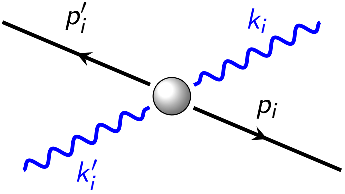
Measuring the Boiling Point of the Vacuum of Quantum Electrodynamics [arXiv:1807.10670]



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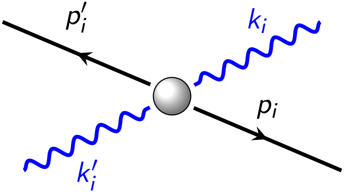

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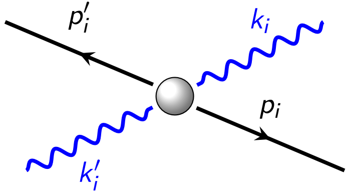

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- Non-linear QED
- The rate (per unit volume V) of spontaneous electron-positron pair production (SPP) in a strong static electric field E

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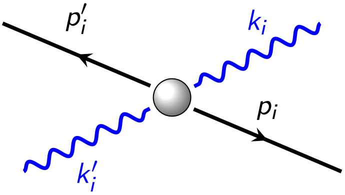

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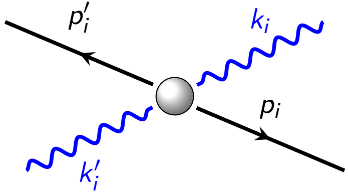

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$$\Gamma_{\mathrm{SPP}} \propto \exp\left(-\pi\frac{m_e^2}{e|\mathbf{E}|}\right)$$

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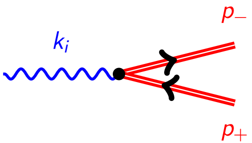
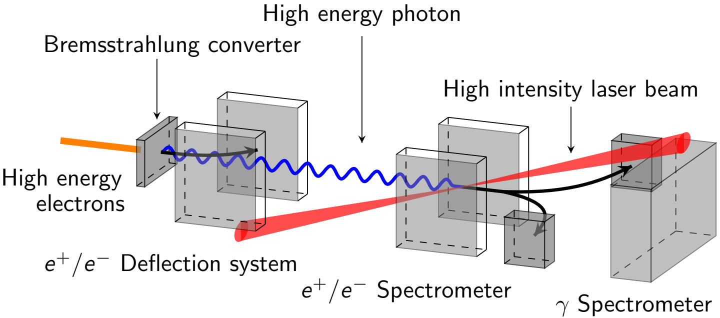


- One Photon Pair Production (OPPP)



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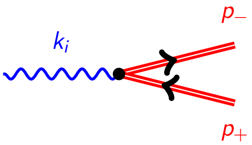
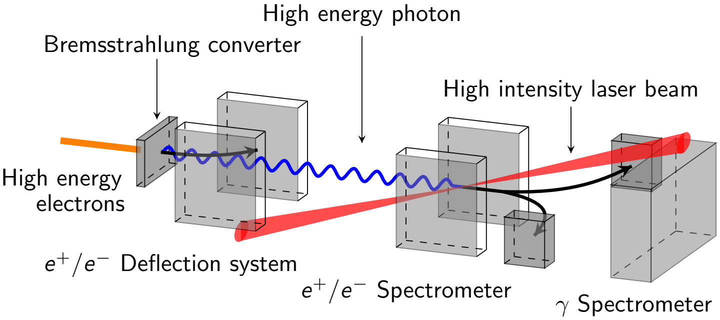


- One Photon Pair Production (OPPP)
- Bremsstrahlung spectrum



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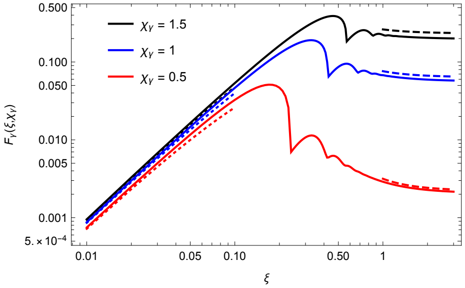


$$n_0 \equiv \frac{2\xi\left(1+\xi^2\right)}{\chi_\gamma},$$

$$z_v \equiv \frac{4\xi^2\sqrt{1+\xi^2}}{\chi_\gamma}\left[v\left(v_n - v\right)\right]^{1/2},$$

$$v_n \equiv \frac{\chi_\gamma\, n}{2\xi(1+\xi^2)} = \frac{n}{n_0},$$

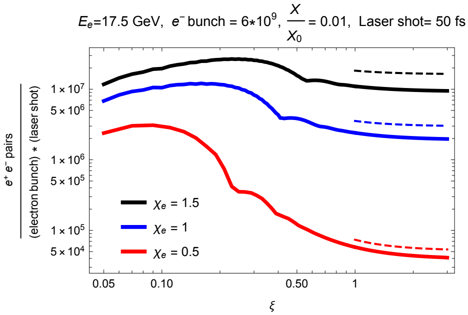
$$F_\gamma(\xi,\chi_\gamma) = \sum_{n>n_0}^{\infty}\int_1^{v_n}\frac{dv}{v\sqrt{v(v-1)}}\left\{2J_n(z_v)^2 + \xi^2(2v-1)\left[J_{n-1}(z_v)^2 + J_{n+1}(z_v)^2 - 2J_n(z_v)^2\right]\right\}$$

$$F_\gamma(\xi,\chi_\gamma) = \frac{3}{4}\sqrt{\frac{3}{2}}\,\chi_\gamma\,\mathrm{e}^{\left[-\frac{8}{3\chi_\gamma}\left(1-\frac{1}{15}\xi^{-2}+\mathcal{O}(\xi^{-4})\right)\right]}\quad \text{for } \xi \gtrsim \frac{1}{\sqrt{\chi_\gamma}} \gg 1, \quad \xi = \frac{e\,|\mathbf{E}|}{\omega m_e} = \frac{m_e}{\omega}\frac{|\mathbf{E}|}{\mathrm{E}_c}.$$

Measuring the Boiling Point of the Vacuum of Quantum Electrodynamics

$$\frac{dN}{d\chi_\gamma} = \frac{X}{X_0}\frac{1}{\chi_\gamma}\left[\frac{4}{3} - \frac{4}{3}\frac{\chi_\gamma}{\chi_e} + \left(\frac{\chi_\gamma}{\chi_e}\right)^2\right]$$

$$w(\xi,\chi_\gamma) = \frac{e^2 m_e^2}{16\pi E_e}\frac{\chi_e}{\chi_\gamma}F_\gamma(\xi,\chi_\gamma)$$



$$\int_0^{\chi_e} w(\xi,\chi_\gamma)\frac{dN}{d\chi_\gamma}d\chi_\gamma = \frac{e^2 m_e^2}{16\pi}\frac{\chi_e}{E_e}\int_0^{\chi_e}\frac{1}{\chi_\gamma}F_\gamma(\xi,\chi_\gamma)\frac{dN}{d\chi_\gamma}d\chi_\gamma$$

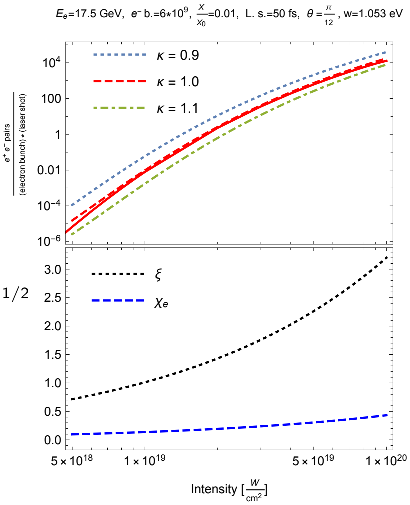
$$\Gamma_{\mathrm{BPPP}} \to \frac{\alpha m_e^2}{E_e}\frac{9}{128}\sqrt{\frac{3}{2}}{\chi_e}^2 e^{-\frac{8}{3\chi_e}\left(1-\frac{1}{15\xi^2}\right)}\frac{X}{X0}$$

$$\Gamma_{\mathrm{BPPP}} \to \frac{9}{128}\sqrt{\frac{3}{2}}\,\alpha\, E_e\,(1+\cos\theta)^2\left(\frac{|\mathbf{E}|}{\mathrm{E}_c}\right)^2 \exp\left[-\frac{8}{3}\frac{1}{1+\cos\theta}\frac{m_e}{\omega_i}\frac{\mathrm{E}_c}{|\mathbf{E}|}\right].$$

Measuring the Boiling Point of the Vacuum of Quantum Electrodynamics

$$\xi = 3.2 \left(\frac{I}{10^{20}\ \frac{\mathrm{W}}{\mathrm{cm}^2}} \right)^{1/2} \left(\frac{1.053\,\mathrm{eV}}{\omega} \right)$$

$$\chi_e = 0.22\,(1 + \cos\theta) \left(\frac{E_e}{17.5\,\mathrm{GeV}} \right) \left(\frac{I}{10^{20}\ \frac{\mathrm{W}}{\mathrm{cm}^2}} \right)^{1/2}$$



Conclusions

- The rates of laser-assisted BPPP, in the limit of for $\xi \gtrsim \frac{1}{\sqrt{\chi_e}} \gg 1$, and SPP, are closely related:

$$\frac{\Gamma_{\mathsf{SPP}}}{V} = \frac{m_e^4}{(2\pi)^3}\left(\frac{|\mathbf{E}|}{\mathrm{E_c}}\right)^2 \sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-n\pi\frac{\mathrm{E_c}}{|\mathbf{E}|}\right)$$

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- At least one pair per electron bunch should be observed for values of the intensity above the $2\,x\,10^{19}\,\frac{\mathrm{W}}{\mathrm{cm}^2}$, which is inside the range described above.

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- The behaviour of the rate in terms of the intensity and also on the value of E_c shows, that a slight variation of the 10% on the last one would affect the rate in approximately one order of magnitude, around $2\,x\,10^{19}\,\frac{\mathrm{W}}{\mathrm{cm}^2}$, with this difference being smaller for higher intensities.