A Stringy Test of the Weak Gravity Conjecture

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Motivation

Quantum Gravity Conjectures testable in string theory
- in a quantitative manner! [Harlow,Ooguri'18]

- Check of swampland conjectures and sharper formulation
- Study manifestations of swampland conjectures in string geometry:
  (Why) Does mathematics know about QG?

This programme will relate various aspects of

- Kähler geometry of elliptic fibrations
- Arithmetic properties of modular forms
  Topological strings, mirror symmetry and BPS invariants
- Black hole physics/Swampland ideas

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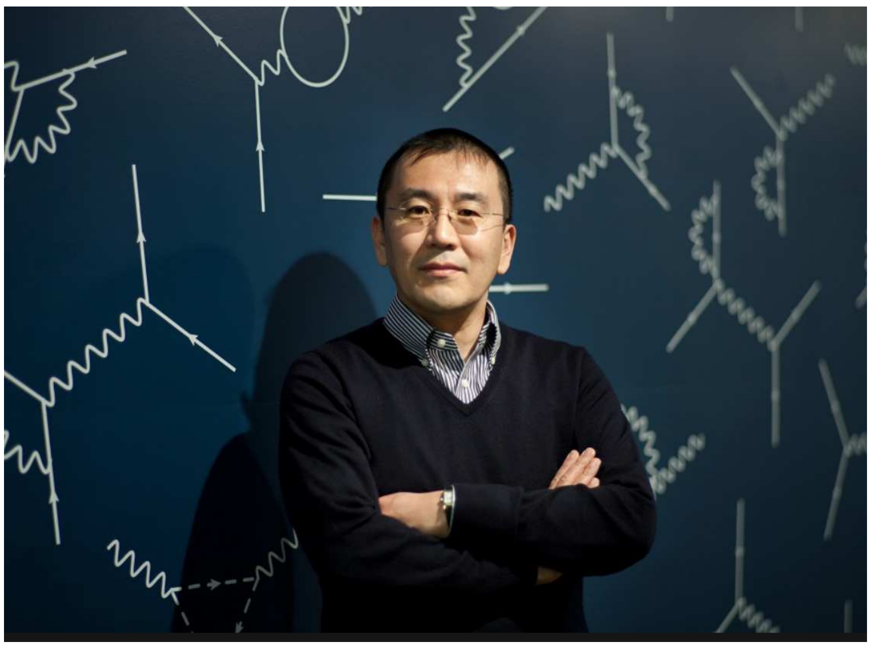
  (Why) Does mathematics know about QG?

This covers many aspects of the physics and math discussed by:



Motivation

Swampland/QG Conjectures in 'open string theories'

1. No Global Symmetries: [Banks,Dixon'88]; [Harlow,Ooguri'18]
   Gauge symmetries cannot become global in presence of gravity.
   What goes wrong if they (seemingly) do?

2. Completeness Conjecture: [Polchinski'03]
   The full charge lattice should be populated.
   Open string charges seem limited - where do the states come from?

3. Weak Gravity Conjecture: [Arkani-Hamed,Motl,Nicolis,Vafa'06]
   (Sub)Lattice of charged particles with $q^2 g_{\mathrm{YM}}^2 \geq \# M^2$
   What is the correct numerical bound and can it be tested?

A kit interpolating between these is the
Swampland Distance Conjecture [Ooguri,Vafa'06]!

Main Result

Consider most general F-theory compactification to 6d with 8 supercharges.

Whenever there exists a geometric limit where $g_{\mathrm{YM}} \to 0$ while M_{Pl} fixed, we

- prove the Sublattice Weak Gravity Conjecture
- including the effect of scalar fields
- and determine the index of the relevant sublattice of **non-BPS states**.

✓ Implies by duality similar result for 6d heterotic string (N=(1,0))

✓ Related to BPS state counting in 5d M-theory

F-theory in 6d

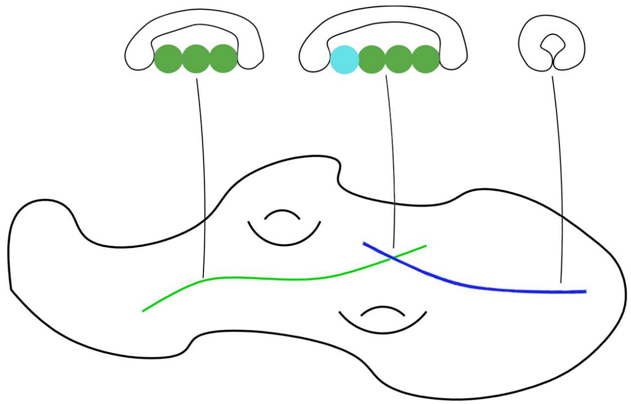
Context: F-theory compactified on elliptic CY$_3$ $Y_3 \to B_2$ [talk by W. Taylor]

Effective theory in $\mathbb{R}^{1,5}$:

$N = (1,0)$ supergravity (8 SUSYs)

base B_2: complex Kähler surface

7-branes on complex curve $C \subset B_2$



Couplings:

$\mathbf{M_{Pl}^4 = 4\pi \mathrm{vol}_J(B_2)}$ $\qquad$ $\frac{\mathbf{1}}{\mathbf{g^2_{YM}}} = \frac{\mathbf{1}}{\mathbf{2\pi}} \mathrm{vol}_{\mathbf{J}}(\mathbf{C})$

- non-abelian gauge algebra $\mathfrak{g}$:
  C contained in discriminant of fibration (wrapped by brane stack)
- abelian gauge algebra $\mathfrak{g} = \mathfrak{u}(1)_A$:
  $C = -\pi_*(\sigma(S_A) \cdot \sigma(S_A))$ (height pairing of rational section S_A)

Gravity and U(1)s

What happens if take $g_{\rm YM} \to 0$ at $M_{\rm Pl}$ finite?

Field theory intuition:

1. Weak Gravity Conjecture [Arkani-Hamed,Motl,Nicolis,Vafa'06], ...
   Gravity is weakest force.
2. In presence of gravity, no global symmetries. [Banks,Dixon'88], ...

General expectation: [Ooguri,Vafa'06], ... [talks by Valenzuela & Shiu]

✓ Offensive limit should be at infinite distance (beyond reach)
✓ Effective theory must break down (quantum gravity censorship)

[Kläwer,Palti'16] [Palti'17] [Grimm,Palti,Valenzuela'18] [Heidenreich,Reece,Rudelius'16/'18]

[Montero,Shiu,Soler'16] [Andriolo,Junghans,Noumi,Shiu'18] [Blumenhagen...'18] [Hebecker...15]

New: Quantum Gravity Conjectures and $U(1)$/gauge symmetries
in 'open string sector'

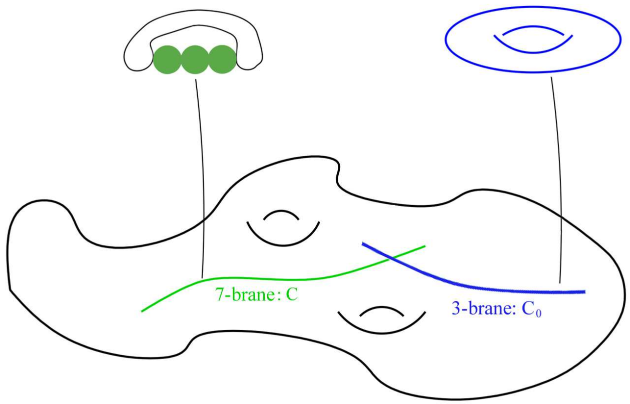
Summary of results

Main results: [Lee,Lerche,TW'18]

1. For fixed M_{Pl}, limit $g_{\mathrm{YM}} \to 0$ lies at infinite distance in Kähler moduli space of base B_2.

2. As $g_{\mathrm{YM}} \to 0$, necessarily **charged tensionless weakly coupled strings** arise in the 6d compactification.

   - **7-brane on C**
     $\mathrm{vol}_J(C) \sim t \sim \frac{1}{g_{\mathrm{YM}}^2} \to \infty$
   - **3-brane on curve C_0 with**
     $C_0 \cdot C \neq 0$
     $\mathrm{vol}_J(C_0) \sim \frac{1}{t} \to 0$



3. The charged tensionless strings imply a breakdown of the effective theory.

Summary of results

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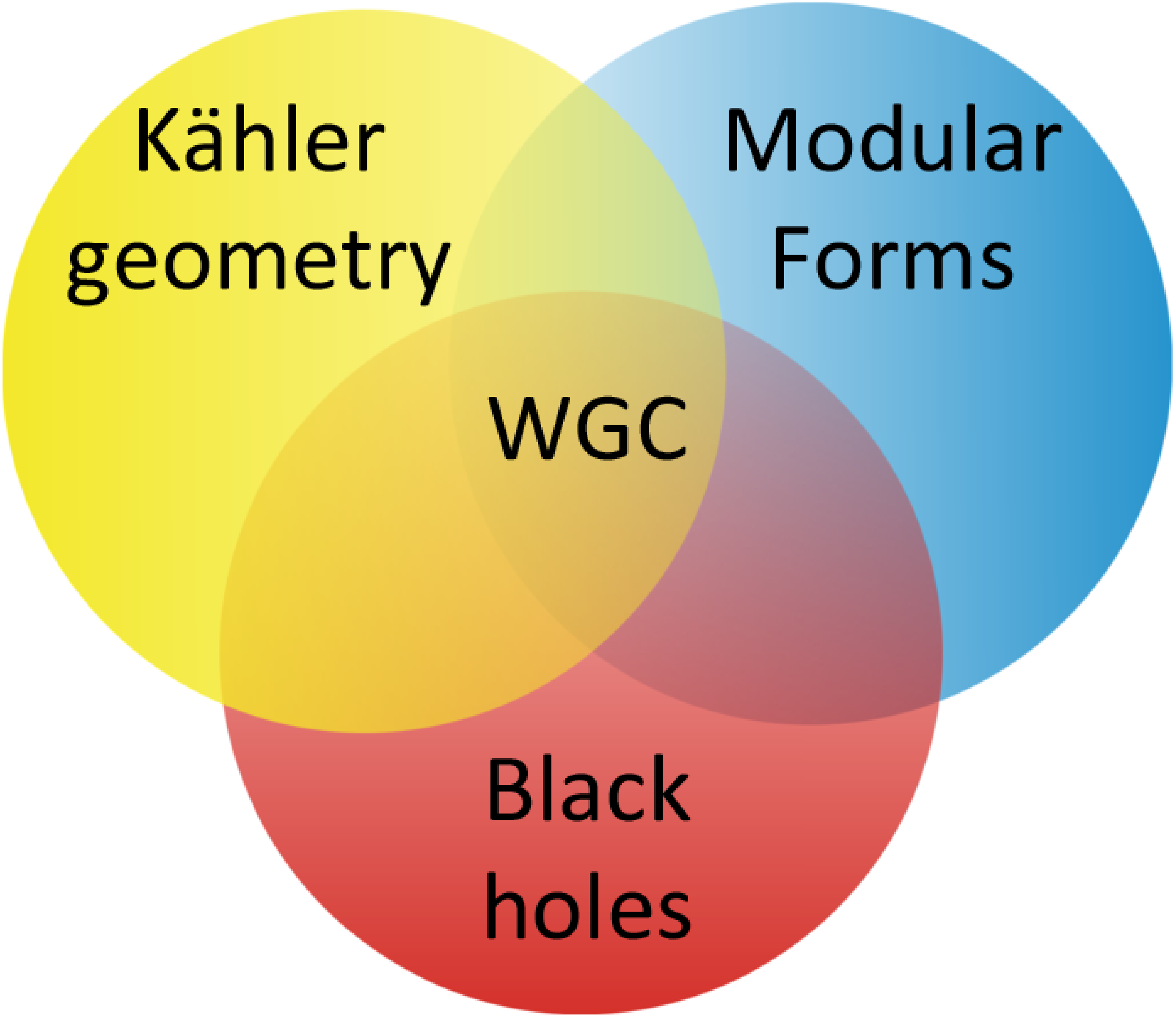
1. For fixed M_{Pl}, limit $g_{\mathrm{YM}} \to 0$ lies at infinite distance in Kähler moduli space of base B_2.
2. As $g_{\mathrm{YM}} \to 0$, necessarily charged tensionless strings arise in the 6d compactification.
3. The charged tensionless strings imply a breakdown of the effective theory.
4. The tensionless string is the critical 6d heterotic string and is weakly coupled in the tensionless limit.

   Its tower of particle excitations can be analyzed quantitatively:

   - Charged string excitations satisfy the Completeness Hypothesis.
   - A sublattice of charged states satisfies the Sublattice Weak Gravity Conjecture bound - including the effect of scalar fields!


Kähler geometry
Modular Forms
WGC
Black holes

I) Weak Coupling Limit
&
Kähler geometry

Global limit in Kähler geometry

[Lee,Lerche,TW'18]

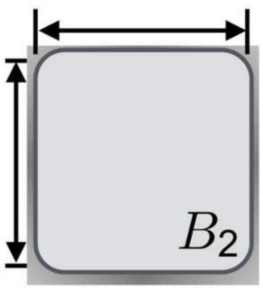
Aim: $\frac{1}{g_{\rm YM}^2} \sim \mathrm{vol}_J(C) \to \infty$ while $M_{\rm Pl} \sim \mathrm{vol}_J(B_2) \equiv 1 \quad (*)$

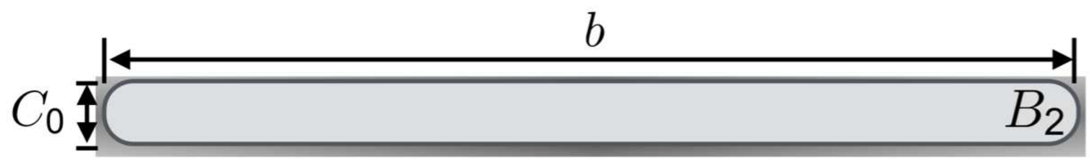
Result: There must exist another curve C_0 with

$$C_0 \cdot C \neq 0 \qquad \text{and} \qquad \mathrm{vol}_J(C_0) \to 0$$

General intuition

"On finite volume surface, if one direction gets big, normal direction must get very small".



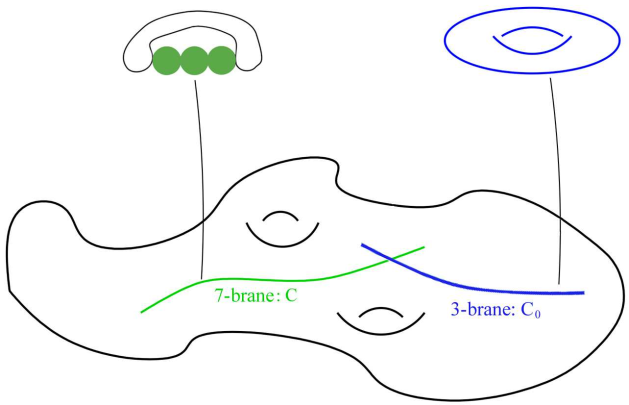


Global limit in Kähler geometry

$$\frac{1}{g_{\mathrm{YM}}^2} \sim \mathrm{vol}_J(C) \to \infty \quad \text{while} \qquad M_{\mathrm{Pl}} \sim \mathrm{vol}_J(B_2) \equiv 1 \quad (*)$$

May not be possible to take for given B_2, but if it can be taken, then it is always of the following form:

- **7-brane on C**
  $\mathrm{vol}_J(C) \sim t \sim \frac{1}{g_{\mathrm{YM}}^2} \to \infty$
- **curve C_0 with $C_0 \cdot C \neq 0$**
  $\mathrm{vol}_J(C_0) \sim \frac{1}{t} \to 0$



Quantum Gravity Conjectures

1) No global symmetries.

$\Rightarrow$ *The limit must be at infinite distance in moduli space.*

Indeed this is the case here.

Result: Limit $t \to \infty$ at distance $\Delta \sim \log(t) \to \infty$

2) Swampland Distance Conjecture:

[Ooguri,Vafa'06] [Kläwer,Palti'16] [Palti'17] [Heidenreich,Reece,Rudelius'17,'18]

[Grimm,Palti,Valenzuela'18] [Andriolo,Junghans,Noumi,Shiu'18]

[Blumenhagen,Kläwer,Schlechter,Wolf'18]

Infinitely many (charged!) states should become massless at exponential rate.

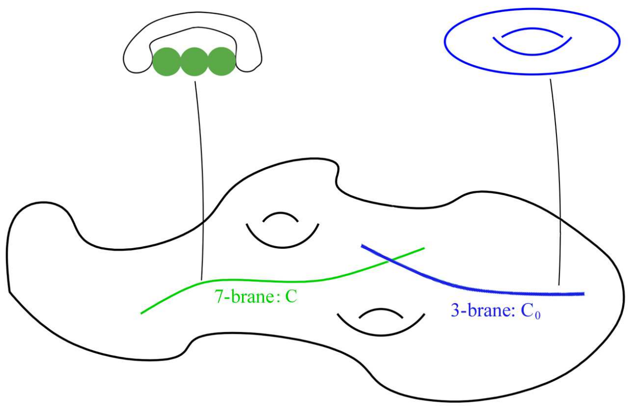
Present case: **Tensionless strings**

$$\Delta \sim \log(t), \qquad \textbf{tension} \sim T \sim \mathrm{vol}_J(C_0) \sim \frac{1}{t} \sim e^{-\Delta}$$

II) Tensionless Strings, Modular Forms & BPS Invariants

Tensionless strings

- 7-brane on C
- D3-brane on $\mathbb{P}^1$ C_0 with $C_0 \cdot C \neq 0$ and $C_0^2 = 0$
  $\Rightarrow$ effective string in 6d



Twisted reduction of $N = 4$ SYM with varying gauge coupling along C_0

[Martucci'14][Haghighat,Murthy,Vafa,Vandoren'15][Lawrie,Schafer-Nameki,TW'16]

- 2d $N = (0, 4)$ effective theory describes worldsheet theory
- At intersection $C_0 \cap C$:
  isolated 3-7 string modes **charged** under 7-brane gauge group

Analysis of worldsheet theory on $C_0 = \mathbb{P}^1$ with $C_0^2 = 0$:

The effective string is <u>the</u> critical heterotic string propagating in 6d.

Tensionless 6d strings

Case 1)

F-theory base B_2 is Hirzebruch

$$\begin{array}{rcccc} p: & \mathbb{P}^1_f = C_0 & \rightarrow & \mathbb{F}_a \\ & & & \downarrow \\ & & & \mathbb{P}^1_b \end{array}$$

Exists perturbative heterotic dual on K3 $\mathcal{K}$

$$\begin{array}{rcccc} r: & T^2 & \rightarrow & \mathcal{K} \\ & & & \downarrow \\ & & & \mathbb{P}^1_b \end{array}$$

$$(g_s^h)^2 = \frac{\mathrm{vol}_J(\mathbb{P}^1_f)}{\mathrm{vol}_J(\mathbb{P}^1_b)} \rightarrow 0 \quad \text{in tensionless limit}$$

Case 2) General base B_2

- Dual heterotic string on $\mathcal{K}$ is in general not perturbative due to presence of extra NS 5-branes on heterotic side
- Working assumption: Away from these defects heterotic string can be treated 'quasi-perturbatively' (justified in examples)

The elliptic genus

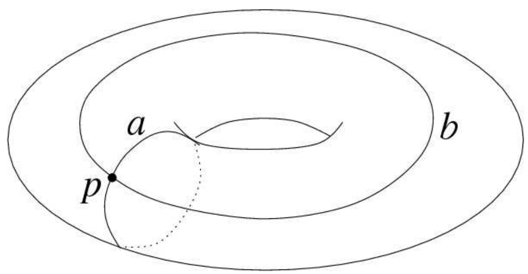
Elliptic genus of 6d het. string $\leftrightarrow$ Subsector of charged non-BPS states

[Schellekens,Warner'87] [Witten'87], ...

$$Z_{\mathcal{K}}(\tau, z) \equiv \mathrm{Tr}_R \left[(-1)^F F^2 q^{H_L} \bar{q}^{H_R} \xi^J \right]$$

$q = e^{2\pi i \tau}$: τ complex structure of worldsheet T^2

$\xi^J = e^{2\pi i z J}$: fugacity w.r.t. flavour symmetry - e.g. $U(1)$



$$Z_{\mathcal{K}}(\tau, z) = q^{-1} \sum_{n \geq 0} N(n, r)\, q^n\, \xi^r$$

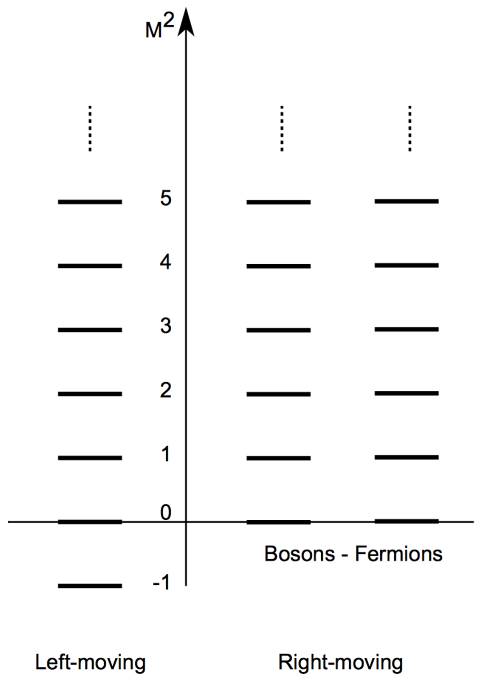
n: excitation level of string r: U(1) charge

The elliptic genus

$$Z_{\mathcal{K}}(\tau, z) = \mathrm{Tr}_R \left[(-1)^F F^2 q^{H_L} \bar{q}^{H_R} \xi^J\right] = q^{-1} \sum_{n \geq 0} N(n, r)\, q^n\, \xi^r$$

- only trace of left-movers
- Level-matched (!) physical states by pairing with right-movers

  $\Rightarrow$ Subsector of particle excitations of the 6d string

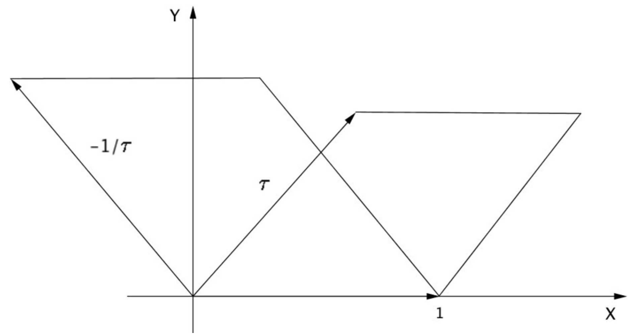


If (dual) heterotic string is perturbative, can use classic results on CFT of heterotic string [Schellekens,Warner'87]

Otherwise use intriguing duality to M-theory and BPS invariants as pioneered in [Klemm,Mayr,Vafa'96]

The elliptic genus

Ell. genus must behave well under $\mathrm{SL}(2,\mathbb{Z})$ trafo of string worldsheet



(Ratio of) **(quasi-)modular forms** of weight -2 and certain $U(1)$ fugacity index m

$$
\begin{aligned}
\varphi_{\mathbf{w},\mathbf{m}}\left(\frac{a\tau+b}{c\tau+d},\frac{\zeta}{c\tau+d}\right) &= (c\tau+d)^{\mathbf{w}} e^{2\pi i\frac{\mathbf{m}\, c}{c\tau+d}\frac{\zeta^2}{2}}\varphi_{\mathbf{w},\mathbf{m}}(\tau,\zeta) \\
\varphi_{\mathbf{w},\mathbf{m}}\left(\tau,\zeta+\lambda\tau+\mu\right) &= e^{-2\pi i\mathbf{m}(\frac{\lambda^2}{2}\tau+2\frac{\lambda\zeta}{2})}\varphi_{\mathbf{w},\mathbf{m}}(\tau,\zeta)
\end{aligned}
$$

- m determined by the geometry - model dependent!
- This suffices to fix structure of the charged subsector of excitations - as a 'function of m'

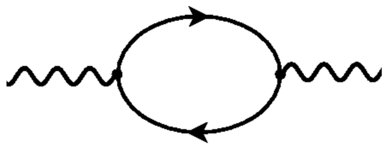
The elliptic genus

Fugacity index m determined by 't Hooft anomalies of string worldsheet

[Schellekens,Warner'86] [Benini,Eager,Hori,Tachikawa'13] [. . .]

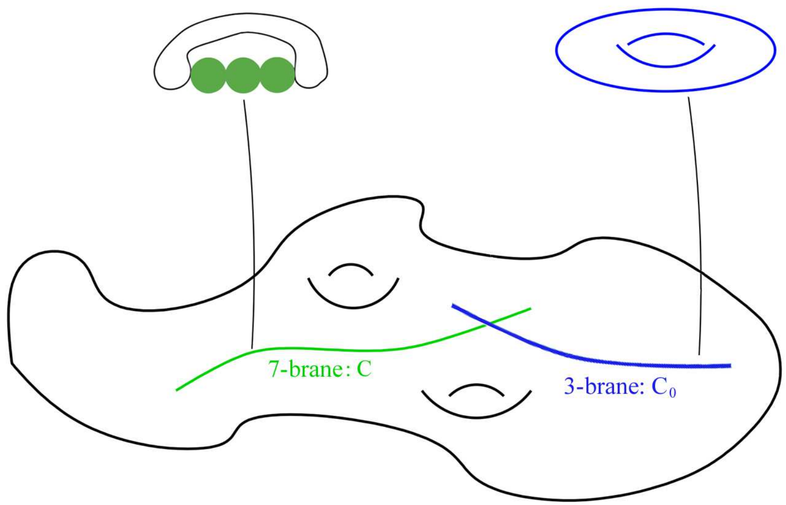
U(1) part of anomaly polynomial computed in [Xu,TW'17]

$$I_4 = -\frac{1}{2}F^2\, m + \ldots$$



with

$$\begin{aligned} m &= -\frac{1}{2}\pi_*(\sigma(S)\cdot\sigma(S))\cdot C \\ &= \frac{1}{2}C_0\cdot C \end{aligned}$$



The elliptic genus

$$Z_{\mathcal{K}}(\tau, z) = \frac{\Phi_{10,m}(\tau, z)}{\eta^{24}(\tau)} = q^{-1} \sum_{n \geq 0} N(n, r)\, q^n\, \xi^r$$

- $\eta^{24}(\tau)$ from zero-point energy
- $\Phi_{10,m}(\tau, z)$: (quasi-)modular Jacobi form of weight $w = 10$

  [Eichler,Zagier]

Every such form can be expanded

$$\begin{aligned} \varphi_{w,m}(\tau, z) &= \sum_{\ell \in \mathbb{Z} \bmod 2m} h_\ell(\tau) \Theta_{m,\ell}(\tau, z) \\ \Theta_{m,\ell}(\tau, z) &= \sum_{k \in \mathbb{Z}} q^{(\ell + 2mk)^2/4m}\, \xi^{\ell + 2mk} \end{aligned}$$

Subsector with $\ell = 0$ contains: $n = mk$ $\qquad$ $r = 2mk \equiv$ U(1) charge $\mathfrak{q}_k$

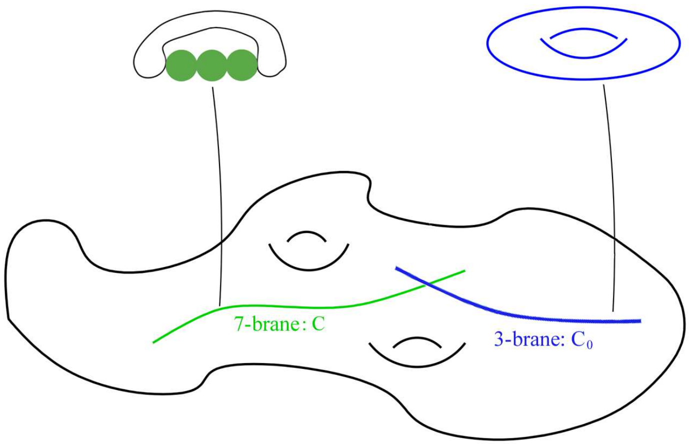
$\Longrightarrow$ sublattice of charge lattice with $\mathfrak{q}_k = 2m\,k\,, k \in \mathbb{Z}$

$$\mathfrak{q}_k^2 = 4m\, n(k)$$ [Lee,Lerche,TW'18]

The elliptic genus

Explicit computation possible via duality with M-theory [Klemm,Mayr,Vafa'96]

[Haghighat,Iqbal,Kozaz,Lockhart,Vafa'13] [Haghighat,Klemm,Lockhart,Vafa'14]



F-theory in $\mathbb{R}^{1,4} \times S^1$

BPS string wrapped on S^1

wrapping number w and

KK momentum k

M-theory in $\mathbb{R}^{1,4}$

BPS particle in 5d

M2 brane on

$wC_0 + k\mathbb{E}_\tau$

Relation: $w\,k = n$ (excitation number) [Klemm,Mayr,Vafa'96]

Index 6d of string non-BPS excitations

Gopakumar-Vafa invariants of 5d BPS states

The elliptic genus

Index 6d string non-BPS excitations

Gopakumar-Vafa invariants of 5d BPS states

$$Z_{\mathcal{K}}(\tau,z) = -q^{-1}\mathcal{F}^{(0)}_{C_0}(\tau,z) = -q^{-1}\sum N^{(0)}_{C_0}(n,r)\, q^n\, \xi^r$$

- $\mathcal{F}^{(0)}_{C_0}(\tau,z)$: genus zero prepotential of topological string
- $N^{(0)}_{C_0}(n,r)$: Gopakumar-Vafa (Gromov-Witten) invariants
  Computable via mirror symmetry for elliptic Y_3
  [Klemm,Mayr,Vafa'96][Klemm,Manschot,Wotschke'12][Huang,Katz,Klemm'15], ...

'Modular bootstrap'

- General Ansatz for quasi-modular form
  $\mathbb{C}[E_2(\tau), E_4(\tau), E_6(\tau); \varphi_{-2,1}(\tau,z), \varphi_{0,1}(\tau,z)]$ [Eichler,Zagier]
  of suitable weight and index
- Match free parameters against explicit BPS numbers via mirror symmetry

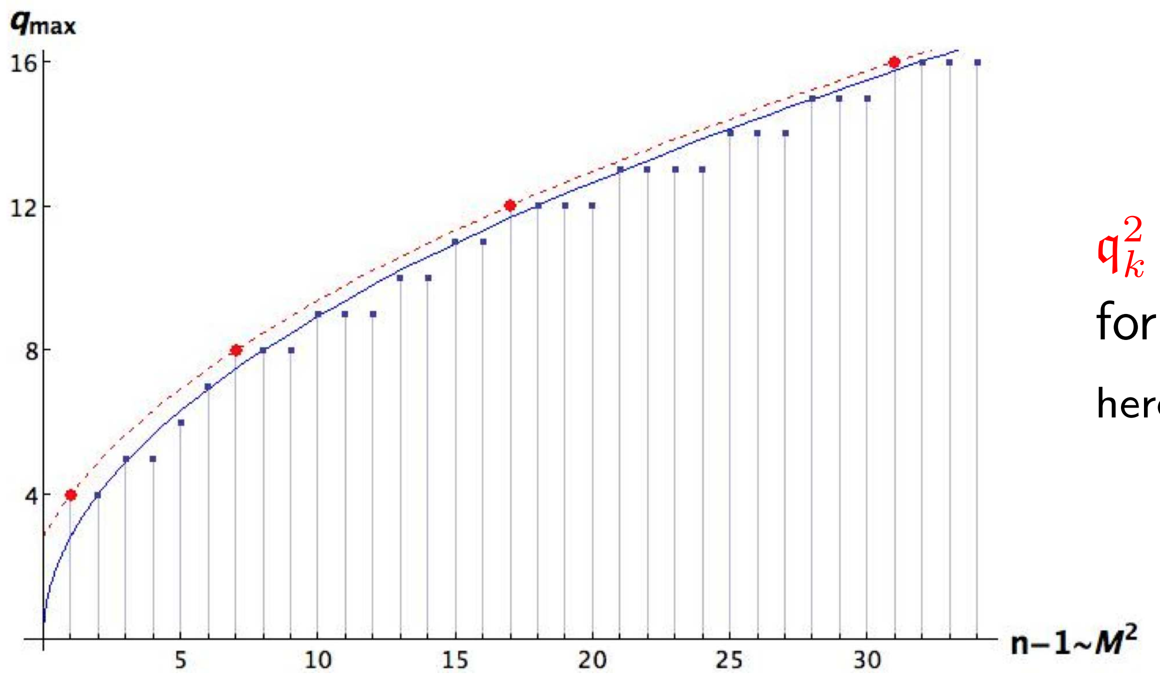
The elliptic genus

Example: $B_2 = dP_2$ [Lee,Lerche,TW'18]

topological type of $U(1)$ model with $m_A = 2$

$$
\begin{aligned}
\mathcal{F}^{(0)}_{C_0}(\tau,\xi) \quad = \quad & \frac{q}{1728\eta^{24}}\Big(-23E_4^2E_6\varphi_{-2,1}^2 + 27E_4^3\varphi_{-2,1}\varphi_{0,1} + 19E_6^2\varphi_{-2,1}\varphi_{0,1} - 23E_4E_6\varphi_{0,1}^2 \\
& +E_2(-E_6^2\varphi_{-2,1}^2 + 2E_4E_6\varphi_{-2,1}\varphi_{0,1} - E_4^2\varphi_{0,1}^2)\Big) \\
= \quad & -2 + \left(252 + 84\xi^{\pm1}\right) q \\
& + \left(116580 + 65164\xi^{\pm1} + 9448\xi^{\pm2} + 84\xi^{\pm3} - 2\xi^{\pm4}\right) q^2 \\
& + \left(6238536 + 3986964\xi^{\pm1} + 965232\xi^{\pm2} + 65164\xi^{\pm3} + 252\xi^{\pm4}\right) q^3 \\
& + \mathcal{O}(q^4)\,.
\end{aligned}
$$

The elliptic genus



$\mathfrak{q}_k^2 = 4m\, n(k)$

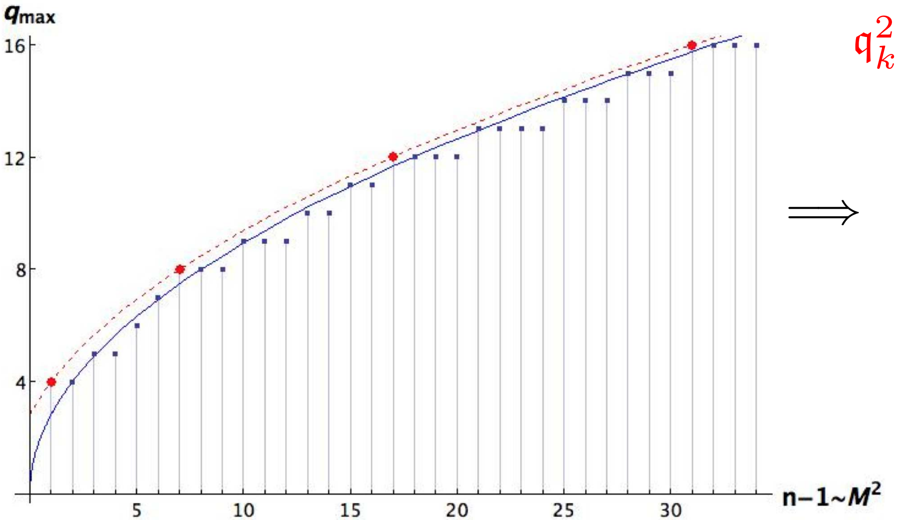
for $\mathfrak{q}_k = 2m\, k$

here: $m = 2$, on dP_2

$\Rightarrow$ **Completeness Hypothesis satisfied**:

- Each charge q is populated by some state
- Note: Not the case for purely open [p,q] string sector!

Scalar Weak Gravity Conjecture



$\mathfrak{q}_k^2 = 4m\,n(k)$ for $\mathfrak{q}_k = 2m\,k$

$$\Longrightarrow \quad \mathfrak{q}_k^2\, g_{\rm YM}^2 = \frac{M_n^2}{M_{\rm Pl}^4} + 4m\, g_{\rm YM}^2$$

$$> \frac{M_n^2}{M_{\rm Pl}^4} \quad \text{as} \quad t \to \infty$$

- Mass shell condition for perturbative string states:

$$M_n^2 = 8\pi T(n-1)$$

Crucial: Valid in pert. limit of het. string = tensionless limit $t \to \infty$

$$M_{\rm Pl}^4 = 4\pi\,{\rm vol}(B_2) \equiv 4\pi\,, \qquad \frac{1}{g_{\rm YM}^2} = \frac{1}{2\pi}{\rm vol}(C)$$

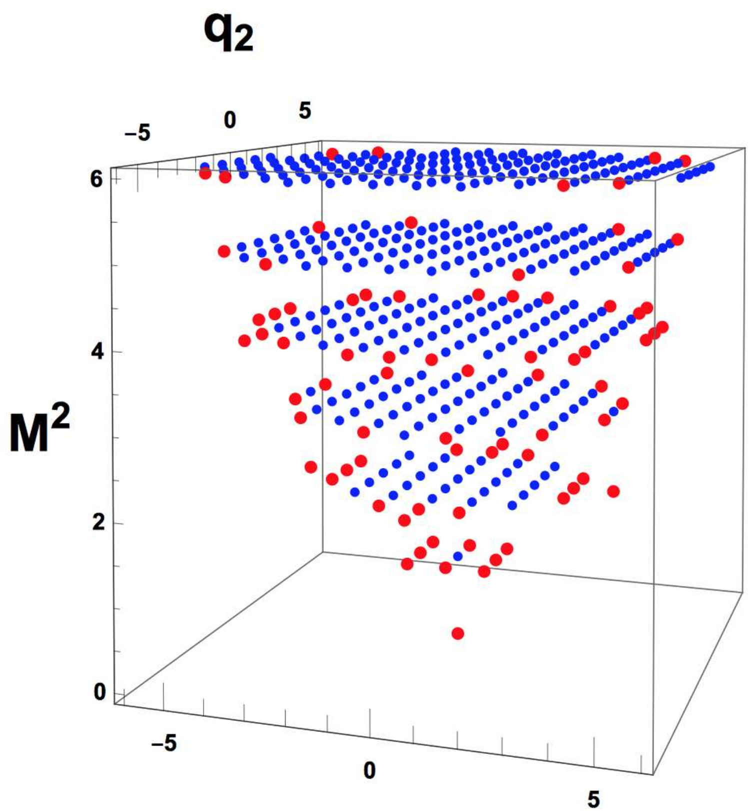
$$T = 2\pi\,{\rm vol}(C_0)\,.$$

with ${\rm vol}(C)\,{\rm vol}(C_0) \to 2m$ as $t \to \infty$

The elliptic genus

Generalisation to several U(1) factors surprisingly complicated: Theory of Lattice quasi-Jacobi forms

- No general generating system is published- active research [Gritsenko,Skoruppa,Zagier to appear]
- Novel examples in [Lee,Lerche,TW'18]



III) Black Hole physics
Scalar Weak Gravity Conjecture

Weak Gravity Conjecture

Black holes must be able to decay [AMNV'06]

$\Rightarrow$ There must exist a state which is 'superextremal' with respect to charged extremal black hole

$$\frac{\mathfrak{q}^2 g_{\mathrm{YM}}^2}{M^2}\Big|_{\mathrm{state}} \overset{!}{\geq} \frac{Q^2 g_{\mathrm{YM}}^2}{M^2}\Big|_{\mathrm{B.H.}}$$

1) Which particle(s) should satisfy it?

- **Sublattice Weak Gravity Conjecture**

  There must exist a sublattice of the charge lattice populated by states satisfying the WGC. [Heidenreich,Reece,Rudelius'15/6] [Montero,Shiu,Soler'16]

- Justification:

  This condition is stable under dimensional reduction.

Challenge for string theory: How coarse is the sublattice?

Weak Gravity Conjecture

$$\frac{\mathfrak{q}^2 g_{\rm YM}^2}{M^2}\Big|_{\rm state} \overset{!}{\geq} \frac{Q^2 g_{\rm YM}^2}{M^2}\Big|_{\rm B.H.}$$

2) Which black holes must be considered?

- In Einstein-Maxwell theory: Extremal Reissner-Nordstrøm black holes
- In SUGRA have extra massless scalar fields and BH solution will involve a profile for scalars

  $\Longrightarrow$ modification of $\frac{Q}{M}\big|_{\rm B.H.}$

Analytic solution for special case of dilatonic black hole: [Gibbons,Maeda'88]

- $S = \int \sqrt{-g}R + \frac{1}{2} d\phi \wedge * d\phi + \frac{1}{4 g_{\rm YM}^2} e^{\alpha\phi} F_{\mu\nu}F^{\mu\nu}$
- WGC bound for decay of (d=6) dilatonic RN black hole:

$$q^2 g_{\rm YM}^2 \overset{!}{\geq} \frac{M^2}{M_{\rm Pl}^{d-2}} \left(\frac{d-3}{d-2} + \frac{\alpha^2}{4} \right)$$ [Heidenreich,Reece,Rudelius'15]

Weak Gravity Conjecture

How relevant is the dilatonic case? [Lee,Lerche,TW'18]

Consider vicinity of weak coupling point $g_{\rm YM} \to 0$

- This must lie at infinite distance in moduli space.
- According to Swampland Distance Conjecture, there must appear a tower of exponentially massless fields

$$M^2(\phi) = M^2\, e^{-c\phi} \quad \text{for a canonically normalised scalar } \phi$$

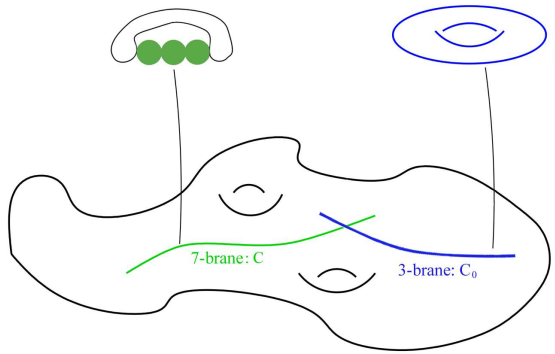
If WGC holds in vicinity of weak coupling point for this tower, then also $g_{\rm YM}(\phi)$ depends exponentially on same ϕ:

$$\frac{1}{g^2_{\rm YM}(\phi)} = \frac{1}{g^2_{\rm YM}} e^{c\phi}$$

$\Rightarrow$ Dilatonic case universally governs WGC near weak coupling point.

Weak Gravity Conjecture

This general prediction is confirmed in F-theory:



- Explicit analysis of 6d (1,0) SUGRA in special limit:

$$S = \int \sqrt{-g}R + \frac{1}{2}d\phi \wedge *d\phi + \frac{1}{4g_{\mathrm{YM}}^2}e^{\alpha\phi}F_{\mu\nu}F^{\mu\nu} \text{ with } \alpha = 1$$

- Resulting QGC bound:

$$q^2 g_{\mathrm{YM}}^2 \overset{!}{\geq} \frac{M^2}{M_{\mathrm{Pl}}^{d-2}}\left(\frac{d-3}{d-2} + \frac{\alpha^2}{4}\right) = \frac{M^2}{M_{\mathrm{Pl}}^{d-2}}$$

Scalar Weak Gravity Conjecture

An a priori different formulation:

Scalar Weak Gravity Conjecture [Palti'17]

There must exist WGC particle(s) such that

$$\text{Coulomb force} \overset{!}{\geq} \text{Gravitational force in d=6} + \text{Yukawa force}$$

$$q^2 g_{\mathrm{YM}}^2 \overset{!}{\geq} \frac{M^2}{M_{\mathrm{Pl}}^{d-2}} \left(\frac{d-3}{d-2} + \frac{1}{M^2} |\partial M|^2 \right)$$

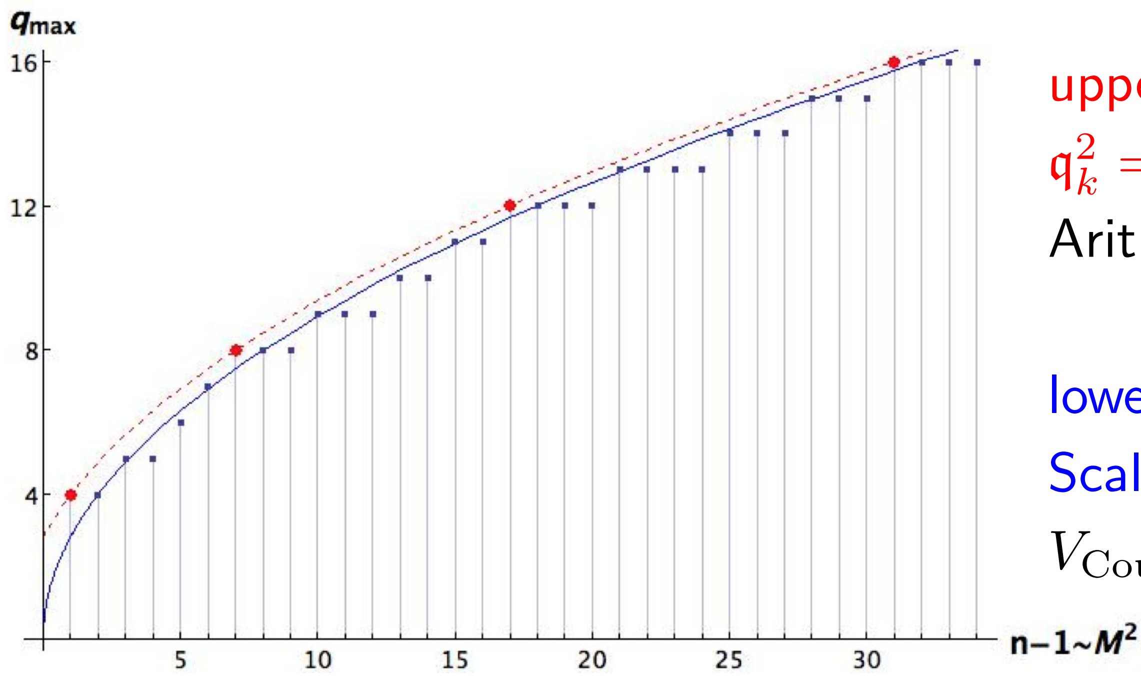
Near weak coupling point:

- $M^2(\phi) = M^2\, e^{-c\phi}$ for a canonically normalised scalar ϕ
- $\frac{1}{g_{\mathrm{YM}}^2(\phi)} = \frac{1}{g_{\mathrm{YM}}^2} e^{c\phi}$

$\Longrightarrow$ Gives same constraint as via decay of black hole!

$$q^2 g_{\mathrm{YM}}^2 \overset{!}{\geq} \frac{M^2}{M_{\mathrm{Pl}}^{d-2}} \left(\frac{d-3}{d-2} + \frac{\alpha^2}{4} \right) \quad \text{for} \quad \alpha = c$$

Scalar Weak Gravity Conjecture



upper line:

$\mathfrak{q}_k^2 = 4m\, n(k)$

Arithmetics of Jacobi forms

lower line:

Scalar WGC bound

$V_{\rm Coul} = V_{\rm Grav} + V_{\rm Yuk}$

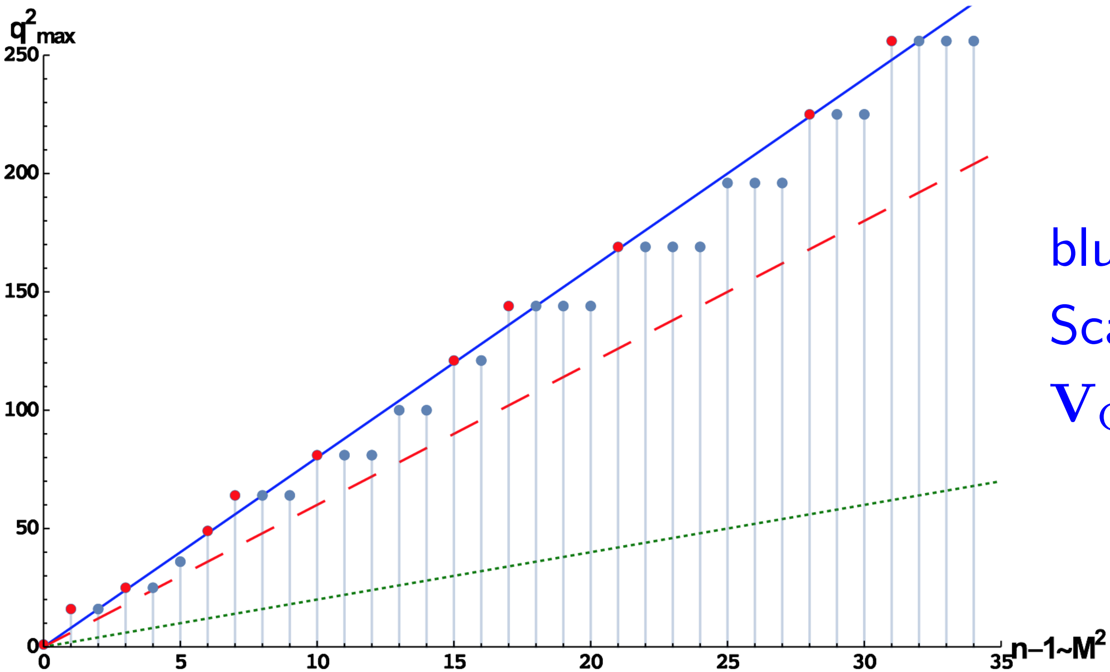
Our derivation valid in limit $g_{\rm YM} \to 0$ = tensionless limit $t \to \infty$

- Validity of $M_n^2 = 8\pi T(n-1)$
- $\mathrm{vol}(C)\mathrm{vol}(C_0) \to 2m$

What about general points in moduli space?

Corrections? talk by G. Shiu

Scalar Weak Gravity Conjecture



blue solid line:

Scalar WGC bound

$\mathbf{V}_{\rm Coul} = \mathbf{V}_{\rm Grav} + \mathbf{V}_{\rm Yuk}$

Our derivation valid in limit $g_{\rm YM} \to 0$ = tensionless limit $t \to \infty$

- Validity of $M_n^2 = 8\pi T(n-1)$
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What about general points in moduli space?

Corrections? talk by G. Shiu

Summary

Systematic study of Quantum Gravity constraints for 6d open string U(1)s

As gauge symmetry becomes global:

Tower of infinitely many charged from tensionless '6d critical heterotic string'

Different from behaviour of SCFT strings ($C^2 < 0$): Strongly coupled!

Beautiful interplay between geometry, arithmetics, CFT and Quantum Gravity constraints

No global symmetries	Kähler geometry of B_2
Swampland Distance Conjecture	Mori's cone theorem
Completeness Hypothesis	Weak Jacobi forms
Scalar Weak Gravity Conj.	and their modular properties

How do (Weak) Jacobi forms know about the Weak Gravity Conjectures?