Anomaly and Symmetry-Protected Critical Phases

Masaki Oshikawa
ISSP, University of Tokyo

7-9 November 2018
Theoretical Physics Symposium 2018
Wolfgang-Pauli Centre, Hamburg, Germany

This talk is based on discussion with many people, in particular collaborations with

Frank Pollmann (now at TU München)
Erez Berg (now at Weizmann Institute)
Ari M. Turner (now at Technion)

Shunsuke Furuya (now at RIKEN)
Yuan Yao (ISSP)
Chang-Tse Hsieh (Kavli IPMU & ISSP)

Hirosi and Me

PRL **108,** 161803 (2012) PHYSICAL REVIEW LETTERS week ending 20 APRIL 2012

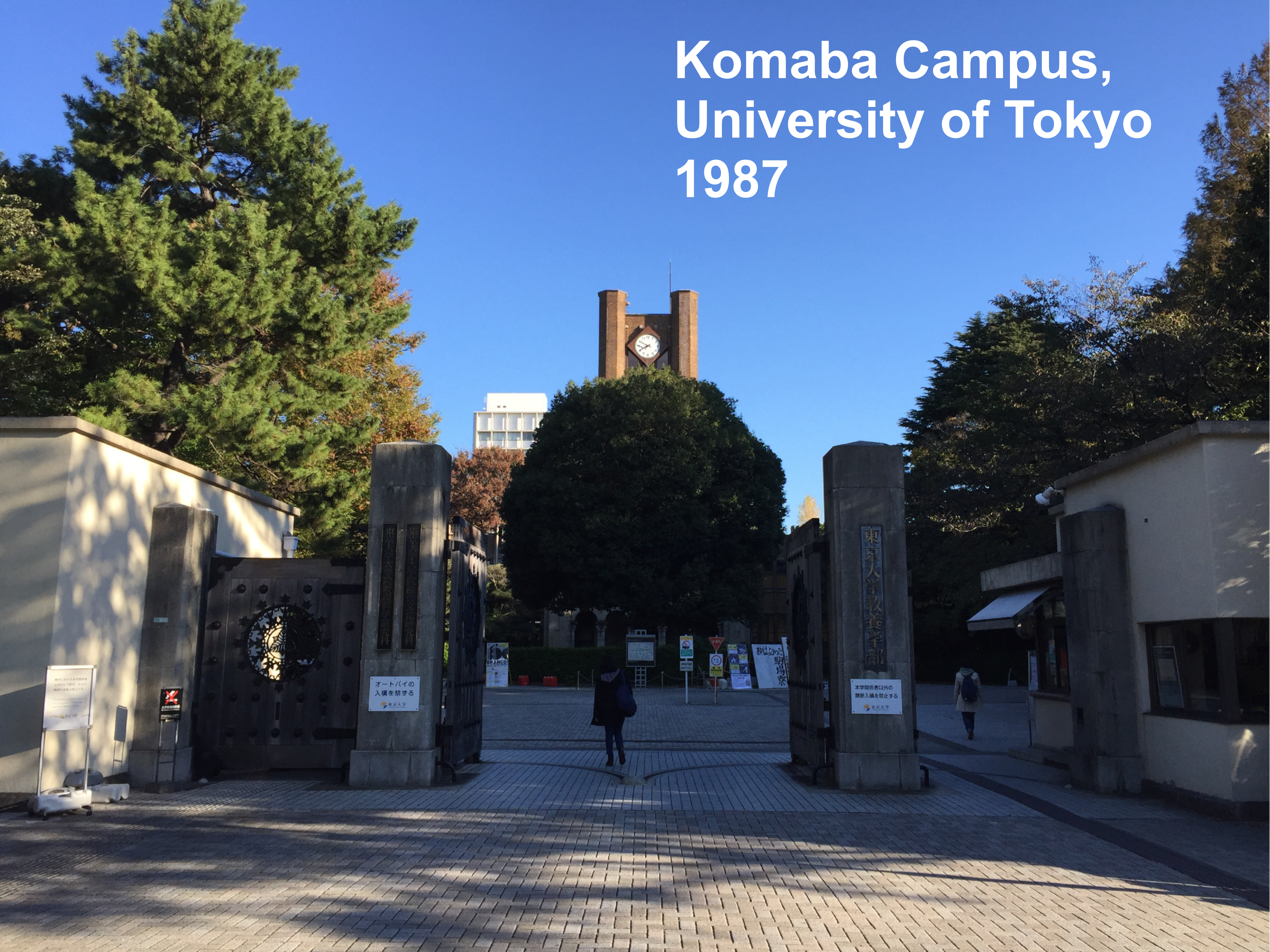
Instability in Magnetic Materials with a Dynamical Axion Field

Hirosi Ooguri[1,2] and Masaki Oshikawa[3]

[1]*California Institute of Technology, 452-48, Pasadena, California 91125, USA*
[2]*Kavli IPMU, University of Tokyo (WPI), Kashiwa 277-8583, Japan*
[3]*Institute for Solid State Physics, University of Tokyo, Kashiwa 277-8581, Japan*
(Received 14 December 2011; published 20 April 2012)

It has been pointed out that axion electrodynamics exhibits instability in the presence of a background electric field. We show that the instability leads to a complete screening of an applied electric field above a certain critical value and the excess energy is converted into a magnetic field. We clarify the physical origin of the screening effect and discuss its possible experimental realization in magnetic materials where magnetic fluctuations play the role of the dynamical axion field.

DOI: 10.1103/PhysRevLett.108.161803 PACS numbers: 14.80.Va, 73.61.−r


Komaba Campus,
University of Tokyo
1987
東京大学教養学部
オートバイの
入構を禁ずる
本学関係者以外の
無断入構を禁止する

Hirosi: obtained Master’s degree from Kyoto Univ. and immediately appointed to “Joshu” position at UTokyo in 1986, without Ph. D. (very rare occurrence in high-energy theory)

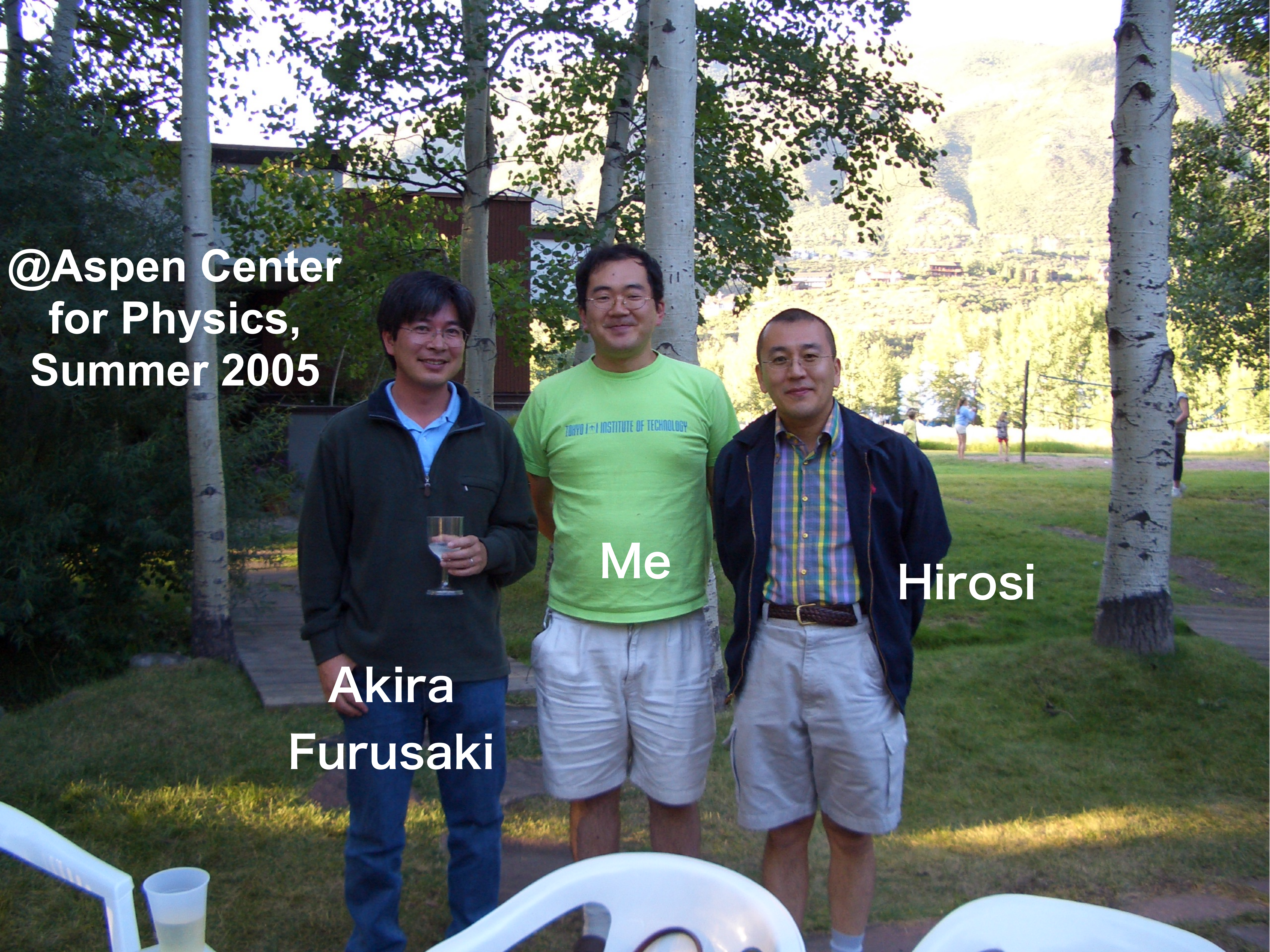
“Joshu” = Research Associate / Assistant Prof. / Wissenschaftlicher Assistent

I: entered UTokyo in April 1986, proceeded to Department of Physics in Summer 1987

Fall Semester 1987:
I was in Hirosi’s class for “Seminar” (problem-solving session) in Komaba Campus!

Me (1990)

Hirosi (1987?)


@Aspen Center
for Physics,
Summer 2005
Me
Hirosi
Akira
Furusaki

PRL **108,** 161803 (2012) PHYSICAL REVIEW LETTERS week ending 20 APRIL 2012

Instability in Magnetic Materials with a Dynamical Axion Field

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DOI: 10.1103/PhysRevLett.108.161803 PACS numbers: 14.80.Va, 73.61.−r

arXiv:1808.10466

Axion instability and non-linear electromagnetic effect

Tatsushi Imaeda, Yuki Kawaguchi, and Yukio Tanaka
Department of Applied Physics, Nagoya University, Nagoya 464-8603, Japan

Masatoshi Sato
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
(Dated: September 3, 2018)

We investigate the instability due to dynamical axion field near the topological phase transition of insulators. We first point out that the amplitude of dynamical axion field is bounded for magnetic insulators in general, which suppresses the axion instability. Near the topological phase transition, however, the axion field may have a large fluctuation, which decreases the critical electric field for the instability and increases the axion induced magnetic flux density. Using two different model Hamiltonians, we report the electromagnetic response of the axion field in details.

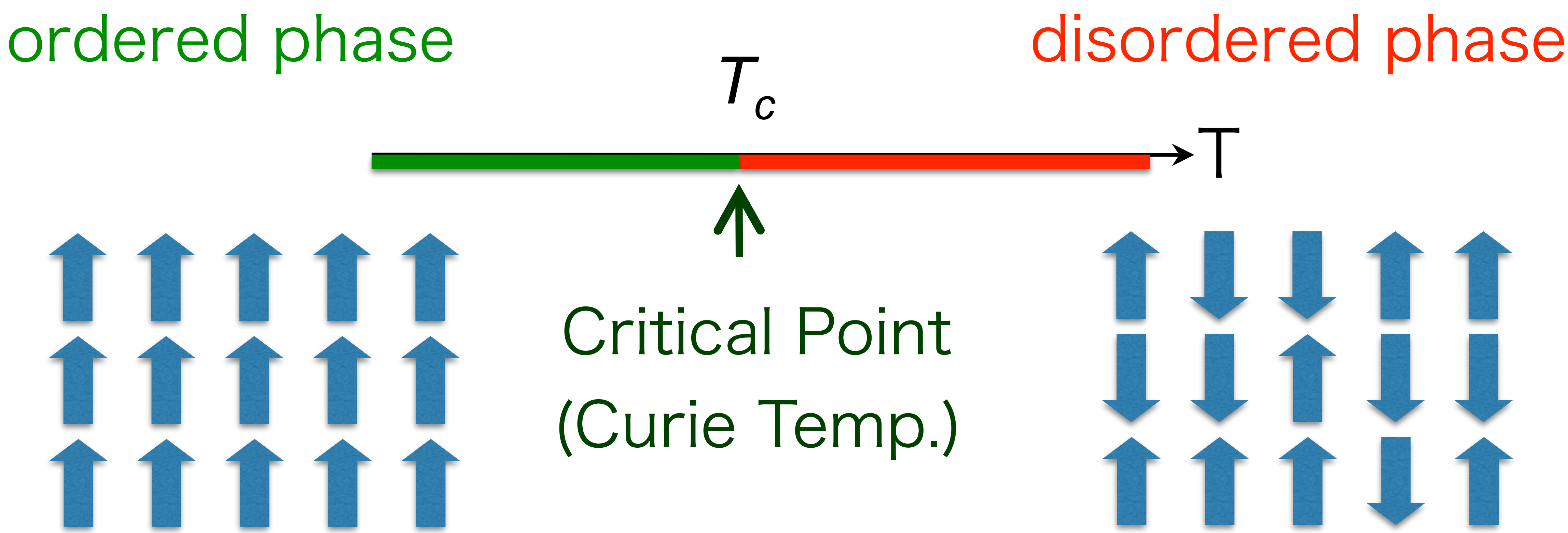
ISSP Public Lectures 2014

Largest turnout ～400 people
in the history of ISSP Public Lectures

Classification of states of matter

= distinction of different phases

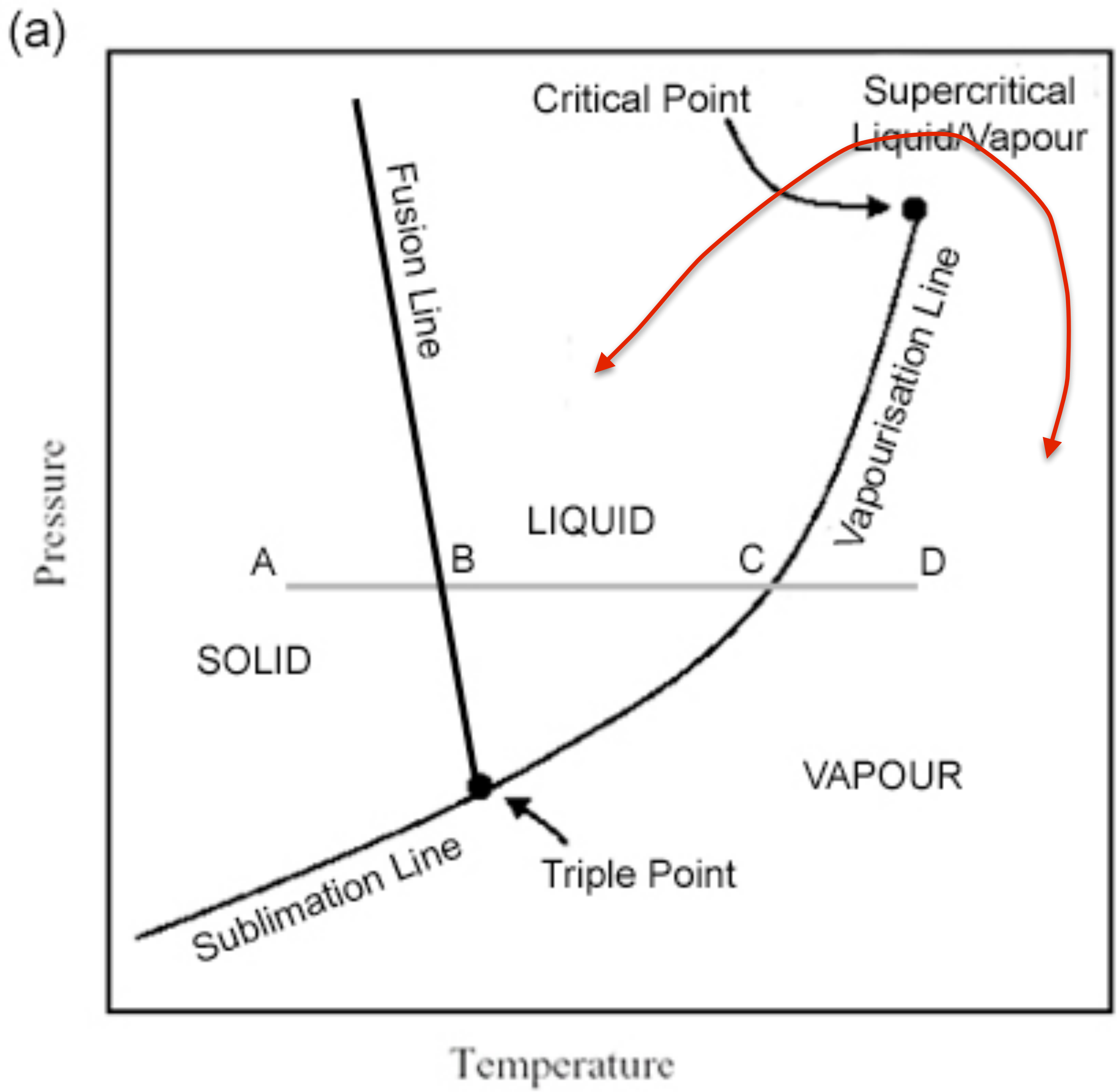
Phase diagram of a ferromagnet



simple model: (classical) Ising model

$$\mathcal{H} = J \sum_{\langle j,k \rangle} \sigma_j^z \sigma_k^z$$

Are liquid and gas different?



Phase transition can be “avoided” by going beyond the critical point

Liquid/gas are “essentially indistinguishable”

Figure from
Sonntag R E, Borgnakke C, Van Wylen G J, “Fundamentals of Thermodynamics”

What about solid?

Can we avoid the phase transition between solid/liquid at, e.g. higher pressures?

NO!

in solid, translation symmetry is spontaneously broken, while it is not in liquid/gas

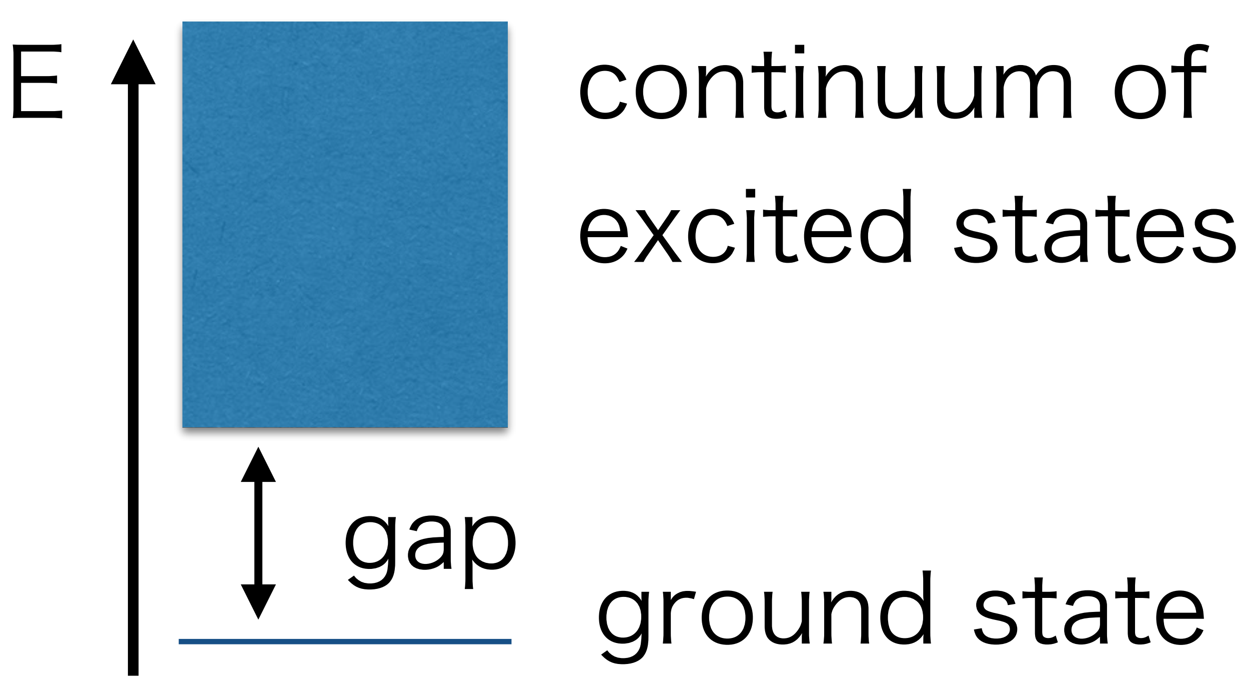
SSB clearly distinguish different phases, implying existence of phase transitions

Quantum Phase Transitions

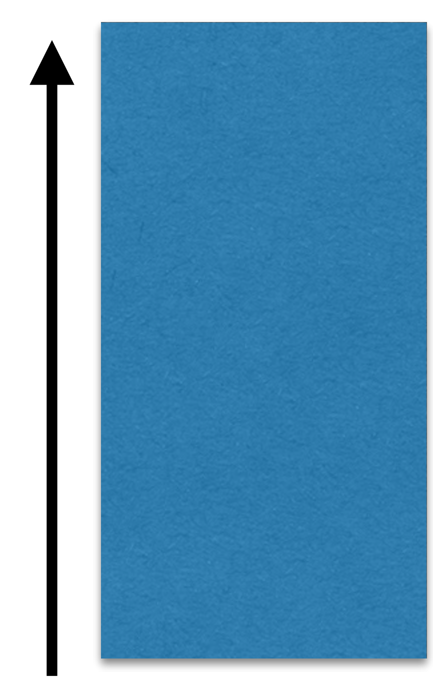
Quantum fluctuations can drive the system at T=0 into different quantum phases, and cause quantum phase transitions between quantum phases

Similarity (and in fact mathematical mapping in many cases) to classical phase transition driven by thermal fluctuations

gapped (off-critical)



gapless (critical)



What distinguishes different phases?

Different orders (or their absence) characterize each phase

Ferromagnet: magnetic order
Superfluid (3D): off-diagonal long-range order
(order of U(1) phase of wavefunctions)

etc.

"order" ⇔ Spontaneous Symmetry Breaking

??

However…………

Recently, it has been recognized that there are many quantum phases that are beyond understanding in terms of conventional orders/spontaneous symmetry breaking

“topological phases”

how to define them?

how to distinguish different phases?

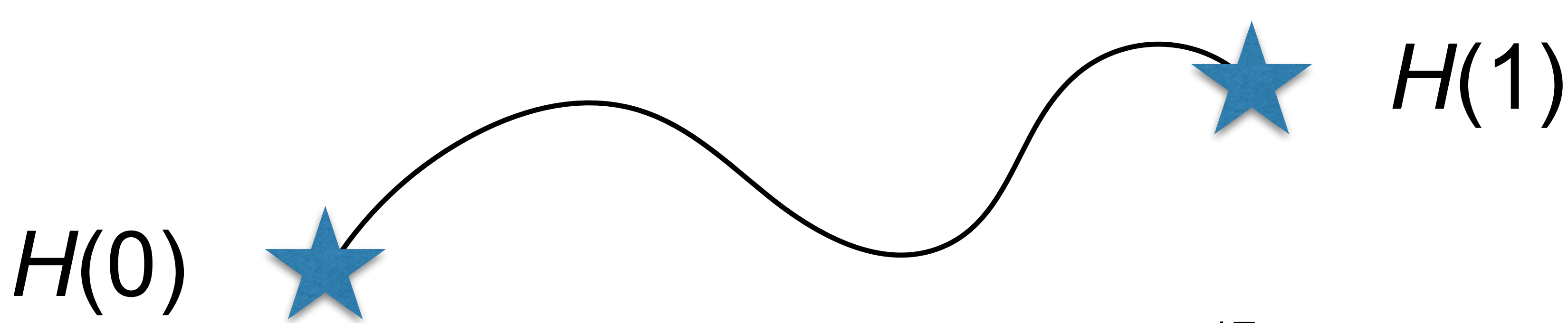
“Operational” definition of phases

A family of Hamiltonians $H(g)$ parametrized by g

Singularity in the ground state of $H(g)$,
as a function of g ⇒ **quantum phase transition**

If the two gapped ground states are connected adiabatically, i.e. if there exists a path $H(g)$ the two Hamiltonians without a quantum phase transition **they belong to the same phase**

Otherwise (if there is no adiabatic path connecting the two) they belong to different phases, even if there is no distinction in terms of SSB



Topological Order in 1D

Any gapped ground state of a local 1D Hamiltonian is connected to a trivial state adiabatically

Chen-Gu-Wen (2011)

Absence of (genuine) "topologically ordered phase" in 1D!

i.e. there is only one, trivial phase in 1D (in the absence of symmetries)

However, there can be more variety of phases if some symmetries are imposed

Imposing Symmetries

For a gapped Hamiltonian with a symmetry

1) the ground state is in a trivial phase
2) the symmetry is spontaneously broken in the ground state (SSB phase)
3) the ground state cannot be adiabatically connected to a trivial (product) state, even if we break the symmetry (topological order, absent in 1D)

4) the symmetry is unbroken, but the ground state cannot be adiabatically connected to a trivial (product) state as long as the symmetry is kept

“SPT phase”

the symmetry is unbroken, but the gapped ground state can **NOT** be **adiabatically connected to a trivial state**, if and only if the **Hamiltonian respects the symmetry**

then the ground state belongs to a Symmetry-Protected Topological Phase

(Generalization of “topological insulators” of free fermions to interacting systems)

Haldane gap

Heisenberg antiferromagnetic chain

$$\mathcal{H} = J \sum_j \vec{S}_j \cdot \vec{S}_{j+1}$$

S=1/2, 3/2, 5/2........

"massless" = gapless, power-law decay of spin correlations

S=1, 2, 3, ...

"massive" = non-zero gap, exponential decay of spin correlations

Haldane *conjecture* (1981)

Lieb-Schultz-Mattis theorem

For **translation** & SU(2) invariant spin chains

if S is integer: no constraint

if S is half-odd-integer:
the system must be gapless,
OR the ground state is at least doubly degenerate

Lieb-Schultz-Mattis 1961 (S=1/2 chain at zero magnetization)
Affleck-Lieb 1986 (arbitrary S chain at zero magnetization)
MO-Yamanaka-Affleck 1997, MO 2000, Hastings 2004, etc etc.

more generally, "filling-enforced constraints"

“Proof” by large gauge invariance

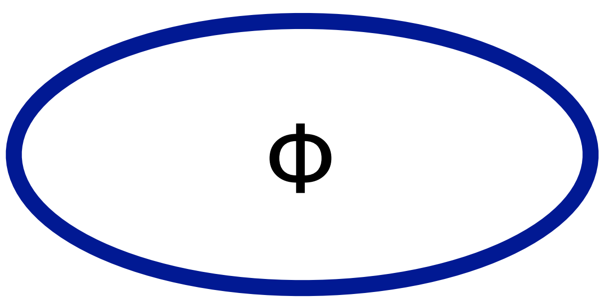
$$T_x|\Psi_0\rangle = e^{iP_x^0}|\Psi_0\rangle$$

LSM, Affleck-Lieb, M.O……

$$|\Psi_0\rangle \xrightarrow[\text{momentum unchanged}]{\text{adiabatic flux insertion}} |\Psi_0'\rangle$$

Large gauge transformation

$$U_x|\Psi_0'\rangle$$



$$T_x\left(U_x|\Psi_0'\rangle\right) = e^{2\pi i\nu}U_x T_x|\Psi_0'\rangle$$
$$= e^{i(P_x^0+2\pi\nu)}\left(U_x|\Psi_0'\rangle\right)$$

momentum shift by $2\pi\nu = 2\pi(S-m)$

the “new” ground state has extra momentum π for half-odd-int *S*

Letters in Mathematical Physics **12** (1986) 57–69.

57

Affleck-Lieb 1986
S: half-odd-integer
→ gapless or
2-fold g.s. degeneracy

A Proof of Part of Haldane's Conjecture on Spin Chains

IAN AFFLECK★ and ELLIOTT H. LIEB★★
Departments of Mathematics and Physics, Princeton University, P.O. Box 708, Princeton, NJ 08544, U.S.A.

(Received: 10 March 1986)

Abstract. It has been argued that the spectra of infinite length, translation and $U(1)$ invariant, anisotropic, antiferromagnetic spin s chains differ according to whether s is integral or $\frac{1}{2}$ integral: There is a range of parameters for which there is a unique ground state with a gap above it in the integral case, but no such range exists for the $\frac{1}{2}$ integral case. We prove the above statement for $\frac{1}{2}$ integral spin. We also prove that for *all s*, *finite length* chains have a unique ground state for a wide range of parameters. The argument was extended to $SU(n)$ chains, and we prove analogous results in that case as well.

was a generalization of
"Lieb-Schultz-Mattis Theorem"

ANNALS OF PHYSICS: **16**, 407–466 (1961)

Two Soluble Models of an Antiferromagnetic Chain

ELLIOTT LIEB, THEODORE SCHULTZ, AND DANIEL MATTIS
Thomas J. Watson Research Center, Yorktown, New York

II. THE XY MODEL

A. FORMULATION

The first model consists of N spin ½'s (N even) arranged in a row and having only nearest neighbor interactions. It is

$$H_\gamma = \sum_i [(1+\gamma) S^x_i S^x_{i+1} + (1-\gamma) S^y_i S^y_{i+1}], \tag{2.1}$$

a's and $a^\dagger$'s do not preserve this mixed set of canonical rules. However, it is possible to transform to a new set of variables that are strictly Fermi operators and in terms of which the Hamiltonian is just as simple.[1] Let

$$c_i \equiv \exp\left[\pi i \sum_1^{i-1} a_j^\dagger a_j\right] a_i$$

Main Result of "LSM" paper:
S=1/2 XY chain is solvable by mapping to fermions

What about the LSM theorem?

APPENDIX B. NONDEGENERACY OF THE GROUND STATE AND ABSENCE OF AN ENERGY GAP IN THE HEISENBERG MODEL

We prove two exact theorems about the ground state and excitation spectrum for a Heisenberg model with nearest neighbor interactions in one dimension.

Appendix….

APPENDIX B. NONDEGENERACY OF THE GROUND STATE AND ABSENCE OF AN ENERGY GAP IN THE HEISENBERG MODEL

We prove two exact theorems about the ground state and excitation spectrum for a Heisenberg model with nearest neighbor interactions in one dimension. The generalization to longer range interactions and higher-dimensional lattices is indicated. A further generalization to particles of spin $\neq \frac{1}{2}$ and a discussion of the ordering of excited state energy levels has been submitted for publication in the *Journal of Mathematical Physics* by Lieb and Mattis.

?!

Perhaps refers to this paper

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 3, NUMBER 4 JULY-AUGUST 1962

Ordering Energy Levels of Interacting Spin Systems

ELLIOTT LIEB AND DANIEL MATTIS
Thomas J. Watson Research Center, International Business Machines Corporation, Yorktown Heights, New York
(Received October 6, 1961)

But no mention is actually made on the generalization of LSM theorem?!

Maybe….

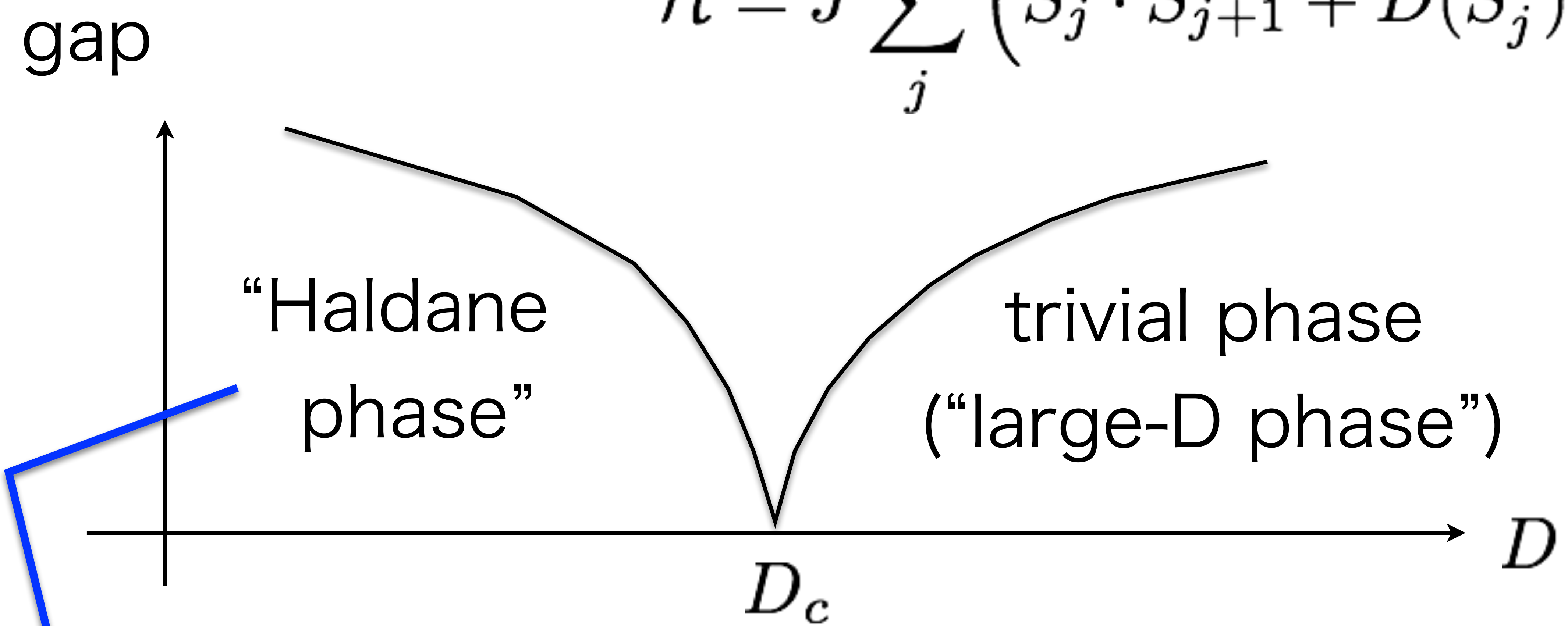
LSM tried to generalize their theorem to general *S*,
but "failed" to prove it for integer *S*

So they scrapped the generalization and never published
(until Affleck-Lieb paper 25 years ago)

…. maybe missing the evidence of the "Haldane gap"??

Haldane Phase and QPT

$$\mathcal{H} = J\sum_j \left(\vec{S}_j \cdot \vec{S}_{j+1} + D(S_j^z)^2\right).$$



$D \to \infty$

$|\mathcal{D}\rangle = |000000\ldots\rangle$

quantum phase transition

No local order parameter, but is a distinct "Symmetry-Protected Topological Phase"

Haldane Phase as a SPT

In the presence of *any one* of the following symmetries, the Haldane phase is separated from a trivial (product) state by a quantum phase transition:

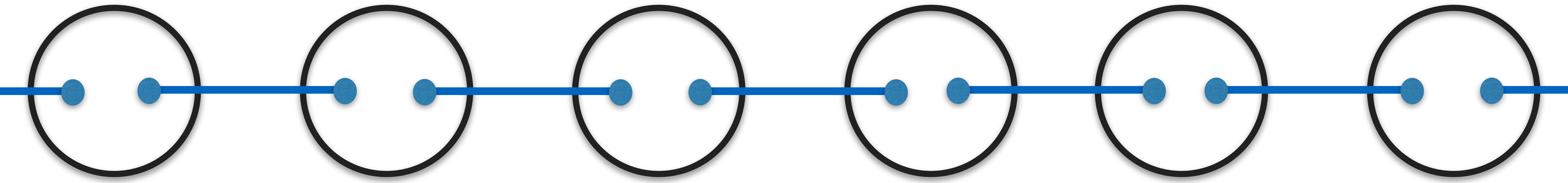
i) time reversal symmetry
ii) dihedral ($Z_2 \times Z_2$) symmetry (π-rotation about x, y, and z axes)
iii) lattice inversion symmetry about a bond center

Gu-Wen (2009)
Pollmann-Turner-Berg-MO (2010)

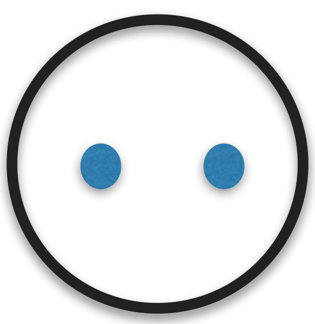
AKLT model/state

$$\mathcal{H} = J\sum_j \left[\vec{S}_j \cdot \vec{S}_{j+1} + \frac{1}{3}\left(\vec{S}_j \cdot \vec{S}_{j+1}\right)^2\right]$$

Exact groundstate: (Affleck-Kennedy-Lieb-Tasaki 1987)



 Singlet pair of two S=1/2's -"valence bonds"

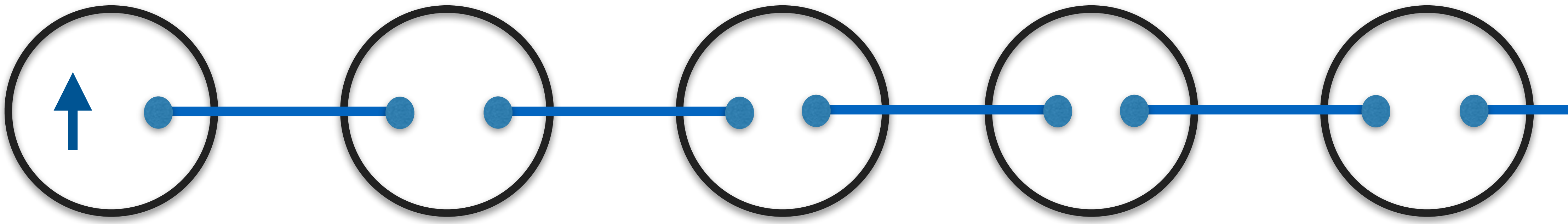
 Symmetrization of two S=1/2's ⇒ S=1

✓non-zero gap, exponential decay of correlations

(supporting the Haldane conjecture)

Why SPT?

Pollmann-Turner-Berg-MO (2009)



Open boundary condition : “edge state” of S=1/2

The ground state is doubly degenerate because of the edge spin (4-fold considering both ends)

This degeneracy is exact under time reversal (Kramers degeneracy):

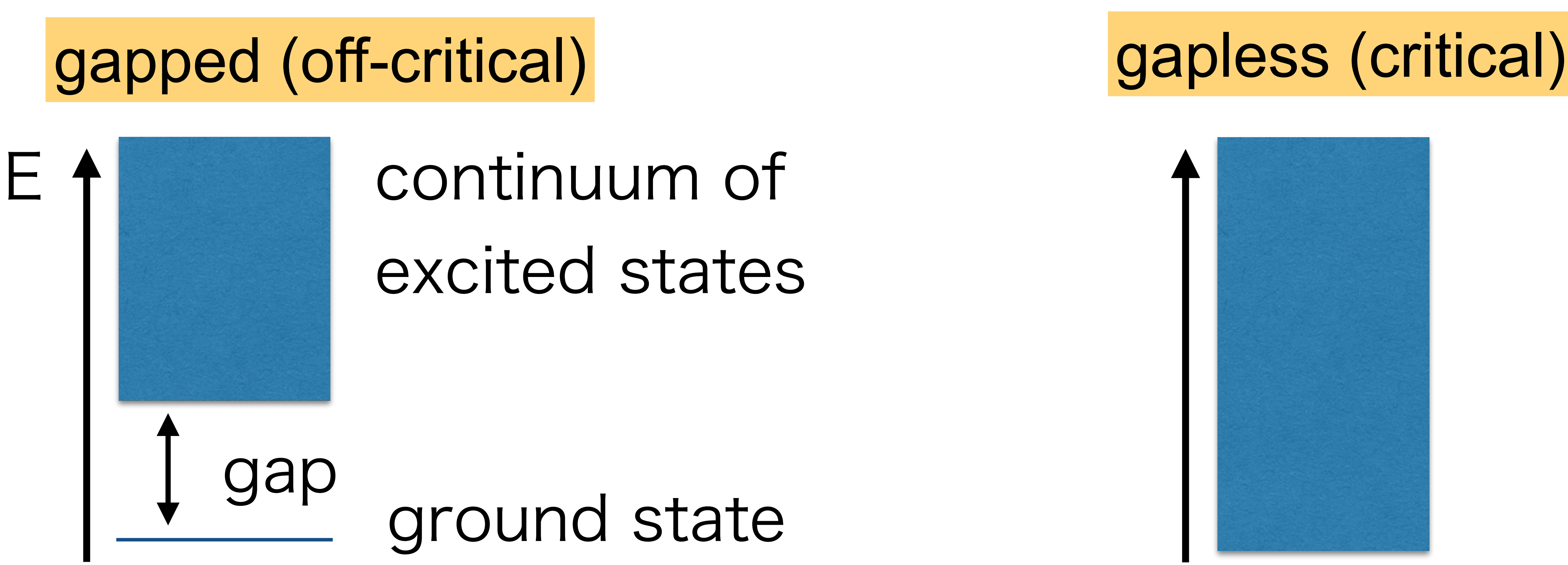
⇒ time reversal must be broken, or there must be

a quantum phase transition to remove the degeneracy!

Quantum Phase Transitions

Quantum fluctuations can drive the system at T=0 into different quantum phases, and cause quantum phase transitions between quantum phases

Similarity (and in fact mathematical mapping in many cases) to classical phase transition driven by thermal fluctuations

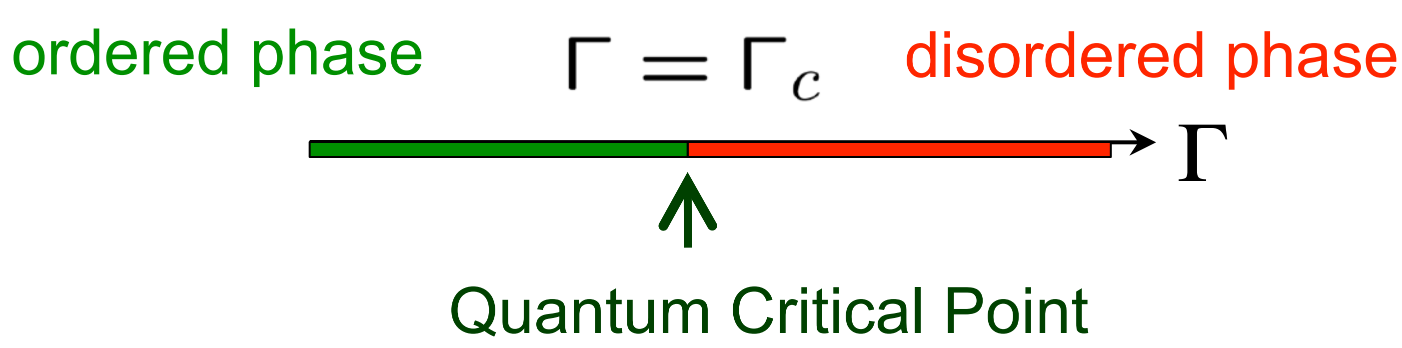


Gapless Quantum Critical Point

Gapless excitations appear at quantum critical points

e.g. (quantum) transverse Ising model

$$\mathcal{H} = -\sum_{\langle j,k \rangle} \sigma_j^z \sigma_k^z - \Gamma \sum_j \sigma_j^x$$



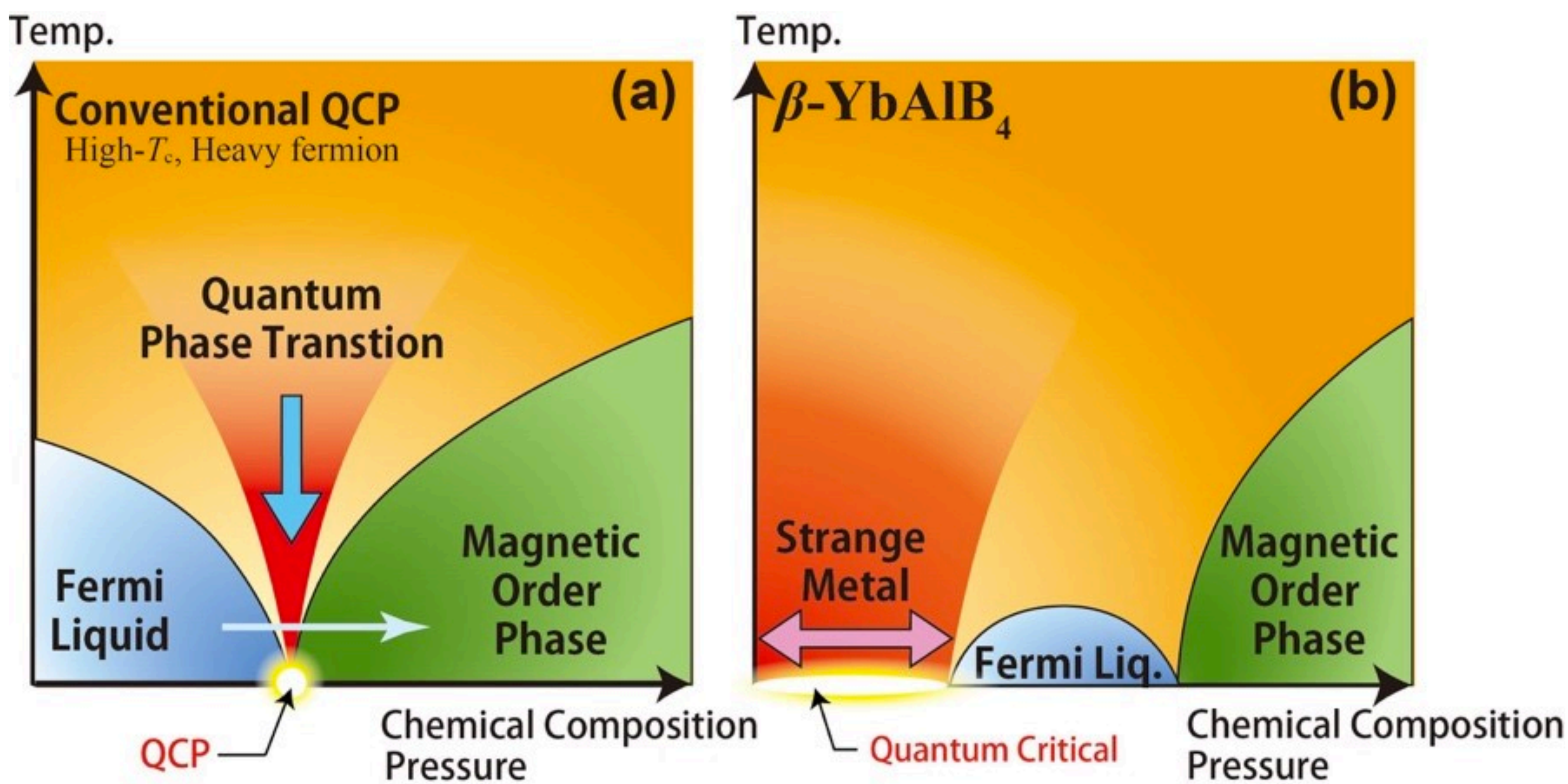
critical point = RG fixed point

relevant perturbation → gap

Gapless Critical Phases

However, quantum critical phases often appear in cond-mat physics without any apparent fine-tuning

- metallic systems
- Dirac/Weyl semimetals
- β-$YbAlB_4$



[Nakatsuji Group, ISSP]

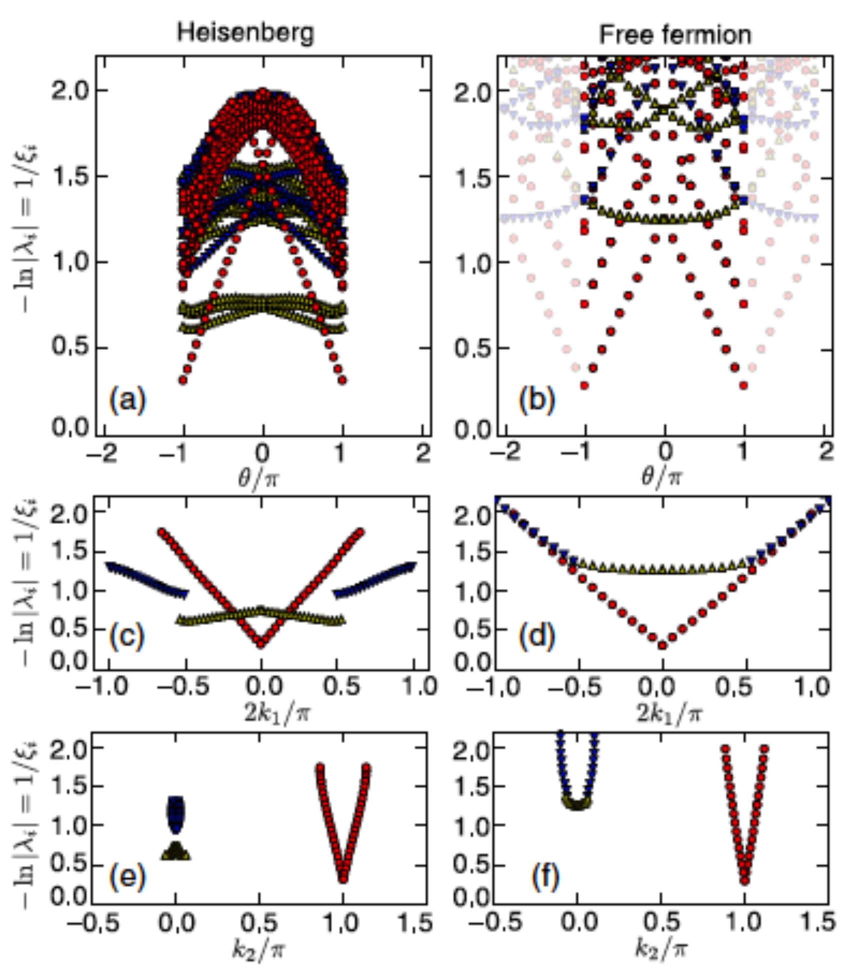
Gapless Critical Phases

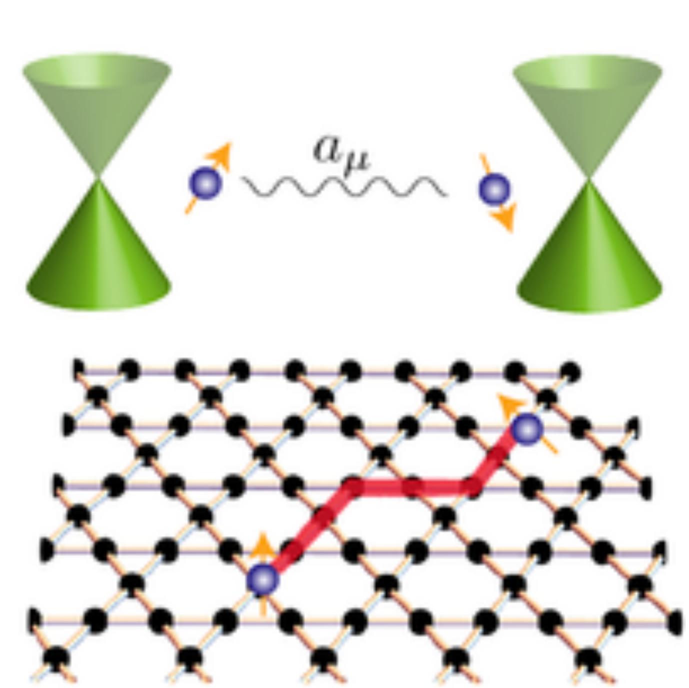
- Kagome spin liquid (*S*=1/2 antiferromagnet):
  Dirac spin liquid?

PHYSICAL REVIEW X **7**, 031020 (2017)

Signatures of Dirac Cones in a DMRG Study of the Kagome Heisenberg Model

Yin-Chen He,[1,2,3] Michael P. Zaletel,[4,3,6] Masaki Oshikawa,[5,3] and Frank Pollmann[1,3,7]





Gapless Critical Phases

- Why are they stable?

- Classification/characterization of these phases

We do have some understanding based on CFT etc.
but we need more!

We can gain some insights from the recent developments
in the classification of gapped topological phases…

Generalization of SPT phases?

I will attempt to extend the notion of
"Symmetry-Protected (Topological) Phases"
to gapless phases

I will discuss an example in 1+1 dimensions
(spin chains, effective CFT)
although the concept can be hopefully generalized
to higher dimensions

S. C. Furuya & M. O. Phys. Rev. Lett. 118, 021601 (2017)

Y. Yao, C.-T. Hsieh, & M. O. arXiv:1805.06885

Our Model

Spin-*S* antiferromagnetic chain with the global SU(2) and lattice translation symmetries

$$\mathcal{H} = \sum_j \left[\vec{S}_j \cdot \vec{S}_{j+1} + J_q \left(\vec{S}_j \cdot \vec{S}_{j+1} \right)^2 + J_2 \vec{S}_j \cdot \vec{S}_{j+2} \cdots \right]$$

Lorentz invariance is expected;

when gapless, low-energy physics should be described by a SU(2) symmetric CFT

$SU(2)_k$ Wess-Zumino-Witten theory

characterized by "level" $k = 1, 2, 3, \ldots$

$$\langle \vec{S}_0 \cdot \vec{S}_r \rangle \propto (-1)^r \left(\frac{1}{r} \right)^{3/(k+2)}$$

Our Claim

In the presence of the SU(2) and
lattice translation (by one site) symmetries,

$$S = 1/2, 3/2, 5/2, \ldots$$

- The system is gapped with a SSB of the translation symmetry (doubly degenerate GS)

OR - The system is gapless, described by **$SU(2)_k$ WZW with an odd k**

$$S = 1, 2, 3, \ldots$$

- The system is gapped (can be without SSB)

OR - The system is gapless, described by **$SU(2)_k$ WZW with an even k**

“Symmetry Protected” gapless phases

SU(2) + Lorentz + lattice translation symmetries

SU(2)$_k$ WZW k: even	**SU(2)$_k$ WZW** k: odd

SU(2) WZW Theory

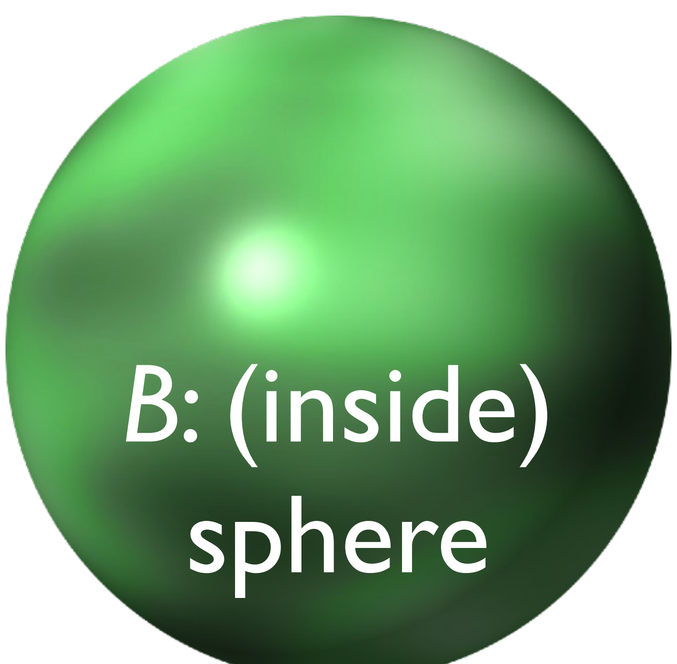
$$S = S_0 + k\Gamma_{WZ}$$

g: SU(2) matrix-valued field

$$S_0 = \frac{1}{2\lambda^2}\int d^2x\ \mathrm{Tr}[(g^{-1}\partial_\mu g)^2]$$

$$\Gamma_{WZ} = \frac{1}{12\pi}\int_B d^3x\ \epsilon^{ijk}\mathrm{Tr}[(g^{-1}\partial_i g)(g^{-1}\partial_j g)(g^{-1}\partial_k g)]$$

original space-time:
surface of the sphere



uniqueness of $k\Gamma_{WZ}$
(modulo 2π)
⇒ *k*: integer

RG has a nontrivial fixed point
if $k \neq 0$ → gapless critical phase

Spin chain and WZW

$$\vec{S}_i \sim \vec{J}_i + \mathrm{const.}(-1)^i \mathrm{tr}(g\vec{\sigma})$$

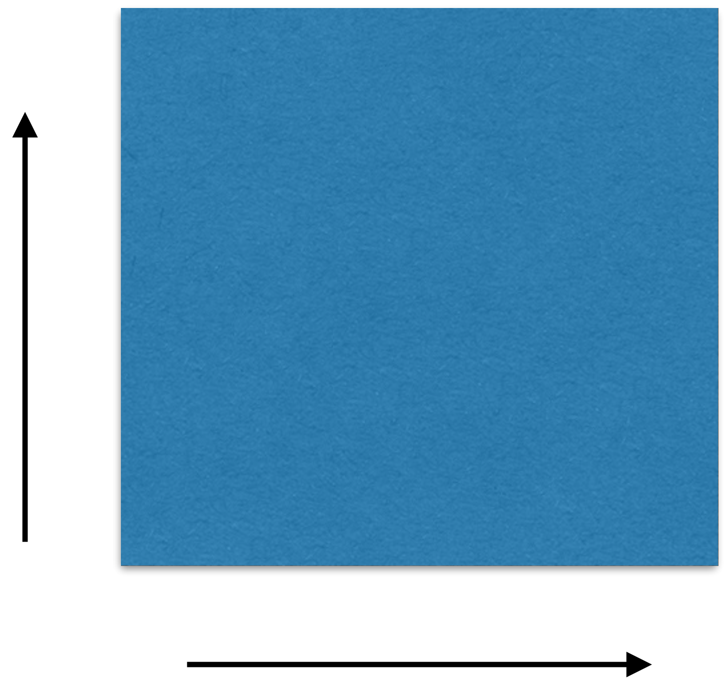
Lattice translation symmetry

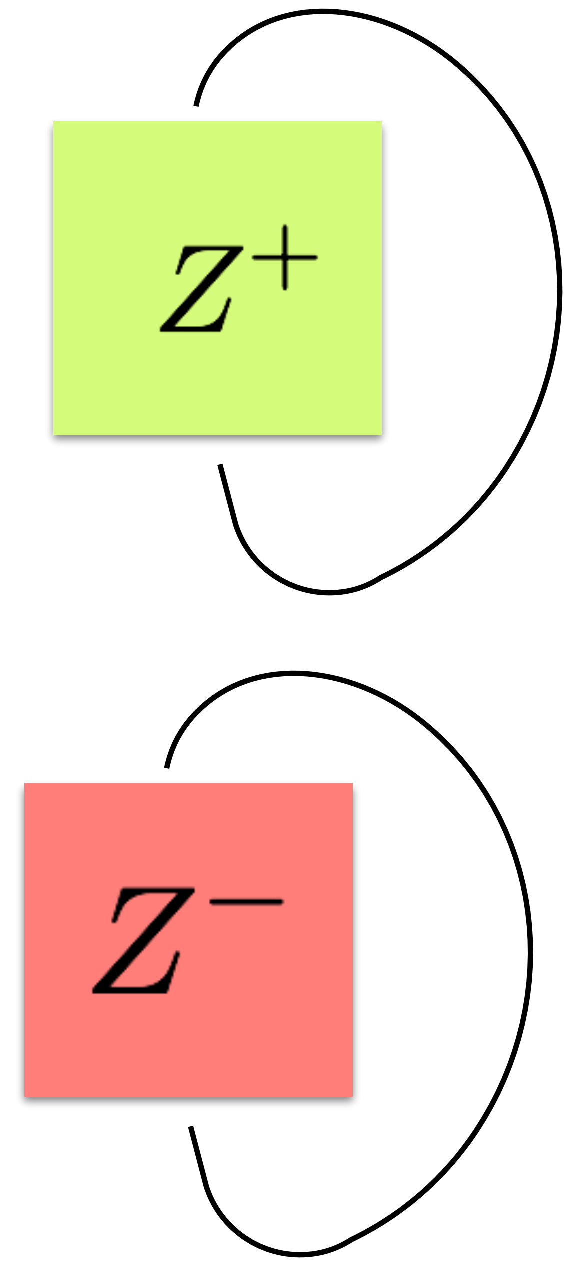
⇔ discrete Z_2 symmetry $g \to -g$

If there is the Z_2 symmetry,

we should be able to consider a projection to Z_2-symmetric subspace?

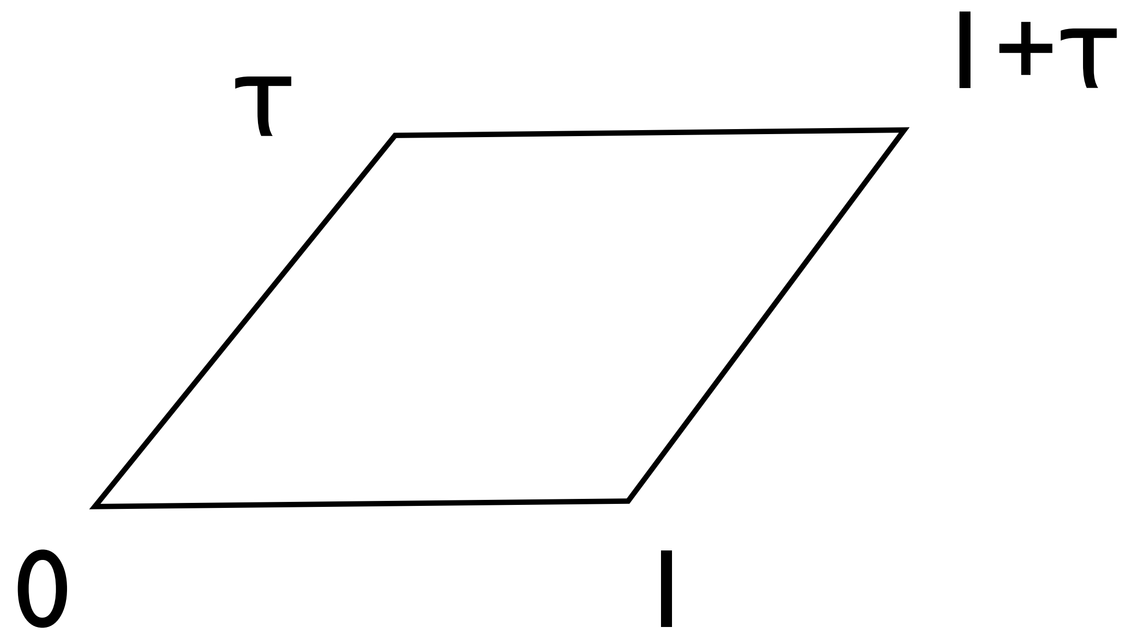
Projection vs. Path Integral





$$Z_+^{\mathrm{proj}} = \mathrm{Tr}[P_+ e^{-\mathcal{H}}] = \frac{1}{2}[Z^+ + Z^-]$$

Modular Invariance



Partition function of a consistent CFT must be invariant under modular transformations generated by

$$\mathcal{S} : \tau \to -1/\tau$$

$$\mathcal{T} : \tau \to \tau + 1$$

Orbifold Construction

The "projected" partition function Z_+^{proj} is not modular invariant by itself — must be supplemented by twisted sectors

$$Z_+ = (1 + \mathcal{S} + \mathcal{T}\mathcal{S}) Z_+^{\mathrm{proj}} - Z_{\mathrm{WZW}}$$

The resulting partition function represents the "Z_2 orbifold" of the original $SU(2)_k$ WZW theory

Global Anomaly

('t Hooft anomaly)

The Z_2 orbifold should be modular invariant by construction — but this is NOT always the case!

The Z_2 orbifold is **modular invariant if *k* is even**, but it is **modular NON-invariant if *k* is odd**

Gepner-Witten 1986

STRING THEORY ON GROUP MANIFOLDS

Doron GEPNER and Edward WITTEN

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544, USA

Received 26 May 1986

What does this mean?

If the orbifold is modular invariant, we can consider projection onto the symmetric sector, and open a gap within that sector to obtain the unique ground state

However, if it is modular non-invariant (ie. k: odd),
we cannot open the gap to obtain a unique ground state within the symmetric sector;
ground states in the symmetric/antisymmetric sectors must be degenerate!

"Lieb-Schultz-Mattis (LSM) constraint" in CFT!

Global anomaly =
"ingappability" in the presence of the symmetry
(S. Ryu et al. on edge theory)

Selection Rule

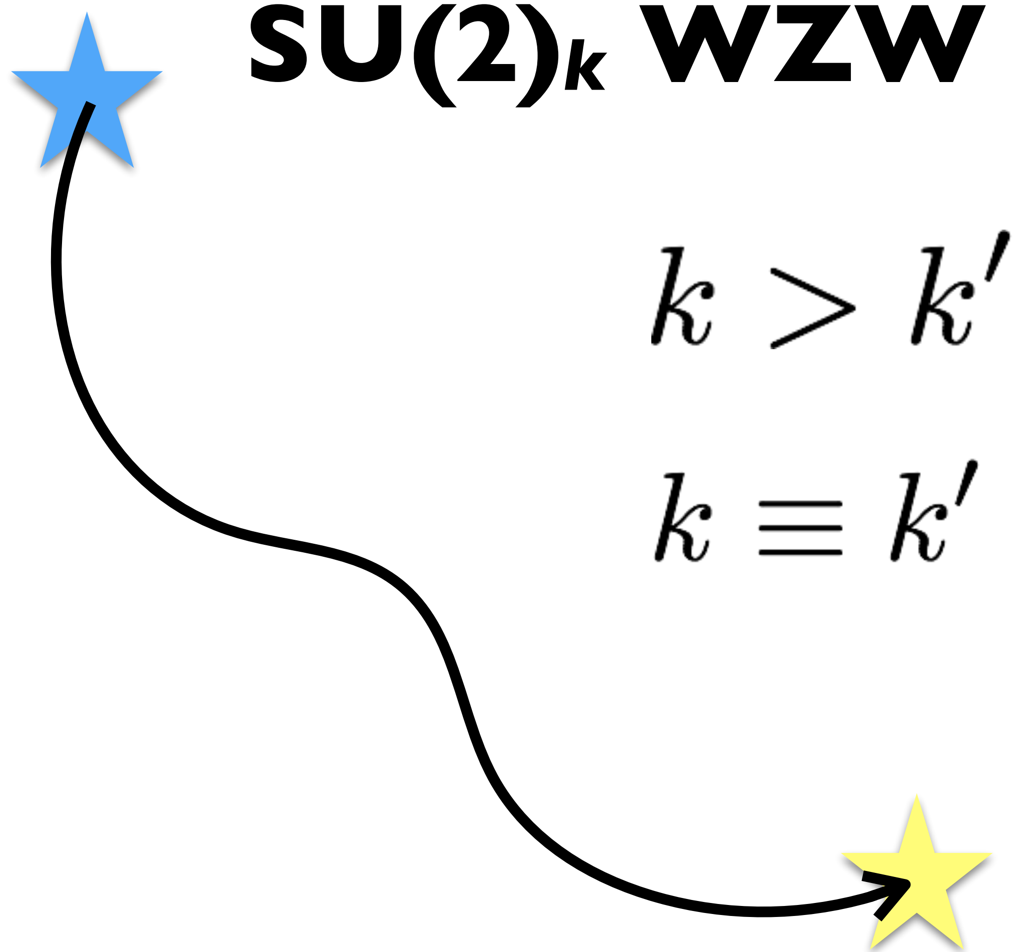
Perturb $SU(2)_k$ WZW with SU(2) and Z_2-symmetric relevant operators; suppose the RG flow reaches $SU(2)_{k'}$ WZW

if k is even, we should be able to consider the projection onto Z_2 symmetric sector; the RG flow can be understood in terms of the Z_2 orbifold → k' is also even

if k is odd, the IR fixed point should also have the global anomaly (otherwise contradicts with LSM) → k' is also odd

"anomaly matching"

In terms of RG…



$SU(2)_k$ WZW

$$k > k'$$ ← c- theorem

$$k \equiv k' \mod 2$$ ← present work

$SU(2)_{k'}$ WZW

$SU(2)_0$ WZW is identified with gapped phase with a unique ground state

Spin Chains and WZW

There is a special integrable (Bethe-ansatz solvable) spin chain model for any *S*, Takhtajan-Babujian (TB) model

e.g. for *S*=1: $$\mathcal{H}_{TB} = \sum_j \left[\vec{S}_j \cdot \vec{S}_j - (\vec{S}_j \cdot \vec{S}_j)^2 \right]$$

Spin-*S* TB model is described by $SU(2)_{2S}$ WZW (k=2S even if *S* is integer, k odd if *S* is half-odd integer)

Other models can be regarded as TB model + perturbations, so
k: even if *S* is integer, k:odd if *S* is half-odd integer
if the one-site translation symmetry is kept

Our Claim

In the presence of the SU(2) and
lattice translation (by one site) symmetries,

$$S = 1/2, 3/2, 5/2, \ldots$$

- The system is gapped with a SSB of the translation symmetry (doubly degenerate GS)

OR - The system is gapless, described by **$SU(2)_k$ WZW with an odd k**

$$S = 1, 2, 3, \ldots$$

- The system is gapped (can be without SSB)

OR - The system is gapless, described by **$SU(2)_k$ WZW with an even k**

Summary & Outlook

"Global $\mathbf{Z}_2$ anomaly" ('t Hooft anomaly) discovered by Gepner and Witten in 1986 can be interpreted, in the condensed matter / lattice context, an inheritance of **Lieb-Schultz-Mattis constraint** in the microscopic model to CFT as low-energy effective field theories

⇒ ***"Symmetry-Protected Critical Phases"***

Lieb-Schultz-Mattis type constraints in higher dimensions and for various symmetries: corresponding anomalies?

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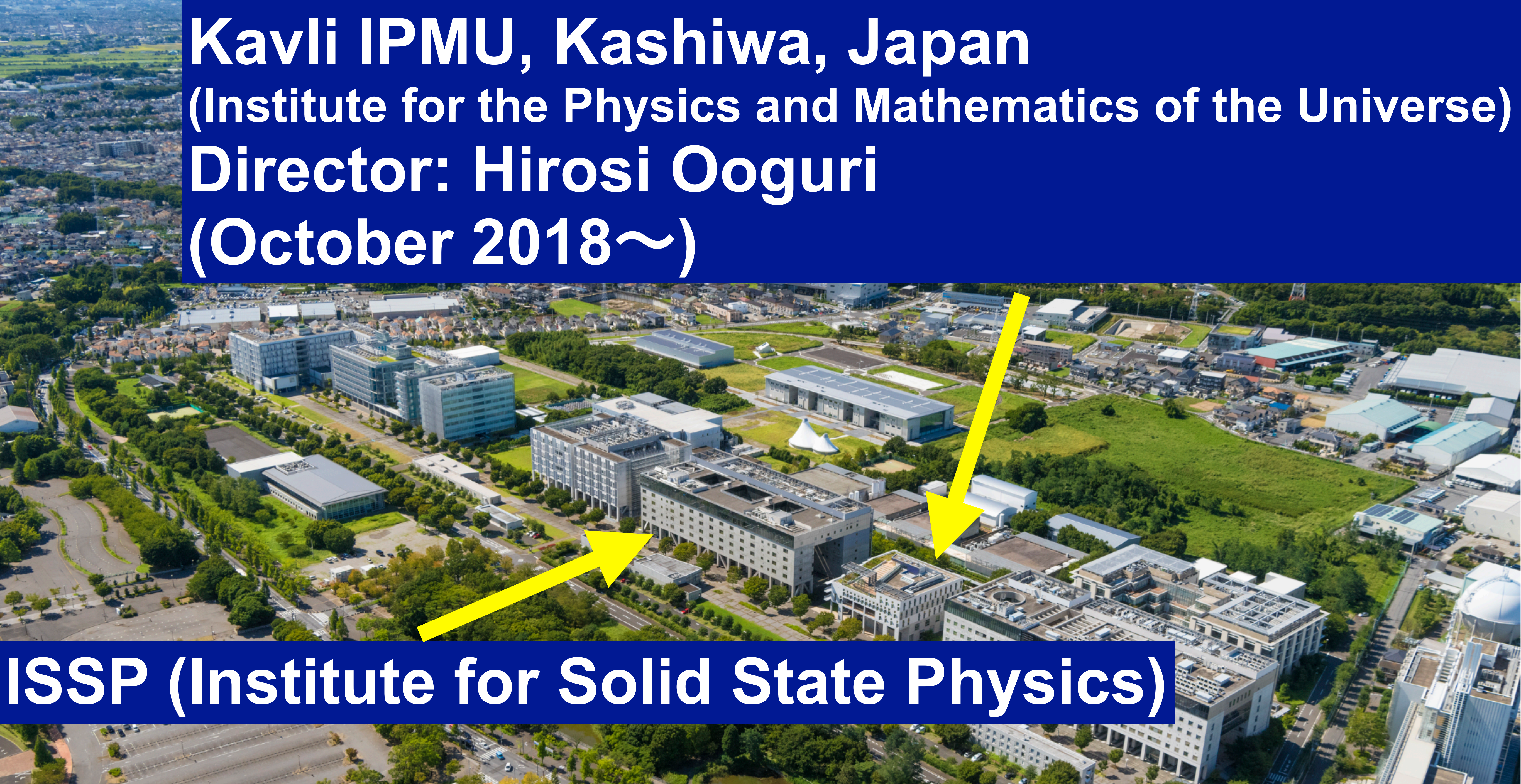

High-energy physics and mathematics are closer to condensed matter physics, than you might think…

We will see more

unexpected and fruitful encounters!