Λ

Nemanja Kaloper, UC Davis

Λ: outline

- The genesis of the problem
- Separating issues
- Another try: here we go again
- What are we in fact testing here?

Cosmological experiment of Archimedes

- Number of galaxies in the visible universe: 10^{11}
- Number of stars in a galaxy: 10^{11}
- Sol: a typical star
- $M_{sol} \sim 10^{30}$ kg $\sim 10^{57}$ GeV
- Hence: visible cosmic mass is $M \sim 10^{78}$ GeV !
- Size of the universe = Age (standard candles) $\sim 2 \times 10^{10}$ light years $\sim 2 \times 10^{33}$ eV^{-1}
- Hence: visible mass density is

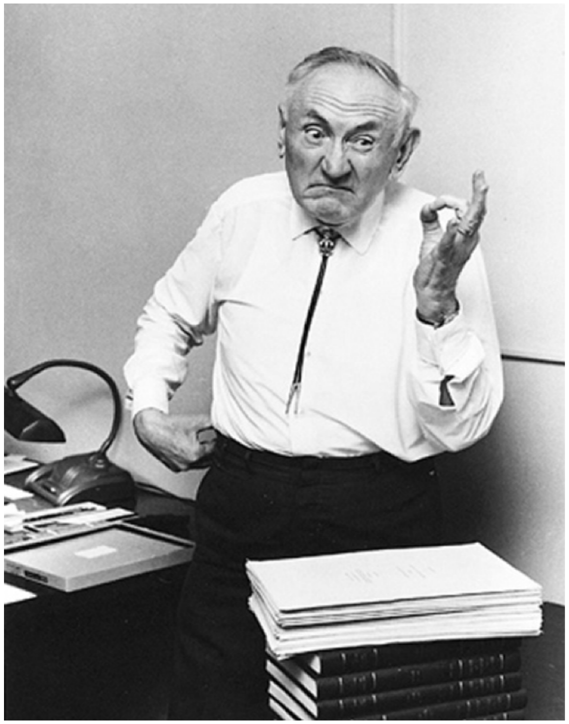
$$\rho_{visible} = \frac{3}{4\pi}\frac{M}{R^3} = \frac{3}{4\pi}\frac{10^{87}}{(2\times 10^{33})^3}\mathrm{eV}^4 \simeq 2\times 10^{-14}\mathrm{eV}^4$$

What about GRAVITY!!!

- Direct measurement of motion of galaxies and clusters showed that the total mass could be greater by as many as two orders of magnitude!

$$\rho_{total} \simeq 10^{-12}\mathrm{eV}^4 \gg \rho_{visible}$$

- Fritz Zwicky, 1930: a simple hypothesis:
- There is INVISIBLE mass in the universe;
- We feel its gravity.
- Betting on the Copernican principle:
- NOTHING special about terrestrial physics!



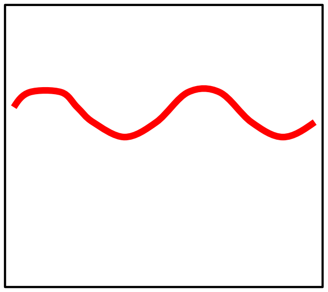
Λ

- Bronstein and Pauli, 1930's - "...the radius of the world would not even reach to the Moon..."
- In the 60's, Sakharov and Zeldovich started to worry about Λ because of Quantum Mechanics: quantum vacuum is a "happening" place!
- Oscillators in a box of size L and lattice spacing a:

$$E(\omega) = \omega\,(n+1/2), \quad E_{tot} \approx \Sigma\,\omega/2$$

$$\omega \sim 1/\lambda_j \sim k_j/L, \qquad 0<k<N=L/a,\ j = 1,2,3$$

$$\mathcal{E} \sim \sum_{k_1,k_2,k_3} \frac{k_1+k_2+k_3}{L} \sim \frac{N^4}{L} = \frac{L^3}{a^4}$$



- Vacuum energy density: Λ = E/Volume

$$\Lambda \sim \frac{1}{a^4}$$

- Λ lives at the UV cutoff - and wants to be BIG! ... it is UV sensitive!
- THUS IT MUST BE RENORMALIZED!

What is vacuum energy?

Consider matter QFT coupled to semiclassical gravity. Renormalize QFT; naively, the cosmological constant is just another coupling in the effective action of gravity:

$$S = \int d^4x \sqrt{g} \left\{ \Lambda + \frac{M_4^2}{2} R - \mathcal{L}_{matter} \right\}$$

Numerically, this looks like

$$S = \int d^4x \sqrt{g} \left\{ \mathcal{O}(10^{-60}) + \mathcal{O}(10^{30})(tiny)\ curvature + \mathcal{O}(1) subnuclear\ physics \right\} TeV^4$$

Appears as a hierarchy problem in quantum field theory...

Renormalizing Λ in GR

- $(\det g)^{1/2}$ is not gauge invariant; but its spacetime integral is.

$$V = \int d^4x \sqrt{-g}$$

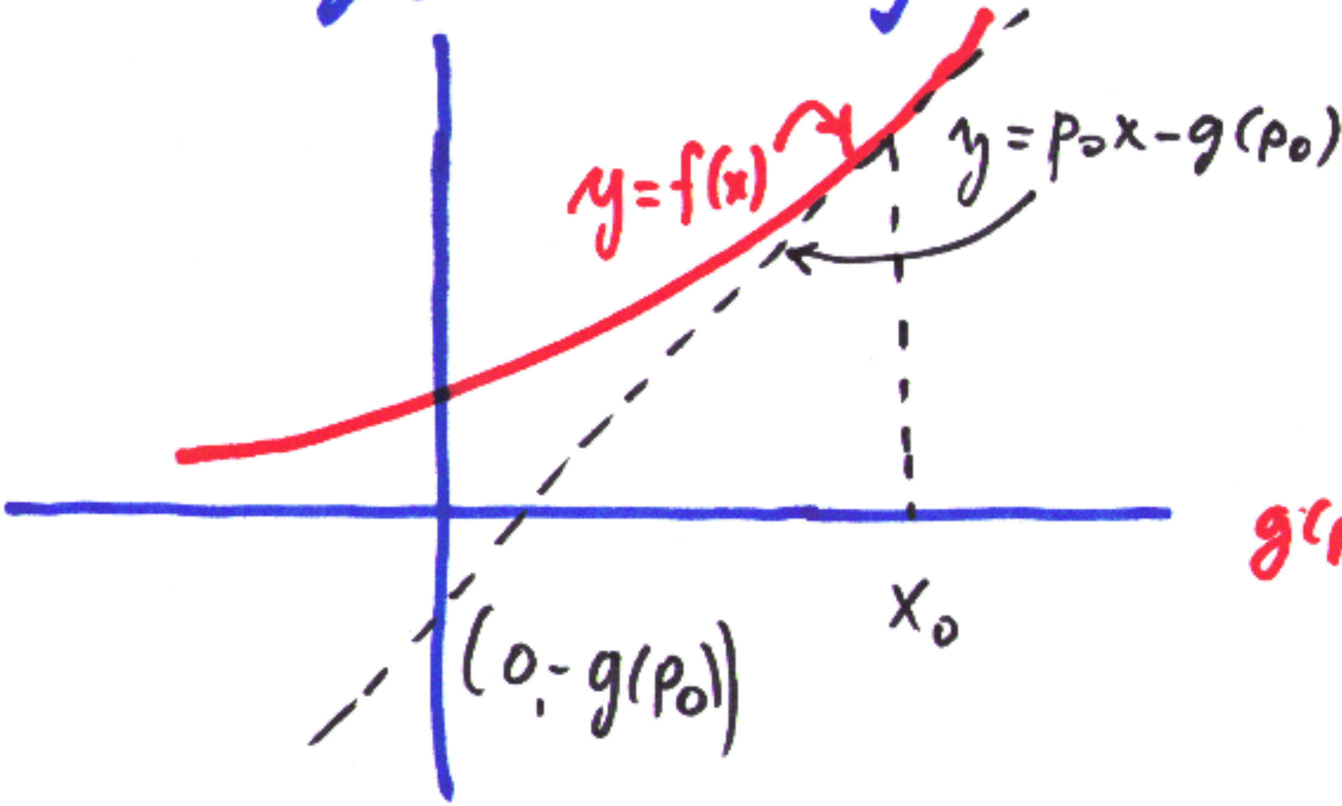
- The term in the action is $V\,\Lambda$: it trades an independent variable V for a new INDEPENDENT variable Λ (Legendre transform)
- This is perfectly reasonable: measured cosmological constant is the SUM of a quantum vacuum energy AND a bare counterterm

$$\Lambda_{total} = \Lambda_{classical} + \Lambda_{quantum}$$

- But since independent V was traded for total Λ: CC is not calculable; bare counterterm and so renormalized CC is totally arbitrary!

Where gravity falls...

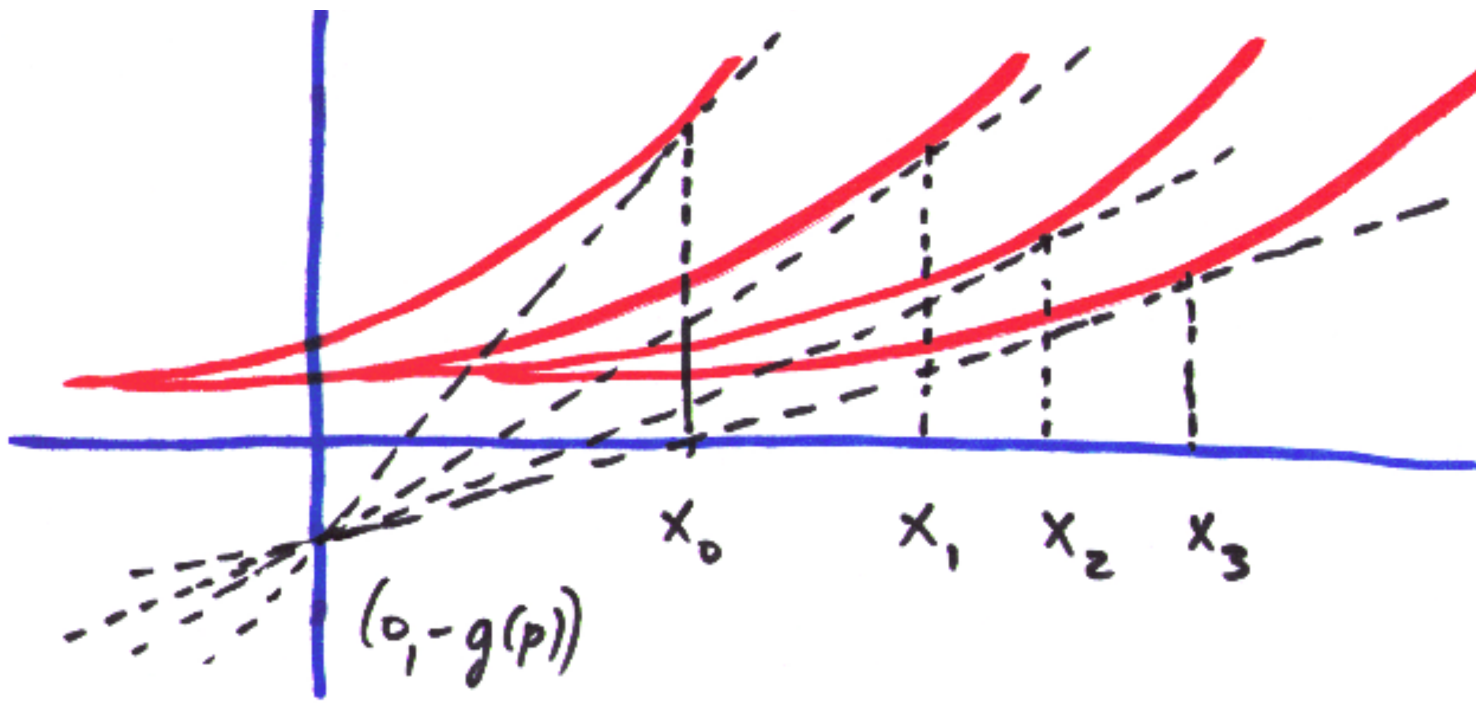
$\int \sqrt{g}\,\Lambda$: a Legendre transf.



$$p = \frac{\partial y}{\partial x}$$

$$g(p) = xp - y$$

Now: forget $f(x)$! Can reconstruct it by solving $g(y') = xy' - y$?



Solution not unique if we don't know x_k ! In GR: $x = (\det g)^{1/2}$ a nonpropagating pure gauge degree of freedom: can be ANYTHING! So: we need a boundary condition! (Einstein, unimodular GR, 1919)

So we can't calculate it, and fitting looks ugly

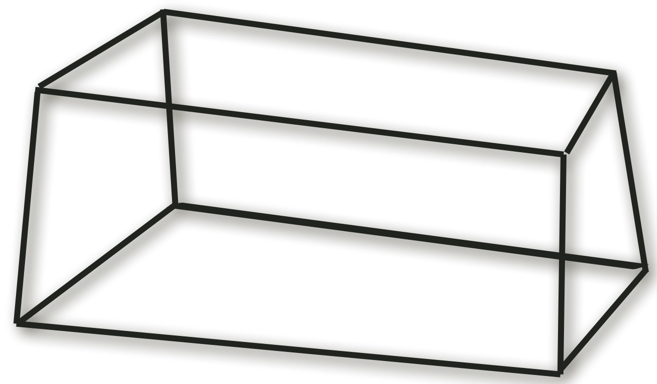
$$S = \int d^4x \sqrt{g} \Big\{ \mathcal{O}(10^{-60}) + \mathcal{O}(10^{30})(tiny)\ curvature + \mathcal{O}(1) subnuclear\ physics \Big\} TeV^4$$

Who cares?

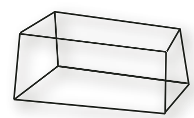
- Is this a problem? Hung jury. Some say "yes, naturalness… ", some say, "nah, landscape, anthropics…"
- The trouble with naturalness is you don't see it where you may need it most (Higgs).
- The trouble with anthropics is, when do you apply it? At the onset of inflation? At reheating? At BBN? At the last scattering surface? At recombination? At first light? … At… now?
- An example for both: a ultralight field (natural?) was a DE yesterday and a DM today; when do you constrain it (when does anthropics kick in)?
- What is it that you want to cancel anyway?
- Take gravity as a spectator; a probe. Just do QFT vacuum energy

Boxes and scales

- To calculate go to local free falling elevator; set its size (background curvature)



- Fix boundary conditions on the sides – a cavity like in E&M
- Calculate away: once you compute the corrections to the box size…



- The incredible renormalized shrunk box – every single time…
- So… prop it back up to large size… etc.
- Your IR physics may not care so much about messing with boundary conditions if it depends on the box size only logarithmically;
- but if it depends through powers… eg Higgs… something may be awry
- The box is a patch of the real early universe. Can't evaluate how fast it expands. Exponential errors the late volume size… Measures uncertain?

Hard to adjust dynamically: the Weinberg no-go

- Work in 4D gravity, finitely many fields, Poincare symmetry

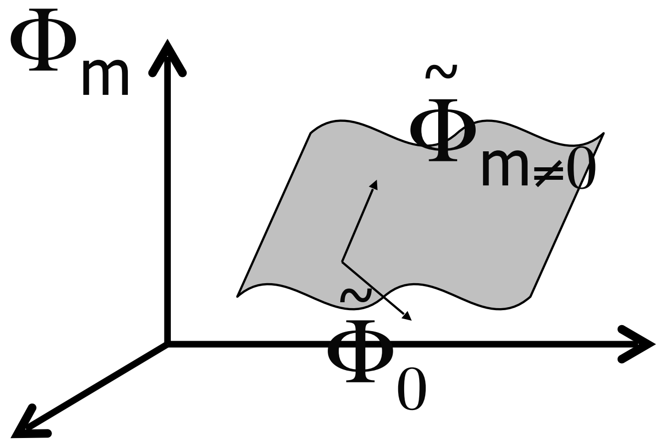
Field Eqs: $$\frac{\delta \mathcal{L}}{\delta \Phi_m} = 0$$

Gravity: $$\frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} = 0$$

- Field eqs are trivial; diffeomorphism invariance then sets $\mathcal{L} = \Lambda \sqrt{\det(g)}$, and gravity demands $\Lambda = 0$
- THAT IS THE FINE-TUNING!

What if gravity eqs are not independent???

What if $$2g^{\mu\nu}\frac{\delta\mathcal{L}}{\delta g_{\mu\nu}} = \sum_m f(\Phi_m)\frac{\delta\mathcal{L}}{\delta\Phi_m}$$?



The logic: replace V in the Einstein eqs by its DERIVATIVE

$$\Lambda \simeq V \quad \rightarrow \quad \Lambda \simeq \partial V$$

But 4D too confining: Weinberg no-go!

- Symmetries require

$$\mathcal{L} = \sqrt{-g}(\Lambda_{bare} + \Lambda_{vacuum})e^{\tilde{\Phi}_0}$$

- So either we set $\Lambda_{bare} + \Lambda_{vacuum} = 0$ (fine-tuning) or we send $\tilde{\Phi}_0 \to -\infty$
- But: radiative stability requires $\mu_{eff} = \mu_0 e^{\tilde{\Phi}_0/4}$ for all mass scales in the theory - in this limit they would all vanish

The cosmological constant problem

- Bare counterterm is a completely free variable replacing the total worldvolume of the universe
- Quantum vacuum energy UV sensitive: there are infinitely many large corrections; counterterm needs to be readjusted order by order
- So: can we tame the "oscillating" series, and make the finite part UV-insensitive?
- If yes, how do we fix its value?

Some Takeaways from Old Attempts I

- Problem: equivalence principle – all energy gravitates
- By symmetries of the cc it can only go into the intrinsic curvature
- In selftuning brane setup, the 4d space was a subspace – so has both extrinsic and intrinsic curvature
- Good thing: can divert vacuum energy in a radiatively stable manner to extrinsic cuvature
- Bad thing: 5d differs imply a conservation law – Gauss law for gravity
- So geometry is either tuned or singular by backreaction

Some Takeaways from Old Attempts II

- 4D normalized action

$$S^T = \frac{\int \sqrt{g}\left(M_{Pl}^2 R/2 - \mathcal{L}_m - \Lambda\right)}{\int \sqrt{g}}$$

- Motivated by a search for a manifestly T-dual target space string action
- Good thing: cancels the classical and tree level cc, hiding them into the Lagrange multiplier sector
- Bad thing: not radiatively stable

$$S^T_{eff} = \frac{S^T_0}{\Omega} + S^T_1 + \Omega S^T_2 + \Omega^2 S^T_3 + \ldots \qquad \Omega = \mu^4 \int d^4x \sqrt{g}$$

- Volume is like $\hbar$: loops change the dependence on volume
- Idea: combine two setups and try to use only good things

A Road to Sequester

- The `run of the mill' way of thinking about the problem is not utilizing the complete arbitrariness of bare Λ
- Since it is an independent gauge invariant parameter of the theory, why not vary with respect to it?

$$-\int d^4x\sqrt{-g}\Lambda$$

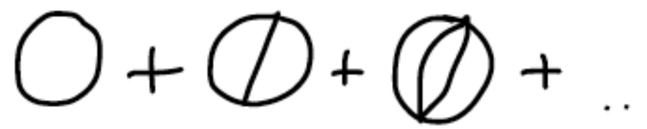
- Naive variation would constrain the metric: $\int d^4x\sqrt{-g} = 0$
- This is bad - not a lot of room to fit a universe in!
- A hint: isoperimetric problem in variational calculus - add a constraint which makes $\int d^4x\sqrt{-g} \neq 0$

$$-\int d^4x\sqrt{-g}\Lambda + \sigma(\frac{\Lambda}{\mu^4})$$

Scaling

- This fixes the worldvolume of the universe in terms of Λ
- How do we fix Λ?

$$\frac{1}{4}\langle T^{\mu\,\mathrm{const}}_{\mu}\rangle_{vac} = \Lambda_{vac} =$$



- Ignore virtual gravitons – tough enough without them!
- All the loops have engineering dimension 4 – because there is no external momenta

$$\Lambda_{vac} = \langle 0|\mathcal{L}_{matter}|0\rangle = \frac{1}{4}\langle 0|T^{\mu}_{\mu}|0\rangle$$

- Vacuum energy is the constant part of matter Lagrangian – which has engineering dimension 4!
- So let's cancel the terms of engineering dimension 4

Vacuum energy sequester

- So wherever we have a matter sector dimensional parameter we introduce a "stiff dilaton" - a spurion

$$S = \int d^4x\sqrt{-g}\Big(\frac{M_{Pl}^2}{2}R - \lambda^4\mathcal{L}(\frac{g^{\mu\nu}}{\lambda^2},\Phi) - \Lambda\Big) + \sigma(\frac{\Lambda}{\lambda^4\mu^4})$$

- Next we promote it into an arbitrary global field like Λ

$$M_{Pl}^2 G^\mu{}_\nu = T^\mu{}_\nu - \Lambda \delta^\mu{}_\nu$$

$$\frac{\sigma'}{\lambda^4\mu^4} = \int d^4x\sqrt{g} \qquad \Lambda\frac{\sigma'}{\lambda^4\mu^4} = \frac{1}{4}\int d^4x\, T^\mu{}_\mu$$

- Out comes

$$\Lambda = \frac{1}{4}\frac{\int d^4x\sqrt{g}\langle 0|T^\mu{}_\mu|0\rangle}{\int d^4x\sqrt{g}} = \frac{1}{4}\langle T^\mu{}_\mu\rangle$$

New gravitational field equations

- Separate vacuum energy from the rest:

$$T^\mu{}_\nu = -\Lambda_{vac}\,\delta^\mu{}_\nu + \tau^\mu{}_\nu$$

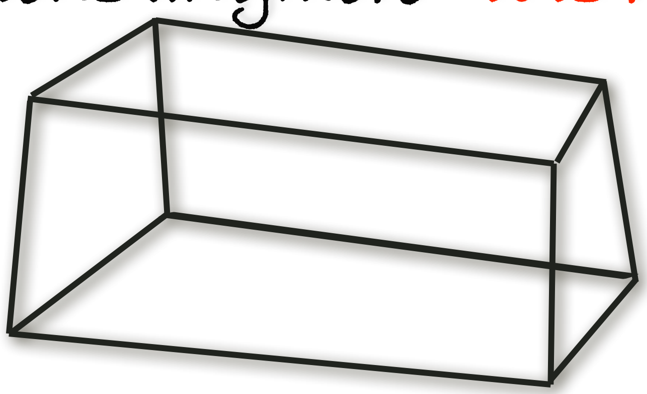
NONCONST PART

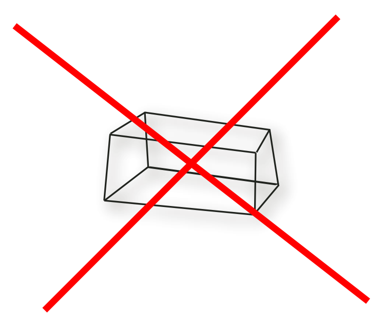
- Plug into gravity eqs using

$$\Lambda = \langle T\rangle/4 = -\Lambda_{vac}$$

$$M_{Pl}^2 G^\mu{}_\nu = \tau^\mu{}_\nu - \frac{1}{4}\langle \tau^\alpha{}_\alpha\rangle \delta^\mu{}_\nu$$

- Vacuum energy completely cancelled from the curvature irrespective of the loop order in perturbation theory!!!
- The geometry does not care about quantum vacuum loop corrections anymore – it is radiatively STABLE!!!





Out of the cauldron, but, … into the fire?…

- This seems easy!!! What's the `damage'?

$$\frac{m_{phys}}{M_{Pl}} = \lambda \frac{m_{bare}}{M_{Pl}}$$

- Since $m_{phys} \neq 0 \to \lambda \neq 0$ and $\lambda^4 \mu^4 = \sigma' / \int d^4x \sqrt{-g}$ the worldvolume MUST be finite! Otherwise we cannot have nonzero rest masses of particles
- To preserve diffeomorsphism invariance and local Poincare the universe must be finite in SPACE and TIME!
- If one accepts this framework, then the fact that we have nonzero weight IMPLIES the universe must END!

Nonlocality? OK, Nonlocality!

- There is residual, nonzero leftover cosmological constant

$$\Lambda_{eff} = \frac{1}{4}\langle \tau^{\mu}{}_{\mu} \rangle = \frac{1}{4}\frac{\int d^4x\sqrt{-g}\,\tau^{\mu}{}_{\mu}}{\int d^4x\sqrt{-g}}$$

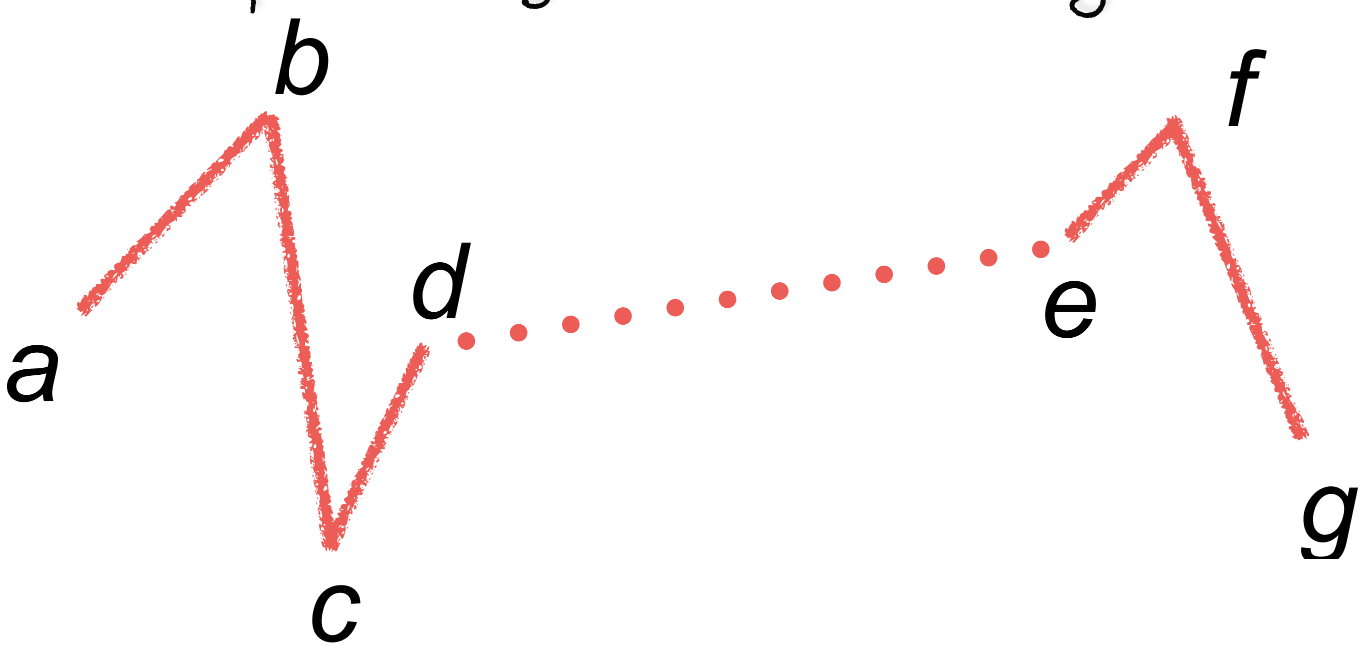
- It is nonlocal! In time, too! This looks scary! But...
- ...a finite part of a UV sensitive observable cannot be calculated but must be measured - and cc is nonlocal!
- Let stress energy obey null energy condition; the integrals dominated by the largest volume
- In big old inflated universes the residual cc is SMALL !!
- Suffices to take integrals to be larger than Hubble

The worst sacrifice: QM calculability

- The action is not additive:

$$S = \int d^4x \sqrt{-g}\Big(\frac{M_{Pl}^2}{2}R - \lambda^4 \mathcal{L}(\frac{g^{\mu\nu}}{\lambda^2}, \Phi) - \Lambda\Big) + \sigma(\frac{\Lambda}{\lambda^4\mu^4})$$

- So the path integral does not really exist in the usual sense



$$\neq Amplitude$$

- The Feynman-Katz-Trotter formula won't work right

A fix: $global = \int local$

- We want to interpret the global constraint as an integral of a local one; $\sigma \to \int \dot{f}$ no new degrees of freedom; we need 1st order eq of motion!
- In spacetime: the integrand must be a space-filling form

$$\sigma(\Lambda) \to \int m\phi(\Lambda) F = \int m\phi(\Lambda) dA$$

- Inverting the $\phi \leftrightarrow \Lambda = \mu^4 V(\phi)$ relationship:

$$\int \left(\sqrt{g}\mu^4 V(m\phi/\mu^2) + \frac{1}{4!} m\phi \epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu\lambda\sigma} \right)$$

- This is the local variant of the global constraint
- Scale-transform the metric & do this twice

Local Sequester

- Theory

$$S = \int d^4x\sqrt{g}\left[\frac{M_{Pl}^2}{2}\mathcal{U}(\frac{M\hat{\phi}}{M_{Pl}^2})R - \mu^4 V(\frac{m\phi}{\mu^2}) - \mathcal{L}_m(g^{\mu\nu}, \Phi) + \frac{\epsilon^{\mu\nu\lambda\sigma}}{4!\sqrt{g}}\left(m\phi F_{\mu\nu\lambda\sigma} + M\hat{\phi}\hat{F}_{\mu\nu\lambda\sigma}\right)\right]$$

- Eoms

$$\partial_\mu\phi = \partial_\mu\hat{\phi} = 0\,, \qquad F_{\mu\nu\lambda\sigma} = \mu^2 V'\sqrt{g}\epsilon_{\mu\nu\lambda\sigma}\,, \qquad \hat{F}_{\mu\nu\lambda\sigma} = -\frac{\mathcal{U}'}{2}R\sqrt{g}\epsilon_{\mu\nu\lambda\sigma}\,,$$

$$M_{Pl}^2\mathcal{U}\,G^\mu{}_\nu = T^\mu{}_\nu - \mu^4 V\delta^\mu{}_\nu\,.$$

- Integrals are tools to extract the counterterms: we have two counterterms, for M_{Pl}, Λ and two fluxes, $\int F, \int \hat{F}$
- In vacuum

$$M_{Pl}^2\mathcal{U}\,G^\mu{}_\nu = \frac{\mu^2}{2}\,M_{Pl}^2\mathcal{U}\,\frac{V'}{\mathcal{U}'}\,\frac{\int\hat{F}}{\int F}\,\delta^\mu{}_\nu$$

- finite cc: $$\Lambda_{residual} = -\frac{\mu^2}{2}\,M_{Pl}^2\mathcal{U}\,\frac{V'}{\mathcal{U}'}\,\frac{\int\hat{F}}{\int F}$$

How does it work?

- Counterterm cancels quantum cc

$$\langle R\rangle = \int\sqrt{g}R/\int\sqrt{g} = (4\mu^4 V - \langle T\rangle)/M_{Pl}^2\mathcal{U} = -2\frac{\mu^2 V'}{\mathcal{U}'}\int\hat{F}/\int F$$

- RHS is radiatively stable since $M_{Pl}\mathcal{U}$ is natural

$$(M_{Pl}^{ren})^2 \simeq M_{Pl}^2\mathcal{U} + \mathcal{O}(N)\,(M_{UV})^2 + \sum_{species}\mathcal{O}(1)m_M^2\ln(M_{UV}/m_M) + \sum_{species}\mathcal{O}(1)m_M^2 + \ldots$$

- The main point: forms DO NOT gravitate since $\int F$ are metric independent: no stress energy
- This violates WEP but at infinite wavelength - OK!
- But... where could such an action ever come from???

Hiding in plain view...

- Irrational axion: aligned monodromy inflation at scales below the inflation mass! (Go on, compare...)
- Same here: flux monodromy inflation at scales below the axion mass (times 2!)

$$\int \left\{ \sqrt{g} \left[\frac{M_{Pl}^2}{2} \mathcal{U}(\frac{M\hat{\phi}}{M_{Pl}^2}) R - \mathcal{L}_m - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\hat{\phi})^2 - \frac{m^2}{2}(\phi + \frac{\mathcal{Q}}{m})^2 - \frac{M^2}{2}(\hat{\phi} + \frac{\hat{\mathcal{Q}}}{M})^2 \right] \right.$$
$$\left. + \frac{1}{6}\epsilon^{\mu\nu\lambda\sigma}\left(\mathcal{Q}\partial_{[\mu}A_{\nu\lambda\sigma]} + \hat{\mathcal{Q}}\partial_{[\mu}\hat{A}_{\nu\lambda\sigma]} \right) + (\frac{\phi}{f} + \theta)\,\frac{g^2}{32\pi^2}\,\epsilon^{\mu\nu\lambda\sigma}\,Tr(G_{\mu\nu}\;G_{\lambda\sigma}) + \ldots \right\} .$$

- Can include quantum corrections to monodromies

$$\frac{m^2}{2}(\phi + \frac{\mathcal{Q}}{m})^2 \quad \rightarrow \quad V(\frac{m\phi + \mathcal{Q}}{\mathcal{M}^2/4\pi}) , \qquad \frac{M^2}{2}(\hat{\phi} + \frac{\hat{\mathcal{Q}}}{M})^2 \quad \rightarrow \quad \hat{V}(\frac{M\hat{\phi} + \hat{\mathcal{Q}}}{\hat{\mathcal{M}}^2/4\pi}) ,$$

- This is good below the cutoff M

Irrationality

- Monodromy-gauge theory coupling periods are incommensurate
- At scales below confinement

$$V_{tot} = V\left(\frac{m\phi + \mathcal{Q}}{\mathcal{M}^2/4\pi}\right) + \lambda^4\left[1 - \cos\left(\frac{\omega\phi}{\mathcal{F}} + \theta\right)\right] + \ldots$$

$$\lambda \simeq \bar{\lambda}\, e^{-S/4}$$

- Fine structure of vacua!
- This theory has two outright "Lagrange multipliers", $\mathcal{Q}, \hat{\mathcal{Q}}$ and a very heavy field $\hat{\phi}$; integrating them out yields sequestration

$$M_{Pl}^2 \mathcal{U}\, G^{\mu}{}_{\nu} = T^{\mu}{}_{\nu} - \frac{1}{4}\langle T\rangle \delta^{\mu}{}_{\nu} - \Big(V_{tot} - \langle V_{tot}\rangle\Big)\delta^{\mu}{}_{\nu} - \frac{M_{Pl}^2}{2}\frac{\mathcal{U}}{\mathcal{U}'}\,\partial_{\mathcal{Q}} V_{tot}\,\frac{\int \hat{F}}{\int F} + \ldots$$

The residual cosmological constant

- UV contributions removed by Fs
- What's left is

$$\Lambda_{residual} = \frac{M_{Pl}^2}{2}\frac{\mathcal{U}}{\mathcal{U}'}\partial_Q V_{tot}\frac{\int \hat{F}}{\int F}, \qquad -\partial_Q V = \frac{\omega\lambda^4}{m\mathcal{F}}\sin(\frac{\omega\phi}{\mathcal{F}}+\theta)$$

- Thus the residual cc is

$$\Lambda_{residual} = -\frac{\omega}{2}\frac{\mathcal{U}}{\mathcal{U}'}\frac{M_{Pl}^2}{m\mathcal{F}}\lambda^4\sin(\frac{\omega\phi}{\mathcal{F}}+\theta)\frac{\int \hat{F}}{\int F}$$

- $\mathcal{U}$, $\mathcal{U}'$, $\frac{\int \hat{F}}{\int F}$ are ~1, thus

$$\Lambda_{residual} \simeq \frac{M_{Pl}^2}{\mathcal{F}m}\lambda^4\sin(\frac{\omega\phi}{\mathcal{F}}+\theta)$$

- This is a LANDSCAPE!

The vacua

- The cosmological constant is stable at scales above and unstable below $\frac{M_{Pl}^2}{m\mathcal{F}}\lambda^4$ but this is a lot lower than the cutoff; if

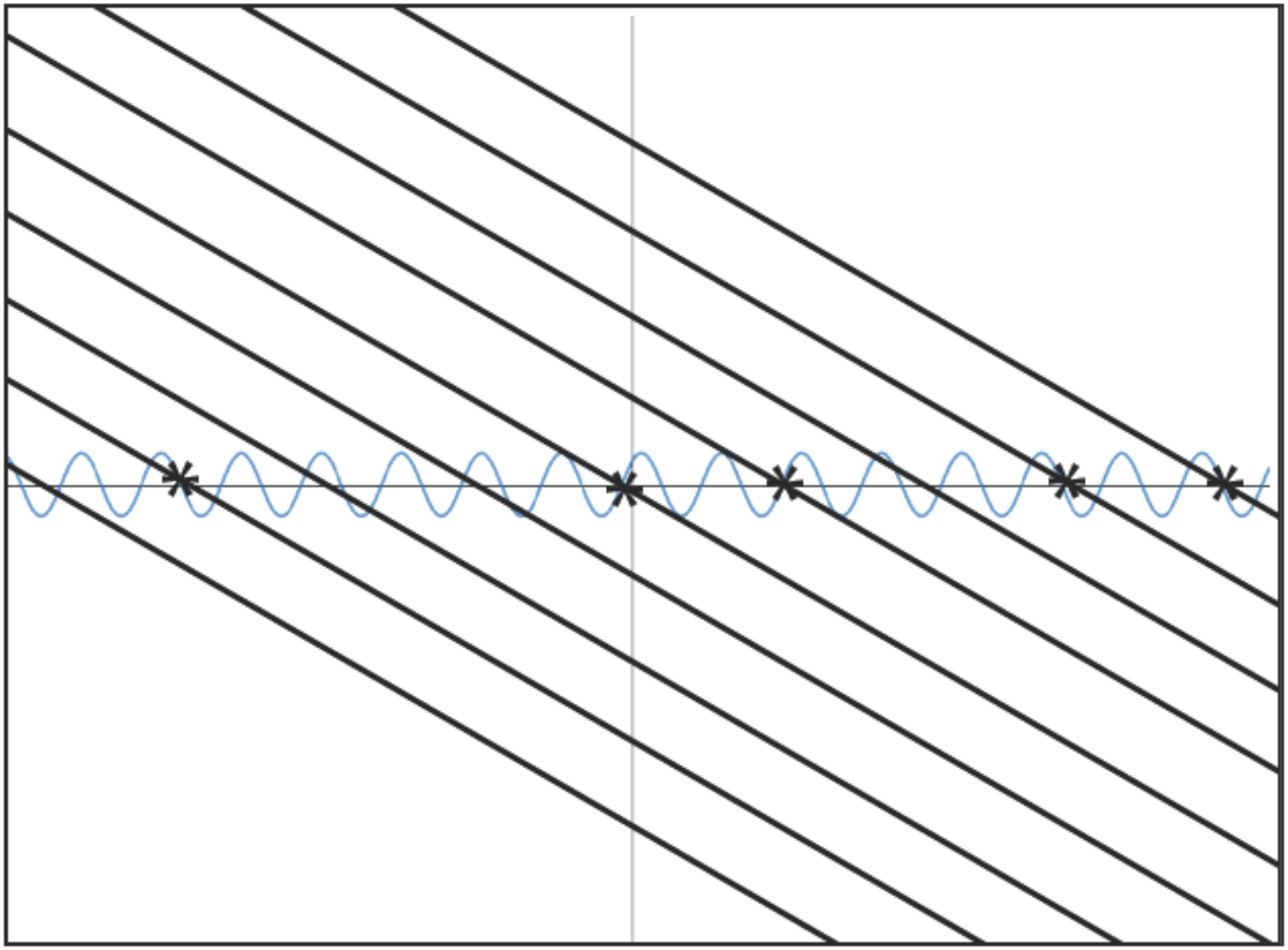
$$\frac{M_{Pl}^2}{m\mathcal{F}}\lambda^4 \simeq \frac{M_{Pl}^2}{m\mathcal{F}}\bar{\lambda}^4 e^{-S} \lesssim 10^{-12}\,\mathrm{eV}^4$$

- problem solved!
- If not, then one needs an alternative = anthropics

$$\Lambda_{residual} = -\frac{M_{Pl}^2}{m\mathcal{F}}\lambda^4 \sin(\omega l + \theta) \quad \rightarrow \quad \Delta\Lambda_{residual} < \frac{M_{Pl}^2}{m\mathcal{F}}\lambda^4\,\epsilon$$

- The vacua with tiny cosmological constant are many due to the irrational factor

Irrational Landscape

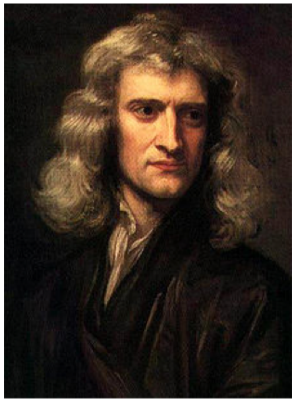


The asterisks are the vacua with very small cosmological constant due to $\omega\phi/\mathcal{F} + \theta \approx 2\pi n$. Alternatively, if $\frac{M_{Pl}^2}{m\mathcal{F}}\lambda^4 \lesssim 10^{-12}\,\mathrm{eV}^4$, the cosmological constant is small in any intersection.

Summary

- Works around Weinberg no-go by utilizing non uniqueness of the measure of action and nongravitating forms
- UV sensitive vacuum energy cancelled in the loop expansion for matter fields - cc may be sensitive to IR physics
- The vacua are a landscape, but a different kind of landscape - controlled by IR
- Step is small regardless of the initial value of cc (in contrast to BP) and UV terms are cancelled (in contrast to BDS)
- What of graviton loops? Work in progress for now...
- Are there non-anthropic means for choosing cc even with large nonperturbative gauge theory contributions? Do landscape and perturbative (dynamical...) naturalness coexist?...

...A resurrection of an ancient dilemma...



We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances.

This best of all possible worlds will contain all possibilities, with our finite experience of eternity giving no reason to dispute nature's perfection.

...Could the answer be ...a compromise?...