NNLO Higgs production via gluon fusion with finite top mass*

Kemal Ozeren

Bergische Universität Wuppertal

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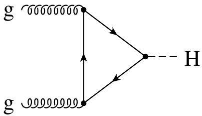
(*work done in collaboration with Robert Harlander)

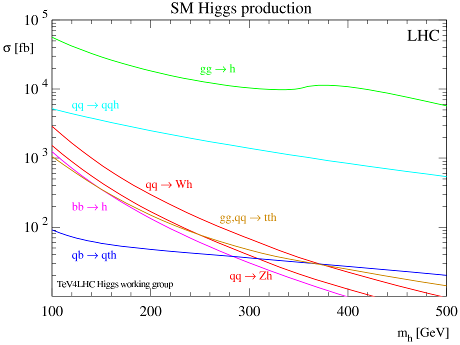
arXiv:0907.2997 [Phys. Lett B 679 (2009) 467]
arXiv:0909.3420 [submitted to JHEP]

Outline

$gg \to H$ in the SM

- production is via a top loop





- LO cross section known

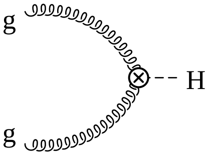
$$\sigma_{LO} = \frac{G_F \alpha_s^2(\mu^2)}{128\sqrt{2}\pi} \tau^2 \, \delta(1-x) |1 + (1-\tau) f(\tau)|^2$$

$$\tau = \frac{4m_t^2}{m_H^2}$$

[Georgi, Glashow, Machacek, Nanopoulos]

The Heavy Top Effective Theory

- If $\frac{m_H}{2m_{top}} \ll 1$ work in effective theory
- Top 'integrated out' of the theory
- ...but leaves its legacy in the form of altered couplings and new vertices



$$\mathcal{L}_{eff} = -\frac{H}{4v} C_1 G_{\mu\nu} G^{\mu\nu}$$

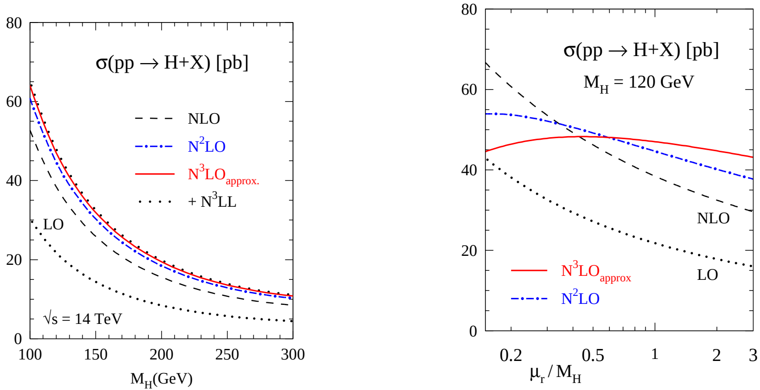
$$C_1 = -\frac{1}{3}\frac{\alpha_s}{\pi}\left\{1 + \frac{11}{4}\frac{\alpha_s}{\pi} + \cdots\right\}$$

Major benefit: reduces number of loops by one

Quantum Corrections

- QCD corrections huge - $\mathcal{O}(100\%)$
  - NLO (effective theory) [Dawson '91]
  - NLO (HIGLU) [Spira,Djouadi,Graudenz,Zerwas '95]
  - NNLO (effective theory) [Harlander,Kilgore] [Anastasiou,Melnikov '02] [Ravindran,Smith,van Neerven '03]
- Electroweak
  [Actis, Passarino, Sturm, Uccirati '08]
- Mixed QCD-Electoweak
  [Anastasiou, Boughezal, Petriello '08]
- NNLO+NNLL - $\mathcal{O}(\%)$
  [Catani, de Florian, Grazzini, Nason '03]
- N^3LO threshold enhanced corrections
  [Moch, Vogt '05], [Laenen, Magnea '05], [Ravindran '05]
  [Kidonakis '05], [Idilbi, Ju, Yuan '05]
- "π^2-resummation" [Ahrens, Becher, Neubert, Yang '08]

Overview of QCD Corrections



Moch, Vogt

- QCD effects well under control
- Residual scale uncertainty $\sim$ 5%
- See also updated analyses

[Anastasiou, Boughezal, Petriello '08] [de Florian, Grazzini '09]

How accurate is the effective theory at NNLO?

Top mass effects

Exact mass dependence known for

- inclusive NLO
  [Spira, Djouadi, Graudenz '93'95]
  [Bonciani, Degrassi, Vicini '07]
- p_t distribution at LO
  [Anastasiou, Bucherer, Kunszt '09]
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- y distribution at NLO
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  (i.e. all NNLO quantities)

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$$\sigma^{HO} = \sigma^{LO}(M_t) \left(\frac{\sigma^{HO}}{\sigma^{LO}}\right)_{M_t \to \infty}$$

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- everything else
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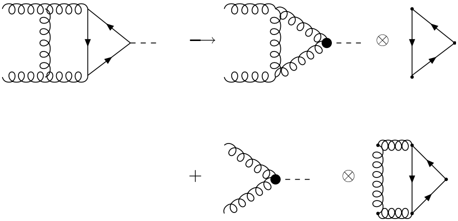
Asymptotic Expansion

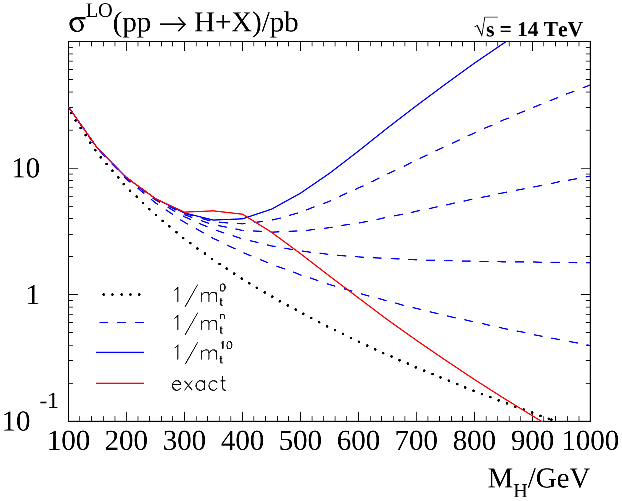
- Full NNLO calculation with top mass not currently feasible
- We perform an asymptotic expansion in $\frac{1}{m_t}$



$$\sigma = \sum_n \left(\frac{m_H^2}{4m_t^2} \right)^n \sigma_n$$

- First term σ_0 is the effective theory result
- First non-leading $1/m_t$ term at NLO known
  [Dawson,Kauffman '93]
- Tools exist to automatize the calculation
  QGRAF, EXP, FORM, MATAD, MINCER

Expansion of σ_{LO}



NNLO ingredients

We have three contributions:

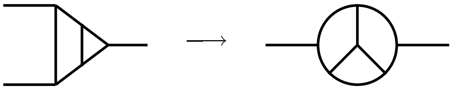
- double virtual
  [Harlander,KJO ’09]
  [Pak,Rogal,Steinhauser ’09]
- single real emission
- double real emission

$$\begin{aligned}\sigma &= \int M^{(3)}_{gg\to H}\left[M^{(1)}_{gg\to H}\right]^* + \left|M^{(2)}_{gg\to H}\right|^2 \\ &+ \iint M^{(2)}_{gg\to Hg}\left[M^{(1)}_{gg\to Hg}\right]^* \\ &+ \iiint \left|M^{(1)}_{gg\to Hgg}\right|^2\end{aligned}$$

- Each integrated over relevant phase space volume

Loop amplitudes - double virtual piece

- After expansion, double virtual part consists of 2-loop 3-point diagrams
- Use Baikov-Smirnov method to map onto known 3-loop 2-point diagrams
  [hep-ph/0001192]



- First used to evaluate the virtual contribution to the Higgs cross section in the effective theory
  [Harlander '00]
- Can treat arbitrary propagator powers

Phase Space Integration (1)

- One particle final states are easy $\longrightarrow \delta(1 - \frac{m_H^2}{\hat{s}})$
- Two particle final states are somewhat less easy
  - One can go a long way with

$$\int_0^1 \mathrm{d}v\ v^{\alpha}(1-v)^{\beta} = \frac{\Gamma(1+\alpha)\Gamma(1+\beta)}{\Gamma(2+\alpha+\beta)}$$

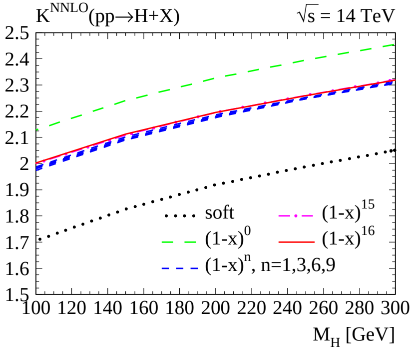
  - Amplitudes come with ${}_2F_1$ hypergeometrics (from the box integrals)
  - Direct integration yields extended hypergeometrics (${}_3F_2,\ {}_4F_3$)

$$\int_0^1 \mathrm{d}v\ v^{\alpha}(1-v)^{\beta}\ {}_2F_1(\ldots, z\ v) \sim {}_3F_2(\ldots, z)$$

- Use HypExp package to expand these in ϵ
  [Huber,Maitre]

Phase Space Integration (2)

- Three particle final states very difficult
- Expand amplitude and phase space in powers of $(1 - \frac{m_H^2}{\hat{s}})$
- Series converges quickly



[Harlander,Kilgore]

- In order to cancel poles must also expand the single real contribution

Structure of the Results

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- $\hat{\sigma}$ is a function only of x

$$\sigma = \int \mathcal{L}(x)\,\hat{\sigma}(x)\,\mathrm{d}x\,, \qquad x = \frac{m_H^2}{\hat{s}}$$

- Partonic threshold is $x \to 1$, leads to singularities

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- ... which cancel for inclusive quantities

$$\frac{1}{(1-x)^{1-2\epsilon}} = \frac{\delta(1-x)}{2\epsilon} + \left(\frac{1}{1-x}\right)_+ + 2\epsilon\left[\frac{\ln(1-x)}{1-x}\right]_+ + \dots$$

- Plus distribution (very useful!)

$$\int f(x)\left(\frac{1}{1-x}\right)_+ \mathrm{d}x = \int \frac{f(x)-f(1)}{1-x}\,\mathrm{d}x$$

Checks

Huge complexity $\longrightarrow$ any check is welcome

- Pole structure [Catani] ✓
- Log terms determined by Ren Group and Factorisation scale invariance ✓
- Agreement of leading term with effective theory ✓
- For single real piece: soft expansion before/after phase space integration ✓

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Strongest check: independent calculation
[Pak,Rogal,Steinhauser (in prep)]

- compute imaginary part of 4-loop diagrams
- see M.Rogal's talk
- full agreement of analytic results ✓

Small x behaviour

The total cross section is

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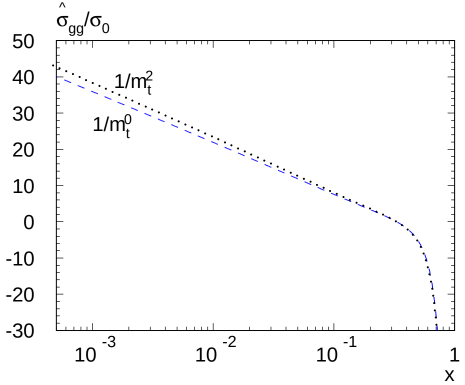

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- Appears disastrous...

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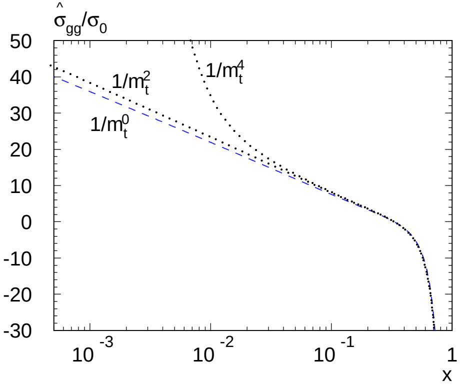

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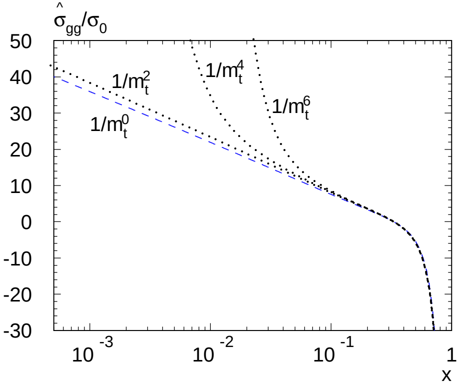

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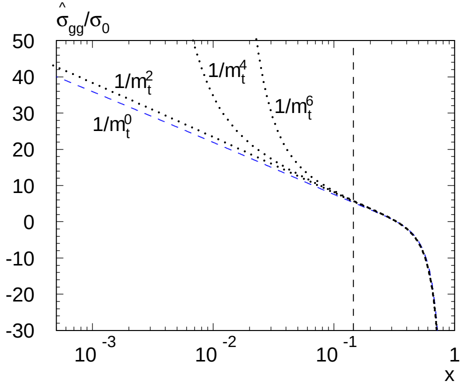

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Small x matching

- Leading small-x behaviour known exactly
  [Marzani,Ball,Del Duca,Forte,Vicini '08]

$$\hat{\sigma}^{(1)}_{gg} = 3\,\sigma_0\,\mathcal{C}^{(1)} + \mathcal{O}(x)$$
$$\hat{\sigma}^{(2)}_{gg} = -9\,\sigma_0\,\mathcal{C}^{(2)}\,\ln x + c + \mathcal{O}(x)$$

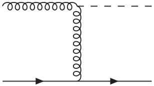
m_H	$\mathcal{C}^{(1)}(y_t)$	$\mathcal{C}^{(2)}(y_t)$
110	5.0447	16.2570
120	4.6873	14.5133
130	4.3568	13.0155
140	4.0490	11.7196
150	3.7607	10.5919
160	3.4890	9.6058
170	3.2318	8.7406

- Coefficients given as table of numerical values

We can therefore improve our result. For example, at NLO we use

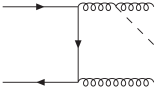
$$\hat{\sigma}^{(1)}_{gg}(x) = \hat{\sigma}^{(1)}_{gg,N}(x) + (1-x)^{N+1}\left[3\,\sigma_0\mathcal{C}^{(1)} - \hat{\sigma}^{(1)}_{gg,N}(0)\right]$$

Sub-channels



In addition to gg, we also have

- qg, $q\bar{q}$ starting at NLO
- qq, qq' at NNLO



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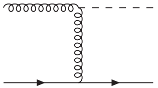
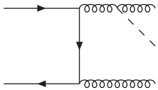
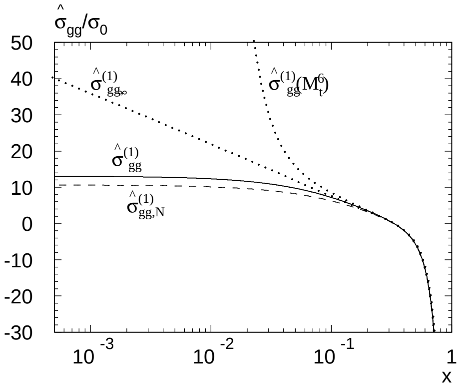




The small-x behaviour of these contributions is unknown.

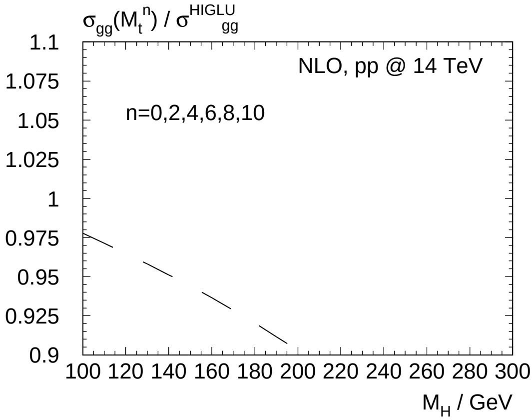
- include $1/M_t$ terms up to a certain depth, beyond which the series deteriorates

NLO Results



- Both effective theory result and our expansion have spurious small x behaviour
- We improve it by incorporating the known exact $x \to 0$ piece

NLO $1/m_t$ expansion - comparison with HIGLU



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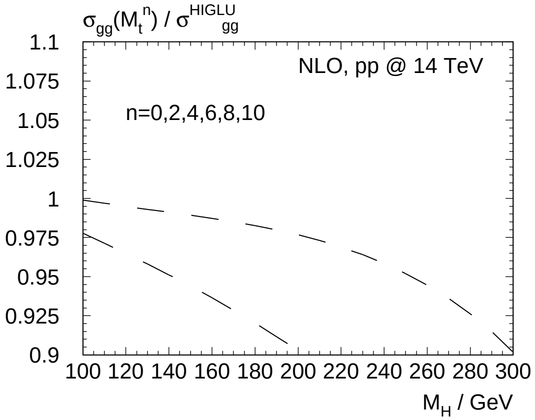

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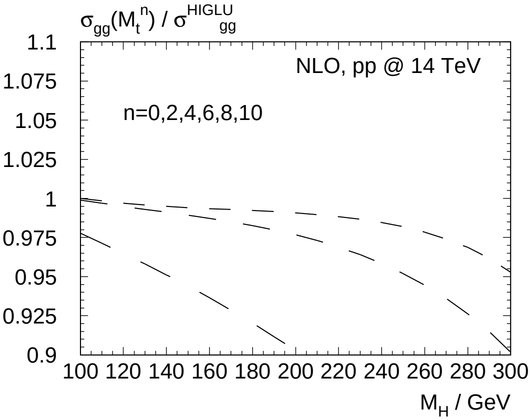

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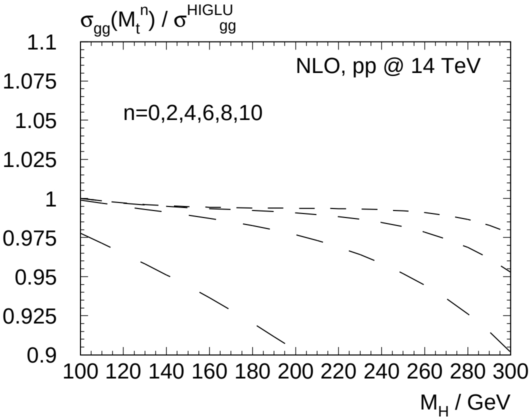

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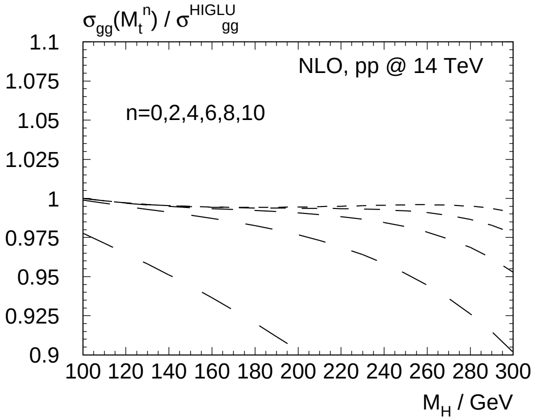

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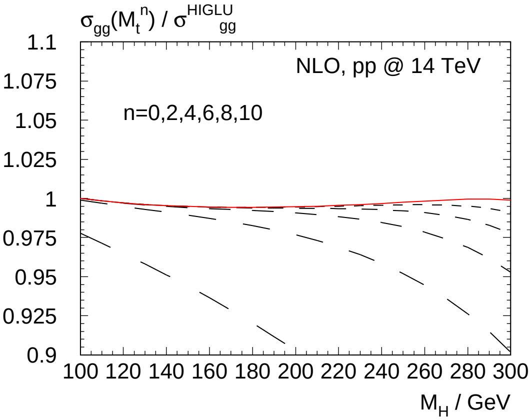

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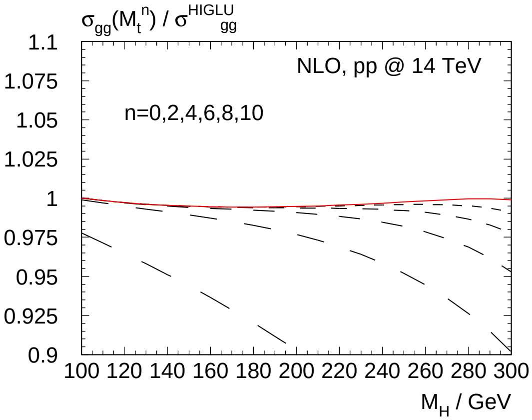
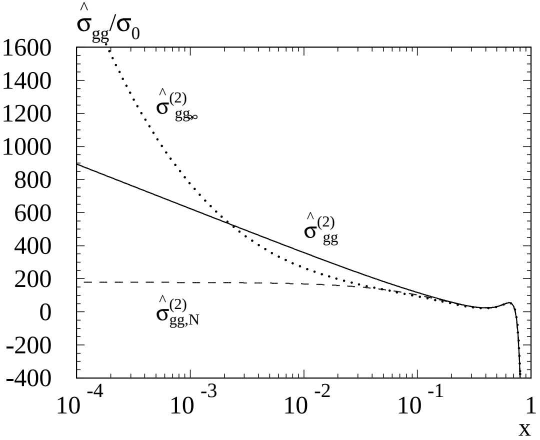


- Excellent agreement
- Use the same approach at NNLO

Results at NNLO - partonic cross-section



- Effective theory result has spurious high energy behaviour
- We match our result onto the exact small x result (solid curve)

NNLO $1/m_t$ expansion - comparison with effective theory

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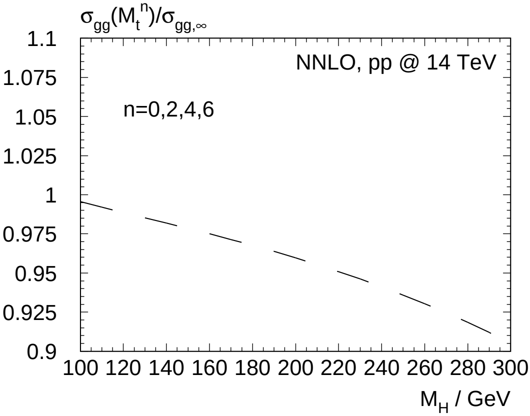

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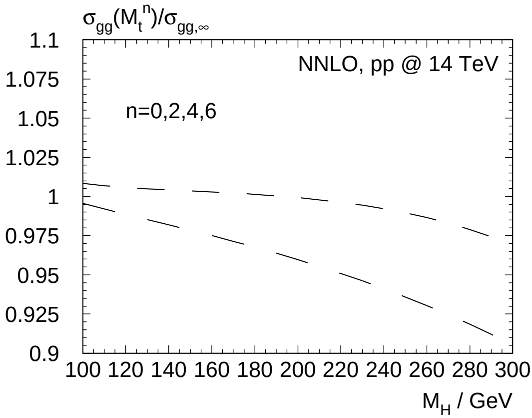

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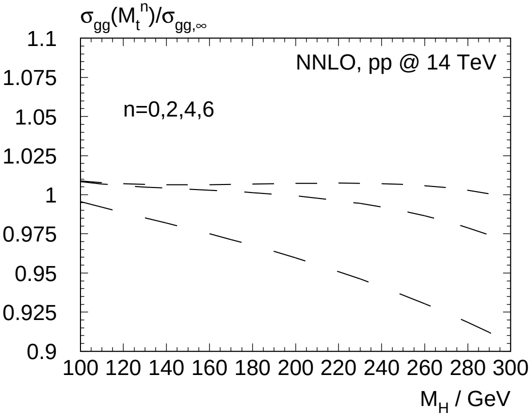

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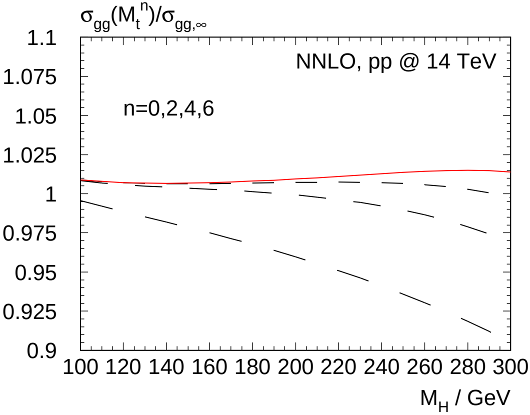

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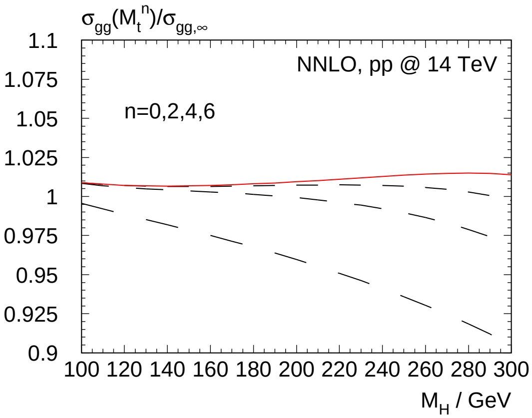

- Small deviations, $\simeq 1\%$

Summary and Outlook

- Long-standing problem: how accurate is the large m_t limit at NNLO?
- We have shown that top mass effects are small - about 1%
- Analytic results confirmed by independent calculation
  [Pak,Rogal,Steinhauser (in prep)]
- Use of effective theory is justified

Next Steps:

- Consider effects on exclusive quantities