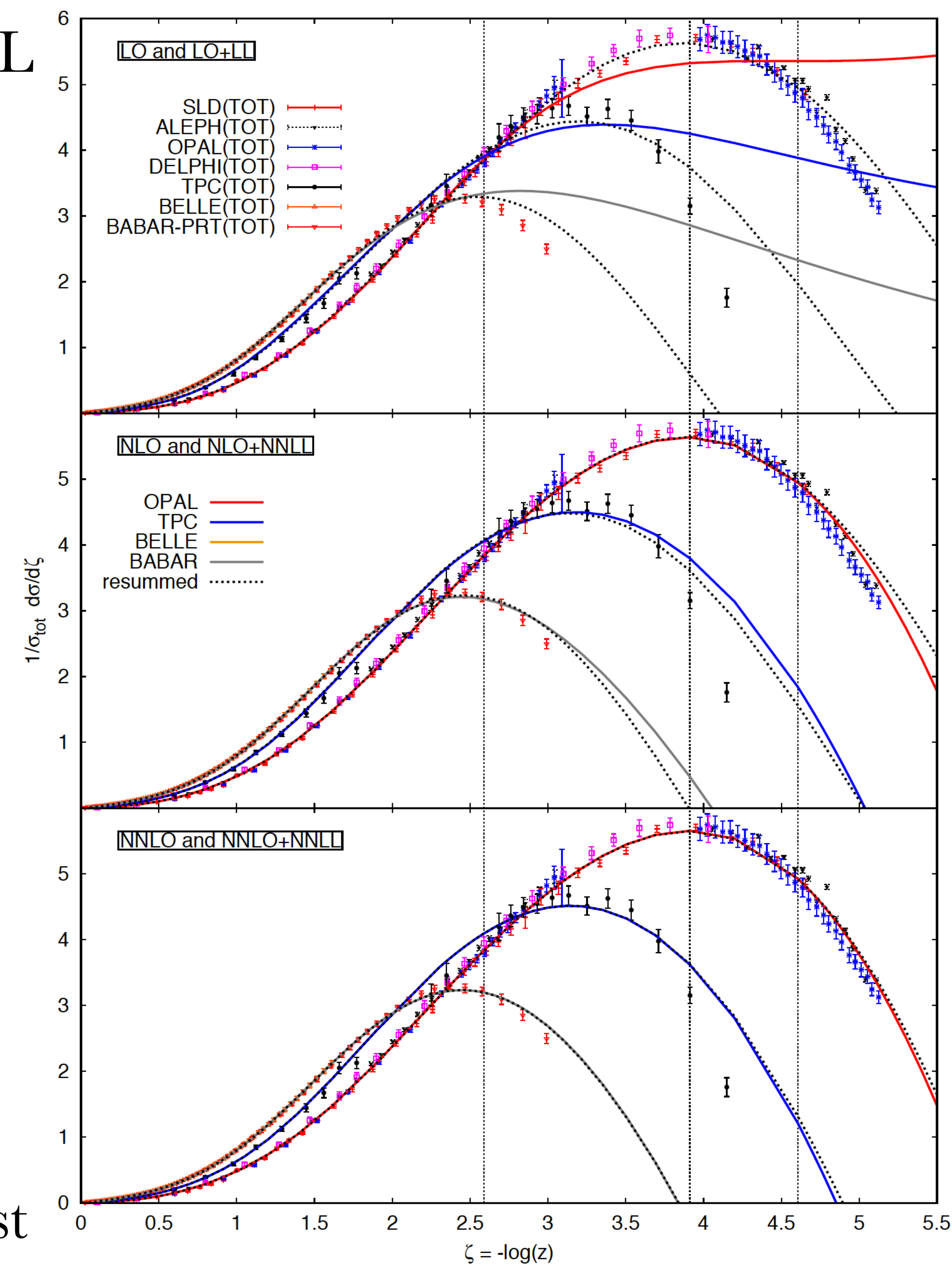
Recent results

The impact of small-z resummation corrections

- **Small-*z* resummation** corrections up to NNLL are implemented in the **SIA** hard cross sections and the **DGLAP** splitting functions.
- **Fits** at the different orders are performed to a variety of **pion** SIA data.

accuracy	χ^2
LO	1260.78
NLO	354.10
NNLO	330.08
LO+LL	405.54
NLO+NNLL	352.28
NNLO+NNLL	329.96

- Beyond LO, resummation provides only a **small improvement** in the kinematic region of interest w.r.t. the to fixed-order (particularly at NNLO).



Anderle *et al.* [ArXiv:1611.03371]

Unidentified charged hadron FFs

A brief overview

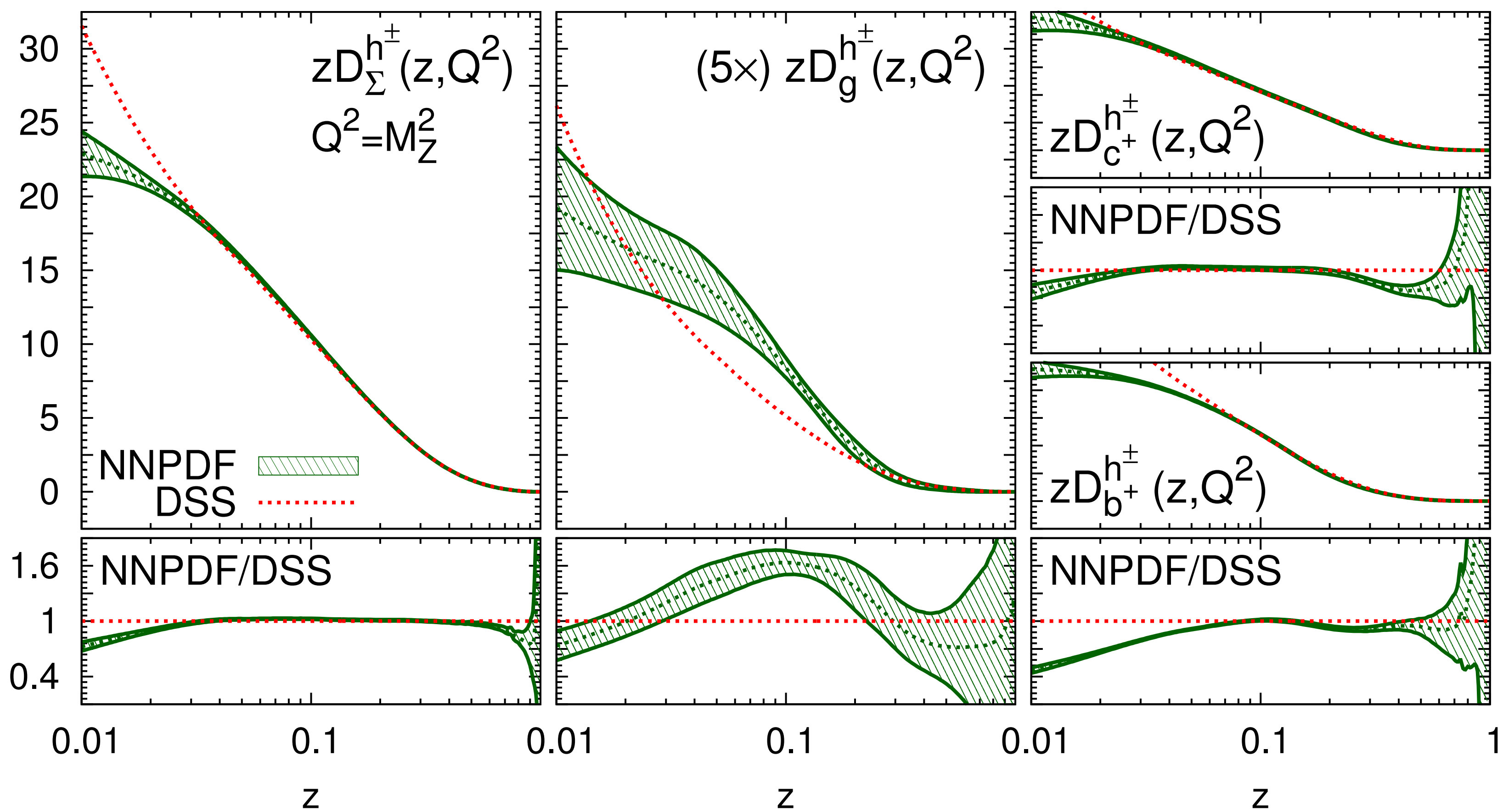
- Many experiments provide data for **charged-hadron** production:
  - this data includes, not only pions, kaons and protons, but **also heavier** (and less abundant) **charged hadrons**.
- Restricting to **SIA experiments**, data is available from:
  - TASSO, TPC, ALEPH, DELPHI, OPAL, SLD.
- Some experiments measure also the **longitudinal** cross section:
  - ALEPH, DELPHI, OPAL.
- Predictions for the longitudinal cross section start at $O(\alpha_s)$:
  - as a consequence it is **not possible to go beyond NLO** (i.e. $O(\alpha_s^2)$) yet.
  - This data provides a strong handle on the **gluon distribution**.

Unidentified charged hadron FFs

The NNFF1.0h analysis

E. Nocera [ArXiv:1709.03400]

- General good description of the entire dataset (χ^2 / N_{dat} = 0.83).
- Particularly good the description of the **longitudinal data**.



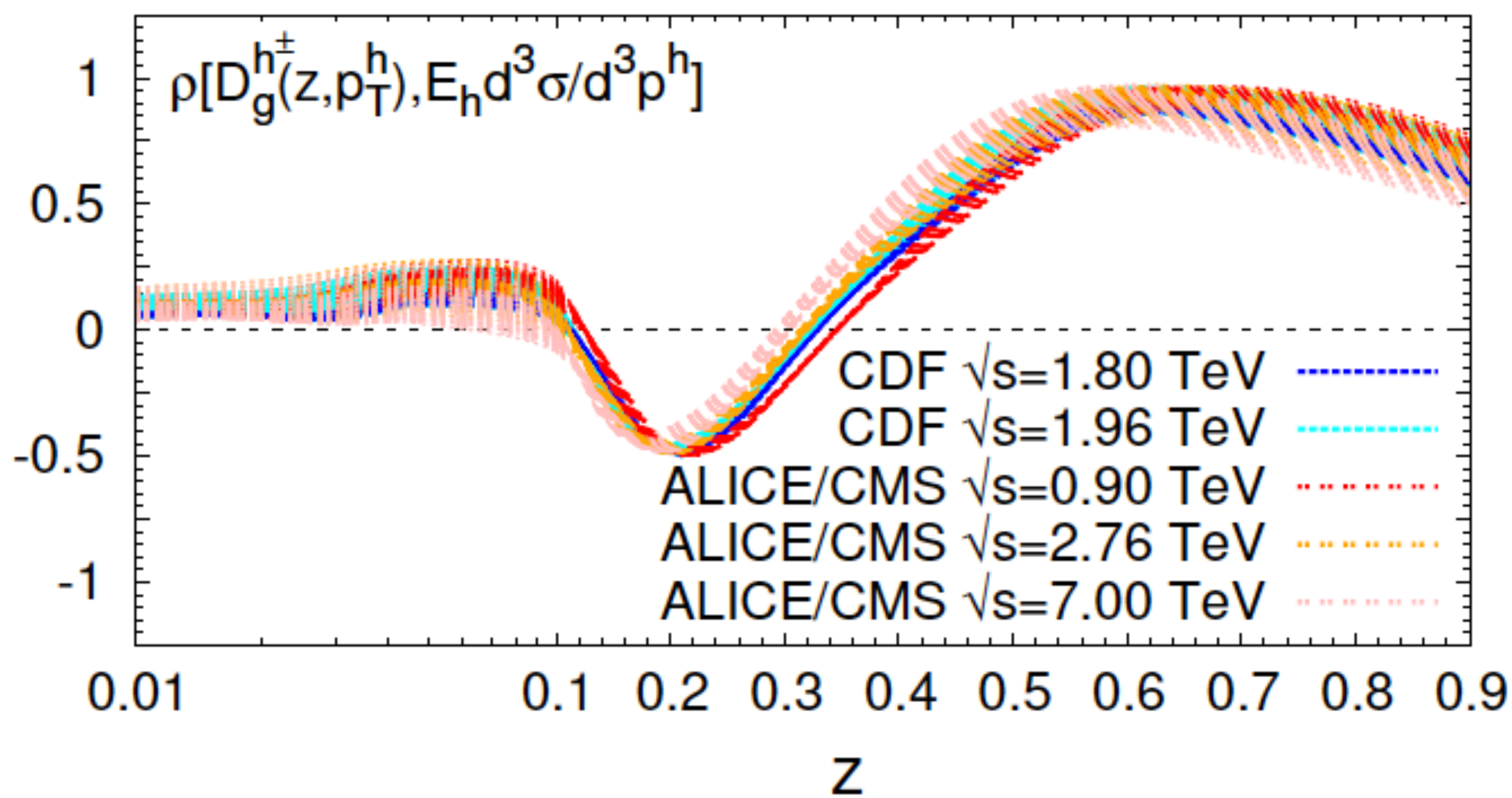
- Significant differences w.r.t. DSS, particularly for the gluon.

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	N_{dat}	χ^2/N_{dat}
TASSO14	[5]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dz}$	14.00	15 (20)	1.23
TASSO22	[5]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dz}$	22.00	15 (20)	0.51
TPC	[6]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dz}$	29.00	21 (34)	1.65
TASSO35	[5]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dz}$	35.00	15 (20)	1.14
TASSO44	[5]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dz}$	44.00	15 (20)	0.68
ALEPH	[7]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dz}$	91.20	32 (35)	1.04
	[7]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma_L^{h^\pm}}{dz}$	91.20	19 (21)	0.36
DELPHI	[8]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dp_h}$	91.20	21 (27)	0.65
	[8]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dp_h}\Big\rvert_{uds}$	91.20	21 (27)	0.17
	[8]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dp_h}\Big\rvert_{b}$	91.20	21 (27)	0.82
	[9]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma_L^{h^\pm}}{dz}$	91.20	20 (22)	0.72
	[9]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma_L^{h^\pm}}{dz}\Big\rvert_{b}$	91.20	20 (22)	0.44
OPAL	[10]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dz}$	91.20	20 (22)	2.41
	[10]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dz}\Big\rvert_{uds}$	91.20	20 (22)	0.90
	[10]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dz}\Big\rvert_{c}$	91.20	20 (22)	0.61
	[10]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dz}\Big\rvert_{b}$	91.20	20 (22)	0.21
	[11]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma_L^{h^\pm}}{dz}$	91.20	20 (22)	0.31
SLD	[12]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dp_h}$	91.28	34 (40)	0.75
	[12]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dz}\Big\rvert_{uds}$	91.28	34 (40)	1.03
	[12]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dz}\Big\rvert_{c}$	91.28	34 (40)	0.62
	[12]	$\frac{1}{\sigma_{\mathrm{tot}}}\frac{d\sigma^{h^\pm}}{dz}\Big\rvert_{b}$	91.28	34 (40)	0.97
Total dataset				471 (527)	0.83

Unidentified charged hadron FFs

Hadroproduction in pp collision data

- **CDF** at the Tevatron, and **CMS** and **ALICE** at the LHC released charged-hadron p_T spectra at different c.o.m. energies:
  - CMS and ALICE at $\sqrt{S} = 0.9$, 2.76, and 7 TeV,
  - CDF at $\sqrt{S} = 0.63$, 1.8, and 1.96 TeV.
  - Sensitivity to the **charged-hadron FFs**, particularly to the **gluon**.



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  - CDF at $\sqrt{S}$ = 0.63, 1.8, and 1.96 TeV.
  - Sensitivity to the **charged-hadron FFs**, particularly to the **gluon**.
- Hard cross sections currently know to **NLO** (i.e. $O(\alpha_s^3)$).
  - **large scale variations** at low p_T. Consider only data with $p_T > 7$ GeV.
  - No CDF data at 0.63 TeV survives.
- Include CMS, ALICE, and CDF data in the NNFF1.0h analysis of charged-hadron FFs by means of **Bayesian reweighting**:
  - start with a sample of **2000** replicas,
  - use NNPDF31_nlo_as_0118 for the PDFs.

Unidentified charged hadron FFs

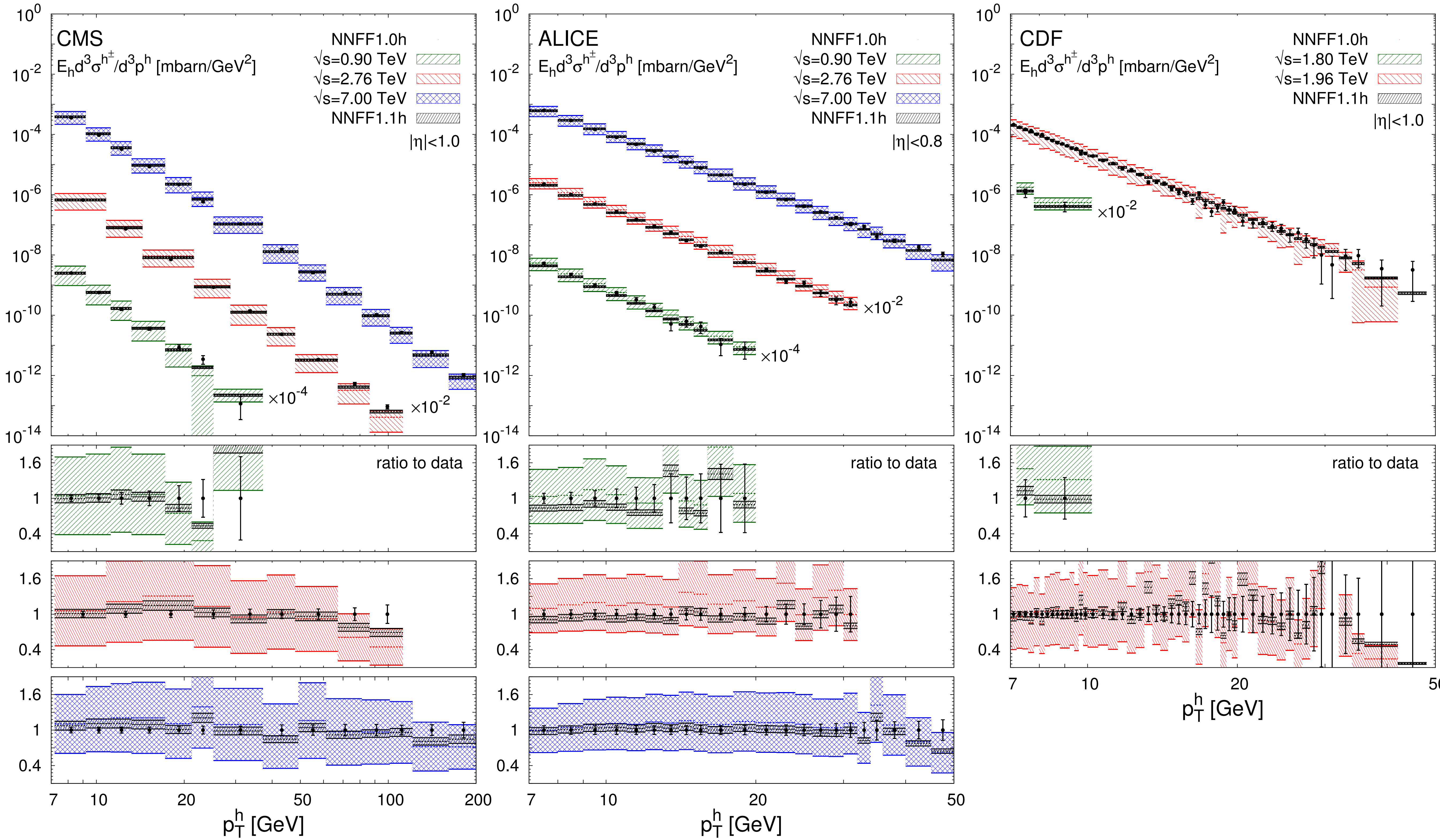
The NNFF1.1h analysis: the fit quality

Process	Experiment	Ref.	$\sqrt{s}$ [TeV]	$N_{\rm dat}$	$\chi^2_{\rm in}/N_{\rm dat}$	$\chi^2_{\rm rw}/N_{\rm dat}$
SIA	various, see Table 1 in [33]			471 (527)	0.83	0.83
pp	CDF	[9]	1.80	2 (49)	3.32	0.20
		[10]	1.96	50 (230)	2.93	1.23
	CMS	[13]	0.90	7 (20)	4.20	0.70
		[14]	2.76	9 (22)	10.6	1.24
		[13]	7.00	14 (27)	12.4	1.64
	ALICE	[15]	0.90	11 (54)	4.94	1.88
		[15]	2.76	27 (60)	13.3	0.82
		[15]	7.00	22 (65)	6.03	0.53
				603 (1054)	6.54	1.11

- **Dramatic** reduction of the χ^2's:
  - **global** χ^2 from 6.54 to 1.11,
  - reduction of all the **single** χ^2's,
  - χ^2 of the SIA data set unchanged $\Rightarrow$ **consistency** with the pp data.

Unidentified charged hadron FFs

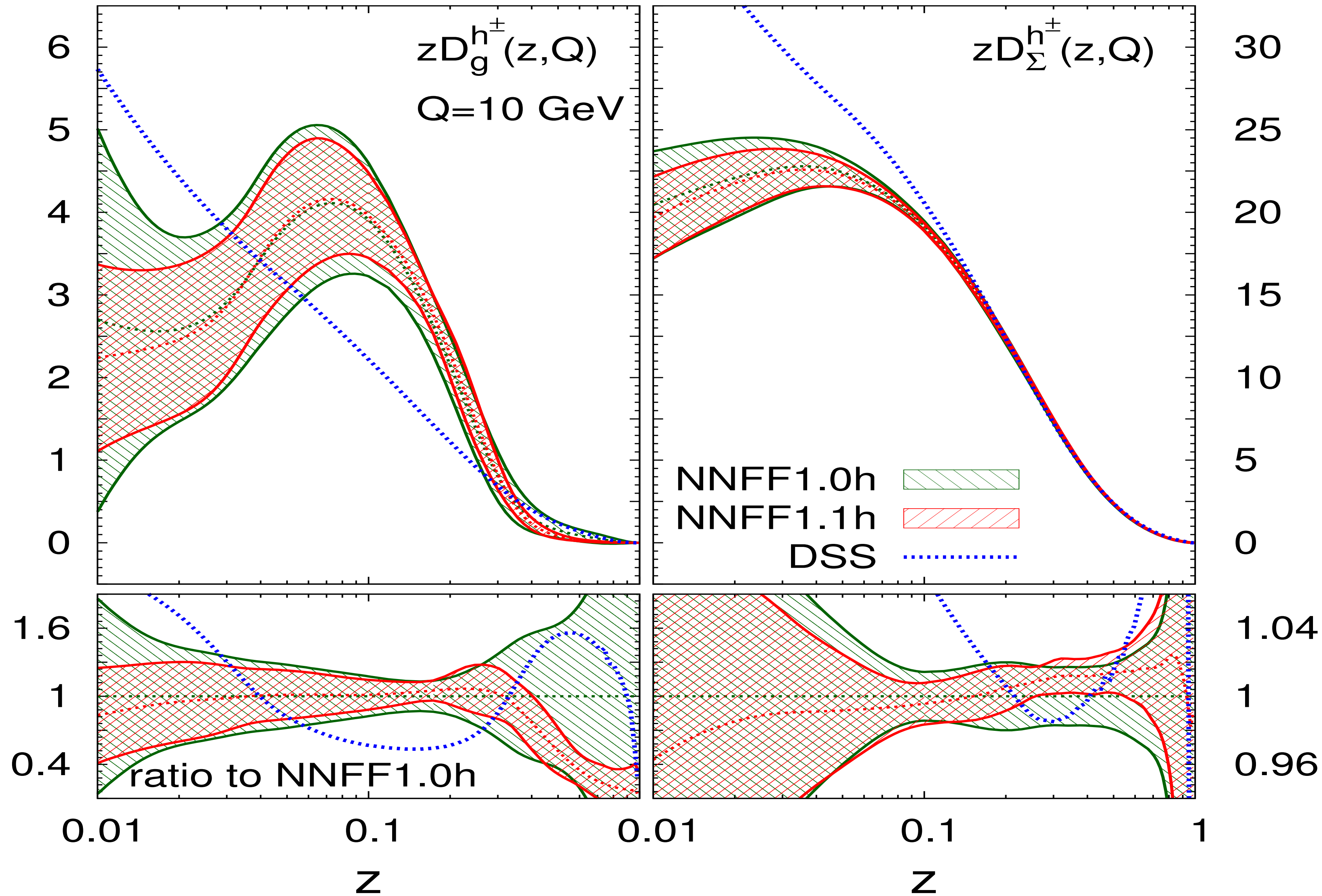
The NNFF1.1h analysis: comparison to pp data



- **Dramatic** reduction of the FF uncertainties going from NNFF1.0h to NNFF1.1h.

Unidentified charged hadron FFs

The NNFF1.1h analysis: the FFs



- **Suppression** of gluon FF at **large** *z*:
  - required to describe the ***pp* data**,
  - pronounced **decrease** of the uncertainty.
- **Singlet FF** not significantly affected in shape:
  - uncertainty reduction are medium values of *z*.

Summary

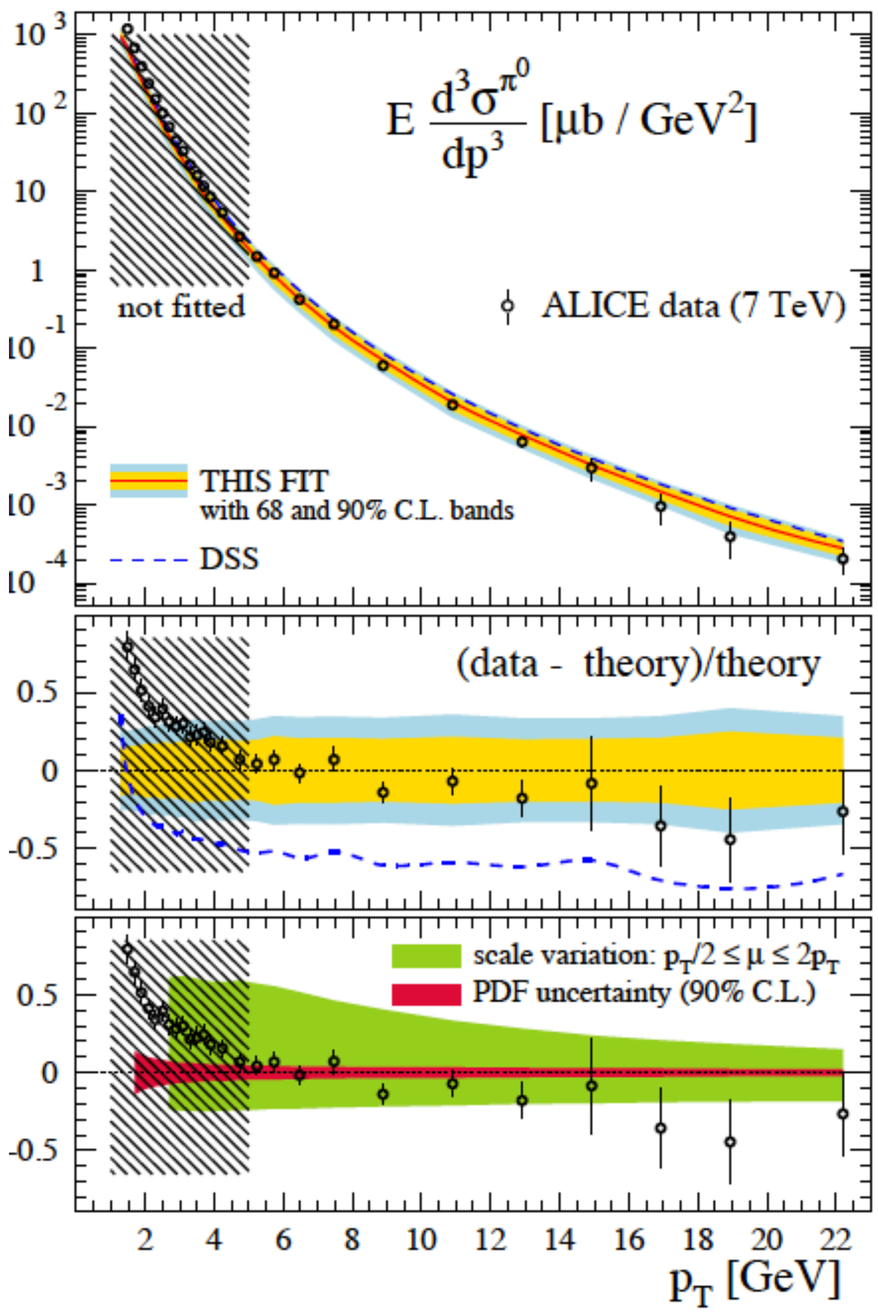
- **Well-established** procedure in the context of **collinear factorisation** to extract non-perturbative quantities from fits to **experimental data**:
  - **PDFs** have driven many of the technical developments in this context but in the last years **FFs** are quickly catching up.
  - An impressive amount of **new results** have been produced recently in the field of FFs (I have only quickly mentioned a few of them).
- The **intrinsic complexity** of the procedure leaves room for different choices that make up most of the differences between the determinations.
- On the **experimental side**, a wealth of new precise data from:
  - BELLE and BABAR (SIA),
  - HERMES and COMPASS (SIDIS),
  - Tevatron, LHC, RHIC (*pp*).
- Many developments on the **theory side**:
  - higher-order corrections,
  - resummation corrections,
  - heavy-quark mass corrections.
- It is our duty to exploit this information to extract **accurate FFs**.

Backup slides

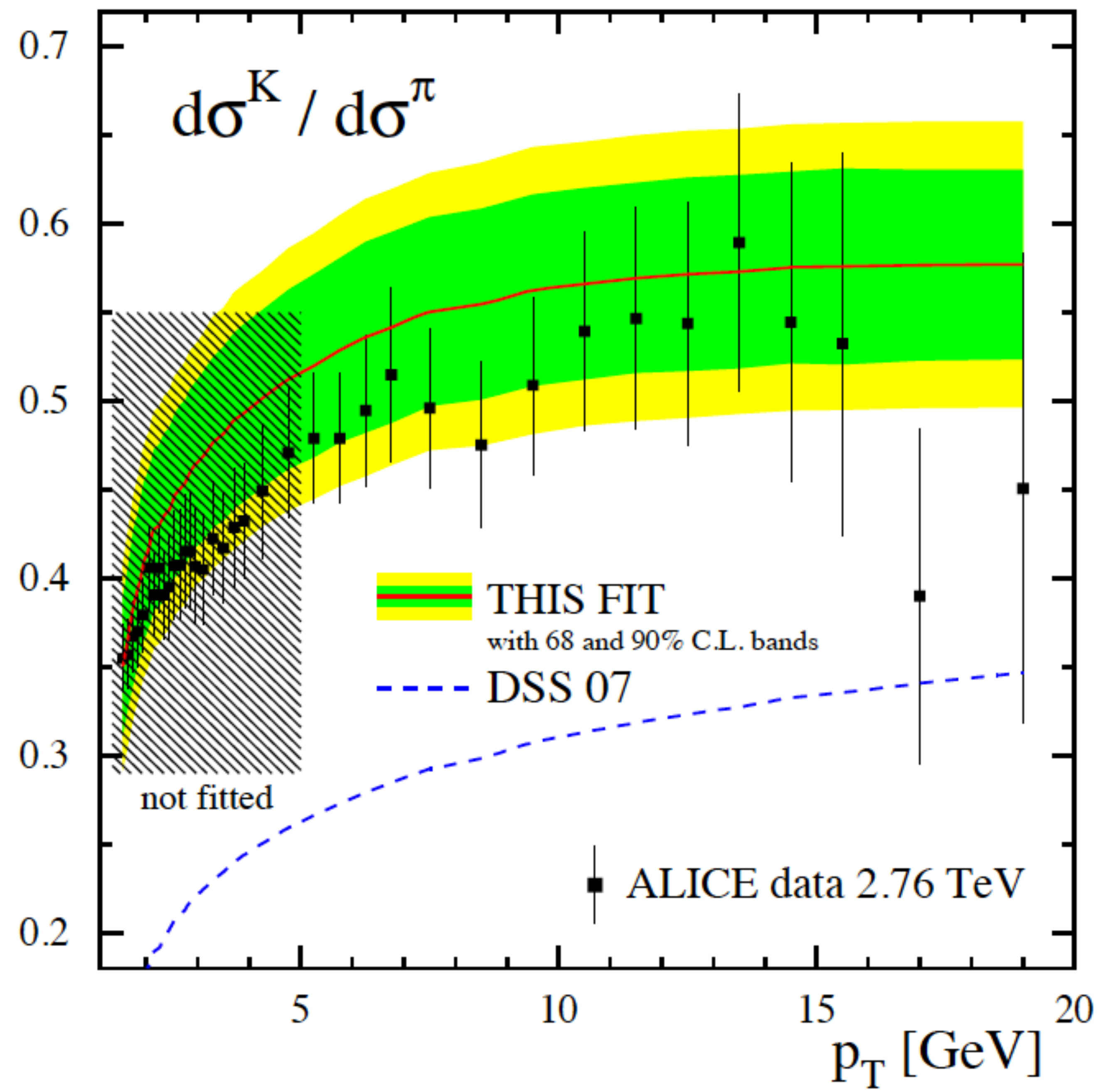
Recent results

The impact of LHC data on FFs

The DSS group included for the first time ALICE data for π^0 production at 7 TeV and K/π ratio data at 2.76 TeV in two separate analyses for **pion** (lefts) and **kaon** (right) FFs.



de Florian *et al.* [ArXiv:1702.06353]

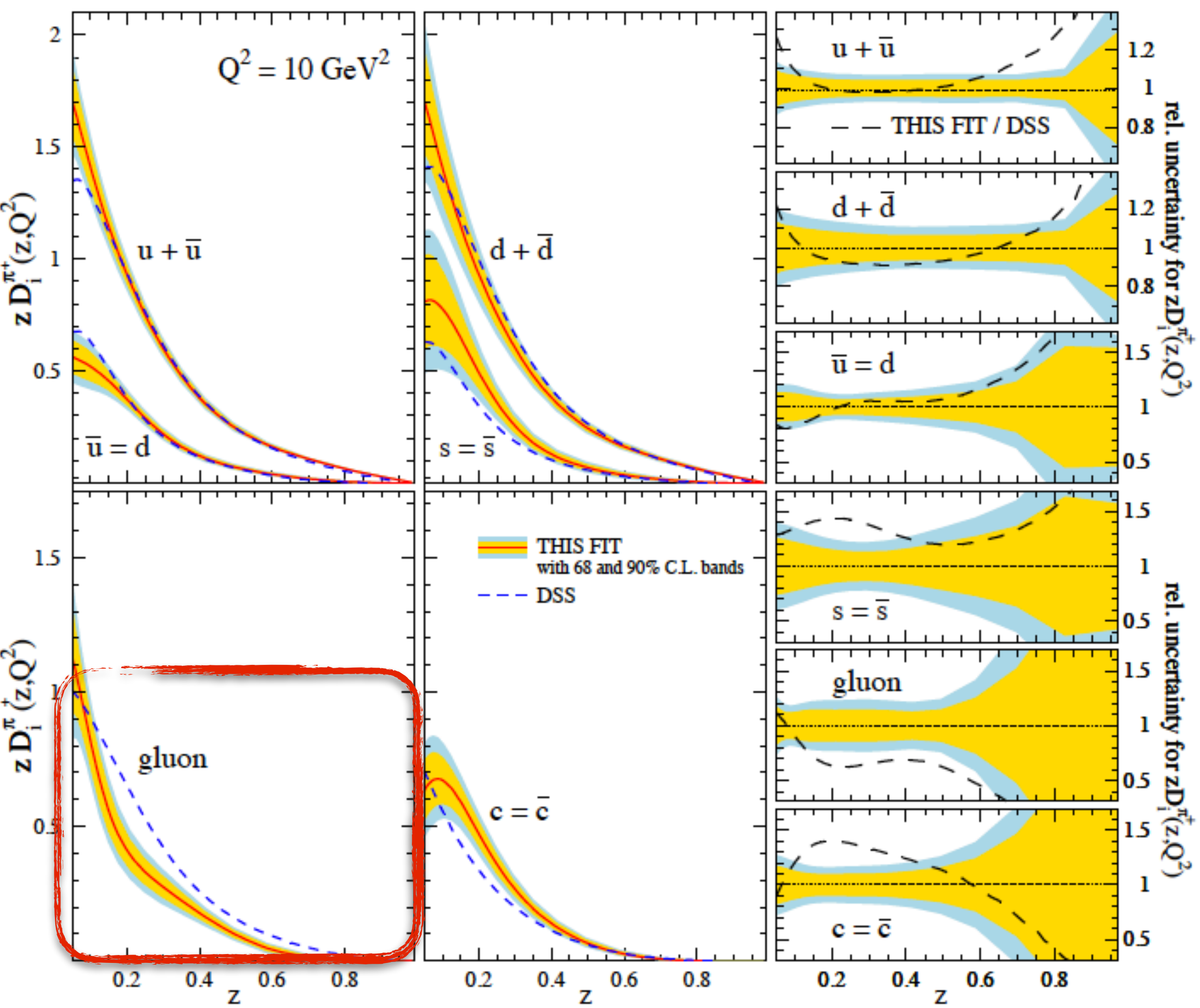


de Florian *et al.* [ArXiv:1410.6027]

These datasets were very **poorly described** by the preceding analysis that did not include them (**DSS07**, dashed blue curve).

Recent results

The impact of LHC data on FFs



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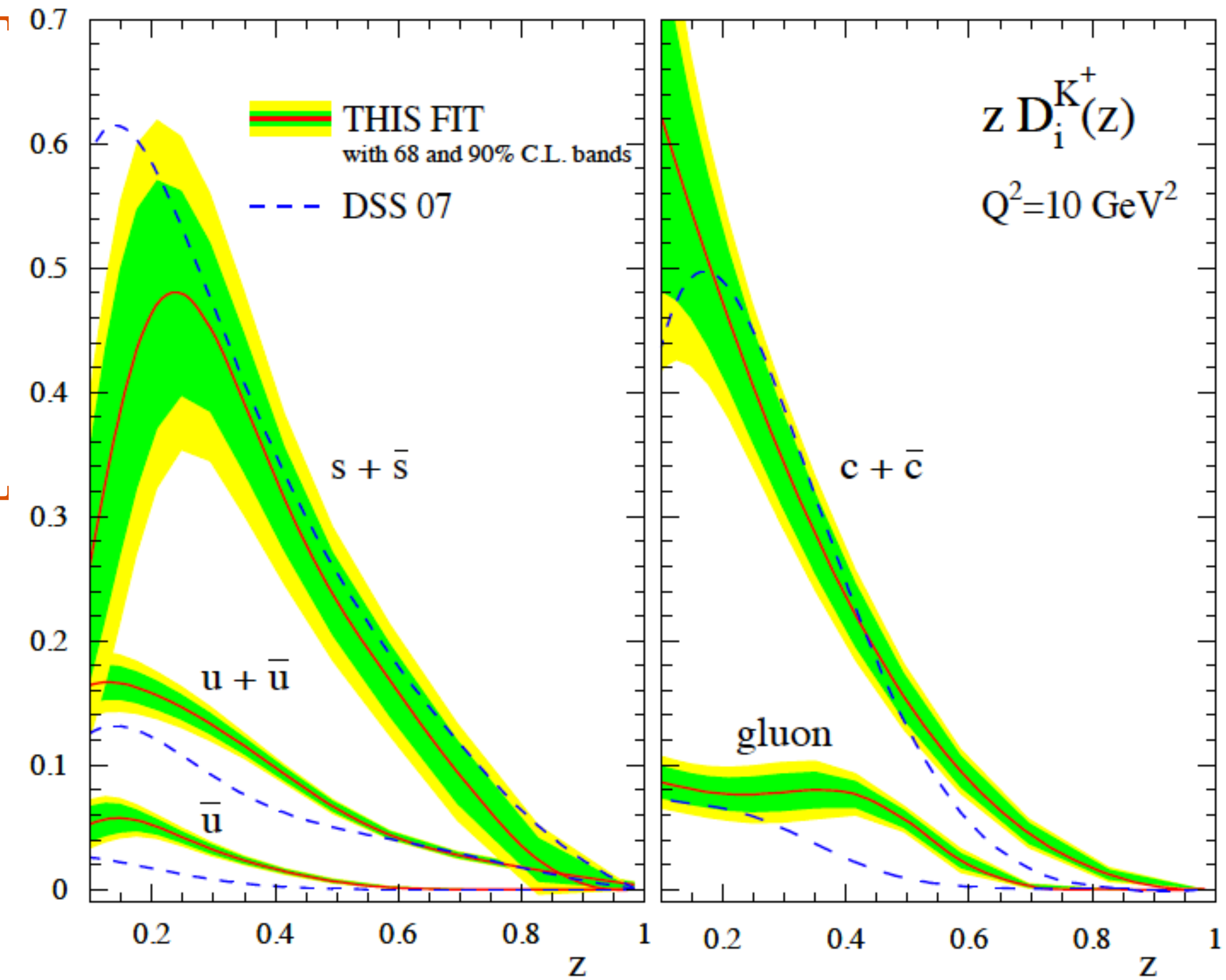


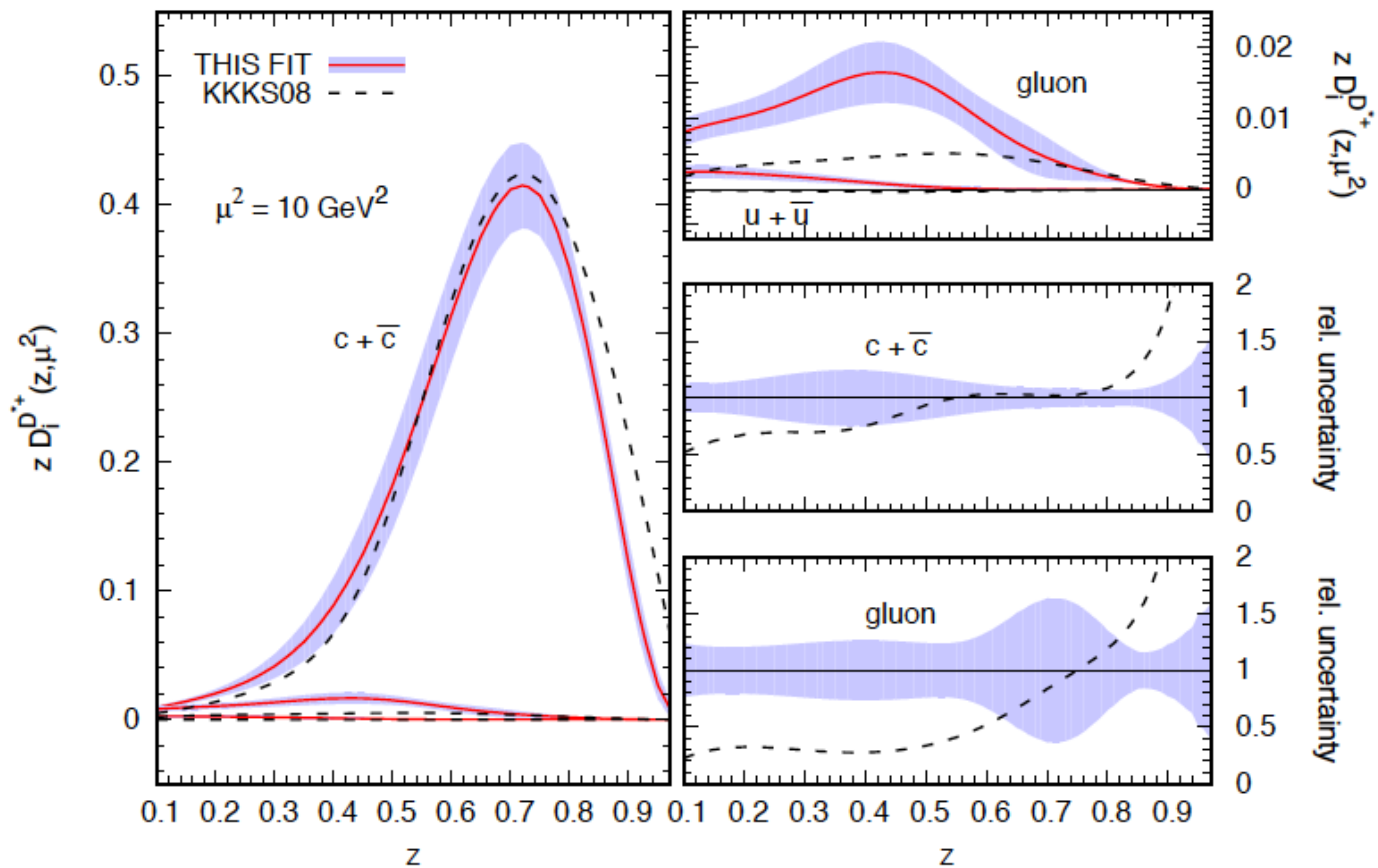
de Florian *et al.* [ArXiv:1410.6027]

- The ALICE data for **pions** (left plot) caused:
  - a strong **suppression** of the **gluon distribution**,
  - a corresponding **enhancement** of the **charm distribution**.
- Larger and more general difference for **kaons** (right plot).

Recent results

A global fit of the FFs of $D^{*\pm}$

- **Global** fit based on data for:
  - single inclusive annihilation,
  - *pp* collision,
  - in-jet fragmentation in *pp* collision.

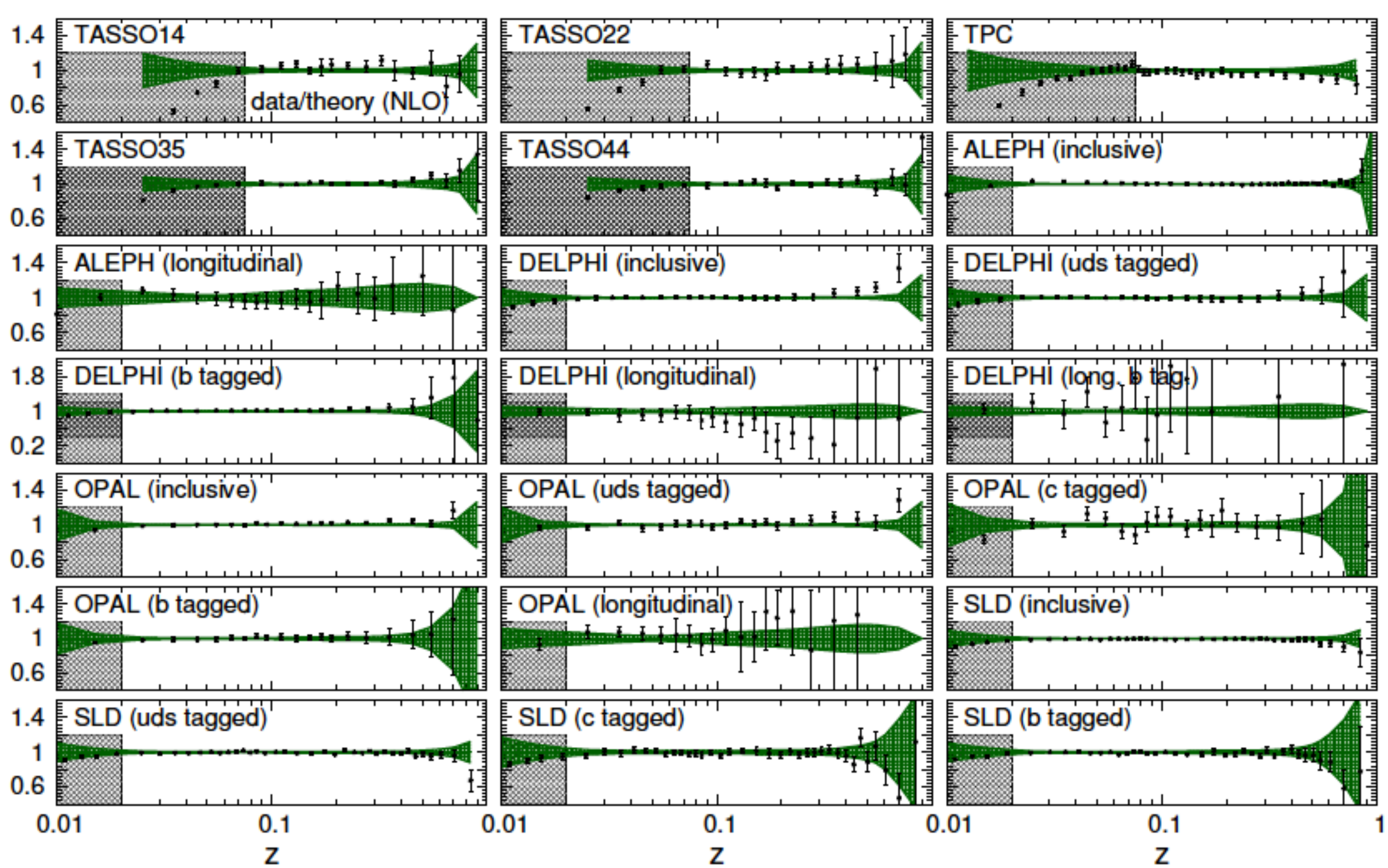


Anderle *et al.* [ArXiv:1706.09857]

Charged hadron FFs

The NNFF1.0h analysis

- General good description of the entire dataset ($\chi^2 / N_{\rm dat} = 0.83$).
- Particularly good the description of the **longitudinal data**.



E. Nocera [ArXiv:1709.03400]

Experiment	Reference	Observable	$\sqrt{s}$ [GeV]	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$
TASSO14	[5]	$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{h^\pm}}{dz}$	14.00	15 (20)	1.23
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TPC	[6]	$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{h^\pm}}{dz}$	29.00	21 (34)	1.65
TASSO35	[5]	$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{h^\pm}}{dz}$	35.00	15 (20)	1.14
TASSO44	[5]	$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{h^\pm}}{dz}$	44.00	15 (20)	0.68
ALEPH	[7]	$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{h^\pm}}{dz}$	91.20	32 (35)	1.04
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OPAL	[10]	$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{h^\pm}}{dz}$	91.20	20 (22)	2.41
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	[10]	$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{h^\pm}}{dz}\Big\vert_{c}$	91.20	20 (22)	0.61
	[10]	$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{h^\pm}}{dz}\Big\vert_{b}$	91.20	20 (22)	0.21
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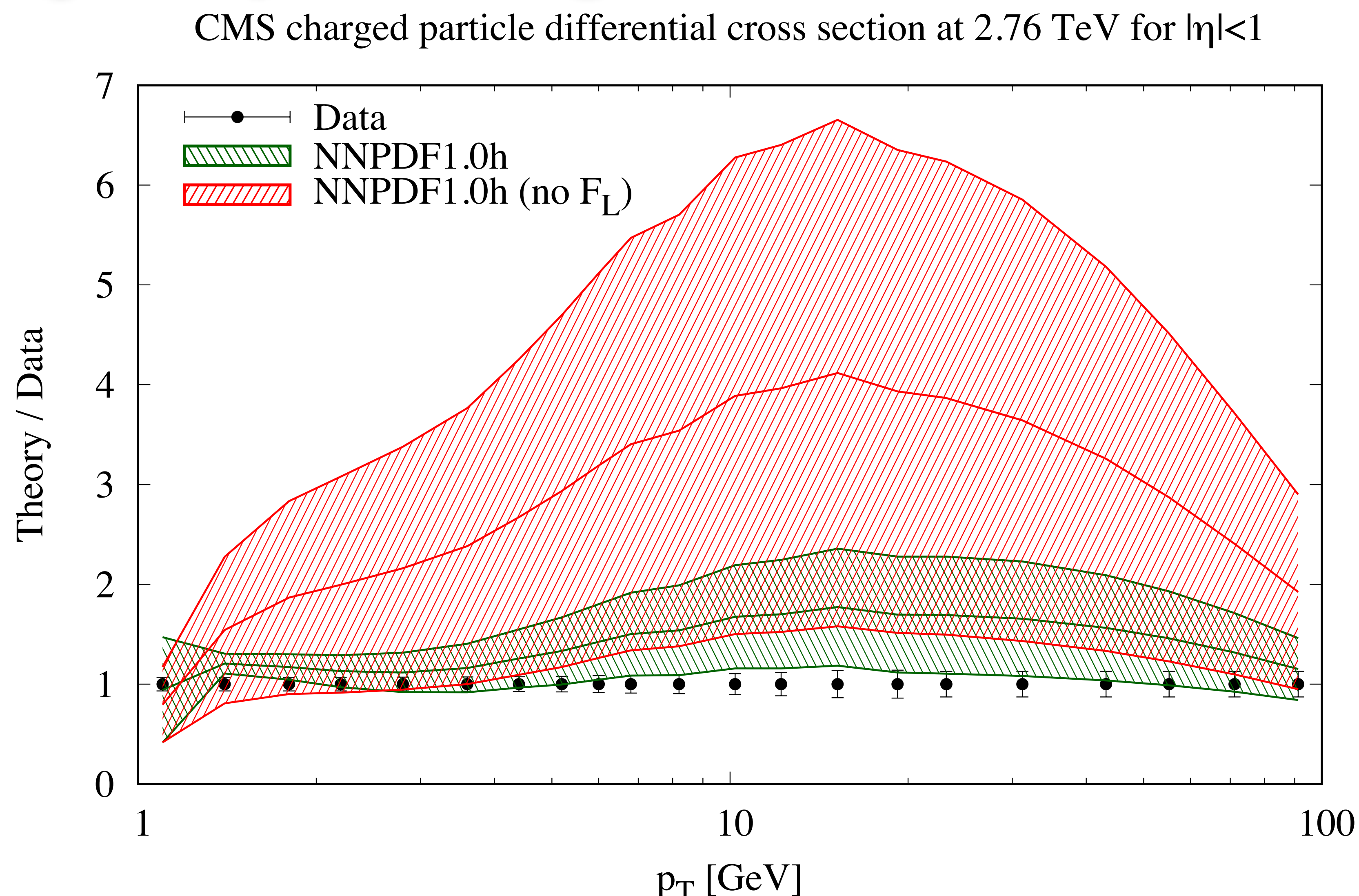
Total dataset				471 (527)	0.83

Unidentified charged hadron FFs

Aside: the impact of the longitudinal data



- Strong **sensitivity to the gluon distribution**.
- Significant impact of the **longitudinal data**:
  - reduction of the **uncertainty**,
  - **better agreement** with CMS data.
- **LHC** and **Tevatron** data expected to have a big impact.

Settings

- **Physical parameters**:

$$\alpha_s(M_Z) = 0.118, \quad \alpha_{\rm em}(M_Z) = 1/127, \quad m_c = 1.51 \text{ GeV}, \quad m_b = 4.92 \text{ GeV}$$

- **Parametrisation scale**:

$$Q_0 = 5 \text{ GeV} \ (> m_c, m_b)$$

  - substantial heavy-quark intrinsic component,
  - heavy-quark FFs parametrised on the same footing as the light FFs.

- **5 independent FFs** for each hadronic species *h*:

$$\{D^h_{u^+}, D^h_{s^+ + d^+}, D^h_{c^+}, D^h_{b^+}, D^h_g\}$$

  - **inclusive SIA data** only constrains three FF combinations,
  - heavy-quark FFs constrained directly by **tagged SIA data**.

- Each FF is parametrised by a **Neural Net** (architecture 2-5-3-1).

- **Kinematic cuts**:

$$z_{\rm min} \leq z \leq z_{\rm max}, \quad z_{\rm min} = \begin{cases} 0.02 & \text{for } \sqrt{s} = M_Z \\ 0.075 & \text{otherwise} \end{cases}, \quad z_{\rm max} = 0.9$$

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- **Kinematic cuts**:

contributions $\propto M_h/ sz^2$ and $\ln(z)$

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- **Kinematic cuts**:

contributions ∝ ln(1 - z)

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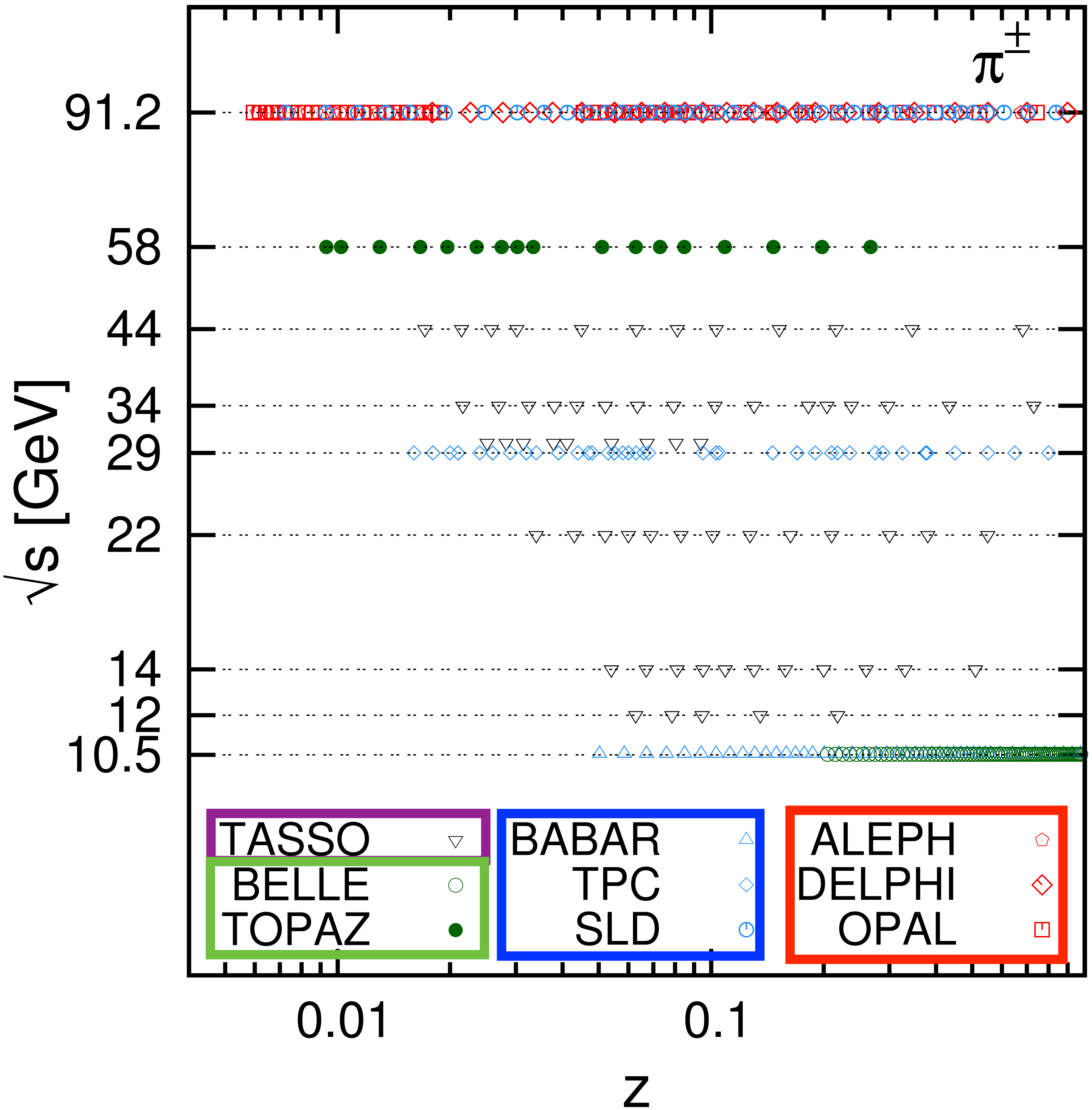
Dataset

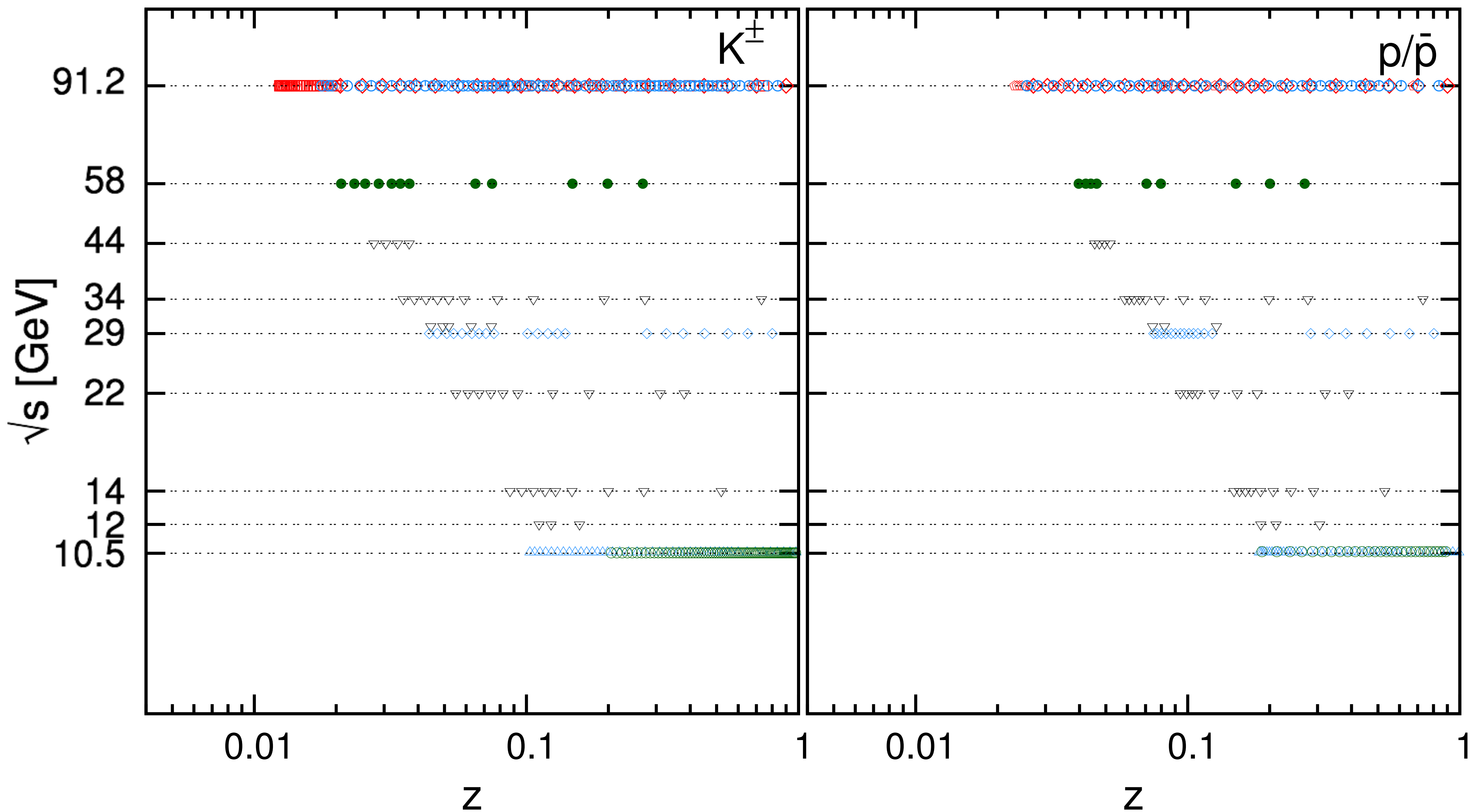
- **Only SIA cross sections** (normalised and absolute) included.



- PETRA (DESY)
  - Phys. Lett. B94 (1980) 444
  - Z.Phys. C17 (1983) 5-15
  - Z.Phys. C42 (1989) 189
- KEK
  - Phys. Rev. Lett. 111 (2013) 062002
  - Phys. Rev. D92 (2015), no. 9 092007
  - Phys. Lett. B345 (1995) 335-342
- SLAC
  - Phys. Rev. D88 (2013) 032011
  - Phys. Rev. Lett. 61 (1988) 1263
  - Phys. Rev. D69 (2004) 072003
  - X.Q. Lu, Ph.D. thesis, Johns Hopkins U., 1986
- LEP (CERN)
  - Z. Phys. C66 (1995) 355-366
  - Eur. Phys. J. C5 (1998) 585-620
  - Z. Phys. C63 (1994) 181-196.
- 428 data points (after cuts)

Dataset

- **Only SIA cross sections** (normalised and absolute) included.
- We have fitted FFs also to $K^{\pm}$ and $p/\overline{p}$ data.



385 data points
(after cuts)

360 data points
(after cuts)

Fit quality

- Fit quality **increasingly better** going from LO to NNLO:
  - substantial from LO to NLO, more moderate from NLO to NNLO.
  - **NNLO** corrections are anyway **beneficial** (particularly for pions).

Exp.	χ^2/N_{dat} ($h = \pi^\pm$) LO	NLO	NNLO	χ^2/N_{dat} ($h = K^\pm$) LO	NLO	NNLO	χ^2/N_{dat} ($h = p/\bar{p}$) LO	NLO	NNLO
BELLE	0.60	0.11	0.09	0.21	0.32	0.33	0.10	0.31	0.50
BABAR	1.91	1.77	0.78	2.86	1.11	0.95	4.74	3.75	3.25
TASSO12	0.70	0.85	0.87	1.10	1.03	1.02	0.69	0.70	0.72
TASSO14	1.55	1.67	1.70	2.17	2.13	2.07	1.32	1.25	1.22
TASSO22	1.64	1.91	1.91	2.14	2.77	2.62	0.98	0.92	0.93
TPC (incl.)	0.46	0.65	0.85	0.94	1.09	1.01	1.04	1.10	1.08
TPC (*uds* tag)	0.78	0.55	0.49	—	—	—	—	—	—
TPC (*c* tag)	0.55	0.53	0.52	—	—	—	—	—	—
TPC (*b* tag)	1.44	1.43	1.43	—	—	—	—	—	—
TASSO30	—	—	—	—	—	—	0.25	0.19	0.18
TASSO34	1.16	0.98	1.00	0.27	0.44	0.36	0.82	0.81	0.78
TASSO44	2.01	2.24	2.34	—	—	—	—	—	—
TOPAZ	1.04	0.82	0.80	0.61	1.19	0.99	0.79	1.21	1.19
ALEPH	1.68	0.90	0.78	0.47	0.55	0.56	1.36	1.43	1.28
DELPHI (incl.)	1.44	1.79	1.86	0.28	0.33	0.34	0.48	0.49	0.49
DELPHI (*uds* tag)	1.30	1.48	1.54	1.38	1.49	1.32	0.47	0.46	0.45
DELPHI (*b* tag)	1.21	0.99	0.95	0.58	0.49	0.52	0.89	0.89	0.91
OPAL	2.29	1.88	1.84	1.67	1.57	1.66	—	—	—
SLD (incl.)	2.33	1.14	0.83	0.86	0.62	0.57	0.66	0.65	0.64
SLD (*uds* tag)	0.95	0.65	0.52	1.31	1.02	0.93	0.77	0.76	0.78
SLD (*c* tag)	3.33	1.33	1.06	0.92	0.47	0.38	1.22	1.22	1.21
SLD (*b* tag)	0.45	0.38	0.36	0.59	0.67	0.62	1.12	1.29	1.33
Total dataset	**1.44**	**1.02**	**0.87**	**1.02**	**0.78**	**0.73**	**1.31**	**1.23**	**1.17**

Fit quality

- Fit quality **increasingly better** going from LO to NNLO:
  - substantial from LO to NLO, more moderate from NLO to NNLO.
  - **NNLO** corrections are anyway **beneficial** (particularly for pions).
- **Tension** between BELLE and BABAR for kaons and protons:
  - **opposite trend** upon inclusion of higher-order corrections.

Exp.	$\chi^2/N_{\rm dat}$ $(h=\pi^\pm)$ LO	NLO	NNLO	$\chi^2/N_{\rm dat}$ $(h=K^\pm)$ LO	NLO	NNLO	$\chi^2/N_{\rm dat}$ $(h=p/\bar{p})$ LO	NLO	NNLO
BELLE	0.60	0.11	0.09	0.21	0.32	0.33	0.10	0.31	0.50
BABAR	1.91	1.77	0.78	2.86	1.11	0.95	4.74	3.75	3.25
TASSO12	0.70	0.85	0.87	1.10	1.03	1.02	0.69	0.70	0.72
TASSO14	1.55	1.67	1.70	2.17	2.13	2.07	1.32	1.25	1.22
TASSO22	1.64	1.91	1.91	2.14	2.77	2.62	0.98	0.92	0.93
TPC (incl.)	0.46	0.65	0.85	0.94	1.09	1.01	1.04	1.10	1.08
TPC (*uds* tag)	0.78	0.55	0.49	—	—	—	—	—	—
TPC (*c* tag)	0.55	0.53	0.52	—	—	—	—	—	—
TPC (*b* tag)	1.44	1.43	1.43	—	—	—	—	—	—
TASSO30	—	—	—	—	—	—	0.25	0.19	0.18
TASSO34	1.16	0.98	1.00	0.27	0.44	0.36	0.82	0.81	0.78
TASSO44	2.01	2.24	2.34	—	—	—	—	—	—
TOPAZ	1.04	0.82	0.80	0.61	1.19	0.99	0.79	1.21	1.19
ALEPH	1.68	0.90	0.78	0.47	0.55	0.56	1.36	1.43	1.28
DELPHI (incl.)	1.44	1.79	1.86	0.28	0.33	0.34	0.48	0.49	0.49
DELPHI (*uds* tag)	1.30	1.48	1.54	1.38	1.49	1.32	0.47	0.46	0.45
DELPHI (*b* tag)	1.21	0.99	0.95	0.58	0.49	0.52	0.89	0.89	0.91
OPAL	2.29	1.88	1.84	1.67	1.57	1.66	—	—	—
SLD (incl.)	2.33	1.14	0.83	0.86	0.62	0.57	0.66	0.65	0.64
SLD (*uds* tag)	0.95	0.65	0.52	1.31	1.02	0.93	0.77	0.76	0.78
SLD (*c* tag)	3.33	1.33	1.06	0.92	0.47	0.38	1.22	1.22	1.21
SLD (*b* tag)	0.45	0.38	0.36	0.59	0.67	0.62	1.12	1.29	1.33
Total dataset	**1.44**	**1.02**	**0.87**	**1.02**	**0.78**	**0.73**	**1.31**	**1.23**	**1.17**

Fit quality

- Fit quality **increasingly better** going from LO to NNLO:
  - substantial from LO to NLO, more moderate from NLO to NNLO.
  - **NNLO** corrections are anyway **beneficial** (particularly for pions).
- **Tension** between BELLE and BABAR for kaons and protons:
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- **Anomalously small** χ^2 for BELLE:
  - possible underestimate of the **uncorrelated systematic** uncertainty.

	χ^2/N_{dat} $(h=\pi^\pm)$			χ^2/N_{dat} $(h=K^\pm)$			χ^2/N_{dat} $(h=p/\bar{p})$		
Exp.	LO	NLO	NNLO	LO	NLO	NNLO	LO	NLO	NNLO
BELLE	0.60	0.11	0.09	0.21	0.32	0.33	0.10	0.31	0.50
BABAR	1.91	1.77	0.78	2.86	1.11	0.95	4.74	3.75	3.25
TASSO12	0.70	0.85	0.87	1.10	1.03	1.02	0.69	0.70	0.72
TASSO14	1.55	1.67	1.70	2.17	2.13	2.07	1.32	1.25	1.22
TASSO22	1.64	1.91	1.91	2.14	2.77	2.62	0.98	0.92	0.93
TPC (incl.)	0.46	0.65	0.85	0.94	1.09	1.01	1.04	1.10	1.08
TPC (*uds* tag)	0.78	0.55	0.49	—	—	—	—	—	—
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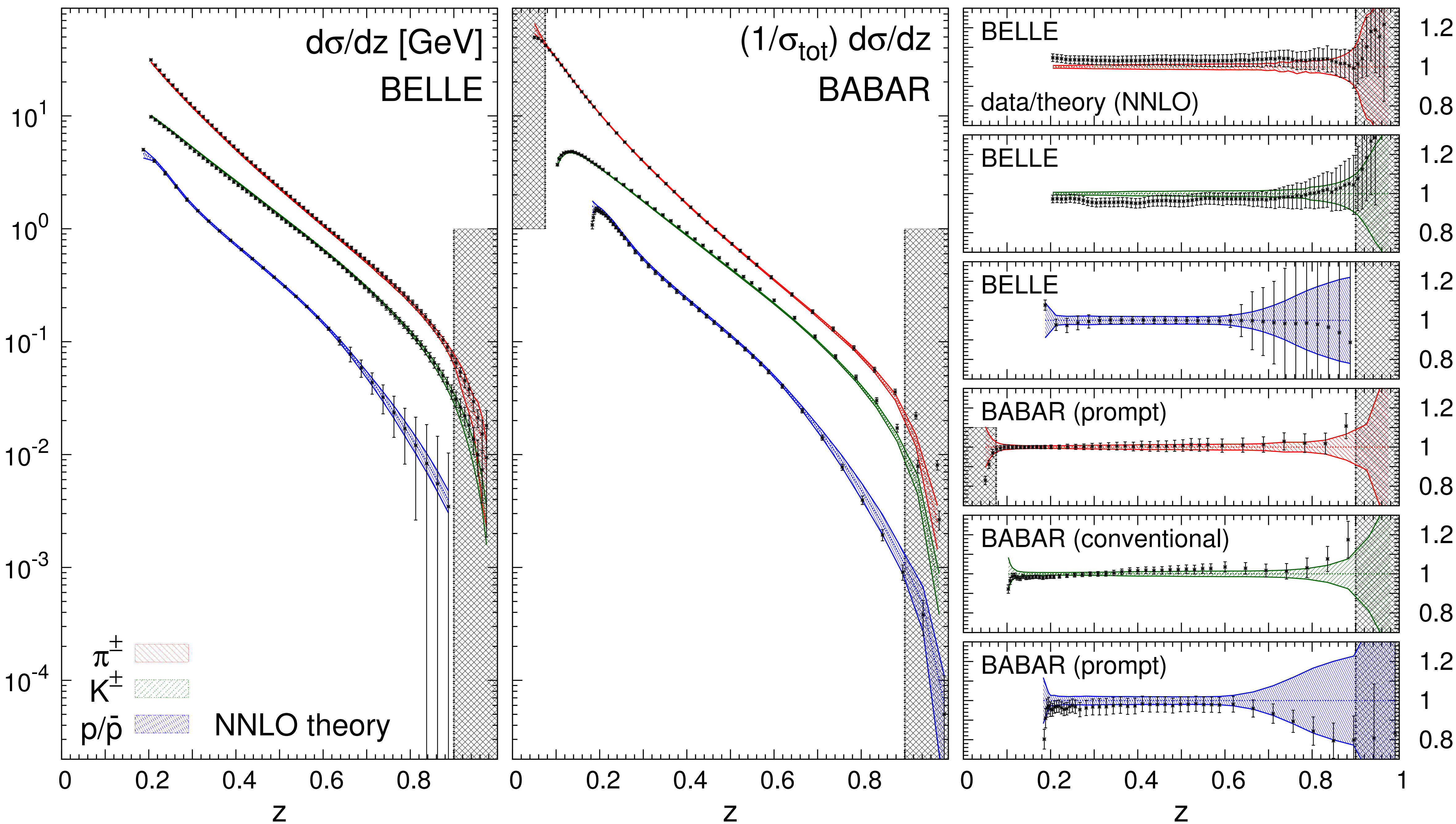
- **Possible tension** also between DELPHI inclusive and the other experiments at M_Z:
  - opposite trend upon inclusion of higher-order corrections.

Description of the data

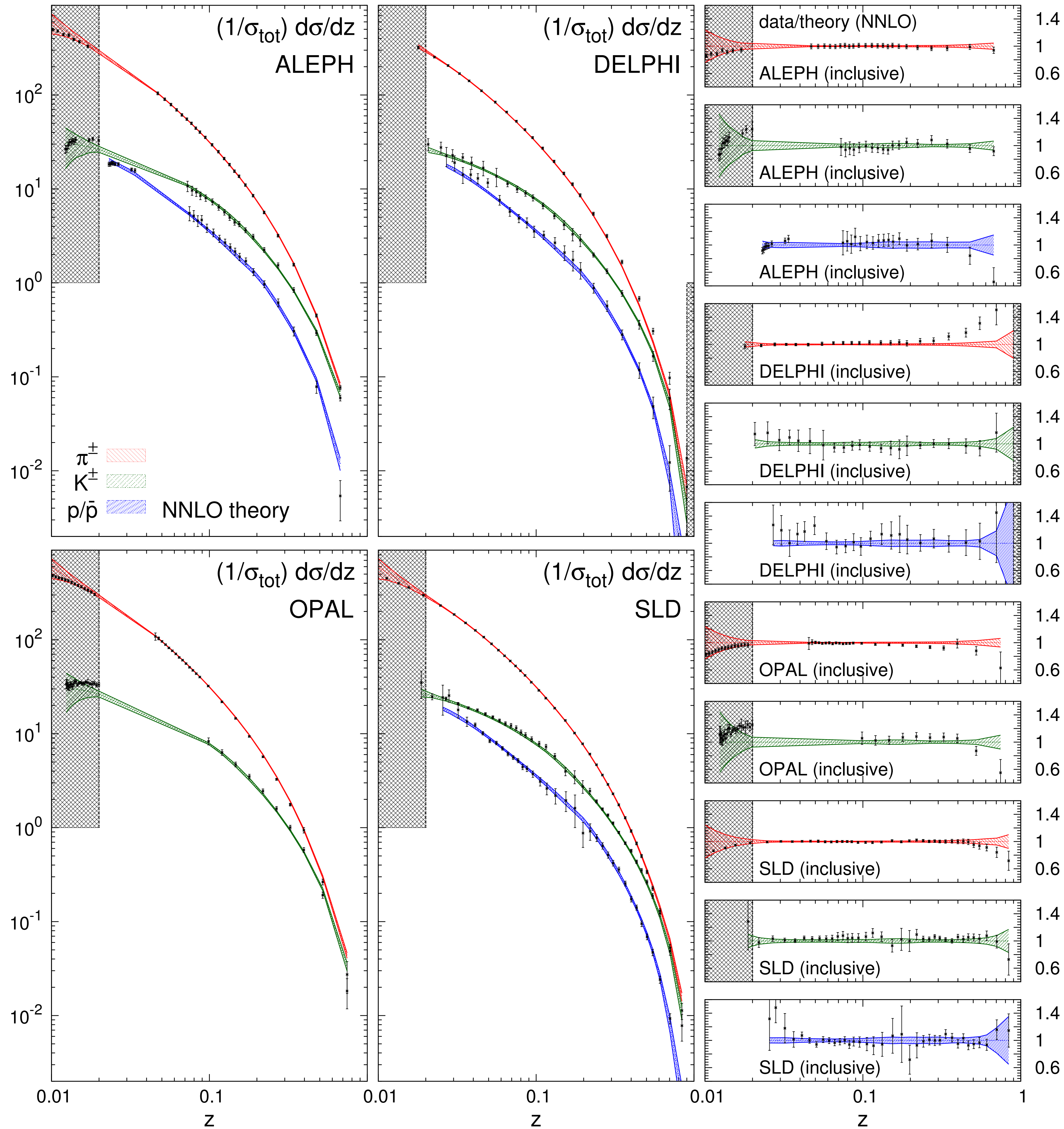
- Data/Theory comparison for **BELLE** and **BABAR** using NNFF1.0 at NNLO:
    - the bands indicate the 1-σ uncertainty.



- Very good description in the region not excluded by the kinematic cuts (shaded areas).
- Different **trend** of the data at **low *z*** for **kaons** and particularly for **protons**:
    - possible reason of the worsening of the χ^2.

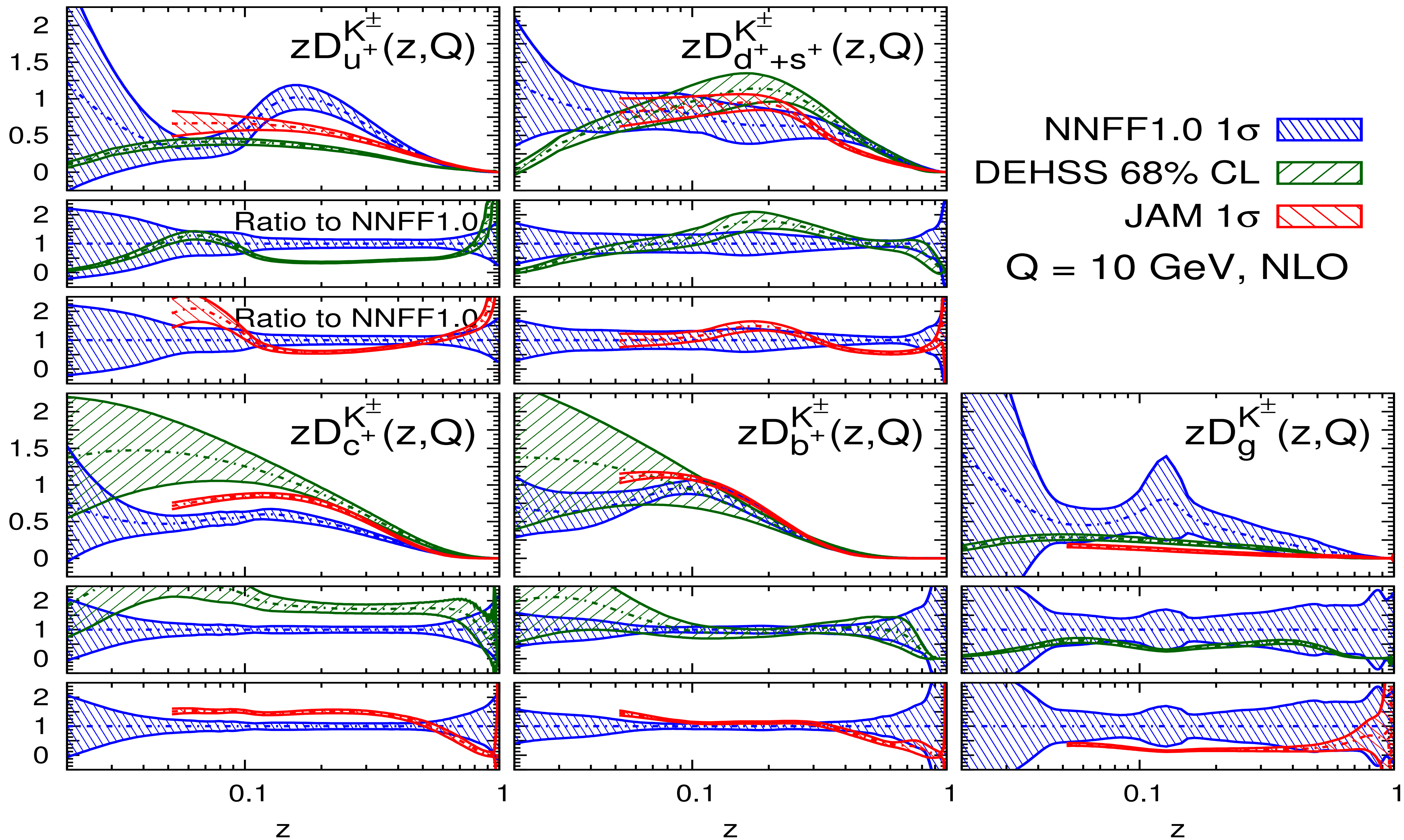
Description of the data

- Data/Theory comparison for the experiments at M_Z using NNFF1.0 at NNLO.
- Very good description in the region allowed by the kinematic cuts.
- Often also the data excluded by the cuts are well described.
- The predictions for pions for **DELPHI** overshoot the data:
    - origin of the worse χ^2 as compared to the other experiments at M_Z.



Most recent determinations

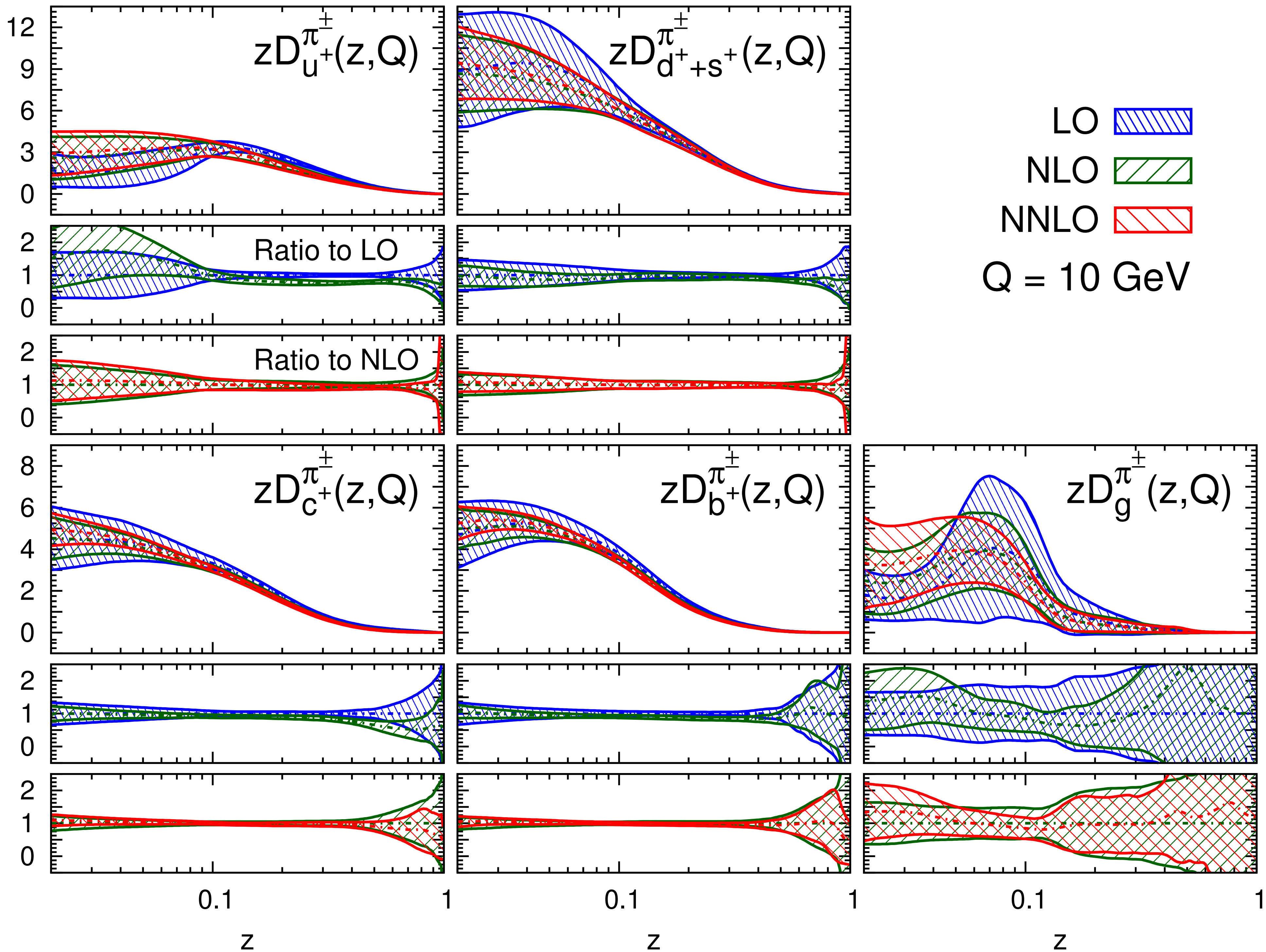
"Global" fits: Comparison for kaons



- More substantial differences all over the board as compared to pions:
  - fair agreement only for b^+,
  - **larger differences in the uncertainties**,
  - particularly marked for the **gluon** distributions.

Fragmentation functions

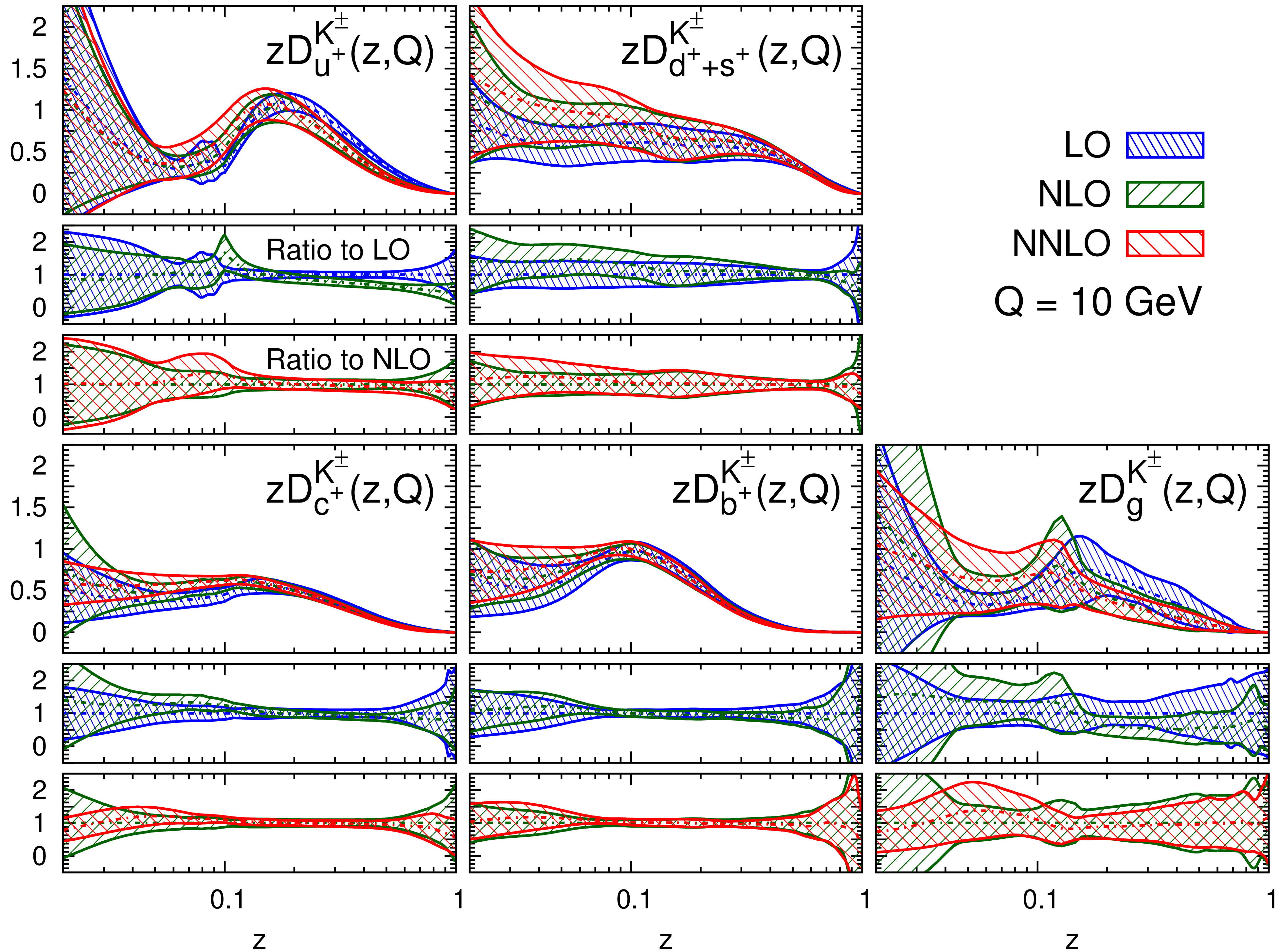
Perturbative stability (Pions)



- **Stabilisation** going from LO to NNLO,
- LO uncertainties slightly larger: poorer theoretical description.

Fragmentation functions

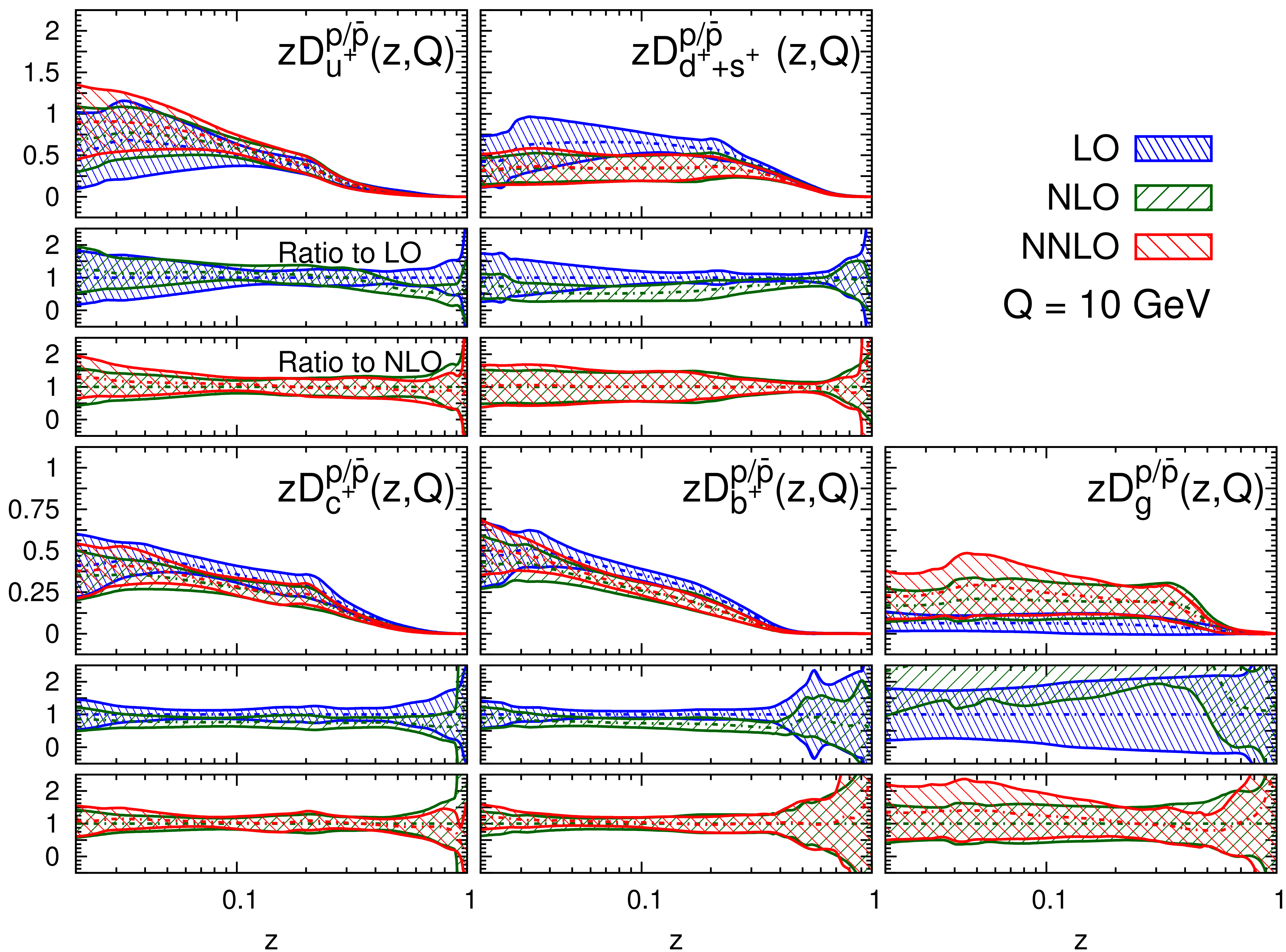
Perturbative stability (Kaons)



- Same for kaons…

Fragmentation functions

Perturbative stability (Protons)



- …and for protons.

Fitting methodology

The NNPDF approach

- In the NNPDF procedure applied to PDFs the parametrisation is:

$$f_i(x) = \boxed{x^{\alpha_i}(1-x)^{\beta_i}} \mathrm{NN}_i(x)$$

Preprocessing function

- The preprocessing function:
  - helps implement **physical constraints** (*e.g.* $f_i(1) = 0$ and integrability),
  - determines the behaviour in the **extrapolation regions**,
  - facilitates the task of the neural network making the **fit easier**.
- The values of α_i and β_i are **iteratively** determined from data.
- For the fits of FFs we remove the preprocessing functions and use:

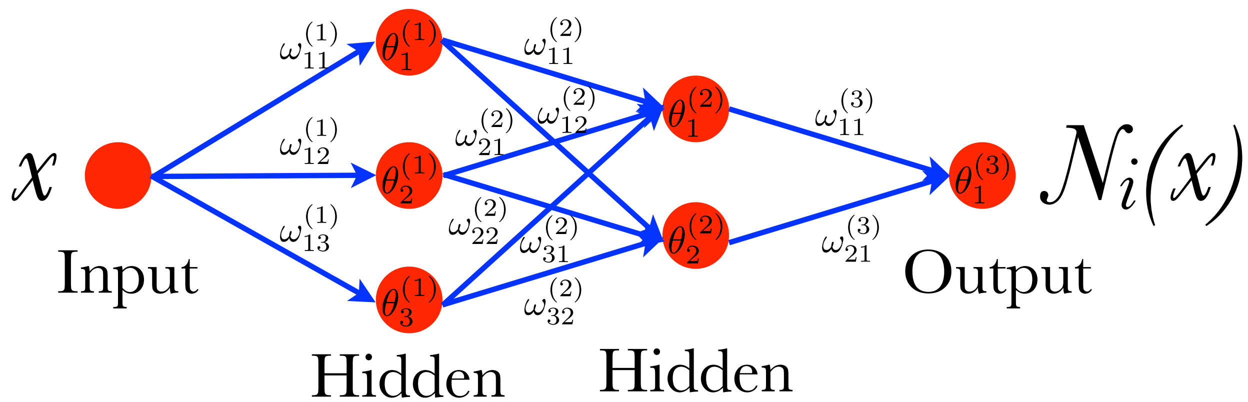
$$f_i(x) = \mathrm{NN}_i(x) - \mathrm{NN}_i(1)$$

- **no need to iterate** to determine α_i and β_i.
- the NN defines the behaviour also in the extrapolation regions.

Fitting methodology

The NNPDF approach

- **Removing the preprocessing** functions requires a proper choice of the **activation function** of the neural networks:



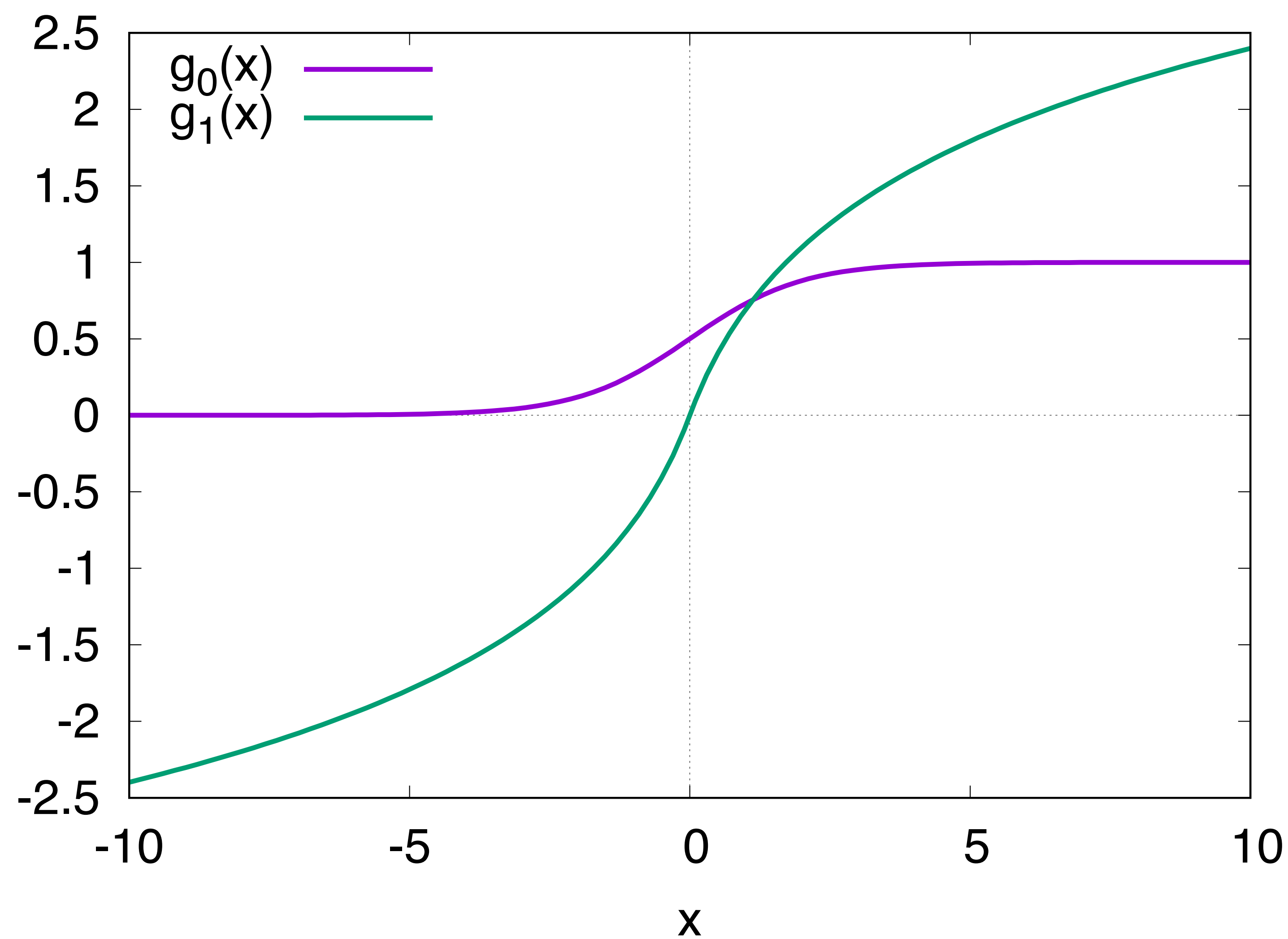
$$\xi_i^{(j)} = g\left(\sum_{k}^{\text{(j-1)th layer}} \xi_k^{(j-1)} \omega_{ki}^{(j)} - \theta_i^{(j)} \right)$$

Saturating function

$$g_0(x) = \frac{1}{1+e^{-x}}$$

Non-saturating function

$$g_1(x) = \mathrm{sign}(x)\ln(|x|+1)$$



Fitting methodology
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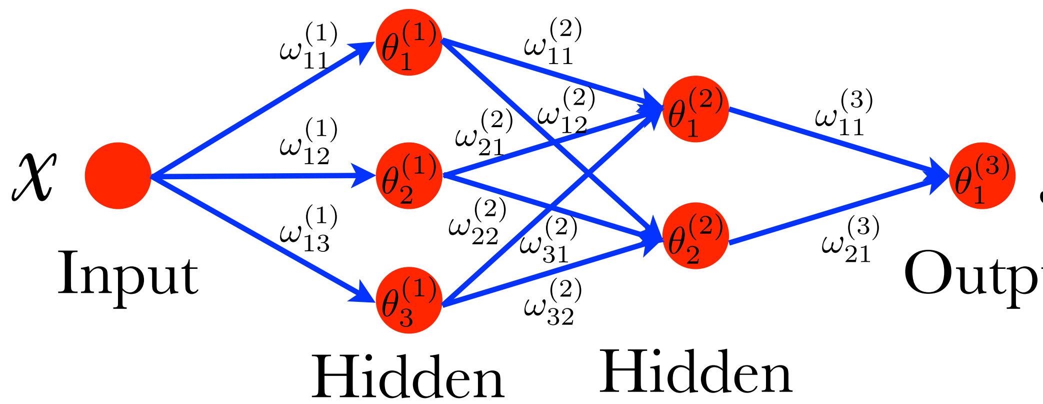

$$\mathcal{N}_i(x) \qquad \xi_i^{(j)} = g\left(\sum_k^{\text{(j-1)th layer}} \xi_k^{(j-1)} \omega_{ki}^{(j)} - \theta_i^{(j)} \right)$$

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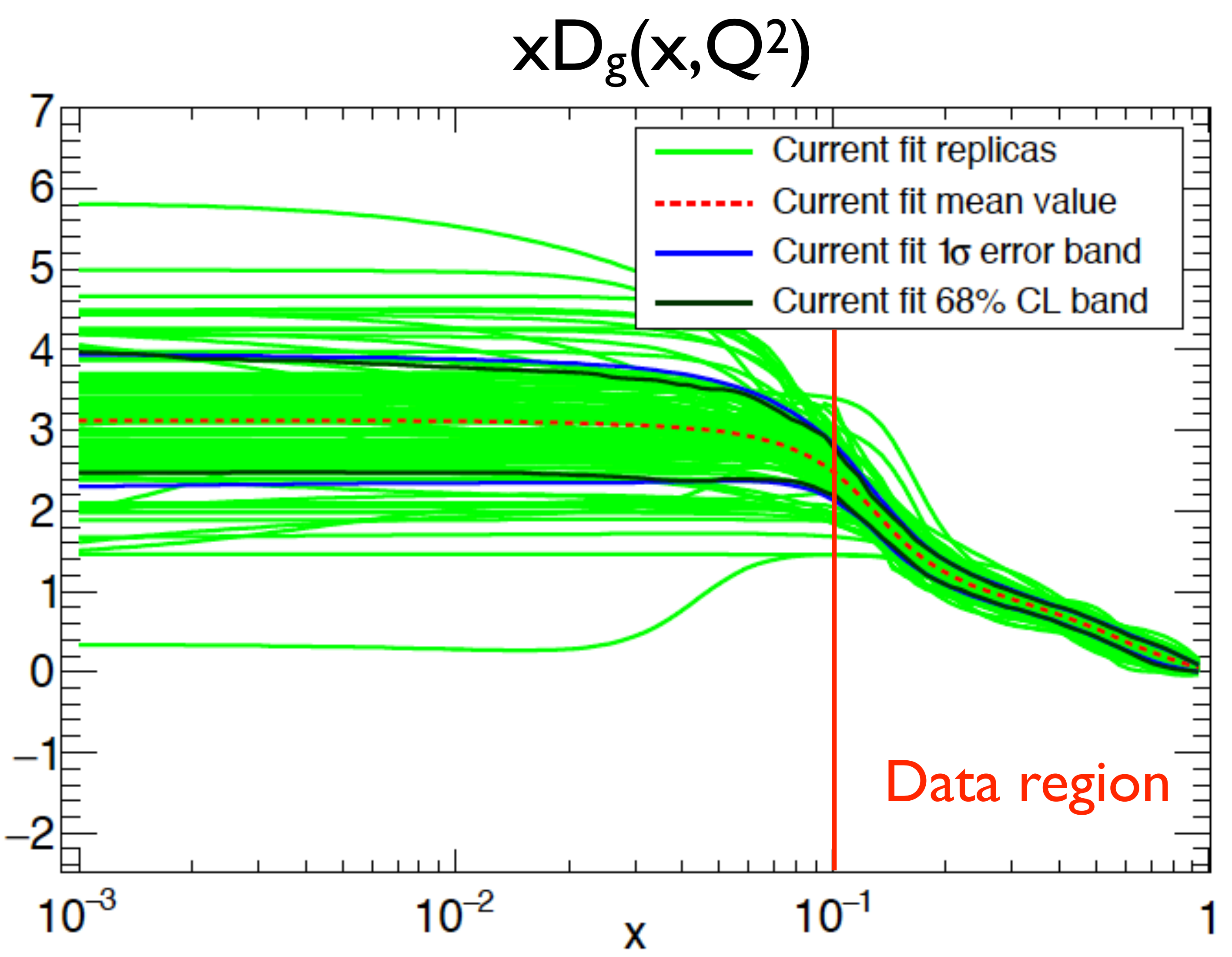

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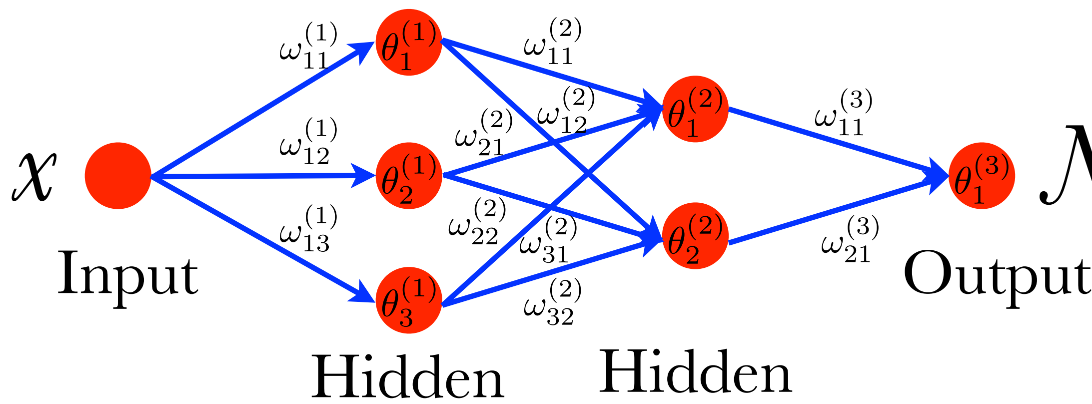

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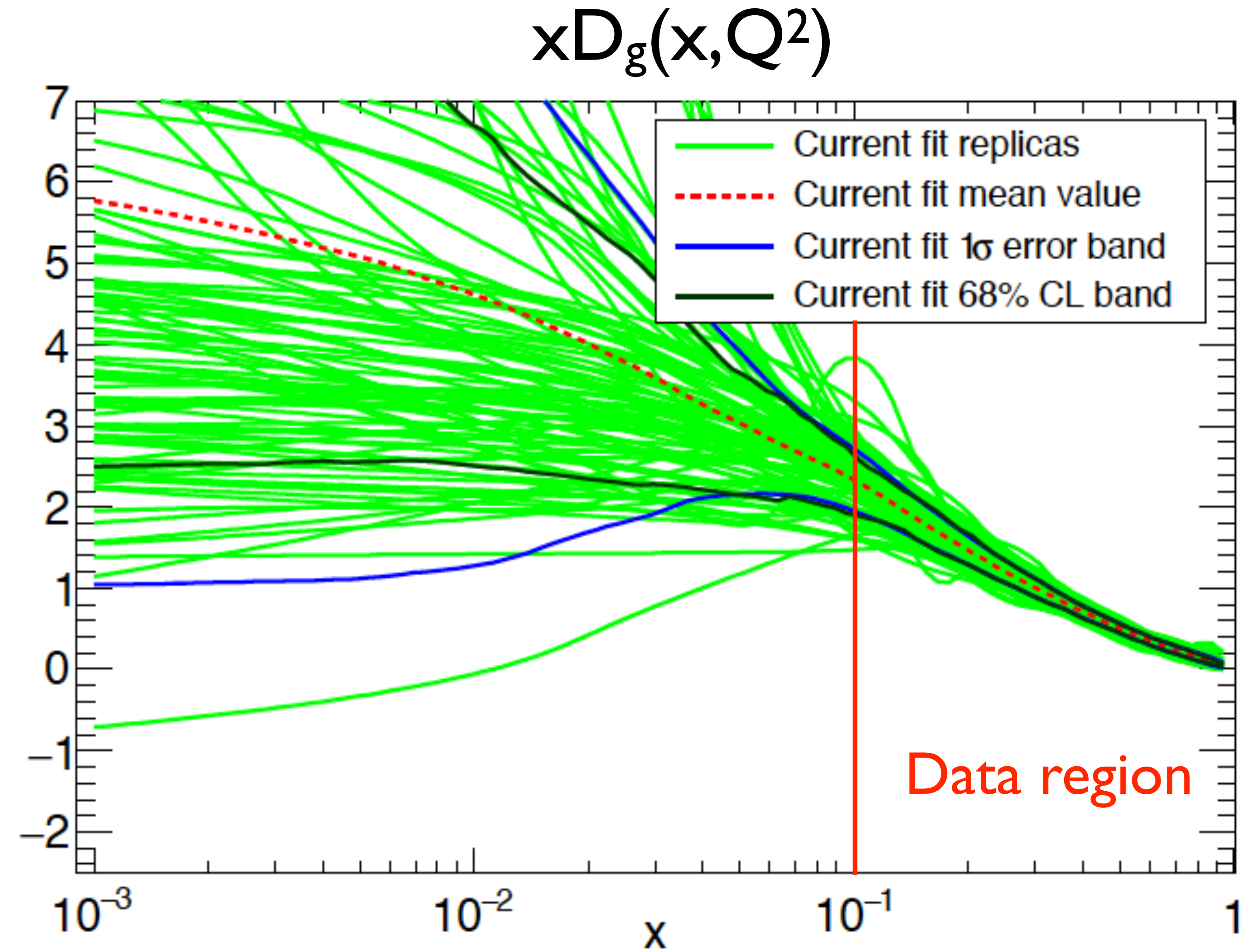
$$g_1(x) = \text{sign}(x) \ln(|x|+1)$$



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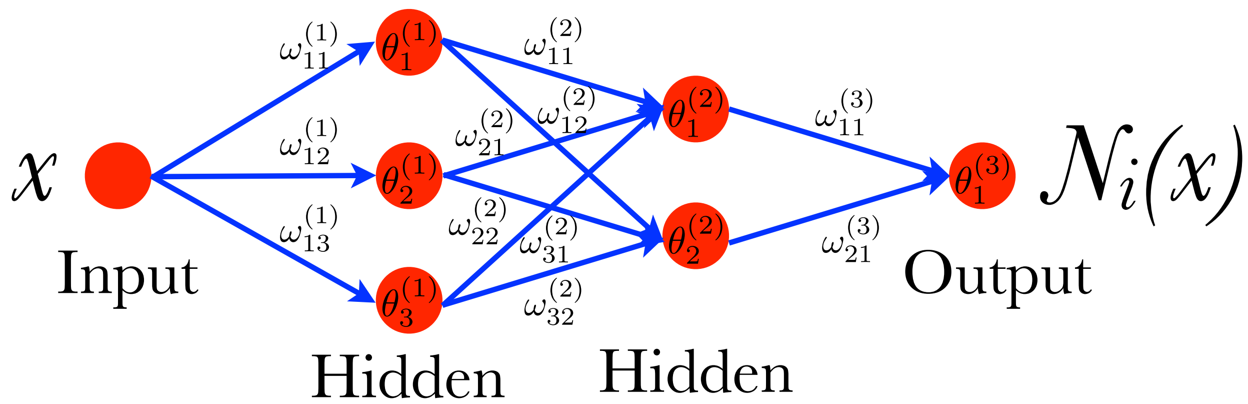

$$\xi_i^{(j)} = g\left(\sum_k^{\text{(j-1)th layer}} \xi_k^{(j-1)} \omega_{ki}^{(j)} - \theta_i^{(j)}\right)$$

Non-saturating function

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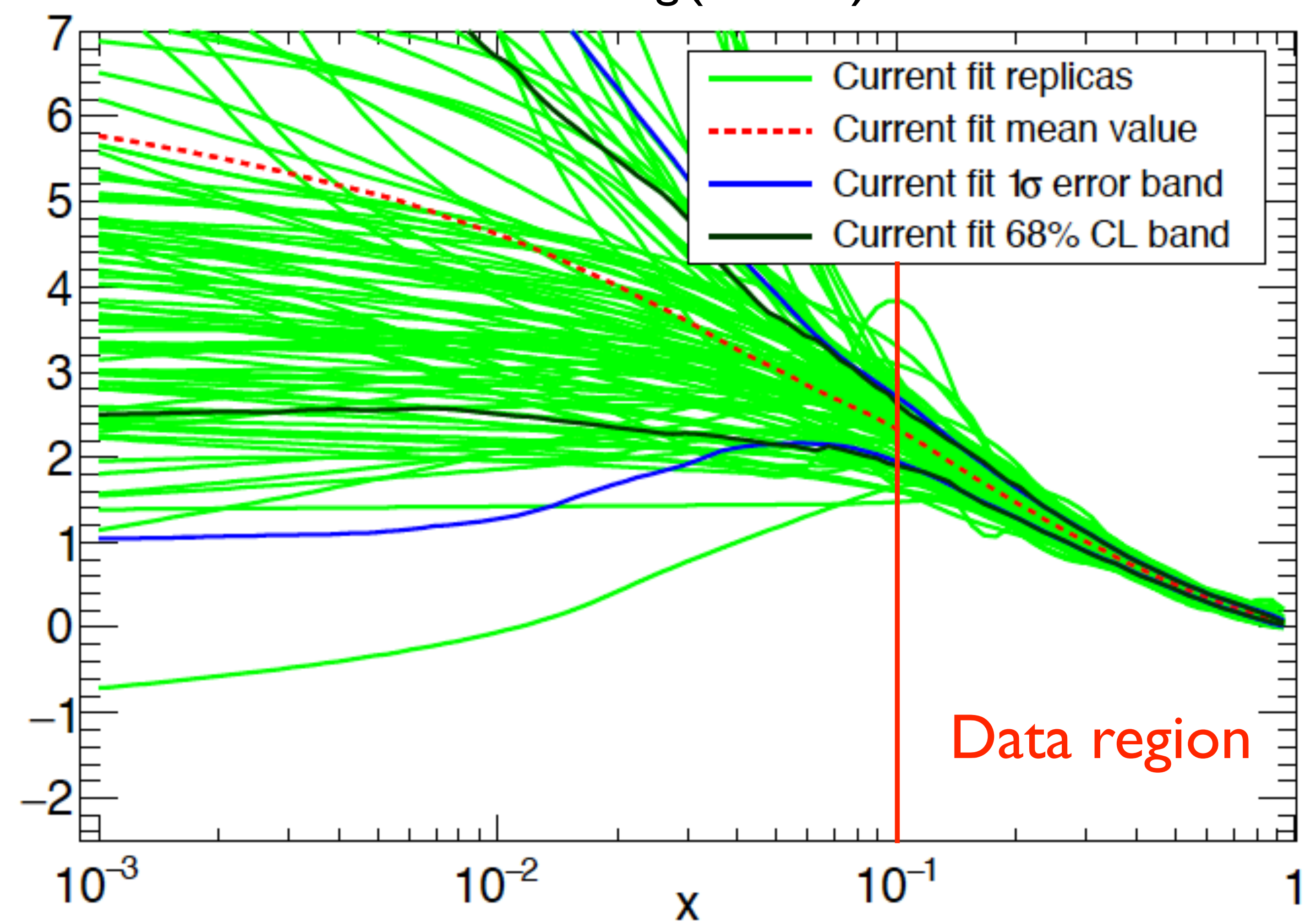
Our choice for the FFs

$xD_g(x,Q^2)$



Closure tests

- How do we know whether our fitting strategy is reliable?

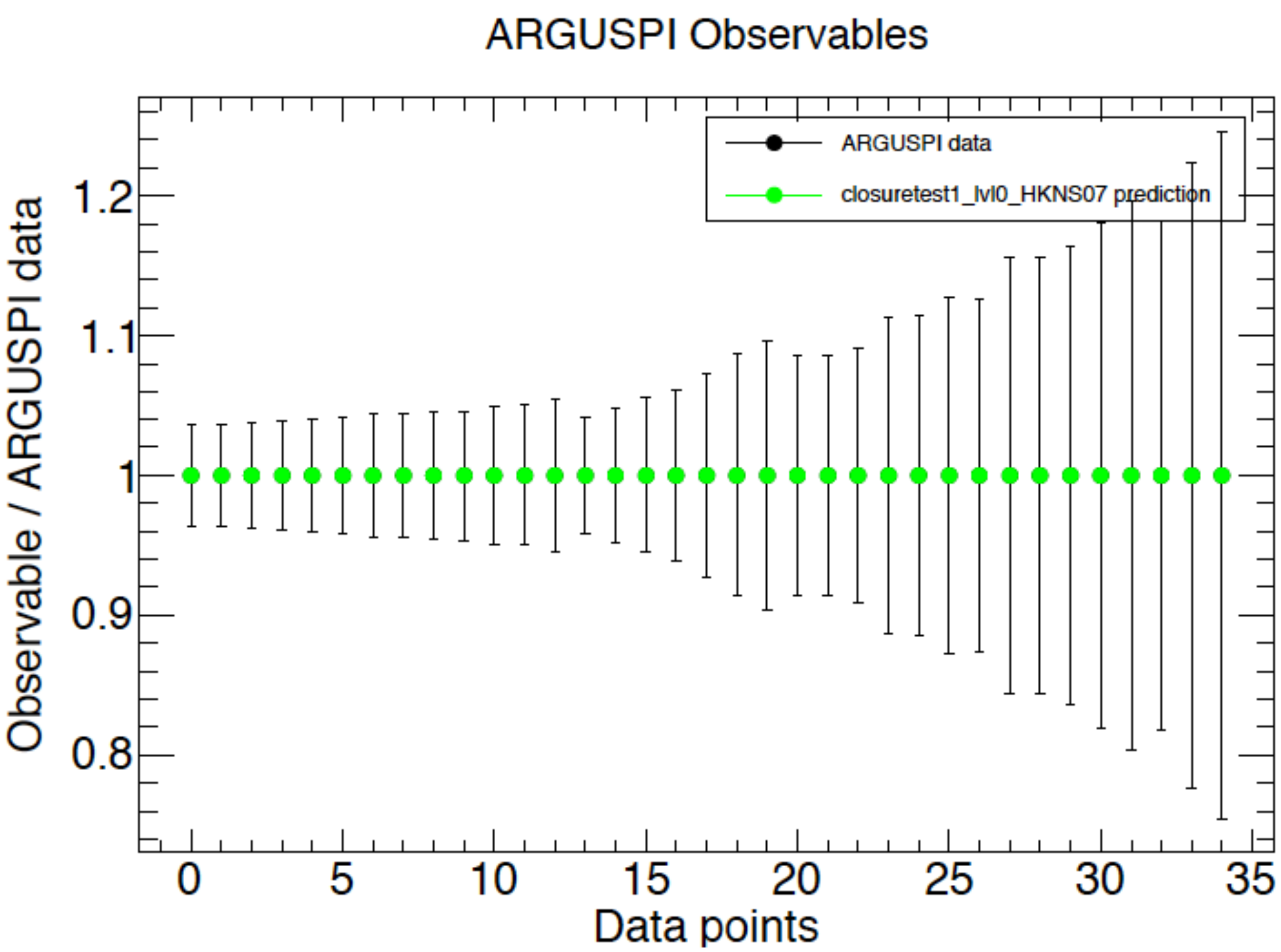
  1) **Assume underlying FFs** are known (*e.g.* HKNS07).
  2) **Generate pseudo-data** with given statistical and correlated systematics.
  3) **Perform a fit** and compare to the "truth".

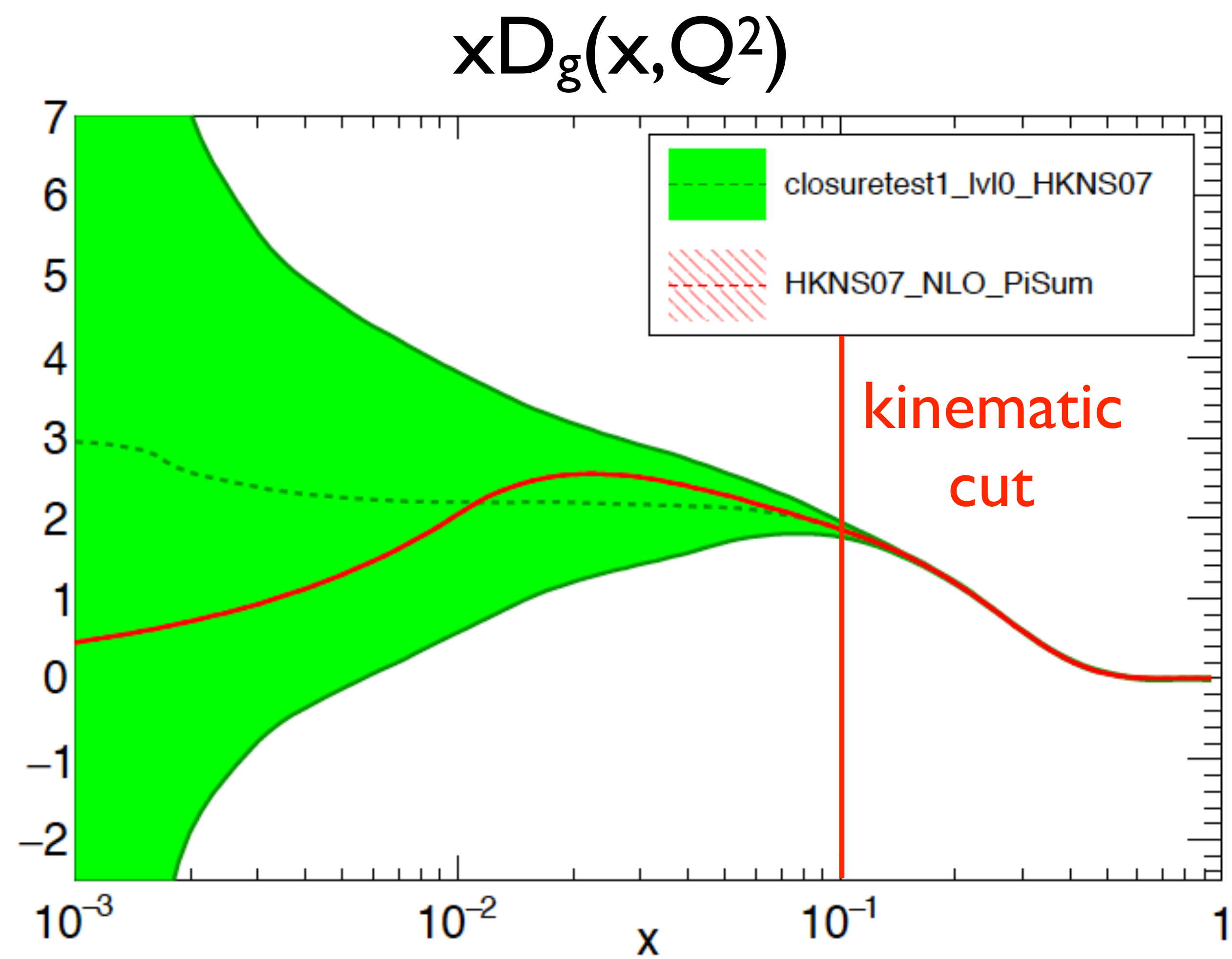
- If needed, use the closure tests to **tune** the fitting algorithm.
- Levels of closure tests: NNPDF Collaboration [arXiv:1410.8849]
  - **level 0**:
    - data point central values equal to the HKNS07 "true" values,
    - uncertainties assumed equal to the experimental ones,
    - we must find $\boldsymbol{\chi^2 \sim 0}$ and that **uncertainty on predictions tends to zero**.
  - **level 2**:
    - data points obtained as random fluctuations with exp. covariance matrix about the "truth",
    - generate Monte Carlo replicas of this data,
    - fit a PDF set to each Monte Carlo replica,
    - we must find $\boldsymbol{\chi^2 \sim 1}$ and that HKNS07 "true" FFs are **within the 1-σ band**.

Closure tests: "level 0"

- We find:

$$\chi^2 \text{ (global)} = 0.00027$$





- Predictions coincide with the data central values.
- Prediction uncertainties shrink to zero.
- FFs in the data region very close to the "truth".
- Uncertainties blow up in the extrapolation region.

Closure tests: "level 0"

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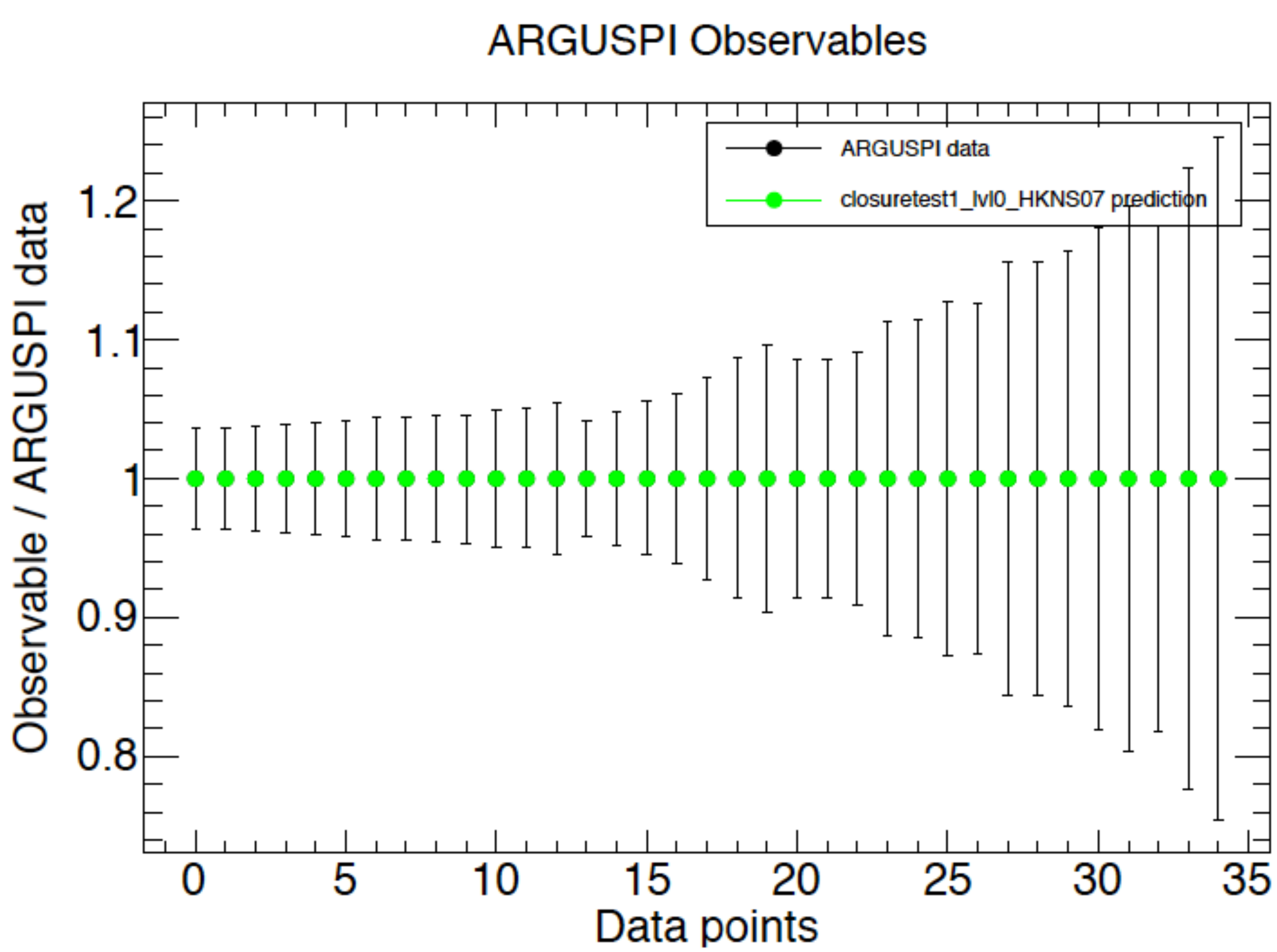

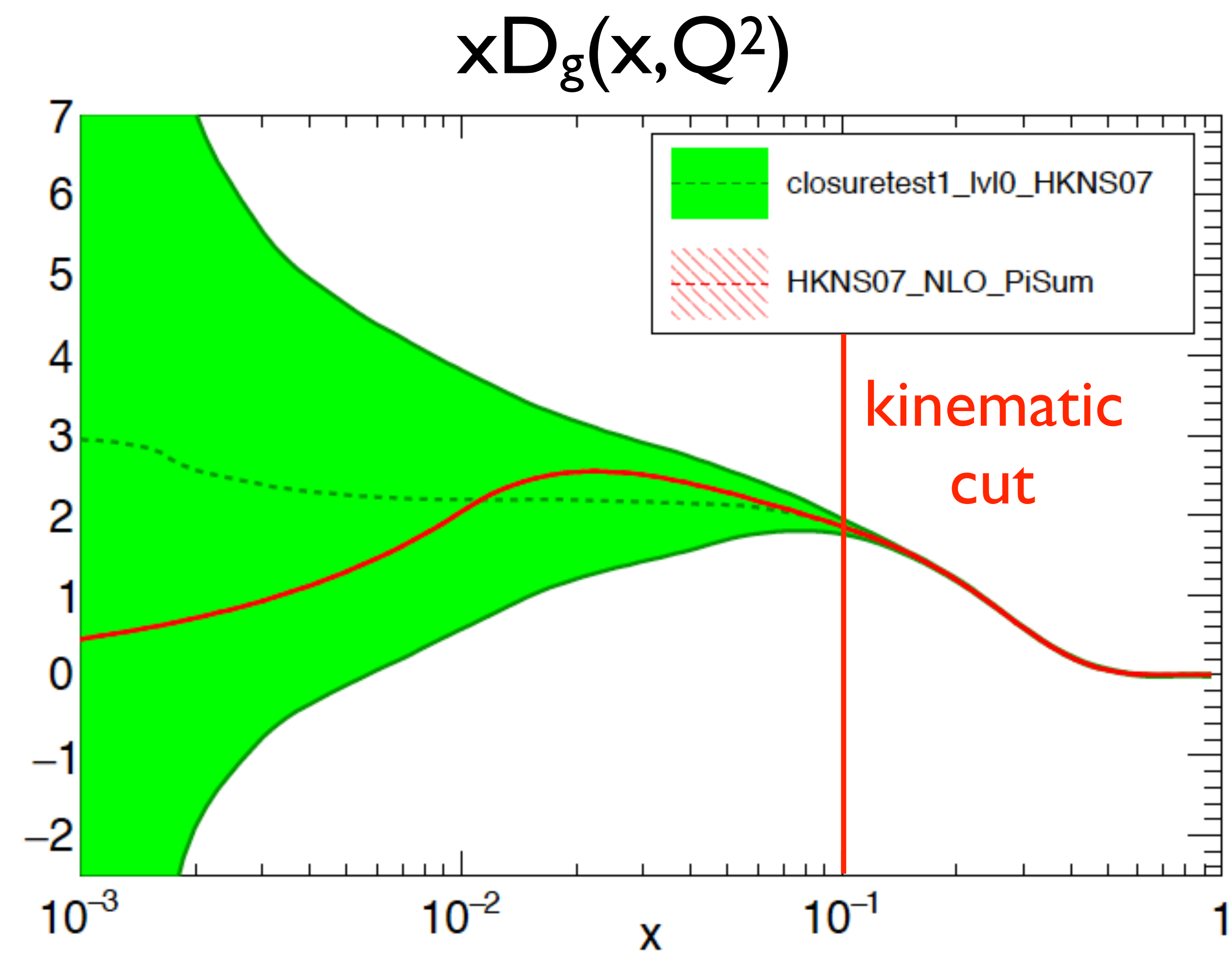

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- Prediction uncertainties shrink to zero.
- FFs in the data region very close to the "truth".
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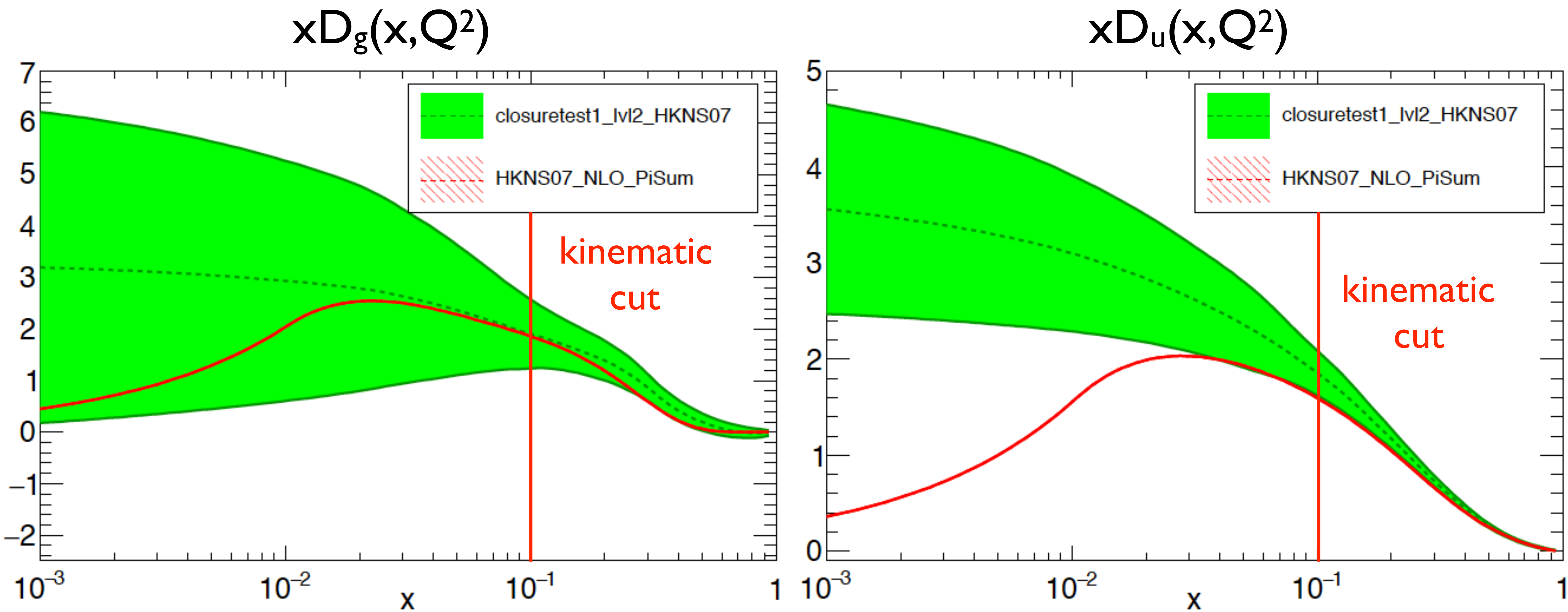
Functional uncertainty

Closure tests: “level 2”

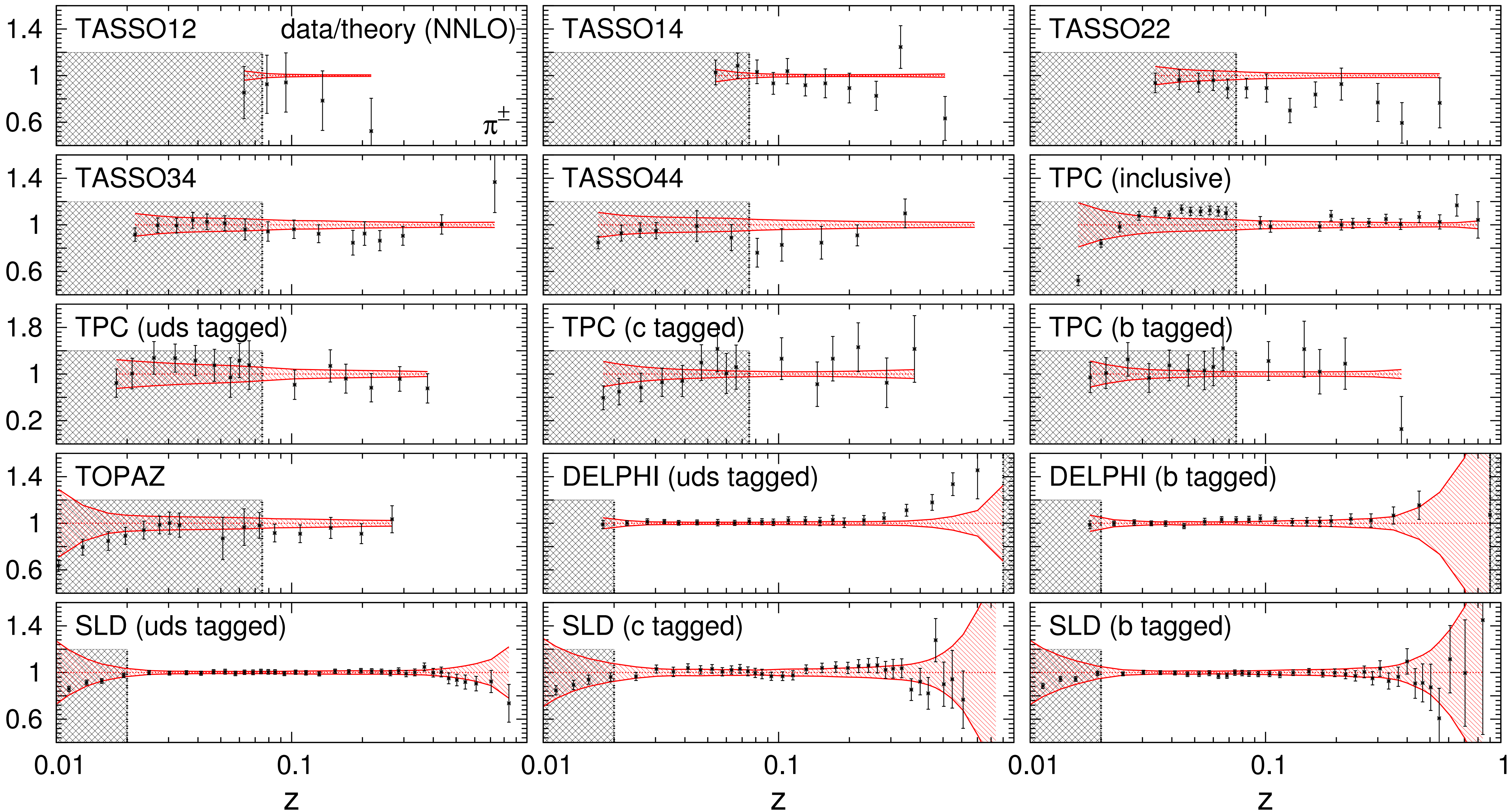
- We find:

$$\chi^2 \text{ (global)} = 0.99307$$

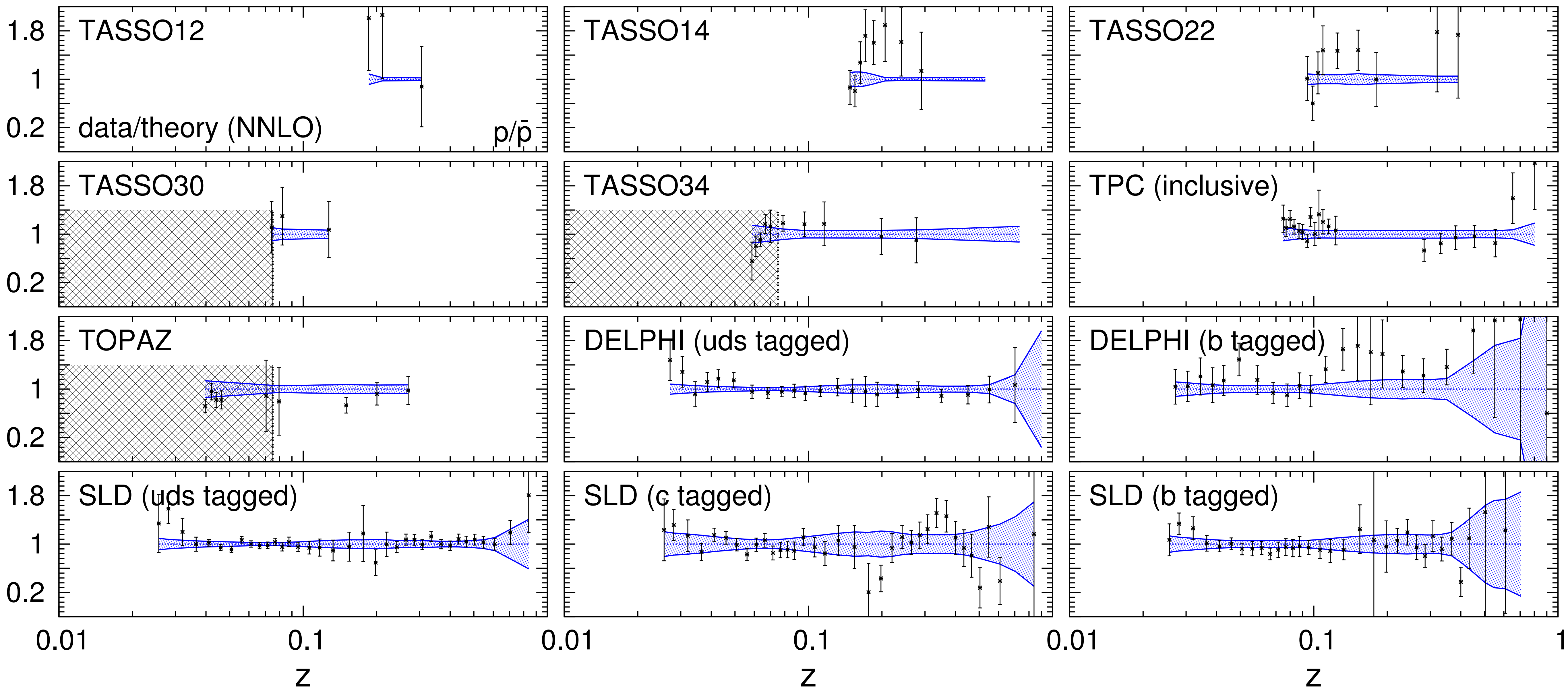


- “True” FFs do fall within the 1-σ band of the fitted FFs (in the data region),

- **Faithful representation** of the experimental uncertainty.



TASSO12
data/theory (NNLO)
$\pi^{\pm}$
TASSO14
TASSO22
TASSO34
TASSO44
TPC (inclusive)
TPC (uds tagged)
TPC (c tagged)
TPC (b tagged)
TOPAZ
DELPHI (uds tagged)
DELPHI (b tagged)
SLD (uds tagged)
SLD (c tagged)
SLD (b tagged)
1.4
1
0.6
1.8
0.2
0.01
0.1
1
z



TASSO12
data/theory (NNLO)
p/p̄
TASSO14
TASSO22
TASSO30
TASSO34
TPC (inclusive)
TOPAZ
DELPHI (uds tagged)
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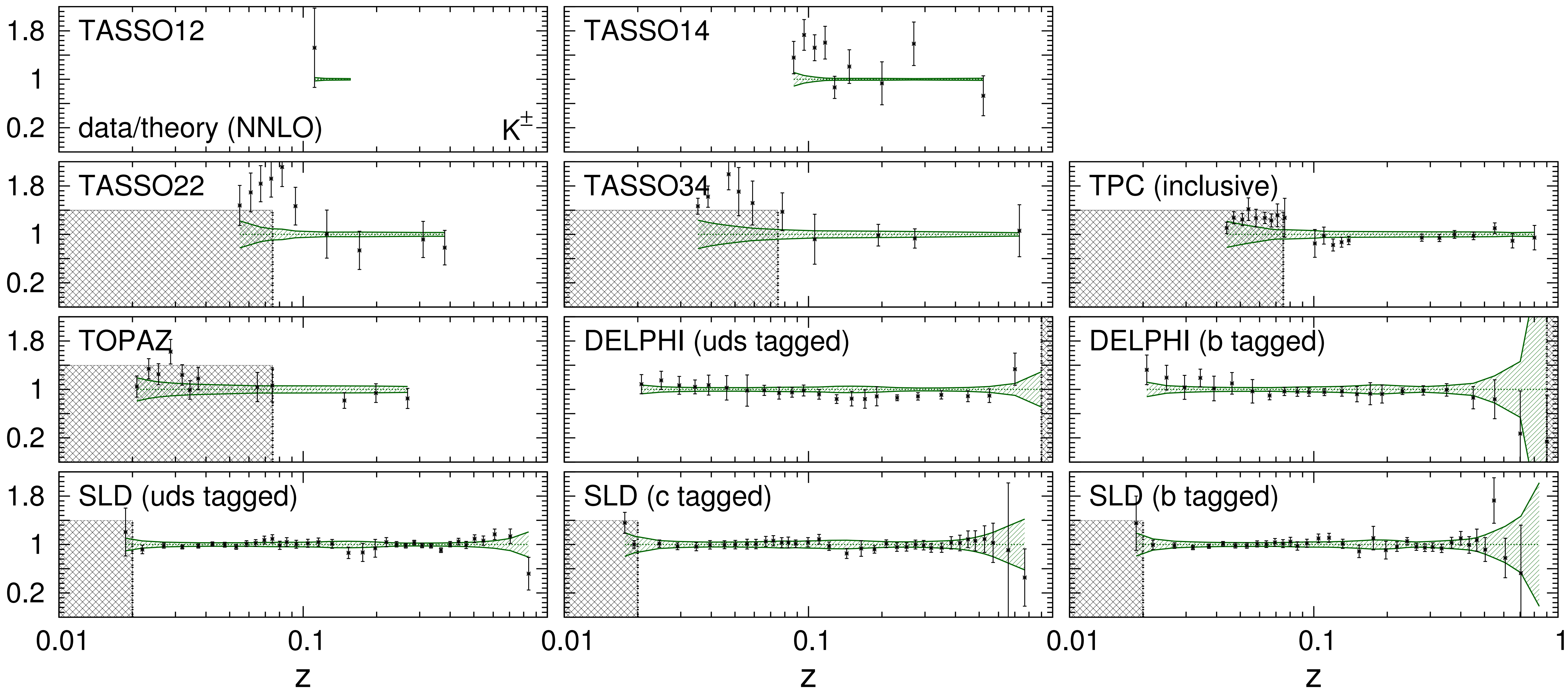
0.2
0.01
0.1
1
z



TASSO12
data/theory (NNLO)
K±
TASSO14
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1
z