NNLL Electroweak Corrections to Gauge Boson Pair Production at LHC

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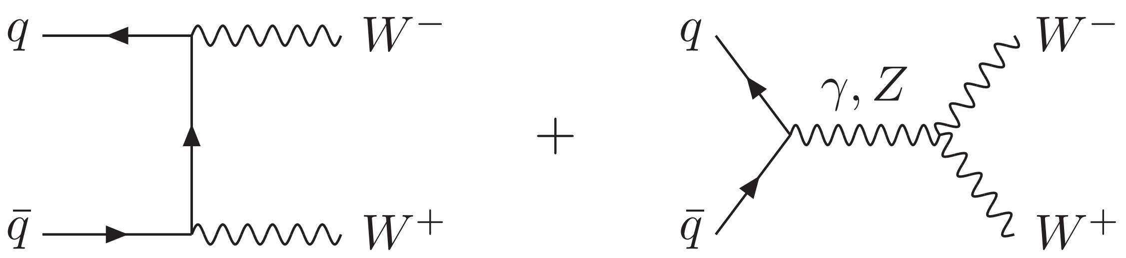
W pair production

- Important test of the vector boson trilinear couplings of the SM
- Background to the Higgs discovery channel $\quad p\,p \to H \to W^+\,W^-$

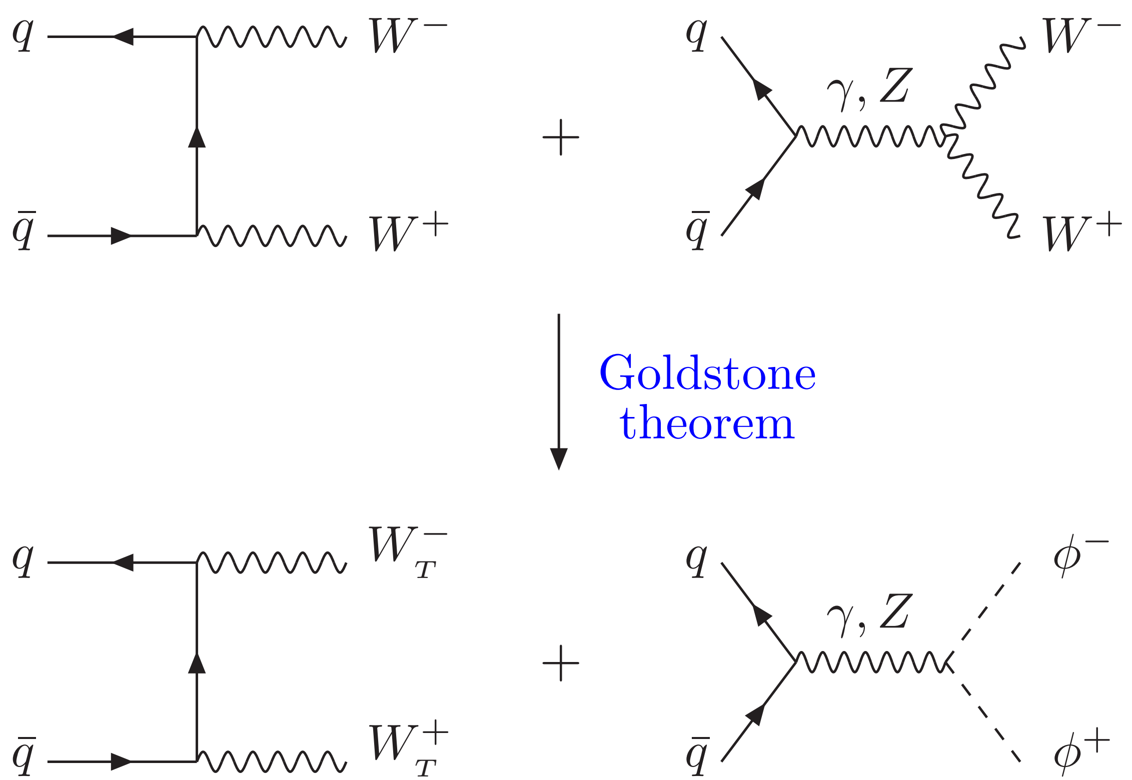
Partonic processes

- $g\,g \to W^+\,W^- \;\to$ 5% of the total cross section
- $q\,\bar{q} \to W^+\,W^-$



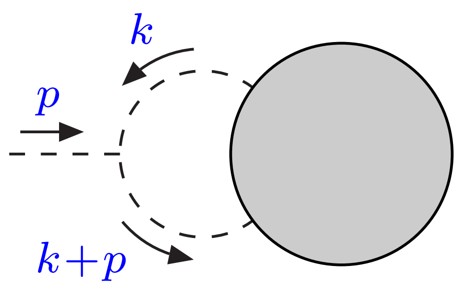
High energy limit

$$s \sim |t| \sim |u| \gg M_W^2$$



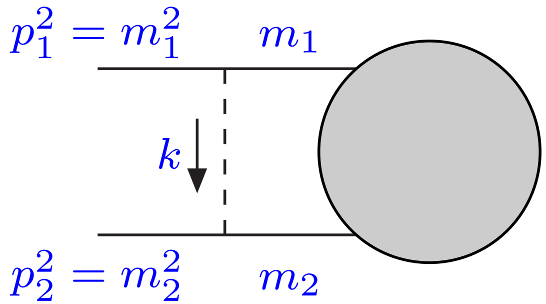
Infrared singularities

Collinear singularities



$k \to \alpha\, p$

Soft singularities



$k \to 0$

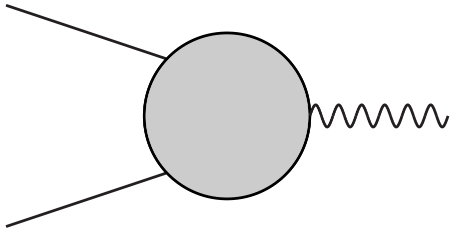
- If m_1 (and/or m_2) is massless $\to$ soft-collinear singularities
- Singularity regularized by a mass M $\to$ $\ln^n \left(\frac{s}{M^2}\right)$

Large logarithms in SU(N)

Collinear and soft-collinear logarithms

They depend just on the external legs → Factorization

One can study a simpler problem: **Scattering in an external field**

$\mathcal{F} \quad =$  $\qquad \mathcal{F} = \mathcal{Z}\mathcal{F}_{\text{Born}}$

$$\frac{\partial}{\partial \log Q^2}\mathcal{Z} = \left\{ \int_{\mu^2}^{Q^2} \frac{dx}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(\mu^2)) \right\}\mathcal{Z} \qquad Q^2 = -s$$

- γ and ζ → universal
- ξ and $\mathcal{Z}_0 \equiv \mathcal{Z}(Q^2 = \mu^2)$ → initial conditions
  They depend on the IR structure and on the definition of μ

$$\mathcal{Z} = \mathcal{Z}_0 \exp\left\{ \int_{\mu^2}^{Q^2} \frac{dx}{x} \left[\int_{\mu^2}^{x} \frac{dx'}{x'} \gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(\mu^2)) \right] \right\}$$

Loop expansion

$$f = \sum_{n=0}^{\infty} \left(\frac{\alpha}{4\pi}\right)^n f^{(n)}, \qquad \gamma^{(0)} = \zeta^{(0)} = \xi^{(0)} = 0 \qquad \mathcal{Z}_0^{(0)} = 1 \qquad L = \log\frac{Q^2}{\mu^2}$$

$$\mathcal{Z}^{(1)} = \frac{1}{2}\gamma^{(1)} L^2 + \left(\zeta^{(1)} + \xi^{(1)}\right) L + \mathcal{Z}_0^{(1)}$$

- By comparing with explicit calculation, we get the 1-loop coefficients

$$\begin{aligned} \mathcal{Z}^{(2)} &= \frac{1}{8}\left[\gamma^{(1)}\right]^2 L^4 + \frac{1}{2}\left[\zeta^{(1)} + \xi^{(1)} - \frac{1}{3}\beta_0\right]\gamma^{(1)} L^3 \\ &+ \frac{1}{2}\left[\gamma^{(2)} + \left(\zeta^{(1)} + \xi^{(1)}\right)^2 - \beta_0\,\zeta^{(1)} + \mathcal{Z}_0^{(1)}\,\gamma^{(1)}\right] L^2 + \mathcal{O}(L) \end{aligned}$$

- Just $\gamma^{(2)}$ has to be computed at 2-loop level

Soft logarithms

"Color" structures in a $2 \to 2$ process

- Process $2_F \to 2_F$: $\quad \mathcal{A} = \mathcal{A}_1\, \boldsymbol{T_a} \times \boldsymbol{T_a} + \mathcal{A}_2\, \mathbf{1} \times \mathbf{1}$
- Process $2_F \to 2_A$: $\quad \mathcal{A} = \mathcal{A}_1\, \boldsymbol{T_a T_b} + \mathcal{A}_2\, \boldsymbol{T_b T_a} + \mathcal{A}_3\, \delta_{ab}\mathbf{1}$

$F, A =$ fudamental, adjoint representation of SU(N)

Vectorial representation of the amplitude:

$$2_F \to 2_F: \; \boldsymbol{\mathcal{A}}^{\rm Born} = A_F^{\rm Born} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad 2_F \to 2_A: \; \boldsymbol{\mathcal{A}}^{\rm Born} = A_A^{\rm Born} \begin{pmatrix} -t/s \\ -u/s \\ 0 \end{pmatrix}$$

$\boldsymbol{\mathcal{A}} = \mathcal{Z}\tilde{\boldsymbol{\mathcal{A}}} \quad \to \quad \mathcal{Z}$ contains the collinear and soft-collinear logs

$$\frac{\partial}{\partial \ln Q^2}\tilde{\boldsymbol{\mathcal{A}}} = \boldsymbol{\chi}(g^2(Q^2))\tilde{\boldsymbol{\mathcal{A}}} \qquad \tilde{\boldsymbol{\mathcal{A}}} = \mathrm{P}\exp\left[\int_{\mu^2}^{Q^2} \frac{dx}{x}\boldsymbol{\chi}(g^2(x))\right]\boldsymbol{\mathcal{A}}_0(g^2(\mu^2))\,.$$

$\mathcal{Z}_0$ incorporated in $\boldsymbol{\mathcal{A}}_0$

Loop expansion

$$\mathcal{A}^{(1)} = \frac{1}{2}\gamma^{(1)}\mathcal{A}^{\text{Born}}L^2 + \left(\zeta^{(1)} + \xi^{(1)} + \chi^{(1)}\right)\mathcal{A}^{\text{Born}}L + \mathcal{A}_0^{(1)},$$

$$\mathcal{A}^{(2)} = \frac{1}{8}\left[\gamma^{(1)}\right]^2\mathcal{A}^{\text{Born}}L^4 + \frac{\gamma^{(1)}}{2}\left[\zeta^{(1)} + \xi^{(1)} + \chi^{(1)} - \frac{\beta_0}{3}\right]\mathcal{A}^{\text{Born}}L^3$$
$$+ \frac{1}{2}\left\{\left[\gamma^{(2)} + \left(\zeta^{(1)} + \xi^{(1)}\right)^2 - \beta_0\,\zeta^{(1)} + \left(\chi^{(1)}\right)^2 - \beta_0\chi^{(1)}\right]\mathcal{A}^{\text{Born}} + \mathcal{A}_0^{(1)}\right\}L^2 + \mathcal{O}(L)$$

- 2-loop coefficients up to NNLL are determined by 1-loop coefficients

The $U(1)$ case: $\mathcal{A}_{U(1)} = \mathcal{U}\,\mathcal{A}^{\text{Born}}$ $\rightarrow$ no vectorial structure

$$\mathcal{U} = \mathcal{U}_0(g^2(M^2))\,\exp\left\{g^2\left[\frac{1}{2}\gamma^{(1)}L^2 + \left(\zeta^{(1)} + \xi^{(1)} + \chi^{(1)}\right)L\right]\right.$$
$$\left. + g^4\left[-\frac{1}{6}\gamma^{(1)}\beta_0\,L^3 + \frac{1}{2}\left(\gamma^{(2)} - \beta_0\,\zeta^{(1)} - \beta_0\,\chi^{(1)}\right)L^2 + \mathcal{O}(L)\right]\right\}$$

Evolution equations in the electroweak SM

Standard Model $\quad\longrightarrow\quad SU(2)_L \times U(1)_Y$ theory

Massless fermions, photon mass λ as infrared regulator

In the high energy limit we have two soft scales:

- Electrweak scale: $M_W \sim M_Z \sim M_H \sim M$
- Photon mass scale: λ

We proceed in four steps:

I) $\sqrt{s} \gg M \sim \lambda$
All gauge bosons lighter than the cut-off $\longrightarrow$ unbroken SU(2)×U(1) theory

$$\boldsymbol{\mathcal{A}}_{\mathrm{unbroken}} = \mathcal{U}_{Y}(M)\,\boldsymbol{\mathcal{A}}_{L}(M).$$

$II)$ $\sqrt{s} \gg M \gg \lambda$

Photonic Sudakov logarithms $\ln(Q^2/\lambda^2)$ are of the form:

$$\boldsymbol{\mathcal{A}}(M,\lambda) = \mathcal{U}_{QED}(M,\lambda)\,\boldsymbol{\mathcal{A}}_0(M) + \mathcal{O}(\lambda/M)$$

$III)$ Matching of the two region at $M = \lambda$:

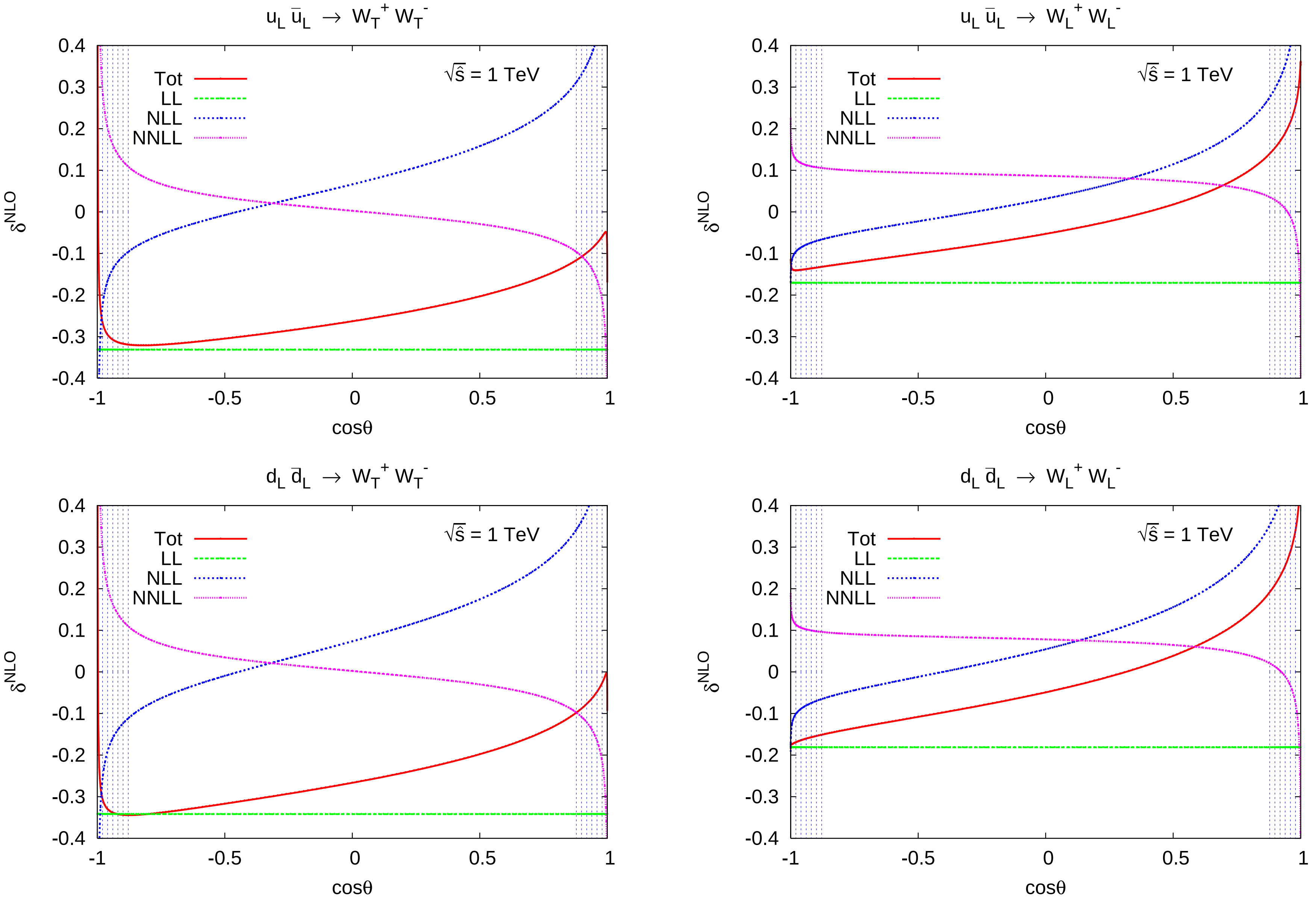
$$\boldsymbol{\mathcal{A}}(M,\lambda) = C(M)\,\mathcal{U}_{QED}(M,\lambda)\,\mathcal{U}^{-1}_{QED}\Big|_{\lambda=M}\,\boldsymbol{\mathcal{A}}_{\text{unbroken}}(M)$$

$IV)$ Separation of the soft-photon contribution:

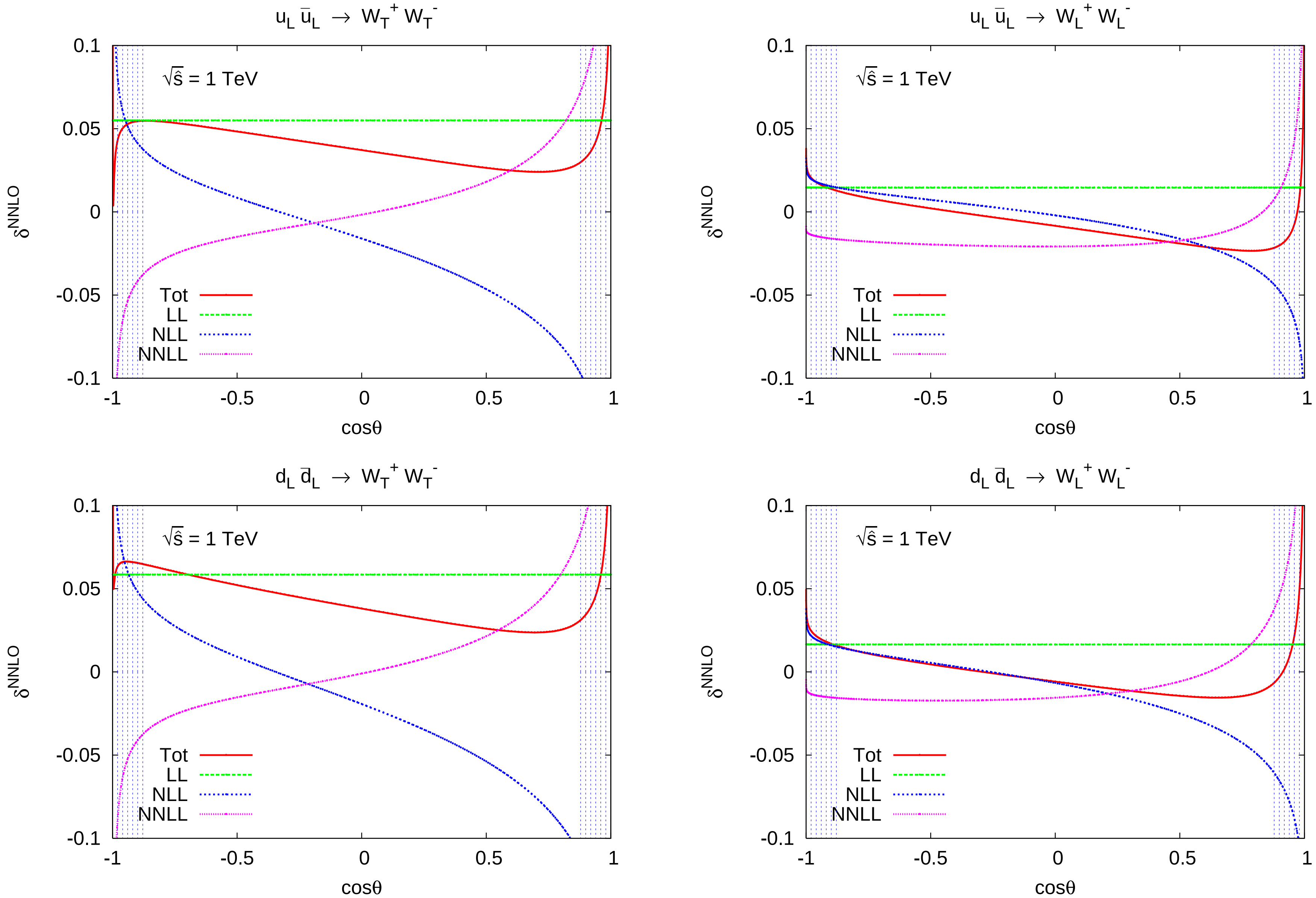
$$\boldsymbol{\mathcal{A}}(M,\lambda) \equiv \mathcal{U}_{\gamma}(M,\lambda)\,\boldsymbol{\mathcal{A}}_{EW}(M)$$

- Collinear and soft logs form initial state radiation $\rightarrow$ $\ln(Q^2/\lambda^2)$
- Just soft logs from final state radiation $\rightarrow$ $\ln(Q^2/\lambda^2)$ and $\ln(Q^2/M_W^2)$

$$\delta^{\mathrm{NLO}} = \frac{d\sigma_{\mathrm{NLO}}}{d\sigma_{\mathrm{LO}}}$$



$$\delta^{\mathrm{NNLO}} = \frac{d\sigma_{\mathrm{NNLO}}}{d\sigma_{\mathrm{LO}}}$$



Hadronic distrbutions

$$\frac{d\sigma}{dp_T} = \frac{1}{N_c^2} \sum_{ij} \int_0^1 dx_1 \int_0^1 dx_2 \, f_{h_1,i}(x_1, \mu_F^2) \, f_{h_2,j}(x_2, \mu_F^2) \, \theta(x_1 x_2 - \tau_{\min}) \, \frac{d\hat{\sigma}_{ij}}{dp_T}$$

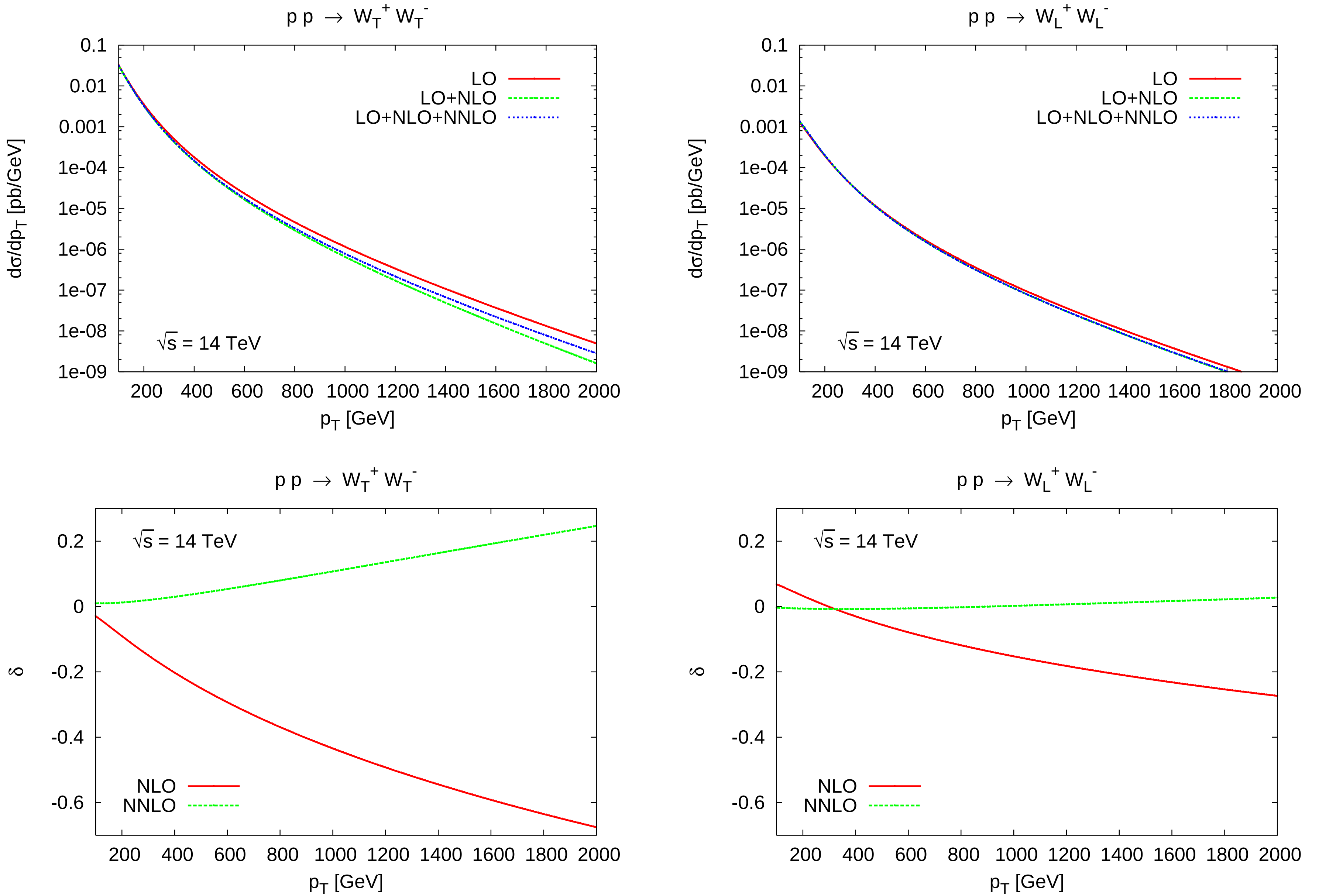
$$\frac{d\hat{\sigma}_{ij}}{dp_T} = \frac{4 p_T}{\sqrt{\hat{s} - 4M_W^2}\sqrt{\hat{s} - s\,\tau_{\min}}} \left[\frac{d\hat{\sigma}_{ij}}{d\cos\theta} + (\hat{t} \leftrightarrow \hat{u}) \right]$$

$$\hat{s} = x_1 \, x_2 \, s \qquad\qquad \tau_{\min} = \frac{4(p_T^2 + M_W^2)}{s}$$

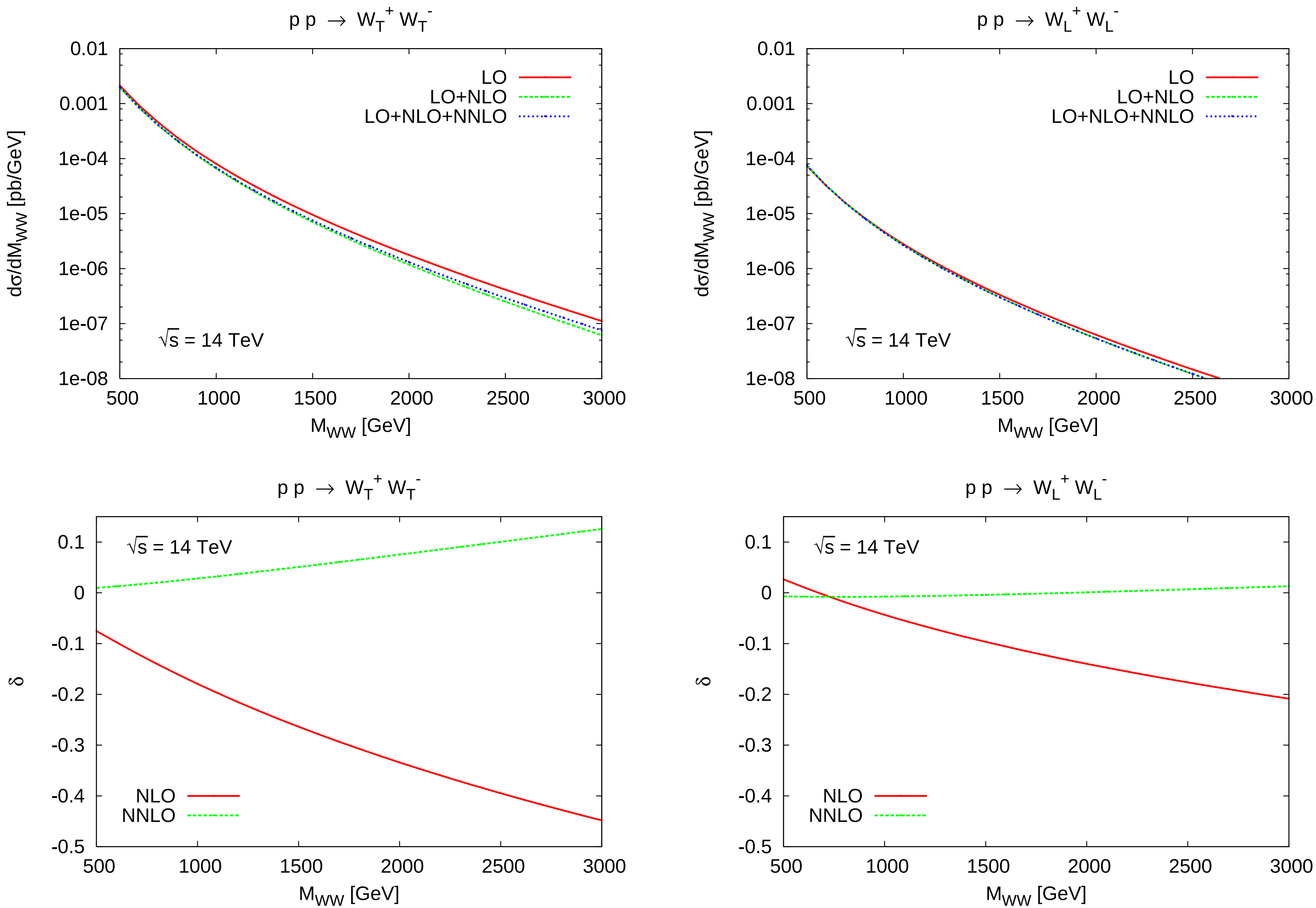
$$\frac{d\sigma}{dM_{WW}} = \frac{1}{N_c^2} \sum_{ij} \int_0^1 dx_1 \int_0^1 dx_2 \, f_{h_1,i}(x_1, \mu_F^2) \, f_{h_2,j}(x_2, \mu_F^2) \, \frac{d\hat{\sigma}_{ij}}{\mathrm{d}M_{WW}}$$

$$\frac{d\hat{\sigma}_{ij}}{dM_{WW}} = \int_{-\alpha}^{\alpha} d\cos\theta \, \frac{d\hat{\sigma}_{ij}}{d\cos\theta} \delta(\sqrt{\hat{s}} - M_{WW}) \qquad \alpha = \cos\theta_{\mathrm{cut}} \qquad \theta_{\mathrm{cut}} \geq 30^o$$

p_T distribution



Invariant mass distribution ($\theta_{\rm cut} = 30^o$)



Conclusions

- NNLL are not negligible with respect to LL and NLL
- NNLO corrections at high energies are important for the production of transversely polarized W’s (10% at 1 TeV for the p_T distribution).
- Next project: ZZ and WZ production.