Low Energy Theorem in SUSY QCD at 2-loop for $h \to b\bar{b}$

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in collaboration with

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Outline

- Motivation
- Status
- Effective theory
- Low energy theorem for QCD and SUSY QCD
- Renormalization
- Numerics
- Summary

Motivation

- Higgs is the only SM particle not yet discovered
  $\rightarrow$ analyze its mass / couplings
- E. g. Hgg and $\mathrm{Hb\bar{b}}$ couplings are essential for various physical processes studied
- Higgs strongly couples to heavy particles
  $\rightarrow$ window for new physics through loop effects
  - $\mathrm{gg} \rightarrow \mathrm{H}$ possible only through quantum loop effects
  - H^{+} decay does not even exist in SM
- ...

SUSY Status

SUSY effects in $h \to gg$

- 1-loop: [Djouadi, Graudenz, Spira, Zerwas ’95]
- 2-loop: stop-contributions

  [Aglietti, Bonciani, Degrassi, Vicini ’06], [Anastasiou, Beerli, Daleo, Kunszt ’06], [Mühlleitner, Spira ’07]
- 2-loop: stop and gluino contributions

  [Anastasiou, Beerli, Daleo ’08], [Harlander, Steinhauser ’04], [Degrassi, Slavich ’08], [Mühlleitner, Rzehak, Spira ’08]

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SUSY effects in $h \to b\bar{b}$

- 1-loop: [Dabelstein '95], [Coarasa, Jimenez, Sola '96], [Eberl, Hidaka, Kraml, Majerotto, Yamada '00]
- 1-loop, large $\tan\beta$: [Guasch, Häfliger, Spira '03] [Carena, Garcia, Nierste, Wagner '00]
- 2-loop: [Noth, Spira '08]

Effective theory approach

Weinberg 1980; Ovrut, Schnitzer 1981

(SUSY) QCD effects:

- Use effective Lagrangian containing
  - Higgs bosons
  - Light fermions
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  - QCD: top quark
  - SQCD: top quark, squarks, gluino

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Full Theory: (SUSY) QCD $\xrightarrow{m_i \to \infty}$ Effective Theory: 5-flavor QCD

Decoupling relations

Bernreuther, Wetzel 1981; Chetyrkin, Kniehl, Steinhauser 1998; SQCD: Bauer, Mihaila, Salomon 2008

Decoupling relations for strong coupling, light quark wave function and mass:

$$\mathrm{g}_\mathrm{s}^{0\prime} = \zeta_\mathrm{g}^0 \mathrm{g}_\mathrm{s}^0 \,, \quad \mathrm{m}_\mathrm{q}^{0\prime} = \zeta_\mathrm{m}^0 \mathrm{m}_\mathrm{q}^0 \,, \quad \psi_\mathrm{q}^{0\prime} = \sqrt{\zeta_2^0}\psi_\mathrm{q}^0$$

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E.g. determine ζ_2^0 from quark field correlator:

$$-\frac{1}{\not p[1+\Sigma_V^0(p^2)]} = i\int dx\, e^{ip\cdot x}\langle T\psi_q^0(x)\bar\psi_q^0(0)\rangle = \frac{i}{\zeta_2^0}\int dx\, e^{ip\cdot x}\langle T\psi_q^{0\prime}(x)\bar\psi_q^{0\prime}(0)\rangle$$

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- Only light degrees of freedom in $\Sigma_V^{0\prime} \Rightarrow \Sigma_V^{0\prime}(0) = 0$ in dim. reg.

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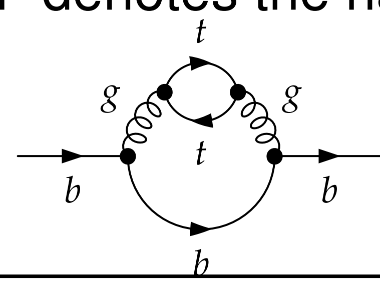
$\Rightarrow$ $\boxed{\zeta_2^0 = 1 + \Sigma_V^{0h}(0)\,,}$ where “h” denotes the hard part of Σ_V^0.

In QCD at 2-loop: $\zeta_2^0 = 1 +$ 

Effective Lagrangian (SM)

Inami, Kubota, Okada 1983; Djouadi, Spira, Zerwas 1991; Chetyrkin, Kniehl, Steinhauser 1997

$$\mathcal{L}_{\mathrm{eff}} = -\frac{\mathsf{H}^0}{\mathsf{v}^0} \sum_{\mathsf{i}=1}^{5} \mathsf{C}_\mathsf{i}^0 \mathcal{O}_\mathsf{i}'$$

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where the bare operators are

$$\mathcal{O}_1' = (G_{\mu\nu}^{0\prime,a})^2 ,$$

$$\mathcal{O}_2' = \sum_{i=1}^{n_l} m_{q_i}^{0\prime} \bar{\psi}_{q_i}^{0\prime} \psi_{q_i}^{0\prime} ,$$

$$\mathcal{O}_3' = \sum_{i=1}^{n_l} \bar{\psi}_{q_i}^{0\prime} \left[\frac{i}{2}(\overrightarrow{\not{D}}^{0\prime} - \overleftarrow{\not{D}}^{0\prime}) - m_{q_i}^{0\prime}\right] \psi_{q_i}^{0\prime} ,$$

plus other unphysical Operators $\mathcal{O}_4', \mathcal{O}_5'$ that inolve ghost interactions.

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\end{aligned}$$

plus other unphysical Operators $\mathcal{O}_4', \mathcal{O}_5'$ that inolve ghost interactions.

$\rightarrow$ Coefficients $C_i^0 = C_i^0(\alpha_s^0, m_t^0, \mu)$ contain effects of heavy particle

Effective Lagrangian (SM)

Renormalization within the EFT Kluberg-Stern, Zuber 1975; Nielsen 1975; Spiridonov 1984

$$[\mathcal{O}_1'] = \left[1 + 2\left(\frac{\alpha_s'\partial}{\partial\alpha_s'} \ln Z_g'\right)\right]\mathcal{O}_1' - 4\left(\frac{\alpha_s'\partial}{\partial\alpha_s'} \ln Z_m'\right)\mathcal{O}_2',$$

$$[\mathcal{O}_2'] = \mathcal{O}_2',$$

$$C_1 = \frac{1}{1 + 2(\alpha_s'\partial/\partial\alpha_s') \ln Z_g'} C_1^0,$$

$$C_2 = \frac{4(\alpha_s'\partial/\partial\alpha_s') \ln Z_m'}{1 + 2(\alpha_s'\partial/\partial\alpha_s') \ln Z_g'} C_1^0 + C_2^0$$

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\end{aligned}
$$

with the EFT renormalization constants for the coupling and the mass

$$
g_s^{0\prime} = \mu^{2\epsilon} Z_g' g_s'\,, \quad m_q^{0\prime} = Z_m' m_q'
$$

Matching the effective theory to the full theory

Consider $\mathrm{Hb\bar{b}}$ correlator in both theories.

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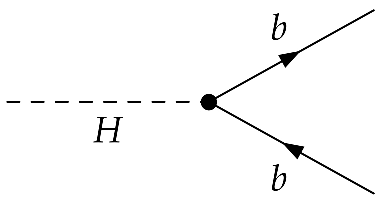
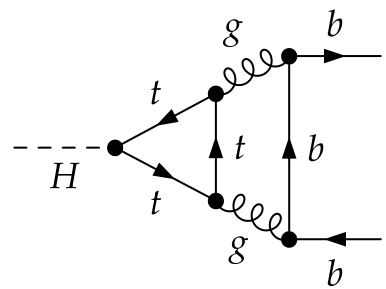
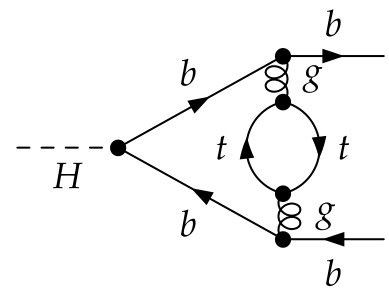
(i) EFT: tree level diagrams generated by $\mathcal{O}_2$, $\mathcal{O}_3$

(ii) Full theory diagrams:

$\Lambda^{0h}_{S,V} =$ 

$+\ 2$ 

$+$ 

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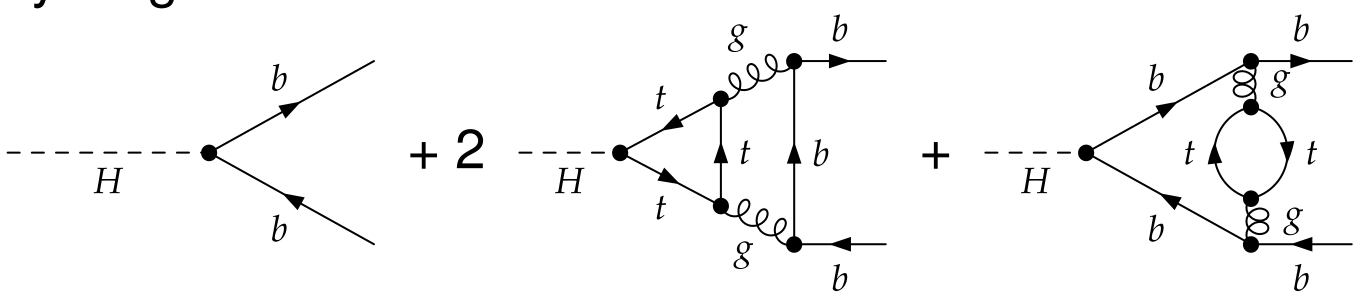

Do matching for scalar and vector part separately:

$$\zeta^0_m \zeta^0_2 (C^0_2 - C^0_3) = 1 + \Lambda^{0h}_S(p_i^2 = 0)$$
$$\zeta^0_2 C^0_3 = \Lambda^{0h}_V(p_i^2 = 0)$$

with

$$\psi^{0\prime}_q = \sqrt{\zeta^0_2}\psi^0_q, \quad m^{0\prime}_q = \zeta^0_m m^0_q$$

Low energy theorem for Higgs interactions

Shifman, Vainshtein, Voloshin, Zakharov 1979; Kniehl, Spira 1995; Kilian 1995;
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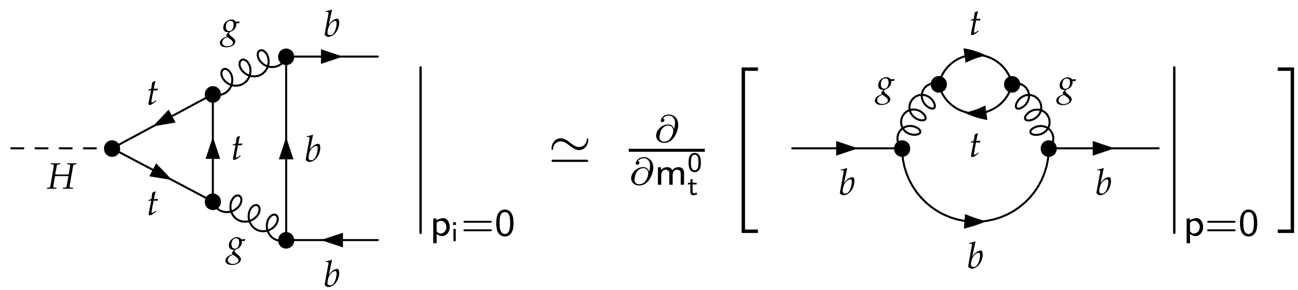
Easiest example:

Exploit that Higgs coupling is proportional to heavy particle mass:



$\Rightarrow$ Vertex amplitudes $\Lambda_S^{0h}(0)$ and $\Lambda_V^{0h}(0)$ can be obtained from light quark propagator via differentiation.

$\Rightarrow$ Limit $M_H \ll m_t$

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In general:

Amplitude of a generic particle configuration X plus Higgs can be obtained from derivatives of X w.r.t. the Higgs field H:

$$\mathcal{A}(\mathrm{X},\mathrm{H})\Big|_{p_\mathrm{H}=0} \simeq \frac{\mathrm{d}}{\mathrm{dH}}\mathcal{A}(\mathrm{X_H})\Big|_{\mathrm{H}=0}$$

where $\mathrm{X_H}$ is an amplitude “depending on H” via its parameters.

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E.g. SM Yukawa Lagrangian: $\mathcal{L}_{Yuk} = -\lambda_t\, \bar{t}^L t^R \phi^{0*}$ with $\phi^0 = \frac{1}{\sqrt{2}}(v + H + i\chi)$

$$\Rightarrow m_t^0(H) = \lambda_t/\sqrt{2}(v + H)$$

$$\frac{d}{dH}\mathcal{A}(X_{m_t^0(H),H})\Big|_{H=0} = \left[\frac{\lambda_t}{\sqrt{2}}\frac{\partial}{\partial m_t^0} + \frac{\partial}{\partial H}\right]\mathcal{A}(X_{m_t^0,H})\Big|_{H=0}$$

SUSY QCD low energy theorem

Degrassi, Slavich, Zwirner 2001

SUSY QCD: (Bare) parameters that depend on neutral Higgs fields H_1^0, H_2^0:

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  $m_{\tilde{t}_{1,2}}^2 = \frac{1}{2}\left[(M_L^2 + M_R^2) \pm \sqrt{(M_L^2 - M_R^2)^2 + 4|\tilde{X}|^2}\right], \;\; m_{\tilde{b}_{1,2}}^2 = \ldots$
  with $M_L^2 = M_Q^2 + \lambda_t^2 |H_2^0|, \;\; M_R^2 = M_U^2 + \lambda_t^2 |H_2^0|, \;\; \tilde{X} = \lambda_t (A_t H_2^0 - \mu_{SUSY} H_1^{0^*})$

- $M_Q, M_{U,D}$: soft SUSY breaking mass parameters
- A_q: trilinear coupling
- μ_{SUSY}: Higgs-Higgsino bilinear coupling

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- Squark mixing angles:
  $\sin 2\theta_t = \frac{2|\tilde{X}|}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}, \; \sin 2\theta_b = \ldots$

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Mass eigenstates h, H from neutral Higgs fields H_1^0, H_2^0:

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix}$$

SUSY QCD low energy theorem

Bare 5-flavor QCD effective Lagrangian:

$$\mathcal{L}_{\mathrm{eff}} = \frac{\mathsf{m}_{\mathsf{b}}^{0\prime}}{\mathsf{v}} \frac{\mathsf{s}_\alpha}{\mathsf{c}_\beta}(\mathsf{C}^0_{2,1} - \frac{1}{\mathsf{tg}\alpha\mathsf{tg}\beta}\mathsf{C}^0_{2,2})\, \mathsf{h}\, \bar{\psi}_{\mathsf{b}}^{0\prime}\psi_{\mathsf{b}}^{0\prime} + \ldots$$

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SUSY QCD coefficients:

$$\mathsf{C}^0_{2,1} = 1 + \hat{\mathsf{D}}_1 \ln \zeta_\mathsf{m}^0 = 1 - \frac{\hat{\mathsf{D}}_1 \Sigma_\mathsf{S}^{0\mathsf{h}}}{1 - \Sigma_\mathsf{S}^{0\mathsf{h}}} - \frac{\hat{\mathsf{D}}_1 \Sigma_\mathsf{V}^{0\mathsf{h}}}{1 + \Sigma_\mathsf{V}^{0\mathsf{h}}}$$

$$\mathsf{C}^0_{2,2} = \hat{\mathsf{D}}_2 \ln \zeta_\mathsf{m}^0 = - \frac{\hat{\mathsf{D}}_2 \Sigma_\mathsf{S}^{0\mathsf{h}}}{1 - \Sigma_\mathsf{S}^{0\mathsf{h}}} - \frac{\hat{\mathsf{D}}_2 \Sigma_\mathsf{V}^{0\mathsf{h}}}{1 + \Sigma_\mathsf{V}^{0\mathsf{h}}}$$

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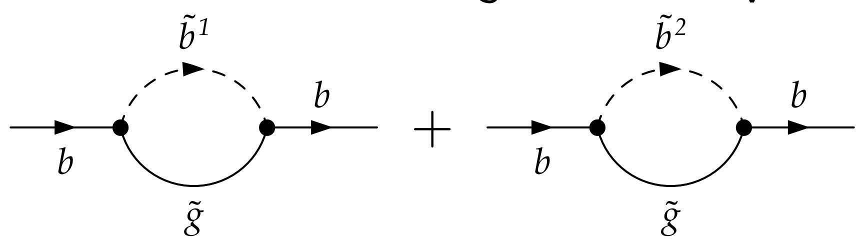
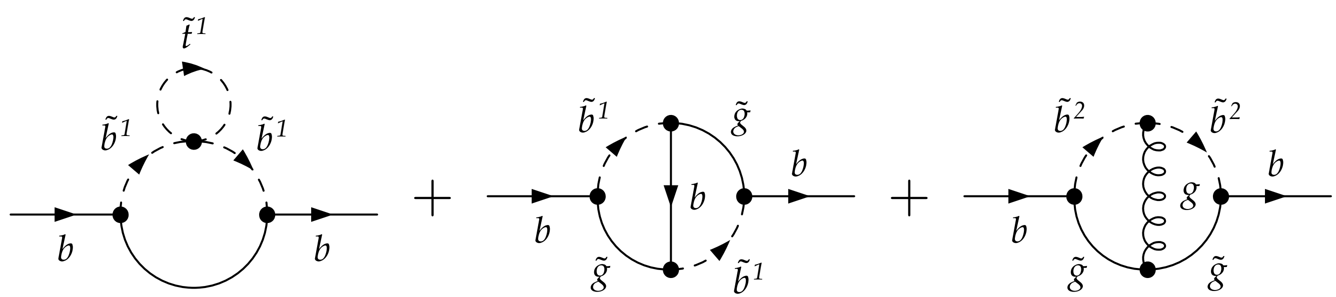
- 1-loop: $\Sigma^{0\mathsf{h},(1\mathsf{l})}_{\mathsf{S},\mathsf{V}} =$



- 2-loop: $\Sigma^{0\mathsf{h},(2\mathsf{l})}_{\mathsf{S},\mathsf{V}} =$



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$$C^0_{2,2} = \hat D_2 \ln\zeta^0_m = -\frac{\hat D_2 \Sigma_S^{0h}}{1-\Sigma_S^{0h}} - \frac{\hat D_2 \Sigma_V^{0h}}{1+\Sigma_V^{0h}}$$

Generalized derivatives:

see Degrassi, Slavich 2008 for $h \to gg$

$$\hat D_1 = [m_b A_b s_{2\theta_b}\hat F_b + 2m_b^2\hat G_b] - [m_t \frac{\mu_{SUSY}}{\tan\beta} s_{2\theta_t}\hat F_t]$$

$$\hat D_2 = -[m_b \mu_{SUSY}(tg\beta) s_{2\theta_b}\hat F_b] + [m_t A_t s_{2\theta_t}\hat F_t + 2m_t^2\hat G_t]$$

$$\hat F_q = \frac{\partial}{\partial m^2_{\tilde q_1}} - \frac{\partial}{\partial m^2_{\tilde q_2}} - \frac{4c^2_{2\theta_t}}{m^2_{\tilde q_1} - m^2_{\tilde q_2}}\frac{\partial}{\partial c^2_{2\theta_q}}\,,\quad \hat G_t = \frac{\partial}{\partial m^2_{\tilde t_1}} + \frac{\partial}{\partial m^2_{\tilde t_2}} + \frac{\partial}{\partial m^2_t}\,,\quad \hat G_b = \frac{\partial}{\partial m^2_{\tilde b_1}} + \frac{\partial}{\partial m^2_{\tilde b_2}}$$

SUSY QCD low energy theorem

Bare 5-flavor QCD effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \frac{\mathsf{m}_\mathsf{b}^{0\prime}}{\mathsf{v}} \frac{\mathsf{s}_\alpha}{\mathsf{c}_\beta} (\mathsf{C}^0_{2,1} - \frac{1}{\mathsf{tg}\alpha\mathsf{tg}\beta} \mathsf{C}^0_{2,2})\, \mathsf{h}\, \bar{\psi}_\mathsf{b}^{0\prime} \psi_\mathsf{b}^{0\prime} + \ldots$$

SUSY QCD coefficients:

$$\mathsf{C}^0_{2,1} = 1 + \hat{\mathsf{D}}_1 \ln \zeta^0_\mathsf{m} = 1 - \frac{\hat{\mathsf{D}}_1 \Sigma_\mathsf{S}^{0\mathsf{h}}}{1 - \Sigma_\mathsf{S}^{0\mathsf{h}}} - \frac{\hat{\mathsf{D}}_1 \Sigma_\mathsf{V}^{0\mathsf{h}}}{1 + \Sigma_\mathsf{V}^{0\mathsf{h}}}$$

$$\mathsf{C}^0_{2,2} = \hat{\mathsf{D}}_2 \ln \zeta^0_\mathsf{m} = -\frac{\hat{\mathsf{D}}_2 \Sigma_\mathsf{S}^{0\mathsf{h}}}{1 - \Sigma_\mathsf{S}^{0\mathsf{h}}} - \frac{\hat{\mathsf{D}}_2 \Sigma_\mathsf{V}^{0\mathsf{h}}}{1 + \Sigma_\mathsf{V}^{0\mathsf{h}}}$$

$\rightarrow$ Check performed by an explicit calculation of associated 2-loop vertex diagrams

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Guasch, Sola, Hollik 1998

$$\delta\theta_b = \frac{\mathrm{Re}\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_1}) + \mathrm{Re}\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_2})}{2(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2})}$$

where $\Sigma_{\tilde{b}_{12}}$ is the non-diagonal sbottom self-energy

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Renormalization within 5-flavor QCD:

$$C_{2,i} = \frac{4(\alpha_s'\partial/\partial\alpha_s')\ln Z_m'}{1+2(\alpha_s'\partial/\partial\alpha_s')\ln Z_g'}C_{1,i}^0 + C_{2,i}^0$$

Numerics

Decay width including (SUSY) QCD effects:

$$\Gamma(h \to b\bar{b}) = \Gamma_{b\bar{b}}^{\text{Born}} \left[(1 + \Delta_b^{\text{5-fl-QCD}})(\tilde{C}_2)^2 + \Xi_b^{\text{5-fl-QCD}}(C_1^{(1l)}) \right]$$

where

$$\Gamma_{b\bar{b}}^{\text{Born}} = \frac{N_c G_F M_h m_b^2}{4\pi\sqrt{2}} \left(1 - \frac{4m_b^2}{M_H^2}\right)^{3/2}$$

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$$\tilde{C}_2 = -\frac{s_\alpha}{c_\beta}\left(C_{2,1} - \frac{1}{\mathrm{tg}\alpha\,\mathrm{tg}\beta}C_{2,2}\right) = -\frac{s_\alpha}{c_\beta}C_2$$

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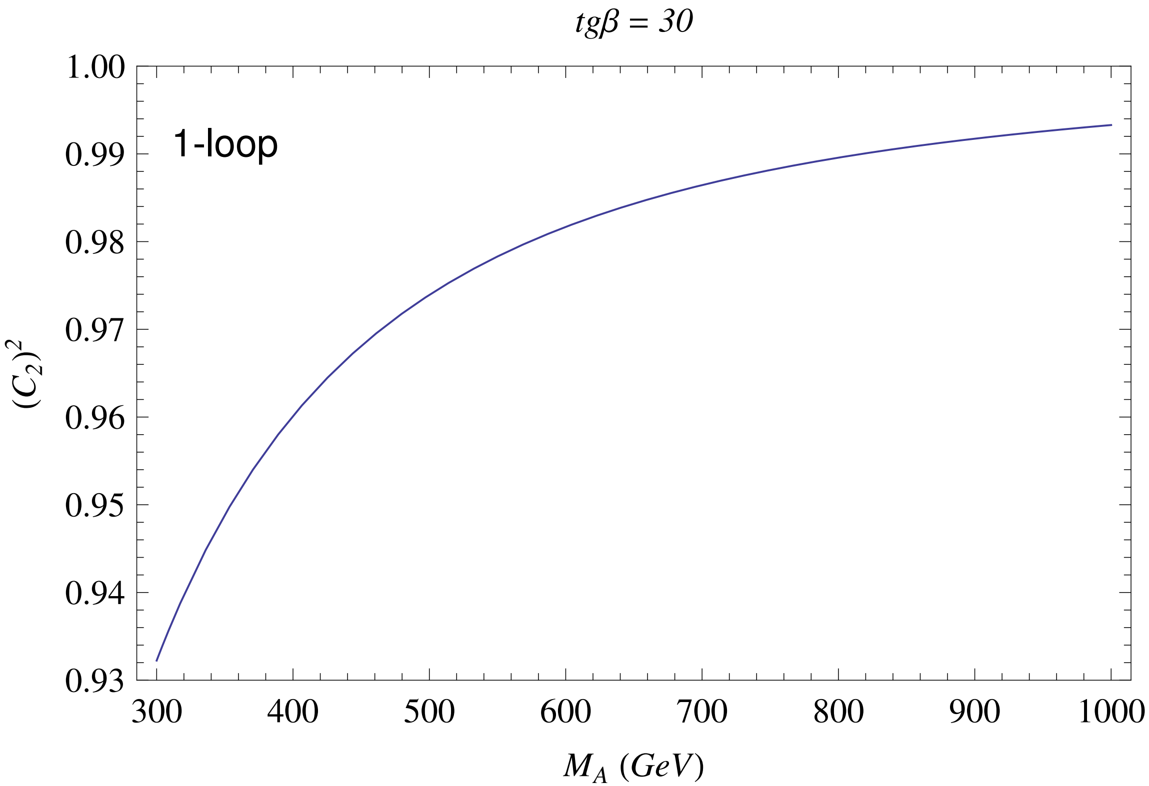

$$A_t = A_b = M_L = M_R = \mu_{SUSY} = 1\,\text{TeV}$$

$$m_{\tilde{g}} = 800\,\text{GeV}, \quad tg\beta = 30, \quad \mu = \frac{1}{3}(m_{\tilde{b}_1} + m_{\tilde{b}_2} + m_{\tilde{g}})$$

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- Low energy theorem at 2-loop allows for a calculation of $\mathrm{Hb\bar{b}}$ coupling from b propagator diagrams
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