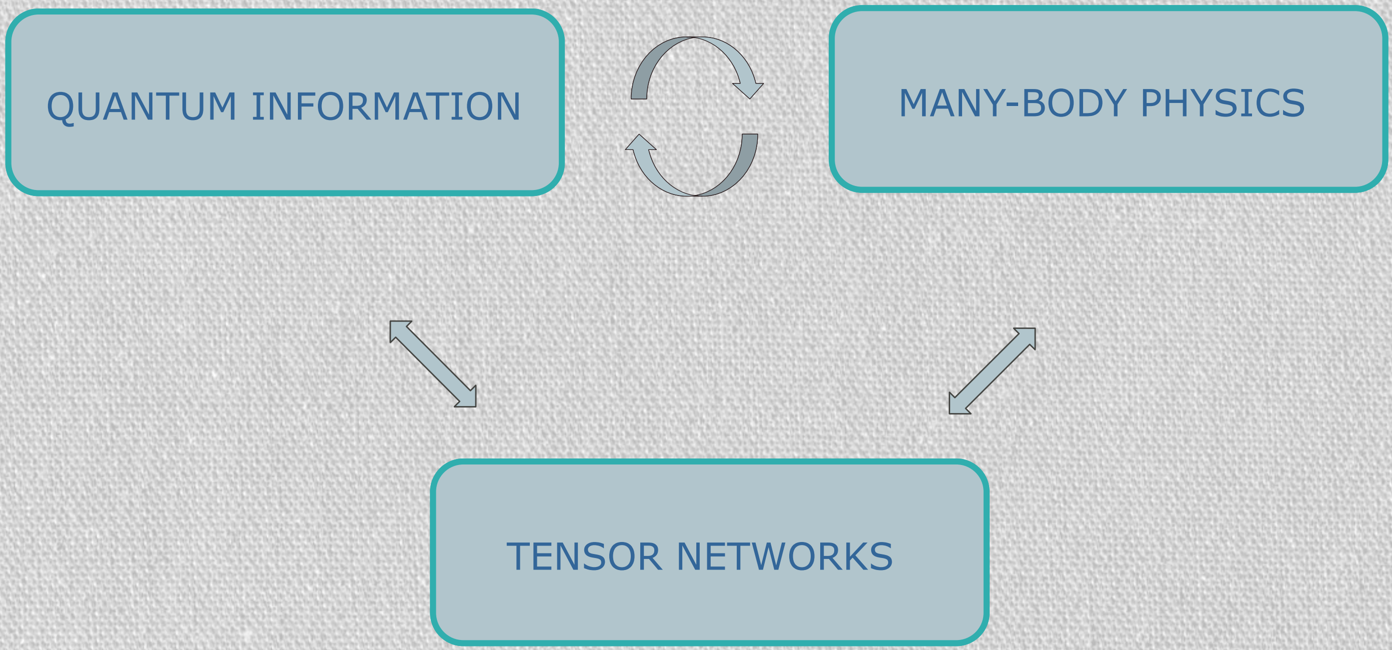
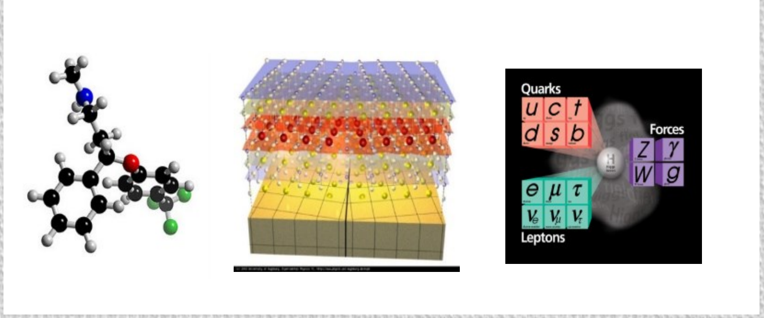
TENSOR NETWORKS
IN
LOW AND HIGH ENERGY PHYSICS

WPC Theoretical Physics Symposium

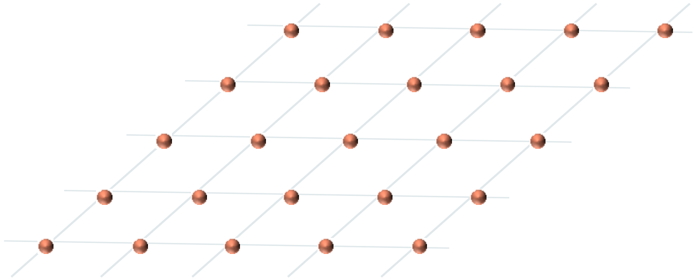
DESY Hamburg
Hamburg, 7 November, 2018


QUANTUM INFORMATION
MANY-BODY PHYSICS
TENSOR NETWORKS

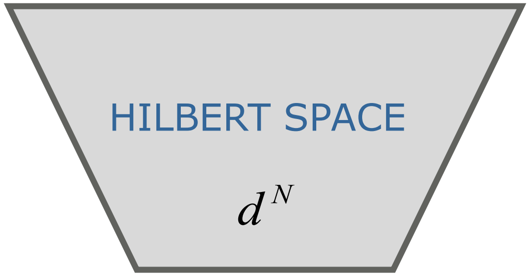
QUANTUM MANY-BODY SYSTEMS



Hamiltonian H



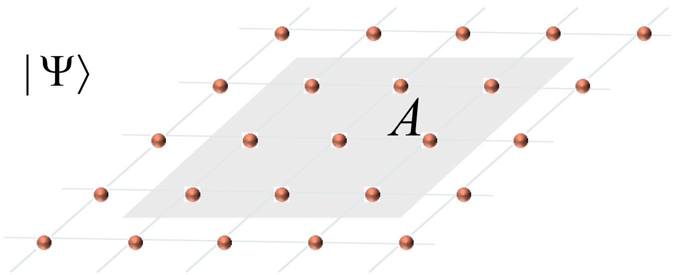
Physics at low temperatures



$$|\Psi\rangle = \sum \psi_{s_1, s_2, \ldots, s_N} \, | s_1, s_2, \ldots, s_N \rangle$$

Exponential problem

ENTANGLEMENT AND AREA LAW



How much entangled?

Lattice in any physical dimension and geometry

Local Hamiltonians: $H = \sum_n h_n$

Thermal equilibrium: $T = 0$

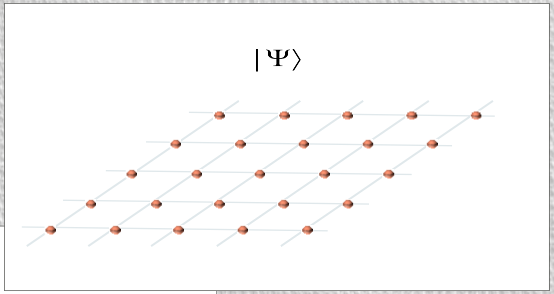
Very little!

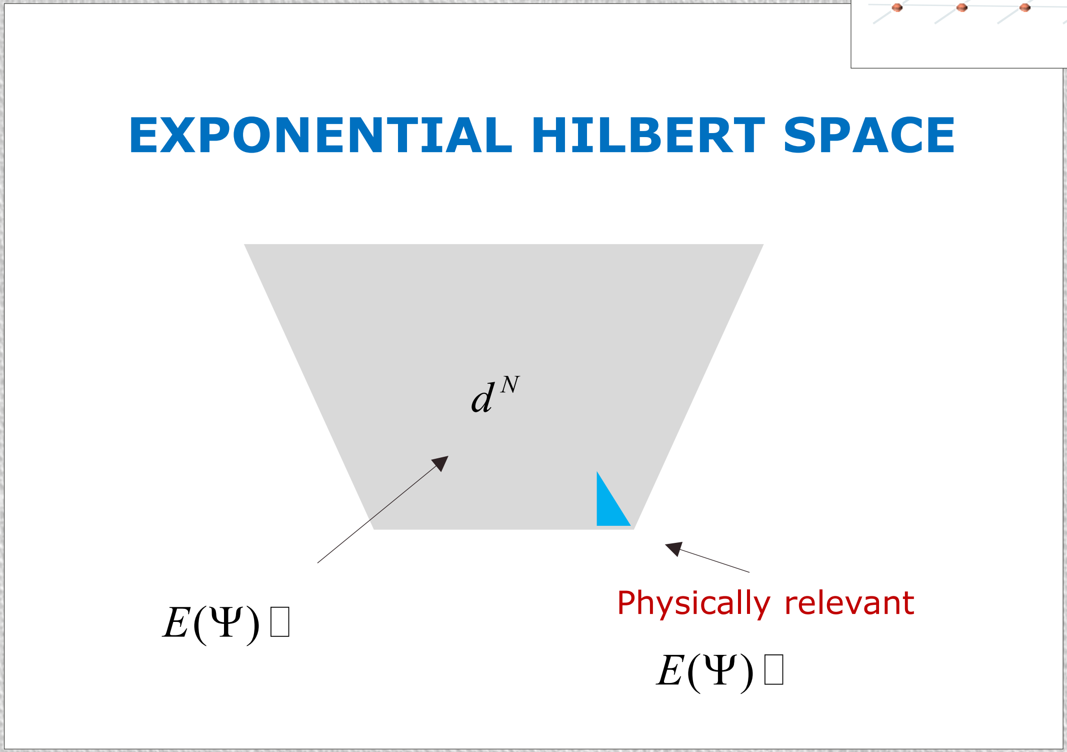
$E(\Psi)$

Area law

ENTANGLEMENT AND AREA LAW

What do we learn?





TENSOR NETWORK STATES

$$|\Psi\rangle = \sum \psi_{s_1,s_2,\ldots,s_N} \, |s_1,s_2,\ldots,s_N\rangle$$

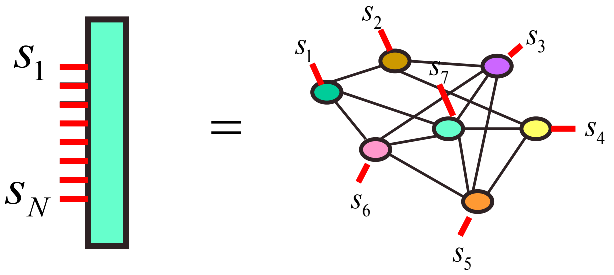
Local
Hamiltonian

Very little entanglement

$$\psi_{s_1,s_2,s_N} \approx \sum_{\alpha,\beta,\ldots} A^{s_1}_{\alpha,\beta,\gamma,\delta} B^{s_2}_{\gamma,\varepsilon,\eta,\mu} C^{s_3}_{\eta,\sigma,\tau,\zeta} \cdots$$

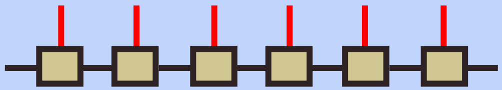
Tensor Networks

$\psi_{s_1,s_2,\ldots,s_N}$



MPS

$|\varphi\rangle$

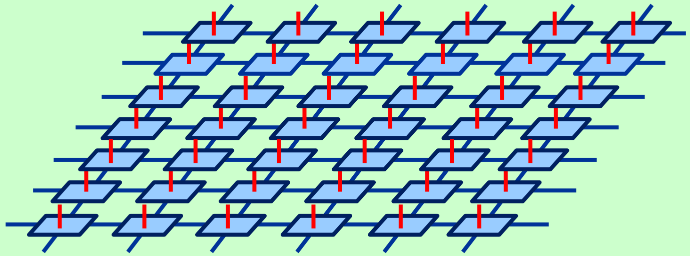


$$\psi_{s_1, s_2, \dots, s_N} = \mathrm{tr}\left[A^{s_1} A^{s_2} \dots A^{s_N} \right]$$

Fannes, Narchtergaele, Werner, CMP **144**, 443 (1992)

PEPS

$|\varphi\rangle$

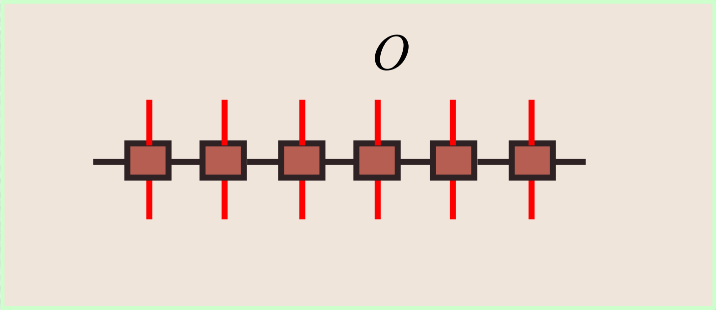


Verstraete, JIC, arxiv:0407066

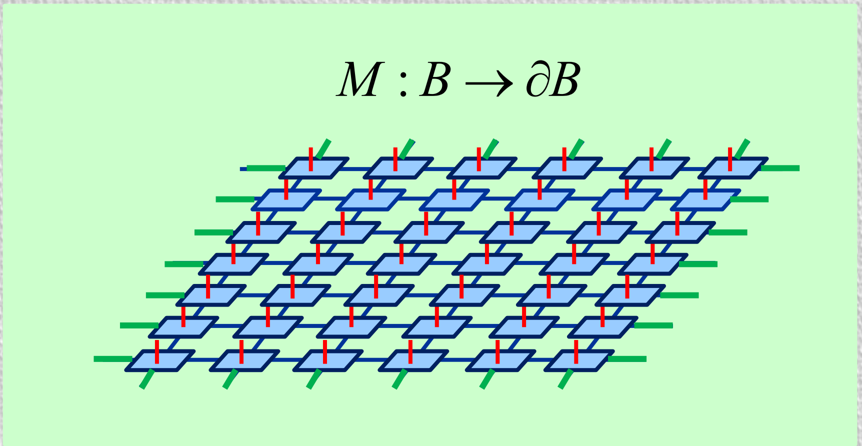
MERA

Vidal, PRL 101, 110501 (2008)

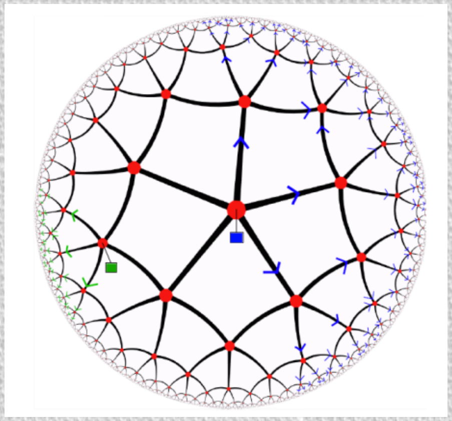
OPERATORS



MAPS

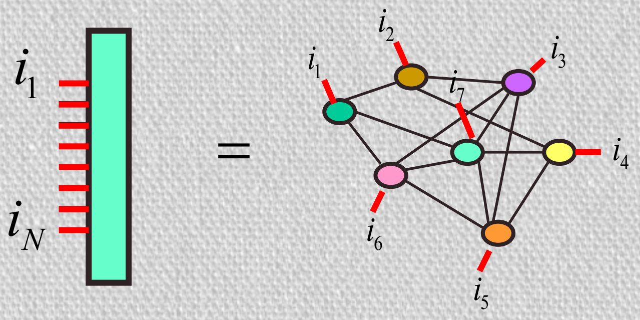


DIMENSION,
GEOMETRY



Fermions, Bosons

TENSOR NETWORKS

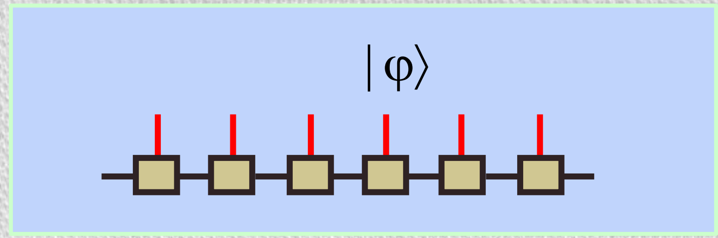


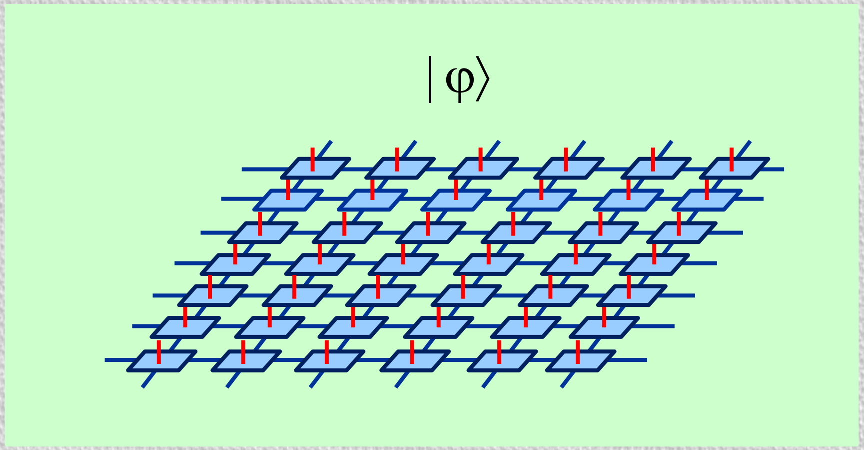
- Condensed matter physics
- Atomic physics / quantum optics
- Lattice gauge theories
- AdS/CFT models

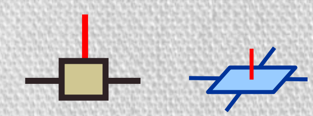
- One-way quantum computing
- Quantum error correcting codes
- Sequentially generated states
- Quantum tomography

- Classical stochastic models
- Applied mathematics
- Image compression
- Machine Learning

TENSOR NETWORKS: HOMOGENEOUS SYSTEMS







a single tensor describes the whole many-body state

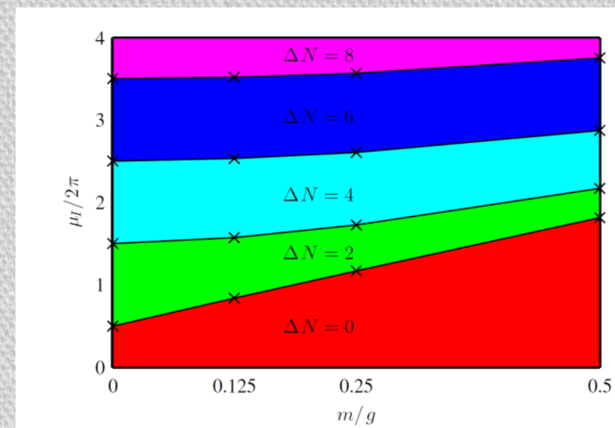
APPLICATION: COMPUTATIONS

HIGH-ENERGY MODELS in 1+1D
SCHWINGER MODEL

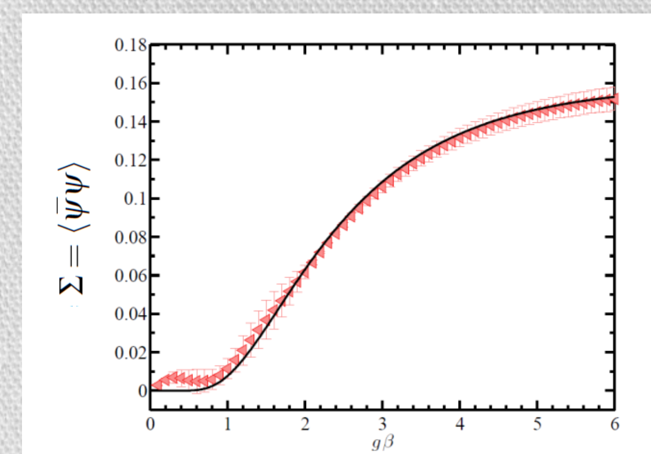
MASS SPECTRUM

	Scalar binding energy		
m/g	MPS with OBC	SCE result [36]	exact
0	1.1279(12)	1.11(3)	1.12838
0.125	1.2155(28)	1.22(2)	-
0.25	1.2239(22)	1.24(3)	-
0.5	1.1998(17)	1.20(3)	-

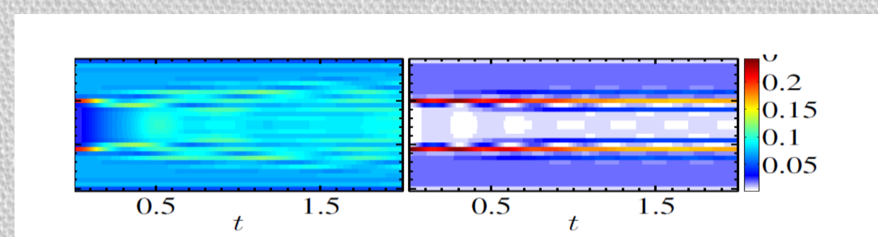
ZERO TEMPERATURE



FINITE TEMPERATURE



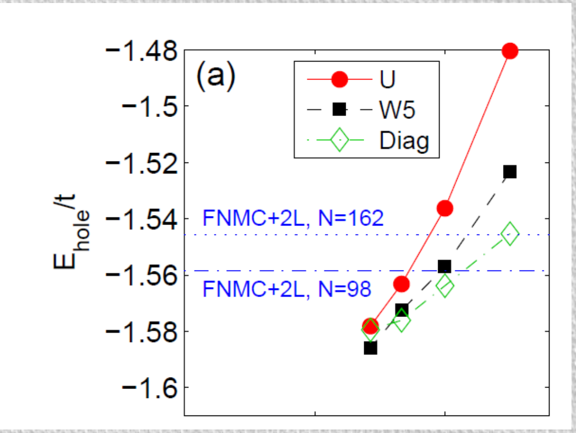
DYNAMICS



Banuls, Cichy, JIC, Jansen, JHEP **11**, 158 (2013)
Banuls, Cichy, JIC, Jansen, Saito PRD **92**, 034519 (2015
Banuls, Cichy, JIC, Jansen, Kühn, PRL **118**, 071601 (2017)

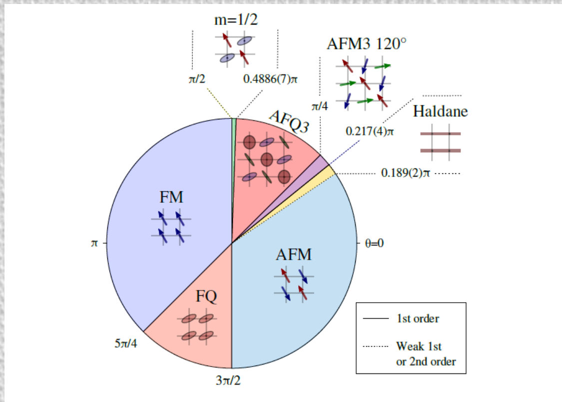
RESULTS 2+1 D
CONDENSED MATTER MODELS

T-J model



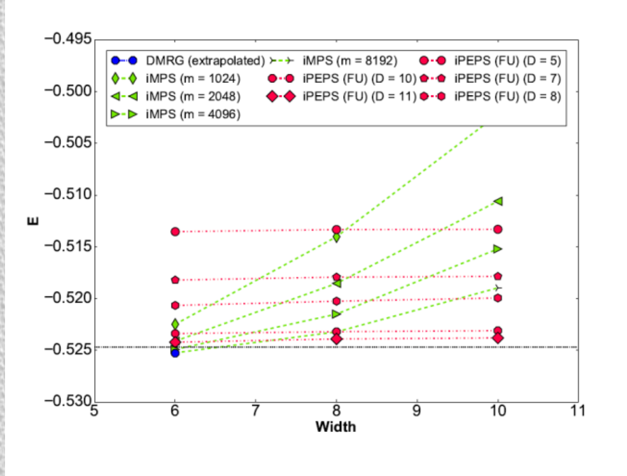
Niesen, Corboz: 1402.2859

Bilinear-biquadratic Heisenberg model



Niesen, Corboz: 1707.01953

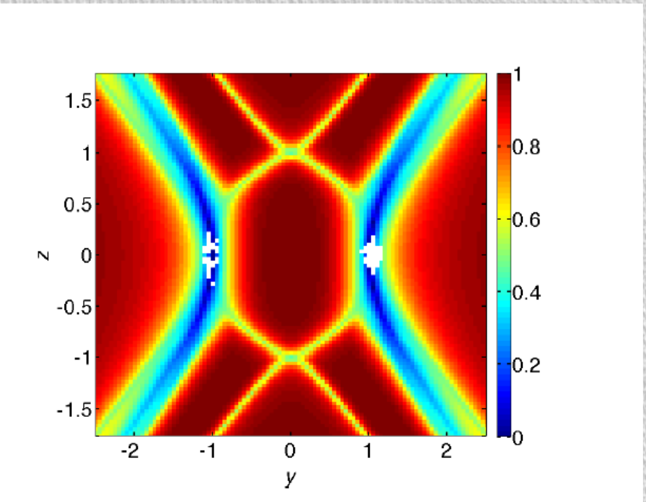
Hubbard model



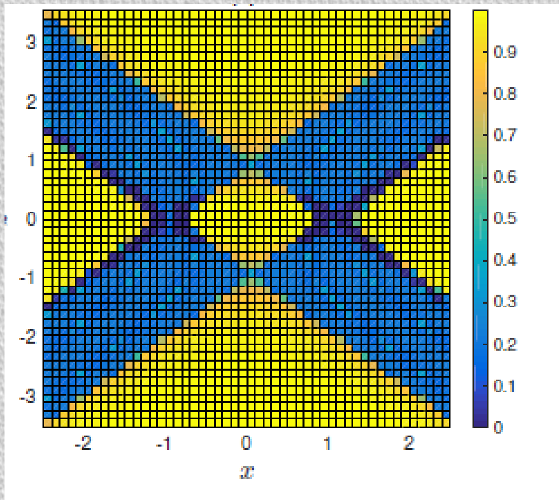
Iregui, Troyer, Corboz, arxiv: 1705.03222

RESULTS 2+1 D
TOY MODELS

U(1) model



SU(2) model



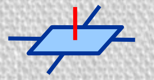
Confinement / deconfinement, screening, etc

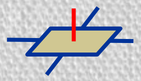
Zohar, Wahl, Burrello, JIC, Ann. Phys **374**, 84 (2016)

FUNDAMENTAL THEOREM

FUNDAMENTAL THEOREM OF TN

Two different tensors may generate the very same state

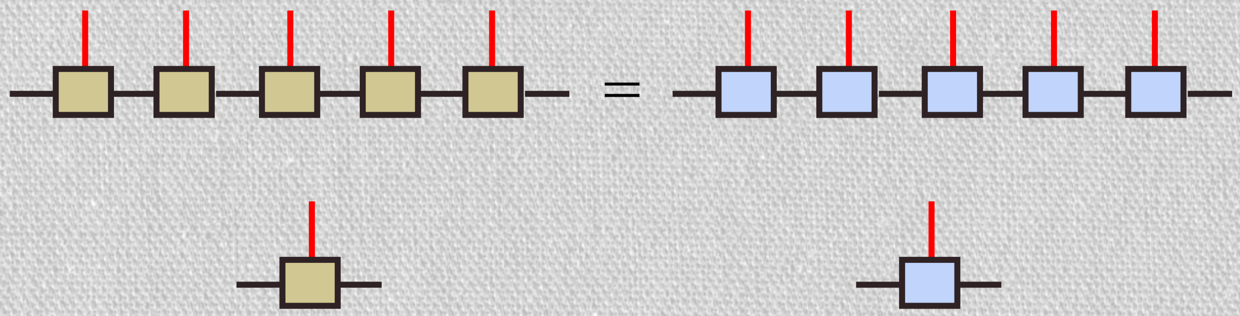




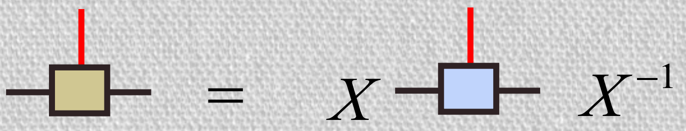
How are the tensors related?

FUNDAMENTAL THEOREM OF TN

1D: MATRIX PRODUCT STATES:



They are related by a gauge transformation

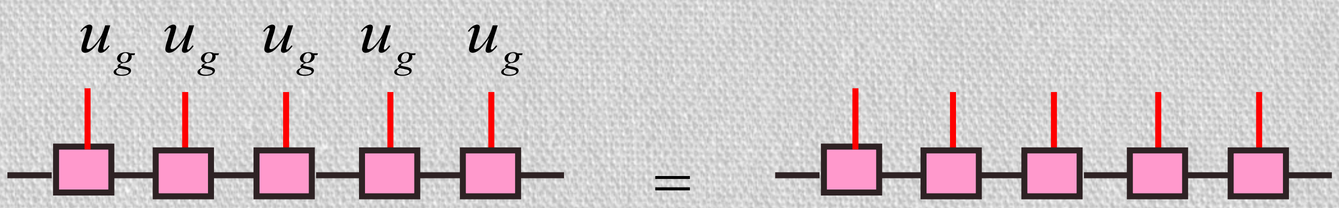


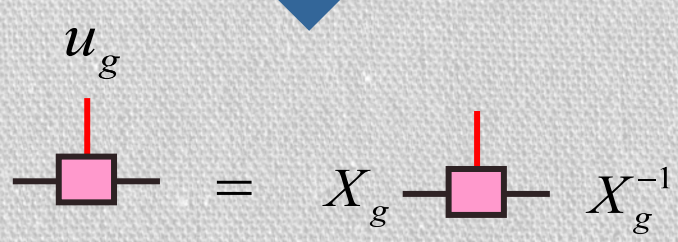
Perez-Garcia, Verstraete, Wolf, JIC, Quant. Inf. Comp. **7**, 401 (2007)

APPLICATIONS IN 1D:

SYMMETRIES:

$$u_g^{\otimes N}\,|\phi\rangle = |\phi\rangle \qquad\qquad g \in G$$





Sanz, Wolf, Perez-Garcia, JIC, PRA **79**, 042308 (2009)

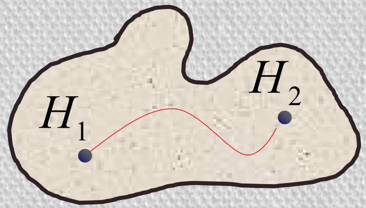
Classification of SPT phases in1D:

Pollman, Turner, Berg, Oshikawa, PRB **81**, 064439 (2010)
Chen, Gu, Wen, PRB **83**, 035107 (2011)
Schúch, Perez-Garcia, JIC, PRB **84**, 165139 (2011)

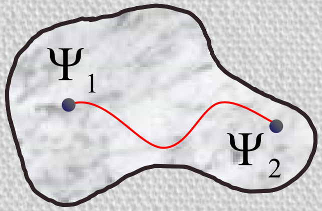
CLASSIFICATION OF PHASES:

GAPPED HAMILTONIANS

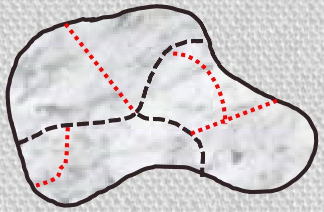
Hamiltonians



States



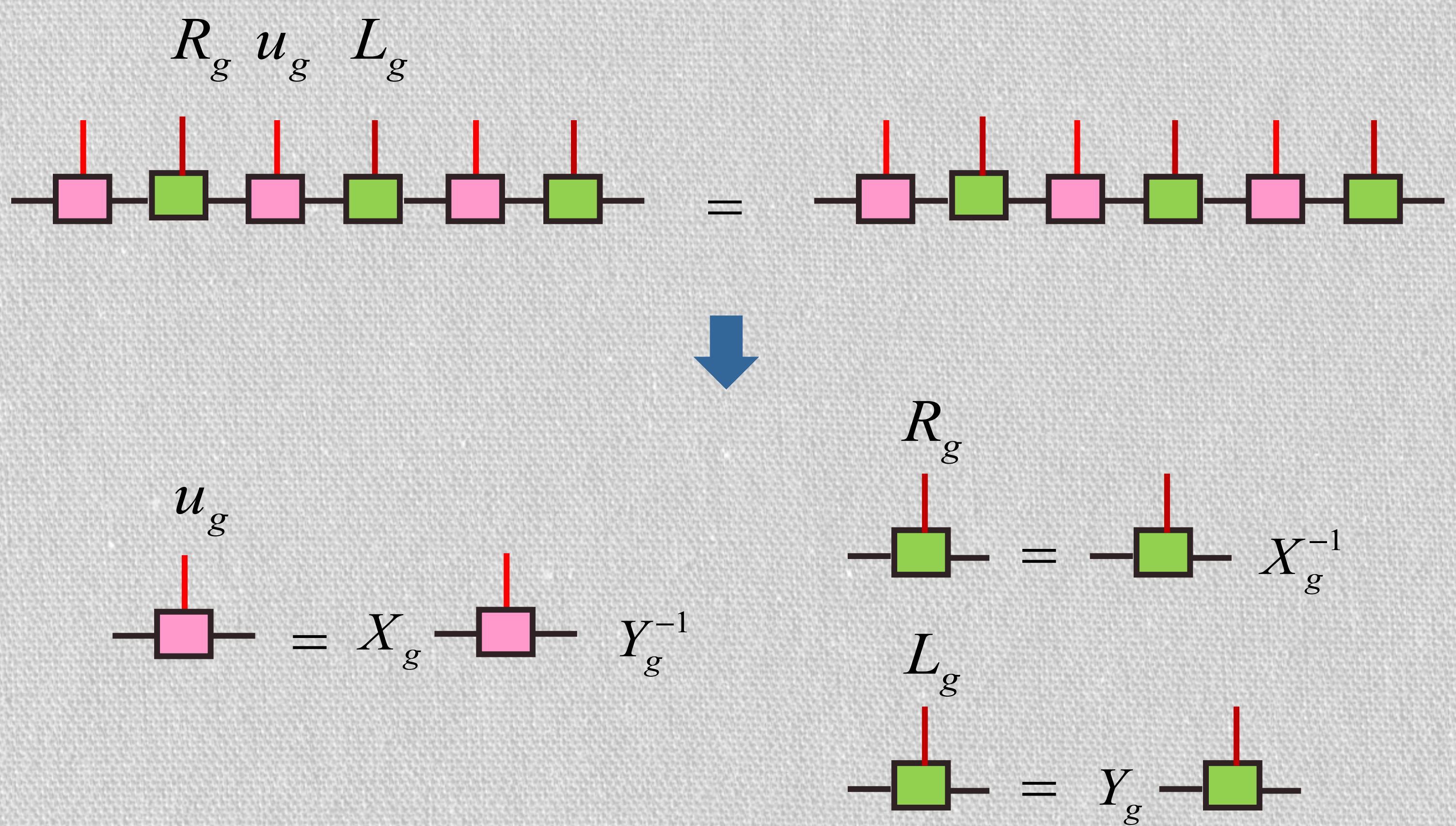
PHASES



- No symmetries
- Symmetries

APPLICATIONS IN 1D:

GAUGE SYMMETRIES:

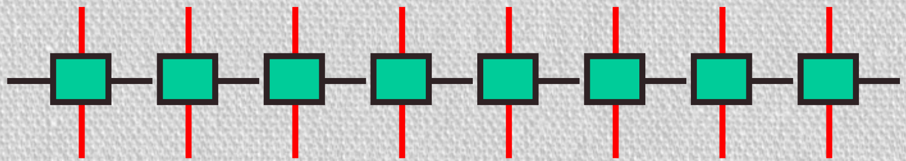


Kull, Molnar, Zohar, JIC; Ann. Phys. **386**, 199 (2017)

APPLICATIONS IN 1D:

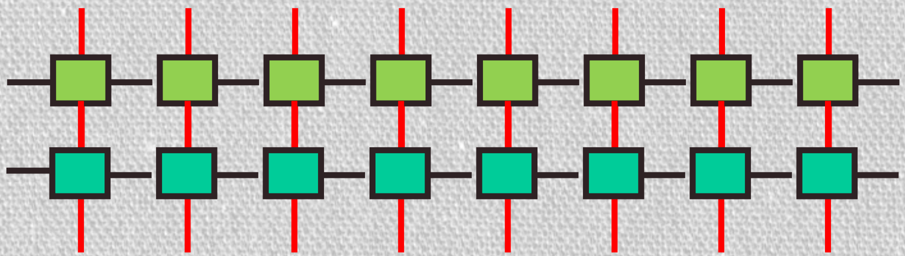
UNITARY OPERATORS:

$$U =$$



is unitary

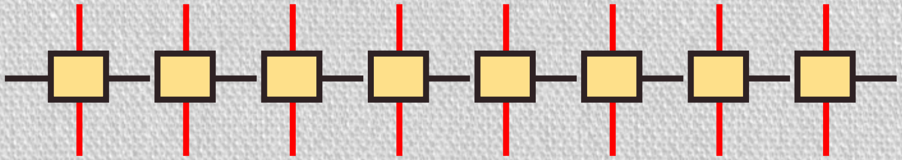
$$1 = UU^{\dagger} =$$



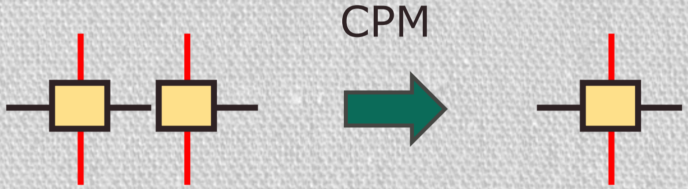
JIC, Perez-Garcia, Schuch, Verstraete, JIC, J. Stat. Mech. **2017**, 083105 (2017)
Gross, Nesme, Vogts, Werner, Commun. Math. Phys. **310**, 419 (2012)

APPLICATIONS IN 1D:

DENSITY OPERATORS:

$\rho =$ 

Renormalization

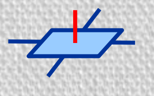


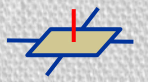
Classification of 2D (non-chiral) topological phases

JIC, Pérez-García, Schuch, Verstraete, Ann. Phys. **378**, 100- (2017).

HIGHER DIMENSIONS:

Two different tensors may describe the very same state





How are the tensors related?

IMPOSSIBLE:

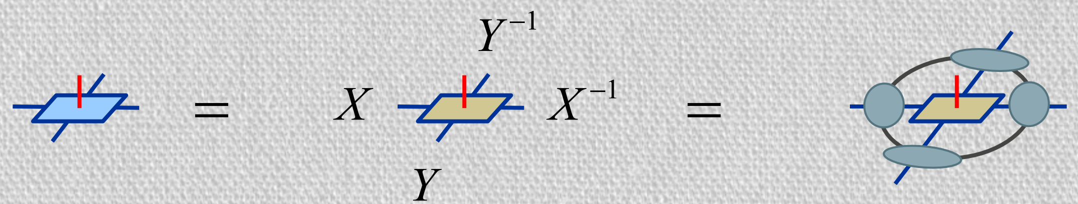
One has to impose restrictions

Scarpa, Molnar, Ge, Garcia-Ripoll, Schuch, Perez-Garcia, Iblisdir, arXiv:1802.08214

FUNDAMENTAL THEOREM OF TN

2D: PEPS

Under some conditions:



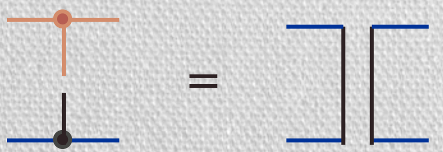
- Topological order
- Classification of SET phases

Chen, Gu, Liu, Wen, PRB **87**, 155114 (2013)
Schuch, JIC, Perez-Garcia, Ann. Phys. **325**, 2153 (2010)
Butlinck, Marien, Williamson, Sahinoglu, Haegeman, Verstraete, Ann. Phys. **378**, 183 (2017)

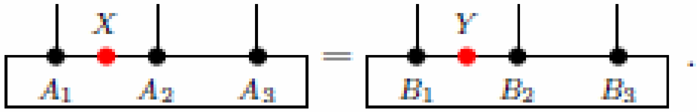
FUNDAMENTAL THEOREM: IDEA OF THE PROOF

Molnar, Garre-Rubio, Perez-Garcia, Schuch, JIC: 1804.04964

IDEA OF THE PROOF:

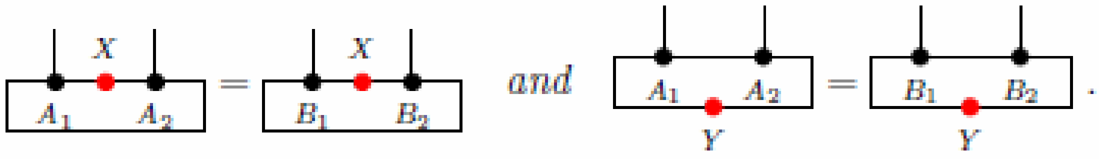


Lemma 1: Suppose A_i, B_i (i=1,2,3) are injective and generate the same MPS. Then, for every edge, there exists a unique invertible Z such that for all X



with $Y=Z\,X\,Z^{-1}$

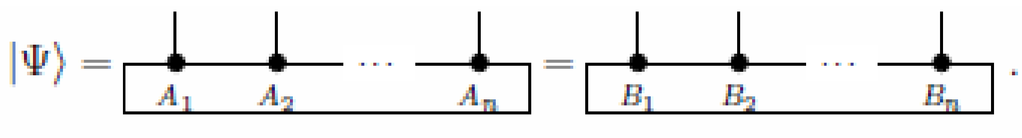
Lemma 2: Suppose A_i, B_i (i=1,2) are injective and generate the same MPS and for all X,and Y



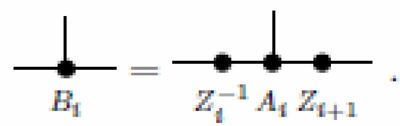
Then $A_1 = \lambda\, B_1$ and $A_2 = \lambda^{-1}\, B_2$

IDEA OF THE PROOF:

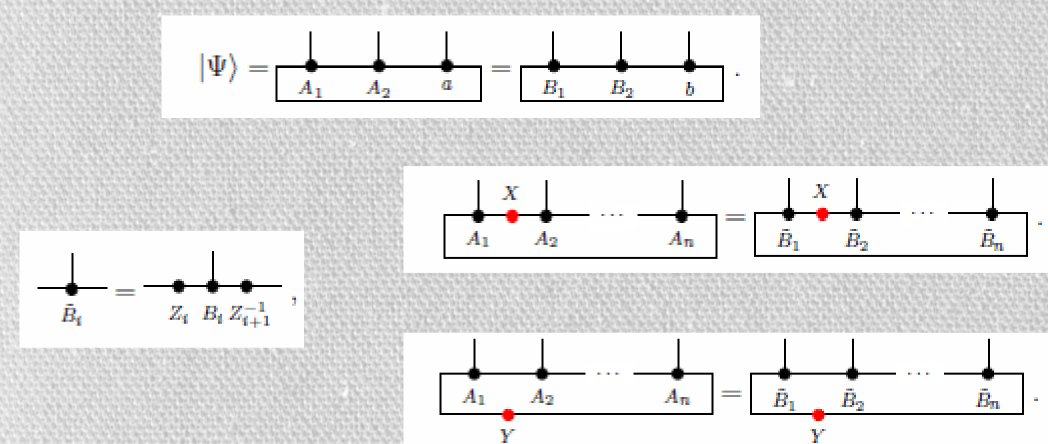
Theorem: Suppose A_i, B_i (i=1,2,…,n) are injective and generate the same MPS



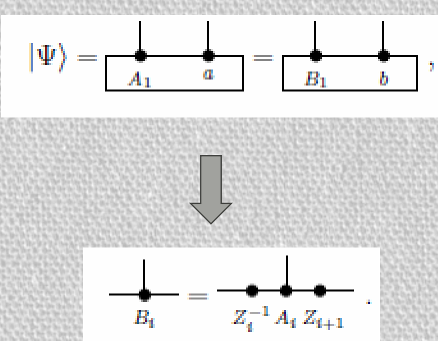
Then there are some invertible Z_i (i=1,2,…,n) such that



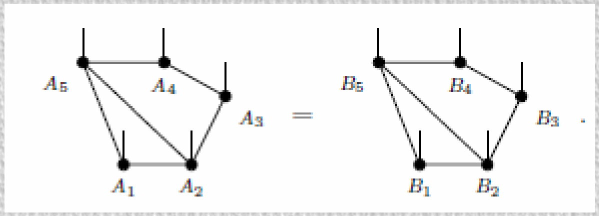
Lemma 1:

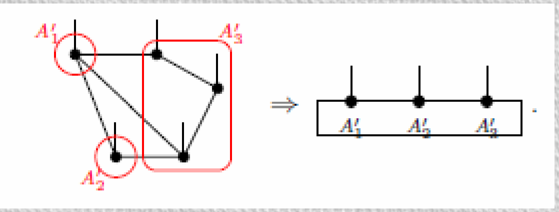


Lemma 2:



IDEA OF THE PROOF:





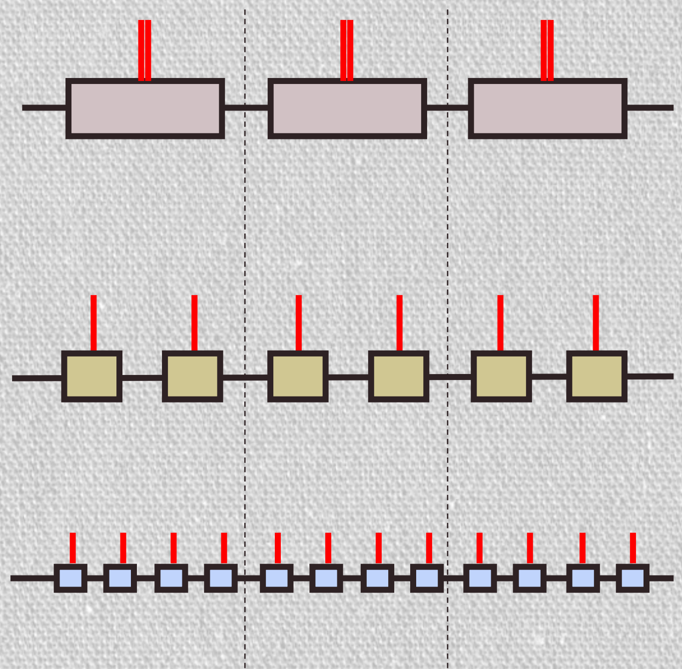
TENSOR NETWORKS
FOR QUANTUM FIELD THEORY

Verstraete and Cirac, PRL 104, 190405 (2010)
Tilloy, JIC, arxiv:1808.00976

see also Hu, Rubio, Vidal, arxiv: 1809.05176

RENORMALIZATION

1D: MATRIX PRODUCT STATES:



$$\mathrm{tr}\left[B^{i_1 i_2} B^{i_3 i_4} \ldots B^{i_{N-1} i_N} \right]$$

$$\mathrm{tr}\left[A^{i_1} A^{i_2} \ldots A^{i_N} \right]$$

$$\mathrm{tr}\left[C^{j_1} C^{j_2} \ldots C^{j_{2N}} \right]$$

Under some conditions, one can take the limit

RENORMALIZATION

1D: CONTINUOUS MATRIX PRODUCT STATES:

$$|\Psi\rangle = \mathrm{Tr}_{\mathrm{aux}}\left[P e^{\int_0^L dx[Q\otimes 1 + R\otimes\psi(x)^\dagger]} \right] |\,\mathrm{vac}\rangle$$

$$= \mathrm{Tr}_{\mathrm{aux}}\left[e^{\int_0^L dxQ} \right] |\,\mathrm{vac}\rangle + \int_0^L dx \mathrm{Tr}_{\mathrm{aux}}\left[e^{\int_X^L dxQ} R e^{\int_0^x dxQ} \right] \psi(x)^\dagger |\mathrm{vac}\rangle + \ldots$$

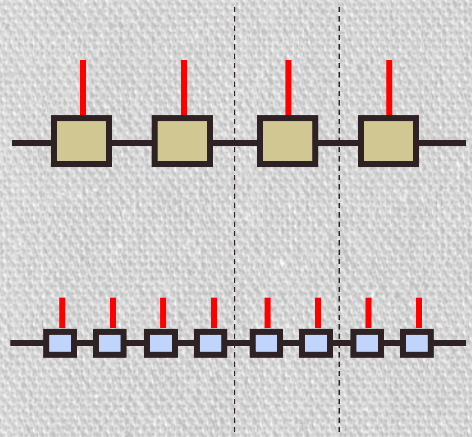
Lieb-Liniger Model

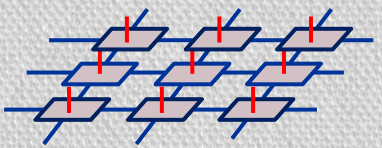
Gross-Niveau Model

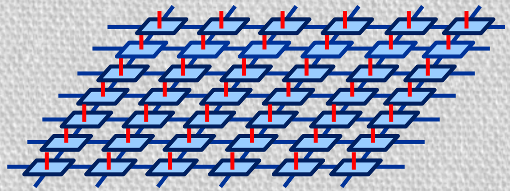
Verstraete and JIC, PRL 104, 190405 (2010)
Haegeman, JIC, Osborne, Verschaelde, Verstraete, POS (2011)

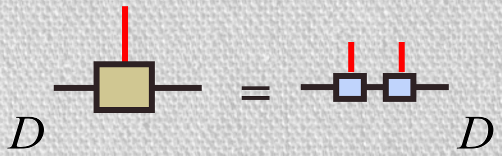
RENORMALIZATION

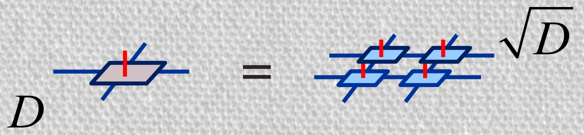
2D: PEPS:





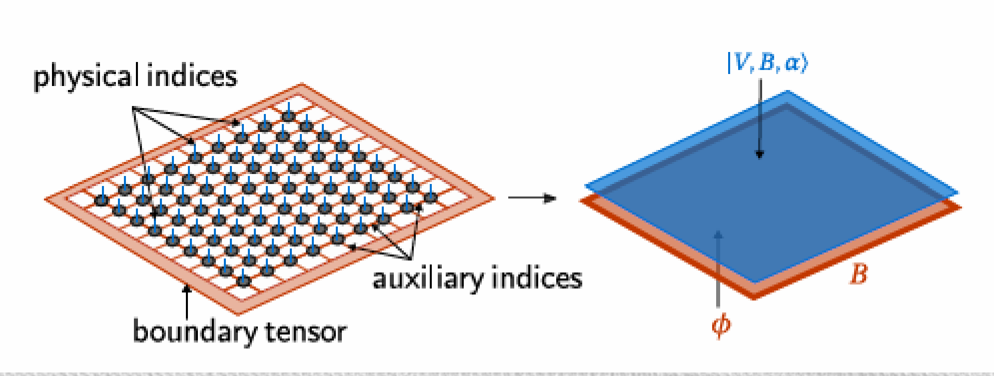






RENORMALIZATION

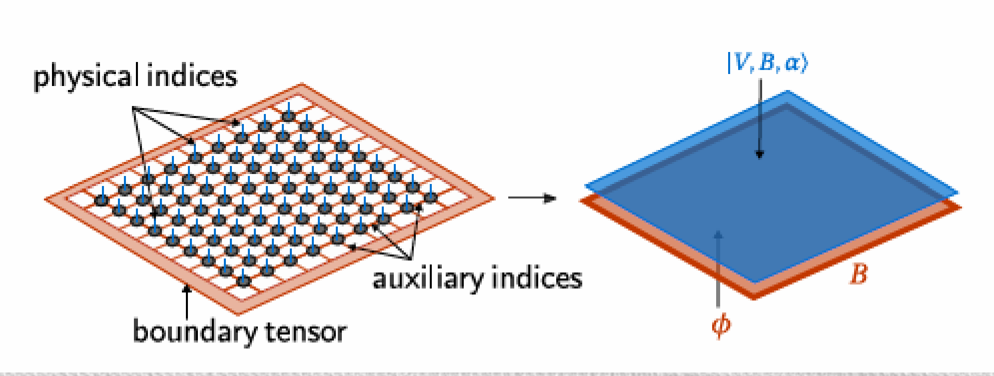
CONTINUOUS PEPS:



$$|\Psi\rangle = \int D[\phi] e^{-\int_\Omega d^d x \left[|\nabla \phi(x)|^2 + V[\phi(x)] - \alpha[\phi(x)] \psi(x)^\dagger \right]} |\text{vac}\rangle$$

RENORMALIZATION

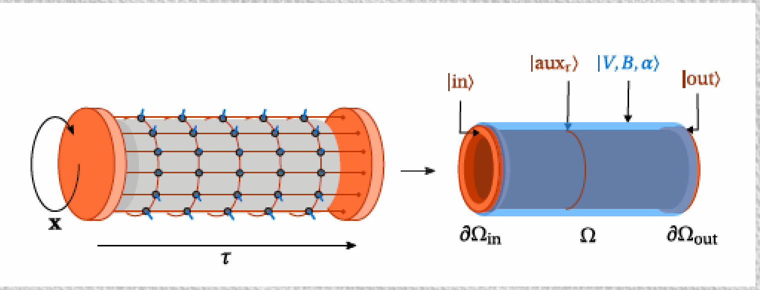
CONTINUOUS PEPS:



$$|\Psi\rangle = \int D[\phi] e^{-\int_{\Omega} d^{\mathrm{d}}x \left[|\nabla\phi(x)|^2 + V[\phi(x)] - \alpha[\phi(x)]\psi(x)^{\dagger} \right]} |\,\mathrm{vac}\rangle$$

OPERATOR REPRESENTATION:

$$|\Psi\rangle = \mathrm{Tr}_{\mathrm{aux}} \left[P e^{\int_0^L dx [Q \otimes 1 + R \otimes \psi(x)^{\dagger}]} \right] |\,\mathrm{vac}\rangle$$



WORK IN PROGRESS

LATTICE GAUGE THEORIES

- Tensor Networks + Monte Carlo
- Elimination of fermionic degrees of freedom
- Constraint-free lattice gauge theories

FUNDAMENTAL THEOREM

- Extensions

CONTINOUS TENSOR NETWORKS

- Continous MERA
- AdS/CFT

SUMMARY

- Motivated by entanglement
- Provide efficient descriptions (thermal equilibrium)
- Computational methods: hard in high dimensions
- Fundamental Theorem in tensor networks:
  - Symmetries
  - Lattice gauge theories
  - Boundary theories
  - Classification
  - Renormalization
- Continuous Tensor Networks for Quantum Field Theories

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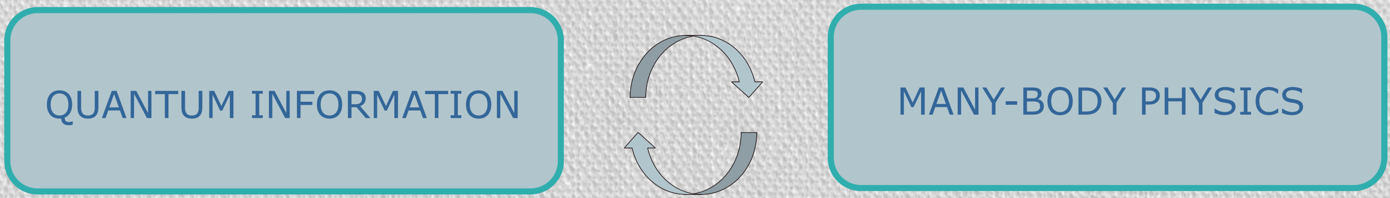

- Quantum Optics
- Solid State

- Atomic Physics
- Condensed Matter Physics
- High-Energy Physics

(Quantum Gravity / Chemistry)