Lessons from six dimensions for geometry, the string landscape and the nature of matter

Hamburg prize symposium, WPC

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Washington (Wati) Taylor, MIT

Based in part on papers written in collaboration with various subsets of:
L. Anderson, M. Cvetic, A. Grassi, J. Gray, T. Grimm, J. Halverson, Y. Huang, S. Johnson, D. Klevers, V. Kumar, G. Martini, D. Morrison, D. Park, H. Piragua, N. Raghuram, J. Shaneson, A. Turner, Y. Wang

in particular,

arXiv: 1805.05907, arXiv: 1809.05160, arXiv: 1811.nnnnn	Y-C. Huang, WT
arXiv: 1803.04447, 1811nnnnn	WT, A. Turner

Hirosi Ooguri

Hirosi has made important contributions
to many aspects of string theory and mathematical physics

Some themes of his work that this talk will touch on:

Calabi-Yau manifolds

Mirror symmetry

The string landscape/swampland

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Outline

Prologue

Q: How do we simplify the problem of unifying gravity with a quantum theory of gauge fields and matter? A: add symmetry

1. Add supersymmetry (SUSY)

– Symmetry Q : bosons $\leftrightarrow$ fermions, extends Poincaré

2. Add dimensions

– Classically, unifies GR with forces like EM (Kaluza-Klein, Einstein, $\sim$ 1921)
– Increased symmetry (larger Poincaré group), increased mathematical control

$D > 11$: No SUSY theories without massless spin > 2 fields (problematic)

$D = 11$: unique theory of pure supergravity (SUGRA)
- 32 Q's ($\mathcal{N} = 1$ SUSY)
- Theory of membranes in 11D, "M-theory"/"M(atrix) theory"
- Long thought alternative to string theory; 1995: dual to ST

$D = 10, 32$ Q's ($\mathcal{N} = 2$): 2 pure SUGRA theories: IIA, IIB (both ST's)

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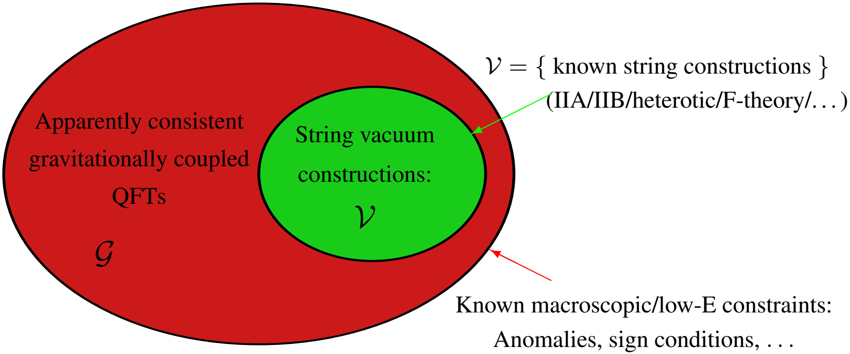

For given dimension D, SUSY $\mathcal{N}$:

- Identify $\mathcal{G}, \mathcal{V}$, theories $x \in \mathcal{G} \setminus \mathcal{V}$ ("swampland"), $\Rightarrow$ new constraints/vacua
- Relate UV/string constraints to macroscopic physics
- If $\mathcal{G} = \mathcal{V}$, QG = string theory in D dimensions with $\mathcal{N}$ supersymmetries

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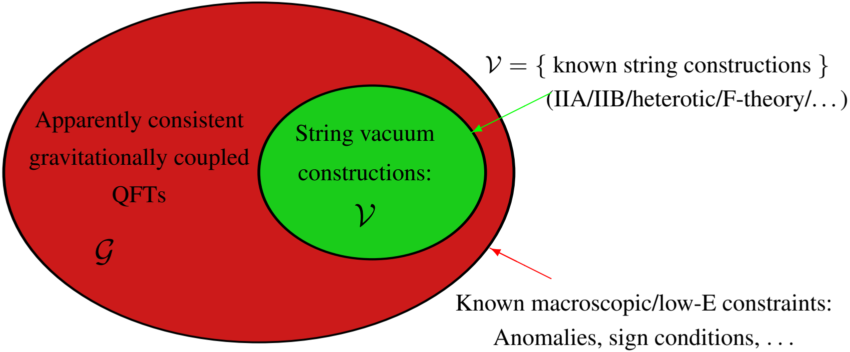

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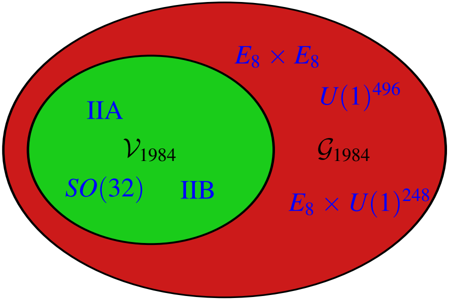
Example: supergravity/string theory in 10 dimensions



1984: Green-Schwarz anomaly cancellation

1985: Heterotic string discovered [Gross/Harvey/Martinec/Rohm]

2010: In 10D, string constraints = low-energy constraints [Adams/DeWolfe/WT]

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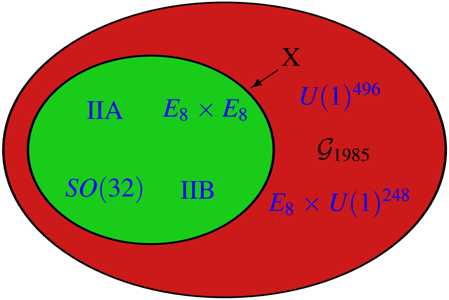

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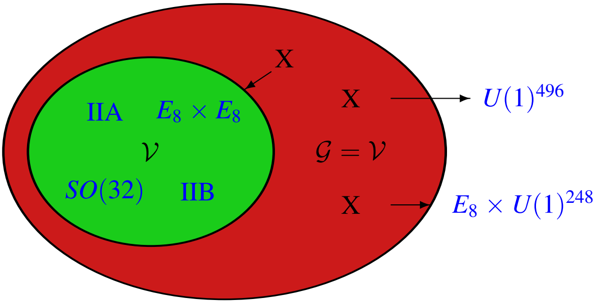

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Six dimensions: tractable but interesting

- Largest dimension with non-adjoint supersymmetric matter
- Strongly constrained from gravitational anomalies
- One big moduli space: connected branches w/ discrete labels

6D supergravity: field content (+ SUSY)

Gravity (metric $g_{\mu\nu}$)
T antisymmetric tensor fields $B_{\mu\nu}$
G gauge symmetry (gauge bosons A_μ)
M matter fields (charged under G or not)

Example anomaly constraint: $M - \dim G = 273 - T$

Strong constraints on $\{\text{consistent theories}\} = \mathcal{G}$
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1. F-theory and Calabi-Yau manifolds

F-theory:

F-theory = dictionary: geometry $\leftrightarrow$ physics of gauge group + matter + SUGRA

- Gives a global picture of "landscape" of vacua
- Geometry: "elliptic" Calabi-Yau manifolds
- Gauge groups, matter encoded in singularities

Calabi-Yau threefolds: manifolds used in superstring compactification

physically:

- Ricci flat: $R_{\mu\nu} = 0$ (solve vacuum Einstein equations)
- Kähler manifolds (complex structure compatible with SUSY)

mathematically: trivial canonical class $K = 0$ (up to torsion)

Long studied by mathematicians and physicists

— Used in compactification of string theory: 10D $\rightarrow$ 4D, 6D, ...

Largest class of known Calabi-Yau threefolds: toric hypersurface CY3's
Kreuzer/Skarke: Classified 473.8M reflexive 4D polytopes

Open Question:
Are there a finite number of topological types of Calabi-Yau threefolds?

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Elliptic and genus one-fibered CY threefolds

An *elliptic* or *genus one fibered* CY3 X:
A torus (fiber) at each $p \in B_2$
$\pi : X \to B_2$

Elliptic: $\exists$ section $\sigma : B_2 \to X, \pi\sigma = \text{Id}$

Elliptic Calabi-Yau threefold has Weierstrass model

$$y^2 = x^3 + fx + g, \quad f, g \text{ 'functions' on } B_2$$

Used for 6D F-theory construction (fiber τ = 10D axiodilaton)

Finite number of topological types of elliptic Calabi-Yau threefolds
[Grassi, Gross]

Constructive proof: (using principles of F-theory)
Bases B_2: "blowup" points of $\mathbb{F}_m, \mathbb{P}^2$ (Grassi);
Finite number of distinct strata in space of B_2 W. models

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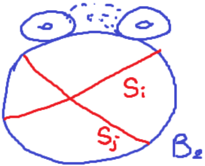
Gauge groups and matter in F-theory

Gauge groups:

7-branes on curve (divisor) S_i
$\rightarrow$ gauge factor G_i
Singularity type: $\text{ord}_{S_i} f, g$ in Weierstrass
(Kodaira classification)

For certain B_2, G nontrivial $\forall$ moduli
("non-Higgsable clusters")



Matter:

Primarily encoded in singularities at complex codimension two loci
(e.g. $S_i \cap S_j \rightarrow$ bifundamental matter)

Matter singularities not fully classified: current research

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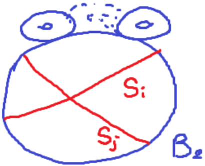

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2. Calabi-Yau threefold geometry

Primary new results on Calabi-Yau geometries

- Can systematically construct elliptic CY3's
  (w/ some remaining technical issues for explicit complete list)

- Most known CY3's are elliptic!

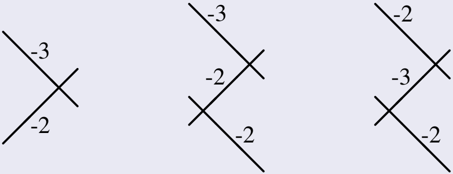
- Mirror symmetry factorizes for many elliptic-elliptic pairs

Constructing and classifying elliptic CY3's (Tools I):

Classification of 6D "Non-Higgsable Clusters" (NHC's) [Morrison/WT]:
Combinations of curves of negative self-intersection → force gauge groups

$-m$

$(m = 3, 4, 5, 6, 7, 8, 12)$



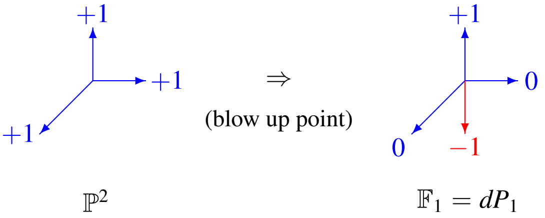
$\mathfrak{su}(3), \mathfrak{so}(8), \mathfrak{f}_4$
$\mathfrak{e}_6, \mathfrak{e}_7, \mathfrak{e}_8$

$\mathfrak{g}_2 \oplus \mathfrak{su}(2)$ | $\mathfrak{g}_2 \oplus \mathfrak{su}(2)$ | $\mathfrak{su}(2) \oplus \mathfrak{so}(7) \oplus \mathfrak{su}(2)$

- Any other combo w/ $\leq -3 \Rightarrow$ non-Kodaira (4, 6) singularity
- Limits complexity of base

Constructing and classifying elliptic CY3's (Tools II):

Toric geometry: simple combinatoric version of algebraic geometry



Toric variety: characterized by toric divisors $D_i \leftrightarrow$ rays $v_i \in \mathbb{Z}^d$

Anomalies + geometry:

Can construct topological Hodge numbers from base + G + matter

e.g. $h^{1,1}(X) = h^{1,1}(B) + \text{rank } G + 1$

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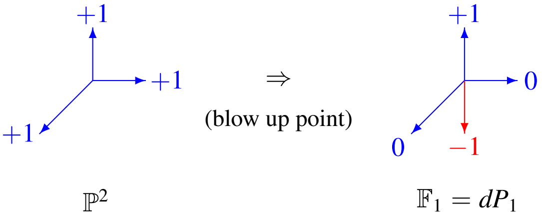

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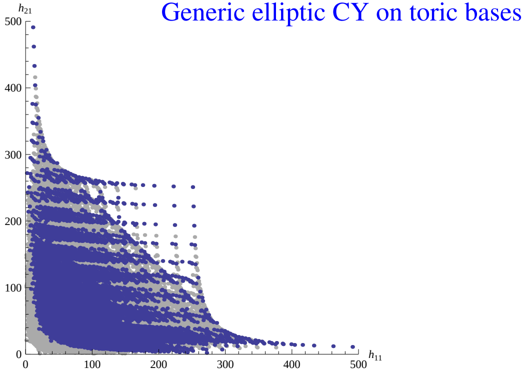
Classification of base surfaces B_2: start with $\mathbb{P}^2, \mathbb{F}_m$, blow up to get all bases B_2

- 61,539 toric bases (some not strictly toric: -9, -10, -11 curves) [Morrison/WT]



- All bases (including non-toric) for EF CY3 w/ $h^{2,1}(X) \geq 150$ [WT/Wang]
- For each B_2, finite number of "tunings" of Weierstrass; increase G.
- Tunings on toric bases: all KS data w/ $h^{1,1} \geq 240$ or $h^{2,1} \geq 240$. [Huang/WT]

Toric hypersurface construction [Batyrev, Kreuzer/Skarke]

Anti-canonical class $-K = \sum_i D_i$ (never compact CY)

Anti-canonical hypersurface $\Rightarrow$ CY by adjunction

∇ polytope: convex hull of vertices v_i

$\{$monomials$\}$ $\leftrightarrow$ lattice points in *dual polytope* $\nabla^* = \{w : w \cdot v \geq -1\}$

Batyrev: $\nabla = \nabla^{**}$ reflexive $\leftrightarrow$ 1 interior point
$\leftrightarrow$ hypersurface CY generically smooth (avoids singularities)

∇, Δ describe mirror Calabi-Yau threefolds $h^{1,1} \leftrightarrow h^{2,1}$

Symmetry in toric hypersurface construction early evidence for mirror symmetry

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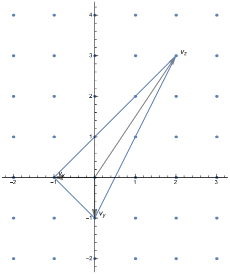
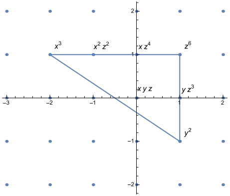
Example: Batyrev for generic elliptic curve

∇





Δ

Gives general Weierstrass ("Tate form") model for elliptic curve:

$$y^2 + a_1 yx + a_3 y = x^3 + a_2 x^2 + a_4 x + a_6$$

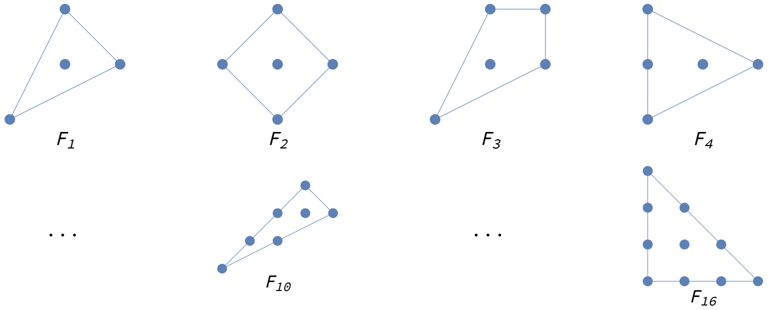
Completing square, cube $\rightarrow$ standard (short) Weierstrass form

$$y^2 = x^3 + fx + g$$

Simple toric fibrations:

$\nabla_2 \subset \nabla$, ∇_2 reflexive

Only 16 reflexive ∇_2's (e.g. F-theory fibers:
[Braun, Braun/Grimm/Keitel, Klevers/Mayorga Pena/Oehlmann/Piragua/Reuter])

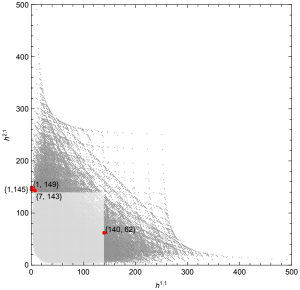


-1 curve $C = D_i^{(2)}$: satisfies $-K \cdot C = C \cdot C + 2 = 1$

All but $F_1 = \mathbb{P}^2, F_2 = \mathbb{F}_0 = \mathbb{P}^1 \times \mathbb{P}^1, F_4 = \mathbb{F}_2$ have -1 curves $\Rightarrow$ toric sections

Growing evidence: most known Calabi-Yau threefolds are elliptic or g1 fibered!
[Candelas/Constantin/Skarke, Gray/Haupt/Lukas, Anderson/Gao/Gray/Lee, . . .]

Check explicitly for fibrations:
all KS polytopes giving CY w/ $h^{1,1} \geq 140$ or $h^{2,1} \geq 140$ [Huang/WT]



Only 4 (of 495515) lack genus one fibers:

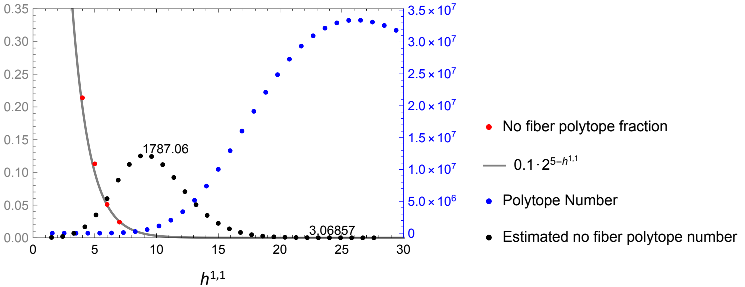
$$(h^{1,1}, h^{2,1}) = (1, 149), \quad (1, 145), \quad (7, 143), \quad (140, 62)$$

When $h^{1,1} = 1$, clearly no fiber (Shioda-Tate-Wazir)

Asymptotics at small $h^{1,1}$:

Probability that a CY3 is not g1/elliptic fibered decreases as $2^{-h^{1,1}}$ for $h^{1,1} > 1$

$h^{1,1}$	2	3	4	5	6	7
# without fiber ∇_2	23	91	256	562	872	1202
Total #	36	244	1197	4990	17101	50376
%	0.639	0.373	0.214	0.113	0.051	0.024



Why exponentially unlikely to not have fiber?

Theorem (Oguiso/Wilson):
A Calabi-Yau 3-fold X, X is genus one (or elliptically) fibered iff there exists a divisor $D \in H^2(X, \mathbb{Q})$ that satisfies $D^3 = 0, D^2 \neq 0$, and $D \cdot C \geq 0$ for all algebraic curves $C \subset X$.

Assuming "random" data for triple intersection form C_{ijk},
how likely is this to occur?

Possible obstructions:

A) Number theoretic (no solution to $C_{ijk}d_id_jd_k = 0$ over integers)

B) Cone obstruction, no solution over reals when $D \subset$ positive cone

Consider each in turn

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Number theoretic obstructions

For example:

$$x^3 + x^2y + y^3 + 2z^3 + 4w^3 = 0$$

has no solutions over the integers $\mathbb{Z}$ (or over $\mathbb{Q}$); ($\mathbb{Z}_2$ obstruction)

Mordell (1937) identified homogeneous degree d polynomial in d^2 variables with obstruction

Subsequent conjectures: d^2 is maximum number of variables with obstruction

Proven for $d = 1, 2$

Counterexample: quartic with 17 variables has obstruction!

Heath-Brown (1983): every non-singular cubic in ≥ 10 variables with rational coefficients has nontrivial rational zero.

Also proven for general cubic in ≥ 16 variables

Upshot: no number-theoretic obstruction when $h^{1,1}(X) > 15$ (likely 9)

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Cone obstructions: apparently exponentially suppressed

Simple heuristic argument:

Assume cone has $D = \sum_i d_i D_i, d_i \geq 0$

Look for positive solution of cubic $\sum_{i,j,k} C_{ijk} d_i d_j d_k = 0$

Proceed by induction:

First, check $M = 2, \sum_{i,j,k}^{M} C_{ijk} d_i d_j d_k = 0$
$\sim$ cubic in two variables, has ≥ 1 real solution; 50% chance in cone

Add one variable: pick random other numbers in cone; probability solution in last variable is positive: 1/2, ...

$\Rightarrow$ suggests probability $\leq \sim 2^{-h^{1,1}}$ that no fiber exists

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Strong evidence: almost all known CY3's have elliptic/g1 fibers

Supported by other recent work, particularly Anderson + Gray + collaborators

E.g. all CICY threefolds with $h^{1,1} > 4$ have g1/elliptic fibers
[Anderson, strings 2018 talk]

If most Calabi-Yau threefold are elliptic/g1 fibered
+ finite number of elliptic/g1 fibered CY threefolds
$\Rightarrow$ would prove finite number of CY threefolds!

Classification of elliptic/g1 CY threefolds $\Rightarrow$ CY3's,
non-fibered threefolds $\sim$ special cases

Note: all elliptic CY's connected by extremal transitions $\rightarrow\sim$ Reid's fantasy?

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Mirror symmetry

Mirror symmetry factorizes for many toric hypersurfaces!

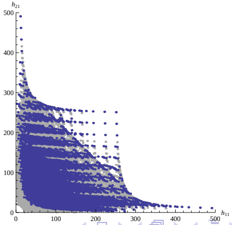
If $F = \nabla_2 \subset \nabla$ is a slice and $\tilde{F} = \Delta_2 \subset \Delta$ is also a slice
$\Rightarrow$ Mirror symmetry factorizes

Simplest factorization:
Standard stacking on $\mathbb{P}^{2,3,1} \leftrightarrow$ Tate form Weierstrass model

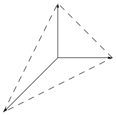
Mirror of generic elliptic fibration over B = ef over $\tilde{B}$ (may be tuned):

$$B \to \tilde{B} \sim \Sigma(-6K_B), \nabla_2 = \Delta_2 = \mathbb{P}^{2,3,1}$$

(65k examples in KS database)



Example: generic elliptic fibration on $\mathbb{P}^2$ (2, 272)



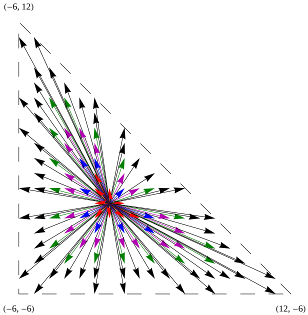
Hodge numbers (2, 272)

$$h^{1,1}(B) = 1$$

$$G = 1$$

$$h^{1,1}(X) = h^{1,1}(B) + \text{rk}\, G + 1 = 2$$

$$h^{2,1}(X) = 301 - 29h^{1,1}(B) - \text{dim}M_{nh} = 272$$



Hodge numbers (272, 2)

(toric rays: $\vec{w} \cdot \vec{v} \geq -6,\ \forall \vec{v} \in \Sigma_B, \vec{w}$ primitive)

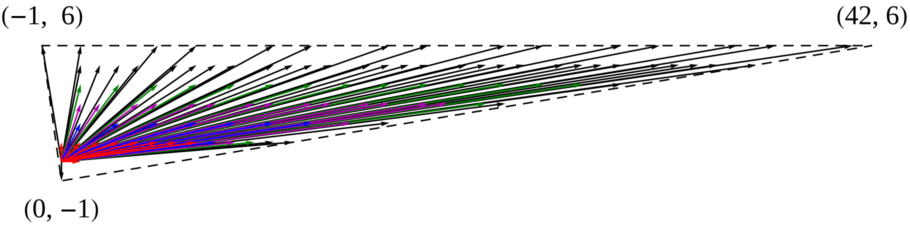
$$h^{1,1}(B) = 106 + 3 = 109$$

$$G = E_8{}^9 \times F_4{}^9 \times (G_2 \times SU(2))^{18}$$

$$h^{1,1}(X) = h^{1,1}(B) + \text{rk}\, G + 1 = 272$$

$$h^{2,1}(X) = 301 - 29h^{1,1}(B) + \dim G - \text{dim}M_{nh} = 2$$

Example: self-mirror generic elliptic fibration (251, 251)



Toric self-intersections:

$(0, 6, -12// -11// -12// -12// -12// -12// -12// -12// -12)$

$(// = (-1, -2, -2, -3, -1, -5, -1, -3, -2, -2, -1))$

$h^{1,1}(B) = 97 + 1 = 98$

$G = E_8{}^9 \times F_4{}^8 \times (G_2 \times SU(2))^{16}$ (rank = 152)

$h^{1,1}(X) = h^{1,1}(B) + \text{rk}\, G + 1 = 251$

$h^{2,1}(X) = 301 - 29h^{1,1}(B) + \dim G - \dim M_{nh} = 251$

Factorized mirror symmetry: more general structures

- Also works for "tuned" Tate models $\leftrightarrow$ reduction on Δ
- Works for other fibers, bundle structures

e.g. $B = \mathbb{P}^2, F = F_2$; base stacked over vertex: $H = (4, 94)$
$\tilde{B} \sim -2K_B, \tilde{F} = F_{15}; H = (94, 4)$
$(\tilde{B} : (-3, -1, -4, -1, -4, -1, -3, -1, -4, -1, -4, -1, -3, -1, -4, -1, -4, -1))$

$B = \mathbb{P}^2$

[mirror symmetry of fibers:
discussed in Klevers/Mayorga Pena/Oehlmann/Piragua/Reuter]

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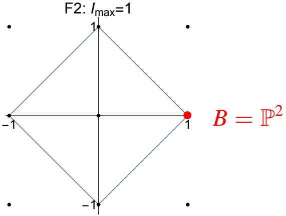

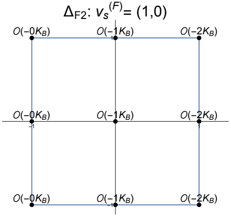

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Factorize mirror symmetry for CY fourfolds

Same story for fourfolds:

$F = F_i \to \tilde{F} = F_{17-i}$,
$B \to \tilde{B} \sim -nK_B$ for vertex stacking,

Example: $B = \mathbb{P}^3$ standard stacking ($F = \mathbb{P}^{2,3,1} = F_{10}$)

Rays in $\tilde{B}$: primitive lattice points in tetrahedron:
w/vertices (-6, -6, -6), (18, -6, -6), (-6, 18, -6), (-6, -6, 18)

$G = E_8^{34} \times F_4^{96} \times G_2^{256} \times SU(2)^{384}$

- (Exponentially) many triangulations
- Note: common endpoint from random blow-up sequence [WT/Wang]

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3. The 6D F-theory landscape and the swampland

Primary results on 6D landscape

- F-theory provides global picture of 6D landscape
- Essentially one theory with many connected branches;
  No potential $\rightarrow$ one big moduli space
- Unbroken gauge symmetries, matter are generic features
  (in 4D as well as 6D!)
- Generic structure:
  Many group + matter "clusters" interacting only gravitationally
  (suggestive for dark matter in 4D)
- F-theory geometry ties closely to 6D anomaly constraints
- Gauge groups, matter strongly constrained in F-theory

Global picture of 6D and 4D F-theory landscape:
Story parallel in many ways

– Compactify on elliptic Calabi-Yau fourfold, base B_3 = complex threefold

– Empirical data suggest similar structure (though less complete for CY4's)

4D theories significantly more subtle:
- Minimal models (Mori theory) more subtle
- Fluxes, superpotential, seven-brane dynamics not completely understood
  e.g. T-branes [Cecotti/Cordova/Heckman/Vafa, Anderson/Heckman/Katz, . . .]

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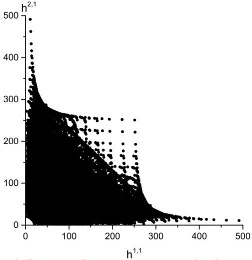

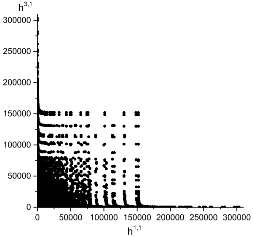

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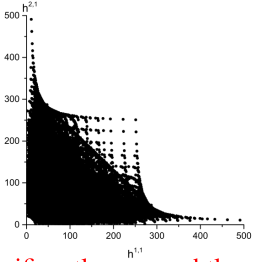

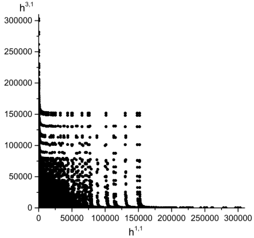

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Overview of 6D, 4D landscape

- Geometric gauge group dominated by $E_8, F_4, G_2, SU(2)$ factors
  (may be broken by fluxes in 4D)
  SU(5) for example requires fine tuning
- Many separate clusters, connected only by gravity (dark matter?)

Considering only toric threefold bases, many possibilities

Explicit constructions: $\sim 10^{755}$ bases [Halverson/Long/Sung]

Monte Carlo analysis: $>\sim 10^{3000}$ bases [WT/Wang]

These include (4, 6) codimension 2 SCFT sectors

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Physics: F-theory flux vacua (w/ Y. Wang)

Can we identify the F-theory geometry with most flux vacua?

Conventional wisdom (Ashok-Denef-Douglas): $\Rightarrow$ in regime $h^{1,1} \ll h^{3,1}$

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$N(\mathcal{M}_{\max}) \sim 10^{272,000}$ non-Higgsable $G_{\max} = E_8^9 \times F_4^8 \times (G_2 \times SU(2))^8$

Circumstantial evidence: $\sum_{X \neq \mathcal{M}_{\max}} N(X) < 10^{-3000} N(\mathcal{M}_{\max})$

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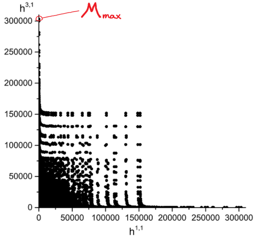

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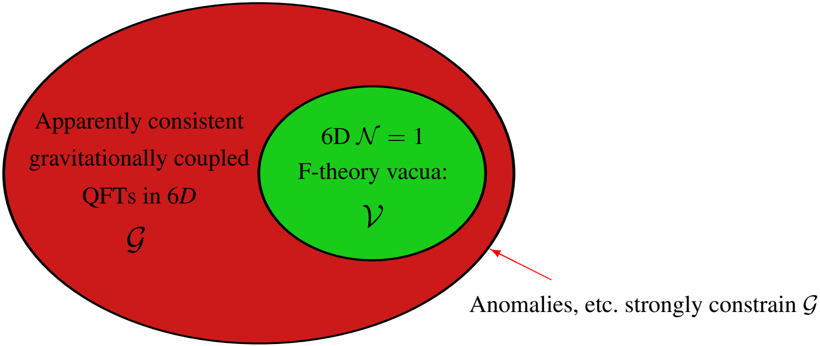
Goal: Map the landscape and swampland for 6D $\mathcal{N} = 1$ SUGRA



Program: systematically analyze $\mathcal{G}$ for 6D $\mathcal{N} = 1$ SUGRA

Find "swampland" of apparently consistent theories w/o F-theory realization

If $x \in \mathcal{G} \setminus \mathcal{V}$, must indicate one of $\begin{cases} \text{a) new string construction: } \mathcal{V}_* \supset \mathcal{V} \\ \text{b) new low-E constraint: } \mathcal{G}_* \subset \mathcal{G} \\ \text{c) true stringy constraint } \mathcal{V}_* \subset \mathcal{G}_* \end{cases}$

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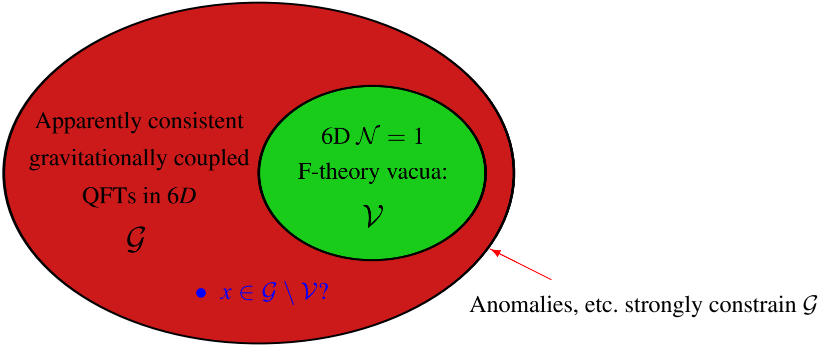

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4. Matter and representations

F-theory only realizes restricted types of matter representations

- Certain matter fields are *generic* in a well-defined way
  — other exotic matter fields realized through more complicated singularities
  — still other matter representations may go beyond conventional F-theory
  — other representations are simply disallowed

- Infinite swampland still for U(1) charges

- Many conclusions generalize to 4D
  Standard model matter is *generic* if $G = SU(3) \times SU(2) \times U(1)/\mathbb{Z}_6$!

Generic matter

In 6D, we define *generic matter* as the set of matter representations that arise on the moduli space branch of highest dimension, for a given gauge group and anomaly coefficients

• Examples: U(1) $\rightarrow$ generic matter $q = 1, 2$
SU(2) $\rightarrow$ generic matter fundamental + adjoint

• For simple groups, $SU(N) \times U(1)$, etc., generic matter fields match anomaly conditions, uniquely determined

• For groups with more factors (e.g. $U(1)^3$), generic matter not unique

• Generic matter is produced by the simplest F-theory tunings, simplest codimension two singularities given G

e.g. Tate tuning [Bershadsky et al., Katz/Morrison/Schafer-Nameki/Sully]

$$y^2 + a_1 yx + a_3 y = x^3 + a_2 x^2 + a_4 x + a_6$$

fix vanishing orders of $a_n \rightarrow SU(N)$ gives generic matter

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Generic Matter for $\mathrm{SU}(N_1) \times \cdots \times \mathrm{SU}(N_r) \times \mathrm{U}(1)^s$, $s \leq 3$

Representations	Number	Representations	Number
$\square_0$	r	$\mathbf{1}_0$	1
$\begin{smallmatrix}\square\\\square\end{smallmatrix}_0$	$r_{\geq 4}$	$\mathbf{1}_1$	s
Adj_0	r	$\mathbf{1}_2$	s
$(\square, \overline{\square})_0$	$\binom{r}{2}$	$\mathbf{1}_{(1,1)}$	$\binom{s}{2}$
$\square_1$	rs	$\mathbf{1}_{(1,-1)}$	$\binom{s}{2}$
$\square_{-1}$	$(r_3 + r_{\geq 4})s$	$\mathbf{1}_{(2,-1)}$	$2\binom{s}{2}$
$\square_{(1,1)}$	$r\binom{s}{2}$	$\mathbf{1}_{(1,1,-1)}$	$3\binom{s}{3}$

6D Anomaly Cancellation

Gravitational

$$273 = H - V + 29T$$

$$a \cdot a = 9 - T$$

Nonabelian

$$a \cdot b_\kappa = -\frac{\lambda_\kappa}{6}\left(\sum_{R_\kappa} x_{R_\kappa} A_{R_\kappa} - A_{\mathrm{Adj}_\kappa}\right)$$

$$0 = \sum_{R_\kappa} x_{R_\kappa} B_{R_\kappa} - B_{\mathrm{Adj}_\kappa}$$

$$b_\kappa \cdot b_\kappa = \frac{\lambda_\kappa^2}{3}\left(\sum_{R_\kappa} x_{R_\kappa} C_{R_\kappa} - C_{\mathrm{Adj}_\kappa}\right)$$

$$b_\kappa \cdot b_\mu = \lambda_\kappa \lambda_\mu \sum_{R_\kappa, R_\mu} x_{R_\kappa, R_\mu} A_{R_\kappa} A_{R_\mu}$$

Mixed

$$b_\kappa \cdot b_{ij} = \lambda_\kappa \sum_{R_\kappa, q_i, q_j} x_{R_\kappa, q_i, q_j} A_{R_\kappa} q_i q_j$$

$$0 = \sum_{R_\kappa, q_i} x_{R_\kappa, q_i} E_{R_\kappa} q_i$$

Abelian

$$a \cdot b_{ij} = -\frac{1}{6} \sum_{q_i, q_j} x_{q_i, q_j} q_i q_j$$

$$3 b_{(ij} \cdot b_{k\ell)} = \sum_{q_i, q_j, q_k, q_\ell} x_{q_i, q_j, q_k, q_\ell} q_i q_j q_k q_\ell$$

Definitions

H = # of hypermultiplets

V = # of vector multiplets

T = # of tensor multiplets

x_R = # of hypers in rep R

$$a, b_\kappa, b_{ij} \in \Gamma^{1,T}$$

$$\lambda_\kappa = 2c_\kappa^\vee / A_{\mathrm{Adj}_\kappa}$$

$$\mathrm{tr}_R F^2 = A_R \, \mathrm{tr}\, F^2$$

$$\mathrm{tr}_R F^4 = B_R \, \mathrm{tr}\, F^4 + C_R (\mathrm{tr}\, F^2)^2$$

$$\mathrm{tr}_R F^3 = E_R \, \mathrm{tr}\, F^3$$

Exotic $SU(N)$ matter

Generic $SU(N)$ matter: from Katz-Vafa rank 1 enhancement at codimension 2:
$A_{N-1} \to A_N : \ydiagram{1}, \to D_N : \ydiagram{1,1}$

Exotic matter types in conventional F-theory

$$\ydiagram{1,1,1} : SU(6), SU(7), SU(8)$$

$$\ydiagram{2} : SU(N)$$

$$\ydiagram{3} : SU(2)$$

Organizing principle: $1 + \frac{1}{2}(a \cdot b + b \cdot b) = \sum(g_R = \frac{1}{12}(2C_R + B_R - A_R))$ [KPT]
(From anomalies; F-theory: arithmetic genus contribution of singular curve)

$g > 0$ realized by singularities over singular 7-branes
[Klevers/Morrison/Raghuram/WT]

Some possibility beyond conventional F-theory: $SU(2)\ydiagram{4}$?
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Exotic matter types in conventional F-theory

$\begin{array}{c}\square\\[-4pt]\square\\[-4pt]\square\end{array} : SU(6), SU(7), SU(8)$	$g = 0$
$\square\!\square : SU(N)$	$g = 1$
$\square\!\square\!\square : SU(2)$	$g = 3$

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U(1) models with charges $q_1, q_2, \ldots$

Compare, for nonabelian groups, $T < 9$:

— Finite number of anomaly-free spectra,

— Good fraction from F-theory, under relatively good control

— Some issues with exotic matter

U(1) anomaly conditions ($a, \tilde{b}$ anomaly coefficients for BRR, BFF)

$$\begin{aligned} -a \cdot \tilde{b} &= \frac{1}{6} \sum q_i^2 \\ \tilde{b} \cdot \tilde{b} &= \frac{1}{3} \sum q_i^4 \end{aligned}$$

For $T = 0$ models: very simple Diophantine equations

$$\begin{aligned} 18\tilde{b} &= \sum q_i^2 \\ 3\tilde{b}^2 &= \sum q_i^4 \end{aligned}$$

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Generic matter: $T = 0, q = 1, 2$

U(1) models:

$$\tilde{b}\left(24 - \tilde{b}\right) \times (\pm\mathbf{1}) + \frac{1}{4}\tilde{b}\left(\tilde{b} - 6\right) \times (\pm\mathbf{2})$$

where $6 \leq \tilde{b} \leq 24, \tilde{b} \in 2\mathbb{Z}$

Compare SU(2) models with fundamentals, ≥ 1 adjoint

$$2b(12 - b) \times \square + \frac{1}{2}(b-1)(b-2) \times \square\square$$

- 1-1 match, SU(2) $\to$ U(1) by Higgsing, $\tilde{b} = 2b$
- All U(1) models from Morrison-Park
- All SU(2) models from simple Tate/UFD construction on degree b

$\Rightarrow T = 0, q = 1, 2$ models have no swamp, $F = \{\text{consistent}\}$

Generic matter: $T = 0, q = 1, 2$

U(1) models:

$$\tilde{b}\left(24 - \tilde{b}\right) \times (\pm\mathbf{1}) + \frac{1}{4}\tilde{b}\left(\tilde{b} - 6\right) \times (\pm\mathbf{2})$$

where $6 \leq \tilde{b} \leq 24, \tilde{b} \in 2\mathbb{Z}$

Compare SU(2) models with fundamentals, ≥ 1 adjoint

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Matching for larger q?

No! Exist infinite families of anomaly-free solutions.

$$54 \times (\pm q) + 54 \times (\pm r) + 54 \times (\pm(q+r)), \quad \tilde{b} = 6\left(q^2 + qr + r^2\right), \quad q, r \in \mathbb{Z}$$

Another family:

$$54 \times (\pm a) + 54 \times (\pm b) + 54 \times (\pm c) + 54 \times (\pm d), \quad \tilde{b} = 12\left(m^2 - mn + n^2\right)^2$$

$$\begin{aligned} a &= m^2 - 2mn, \\ b &= 2mn - n^2, \\ c &= m^2 - n^2, \\ d &= 2\left(m^2 - mn + n^2\right). \end{aligned}$$

Asymptotics: $\tilde{b} \sim \sum q^2, \tilde{b}^2 \sim \sum q^4 \to \mathcal{O}(\tilde{b}^{(m-4)/2})$ w/ m distinct q's

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What is the largest U(1) charge in F-theory?

– Must be finite, since finite # F-theory models, elliptic CY3's

– No U(1) analogue of Kodaira bound ($-12a \geq Nb$ for SU(N)) on anomaly coefficient $\tilde{b}$.

Explicit Weierstrass models: only up to $q = 4$ [Raghuram]

Some suggestions: standard F-theory constructions bounded at $q = 6$

Indirect construction: [Raghuram/WT]

Tune Weierstrass $SU(5) \times SU(4)$ on genus 2 curve in $\mathbb{F}_3$,
matter content including (10, 4) + (5, 6) + (5, $\bar{4}$) hypermultiplets

Higgs on adjoint fields $\rightarrow U(1) \times U(1)$,
matter content including (3, 3) and (4, -3) charges

Higgs on (4, -3) $\rightarrow$ charge $q = 3q_1 + 4q_2 = 21$

$q = 21$ largest known possible U(1) charge. Bigger possible?

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Generic matter and 4D physics

- Dimension, anomaly arguments from 6D don't work in 4D.
  But generic F-theory constructions give same generic matter types!

- Standard model has non-generic matter for $G = SU(3) \times SU(2) \times U(1)$
  But matter is generic for $G = (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$!

Summary

- We have a good systematic handle on elliptic Calabi-Yau threefolds
  — Classify bases, tunings; most known CY3 elliptic, mirror factorizes
  — Open questions regarding codimension 2 fiber singularities (matter)

- Good global picture of 6D F-theory vacua.
  — Single connected moduli space
  — Multiple (SUSY) non-Higgsable gauge factors dominate
  — Nonabelian G, matter: limited swampland
  — Abelian $G = U(1)^k$, charged matter: infinite swampland

- Many 6D features have natural parallels in 4D.
  — Certain generic geometric gauge factors
  — Multiple clusters: dark matter?
  — Generic matter suggests $G = (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$

Congratulations Hirosi!

On the well-deserved
2018 Hamburg prize for theoretical physics