NNLO QCD corrections to $B \to \pi\pi$ decays

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Outline

- Introduction and theoretical framework of non-leptonic B decays
- Motivation for NNLO calculation
- Two-loop techniques in a nut-shell
- Results on tree-dominated $B \to \pi\pi$, $\pi\rho$, $\rho\rho$ decays
- Conclusion

Introduction

- Non-leptonic B decays offer a rich and interesting phenomenology
  - Large data sets from B-factories, in the future from LHCb, possibly SuperB
  - $\mathcal{O}(100)$ final states. Numerous observables: BR, CP asymetries, polarisations ...
  - Test of CKM mechanism (CP violation), New Physics?

Theory (here QCDF @ NLO)

$$\mathcal{B}(B^- \to \pi^-\pi^0) = (5.5 \pm 1.0) \times 10^{-6}$$

$$\mathcal{B}(\bar{B}^0 \to \pi^+\pi^-) = (5.0 \pm 1.2) \times 10^{-6}$$

$$\mathcal{B}(\bar{B}^0 \to \pi^0\pi^0) = (0.73 \pm 0.54) \times 10^{-6}$$

[Beneke, Jäger'05]

$$\mathcal{B}(\bar{B}^0 \to \rho^0\rho^0) = (0.9 \pm 1.4) \times 10^{-6}$$

[Beneke, Rohrer, Yang'06]

$$\mathcal{A}_{CP}(\bar{B}^0 \to \pi^+\pi^-) = 0.103$$

$$\mathcal{A}_{CP}(\bar{B}^0 \to \pi^0\pi^0) = -0.190$$

[Beneke, Neubert'03]

Experiment

$$\mathcal{B}(B^- \to \pi^-\pi^0) = (5.59^{+0.41}_{-0.40}) \times 10^{-6}$$

$$\mathcal{B}(\bar{B}^0 \to \pi^+\pi^-) = (5.16 \pm 0.22) \times 10^{-6}$$

$$\mathcal{B}(\bar{B}^0 \to \pi^0\pi^0) = (1.55 \pm 0.19) \times 10^{-6}$$

$$\mathcal{B}(\bar{B}^0 \to \rho^0\rho^0) = (0.73^{+0.27}_{-0.28}) \times 10^{-6}$$

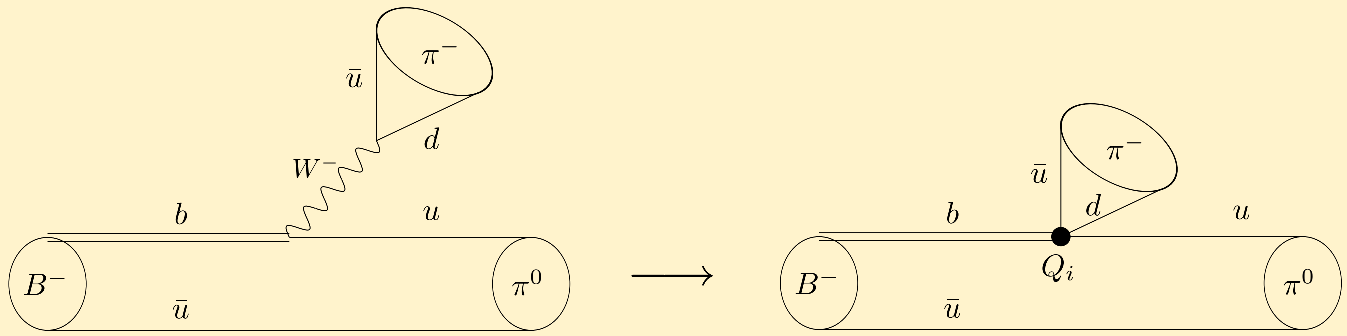
$$\mathcal{A}_{CP}(\bar{B}^0 \to \pi^+\pi^-) = 0.38 \pm 0.06$$

$$\mathcal{A}_{CP}(\bar{B}^0 \to \pi^0\pi^0) = 0.43^{+0.25}_{-0.24}$$

[PDG'08, HFAG'09]

- Problems with "colour-suppressed" tree-dominated decays (e. g. $\bar{B}^0 \to \pi^0\pi^0$).

Effective theory for B decays



- Effective Hamiltonian: *[Buras,Buchalla,Lautenbacher'96; Chetyrkin,Misiak,Münz'98]*

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{k=3}^{6} C_k Q_k + C_8 Q_8 \right] + \text{h.c.}$$

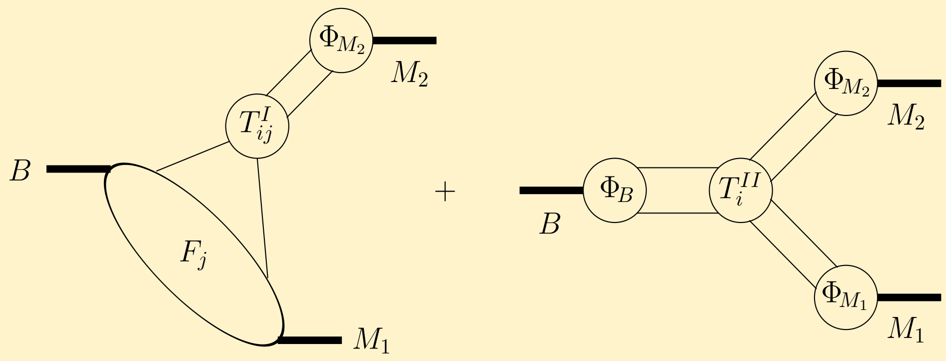
$$Q_1^p = (\bar{d}_L \gamma^\mu T^a p_L)(\bar{p}_L \gamma_\mu T^a b_L) \qquad Q_4 = (\bar{d}_L \gamma^\mu T^a b_L) \sum_q (\bar{q} \gamma_\mu T^a q) \qquad Q_8 = -\frac{g_s}{16\pi^2} m_b \, \bar{d}_L \, \sigma_{\mu\nu} G^{\mu\nu} b_R$$

$$Q_2^p = (\bar{d}_L \gamma^\mu p_L)(\bar{p}_L \gamma_\mu b_L) \qquad Q_5 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q)$$

$$Q_3 = (\bar{d}_L \gamma^\mu b_L) \sum_q (\bar{q} \gamma_\mu q) \qquad Q_6 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho T^a b_L) \sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^a q) \qquad \lambda_p = V_{pb} V_{pd}^*$$

- To be supplemented by evanescent operators (vanish in 4 dim., but not in D dim.)
  - Required to make the system closed under renormalisation
- Can use naïvely anticommuting γ_5 in dim. reg. in CMM basis

QCD factorisation



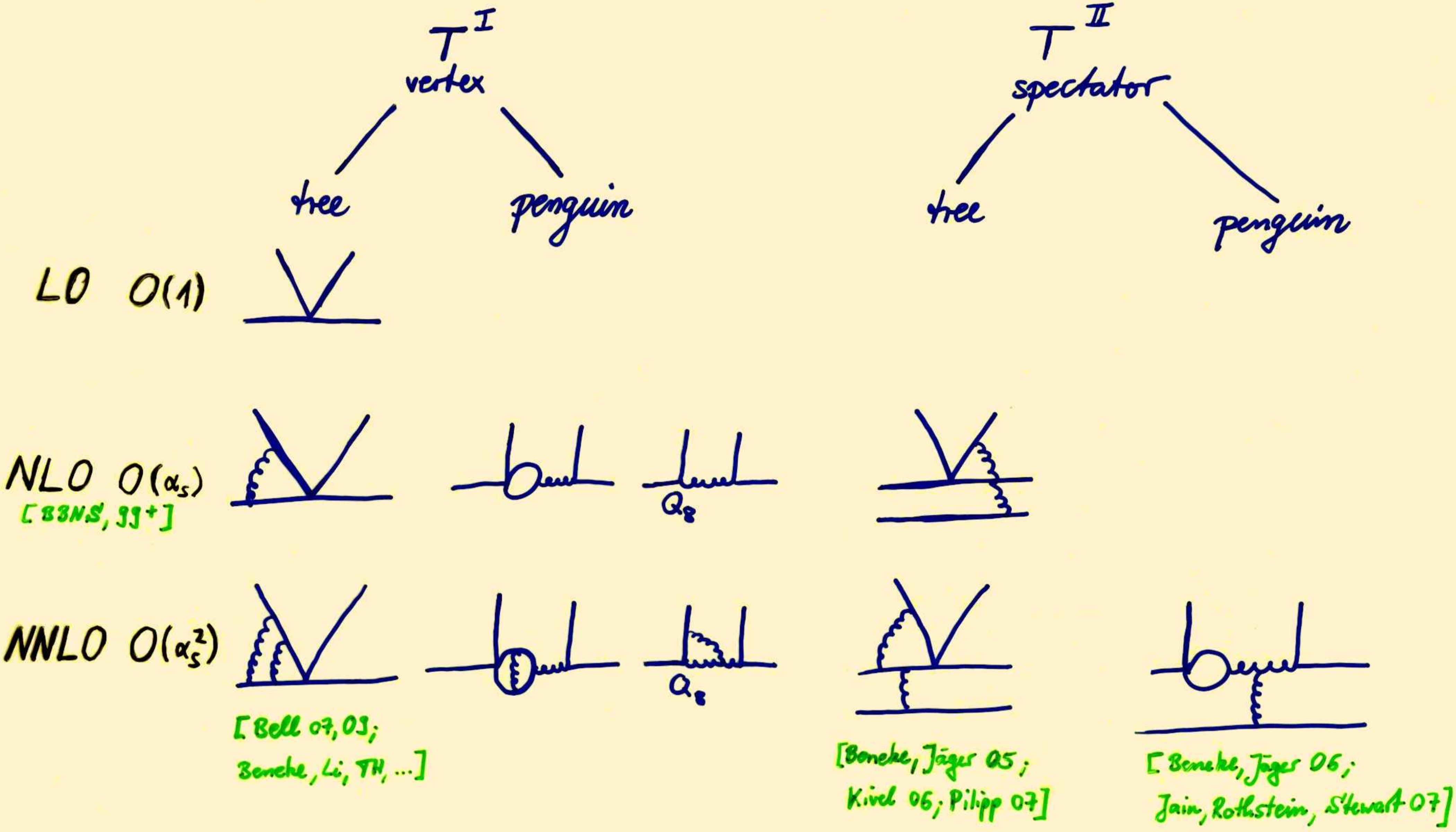
- Theoretical description of non-leptonic B decays difficult due to complicated QCD effects in the purely hadronic final state
- Simplification in the limit $m_b \gg \Lambda_{\text{QCD}}$ *[Beneke,Buchalla,Neubert,Sachrajda'99-'04]*

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2 \, F_+^{B\to M_1}(0) \, f_{M_2} \int_0^1 du \;\; T_i^I(u) \, \phi_{M_2}(u)$$

$$+ f_B \, f_{M_1} \, f_{M_2} \int_0^1 d\omega dv du \;\; T_i^{II}(\omega, v, u) \, \phi_B(\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u)$$

- $T^{I,II}$: Hard scattering kernels, perturbatively calculable. $T^{II} = \mathcal{O}(\alpha_s)$
- F_+ : $B \to M$ form factor
  f_i : decay constants
  ϕ_i : light-cone distribution amplitudes

QCD factorisation



moreover: „right" vs. „wrong" insertion

QCD factorisation, motivation for NNLO

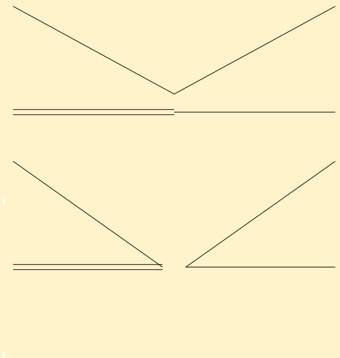
$$\sqrt{2}\,\langle \pi^- \pi^0 | \,\mathcal{H}_{eff}\, | B^- \rangle = \lambda_u \big[\alpha_1(\pi\pi) + \alpha_2(\pi\pi) \big]\; A_{\pi\pi}$$

$$\langle \pi^+ \pi^- | \,\mathcal{H}_{eff}\, | \bar{B}^0 \rangle = \big\{ \lambda_u \big[\alpha_1(\pi\pi) + \alpha_4^u(\pi\pi) \big] + \lambda_c\, \alpha_4^c(\pi\pi) \big\}\; A_{\pi\pi}$$

$$-\,\langle \pi^0 \pi^0 | \,\mathcal{H}_{eff}\, | \bar{B}^0 \rangle = \big\{ \lambda_u \big[\alpha_2(\pi\pi) - \alpha_4^u(\pi\pi) \big] - \lambda_c\, \alpha_4^c(\pi\pi) \big\}\; A_{\pi\pi}$$

[Beneke,Neubert'03]

- α_1 : colour-allowed tree amplitude, "right insertion"
- α_2 : colour-suppressed tree amplitude, "wrong insertion"



- NLO results

$$\alpha_1(\pi\pi) \;=\; 1.009 + [0.023 + 0.010\, i]_{\text{NLO}} - \left[\tfrac{r_{\text{sp}}}{0.445}\right] \Big\{ [0.014]_{\text{LOsp}} + [0.008]_{\text{tw3}} \Big\} = 1.010 + 0.010i$$

$$\alpha_2(\pi\pi) \;=\; 0.220 - [0.179 + 0.077\, i]_{\text{NLO}} + \left[\tfrac{r_{\text{sp}}}{0.445}\right] \Big\{ [0.114]_{\text{LOsp}} + [0.067]_{\text{tw3}} \Big\} = 0.222 - 0.077i$$

[Beneke,Buchalla,Neubert,Sachrajda'99,'01; Beneke,Neubert'03; Beneke,Jäger'05,'06; Kivel'06; Pilipp'07; Bell'07]

[Hill,Becher,Lee,Neubert'04; Becher,Hill'04; Kirilin'05; Beneke,Yang'05]

QCD factorisation, motivation for NNLO

- NLO results

$$\alpha_1(\pi\pi) = 1.009 + [0.023 + 0.010\,i]_{\rm NLO} - \left[\frac{r_{\rm sp}}{0.445}\right]\Big\{[0.014]_{\rm LOsp} + [0.008]_{\rm tw3}\Big\} = 1.010 + 0.010i$$

$$\alpha_2(\pi\pi) = 0.220 - [0.179 + 0.077\,i]_{\rm NLO} + \left[\frac{r_{\rm sp}}{0.445}\right]\Big\{[0.114]_{\rm LOsp} + [0.067]_{\rm tw3}\Big\} = 0.222 - 0.077i$$

[Beneke,Buchalla,Neubert,Sachrajda'99,'01; Beneke,Neubert'03; Beneke,Jäger'05,'06; Kivel'06; Pilipp'07; Bell'07]

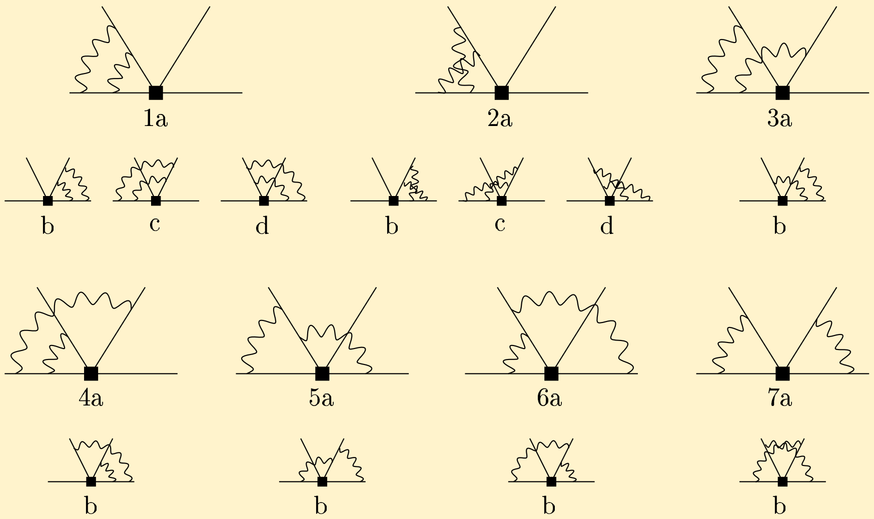
[Hill,Becher,Lee,Neubert'04; Becher,Hill'04; Kirilin'05; Beneke,Yang'05]

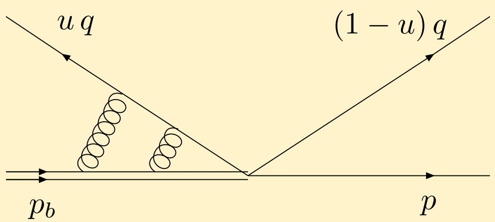
- Large cancellation in LO + NLO in α_2, particularly sensitive to NNLO
- Direct CP asymmetries start at $\mathcal{O}(\alpha_s)$, NNLO is only the first correction
- Q: Does factorization hold? Does NNLO QCDF tend toward the right direction?

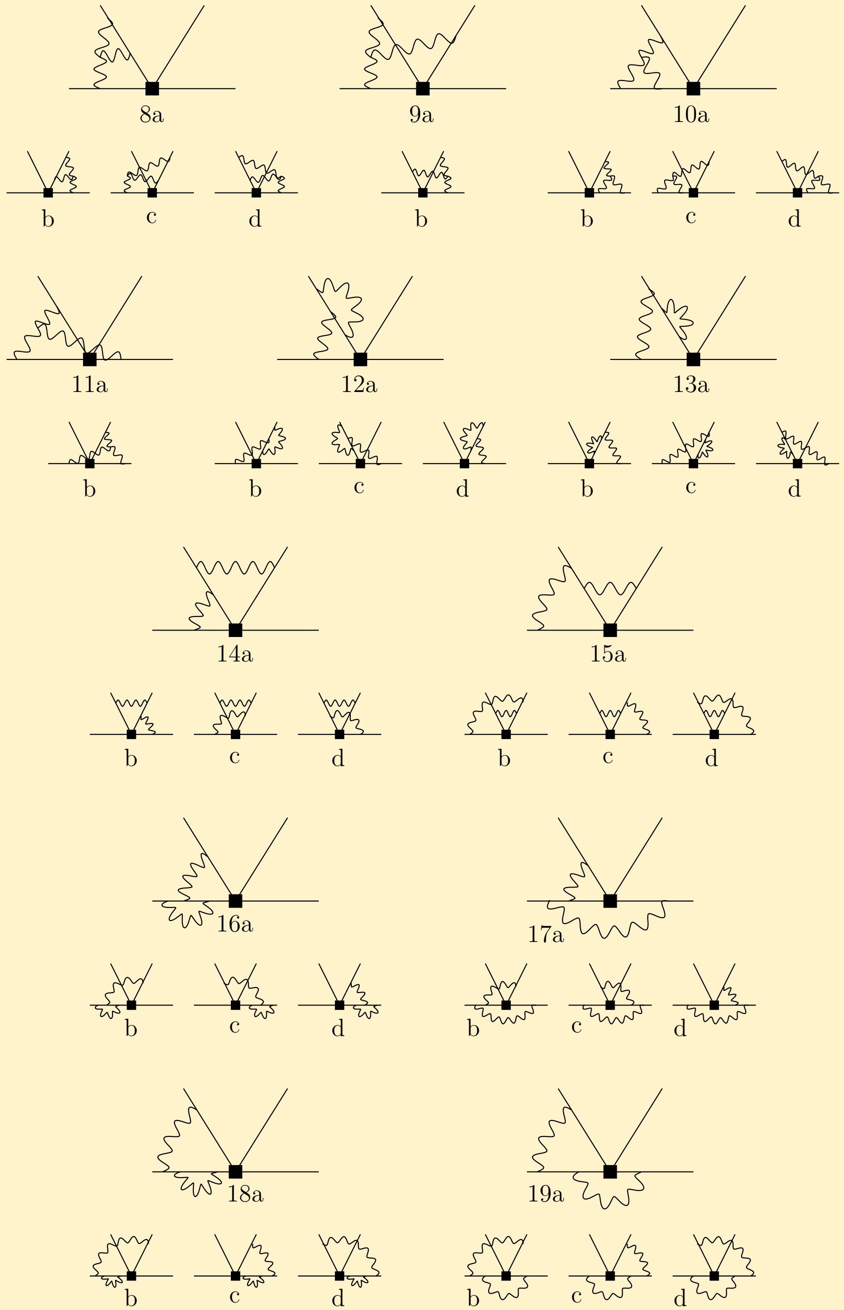
- Goal: $\mathcal{O}(\alpha_s^2)$ vertex corrections to α_1 and α_2 $\Leftrightarrow$ 2-loop matrix elements of Q_1, Q_2

Two-loop diagrams

- Non-factorizable two-loop diagrams for non-leptonic B-decays *[Beneke,Buchalla,Neubert,Sachrajda'00]*



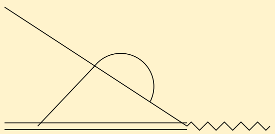




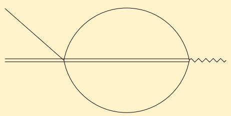
- Kinematics: $p_b^2 = m_b^2$, $q^2 = 0$, $p^2 = 0$ or $p^2 = m_c^2$

Multi-loop techniques in a nut-shell

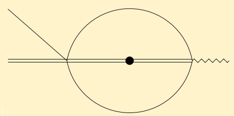
- Dimensional regularisation with $D = 4 - 2\epsilon$ regulates UV and IR. Poles up to $1/\epsilon^4$.
- Passarino-Veltman reduction of tensor integrals to scalar integrals *[Passarino,Veltman'79]*
- Reduction of scalar integrals to a small set of master integrals
  - Integration-by-parts and Lorentz-invariance identities *[Tkachov'81; Chetyrkin,Tkachov'81; Gehrmann,Remiddi'99]*
  - System of equations solved by Laporta algorithm *[Laporta'01;Anastasiou,Lazopoulos'04;Smirnov'08]*



$$= \frac{(8-3D)(7uD-8D-24u+28)}{3(D-4)^2\, m_b^4\, u^3}$$



$$- \frac{2[u^2(D-4)+(16D-56)(1-u)]}{3(D-4)^2\, m_b^2\, u^3}$$

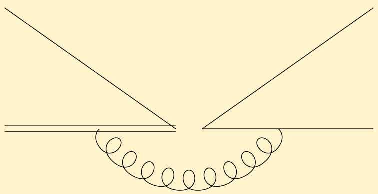


- Techniques for the evaluation of the 42 master integrals
  - Hypergeometric functions, ϵ-expansion in Mathematica or Form *[Moch,Uwer'05; Maitre,TH'05,'07]*
  - Differential equations *[Kotikov'91; Remiddi'97]*
  - Mellin-Barnes representations *[Smirnov'99; Tausk'99; Czakon'05; Gluza,Kajda,Riemann'07]*

Master formula, hard scattering kernel for α_2

$$
\begin{aligned}
\widetilde{T}_i^{(1)} &= \widetilde{A}_{i1}^{(1),nf} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)} \\
&\quad + \underbrace{\widetilde{A}_{i1}^{(1),f} - A^{(1),f}\,\widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{T}_i^{(2)} &= \widetilde{A}_{i1}^{(2),nf} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\,\widetilde{A}_{i1}^{(1),nf} \\
&\quad + (-i)\,\delta m^{(1)}\,\widetilde{A}_{i1}^{\prime(1),nf} \\
&\quad + (Z_{ext}^{(1)} + \xi_{45}^{(1)})\,[\widetilde{A}_{i1}^{(1),nf} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}] \\
&\quad - \widetilde{T}_i^{(1)}\,[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1} \widetilde{H}_{ib}^{(1)}\,\widetilde{Y}_{b1}^{(1)} \\
&\quad + [\widetilde{A}_{i1}^{(2),f} - A^{(2),f}\,\widetilde{A}_{i1}^{(0)}] \\
&\quad + (-i)\,\delta m^{(1)}\,[\widetilde{A}_{i1}^{\prime(1),f} - A^{\prime(1),f}\,\widetilde{A}_{i1}^{(0)}] \\
&\quad + (Z_{\alpha}^{(1)} + Z_{ext}^{(1)} + \xi_{45}^{(1)})[\widetilde{A}_{i1}^{(1),f} - A^{(1),f}\,\widetilde{A}_{i1}^{(0)}] \\
&\quad - C_{FF}^{(1)}\,\widetilde{A}_{i1}^{(0)}[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)}
\end{aligned}
$$



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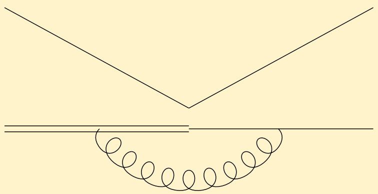


$$
\begin{aligned}
T_1^{(2),re} &= \frac{(47u^5 - 278u^4 + 1223u^3 - 2316u^2 + 2036u - 652)\ln^4(1-u)}{162(u-1)^2u^3} \\
&- \frac{(2u^3 + 4u^2 + 173u + 16)\ln^3(1-u)}{81u} - \frac{(4u^3 - 61u^2 - 436u + 16)\ln^2(1-u)}{54u^2} \\
&+ \frac{2\,(73u^5 + 38u^4 - 1103u^3 + 2316u^2 - 2036u + 652)\ln(u)\ln^3(1-u)}{81(u-1)^2u^3} \\
&- \frac{(17u^3 + 300u^2 - 1098u + 978)\ln^2(u)\ln^2(1-u)}{27u^3} \\
&- \frac{\pi^2\,(9u^5 + 166u^4 - 1167u^3 + 2316u^2 - 2036u + 652)\ln^2(1-u)}{81(u-1)^2u^3} \\
&+ \frac{(2u^5 - 20u^3 + 125u^2 - 76u - 52)\ln(u)\ln^2(1-u)}{27(u-1)^2u} + \frac{2}{9}\ln^3(u)\ln(1-u) \\
&+ \frac{7(u-2)^2\ln(2-u)\ln^2(1-u)}{9(u-1)^2} + \frac{16}{9}\mathrm{Li}_2(u)\ln^2(1-u) \\
&+ \frac{(2u^6 + 4u^5 - 191u^4 - 167u^3 + 1022u^2 - 646u - 6)\ln^2(u)\ln(1-u)}{27(u-1)u^3} \\
&- \frac{\pi^2\,(2u^5 + 355u^3 - 623u^2 + 385u - 140)\ln(1-u)}{81(u-1)^2u} \\
&- \frac{(4u^4 - 638u^3 + 1487u^2 - 1597u + 664)\ln(u)\ln(1-u)}{27(u-1)u^2} \\
&+ \frac{14(u-2)^2\mathrm{Li}_2(u-1)\ln(1-u)}{9(u-1)^2} + \frac{16\,(6u^2 - 16u - 5)\,\mathrm{Li}_3(u)\ln(1-u)}{27(u-1)^2} \\
&- \frac{2\,(94u^3 - 271u^2 + 166u + 32)\,\mathrm{Li}_2(u)\ln(1-u)}{27(u-1)^2u} + \frac{(1601u - 1172)\ln(1-u)}{54u} \\
&+ \frac{4\,(4u^3 - 50u^2 + 183u - 163)\ln(u)\mathrm{Li}_2(u)\ln(1-u)}{27u^3} \\
&+ \frac{(2u^3 - 436u^2 + 657u - 332)\ln^2(u)}{27(u-1)u} - \frac{8\,(3u^2 - 14u - 19)\,\zeta(3)\ln(1-u)}{27(u-1)^2} \\
&+ \frac{2\,(20u^5 - 94u^4 + 292u^3 - 579u^2 + 509u - 163)\,\mathrm{Li}_2(u)^2}{27(u-1)^2u^3} \\
&+ \frac{\pi^2\,(4u^4 - 435u^3 + 3174u^2 - 5346u + 2688)}{162(u-1)u^2} + \frac{64}{9}\mathrm{Li}_3(1-u)\ln(1-u)
\end{aligned}
$$

+ … (five pages)

Numerical Results

- Convolution of hard scattering kernels with pion LCDA yields topological tree amplitudes $\alpha_1(\pi\pi)$ and $\alpha_2(\pi\pi)$ to NNLO

- Have expressions for $\alpha_1(\pi\pi)$ and $\alpha_2(\pi\pi)$ completely analytically, including m_c dependence

$$\alpha_1(\pi\pi) \supset \ldots + 8194\zeta_5 - 2028\pi^2\zeta_3 - \ln^3\left(\frac{4z}{\left(\sqrt{4z+1}+1\right)^2}\right) - 12\,\mathrm{Li}_3\left(\frac{4z}{\left(\sqrt{4z+1}+1\right)^2}\right) + 2\,\mathrm{Li}_3\left(\frac{2\sqrt{z}}{\sqrt{z}+1}\right) + \ldots (3\,\text{pages})$$

- We find complete agreement (numerically) with G. Bell *[G. Bell'09]*

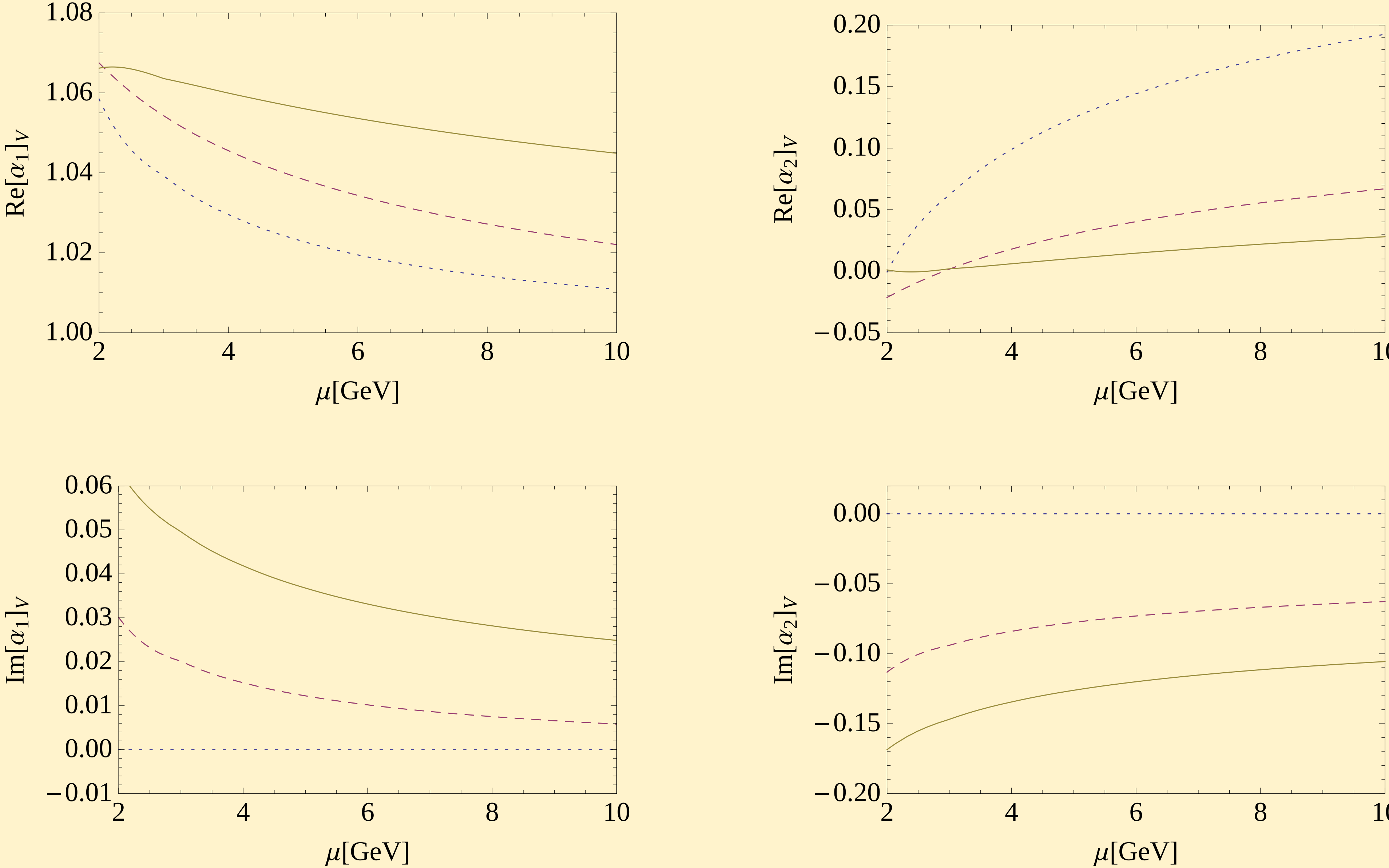
$$\begin{aligned}
\alpha_1(\pi\pi) &= 1.009 + [0.023 + 0.010\,i]_{\mathrm{NLO}} + [0.026 + 0.028\,i]_{\mathrm{NNLO}} \\
&\quad - \left[\frac{r_{\mathrm{sp}}}{0.445}\right]\left\{[0.014]_{\mathrm{LOsp}} + [0.034 + 0.027\,i]_{\mathrm{NLOsp}} + [0.008]_{\mathrm{tw3}}\right\} \\
&= 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})\,i
\end{aligned}$$

$$r_{\mathrm{sp}} = \frac{9 f_{M_1} \hat{f}_B}{m_b \lambda_B f_+^{B\pi}(0)}$$

$$\begin{aligned}
\alpha_2(\pi\pi) &= 0.220 - [0.179 + 0.077\,i]_{\mathrm{NLO}} - [0.031 + 0.050\,i]_{\mathrm{NNLO}} \\
&\quad + \left[\frac{r_{\mathrm{sp}}}{0.445}\right]\left\{[0.114]_{\mathrm{LOsp}} + [0.049 + 0.051\,i]_{\mathrm{NLOsp}} + [0.067]_{\mathrm{tw3}}\right\} \\
&= 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})\,i
\end{aligned}$$

- NNLO corrections to vertex and spectator terms significant but tend to cancel! ☹

Renormalization scale dependence



Factorisation test

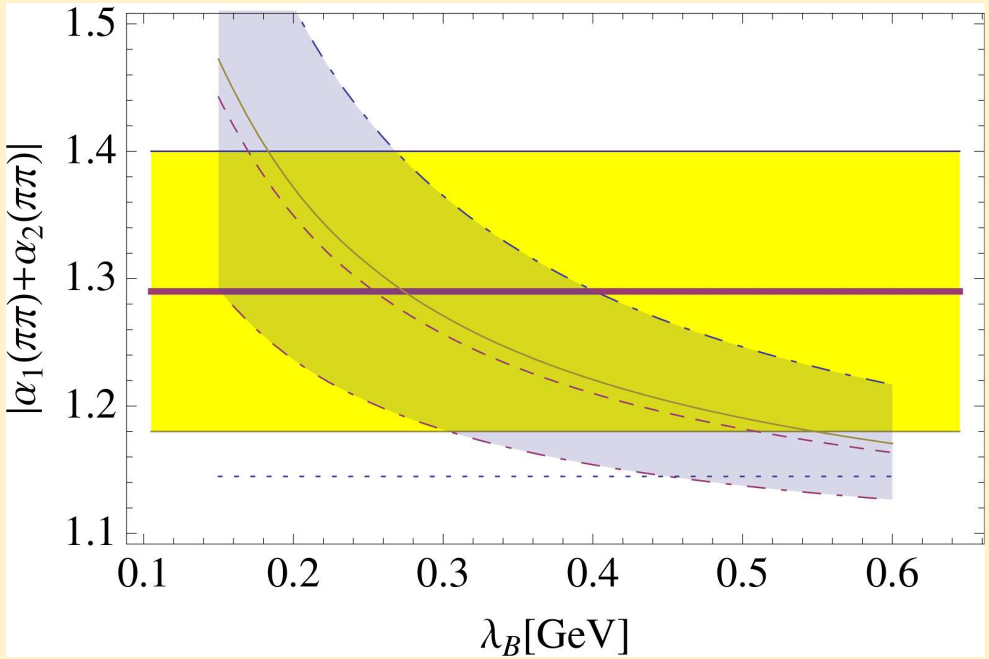
$$R \equiv \frac{\Gamma(B^- \to \pi^-\pi^0)}{d\Gamma(\bar{B}^0 \to \pi^+\ell^-\bar{\nu})/dq^2\big|_{q^2=0}} = 3\pi^2 f_\pi^2\, |V_{ud}|^2\, |\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|^2$$

- From semi-leptonic data
  [cf. Becher,Hill'05; Ball'06; BaBar'06]

  $$|V_{ub}| f_+^{B\pi}(0) = (9.1 \pm 0.7) \times 10^{-4}$$

  equivalent to

  $$|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|_{\mathrm{exp}} = 1.29 \pm 0.11$$

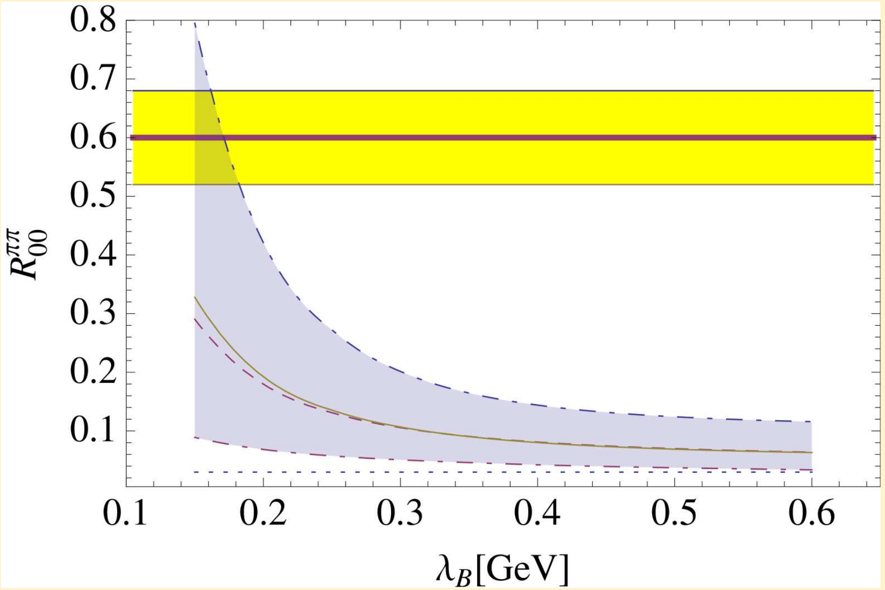


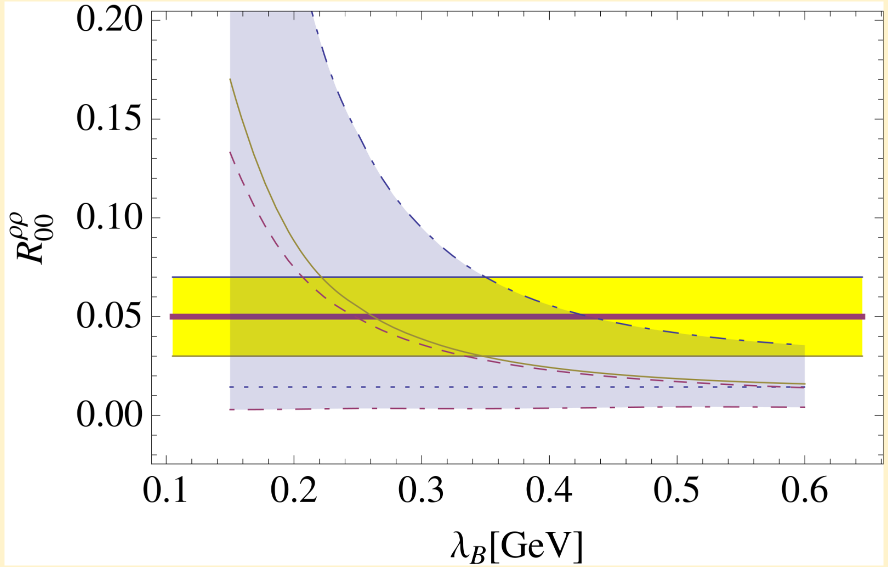
- Good agreement with theory supports QCDF approach

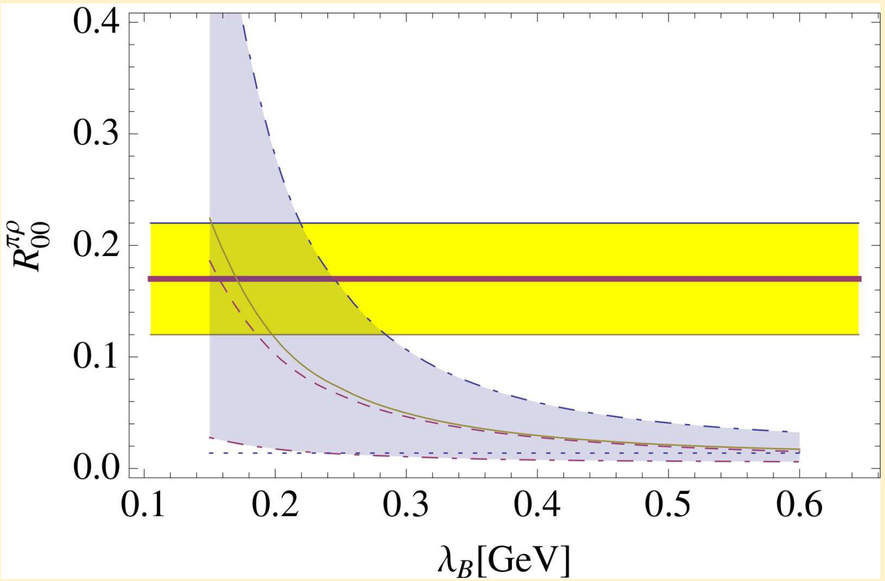
  $$|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|_{\mathrm{th.}} = 1.24^{+0.16}_{-0.10}$$

- Central exptl. value allows $\lambda_B \in [150, 400]$ MeV (on lower side of expectations).

Ratios involving colour-suppressed decays







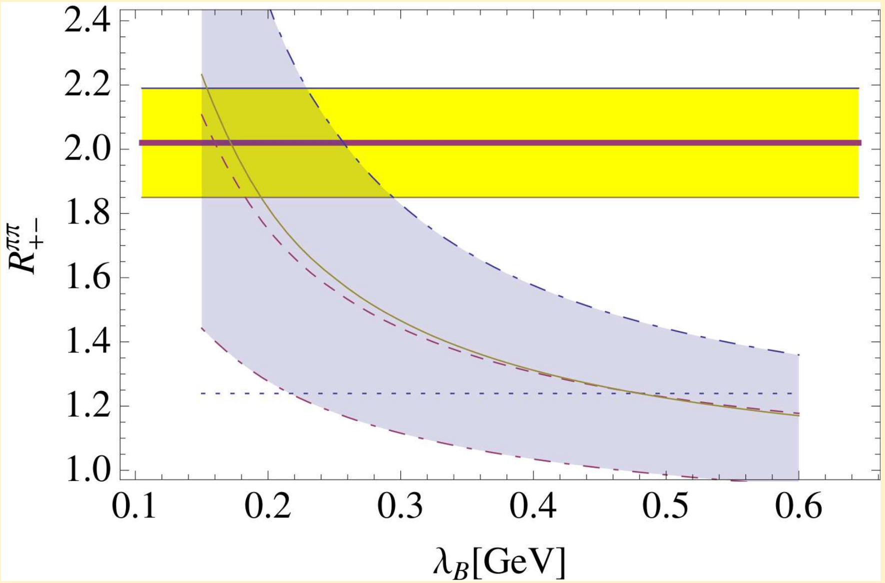
$$R_{00}^{\pi\pi} = 2\,\frac{\Gamma(B^0 \to \pi^0\pi^0)}{\Gamma(B^0 \to \pi^+\pi^-)}$$

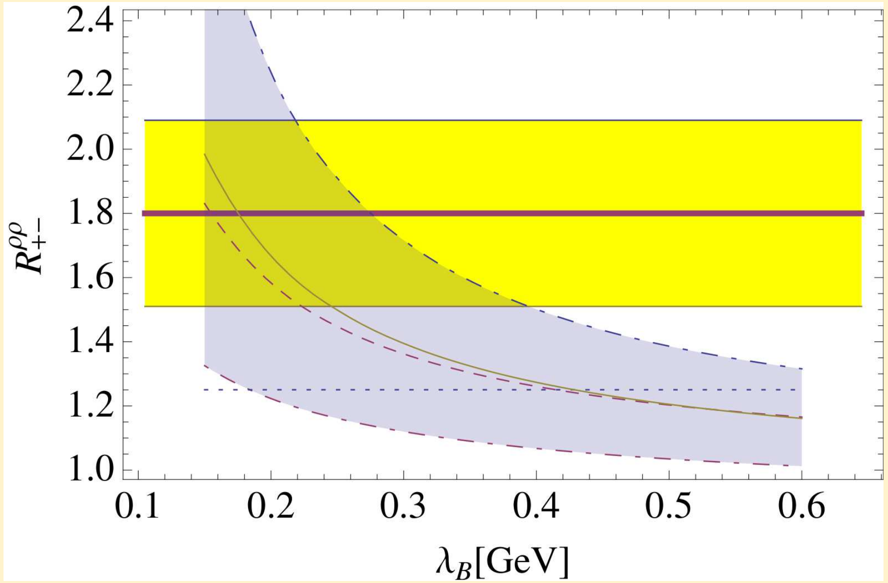
$$R_{00}^{\rho\rho} = 2\,\frac{\Gamma(B^0 \to \rho_L^0\rho_L^0)}{\Gamma(B^0 \to \rho_L^+\rho_L^-)}$$

$$R_{00}^{\pi\rho} = \frac{2\,\Gamma(B^0 \to \pi^0\rho^0)}{\Gamma(B^0 \to \pi^+\rho^-) + \Gamma(B^0 \to \pi^-\rho^+)}$$

Preference for small λ_B, i.e. strong spectator scattering, as already found at NLO in *[Beneke,Neubert'03]*

More ratios

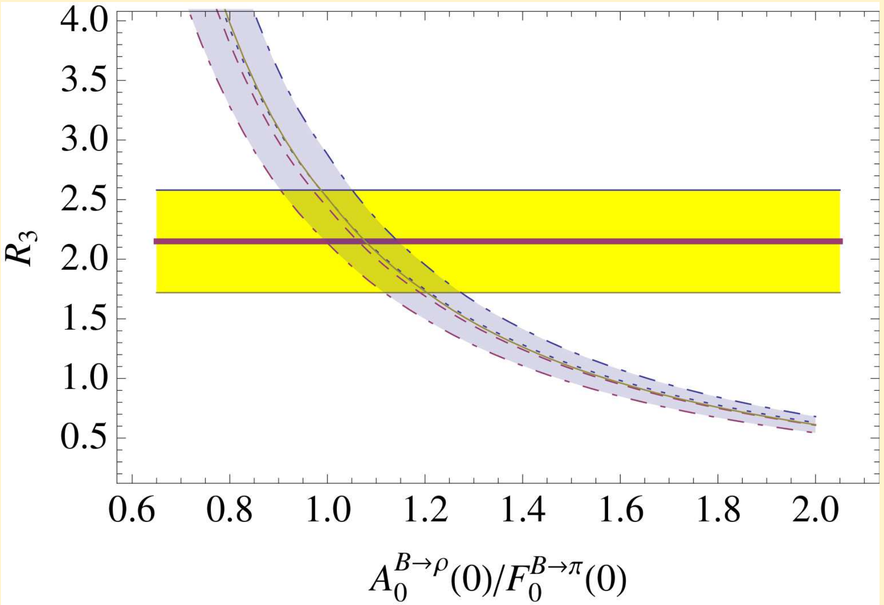


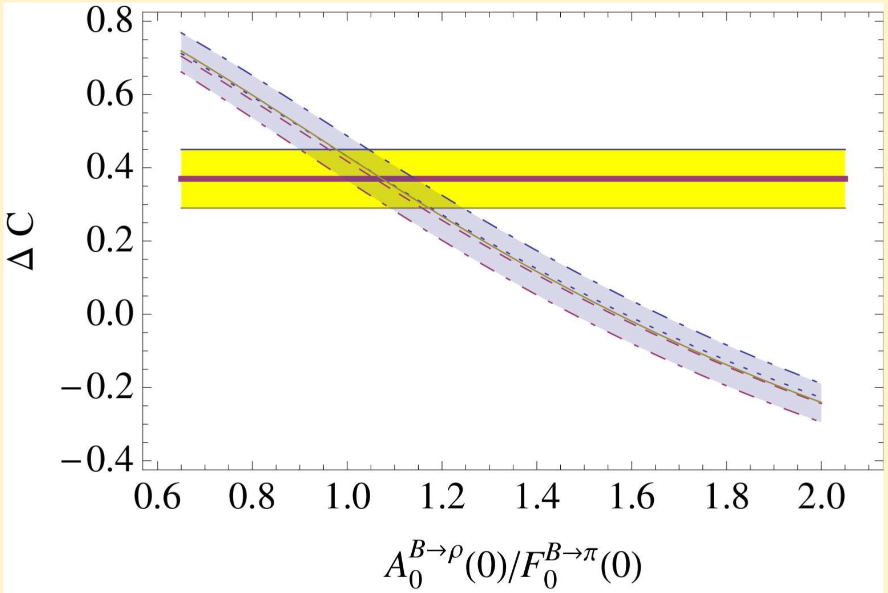


$$
\begin{aligned}
R_{+-}^{\pi\pi} &= 2\,\frac{\Gamma(B^{\pm}\to\pi^{\pm}\pi^{0})}{\Gamma(B^{0}\to\pi^{+}\pi^{-})} \\
R_{+-}^{\rho\rho} &= 2\,\frac{\Gamma(B^{\pm}\to\rho_L^{\pm}\rho_L^{0})}{\Gamma(B^{0}\to\rho_L^{+}\rho_L^{-})}
\end{aligned}
$$

Also here: Preference for small $\lambda_B \simeq 200$ MeV

Dependence on form factors





$$R_3 = \frac{\Gamma(\bar{B}^0 \to \pi^+\rho^-)}{\Gamma(\bar{B}^0 \to \pi^-\rho^+)}$$

$$\Delta C = \frac{1}{2}\left[C(\pi^-\rho^+) - C(\pi^+\rho^-)\right]$$

- Default values
  - $A_0^{B\to\rho}(0)/F_0^{B\to\pi}(0) = 1.2$
  - $F_0^{B\to\pi}(0) = 0.25$ (fixed)
- Agreement excellent for $A_0^{B\to\rho}(0)/F_0^{B\to\pi}(0) \in [1.0, 1.2]$

Final numerical results

	Theory I	Theory II	Exp.
$B^- \to \pi^-\pi^0$	$5.43^{+0.06\,+1.45}_{-0.06\,-0.84}$	$5.82^{+0.07\,+1.42}_{-0.06\,-1.35}$	$5.59^{+0.41}_{-0.40}$
$\bar{B}^0_d \to \pi^+\pi^-$	$7.37^{+0.86\,+1.22}_{-0.69\,-0.97}$	$5.70^{+0.70\,+1.16}_{-0.55\,-0.97}$	5.16 ± 0.22
$\bar{B}^0_d \to \pi^0\pi^0$	$0.33^{+0.11\,+0.42}_{-0.08\,-0.17}$	$0.63^{+0.12\,+0.64}_{-0.10\,-0.42}$	1.55 ± 0.19
$B^- \to \pi^-\rho^0$	$8.68^{+0.42\,+2.71}_{-0.41\,-1.56}$	$9.84^{+0.41\,+2.54}_{-0.40\,-2.52}$	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0\rho^-$	$12.38^{+0.90\,+2.18}_{-0.77\,-1.41}$	$12.13^{+0.85\,+2.23}_{-0.73\,-2.17}$	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^\pm\rho^\mp$	$28.08^{+0.27\,+3.82}_{-0.19\,-3.50}$	$21.90^{+0.20\,+3.06}_{-0.12\,-3.55}$	23.0 ± 2.3
$\bar{B}^0 \to \pi^0\rho^0$	$0.52^{+0.04\,+1.11}_{-0.03\,-0.43}$	$1.49^{+0.07\,+1.77}_{-0.07\,-1.29}$	2.0 ± 0.5
$B^- \to \rho^-_L\rho^0_L$	$18.42^{+0.23+3.92}_{-0.21-2.55}$	$19.06^{+0.24+4.59}_{-0.22-4.22}$	$22.8^{+1.8}_{-1.9}$
$\bar{B}^0_d \to \rho^+_L\rho^-_L$	$25.98^{+0.85+2.93}_{-0.77-3.43}$	$20.66^{+0.68+2.99}_{-0.62-3.75}$	$23.7^{+3.1}_{-3.2}$
$\bar{B}^0_d \to \rho^0_L\rho^0_L$	$0.39^{+0.03+0.83}_{-0.03-0.36}$	$1.05^{+0.05+1.62}_{-0.04-1.04}$	$0.55^{+0.22}_{-0.24}$
$R^{\pi\pi}_{+-}$	$1.38^{+0.12+0.53}_{-0.13-0.32}$	$1.91^{+0.18+0.72}_{-0.20-0.64}$	2.02 ± 0.17
$R^{\pi\pi}_{00}$	$0.09^{+0.03+0.12}_{-0.02-0.04}$	$0.22^{+0.06+0.28}_{-0.05-0.16}$	0.60 ± 0.08
$R^{\rho\rho}_{+-}$	$1.32^{+0.02+0.44}_{-0.03-0.27}$	$1.72^{+0.03+0.64}_{-0.03-0.53}$	$1.80^{+0.28}_{-0.29}$
$R^{\rho\rho}_{00}$	$0.03^{+0.00+0.07}_{-0.00-0.03}$	$0.10^{+0.01+0.19}_{-0.01-0.11}$	0.05 ± 0.02
$R^{\pi\rho}_{00}$	$0.04^{+0.00+0.09}_{-0.00-0.03}$	$0.14^{+0.01+0.20}_{-0.01-0.13}$	0.17 ± 0.05
R_3	$1.73^{+0.13+1.12}_{-0.12-0.82}$	$1.69^{+0.13+0.72}_{-0.12-0.59}$	2.15 ± 0.43

- Theory II: With lower λ_B, V_{ub}, and form factors. Our preferred scenario.

Conclusion

- The colour-allowed and colour-supressed tree amplitudes have been computed completely analytically to NNLO
- The two-loop computation requires sophisticated computational techniques
- The NNLO corrections are very small. Accidental cancellation between vertex and spectator term
- QCDF beyond naive factorization describes data well, especially for low λ_B, V_{ub}, and form factors. Exceptions are observables with $\pi^0\pi^0$ final state
- To do: Two-loop penguin amplitudes, CP asymmetries at NLO