Lorentz force induced oscillations

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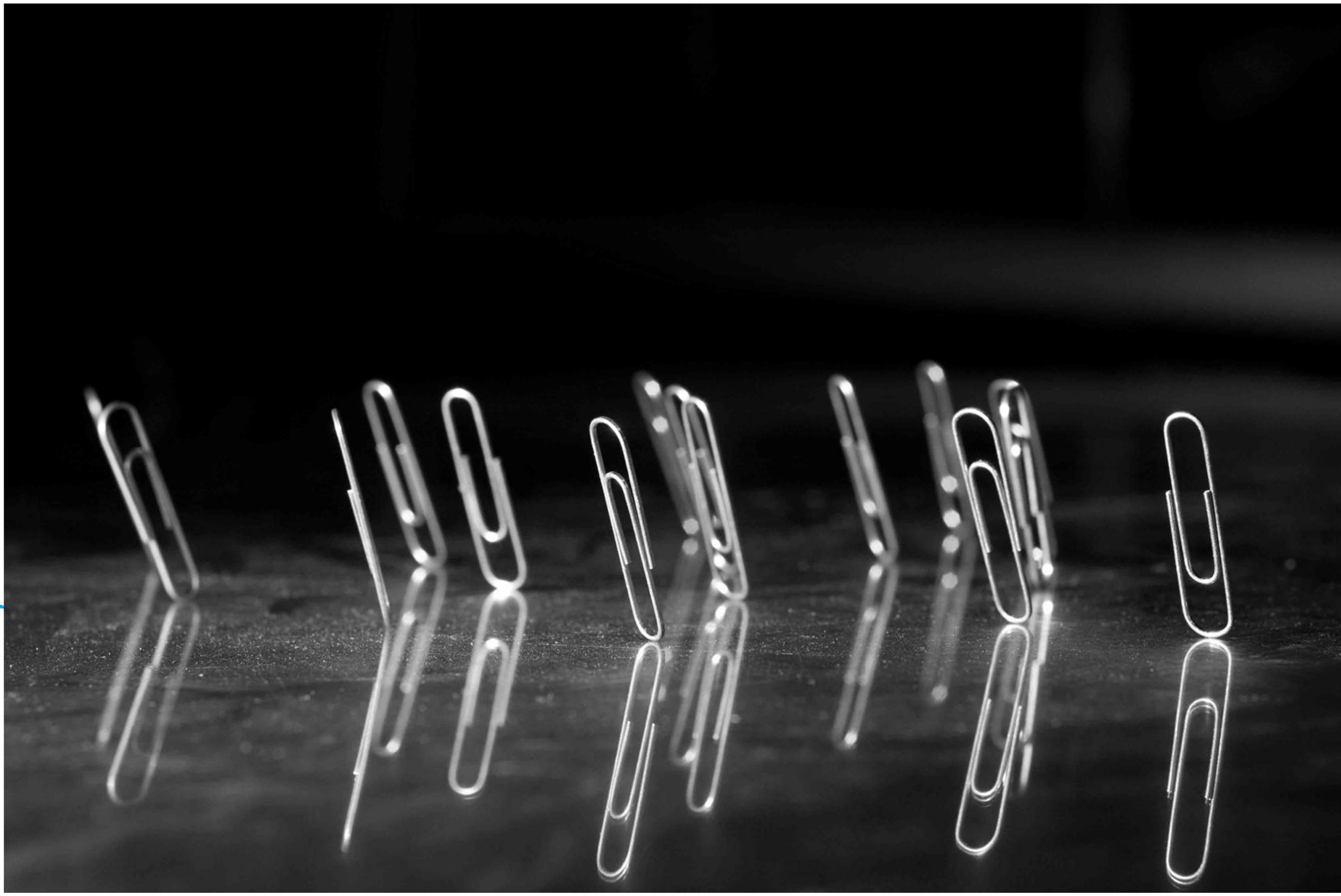
TRIUMF



TTC meeting 2019

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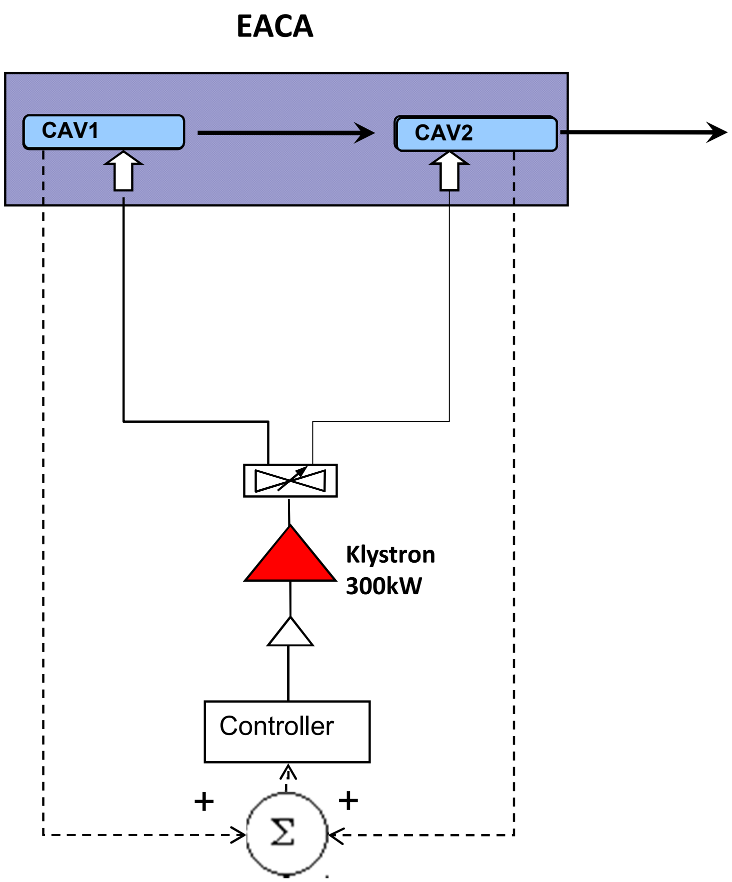
Overview

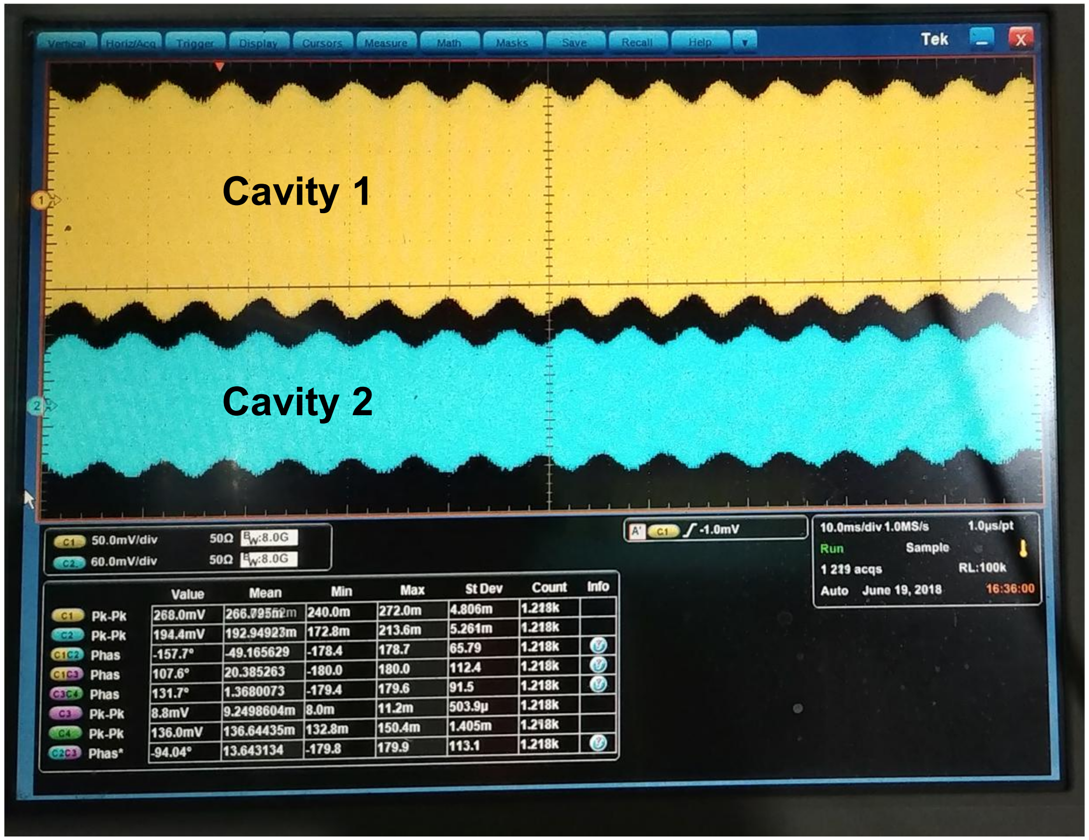
- TRIUMF's e-linac driving configuration
- First operational experiences
- Mathematical formulation for Lorentz force oscillation on a single cavity
- Stability analysis/ oscillation growth rate (linearized system)
- Limit cycle analysis
- Nonlinear Lyapunov stability
- Simulations
- What can we do with these information?
- Conclusions

TRIUMF's e-linac driving configuration



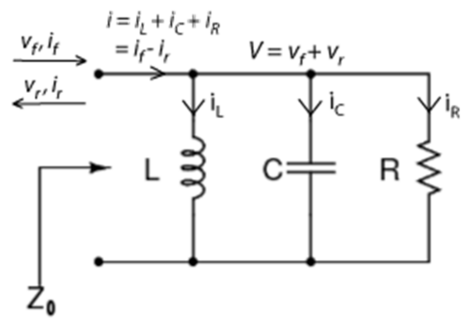
- TRIUMF's e-Linac acceleration cryomodule, consists of 2 TESLA type cavities and is operated with a single klystron in CW mode and vector sum control.

Operational experience



- Amplitude oscillation in both cavities (operational gradient dependent)
  - Vector sum perfectly stable
  - Oscillation frequency $\approx 160\ Hz$
  - Cavity bandwidth $\approx 300\ Hz$
  - Time to grow oscillations $\approx 6 - 10\ sec$

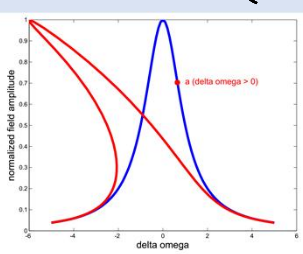
Lorentz force on a single cavity, no feedback



Mathematical problem formulation

- Lorentz force: $F = -K_L {V_{acc}}^2$
- Mechanical system: $\ddot{x} + x = F$
  simplified, damping coefficient = 0
- Equation of motion:

$$\ddot{\boldsymbol{x}} + \boldsymbol{x} = -\boldsymbol{\Lambda}\left(\boldsymbol{v}^2 - \boldsymbol{v}_0^2\right)$$



Electrical part

- Voltage at the cavity:

High quality factor, long time constant τ, voltage at capacitor does not rise instantaneously

$$\frac{di}{dt} = \frac{1}{R}\frac{dV}{dt} + \frac{d^2C}{dt^2}V + 2\frac{dC}{dt}\frac{dV}{dt} + \frac{d^2V}{dt^2}C + \frac{1}{L}V = \frac{2}{Z_0}\frac{dv_f}{dt}$$

$$\tau\dot{v} = (1 - ja)v = v_f$$

$$v^2 - v_0^2 = -2v_f\frac{a + x}{(1 + a^2)[(s\tau + 1)^2 + (a + x)^2]}x$$

Lorentz force on a single cavity, no feedback

Mechanical system

- No damping and no external force
- $\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$
  - Eigenvalues $\pm i$, circle with radius 1 in the phase space
- Adding perturbation, Lorentz force
- $\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} + \begin{pmatrix}F = 0\\ -G\end{pmatrix}$
- Or more general

$$\dot{x} = f(x,y) = -y$$
$$\dot{y} = g(x,y) = x - G$$

System linearization

- Calculating the Jacobian and evaluate at x,y=0
- $\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix}0 & -1\\ 1 & -\frac{dG}{\partial y}_{x,y=0}\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$
  - Perturbation modifies the trajectory and becomes either a stable or unstable spiral
- $\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix}0 & -1\\ 1 & -\mu\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$
  - $\mu = \Lambda 4 v_f^2 \frac{a}{1+a} \frac{\widetilde{\omega}^3}{(\widetilde{\omega}^2+1)^2}$

$\widetilde{\omega} = ratio\ of\ the\ electrical\ bandwidth\ to\ the\ mechanical\ oscillation\ frequency$

Stability analysis of the linearized system

Eigenvalue analysis

- ***System is stable, if real part of eigenvalues is negative***

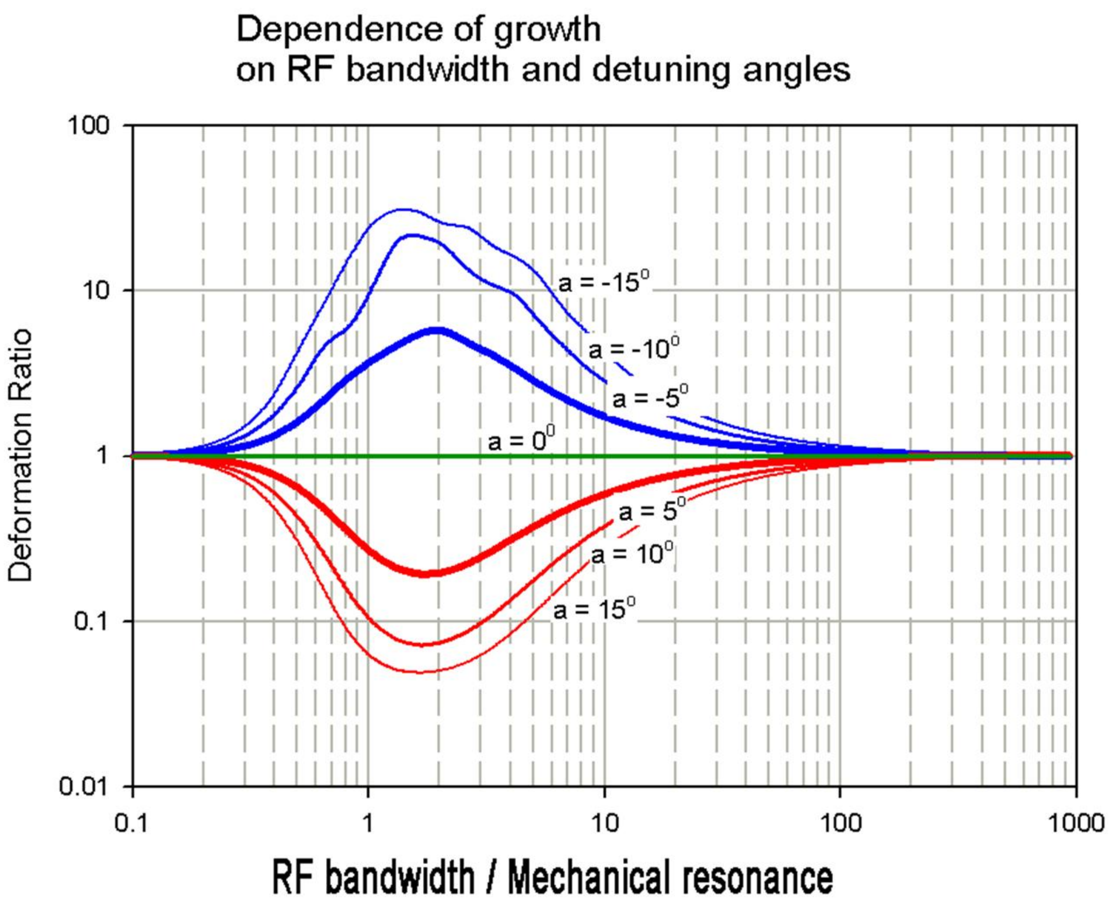
$$\lambda_{1,2} \approx -2\Lambda v_f^2 \frac{a}{1+a^2} \frac{\tilde{\omega}^3}{(\tilde{\omega}^2+1)^2} \pm i$$

- Linearized system has an unstable spiral center at (0,0) for $a < 0$ and a stable spiral center for $a > 0$

Growth rate

- $R \cong -2\Lambda v_f^2 a \frac{\tilde{\omega}^3}{(\tilde{\omega}^2+1)^2}$
  - Lorentz force constant Λ
  - Squared driving voltage v_f^2
  - Amount of detuning $a = \frac{\omega_0 - \omega}{\tilde{\omega}}$
- Max. growth rate $\tilde{\omega} = \sqrt{3} \cong 1.7$

Oscillation growth\decay rate of the nonlinear system



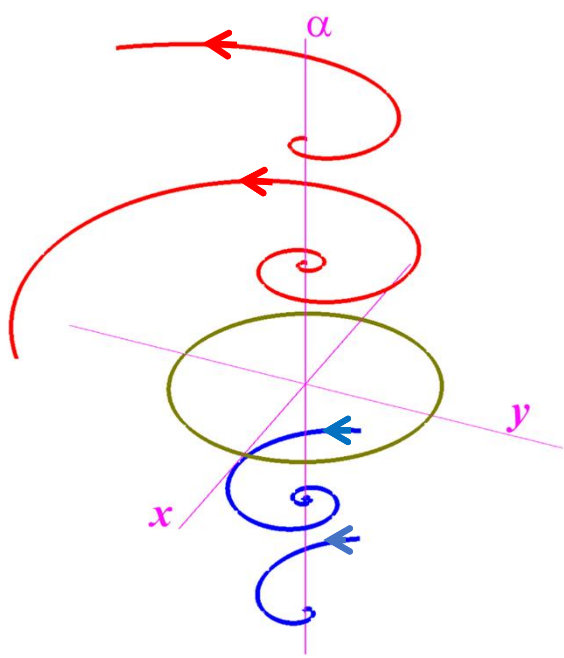
simulation results of the nonlinear system

Limit cycle existence of the nonlinear system by proof of Poincare-Andronov-Hopf bifurcation

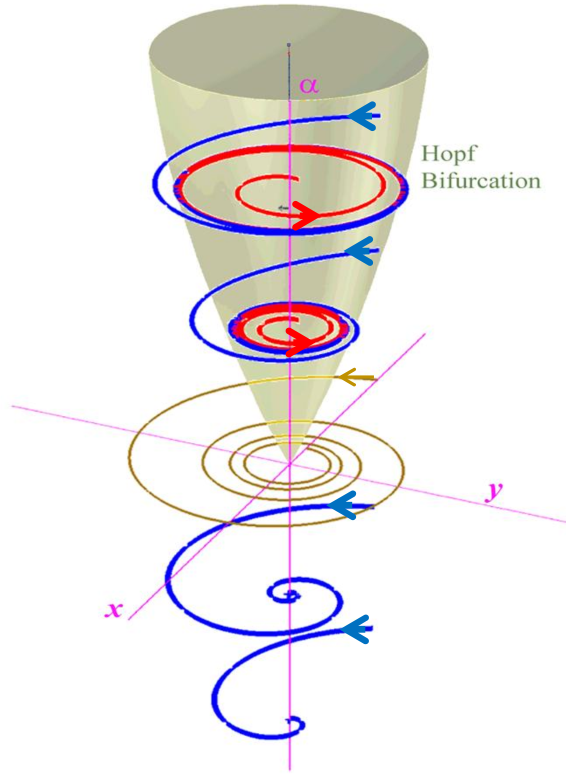
Conditions for limit cycle existence	Lorentz force system
1. Has a pair of purely imaginary eigenvalues	1. $\lambda_{1,2} \approx -2\Lambda v_f^2 \frac{a}{1+a^2} \frac{\tilde{\omega}^3}{(\tilde{\omega}^2+1)^2} \pm i$ $\lambda_{1,2} \approx -\alpha \pm i\beta$ for $\alpha = 0 \quad a = 0$
2. During a parametric change, the critical point changes from a stable to an unstable spiral or vice versa	2. $\frac{d\alpha}{da}_{a=0} = d \neq 0; \quad d = -2\Lambda v_f^2 \frac{\tilde{\omega}^3}{(\tilde{\omega}^2+1)^2}$
3. Whether the bifurcation is sub or supercritical is determined by the sign of the first Lyapunov coefficient	3. $L1\,\|_{a=0} = -\left(\Lambda v_f^2\right)^2 \frac{\tilde{\omega}^3}{(\tilde{\omega}^2+4)} < 0$ **Supercritical, stable limit cycle**

Supercritical Hopf Bifurcation with a stable limit cycle

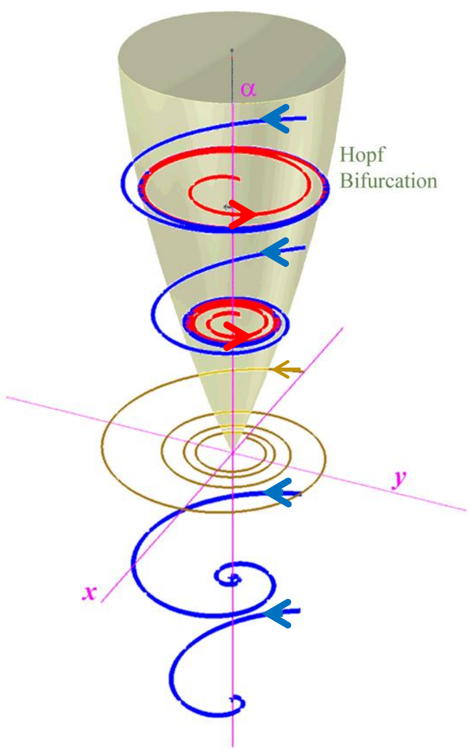
Linearized system



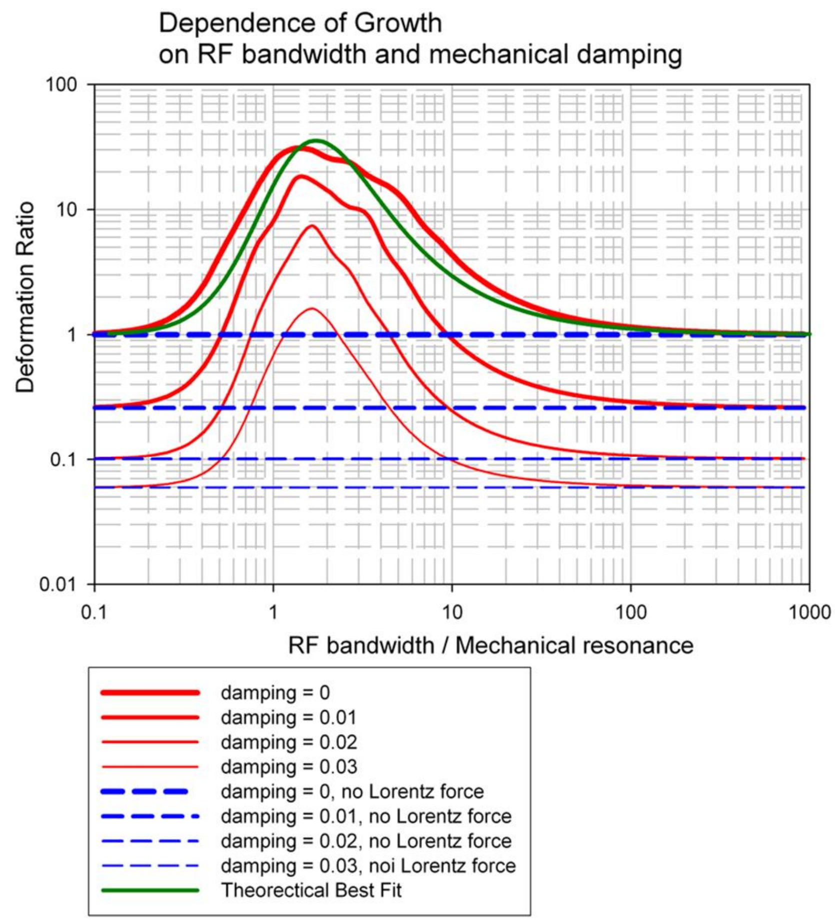
Nonlinear system



Supercritical Hopf Bifurcation with a stable limit cycle

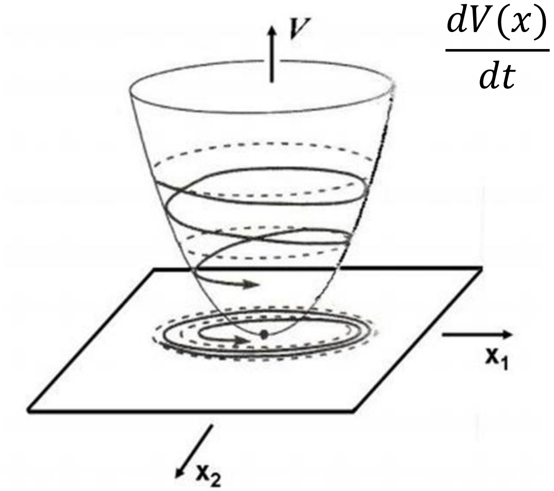


- Unstable within the cone $(a > 0)$, small initial conditions
- Considering damping
  - Cone position moves with increasing damping coefficient along the 'a-Axis'
  - Increases system stability



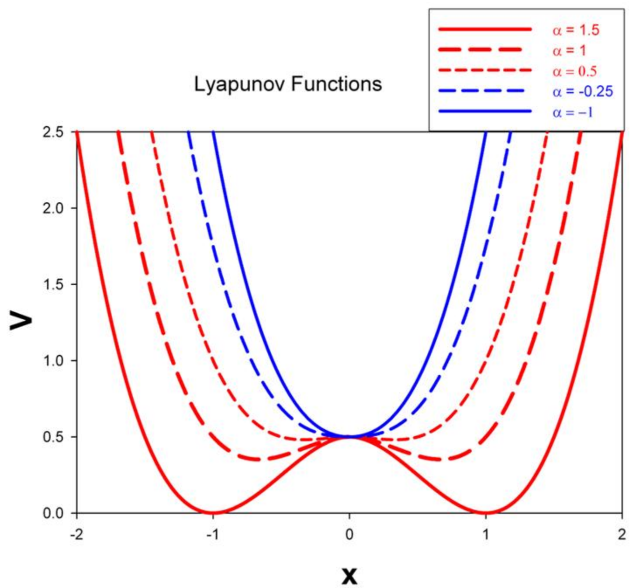
Lyapunov stability/ nonlinear system
(Lyapunov functions may be considered as energy functions)

General stable system

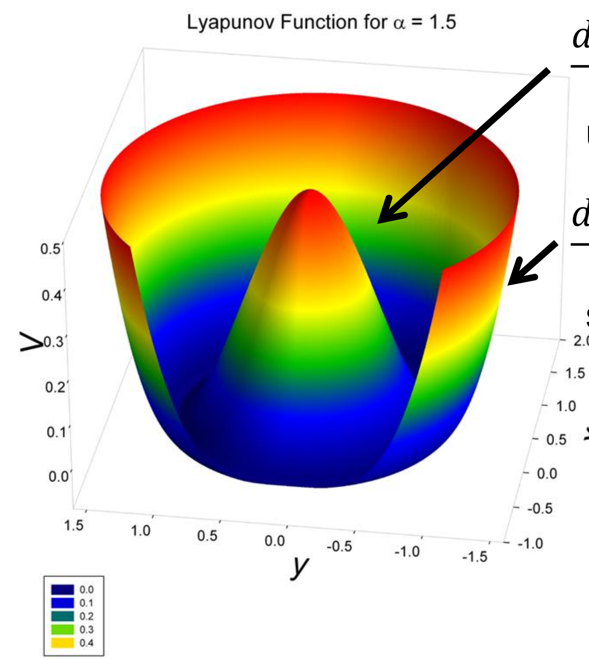


$$\frac{dV(x)}{dt} \leq 0$$

Mechanical cavity system with Lorentz force/ different α



Specific case $\alpha = 1.5°$



$$\frac{dV(x)}{dt} > 0$$

unstable

$$\frac{dV(x)}{dt} < 0$$

stable

Simulations

Matlab/simulation equations

$$\ddot{x} + c\dot{x} + \Omega^2 x = -\Lambda\left(v^2 - {v_0}^2\right)$$

$$\dot{x} = -y$$

$$\dot{y} = cy - \Omega^2 x - \Lambda\left(v_i^2 + v_q^2 - {v_0}^2\right)$$

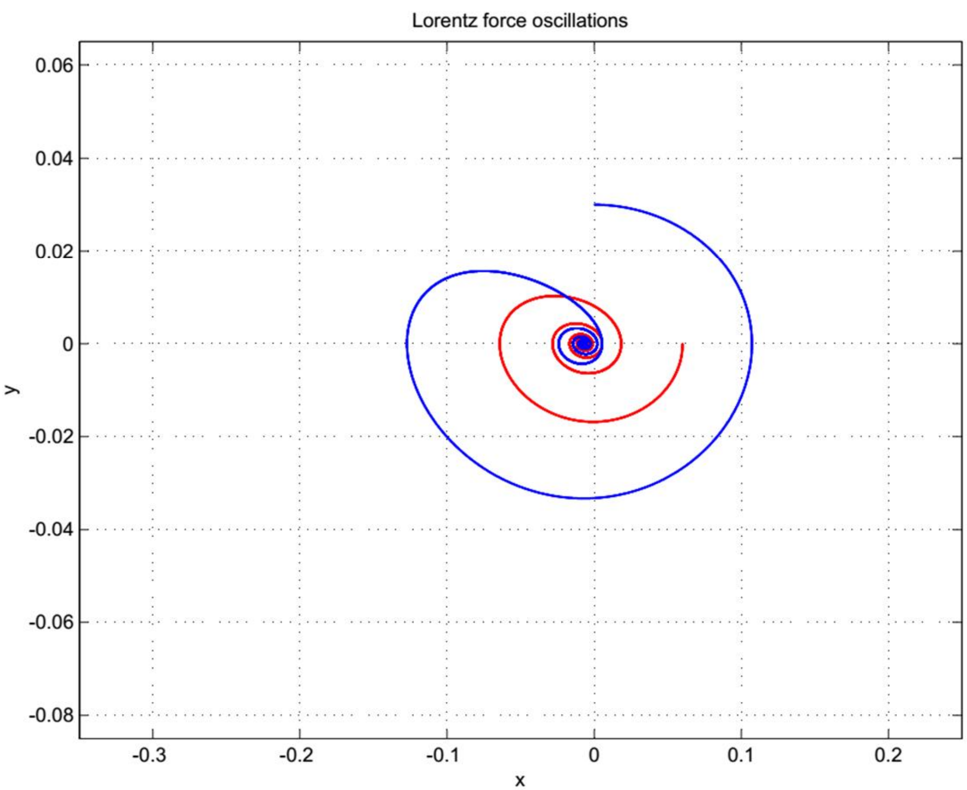
$$\dot{v_i} = -\widetilde{\omega}\left(v_i - (a + x)v_q - v_i\right)$$

$$\dot{v_q} = -\widetilde{\omega}\left((a + x)v_i + v_q\right)$$

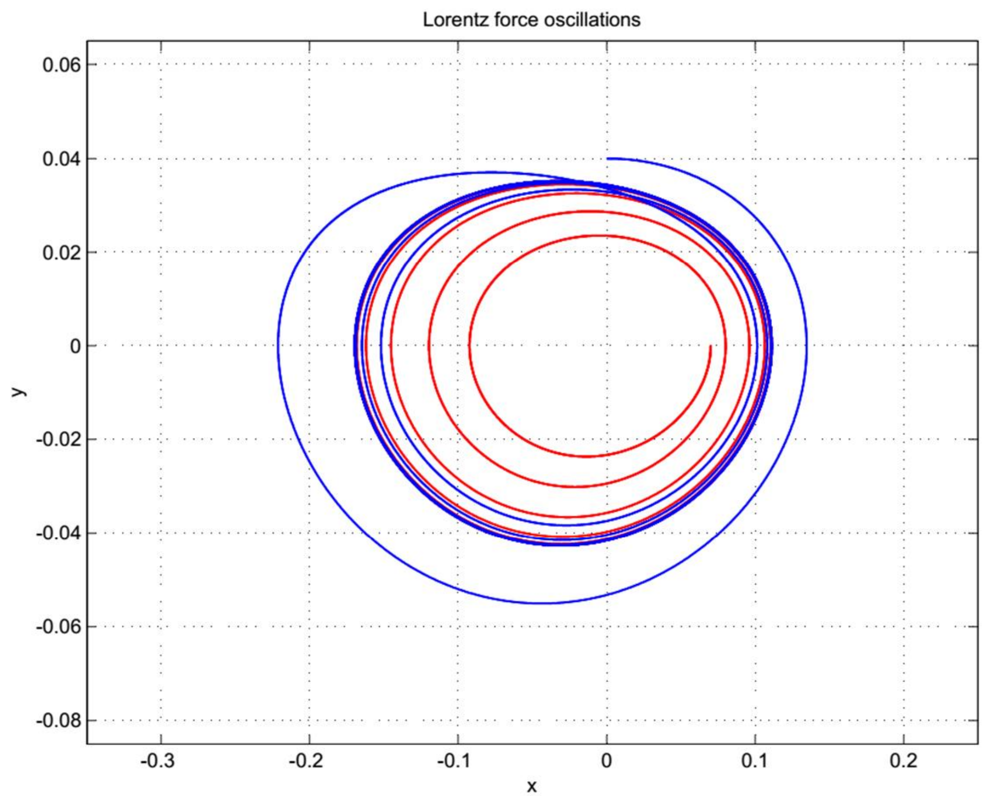
- Damping included
- 4 state variables
  - Voltage signal split into in phase and in quadrature

Simulation results

Stable: $\boldsymbol{\alpha < 0}$, $detuning\ angle\ -1.5°$



Unstable: $\boldsymbol{\alpha > 0}$, $detuning\ angle\ 1.5°$



Conditions for Oscillations in RF cavity due to Lorentz force

• High electric field $(\sim 10 MV/m)$	• High Q cavity ✓
• Insufficient rigidity in RF resonator	• Nb ✓
• Low mechanical damping	• Metal ✓
• Bandwidth of RF roughly double of mechanical mode oscillation	• 300Hz/160Hz ✓
• Poor voltage regulation	• Vector sum ✓
• Driven frequency > RF resonance frequency	
• Slow growth rate	• CW-operation ✓
• Non zero initial conditions	• Microphonics ✓

What can we do to suppress these instabilities?

• High electric field $(\sim 10MV/m)$	• Nothing ✗
• Insufficient rigidity in RF resonator	• Strengthen mechanically ✓
• Low mechanical damping	• Add damping ✓
• Bandwidth of RF roughly double of mechanical mode oscillation	• Nothing ✗
• Poor voltage regulation	• Avoid vector sum ✓ ✗
• Driven frequency > RF resonance frequency	• Tune for $\omega < \omega_0$ ✓
• Slow growth rate	• Pulsed-operation ✓ ✗
• Non zero initial conditions	• Active Microphonics feedback ✓
	• Active Lorentz force oscillation suppression 

Conclusion and lookout

- Field oscillation can occur at high field gradients
- It has a slow rise time
- First observed at TRIUMF's e-LINAC in summer 2018
- It is suppressed under certain conditions
- Active Lorentz force oscillation suppression feasible?
  - Lorentz force affects the cavity acceleration (not the position)
    - What variable could be measured?
  - Phase lag between the mechanical detune and the electrical response
  - $\begin{matrix} \dot{x} = y + \dot{F} \\ \dot{y} = x + G \end{matrix}$, $F = feedback$,

    piezo is affecting the position, not a trivial cancellation feedback problem
  - A deeper analysis of the presented results with respect to a feedback tuning system will be necessary

Thank you for your consideration!

Simulation results (increased detuning angle)

Unstable: $\boldsymbol{\alpha > 0}$ $\quad detuning\ angle\ 5°$