The Superconformal Bootstrap

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I will be discussing aspects of a research program of the past ~5 years, undertaken with many collaborators

Frederico Bonetti
Madalena Lemos
Pedro Liendo
Carlo Meneghelli

Wolfger Peelaers
Leonardo Rastelli
Balt van Rees
...many more I haven't worked w/directly...

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The program has multiple strands –

this talk * ◦ Characterizing algebraic structures associated to various classes of SCFTs.

◦ Using these to constrain/classify spaces of SCFTs.

◦ Combining tools to "solve" to whatever extent possible specific models of interest.

Plan of this talk

- CFT and the conformal bootstrap
- Superconformal theories and their challenges
- Hidden mathematical structures
  - Associated VOAs
  - Superconformal deformation quantization
- Conclusions / future directions

Conformal Field Theory (CFT)

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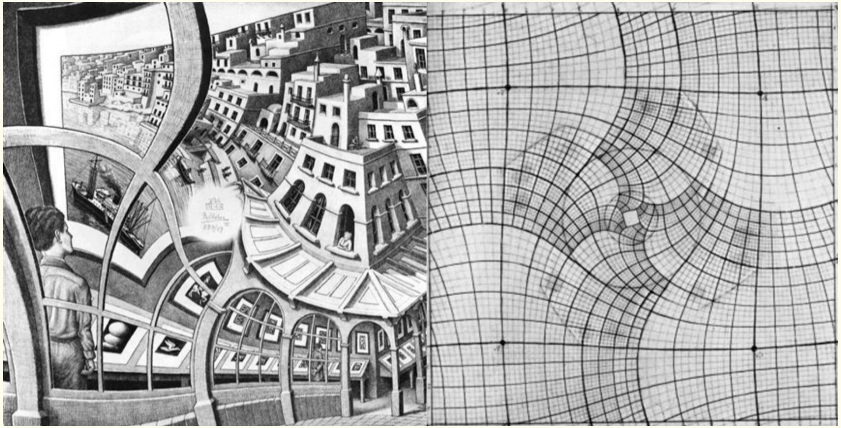
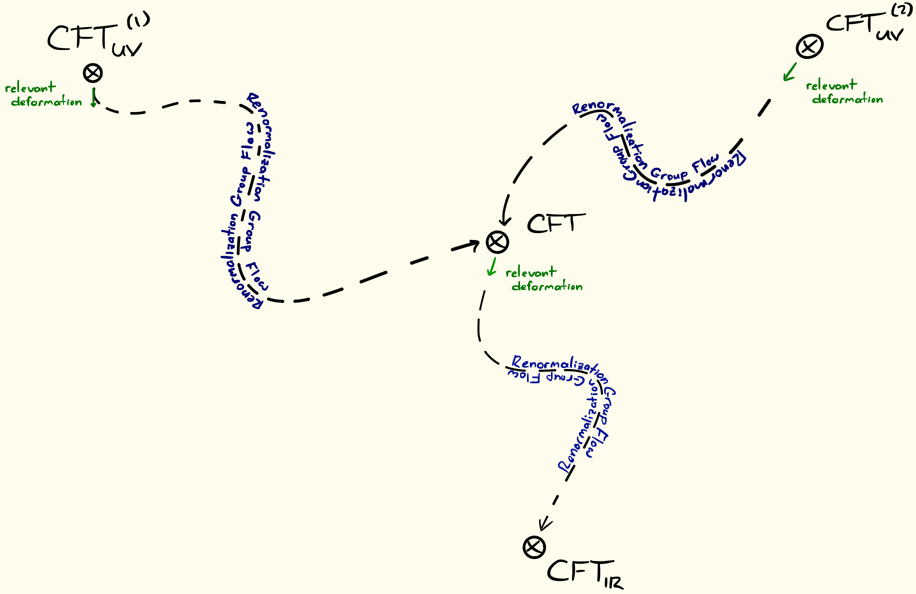


Conformal transformations act as local rotation + rescaling.

Conformal Field Theory (CFT)

CFTs punctuate the space of QFTs –



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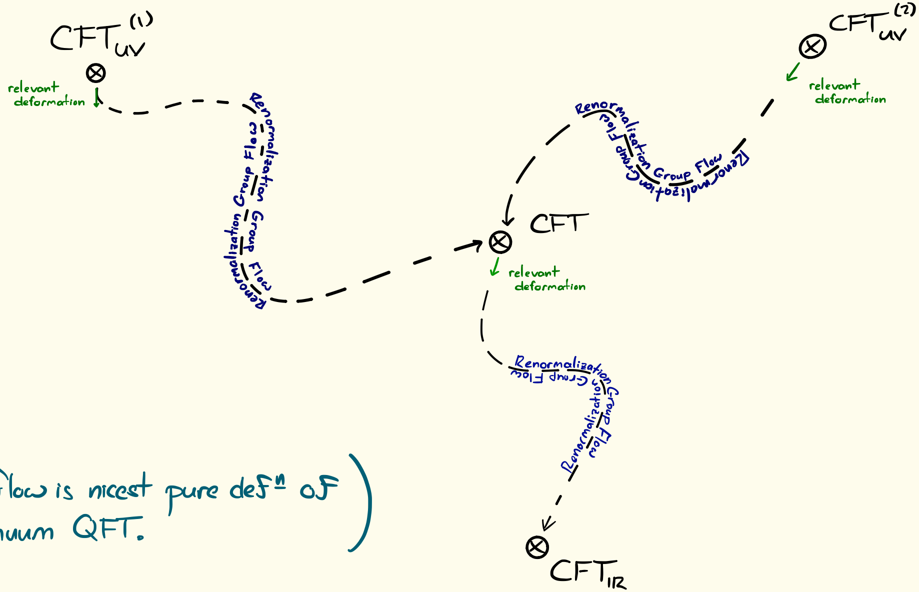

(UV ⟶ IR RG flow is nicest pure def^{n} of continuum QFT.)

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CFTs are generically strongly coupled when formulated via, say, a path integral —

$$\int [D\varphi] \exp\left(\frac{i}{\hbar}\int d^dx\, \mathcal{L}[\varphi]\right)$$

$$\mathcal{L}[\varphi] = \frac{1}{2}(\partial\varphi)^2 - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4 - \dots$$

Coupling constants all $O(1)$; hard to compute! (scale invariance obscured)

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This motivates us to seek an approach that leverages conformal invariance nonperturbatively

CFTs as Algebraic Structures

Local operators form a Hilbert space $\mathcal{H}[\mathbb{S}^{d-1}]$ due to conformal symmetry.

$$\left\{ \text{e.g., } \mathcal{L} = |\partial\varphi|^2 - V(\varphi) \text{ theory} : \varphi(x),\ \varphi^2(x),\ \ldots,\ \varphi \Box^k \partial_{\mu_1} \cdots \partial_{\mu_\ell} \varphi \right\}$$

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Product structure: $$\mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2) = \sum_k C_{12}{}^{k}(x_1 - x_2)\,\mathcal{O}_k(x_2)$$

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"Operator Product Expansion" (OPE)

converges in correlation functions

(in contrast to similar expansions w/out conformality)

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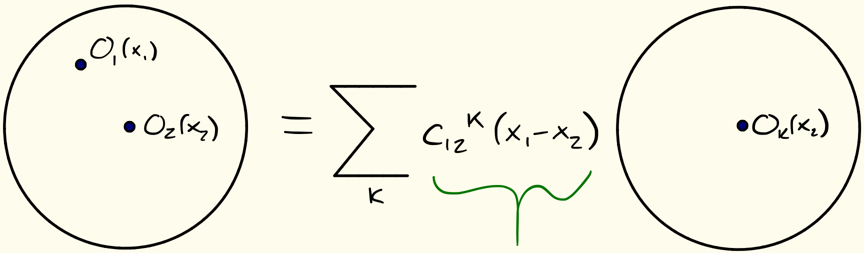

Determined by conformal symmetry
up to overall normalization, "OPE coefficients"

CFTs as Algebraic Structures

All correlation $f^{\underline{n}}$'s fixed by OPE, plus vanishing of one-point functions

$$\langle \mathcal{O}(x) \rangle_{\mathbb{R}^d} = 0 \quad \text{unless} \quad \mathcal{O}(x) \equiv \mathbb{1}$$

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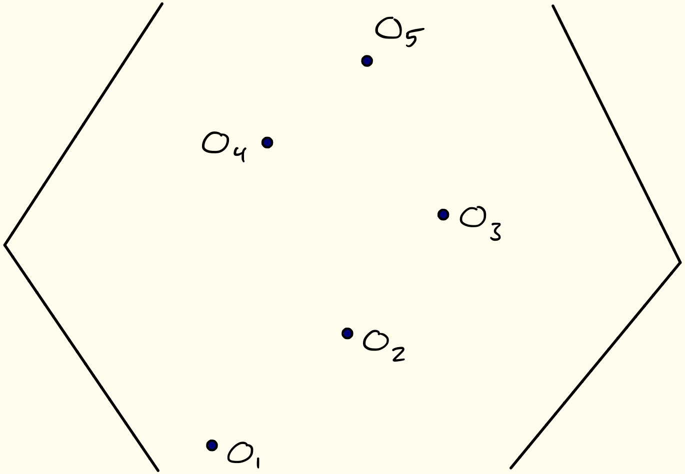

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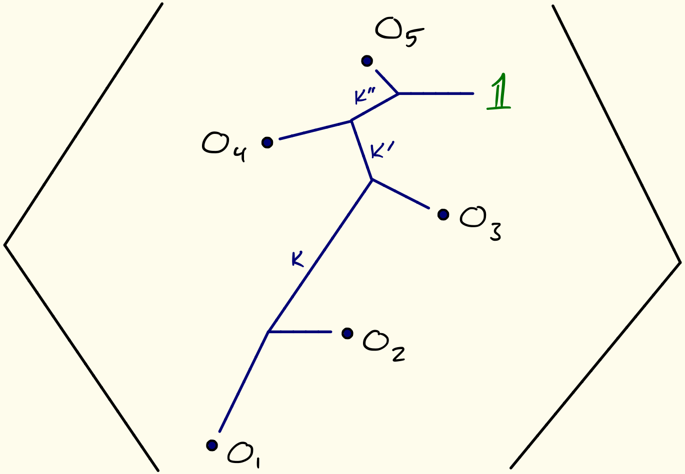
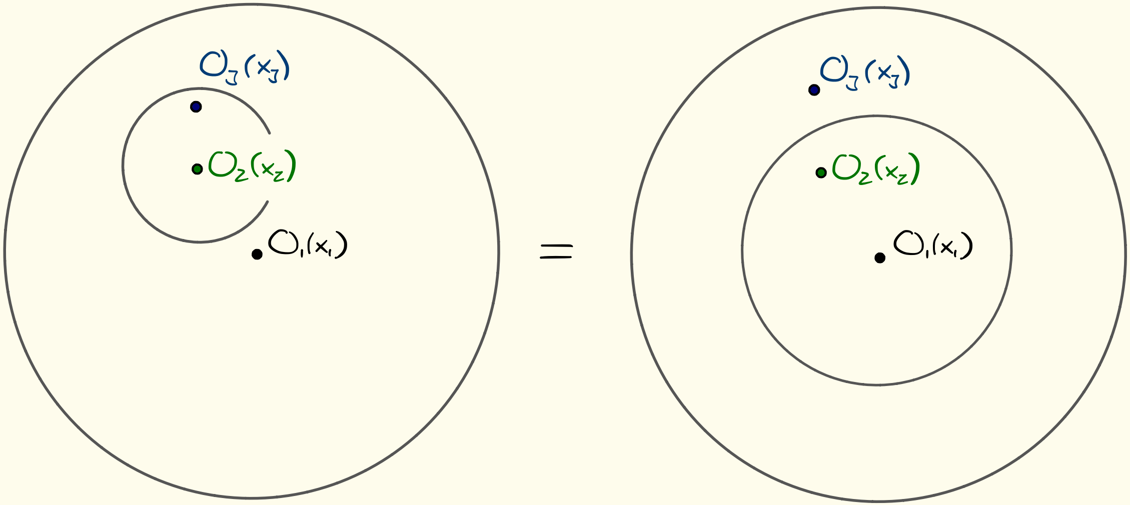


$$= \sum_{k,\, k',\, k''} C_{12}{}^{k}(x_{21})\, C_{k3}{}^{k'}(x_{31})\, C_{k'4}{}^{k''}(x_{41})\, C_{k''5}{}^{\mathbb{1}}(x_{51})$$

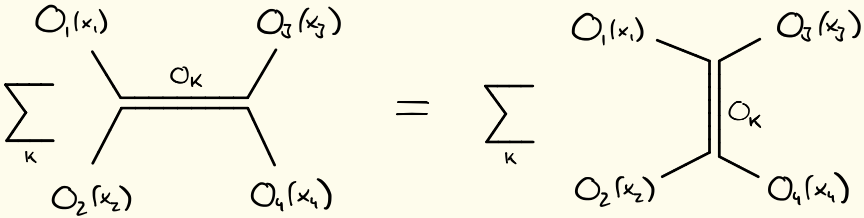
CFTs as Algebraic Structures

Self-consistency/well-definedness of correlators imposes associativity conditions on OPE —



The Bootstrap

It is enough to ensure consistency of 4-pt. functions: $\langle O_1(x_1) O_2(x_2) O_3(x_3) O_4(x_4) \rangle$



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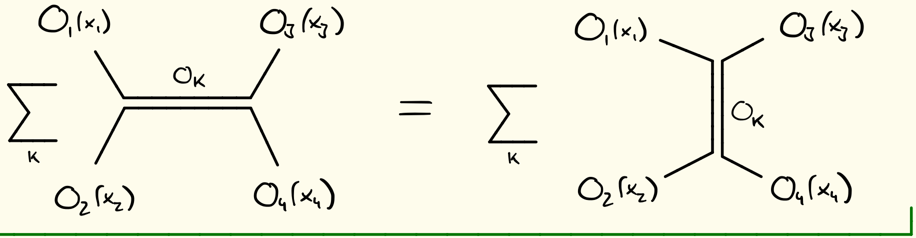

"Crossing Symmetry"

Old aspiration (circa 1970's: Ferrara-Gatto-Grillo, Polyakov):

"Solve CFTs by solving crossing symmetry"

(i.e., self-consistency should be enough)

Remarks on the Conformal Bootstrap

▷ Generally, an infinite set of non-linear, coupled functional equations in $\underbrace{\text{infinitely many unknowns}}_{\text{Operator scalings, spins, and OPE coefficients}}$.

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- Special situation when d = 2: $so(2,2) \subset \text{Virasoro} \times \overline{\text{Virasoro}}$

  infinite dimensional local conformal transformations

In this case, crossing can reduce to a finite system of equations, leading to notion of

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- In d > 2 spacetime dimensions, no such simplification possible, held up progress for a long time.

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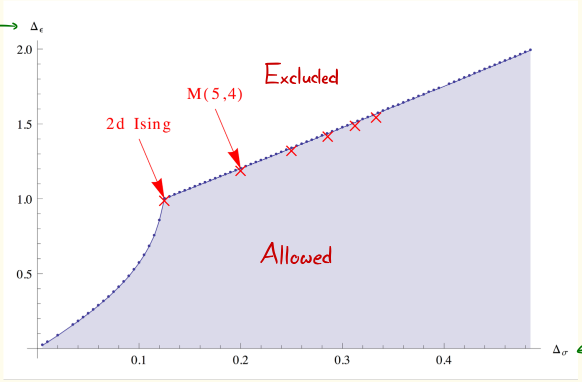

operator scaling dimension

- If a CFT correlator is "extremal" (saturates bounds) it can in principle be reconstructed from the same data that gave rise to the bound.

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$d=5$: $so(5,2) \longrightarrow f(4)$

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Extra fermionic spacetime symmetry generators $\{Q, S\}$, spacetime spinors.

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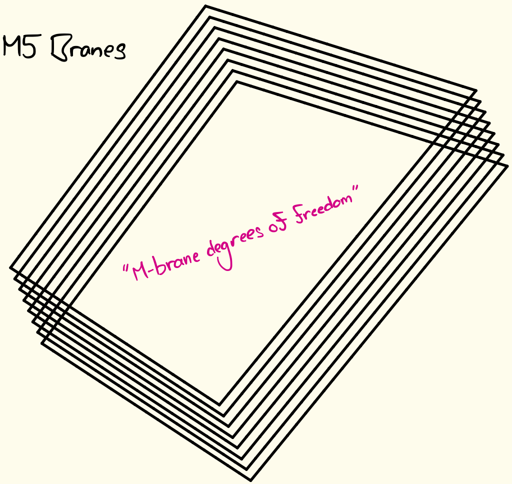
- Many SCFTs in $d=5$ & $d=6$ – predicted by string/M-theory – defy conventional wisdom!

Surprising Predictions from String/M-theory

N × M5 Branes



$E \ll M_{pl}^{(11d)}$

$(2,0)_N$ Superconformal Field Theory

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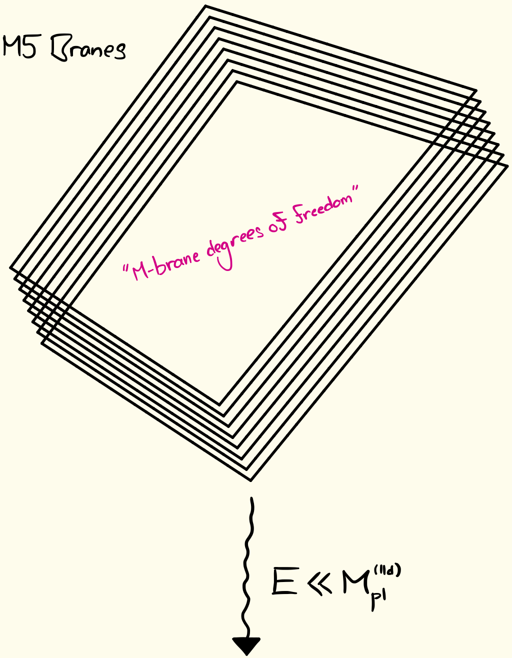

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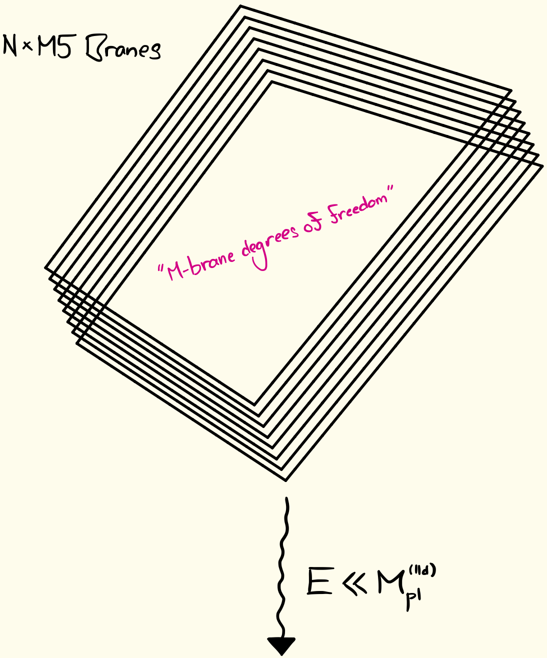

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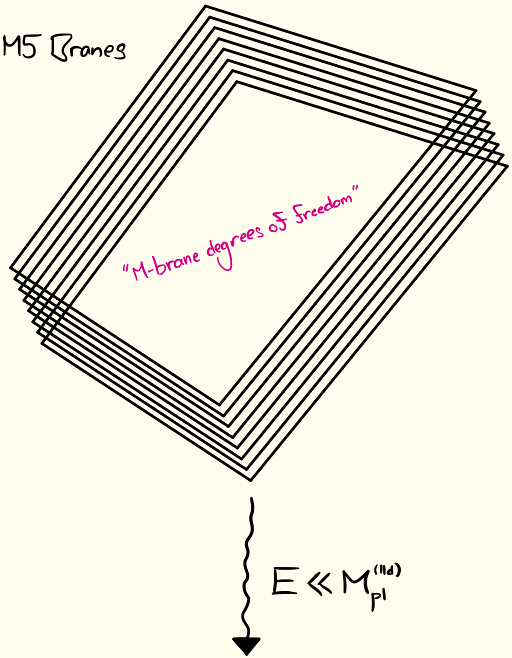

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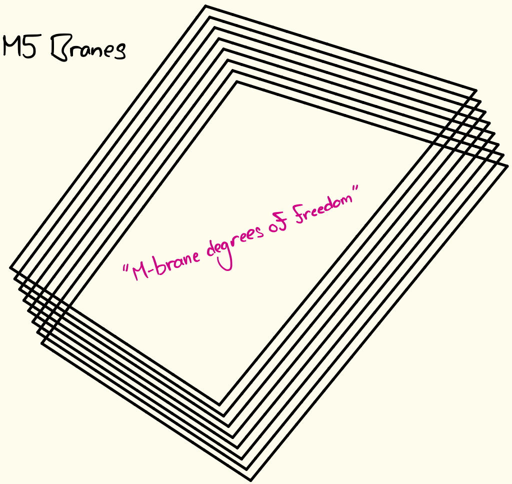

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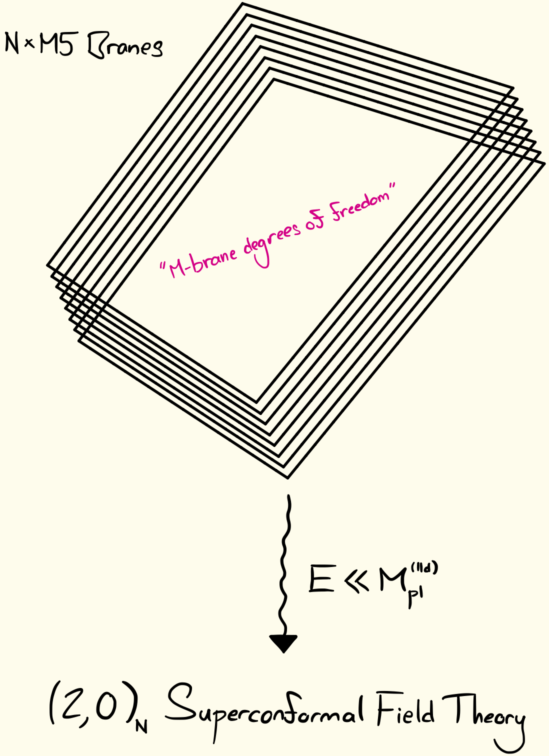

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- Maximum superconformal symmetry in top spacetime dimension.

Algebraic Consequences of Supersymmetry

Fermionic symmetry generators act as differentials compatible with OPE:

$$\delta : H[\mathbb{S}^{d-1}] \longrightarrow H[\mathbb{S}^{d-1}] \qquad \left(\delta := \delta_Q \text{ or } \delta_S\right)$$

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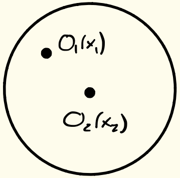
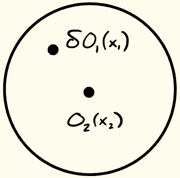
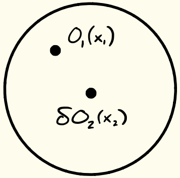
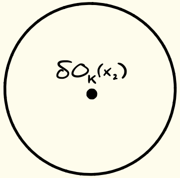
δ 

$=$ 

$+$ 

$= \sum_{O_k \in O_1 \times O_2} C_{12}{}^{k}(x_1 - x_2)$ 

Cohomological Reductions of Superconformal OPE Algebras

Associative OPE inherited by cohomological quotient —

$$H^*\left(\mathcal{H}(\mathbb{S}^{d-1})\right) \cong \frac{\mathrm{Ker}\left(\delta : \mathcal{H}(\mathbb{S}^{d-1}) \to \mathcal{H}(\mathbb{S}^{d-1})\right)}{\mathrm{Im}\left(\delta : \mathcal{H}(\mathbb{S}^{d-1}) \to \mathcal{H}(\mathbb{S}^{d-1})\right)}$$

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Algebraic structure can be dramatically simpler here —

$$\{Q, \tilde{Q}\} \sim H_b \Longrightarrow \delta_{H_b} [\mathcal{O}(x)]_Q = 0$$

Here H_b is a bosonic symmetry generator, e.g., translations, rotations, dilatations.

Simplest Case

If $\{H_b\} = \{\text{spacetime translations}\}$...

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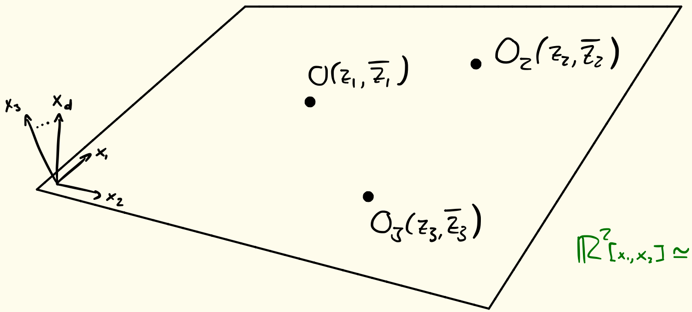
Very important, but not immediately useful for bootstrap purposes.

Meromorphic Reduction [CB, Lemos, Liendo, Peelaers, Rastelli, van Rees (2013)]

$d=4$ $\mathcal{N}\geq 2$ SUSY

– or –

$d=6$ $\mathcal{N}\geq(2,0)$ SUSY



$\mathbb{R}^2_{[x_1,x_2]}\simeq\mathbb{C}_{[z,\bar{z}]}\subset\mathbb{R}^{d=4,6}$

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– or –

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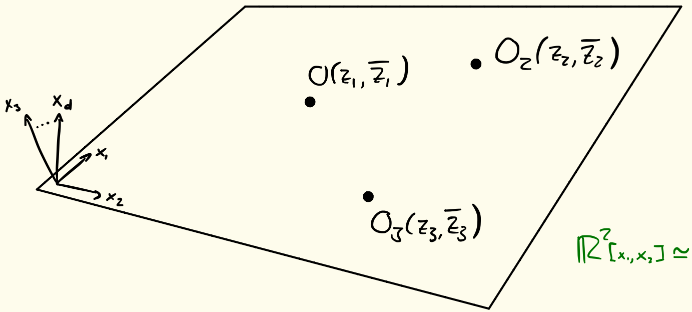

$\mathbb{R}^2[x_1,x_2] \simeq \mathbb{C}[z,\bar{z}] \subset \mathbb{R}^{d=4,6}$

Can identify cohomological reduction with $Q=Q_\chi$ s.t.

- $H_b = \frac{\partial}{\partial \bar{z}} \longrightarrow [\mathcal{O}_i(z_i,\bar{z}_i)]_{Q_\chi} = [\mathcal{O}_i(z_i)]_{Q_\chi}$

- $\delta_{Q_\chi}\mathcal{O}(x)=0 \longrightarrow x_3,\cdots,x_d=0$

Meromorphic Reduction [CB, Lemos, Liendo, Peelaers, Rastelli, van Rees (2013)]

Thus the relevant operators & OPE coefficients define a 2d meromorphic OPE algebra

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Often can identify VOA associated to a given SCFT with very little input (due to rigidity of VOAs).

In other cases, it gets more complicated...

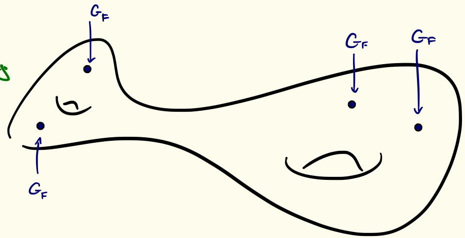
Class $\mathcal{S}_{(ix)}$ Theories [Gaiotto, Gaiotto-Moore-Neitzke]

$(2,0)_{\mathfrak{g}}$ SCFT on $\mathbb{R}^4 \times C$

$E \ll 1/(\mathrm{Vol}(C))^{1/2}$

4d $\mathcal{N}=2$ SCFT on $\mathbb{R}^4$

Punctured Riemann Surface



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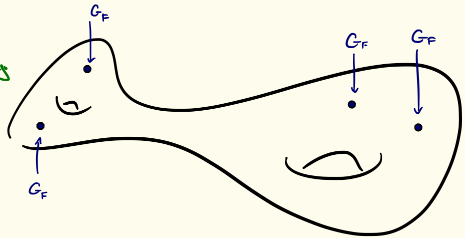

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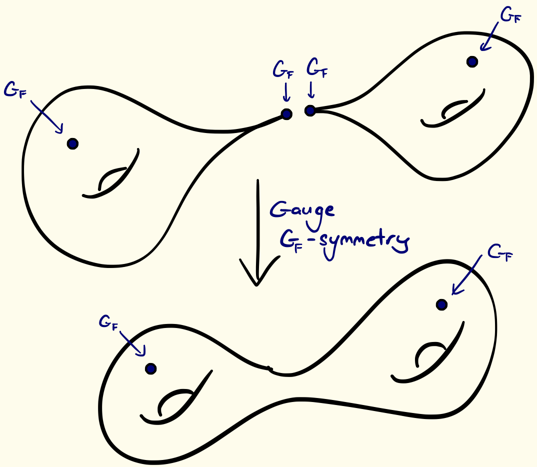

Gluing along punctures

$\cong$

Gauging global symmetries

Class $\mathcal{S}_{(ix)}$ Chiral Algebras [CB, Peelaers, Rastelli, van Rees (2014)]

SCFT-OPE algebras parameterized by moduli space of Riemann Surfaces (generically non-Lagrangian).

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A triumph of mathematical physics!

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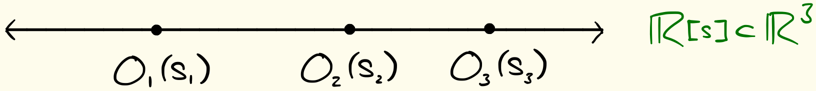
$d=3$ $\mathcal{N}\geq 4$ theories:



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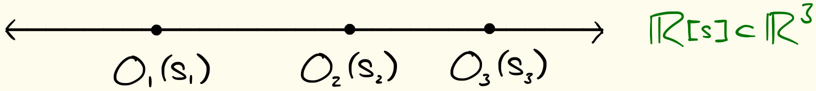

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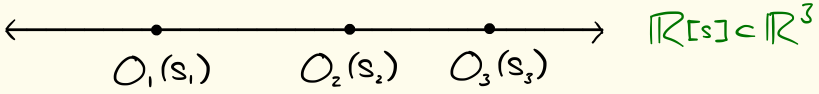

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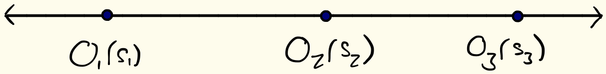

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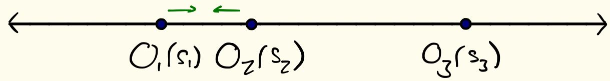

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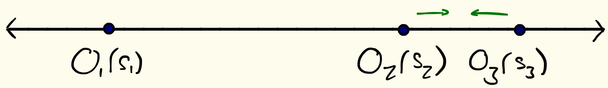

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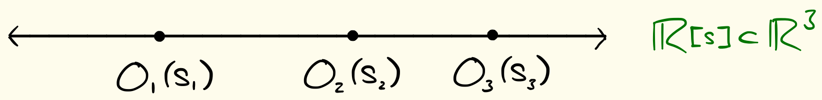

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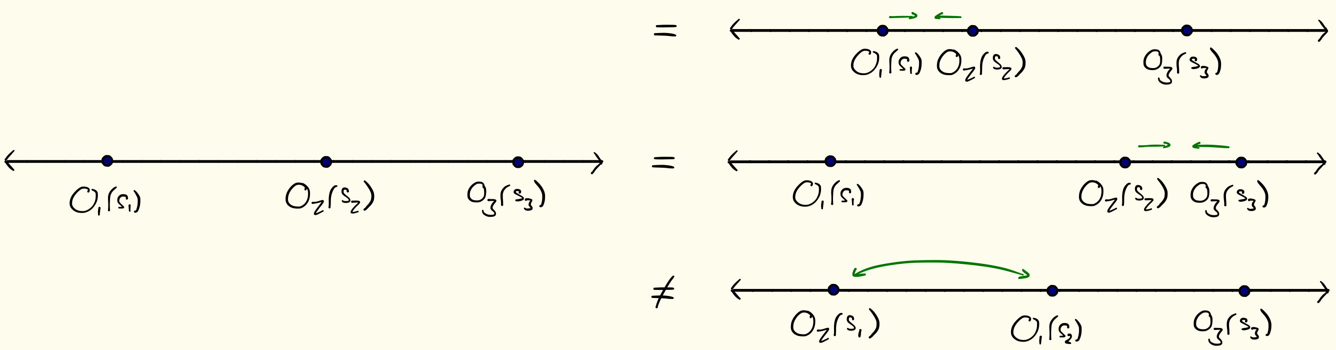

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Deformation quantization: $(\mathcal{A}, \{\ ,\ \})$ Poisson algebra $\left[\begin{array}{l} (f,g) \to fg = gf \\ (f,g) \to \{f,g\} = -\{g,f\} \\ \{f,gh\} = \{f,g\}h + g\{f,h\} \end{array}\right]$

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Solutions to associativity for $\mathcal{A}_\hbar$ classified [Kontsevich (1997)] up to (infinite-dimensional) "gauge" freedom to change basis:

$$f \mapsto f + \hbar f^{(1)} + \hbar^2 f^{(2)} + \cdots \quad \text{for each } f \in \mathcal{A}$$

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(In simplest cases, these extra conditions uniquely determine the algebra! Recent work of Etingof/Rains suggests this may be more general.)

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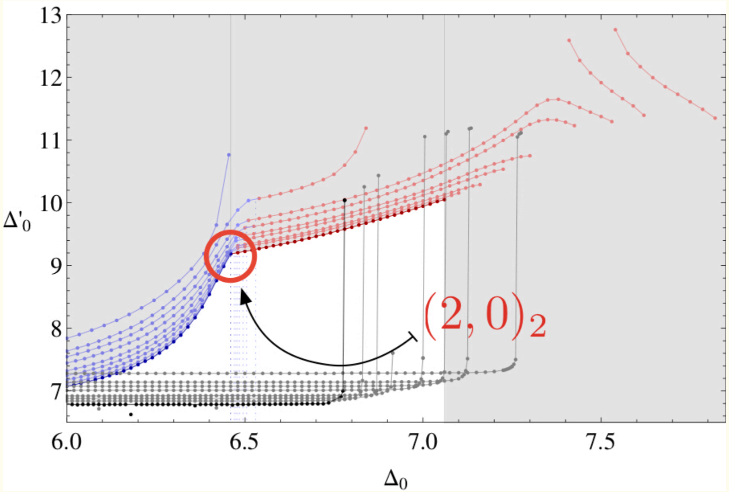

(2,0) theory appears to be "extremal". How to turn that into analytic strategy is the question of the moment.

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(is the (2,0) algebra solvable?)

Danke Sehr!