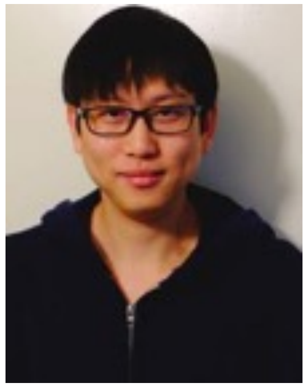
Topology, quantum criticality, and duality

T. Senthil (MIT)



Most recent collaborator: **Zhen Bi (MIT)**

arXiv:1808.07465

Thanks: Max Metlitski (MIT), Nati Seiberg (IAS)

Primer on quantum condensed matter physics

Known UV degrees of freedom   (eg, electrons/spins/bosons)

↓

IR: phases/phase transitions

Orientation

Conventional ordered phases of matter:

Concepts of broken symmetry/ Long Range Order (LRO)

Characterize by Landau order parameter.

Examples

$$| \uparrow\uparrow\uparrow\uparrow \,\ldots\ldots\ldots \rangle \qquad\qquad | \uparrow\downarrow\uparrow\downarrow \,\ldots\ldots\ldots \rangle$$

Ferromagnet Antiferromagnet

Landau Order

Known for several millenia

Non-Landau order I: Topological quantum matter

Low energy effective theory: a topological quantum field theory

Known since 1980s

Non-Landau order I: Topological quantum matter

Low energy effective theory: a topological quantum field theory

Known since 1980s

Examples

(i) (Fractional) Quantum Hall effect

Phenomena: Quantization of Hall conductance, emergence of anyons with fractional quantum numbers, gapless spatial boundaries,……

Low energy effective theory: Chern-Simons theory

Non-Landau order I: Topological quantum matter

Low energy effective theory: a topological quantum field theory

Known since 1980s

Examples

(i) (Fractional) Quantum Hall effect

Phenomena: Quantization of Hall conductance, emergence of anyons with fractional quantum numbers, gapless spatial boundaries,……

Low energy effective theory: Chern-Simons theory

(ii) Symmetry Protected Topological (SPT) phases of matter

Phenomena: Gapped ground state with `trivial' excitations (no anyons), interesting physics at spatial boundaries.

Distinction with completely trivial ground state is protected by a global symmetry.

Non-Landau order II: Beyond topological order.
Gapless phases

Most familiar: Landau fermi liquid (!)

Interesting variants: Dirac, Weyl,…… materials

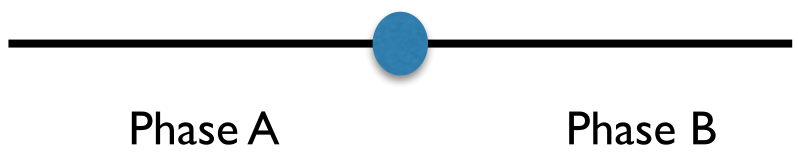
Even more striking: non-fermi liquid metals, ,,,,,,,

- no quasiparticle description of excitation spectrum!

Slowly evolving understanding in last 25 years.

Critical quantum matter (focus of this talk)

Continuous T = 0 quantum phase transitions

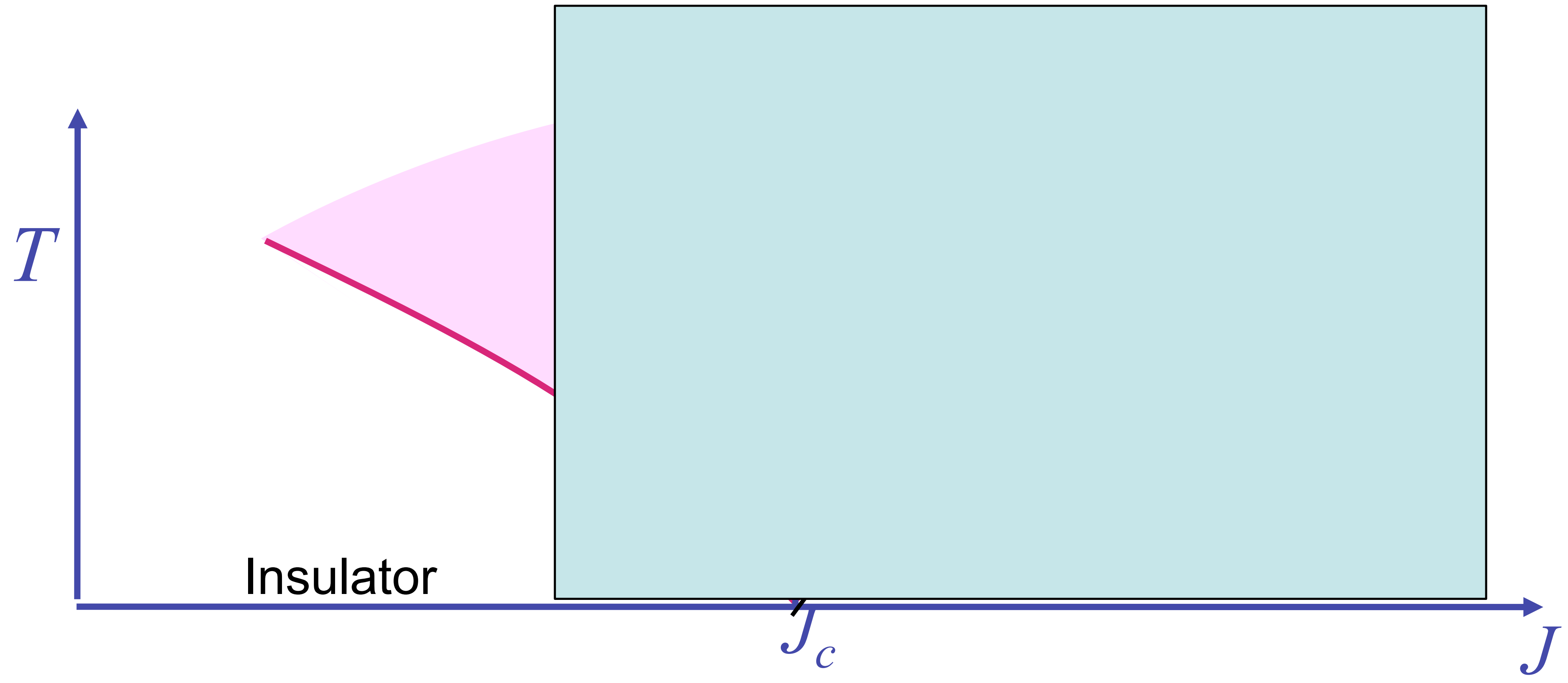


Qualitative change in nature of many body ground state as a function of a tuning parameter.

Typical examples: lose quasiparticle description at the quantum critical point.

Quantum Phase Transitions: Generalities

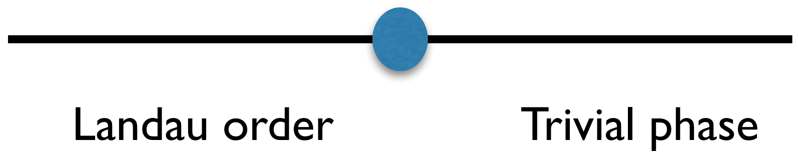
- Universal critical singularities similar to thermal phase transitions
- Continuous quantum phase transitions can control the finite temperature physics in a region.



Quantum criticality in condensed matter/field theory

Our intuition for what kinds of continuous quantum phase transitions are possible and their description is very poor.

Textbook examples:



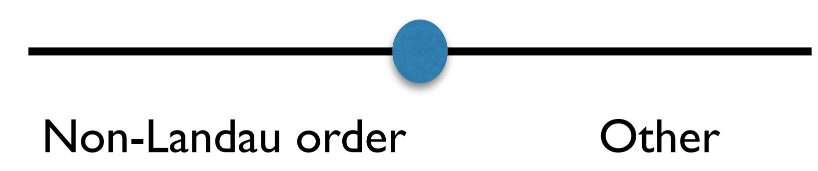
Universal critical singularities: Long wavelength, long time fluctuations of Landau order parameter.

Describe by continuum quantum field theory at zero temperature (may or may not be a CFT).

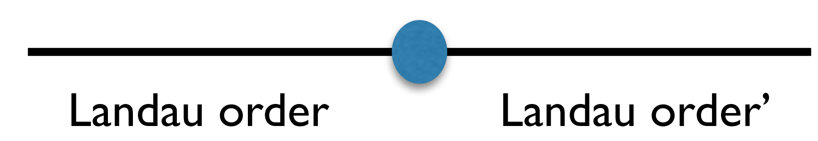
Quantum Landau-Ginzburg-Wilson (LGW) theory of fluctuating order parameter

Quantum criticality beyond the Landau paradigm

Eg: 1. One or both phases have non-Landau order



2. More surprising: Landau-forbidden continuous phase transitions between Landau allowed phases



TS, Vishwanath, Balents, Fisher, Sachdev, 2004

Phase transitions in quantum magnets

Spin-1/2 magnetic moments on a square lattice

Model Hamiltonian

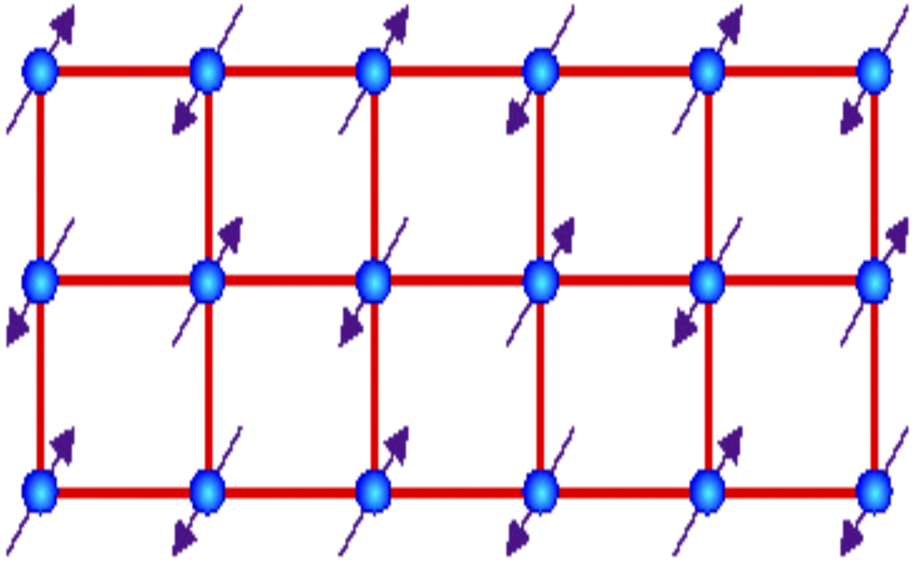
$$H_0 = J \sum_{<rr'>} \vec{S}_r \cdot \vec{S}_{r'} + \cdots$$

$\cdots$ = additional interactions to tune quantum phase transitions

Usual fate: Neel antiferromagnetic order

Breaks SO(3) spin rotation symmetry.

Neel order parameter: SO(3) vector

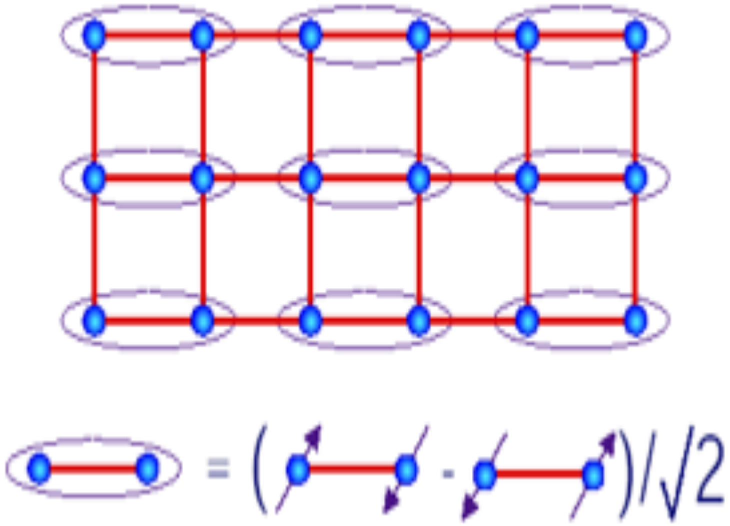


An SO(3) symmetry preserving phase (``quantum paramagnet")

With suitable additional interactions, obtain other phases that preserve spin rotation symmetry.

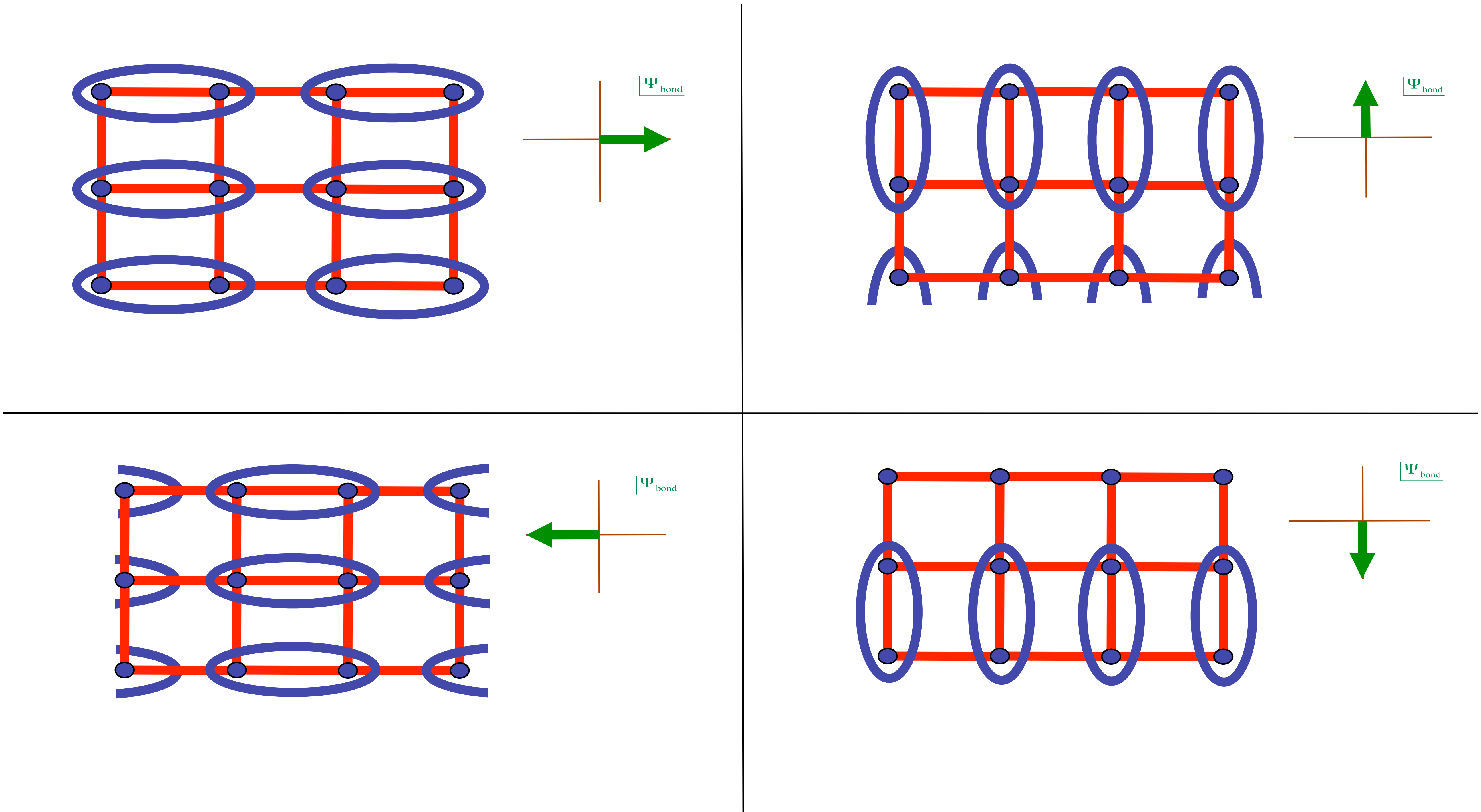
Focus on a particular such phase called a Valence Bond Solid (VBS) that breaks lattice symmetries.

Z_4 order parameter associated with four patterns of VBS ordering.

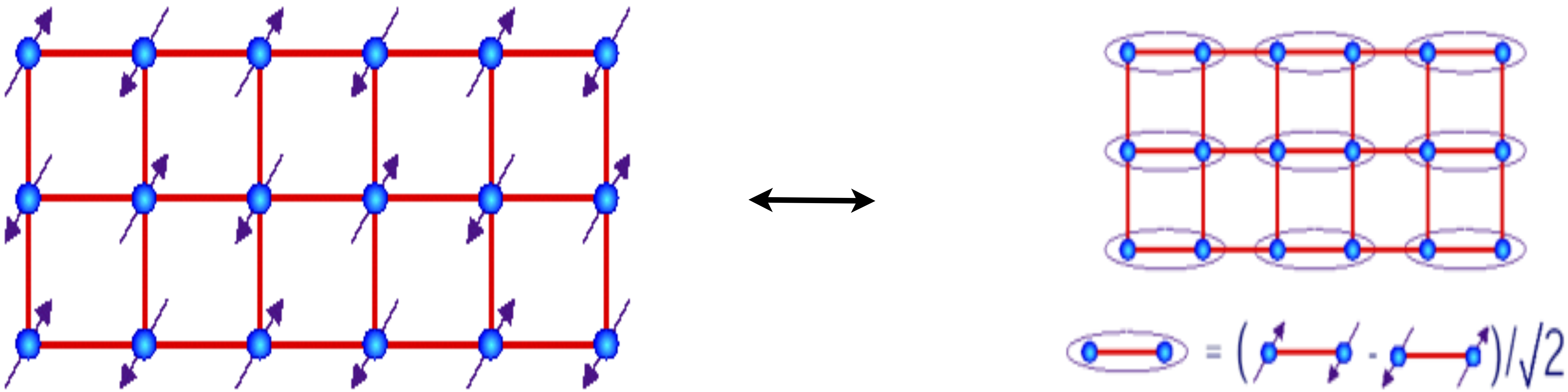


VBS Order Parameter

- Associate a Complex Number Ψ_{bond}



The Neel-VBS quantum phase transition



Naive Landau expectation: Two independent order parameters - no generic direct second order transition.

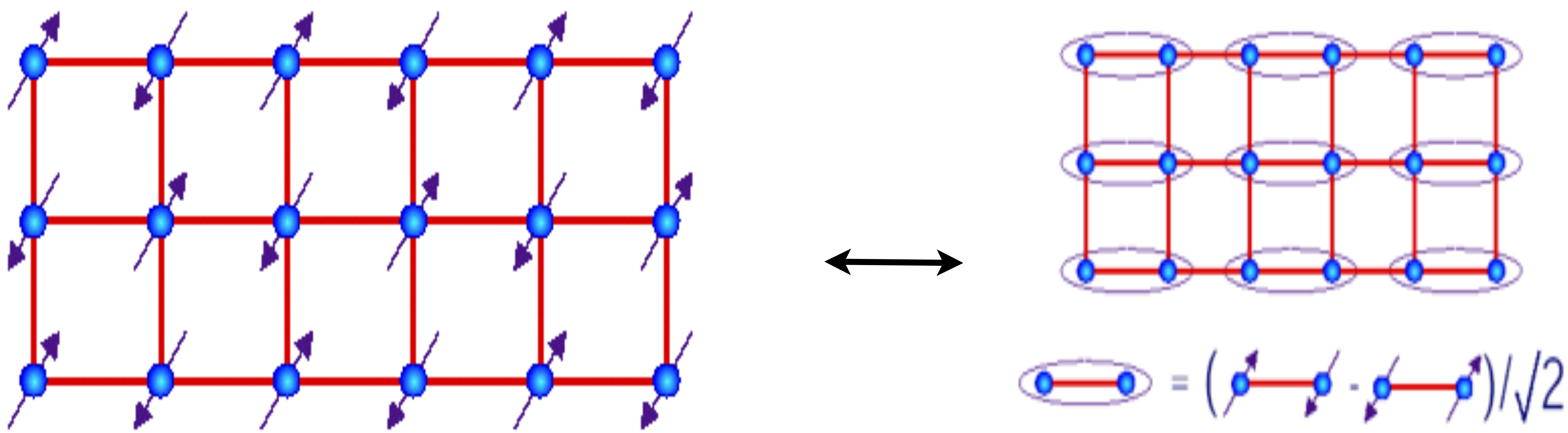
Naive expectation is incorrect: Possibility of a continuous Landau-forbidden phase transition between Landau allowed phases

TS, Vishwanath, Balents, Sachdev, Fisher 2004

The Neel-VBS transition

TS, Vishwanath, Balents, Sachdev, Fisher 2004

Possible Landau-forbidden continuous transitions between Landau allowed phases



Field theoretic framework:

$$\mathcal{L} = \sum_{\alpha=1,2} |D_b z_\alpha|^2 + V(|z|^2) + \cdots$$

z_α: $SU(2)$ doublet ("spinon")

b: dynamical $U(1)$ gauge field.

$\cdots$: all allowed local operators consistent with symmetries of lattice magnet.

Comments

$$\mathcal{L} = \sum_{\alpha=1,2} |D_b z_\alpha|^2 + V(|z|^2) + \cdots$$

- Theory known as "Non-compact CP^1 model" $(NCCP^1)$

  Monopole operators in b not added to action

- Neel order parameter $\vec{N} = z^\dagger \vec{\sigma} z$

  VBS order parameter $\psi_{VBS} = \mathcal{M}_b$ (monopole operator)

Read, Sachdev, 89; Haldane 88

Theory not in terms of natural order parameters but in terms of `fractional spin' fields z + gauge fields.

``Deconfined quantum critical point"

TS, Vishwanath, Balents, Sachdev, Fisher 2004

Deconfined quantum criticality

TS, Vishwanath, Balents, Fisher, Sachdev, 2004

Emergence of field theory in terms of `deconfined' degrees of freedom between two phases with conventional `confined' excitations.

Many proposed examples by now in 2+1-D.

Active area of research: input from many different directions

- numerical simulations, conformal bootstrap, field theory dualities,……

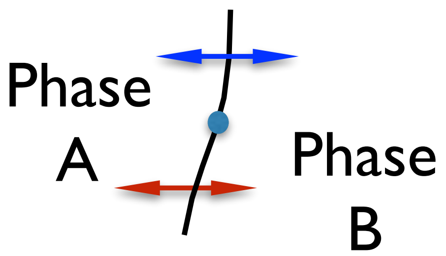
This talk

(Zhen Bi, TS, arXiv, 1808:**07465**)

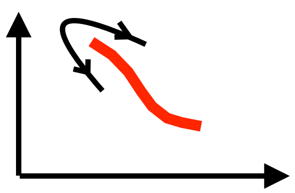
A number of surprising quantum critical phenomena (no or few previous prior examples)

1. (Solvable) Deconfined quantum criticality in 3+1-dimensions

2. Phase transitions described by multiple universality classes



3. Unnecessary continuous phase transitions



4. Band-theory-forbidden quantum criticality between band insulators

Bonus: A striking possible duality of fermions in 3 +1-D.

Outline

Focus on theories in 3+1-D.

1. Preliminaries: the free Dirac fermion

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2. Massless SU(2) Yang-Mills theory with matter: interpretation as deconfined quantum critical points

    - some generalizations

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Focus on theories in 3+1-D.

1. Preliminaries: the free Dirac fermion

2. Massless SU(2) Yang-Mills theory with matter: interpretation as deconfined quantum critical points

   - some generalizations

3. Possible duality in 3+1-D

A gauge theory ⟷ A free theory + a gapped TQFT

Similar example in 2+1-D: Gomis, Komargodski, Seiberg, 2017

Free Dirac fermion in 3+1-D

$$\mathcal{L} = \bar{\psi}\left(-i\partial\!\!\!/ + A\!\!\!/\right)\psi + \ldots.$$

4-component fermion

external background U(1) gauge field(*)

Also allow

(1) a mass term $m\bar{\psi}\psi$

(2) placing on arbitrary smooth oriented space-time manifold with metric g.

Symmetries: U(1) x T

Charge conservation

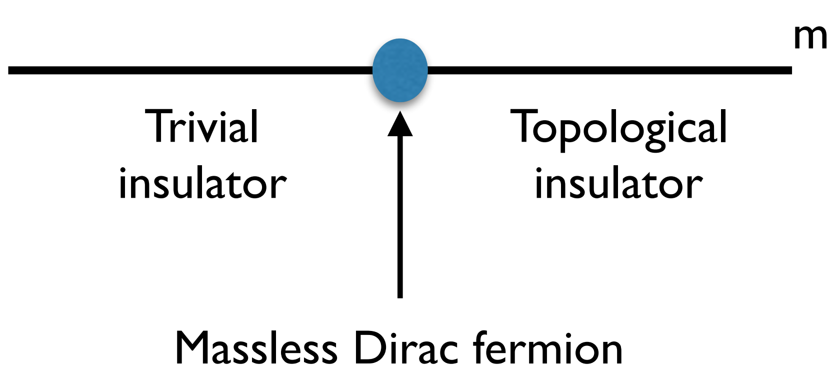
Time reversal

With this choice of T, electric charge is T-reversal odd (could also have made the more standard choice).

(*) Strictly speaking, A is a Spin_c connection.

The massless Dirac fermion as a quantum critical point

As sign of mass is changed there is a phase transition between a trivial insulator and a topological insulator of these fermions at m = 0.



Understand

(i) Physical: Study spatial domain wall between the 2 phases

(ii) Formal: Derive change (between two signs of m) in theta term in response to background gauge fields (A,g).

Sketch of the formal derivation

Similar methods powerful to derive all the results in the more complex examples studied later in the talk.

See, eg, recent review: Witten RMP 2016

Partition function of free Dirac fermion of mass m

$$Z[m; A, g] = det(D + m) = \prod_i (i\lambda_i + m)$$

(λ_i are eigenvalues of Hermitian Dirac operator $-iD$.)

As $\{\gamma_5, D\} = 0$ non-zero eigenvalue come in pairs $(\lambda_i, -\lambda_i)$

Ratio of partition functions

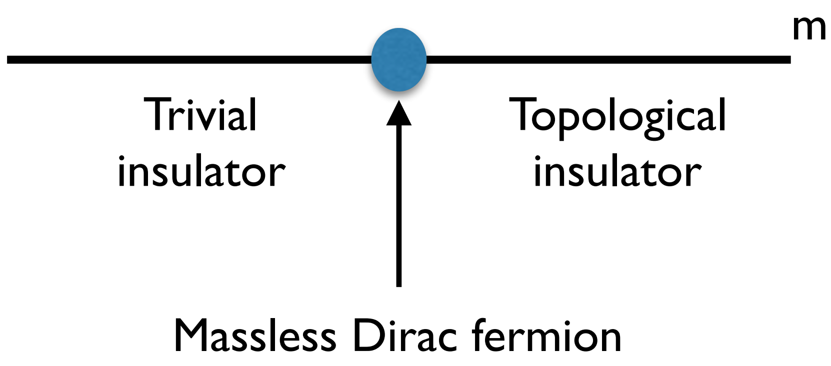
$$\frac{Z[m]}{Z[-m]} = \frac{\prod_i (i\lambda_i + m)}{\prod_i (i\lambda_i - m)}$$

All non-zero eigenvalues cancel out and

$$\frac{Z[m]}{Z[-m]} = (-1)^J$$

J = index of Dirac operator -iD
= topological invariant

Sketch of the formal derivation (cont'd)



$$\frac{Z(m)}{Z(-m)} = (-1)^J$$

J = index of Dirac operator -iD
= topological invariant

By Atiyah-Singer index theorem, this gives a $\theta = \pi$ axion angle for one sign of mass relative to other:

$$J = \frac{1}{2} \int d^4x \frac{dA}{2\pi} \wedge \frac{dA}{2\pi} + \text{gravitational theta term}$$

Symmetry Protected Topological (SPT) insulator: response to background gauge fields has a theta term with quantized coefficient

Comments on the massless point

Massless Dirac theory has more symmetries than massive case.

Eg: chiral rotation of the two Weyl fermions

We regard them as emergent - they survive in the IR when weak interactions are added.

These emergent symmetries are anomalous ('t Hooft anomalies).

A simple generalization

N free Dirac fermions = 2N free Majorana fermions

Symmetry SO(2N) x T.

Taking $m < 0$ theory to be trivial, the $m > 0$ theory has a calculable theta term for background SO(2N) gauge field and metric g.

Massless point: quantum criticality of trivial-topological phase of fermions with SO(2N) X T symmetry.

SU(2) gauge theory with matter

Consider theories with N_f flavors of fermionic matter fields.

Two distinct cases.

(i) matter fields in fundamental (S = 1/2) representation

(ii) matter fields in adjoint (S = 1) representation

These are very different theories!

SU(2) gauge theory with fundamental matter

$$\mathcal{L} = \bar{\psi}\left(-i\gamma^{\mu}(\partial_{\mu} - ia_{\mu}) + m\right)\psi + \frac{1}{2g^2} tr\left(f_{\mu\nu}^2\right)$$

SU(2) gauge field

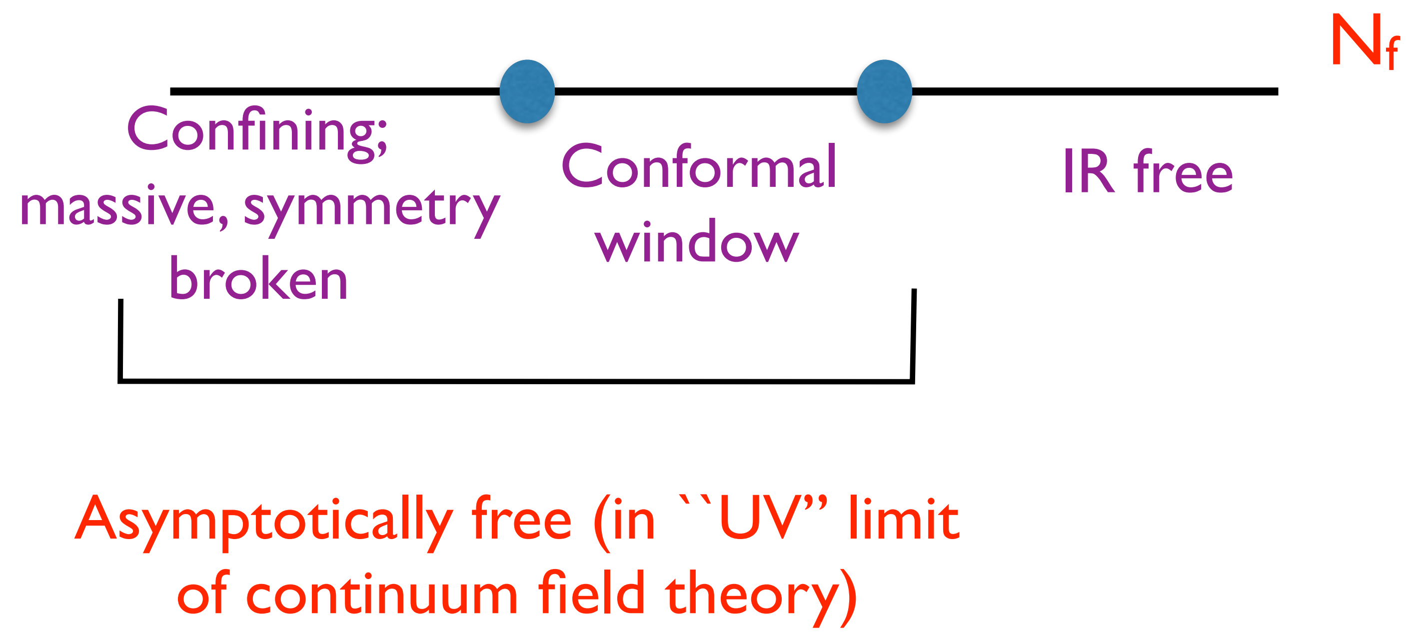
Despite appearances, this is a theory of bosons!

All local operators (baryons, mesons,....) are bosonic.

N_f flavors: can show theory has global symmetry $\frac{Sp(N_f)}{Z_2} \times T$.

View this gauge theory as the IR description of some UV system of interacting gauge-invariant bosons with this global symmetry.

Some well known properties

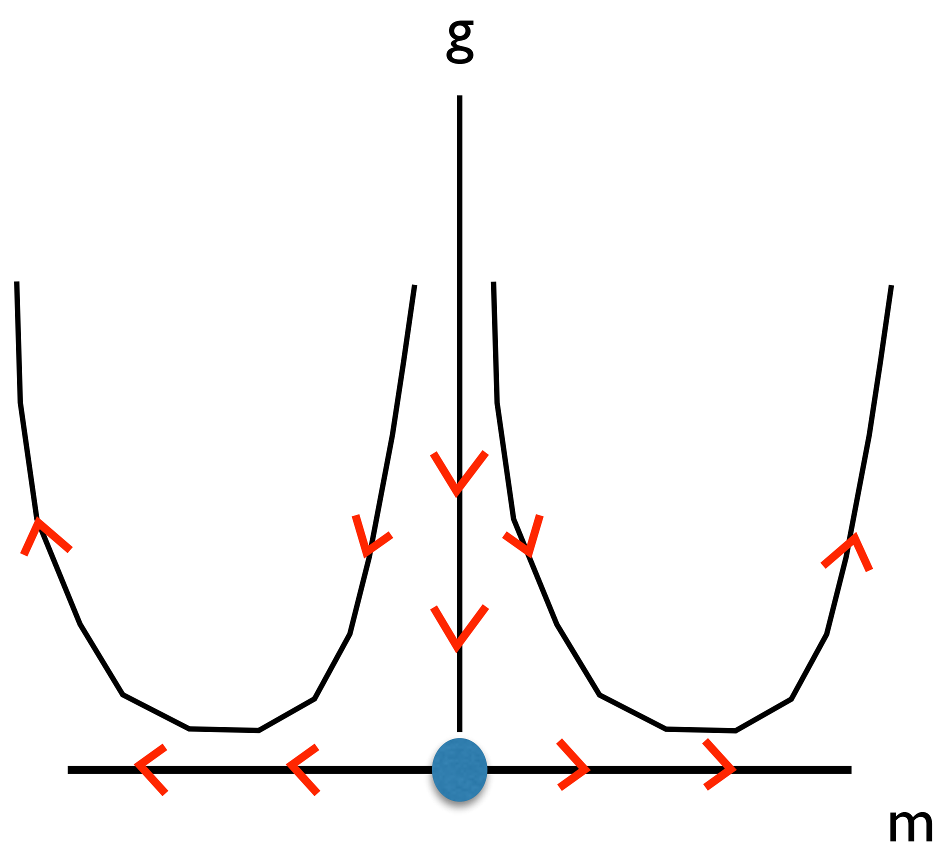


Upper boundary of conformal window known from perturbative RG.
Lower boundary: many numerical studies, controversial.

Though the theories in the conformal window are interesting,
to keep things simple I will mostly focus on the IR-free theories in this talk.

Q: What kind of criticality do these theories describe??

RG flow structure for large N_f



Massless (weakly coupled) fixed point separates two strongly coupled phases

Nature of the two massive phases

$m < 0$: Trivial symmetric gapped phase.

$m > 0$: Dynamical SU(2) gauge field has a theta response at $\theta = N_f \pi$.

N_f odd - (unknown) fate of SU(2) gauge theory at $\theta = \pi$

N_f even - standard SU(2) gauge theory => trivial symmetric gapped phase but could be in a different SPT phase.

Stick to even N_f.

Massless point is deconfined though both phases are confined (deconfined quantum criticality)

Nature of the two massive phases (cont'd)

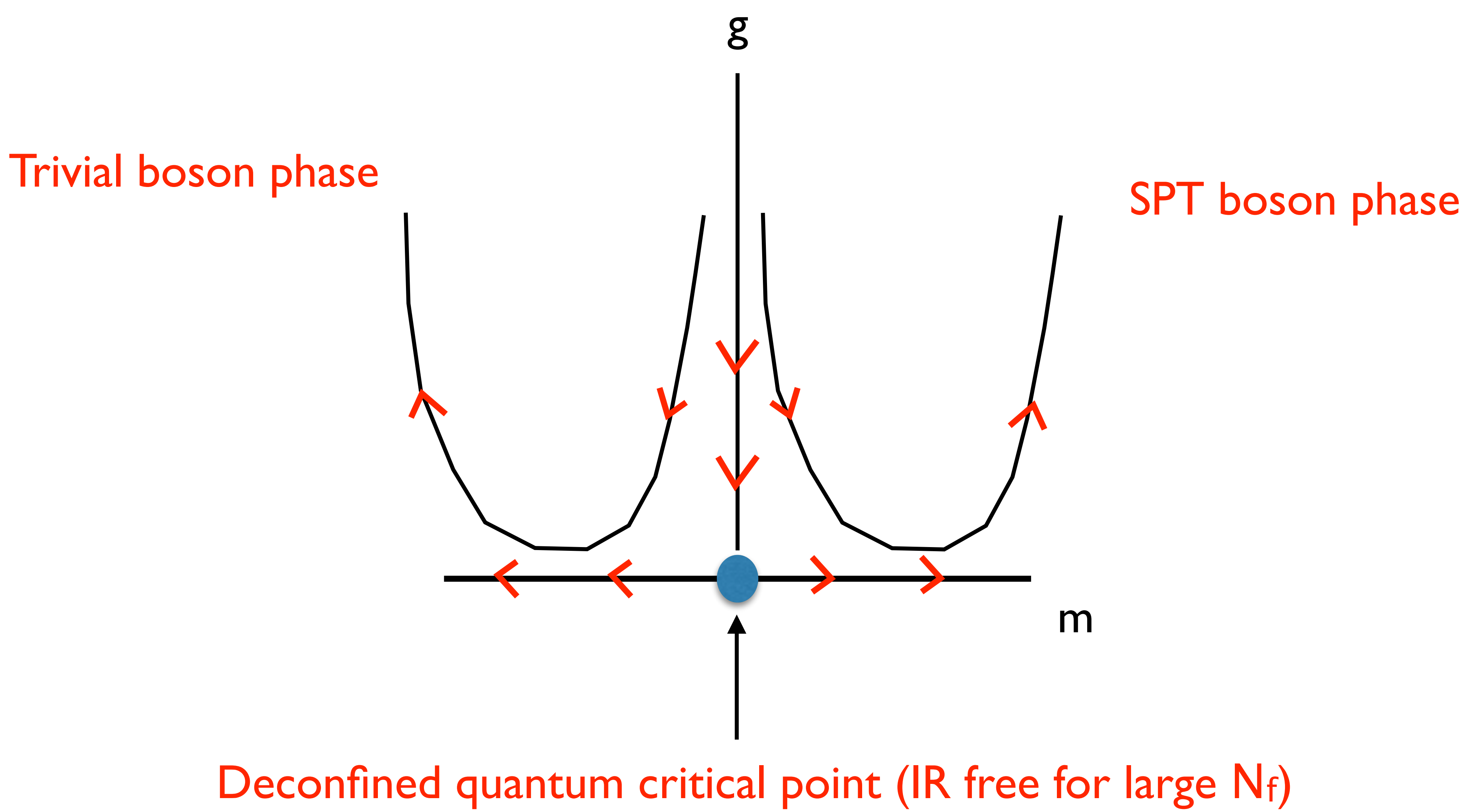
Start with theory of $4N_f$ Majorana fermions with $SO(4N_f) \times T$ symmetry, and calculate ratio of partition functions and associated theta terms for background $SO(4N_f)$ gauge fields.

Make dynamical an SU(2) subgroup to construct needed theory.

Can then get theta term for background global symmetry.

Distinct theta terms depending on the value of $N_f/2$ mod 4 => distinct SPT phases.

Bosonic topological phase transition in 3+1-D

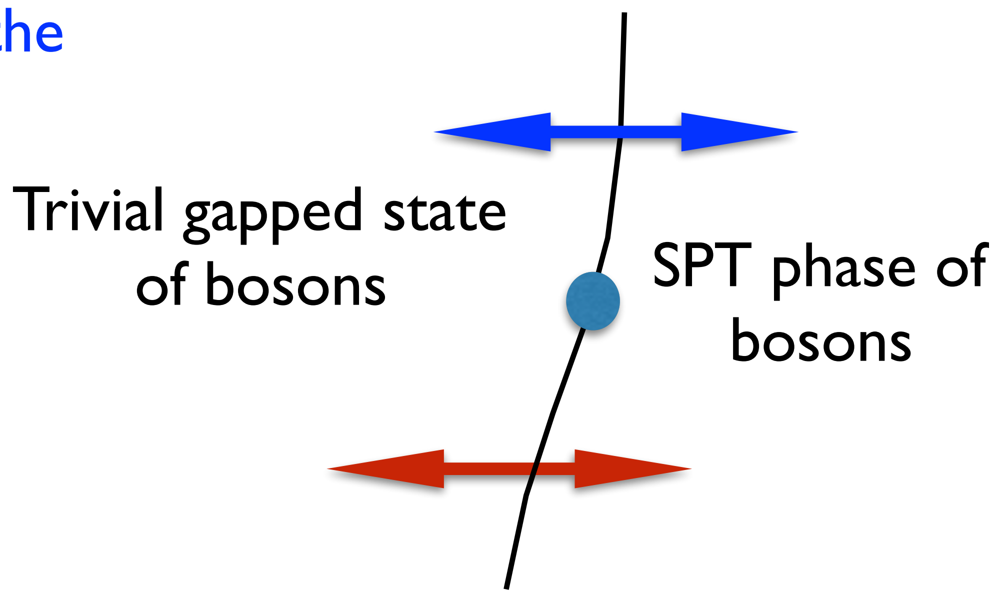


Deconfined critical SU(2) gauge theory with fundamental fermions describes phase transition between Trivial and SPT phases of bosons with $\frac{Sp(N_f)}{Z_2} \times T$ symmetry.

A generalization and some interesting phenomena

Sp(N_c) gauge theories with N_f fundamental fermions: also describe UV bosonic systems with same global symmetry.

These provide a large set of ***IR-distinct*** field theories for the same set of trivial-SPT phase transitions of these bosons.



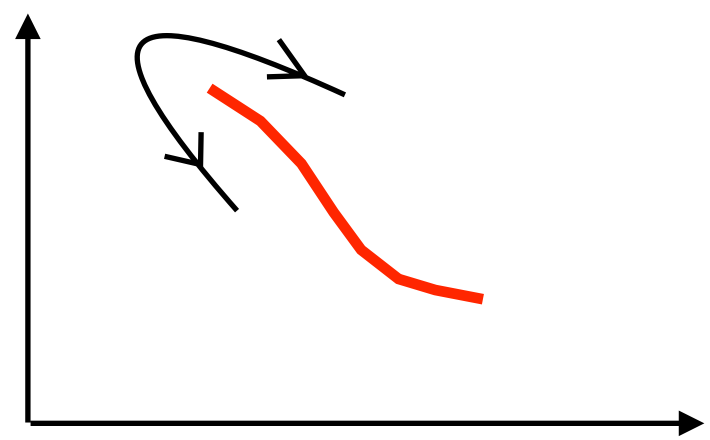
Multiple universality classes for the same phase transition.

These different theories are ``weakly dual'' (have the same local operators, the same global symmetry, and phase diagram) but are not ``strongly dual''.

Other interesting phenomena: Unnecessary phase transitions

Quantum critical points usually separate two distinct phases of matter.

However we find examples where there is a quantum critical line living inside a single phase.



$N_f = 0 \pmod 4$, $N_c = 0 \pmod 4$ (and N_f big enough)

``Unnecessary continuous phase transition"

(can go around the transition analogous to liquid-gas but here the transition is continuous!)

Other examples can be constructed without emergent gauge fields.

SU(2) gauge theory with N_f flavors of adjoint fermionic matter

$$\mathcal{L} = \bar{\psi}\left(-i\gamma^{\mu}(\partial_{\mu} - ia_{\mu}) + m\right)\psi + \frac{1}{2g^2} tr\left(f_{\mu\nu}^2\right) \;\; (+\mathcal{L}_M[z,a])$$

↑
adjoint

This describes a theory with local fermions!

$c \sim \epsilon_{ijk}(\bar{\psi}_i \psi_j)\psi_k$ is a gauge invariant fermion.

Important to add `heavy' (bosonic) spectator matter fields z in fundamental representation.

Global symmetry SO(2 N_f) x T (with c in vector representation)

View this gauge theory as IR description of some UV system of fermions with global SO(2 N_f) x T symmetry.

Remarks on adjoint SU(2) gauge theory

m = 0: The conformal window with adjoint matter occurs at lower N_f than with fundamental matter.

Asymptotic freedom lost at $N_f \geqq 3$.

In absence of spectator fundamental scalars, theory has unbreakable electric strings in fundamental representation

Corresponding ``one-form" symmetry (Gaiotto, Kapustin, Seiberg, Willett, 2015).

Important: spectator fundamental scalars explicitly break the 1-form symmetry

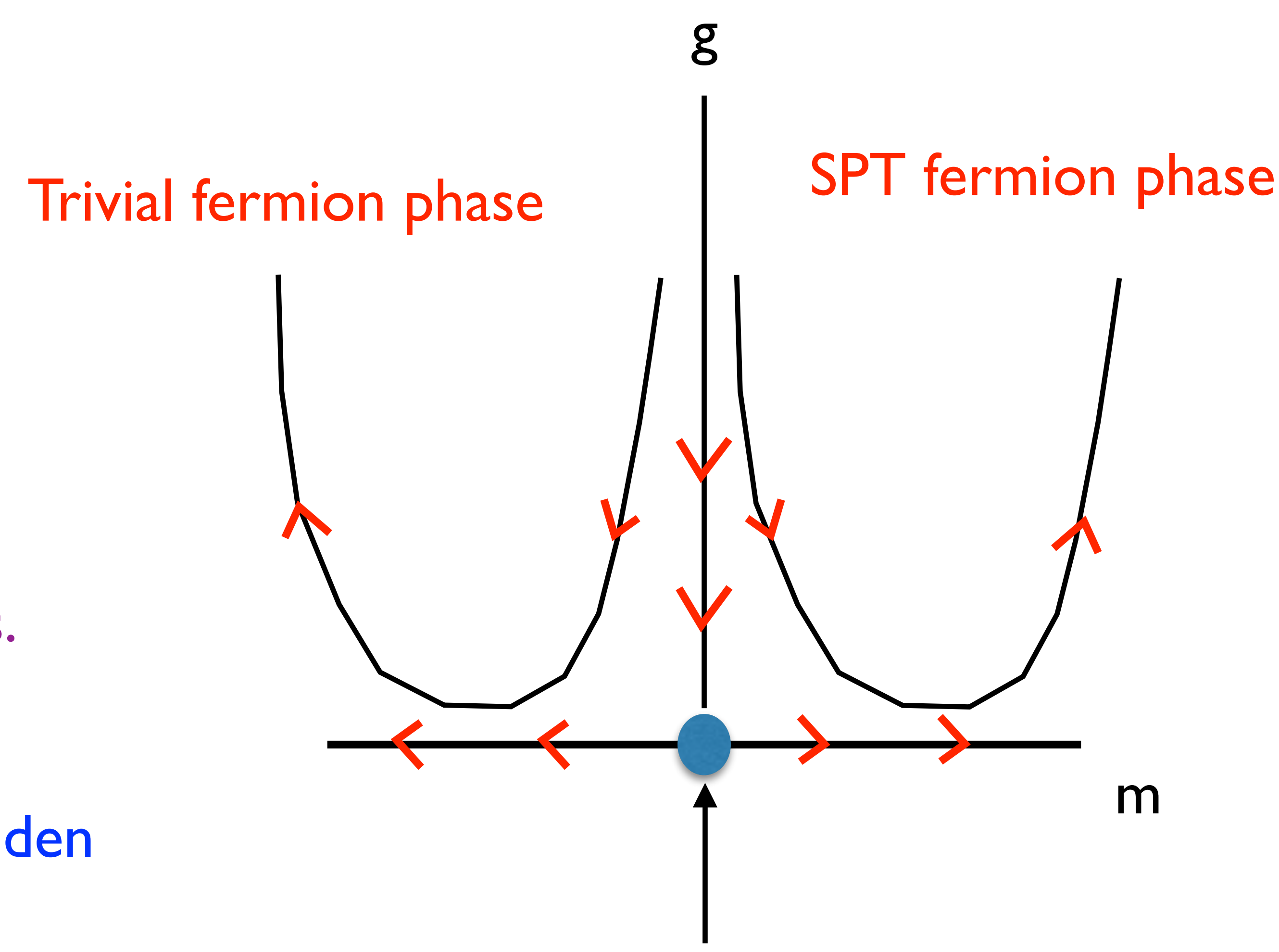
To completely specify the theory, must specify action of global symmetries on spectators.

Large N_f

Story similar to previous examples.

Massless, IR-free theory: deconfined
quantum critical point between
between trivial and SPT phases of fermions.

Interesting examples of band-theory-forbidden
criticality between band insulators.



Deconfined quantum critical point (IR free
for large N_f)

$N_f = 1$

Important theory in both condensed matter and high energy physics

Condensed matter: Physical fermions with U(1) x T symmetry

- a familiar much-studied system

Topological superconductor (``class A III") of importance in many other problems

$N_f = 1$

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Condensed matter: Physical fermions with U(1) x T symmetry

- a familiar much-studied system

Topological superconductor (``class A III") of importance in many other problems

High-energy: Gauge theory is a deformation of famous N = 2 Seiberg-Witten theory

Recent papers: Anber and Poppitz; Cordova and Dumitrescu; Bi and TS.

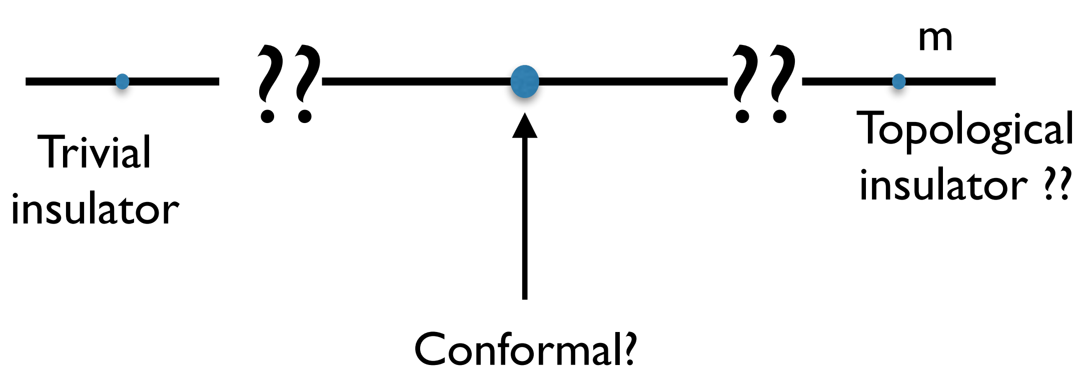
IR physics of SU(2) YM with $N_f = 1$ adjoint fermion

.

m = 0: Possibly conformal from existing numerics (eg, Athenodorou, Bennett, Bergner, Lucini, 2015) .

m ≠ 0, large: Expect confined, symmetry preserving, phases (no induced theta term for dynamical gauge field).

Topological distinction between two ``trivial'' phases at large |m| ??



Phase diagram of SU(2) YM + N_f = 1 adjoint fermion

SPT phases of fermions with $U(1) \times T$ are classified by $Z_8 \times Z_2$.

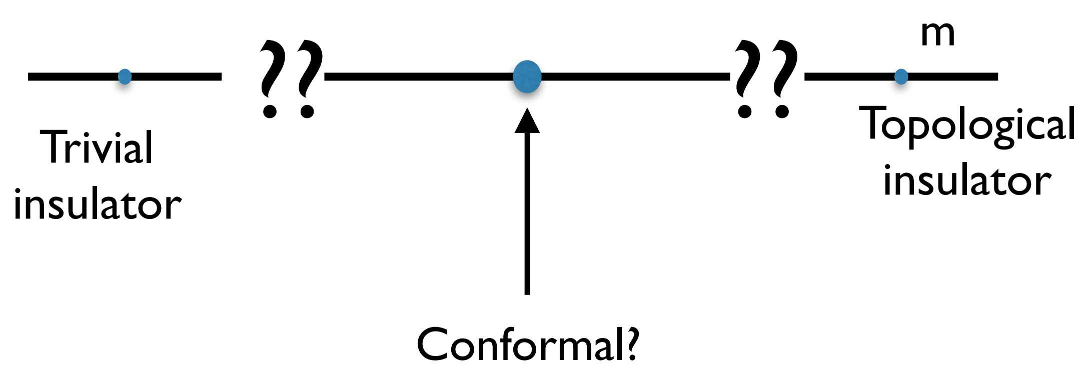
C. Wang, TS, 2014
Freed, Hopkins, 2016

Label by (k,s) with k = (0,1,2,…..,7) and s = (0,1) .

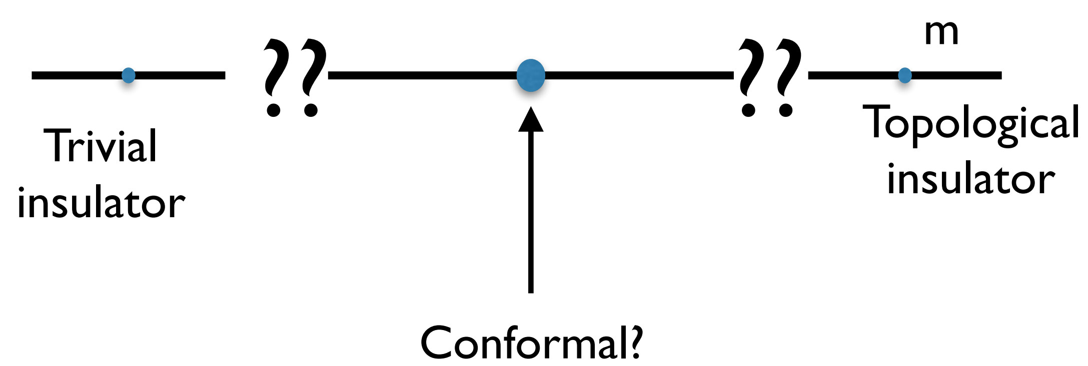
In gauge theory calculating partition function ratio shows that phase with m > 0 is (k,0) with k odd.

Precise T-implementation (including on heavy z bosons) determines which odd k.

Choose z a Kramers doublet => get k = -1.



Completing the phase diagram

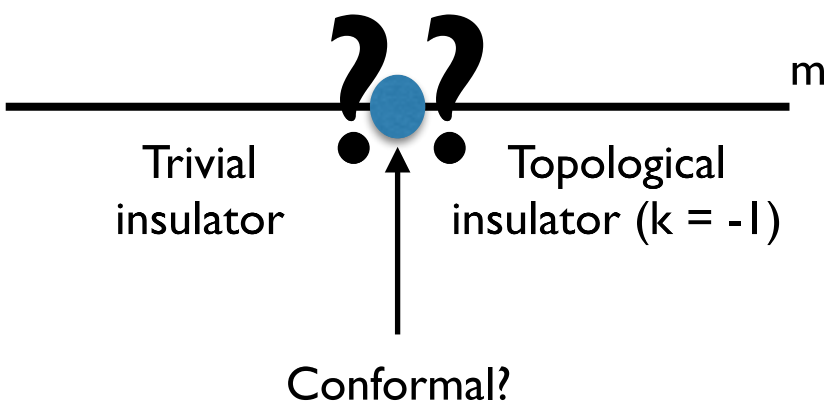


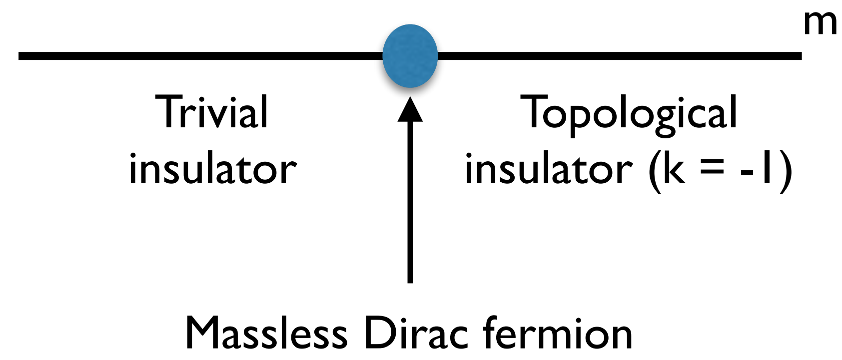
Gauge theory description: one possible evolution from trivial to topological insulator.

Free fermion theory: another possible evolution between same two phases.

Topological quantum criticality of fermions

Could gauge theory and free fermion descriptions be the same??





$$\bar{\psi}\left(\gamma^\mu(\partial_\mu - i a_\mu) + m\right)\psi + \frac{1}{2g^2} tr(f_{\mu\nu})^2$$

$$\bar{\chi}\left(\gamma^\mu \partial_\mu + m\right)\chi$$

The two massless theories have same local operators, and (almost) the same ordinary global symmetries.

``Wild” possibility: Perhaps they are the same theory in the IR?

Could these two 3+1-D theories really be IR dual?

How to tell?

At the very least check that emergent symmetries and their anomalies match at massless point.

Must include both ordinary (0-form) and 1-form global symmetries.

Emergent symmetries: massless free Dirac fermion

Single Dirac fermion = 2 Weyl fermions

Emergent symmetry $\frac{SU(2)\times U(1)}{Z_2}$

$SU(2)$ rotates the two Weyl fermions
U(1): axial rotation

Several anomalies (chiral anomaly for U(1), and Witten anomaly for SU(2))

(+ discrete symmetries: T, P, C)

Emergent symmetries: massless SU(2) YM + $N_f = 1$ adjoint Dirac fermion

Quantum effects reduce axial symmetry to Z_8.

Emergent 0-form symmetry: $\frac{SU(2)\times Z_8}{Z_2}$

$+$ 1-form symmetry

(Unbreakable electric loops in spin-1/2 representation)

Compare with free massless Dirac fermion: Z_8 is replaced by U(1) and no 1-form symmetry.

Can match 0-form symmetries/anomalies if Z_8 is dynamically enhanced to U(1) in IR

Could these two 3+1-D theories really be IR dual?

How to tell?

At the very least check that emergent symmetries and their anomalies match at massless point.

Must include both ordinary (0-form) and 1-form global symmetries.

Good news: If Z_8 of gauge theory is dynamically enhanced to U(1) in IR, then free Dirac fermion can match 0-form symmetries and anomalies.

Could these two 3+1-D theories really be IR dual?

How to tell?

At the very least check that emergent symmetries and their anomalies match at massless point.

Must include both ordinary (0-form) and 1-form global symmetries.

Good news: If Z_8 of gauge theory is dynamically enhanced to U(1) in IR, then free Dirac fermion can match 0-form symmetries and anomalies.

Bad news: Extra anomalies involving the 1-form symmetry (mixed anomaly with Z_8, and with gravity) - no analog in free Dirac theory.

Cordova, Dumitrescu, 2018

Implications

Massless SU(2) YM + N_f = 1 adjoint Dirac fermion cannot just flow to free massless Dirac fermion.

A better alternate:

Match the 1-form anomalies by augmenting the free Dirac fermion with a gapped topological sector that has the right 1-form anomalies.

Massless SU(2) YM theory + N_f = 1 adjoint Dirac fermion ⟷ A free Dirac theory + a gapped TQFT

Suggestion for a specific TQFT in our paper: `loop fractionalized' fermionic Z_2 gauge theory enriched by Z_8, 1-form symmetries

Other candidate phases: Cordova, Dumitrescu

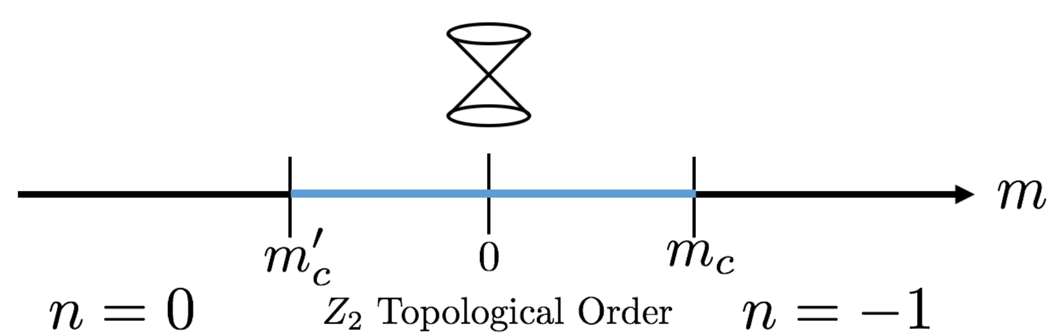
Adding in spectator boson

Massless SU(2) YM theory +
N_f = 1 adjoint Dirac fermion ⟷ A free Dirac theory + a gapped TQFT

Spectator boson breaks 1-form symmetry.

But in the TQFT, the loops have `fractionalized' => topological order survives even when 1-form symmetry is broken, or if a small mass is added.

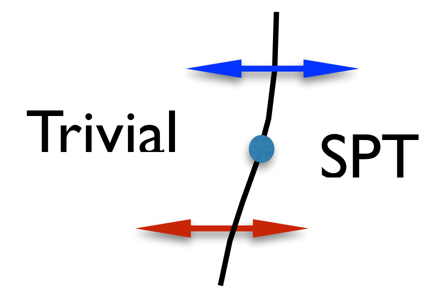
Gauge theory phase diagram
if duality is right



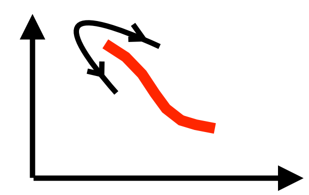
Summary

Simple examples illustrating many surprising quantum critical phenomena.

1. Deconfined quantum criticality in 3+1-dimensions



2. Phase transitions described by multiple universality classes



3. Unnecessary continuous phase transitions

4. Band-theory-forbidden critical points between band insulators

Bonus: A striking possible duality of fermions in 3 +1-D.

Massless SU(2) YM theory + N_f = 1 adjoint Dirac fermion	⟷	A free Dirac theory + a gapped TQFT